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MANUFACTURER'S PATENT LICENSING STRATEGIES BASED ON CO-OPETITION

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Abstract. In the context of co-opetition (cooperation and competition), technological innovation endows manufacturers with new competitiveness and vitality. Through patent technology licensing, manufacturers can achieve technology upgrading and efficiency improvement. To study the patent licensing decisions of manufacturers based on co-opetition, this paper constructs a production patent licensing game model that comprises two manufacturers that cooperate in the upstream component market and compete in the downstream consumer market. We conduct a comparative analysis of the game equilibrium of three patent licensing scenarioscompetition (no licensing), unilateral licensing and cross-licensingand analyse the impact mechanism of key parameters, such as market share and cost advantage (cost burden). The research results are as follows. (1) When the product substitutability (competition intensity) is not very high and the market share of manufacturer 1 is relatively small or the product substitutability (competition intensity) is large and the cost advantage of both manufacturers 1 and 2 is moderate, Pareto improvement under cross-licensing scenario is better than that under unilateral licensing and competition scenarios. (2) The Pareto improvement in the unilateral licensing scenario is always better than that in the other two scenarios. (3) When the cost advantage of manufacturer 1 is small or large, the Pareto improvement in the competition scenario is better than that in the other two licensing scenarios. The paper also analyses consumer surplus in three licensing scenarios. The conclusions can provide a decision-making basis and support for manufacturers to select appropriate licensing strategies and government participation to guide enterprise technology upgrading under different situations.

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1. Introduction

With the rapid progress of technology and the development of emerging economies, firms have realised the importance of cooperation. Factors such as efficiency improvement driven by technology, the demand for productivity, the increasing pressure to integrate the global value chain, the rapid progress of technology, high R&D costs and fierce competition continue to drive manufacturers to cooperate with competitors in upstream activities, such as R&D and production resources [24]. The relationship between manufacturers has gradually

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changed from the traditional upstream and downstream competition relationship to an upstream cooperation (patent licensing) and downstream competition model, which is prevalent in the ICT and automobile industries. In these industries, firms purchase components or raw materials from upstream manufacturers (suppliers) while competing with these same manufacturers (suppliers) in the downstream market.

In the ICT industry, Apple and Samsung purchase their patented components from each other but compete in the downstream consumer market. Huawei released the 2020 version of Huawei Innovation and Intellectual Property White Paper, pointing out that Huawei has reached more than 100 patent licensing agreements (including unilateral and cross-licensing) through friendly negotiations with major global ICT firms in history, including Nokia, Ericsson, Qualcomm, Nortel Networks, Siemens, Alcatel, BT, NTT Docomo, AT&T, Apple and Samsung. Ericsson and Samsung reached a global settlement agreement and signed a multi-year global patent licensing. The global cross-licensing of patents is related to cellular technology, including 5G technology. This cross-licensing agreement covers the network infrastructure and mobile phone sales from January 1, 2021. Huawei's terminal equipment uses BOE's screens. As Samsung and Huawei are competitive in the terminal market, Samsung does not provide Huawei with its top screens to reduce competitive pressure. However, Samsung and Huawei reached a framework Patent licensing Agreement on cross-licensing of standard essential patents worldwide in 2019. Xiaomi uses Samsung's licensed patent screens and other components unilaterally in consumer terminals. Samsung and Xiaomi have a unilateral licensing partnership in the upstream market, but a competitive relationship in the downstream consumer market. In the ICT industry, manufacturers compete (no licensing of components or patent technologys) and cooperate (cross-licensing and unilateral licensing of components or patent technologys) in the upstream market and compete in the downstream consumer market. Thus, patent licensing in the competitive cooperation mode is common in the ICT industry.

Similar situations also exist in the automotive industry. On June 20, 2018, Hyundai Motor Group (Hyundai) and Audi AG (Audi) announced that they had signed a multi-year patent cross-licensing agreement on hydrogen fuel cell electric vehicle (FCEV) components and technologies. Although both parties are competitors in the downstream consumer market, they still signed patent cross-licensing agreements on components or technologies in the upstream market. Hyundai can use the relevant components or technologies of Audi (including new energy), and Audi can use the FCEV components and mature supply chains of Hyundai, such as patent licensing. This practice is common in the automobile industry.

From the perspective of consumer welfare, the cooperation between manufacturers and competitors can effectively utilise resources and improve economic welfare [25]. From the manufacturer's perspective, the manufacturer benefits from patent licensing, and the licensee uses the licensed patent to reduce the component cost and can focus on its advantageous business. Patent licensing cooperation can improve production efficiency or increase income streams to enhance the competitiveness of each enterprise. The licensor gets licensing benefits and promotes its patent R&D (research and development). If the licensor and licensee are cooperative in the upstream market and competitive in the downstream market, the decision-making process becomes complex. After being licensed, the licensee will enhance its competitive advantage and compete with the licensor in the downstream consumer market, which will inhibit the competitive advantage of the licensor and have a negative impact on it. For the licensees, although they have paid patent licensing fees, the cost of components reduces, and their competitive advantage increases. For the licensors, although they have received patent licensing fees, they will intensify competition in the downstream consumer market. Therefore, in the context of competition and cooperation, how licensors and licensees balance the advantages and disadvantages is a complex operational decision [8].

To help firms make optimal operational decisions, this paper studies the patent licensing of manufacturers in the context of co-opetition. Should firms licence patent technology of components to competitors? How should the optimal pricing and patent licensing fees be determined when licensing component patent technology to competitors? Which type of patent licensing fee is the best for firms? How do different patent licensing models affect consumer surplus? To study these problems, this paper considers two manufacturers that produce partially substitutable products and compete with each other in the final customer market. It is assumed that alternative products (such as smartphones or tablets) consist of two key components (such as chips or displays). The two key components are produced by either of the two manufacturers at different production costs. Each manufacturer has a patent for one component, which is produced with high efficiency and low cost. Manufacturers can buy components from market competitors at a high price or pay patent licensing fees to market competitors. They can also use their competitors' patented technologies to produce components at a lower cost. The two manufacturers purchase each other's components or license each other's components patents in the components market; this is a cooperation relationship. Conversely, competing for consumer demand in the downstream consumer market is a competition relationship. This paper attempts to understand how manufacturers choose the optimal patent licensing strategies in a co-opetition scenario by analysing the competition scenario (no licensing), the equilibrium of unilateral licensing and cross-licensing scenarios, and manufacturers' pricing and patent licensing strategies.

The rest of this paper is organised as follows. Section 2 provides a literature review, and Section 3 describes the model development. Section 4 analyses the equilibrium decisions of different modes. Section 5 discusses consumer surplus based on numerical analysis. Finally, the key results and directions for future research are summarised in Section 6.

2. Literature review

Currently, under the pressure of a complex international environment, the prevalence of trade protectionism and technology monopoly, studies on patent licensing and co-opetition have become the focus and a challenge for business and academics. The literature related to this paper mainly includes two aspects. One is the research on supply chain decision-making from the perspective of patent licensing, and the other is the research on supply chain decision-making and coordination from the perspective of co-opetition.

For the literature related to patent licensing, Khoury *et al.* [14] studied non-exclusive technology licensing and discussed how the experience and transaction costs of the licensee's prior participation in external and internal licensing transactions affect technology licensing. Aulakh *et al.* [2] studied the structure of exclusive patent licensing, considered the number of licenses and the exclusive rights of the licensee and studied the determinants of licensing based on the monopoly rents and transaction costs of different types of licensing contracts. Li *et al.* [19] discussed the impact of parallel imports on market participants' profits and unit licensing fees under a monopoly or oligopoly and analysed the dynamic relationship between patent technology licensing and parallel imports. Hong *et al.* [11] studied the technology licensing decision of existing patents using quality improvement technology under the duopoly model with heterogeneous consumers, as well as the quality information disclosure strategies of licensors and licensees competing in quantity. In addition, many studies have analysed patent technology licensing from different perspectives, such as remanufacturing/closed-loop supply chain [10] and uncertainty of R&D and market [38]. Some have also studied patent licensing fees and licensing methods. For example, Kim and Lee [15, 17] studied different patent licensing agreements, such as royalties, fixed fees and auction licensing agreements, and found that non-exclusive contracts increase benefits. By introducing transaction costs, we further study two components of patent licensing agreements, including patent fees and fixed fees [6, 16]. Only a few papers have comprehensively analysed the unilateral and cross-licensing modes of patent licensing.

The research related to co-opetition is mainly concentrated in the fields of manufacturing, information technology and services. Wang *et al.* [33] adopted the Cournot competition model to study the production outsourcing relationship between original equipment manufacturers and contract manufacturers. Yang *et al.* [35] also adopted the concept of co-opetition and the Cournot competition model to analyse the optimal allocation strategies of suppliers with limited supply capacities when selling to competitive buyers. Huang *et al.* [13] studied the formation of alliances between upstream suppliers serving the same downstream manufacturer. They demonstrated that alliances can help ease competition, but the reduction effect of the competition itself does not promote the formation of cooperative alliances. Guo and Wu [9] studied the cooperation and competition between two manufacturing firms sharing production capacity through the random allocation rule. Liang *et al.* [21] also studied the problem of crowdsourcing task allocation with co-opetition. By building a new three-way decision, they designed a two-stage cooperation and competition model, which not only protects the interests of candidate tasks but also optimises the total interests. Luo *et al.* [23] and Zhang and Frazier [37] studied Yahoo and Microsoft, who formed an alliance to use the Bing search engine practice, but the two companies compete with each other to sell search ads, so they used a multi-period cooperative game model to study the formation of dynamic alliances between competing companies. Yuan *et al.* [36] used the queuing theory to build a co-opetition model for service clusters with public space and studied the cooperative competition problem in the service environment.

Considering patent technology licensing and competition between firms, Moreira *et al.* [26] studied the relationship between innovation and technology licensing by tracking the patent licensing entry and vertical design of technological innovation of firms in the global biopharmaceutical industry. The results revealed that licensing entry is driven by competitive pressure. In areas where competitors exert pressure, especially in the case of accumulated R&D investment, licensing entry can improve the innovation capability of firms. Taking technology licensing as a key organisational action can help to better understand the relationship between competition and innovation. Chen *et al.* [5] studied the strategic choice of competition and cooperation between two competitive manufacturers that produce alternative products and cooperate in components production through wholesale (wholesale cooperation) or licensing (licensing cooperation). At present, research that considers the background of co-opetition lacks an analysis of cross patent licensing and choosing between unilateral and cross-licensing.

The research on co-opetition or patent licensing is very mature, but due to complex international situations and firms' practice challenges, research on patent licensing under the background of co-opetition is relatively lacking, especially cross-licensing. Whether to choose a unilateral or cross-licensing is an urgent operation management problem that needs to be solved in the enterprise practice process. The existing research has not systematically analysed the competition scenario (no licensing) and the equilibrium results of unilateral and cross-licensing scenarios under the background of competition and cooperation, and a comparative analysis of the three models has not been conducted. Therefore, this paper takes competition and cooperation as the background to explore the optimal decisions of manufacturers under different scenarios and the equilibrium between different modes.

3. Model development

This paper considers two manufacturers 1 and 2 that have a cooperative relationship in the upstream components market and a competitive relationship in the downstream consumer market. The specific competitive cooperation structure is depicted in Figure 1. Manufacturers 1 and 2 use components 1 and 2 to produce alternative products 1 and 2, respectively, and then sell their products to the consumer market. Manufacturer 1 (2) has the production technical patent of component 1 (2). The research on patent technology licencing in this article mainly focuses on how patent technology can enable manufacturers to gain significant production cost advantages in component production. For example, patented component technology can become a new technological solution for upgrading and improving manufacturing processes, thereby obtaining utility model patents; It can also be an innovative technological solution in the manufacturing process of components, replacing the complex offset printing production technology, significantly reducing production costs in practicality and industrialization, and thus forming invention patents. Manufacturers 1 and 2 can both produce their patented components at lower production costs, while also manufacturing non patented components at higher production costs.

Due to Manufacturer 1(2) having a production patent technology for Component 1(2), Manufacturer 1(2) can produce Component 1(2) at a lower production cost. Due to the fact that Manufacturer 1(2) does not have a production patent technology of Component 2 (1), Manufacturer 1(2) can choose to produce Component 2 (1) at a higher production cost, or choose to obtain a production patent technology licencing for Component 2 (1), in order to produce Component 2 (1) at a lower cost. In this context, the paper considers three scenarioscompetition (no licensing), unilateral and cross-licensingand analyses the optimal decision and equilibrium results under different scenarios. The patent technology referred to in the following section refers to the patent technology for component production.

Assumption 3.1. *The alternative product 1 and 2 manufactured by manufacturers 1 and 2 comprise components 1 and 2. Manufacturer 1 has the patent technology for component 1, and manufacturer 2 has the patent technology for component 2. There are cost differences between components 1 and 2. We assume that manufacturers 1 and 2 have production costs (C_1 and C_2) of components. If manufacturer 1 (2) chooses to produce component 2 (1) on their own, manufacturer 1 (2) will need to pay more production costs than manufacturer 2 (1). This additional production cost is defined as $c_2(c_1)$. So, we can obtain that manufacturer 1 (2) produces component 2 by themselves, the cost will be C_2+c_2 (C_1+c_1). c_1 and c_2 endow manufacturer 1 and 2 with a cost advantage per unit of component. The cost advantages of components 1 and 2 characterizes the patent technology advantages of both manufacturers. This difference in technical advantages leads to different cost advantages, so c_1 and c_2 are different. Our article mainly wants to study how the technology cost advantage affects the manufacturer's choice of licensing strategy. So, we assume that the production cost $C_1 = C_2 = 0$, this practice widely exists in the existing research (e.g., [1, 22]).*

Assumption 3.2. *When manufacturer 1 (2) licenses manufacturer 2 (1) to use component 1 (2), a patent licensing fee is charged. There are two ways to charge patent licensing fees: only a fixed-fee or a combination of a fixed-fee and a royalty rate. These forms of the licensing arrangement are common in practice [28]. Some studies have analyzed the impact of the charging way of patent licensing fees and found that the impact of fixed-fee is not significant [6]. Our research focuses on the impact of royalty. If manufacturer 1(2) can obtain the patent technology of component 2 (1), manufacturer needs to pay a royalty $\varphi_2(\varphi_1)$ for every unit it produces. Regardless of the disturbance of demand, the quantity produced and sold by manufacturers are consistent, meaning that demand and supply are the same. So, the demand function of products is given by $d_1 = q_1 = a - p_1 + \beta p_2$ and $d_2 = q_2 = 1 - a - p_2 + \beta p_1$ (Refer to Assumption 3.4 for a detail explanation). If manufacturer 1 licence the patent technology of component 1to manufacturer 2, manufacturer 1 can obtain the revenues of patent technology licencing of component, which can be given by $\varphi_1 d_2 = \varphi_1 q_2$. This portion of revenue can be understood as the product of licencing fees and sales volumes.*

Manufacturers 1 and 2 can gain a cost advantage (technical advantage) over their competitors' patented components by obtaining a license for those components. Assumption 3.1 states that the production costs for both manufacturers' components are the same and equal to 0. Therefore, after obtaining the patent licensing, the production cost for the competitors also becomes 0. The competitors only need to pay the unit license fee to gain the advantage of using the patented components.

Assumption 3.3. *For the convenience of calculation and expression, the paper uses u for unilateral licensing and c for cross-licensing.*

Assumption 3.4. *To ensure that manufacturers 1 and 2 are incentivized to license patents on components 1 and 2, $c_1 > \phi_1$ and $c_2 > \phi_2$ need to be constrained.*

Manufacturers are competitive in the consumer market. The paper considers a supply chain with two competition manufacturers (indexed by 1 or 2) selling substitutable products by their own sales channels. The demand function of products is given by $d_1 = a - p_1 + \beta p_2$ and $d_2 = 1 - a - p_2 + \beta p_1$ (see [20, 29]). a represents a firm's share of the consumer market. β represents the cross-price elasticity of demand (product substitutability or product competition intensity, larger β means more intense competition). $0 < \beta < 1$ corresponds to products 1 and 2 are partial substitutes. $p_1$and p_2 represent the product price of manufacturers 1 and 2, respectively. Linear demand functions have been extensively used in the economics (*e.g.*, [32]) and operations management literatures (*e.g.*, [7, 31]).

The demand functions of products are given by $d_1 = a + \varepsilon_1 - p_1 + \beta p_2$ and $d_2 = 1 - a + \varepsilon_2 - p_2 + \beta p_1$. We assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $\varepsilon \sim U(0, b)$ [30, 34]. The expectations of the demands are $E(d_1) = a - p_1 + \beta p_2 + b/2$ and

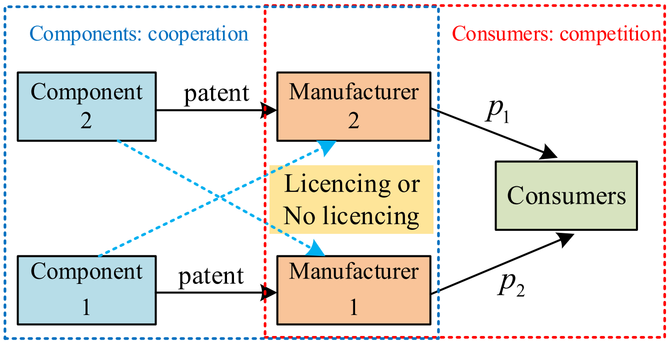


FIGURE 1. Co-opetition structure.

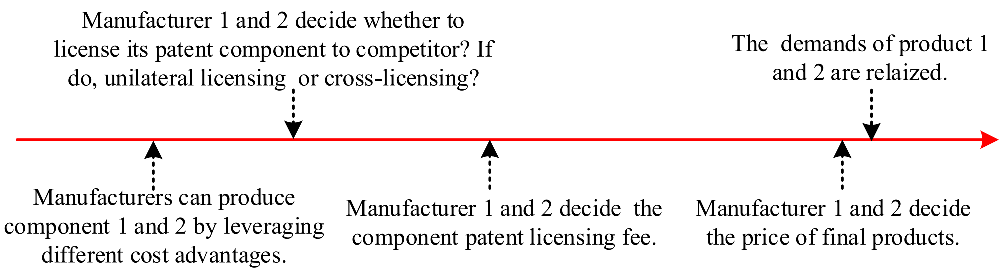


FIGURE 2. Decision-making process.

$E(d_2) = 1 - a - p_2 + \beta p_1 + b/2$. Through the analysis of the model and the optimal results, we have determined that demand uncertainty does not have a significant impact on the original conclusions (see Appendix B).

Based on business operation practice, three typical modes are summarised, as depicted in Figure 1.

Both manufacturers 1 and 2 possess the production capacity for component 1 and 2. However, they hold technical patents only for their respective components, they must acquire the rival component, which have already been assembled into products, in order to compete in the final product market. Both manufacturers are driven to obtain license for their opponent's patented component, necessitating a decision on the mode of patent licensing based on factors such as potential market size and the cost advantage of the patented component. Consequently, the two manufacturers have the option to purchase each other's component.

The process begins with manufacturers 1 and 2 selecting the mode to license their patent technology for their respective component. Subsequently, the two manufacturers determine the patent licensing fee. Next, the two manufacturers determine the pricing for the final product. Finally, the consumer market demand for the two end products is realized, leading to profit generation for both manufacturers. For a more detailed understanding, please refer to Figure 2 for a specific depiction of the process.

3.1. Competition scenario (without licensing)

In the competition scenario, manufacturers 1 and 2 purchase components 2 and 1 in the upstream components market, respectively, do not license technical patents to each other and use their own cost advantages to compete in the downstream consumer market for product prices. The profit functions and decision-making problems of manufacturers 1 and 2 are as follows:

$$\begin{cases} \pi_1(p_1) = (p_1 - c_2)(a - p_1 + \beta p_2) \\ \pi_2(p_2) = (p_2 - c_1)(1 - a - p_2 + \beta p_1) \end{cases} \tag{1}$$

Based on equation (1), the firms decide noncooperatively and simultaneously over prices p_1 and p_2: Manufacturer 1 must find the maximum of the quadratic function

$$p_1 \mapsto (p_1 - c_2)(a - p_1 + \beta p_2),$$

parametrized in p_2, while Manufacturer 2 must find the maximum of

$$p_1 \mapsto (p_1 - c_2)(a - p_1 + \beta p_2),$$

parametrized in p_1. Under some conditions, these maxima are unique, giving rise to best-reply functions $p_2 \mapsto R_1(p_2)$ and $p_1 \mapsto R_2(p_1)$. These cross, one can determine a unique NE by solving the system $\begin{cases} R_1(p_2) = p_1, \\ R_2(p_1) = p_2, \end{cases}$. So, we can get the optimal price (see Tab. 1).

3.2. Unilateral licensing

In the unilateral licensing scenario, as manufacturers 1 and 2 are symmetrical, this paper takes manufacturer 1 as the licensor who licenses the patent of component 1 to manufacturer 2. Here, manufacturer 2 obtains the cost advantage of component 1, and manufacturer 1 has the licensing benefit, but manufacturer 1 still does not have the cost advantage of component 2. The profit functions and decision-making problems of manufacturers 1 and 2 are as follows.

$$\begin{cases} \pi_1^u(p_1, \varphi_1) = (p_1 - c_2)(a - p_1 + \beta p_2) + \varphi_1(1 - a - p_2 + \beta p_1) \\ \pi_2^u(p_2) = (p_2 - \varphi_1)(1 - a - p_2 + \beta p_1). \end{cases} \tag{2}$$

In the second stage, the firms decide noncooperatively and simultaneously over prices p_1 and p_2: Manufacturer 1 must find the maximum of the quadratic function

$$p_1 \mapsto (p_1 - c_2)(a - p_1 + \beta p_2) + \varphi_1(1 - a - p_2 + \beta p_1),$$

parametrized in p_2 and φ_1, while Manufacturer 2 must find the maximum of

$$p_2 \mapsto (p_2 - \varphi_1)(1 - a - p_2 + \beta p_1),$$

parametrized in p_1 and φ_1. Under some conditions, these maxima are unique, giving rise to best-reply functions $p_2 \mapsto R_1(p_2, \varphi_1)$ and $p_1 \mapsto R_2(p_1, \varphi_1)$. If these cross, one can determine a unique NE for the second stage: say $\{\hat{p}_1(\varphi_1); \hat{p}_2(\varphi_1)$ solves the system

$$\begin{cases} R_1(p_2, \varphi_1) = p_1, \\ R_2(p_1, \varphi_1) = p_2, \end{cases}$$

Manufacturer 1's problem in the first stage is then: to find a φ_1 that maximizes $\varphi_1 \mapsto (\hat{p}_1(\varphi_1) - c_2)(a - \hat{p}_1(\varphi_1) + \beta\hat{p}_2(\varphi_1)) + \varphi_1(1 - a - \hat{p}_2(\varphi_1) + \beta\hat{p}_1(\varphi_1))$.So, there is an optimal price and a licensing fee. (see Tab. 1).

3.3. Cross-licensing

In the cross-licensing scenario, manufacturer 1 licenses the patent of component 1 to manufacturer 2, and manufacturer 2 authorises the patent of component 2 to manufacturer 1. Here, both manufacturers have the cost advantage of both components, and both manufacturers obtain licensing benefits. The profit functions and decision-making problems of manufacturers 1 and 2 are as follows.

$$\begin{cases} \pi_1^c(p_1, \varphi_1) = (p_1 - \varphi_2)(a - p_1 + \beta p_2) + \varphi_1(1 - a - p_2 + \beta p_1) \\ \pi_2^c(p_2, \varphi_2) = (p_2 - \varphi_1)(1 - a - p_2 + \beta p_1) + \varphi_2(a - p_1 + \beta p_2). \end{cases} \tag{3}$$

TABLE 1. Equilibrium results under the three licensing scenarios.

Scenarios	Equilibrium results
Competition	$p_1 = \frac{\beta+2a-\beta a+\beta c_1+2c_2}{4-\beta^2}, p_2 = \frac{2-2a+\beta a+2c_1+\beta c_2}{4-\beta^2},$ $d_1 = \frac{(2-\beta)a+\beta(1+c_1)+(\beta^2-2)c_2}{4-\beta^2}, d_2 = \frac{2+(\beta-2)a+(\beta^2-2)c_1+\beta c_2}{4-\beta^2},$ $\pi_1 = \frac{(a+\beta-a\beta+\beta c_1+(\beta^2-2)c_2)^2}{(4-\beta^2)^2}, \pi_2 = \frac{2-2a+a\beta+(\beta^2-2)c_1+\beta c_2)^2}{(4-\beta^2)^2}.$
Unilateral licensing	$p_1^u = \frac{10\beta-\beta^3+a(\beta-2)(\beta-1)(4+\beta)-(5\beta^2-8+3\beta^4)c_2}{2(8-7\beta^2-\beta^4)},$ $p_2^u = \frac{12-2\beta^2-\beta^4+a(8\beta+2\beta^2+\beta^3+\beta^4-12)+\beta(3\beta^2-4+\beta^4)c_2}{2(8-7\beta^2-\beta^4)},$ $d_1^u = \frac{2\beta+\beta^3+a(2-\beta)(4+\beta+\beta^2)+(\beta^2-8+\beta^4)c_2}{2(8+\beta^2)}, d_2^u = \frac{(2+\beta^2)(1-a+\beta c_2)}{8+\beta^2},$ $\varphi_1^u = \frac{8+\beta^4-a(8-8\beta-\beta^3+\beta^4)-(\beta^3-\beta^5)c_2}{2(8-7\beta^2-\beta^4)}, \pi_1^u = \frac{1}{4(8-7\beta^2-\beta^4)}((2+\beta^2)^2$ $+a^2(2-\beta)(1-\beta)(6+\beta+\beta^2)-2a(1-\beta)(4-4\beta-\beta^3)$ $-c_2(1-\beta^2)(2(4\beta+\beta^3+8a+a\beta(\beta-4-\beta^2))$ $+c_2(3\beta^2-8+\beta^4))), \pi_2^u = \frac{(2+\beta^2)^2(1-a+\beta c_2)^2}{(8+\beta^2)^2}.$
Cross-licensing	$p_1^c = \frac{6a(1-\beta)+6\beta}{8-7\beta^2-\beta^4}, p_2^c = \frac{6+6a(\beta-1)}{8-7\beta^2-\beta^4}, d_1^c = a - \frac{6a}{8+\beta^2}, d_2^c = \frac{(1-a)(2+\beta^2)}{(8+\beta^2)},$ $\varphi_1^c = \frac{4-\beta^2+a(2\beta+\beta^2+\beta^3-4)}{8-7\beta^2-\beta^4}, \varphi_2^c = \frac{2\beta+\beta^3+a(4-2\beta-\beta^2-\beta^3)}{8-7\beta^2-\beta^4},$ $\pi_1^c = \frac{(2+\beta^2)(4+6a^2(1-\beta)+\beta^2-2a(1-\beta)(4+\beta))}{(1-\beta^2)(8+\beta^2)^2},$ $\pi_2^c = \frac{(2+6a^2(1-\beta)-2a(2-\beta)(1-\beta)+\beta^2)(2+\beta^2)}{(1-\beta^2)(8+\beta^2)^2}.$

In the second stage, the firms decide noncooperatively and simultaneously over prices p_1 and p_2: Manufacturer 1 must find the maximum of the quadratic function

$$p_1 \mapsto (p_1 - \varphi_2)(a - p_1 + \beta p_2) + \varphi_1(1 - a - p_2 + \beta p_1),$$

parametrized in p_2, φ_1 and φ_2, while Manufacturer 2 must find the maximum of

$$p_2 \mapsto (p_2 - \varphi_1)(1 - a - p_2 + \beta p_1) + \varphi_2(a - p_1 + \beta p_2),$$

parametrized in p_1, φ_1 and φ_2. Under some conditions, these maxima are unique, giving rise to best-reply functions $p_2 \mapsto R_1(p_2, \varphi_1, \varphi_2)$ and $p_1 \mapsto R_2(p_1, \varphi_1, \varphi_2)$. If these cross, one can determine a unique NE for the second stage: say $\{\hat{p}_1(\varphi_1, \varphi_2); \hat{p}_2(\varphi_1, \varphi_2)$ solves the system

$$\begin{cases} R_1(p_2, \varphi_1, \varphi_2) = p_1, \\ R_2(p_1, \varphi_1, \varphi_2) = p_2. \end{cases}.$$

Manufacturer 1's problem in the first stage is then: to find a φ_1 that the maximum of the quadratic function $\varphi_1 \mapsto (\hat{p}_1(\varphi_1, \varphi_2) - \varphi_2)(a - \hat{p}_1(\varphi_1, \varphi_2) + \beta p_2(\varphi_1, \varphi_2)) + \varphi_1(1 - a - \hat{p}_2(\varphi_1, \varphi_2) + \beta \hat{p}_1(\varphi_1, \varphi_2))$,while Manufacturer 2 must find a φ_2 that the maximum of $\varphi_2 \mapsto (\hat{p}_2(\varphi_1, \varphi_2) - \varphi_1)(1 - a - \hat{p}_2(\varphi_1, \varphi_2) + \beta \hat{p}_1(\varphi_1, \varphi_2)) + \varphi_2(a - \hat{p}_1(\varphi_1, \varphi_2) + \beta \hat{p}_2(\varphi_1, \varphi_2))$.

Under some conditions, these maxima are unique, giving rise to best-reply functions $\varphi_2 \mapsto \tilde{R}_1(\varphi_2)$and $\varphi_1 \mapsto \tilde{R}_2(\varphi_1)$. These cross, one can determine a unique NE by solving the system

$$\begin{cases} \tilde{R}_1(\varphi_2) = \varphi_1, \\ \tilde{R}_2(\varphi_1) = \varphi_2. \end{cases}.$$

So, there is an optimal price and a licensing fee. (see Tab. 1).

3.4. Sensitivity analysis

According to the equilibrium results under the three licensing scenarios in Table 1, this section conducts model analysis and analyses how the market share (a) and cost advantages ($c_1$and c_2), which are key parameters of the manufacturers, affect their equilibrium results under the three different licensing scenarios.

The sensitivity analysis of key parameters is carried out (Tab. 2), and then corollaries 1 and 2 are obtained.

Corollary 3.5. *Sensitivity analysis of the key parameters to a manufacturer's optimal price and sales volumes.*
(i) $\frac{\partial p_1}{\partial a} > 0, \frac{\partial p_1}{\partial c_1} > 0, \frac{\partial p_1}{\partial c_2} > 0, \frac{\partial p_2}{\partial a} < 0, \frac{\partial p_2}{\partial c_1} > 0, \frac{\partial p_2}{\partial c_2} > 0; \partial p_1^u \partial a > 0, \frac{\partial p_1^u}{\partial c_2} > 0, \frac{\partial p_2^u}{\partial a} < 0, \frac{\partial p_2^u}{\partial c_2} > 0; \frac{\partial p_1^c}{\partial a} > 0, \frac{\partial p_2^c}{\partial a} < 0.$
(ii) $\frac{\partial d_1}{\partial a} > 0, \frac{\partial d_1}{\partial c_1} > 0, \frac{\partial d_1}{\partial c_2} < 0, \frac{\partial d_2}{\partial a} < 0, \frac{\partial d_2)}{\partial c_1} < 0, \frac{\partial d_2}{\partial c_2} > 0; \frac{\partial d_1^u}{\partial a} > 0, \frac{\partial d_1^u}{\partial c_2} < 0, \frac{\partial d_2^u}{\partial a} < 0, \frac{\partial d_2^u}{\partial c_2} > 0; \frac{\partial d_1^c}{\partial a} > 0, \frac{\partial d_2^c}{\partial a} < 0.$

(i) The products prices of manufacturers 1 and 2s are positively related to their market shares and cost differences (advantages). The larger the market share of the manufacturer, the greater the competitive position. Some positive competitive strategies will be adopted such as reducing prices to expand market share. However, with the improvement of competitors' market position, the two manufacturers adopt the competitive strategy of increasing product prices instead of fighting a price war. The final result of the price war is that the price is equal to the marginal cost, which means that both manufacturers are in internal friction. The greater the cost advantage (disadvantage), the larger (smaller) the profit space. Both manufacturers adopt the competitive strategy of increasing the price of the product. The main reason is that the price elasticity is greater than the sales volumes elasticity, and the price increase effect dominates the sales volumes decrease effect, so the manufacturer's optimal competitive strategy is to increase the price.
(ii) The sales volumes are negatively related to the cost advantage of competitors and positively related to its cost advantage and their market shares. The cost advantage of competitors can weaken their sales volumes, and when a competitive strategy of increasing prices is adopted, the superposition of two negative factors leads to a decrease in sales volumes. The cost advantage is the driving factor and promotes the sales volumes. The increase in sales volumes is mainly affected by the positive factors of an increase in cost advantage and the negative factors of an increase in price. The positive effect of an increase in cost advantage dominates the negative effect of an increase in price, leading to an increase in sales volumes.

Corollary 3.6. *Sensitivity analysis of the key parameters to the manufacturer's optimal patent licensing fee.*

(i) $\frac{\partial \varphi_1^u}{\partial a} < 0, \frac{\partial \varphi_1^u}{\partial c_2} < 0.$
(ii) $\frac{\partial \varphi_1^c}{\partial a} < 0; \frac{\partial \varphi_2^c}{\partial a} > 0.$

(i) The manufacturer's patent licensing fee is negatively related to its market share. The larger the market share, the higher the competitive advantage and position. At this time, the manufacturer reduces the patent licensing fee because patent licensing can reduce the cost of the licensee and improve its production efficiency, thereby increasing its sales volumes. It can also increase the revenue of the licensor, but to a certain extent, it intensifies competition. As the sales volumes of the licensed manufacturer increases, it brings more significant licensing benefits to the licensor, which exceeds the negative impact of intensified competition. Therefore, in the case of unilateral and cross-licensing, with an increase in the manufacturer's market share, the two manufacturers may adopt the competitive strategy of increasing the patent licensing fee.
(ii) Manufacturer 1's patent licensing fee is negatively related to its cost disadvantage. As competitors gain a cost advantage, the growing cost disadvantage puts competitive pressure on manufacturer 1. However, manufacturer 1 cannot offset this cost disadvantage by increasing the price of its product. Doing so would weaken its competitive position and reduce its market share. Moreover, if manufacturer 1 raises the patent license fee, it may lead to a decrease in manufacturer 2's sales volumes and their willingness to license the patent for component 1. Therefore, in order to maintain a competitive position, manufacturer 1 should adopt a strategic approach of reducing patent licensing fees to address the growing cost disadvantages.

TABLE 2. Sensitivity analysis of key parameters.

	Competition			Competition			Competition		
	a	c_1	c_2	a	c_1	c_2	a	c_1	c_2
p_1	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$*$	$\uparrow$	$\uparrow$	$*$	$*$
p_2	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$*$	$\uparrow$	$\downarrow$	$*$	$*$
d_1	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$*$	$\downarrow$	$\uparrow$	$*$	$*$
d_2	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$*$	$\uparrow$	$\downarrow$	$*$	$*$
φ_1	$*$	$*$	$*$	$\downarrow$	$*$	$\downarrow$	$\downarrow$	$*$	$*$
φ_1	$*$	$*$	$*$	$*$	$*$	$*$	$\uparrow$	$*$	$*$

Notes. $\uparrow$, increase; $\downarrow$, decrease; $*$, irrelevant.

Corollary 3.7. *Sensitivity analysis of the product substitutability β to a manufacturer's optimal price, sales volumes and optimal patent licensing fee.*

(i) $\frac{\partial p_1}{\partial \beta} = \frac{\partial d_1}{\partial \beta} > 0, \frac{\partial p_2}{\partial \beta} = \frac{\partial d_2}{\partial \beta} > 0.$
(ii) $\frac{\partial p_1^u}{\partial \beta} > 0, \frac{\partial p_2^u}{\partial \beta} > 0; \frac{\partial d_1^u}{\partial \beta} > 0, \frac{\partial d_2^u}{\partial \beta} > 0; \frac{\partial \varphi_1^u}{\partial \beta} > 0.$
(iii) $\frac{\partial p_1^c}{\partial \beta} > 0; \frac{\partial p_2^c}{\partial \beta} > 0; \frac{\partial d_1^c}{\partial \beta} > 0; \frac{\partial d_2^c}{\partial \beta} > 0; \frac{\partial \varphi_1^u}{\partial \beta} > 0; \frac{\partial \varphi_2^u}{\partial \beta} > 0.$

The optimal product price and sales volumes of manufacturers 1 and 2, along with the patent licensing fees for components 1 and 2, all demonstrate an upward trend in response to increasing product substitutability. The increasing substitutability between the two products indicates a higher level of competition. Given the assumption of oligopoly competition between manufacturers 1 and 2 in the article, both manufacturers adopt a price-increase strategy to mitigate the intensified competition between their respective products.

Moreover, it is noteworthy that as competition intensity rises, the increase in patent licensing fees serves as an additional strategy to alleviate competition.

4. Comparison analysis

According to the equilibrium results under the three licensing scenarios in Table 1, this section conducts comparison analysis from and the equilibrium price and profit of manufacturers under the three licensing scenarios are compared and analysed, and their optimal licensing strategies under the background of co-opetition are determined.

4.1. Patent licensing analysis

Different manufacturers may choose different licensing strategies. For example, Huawei and Apple, as well as Apple and Samsung, first reached a unilateral licensing agreement and then reached a cross-licensing agreement. Therefore, this section makes a comparative analysis of prices and profits under different scenarios to explore the optimal patent licensing decisions of manufacturers under different scenarios. It analyses competition (no licensing) and unilateral licensing scenarios; unilateral and cross-licensing scenarios; cross-licensing and competition scenarios; and competition, unilateral licensing and cross-licensing scenarios.

(1) Competition scenario *versus* unilateral licensing scenario

Corollary 4.1. *Price and threshold analysis under competition and unilateral licensing scenarios.*

(i) $if\ 0 < c_1 < c_1^1,\ then\ p_1^u > p_1\ and\ p_2^u > p_2;\ if\ c_1^1 < c_1 < c_1^0,\ then\ p_1^u > p_1\ and\ p_2^u < p_2;\ if\ c_1^0 < c_1,\ then\ p_1^u < p_1\ and\ p_2^u < p_2.$
(ii) $\frac{\partial c_1^0}{\partial c_2} < \frac{\partial c_1^1}{\partial c_2} < 0 \frac{\partial c_1^0}{\partial a} < \frac{\partial c_1^1}{\partial a} < 0.$

When the cost advantage (disadvantage) of manufacturer 1 (2) is small, the prices of products 1 and 2 in the unilateral licensing scenario are higher than those in the competition scenario. When the cost advantage (disadvantage) of manufacturer 1 (2) is moderate, the price of product 1 in the unilateral licensing scenario is higher than that in the competition scenario, and the price of product 2 is lower than that in the competition scenario. When the cost advantages (disadvantages) of manufacturer 1 (2) are large, the prices of products 1 and 2 in the unilateral licensing scenario are lower than those in the competition scenario. As the cost advantage of manufacturer 1 increases, the prices of products 1 and 2 in the competition scenario increase. The price increase of manufacturer 1 is less than that of manufacturer 2. The manufacturer will shift the negative impact of increased costs to consumers. Moreover, the cost increase in component 1 encourages manufacturer 2 to buy the component licensing of manufacturer 1, reducing the burden of cost increase. Therefore, when the cost advantage is large, unilateral licensing can help manufacturers 1 and 2 achieve cost efficiency. When the cost advantage is small and the driving force of licensing is not enough, the product prices in the competition scenario will be lower than that in the unilateral licensing scenario.

The cost advantagec_2 and market share of manufacturer 2 are negatively related to the threshold value (c_1^0, c_1^1) in both cases, and the impact on c_1^0 is greater than that on c_1^1. The cost advantage of manufacturer 2 or the increase in the market share of manufacturer 1 expands the area where the product price in the competition scenario is higher than that in the unilateral licensing scenario. However, it narrows the area where the product price in the competition scenario is lower than that in the unilateral licensing scenario. The increase in manufacturer 2's cost advantage reduces the profits of the two competing manufacturers and leads to an increase in product pricing. The unilateral licensing scenario can mitigate the profit loss through the licensing agreement, so the competition scenario is greatly affected. However, an increase in manufacturer 2's cost advantage increases the driving force of licensing to a certain extent, which means that the area where the product price is lower than the competition in the unilateral licensing scenario is further expanded.

Proposition 4.2. *In the competition scenario, whether the competitive manufacturers should choose a unilateral licensing strategy is as follows:*

(i) $if\ c_1^3 < c_1 < \min(c_1^2, c_1^4),\ then\ \pi_1^u > \pi_1\ and\ \pi_2^u > \pi_2;\ if\ \max(c_1^2, c_1^4) < c_1,\ then\ \pi_1^u < \pi_1\ and\ \pi_2^u < \pi_2.$
(ii) $\frac{\partial c_1^3}{\partial c_2} < 0, \frac{\partial c_1^4}{\partial c_2} > 0, 0 > \frac{\partial c_1^3}{\partial a} > \frac{\partial c_1^4}{\partial a}.$

When the cost advantage (c_1) of manufacturer 1 is not relatively small, the unilateral licensing scenario is better than the competition scenario, and there is Pareto improvement for manufacturers 1 and 2, as depicted in area III ($\pi_1^u > \pi_1\ and\ \pi_2^u > \pi_2$) in Figure 3. As the cost advantage of manufacturer 1 increases, the total profits of both manufacturers change from bad to good relative to the competition scenario, and the profit of manufacturer 2 is gradually eroded by manufacturer 1 in the competition scenario. Due to obtaining the patented component 1, the profits of manufacturer 2 in the unilateral licensing scenario have gradually improved compared to the competitive scenario. With an increase in manufacturer 1's cost advantage, the total profit of the supply chain gradually changes from being better in the competition scenario to being better in the unilateral licensing scenario. So, the transition from a competitive scenario to a unilateral licensing scenario will also result in Pareto inefficiency to Pareto improvement (as in Region III).

Finally, there is a Pareto improvement area for both manufacturers. The main reason is that when the cost advantage is small, the driving force of patent technology licensing is insufficient. For manufacturer 1, the revenue from patent licensing is not enough to compensate for the profit loss caused by the competitive pressure brought by manufacturer 2 after getting manufacture 1s patent licensing. With the gradual expansion of the cost advantage, Manufacturer 2 can attain more benefits by reducing costs and enhancing efficiency in the unilateral licensing scenario. As depicted in Region II of Figure 3, the total profit has improved under the

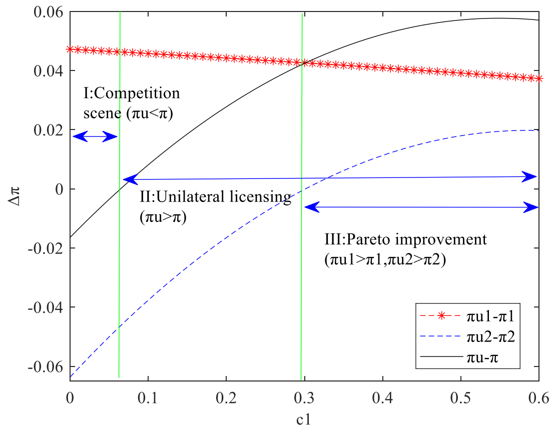


Figure 3. Unilateral licensing and competition scenarios.

unilateral licensing scenario. However, competition between the two manufacturers continues to overshadow the cooperative benefits under post-licensing. Consequently, the unilateral licensing remains ineffective. Therefore, when the cost advantage is large, the competitive position of manufacturer 2 improves after the patent is licensed, and the cooperative benefits under post-licensing is more than competition loss between the two manufacturers. In addition, to some extent, the cost advantage of component 2 prevents manufacturer 1 from obtaining profits from its cost savings. So, there is a Pareto improvement between the two manufacturers.

The threshold value (c_1^3, c_1^4) is negatively and positively related to the cost advantage c_2(saving). It is negatively related to the market share (a), and the market share has a greater impact on the threshold c_1^4. When the cost advantage c_2 of manufacturer 2 increases, the Pareto improvement area of both manufacturers can be expanded. When the cost advantage c_2 increases, the competitiveness of manufacturer 2 improves, and the cost burden of manufacturer 1 purchasing component 2 increases. Here, manufacturer 1 transfers part of the cost burden through the patent licensing of component 1, which can make the Pareto improvement area of two manufacturers competing for a winwin cooperation become larger. As the market share of manufacturer 1 increases, the threshold value (c_1^3, c_1^4) decreases, and the market share of manufacturer 2 will not be occupied. Here, the revenue from licensing of manufacturer 1 also decreases. To avoid the reduction in licensing revenue, manufacturer 1 reduces the licensing fees $\frac{\partial \varphi_1}{\partial a} < 0$to ease the competition with manufacturer 2 and implements the winwin competition strategy of making the cake bigger. The Pareto improvement area of both becomes larger.

(2) Unilateral licensing *versus* cross-licensing

Corollary 4.3. *Price, licensing fee and threshold analysis under unilateral and cross-licensing scenarios:*

(i) *if* $0 < c_2 < c_2^0$, *then* $p_1^c > p_1^u$ *and* $p_2^c > p_2^u$; *if* $c_2^0 < c_2 < c_2^1$, *then* $p_1^c < p_1^u$ *and* $p_2^c > p_2^u$; *if when* $c_2 > c_2^1$, *then* $p_1^c < p_1^u$ *and* $p_2^c < p_2^u$.
(ii) $\varphi_1^c < \varphi_2^u$.
(iii) $0 < \frac{\partial c_2^0}{\partial a} < \frac{\partial c_2^1}{\partial a}$.

When the cost advantage $(c_2)$of manufacturer 2 is small, the product pricing of both manufacturers 1 and 2 is higher in the cross-licensing scenario than that in the unilateral licensing scenario. When the cost advantage of manufacturer 2 is relatively large, the opposite is true. When the cost advantage of manufacturer 2 is moderate, the product pricing of manufacturer 1 is lower in the cross-licensing scenario than in the unilateral licensing scenario, and the product pricing of manufacturer 2 is higher in the cross-licensing scenario than in the unilateral

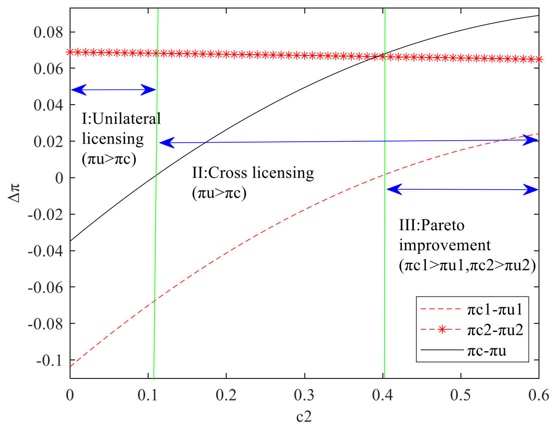


FIGURE 4. Cross and unilateral licensing scenarios.

licensing scenario. As manufacturer 2's cost advantage(c_2) increases, the price of its product decreases faster than that of manufacturer 1. The main reasons are that, first, as the cost advantage (c_2) increases, the competitive position of manufacturer 2 gradually improves. Here, the competitive strategy of manufacturer 2 is to expand market share and increase sales volumes. If manufacturer 1 transfers the cost burden to consumers, it may face profit loss, and its market competitive position will weaken. Therefore, manufacturer 1 also adopts the same competitive strategy. Second, as both competing manufacturers have a patent for one component after cross-licensing, the competition between the two manufacturers slows down to a certain extent. In the case of unilateral licensing, when only one component patent is licensed, the competitive advantage of the manufacturer significantly increases. Therefore, another manufacturer must pass on the cost burden to consumers (increasing product price).

In the cross-licensing scenario, the patent licensing fee of component 1 is lower than that of component 1 in the unilateral licensing scenario. In the cross-licensing scenario, the two competing manufacturers license their own patented technologies to each other, so both can gain cost advantages. Both of them weaken their competitive advantages and ease their competitive situation. In the unidirectional licensing scenario, one party obtains excess revenue through patent licensing, and the other party transfers the cost burden, which intensifies the competition between the two parties. Therefore, the licensing fee should be low in the cross-licensing scenario.

The threshold value (c_2^0, c_2^1) in these two scenarios is positively correlated with the market share of the two competitors. An increase in the market share of these two competitors may lead to different results. When the cost advantage (c_2) is relatively small, the competition area for price increases between the two manufacturers becomes larger and enters into vicious competition. When the cost advantage (c_2) is relatively large, the price competition between the two decreases; the driving force of cross-licensing increases; and the threshold for price decreasing competition becomes higher. Further, when the market share gap between the two is large, the probability of entering into a benign competition is small, but the competition situation of the strong is stronger, the weak is weaker surfaces.

Proposition 4.4. *In the unilateral licensing scenario, whether the competitive manufacturers should choose a cross-licensing strategy is as follows: when* $0 < \beta < 0.735, 0 < a < a_0$, *or* $\beta > 0.735, c_2^2 < c_2 < \min(c_2^3, a)$, *we derive* $\pi_1^c > \pi_1^u$ *and* $\pi_2^c > \pi_2^u$.

Proposition 4.4 reveals that when the product substitutability (price competition intensity) is relatively small, the market share of manufacturer 1 is small, or when the product substitutability (price competition intensity) is relatively large and the cost advantage (c_2) of manufacturer 2 is moderate, both manufacturers can achieve Pareto improvement in the cross-licensing scenario compared with the unilateral licensing scenario, as depicted in area III in Figure 4. Otherwise, the competitor with a relatively strong competitive advantage will capture more total profits of the supply chain than the competitor with a weak competitive advantage (cooperation leads to bigger cake), as depicted in area II in Figure 4. As the cost advantage of manufacturer 2 gradually decreases, the driving force of cross-licensing gradually weakens, resulting in worse cross-licensing than unilateral licensing scenarios such as area I. The main reason lies in the gradual decrease of component 2's cost and the consequent reduction in cost burden for manufacturer 1. This leads to component 1 becoming an advantage for manufacturer 1, thereby weakening the driving force behind manufacturer 1's cross-licensing. When products substitutability is low and manufacturer 1 holds a significant market share, the competition between the two products is relatively moderate. Manufacturer 1 can fully leverage its market share and product competition advantages to generate greater profits. As a result, there is no need for manufacturer 1 to intensify competition or weaken its competitive position through cross-licensing. Consequently, the motivation for cross-licensing is not strong. When the cost advantage is relatively small, the competitive intensity between the two products weakens, thereby strengthening the competitive position of manufacturer 1. Conversely, when the cost advantage is relatively large, both manufacturers 1 and 2 resort to fierce competition and shift the cost burden onto consumers, which is not conducive to Pareto improvement. Cross-licensing can only achieve Pareto improvement when product substitutability is high and the cost advantage is moderate.

(3) Competition scenario *versus* cross-licensing

Corollary 4.5. *Price and threshold analysis in competition and cross-licensing scenarios.*

(i) *if* $2c_2+\beta c_1<\frac{16\beta+\beta^3+\beta^5+a(\beta-1)^2(8-\beta^3)}{8-7\beta^2-\beta^4}$, *then* $p_1^c>p_1$; *otherwise* $p_1^c<p_1$.
(ii) *if* $2c_1+\beta c_2<\frac{2(2+\beta^2)^2+a(\beta-1)^2(8-\beta^3)}{8-7\beta^2-\beta^4}$, *then* $p_2^c>p_2$; *otherwise* $p_2^c<p_2$.

When the linear sum of cost advantages (c_1,c_2) of manufacturers 1 and 2 is small, the pricing of products 1 and 2 in the cross-licensing scenario is greater than that in the competition scenario. When the linear sum of cost advantages (c_1,c_2) of manufacturers 1 and 2 is large, the pricing of products 1 and 2 in the cross-licensing scenario is smaller than that in the competition scenario. Because manufacturers 1 and 2 are not cost-efficient in the competition scenario, both competitive manufacturers have patent licensing of components and do not license each other. They bear the cost burden of non-patented technical components. Here, the product price has no comparative advantage, and consumers bear the cost burden.

Proposition 4.6. *In the competition scenario, whether the competitive manufacturers should choose a cross-licensing strategy is as follows: when* $c_1^7<c_1<c_1^6$, *we derive* $\pi_1^c>\pi_1$ *and* $\pi_2^c>\pi_2$.

When the cost advantage (c_1) of manufacturer 1 is moderate, manufacturers 1 and 2 have Pareto improvement, as depicted in area III in Figure 5. When the cost advantage of manufacturer 1 is small, the competition scenario is better than the cross-licensing scenario, as depicted in area I in Figure 5. The greater the cost advantage of manufacturer 1, the heavier the cost burden on manufacturer 2, and the stronger the driving force behind manufacturer 2's patent licensing. However, as the cost advantage of component 1 gradually increases, the cost gap with component 2 will narrow over time, potentially leading to intensified competition. Manufacturer 2 can benefit from the advantages brought by the increased cost advantage of component 1, thus experiencing a gradual increase in the motivation for cross-licensing. On the other hand, as the cost advantage of component 1 strengthens for manufacturer 1, the competition between components 1 and 2 gradually intensifies, weakening the driving force behind manufacturer 1's cross-licensing. Therefore, when the cost advantage of component 1 is small, the motivation for cross-licensing is strong for manufacturer 1, while it is weak for manufacturer 2. Conversely, when the cost advantage of component 1 is large, the motivation for cross-licensing weakens for manufacturer 1, but strengthens for manufacturer 2. Consequently, the implementation of cross-licensing is a stable Pareto improvement only when the cost advantage falls within a certain range.

(4) Competition scenario *versus* unilateral licensing *versus* cross-licensing

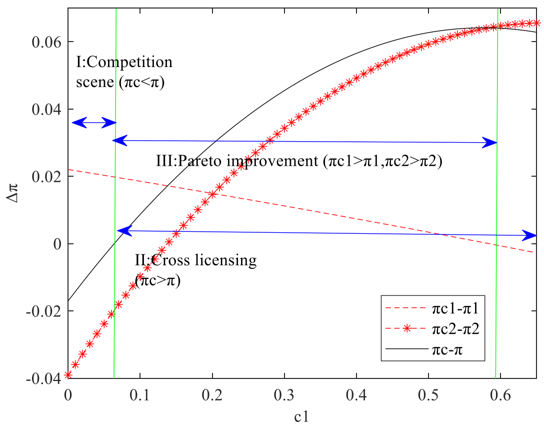


FIGURE 5. Cross-licensing and competition scenarios.

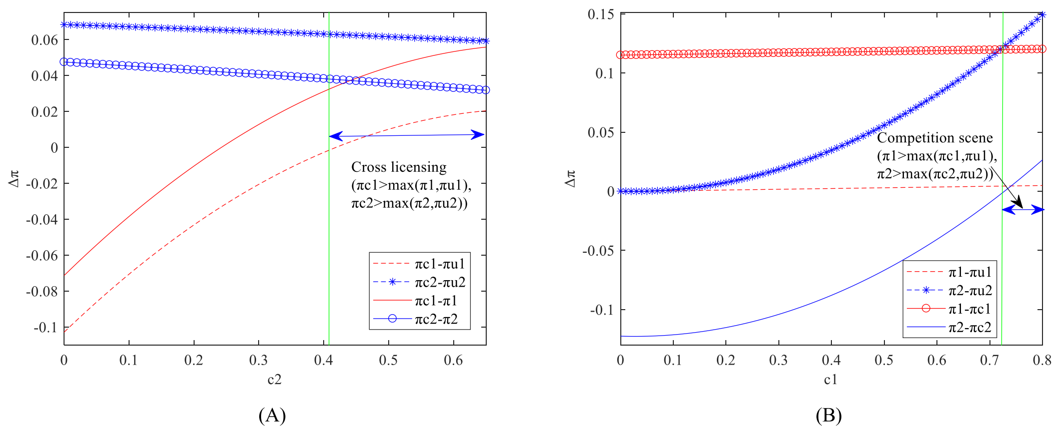


FIGURE 6. Competition, unilateral licensing and cross-licensing scenarios.

Proposition 4.7. *For competitive manufacturers, how to choose the licensing strategies are as follows.*

(i) if $0 < \beta < 0.735, 0 < a < a_0$ *(or* $\beta > 0.735, c_2^2 < c_2 < c_2^3$*), and* $c_1^7 < c_1 < c_1^6$*, then* $\pi_1^c > \max(\pi_1, \pi_1^u)$ *and* $\pi_2^c > max(\pi_2, \pi_2^u)$*;*
(ii) if $0 < c_2 < a$*, then* $\pi_2 > \pi_2^u$*; unilateral licensing is not the optimal licensing strategy at any time.*
(iii) if $c_1 \in (0, min(c_1^3, c_1^7)) \cup (max(c_1^2, c_1^4, c_1^6), 1-a)$*, then* $\pi_1 > max(\pi_1^c, \pi_1^u)$ *and* $\pi_2 > max(\pi_2^c, \pi_2^u)$*.*

By comparing three different forms of licensing scenarios, we can derive the optimal licensing strategies of manufacturers in different scenarios. When the level of product substitutability (competitive intensity) is relatively weak and manufacturer 1 has a relatively small market share, or when the level of product substitutability is high and both manufacturer 1 and manufacturer 2 have moderate cost advantages, the cross-licensing scenario outperforms both the competition scenario and the unilateral licensing scenario. This leads to Pareto improvements for both manufacturers 1 and 2, as illustrated in Figure 6A.

In cases where the substitutability between the two products is weak and manufacturer 1 has a small market share, manufacturer 1's competitiveness is also weak. As a result, manufacturer 1 has a strong motivation to obtain the patent licensing for component 2. The greater the cost advantage of component 1, the less willing

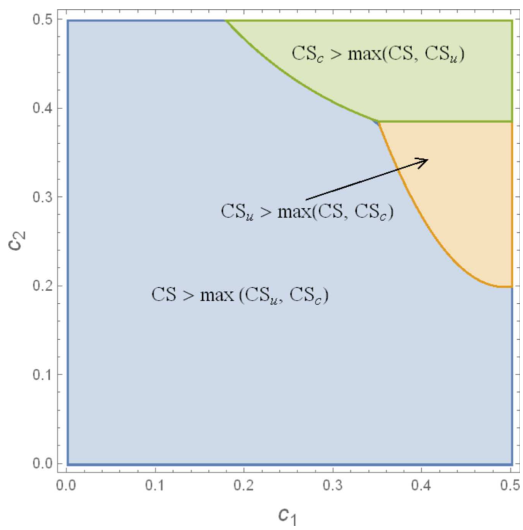


FIGURE 7. Consumer surplus.

manufacturer 1 is to license component 1 to manufacturer 2, while manufacturer 2 becomes more eager to obtain the patent licensing for component 1. Conversely, when the cost advantage of component 1 is smaller, manufacturer 1 is more inclined to license it to manufacturer 2, and manufacturer 2 becomes less interested in acquiring the patent licensing for component 1. Therefore, in such cases, the cross-licensing scenario is likely to result in Pareto improvements only if the cost advantage of either component 1 or component 2 is moderate.

In a unilateral licensing scenario, manufacturer 2 is granted a patent licensing for component 1. However, unless the cost advantage of component 2 is significant and the difference in cost advantages between the two components is small, it leads to intensified competition between manufacturers 1 and 2. Thus, the unilateral licensing scenario does not alleviate the competition.

When manufacturer 1's cost advantage is relatively small or large, the competition scenario may offer better Pareto improvements for manufacturers 1 and 2 compared to the other two licensing scenarios. The incentives for cross-licensing and unilateral licensing scenarios are insufficient for manufacturer 2 or manufacturer 1 when manufacturer 1's cost advantage is small or large.

4.2. Consumer surplus analysis

Analysing the profits of the three licensing scenarios based on the manufacturer's perspective can provide guidance on licensing decisions for co-opetition manufacturers. Because many technological innovations and patent licensing are related to the participation and active guidance of the government, the paper analyses the consumer surplus in different licensing scenarios based on the consumer perspective to provide a decision-making basis for the active guidance of the government. This section analyses the change in consumer surplus in different licensing scenarios in numerical form. This section adopts $CS = \frac{1}{2}(d_1^2 + 2\beta d_1 d_2 + d_2^2)$ [6], indicating consumer surplus.

As depicted in Figure 7, when the cost advantages of manufacturers 1 and 2 are relatively large, the consumer surplus in the cross-licensing scenario is higher than that in the other two scenarios. When the cost advantage of manufacturer 1 is large and that of manufacturer 2 is moderate, the consumer surplus in the unilateral licensing scenario is higher than that in the other two scenarios. When the cost advantage of manufacturer 1 is not great, the consumer surplus in the competition scenario is always superior to that in the other two scenarios. Competition can improve efficiency and consumer surplus, so competition scenarios can achieve more consumer surplus under certain conditions. Although patent licensing can enhance competitiveness and aggravate competition, it can reduce the cost burden of manufacturers and promote benign competition between co-opetition manufacturers. Therefore, under certain conditions, whether unilateral or cross-licensing, it can increase consumer surplus and reduce the cost burden of manufacturers.

5. Conclusion

This paper focuses on co-opetition manufacturers and studies their licensing decisions in three licensing scenarioscompetition (without licensing), unilateral licensing, and cross-licensing. By building a game model, we analyse the equilibrium results of different licensing scenarios and the impact of key parameters on their equilibrium results.

The research results are as follows. (i) When the cost advantage of manufacturer 1 is moderate, the unilateral licensing scenario is better than the competition scenario, and there is Pareto improvement for both manufacturers 1 and 2. When the product substitutability (price competition intensity) is relatively small, the market share of manufacturer 1 is small, or when the product substitutability is relatively large, the cost advantage of manufacturer 2 is moderate. Here, compared with the unilateral licensing scenario, manufacturers 1 and 2 can achieve Pareto improvement in the cross-licensing scenario. When the cost advantage of manufacturer 1 is moderate, manufacturers 1 and 2 have Pareto improvement, and the cross-licensing scenario is better than the competition scenario. (ii) When the product substitutability is not very high and the market share of manufacturer 1 is relatively small or the product substitutability is large and the cost advantage of manufacturers 1 and 2 is moderate, the cross-licensing scenario is better than the unilateral licensing and competition scenarios for Pareto improvement of manufacturers 1 and 2. The Pareto improvement of the unilateral licensing scenario is always better than that of the other two scenarios. When the cost advantage of manufacturer 1 is small or large, the Pareto improvement of manufacturers 1 and 2 in the competition scenario is better than in the other two licensing scenarios.

This study provides the following decision support to firms and governments. (i) Manufacturers 1 and 2 may be in different scenarios and face different licensing options due to the differences in cost advantages and market shares, as well as the substitutability of products and the impact of complex external economic and political environment. If different external environments and internal factors cause manufacturers to make choices only between two licensing modes, manufacturers can make optimal licensing decisions based on their cost advantages, market share, product competition intensity and other factors. Propositions 1, 2 and 3 can serve as the basis for their decisions. (ii) When manufacturers are faced with three types of licensing decisions, they should not choose unilateral licensing agreements. Such agreements are unstable and will not allow co-opetition manufacturers to achieve a winwin situation. For example, Huawei and Apple ultimately chose cross-licensing, and Audi and Hyundai finally chose cross-licensing. The manufacturer can make the optimal licensing decision according to its cost advantage, market share and product competition intensity. For example, when the cost advantage of manufacturer 1 is small or large, the competition scenario (no licensing) is the optimal licensing decision for manufacturers 1 and 2. (iii) Through the analysis of consumer surplus, when the government participates in and guides firms to carry out technological innovation and patent licensing, it can determine whether the licensing mode chosen by firms has greatly damaged or improved the rights and interests of consumers according to the cost advantages of different firms.

Appendix A.

Proof of Corollary 3.5.

(i) $\frac{\partial p_1}{\partial a} = \frac{1}{2+\beta} > 0, \frac{\partial p_1}{\partial c_1} = \frac{-\beta}{-4+\beta^2} > 0, \frac{\partial p_1}{\partial c_2} = \frac{-2}{-4+\beta^2} > 0, \frac{\partial p_2}{\partial a} = \frac{-1}{2+\beta} < 0, \frac{\partial p_2}{\partial c_1} = \frac{-2}{-4+\beta^2} > 0, \frac{\partial p_2}{\partial c_2} = \frac{-\beta}{-4+\beta^2} > 0$; $\frac{\partial p_1^u}{\partial a} = \frac{-(-8+2\beta+\beta^2)}{2(8+8\beta+\beta^2+\beta^3)} > 0$; $\frac{\partial p_1^u}{\partial c_2} = \frac{8+3\beta^2}{16+2\beta^2} > 0$; $\frac{\partial p_2^u}{\partial a} = \frac{-(12+4\beta+2\beta^2+\beta^3)}{2(8+8\beta+\beta^2+\beta^3)} < 0$; $\frac{\partial p_2^u}{\partial c_2} = \frac{\beta(4+\beta^2)}{2(8+\beta^2)} > 0$; $\frac{\partial p_1^c}{\partial a} = \frac{6}{8+8\beta+\beta^2+\beta^3} > 0$; $\frac{\partial p_2^c}{\partial a} = \frac{-6}{8+8\beta+\beta^2+\beta^3} < 0$.

(ii) $\frac{\partial d_1}{\partial a} = \frac{2-\beta}{4-\beta^2} > 0, \frac{\partial d_1}{\partial c_1} = \frac{-\beta}{-4+\beta^2} > 0, \frac{\partial d_1}{\partial c_2} = \frac{2-\beta^2}{-4+\beta^2} < 0, \frac{\partial d_2}{\partial a} = \frac{-1}{2+\beta} < 0, \frac{\partial d_2}{\partial c_1} = \frac{2-\beta^2}{-4+\beta^2} < 0, \frac{\partial d_2}{\partial c_2} = \frac{-\beta}{-4+\beta^2} >$

$0; \frac{\partial d_1^u}{\partial a} = \frac{8-2\beta+\beta^2-\beta^3}{2(8+\beta^2)} > 0, \frac{\partial d_1^u}{\partial c_2} = \frac{-8+\beta^2+\beta^4}{2(8+\beta^2)} < 0, \frac{\partial d_2^u}{\partial a} = \frac{-(2+\beta^2)}{8+\beta^2} < 0, \frac{\partial d_2^u}{\partial c_2} = \frac{\beta(2+\beta^2)}{8+\beta^2} > 0; \frac{\partial d_1^c}{\partial a} = \frac{a(2+\beta^2)}{8+\beta^2} > 0; \frac{\partial d_2^c}{\partial a} = \frac{-(2+\beta^2)}{8+\beta^2} < 0.$

Proof of Corollary 3.6.

(i) $\frac{\partial \varphi_1^u}{\partial a} = \frac{-8+\beta^3}{2(8+8\beta+\beta^2+\beta^3)} < 0, \frac{\partial \varphi_1^u}{\partial c_2} = \frac{-\beta^3}{2(8+\beta^2)} < 0.$
(ii) $\frac{\partial \varphi_1^c}{\partial a} = -\frac{4+2\beta+\beta^2}{8+8\beta+\beta^2+\beta^3} < 0, \frac{\partial \varphi_2^c}{\partial a} = \frac{4+2\beta+\beta^2}{8+8\beta+\beta^2+\beta^3} > 0.$

Proof of Corollary 3.7.

(i) $\frac{\partial p_1}{\partial \beta} = \frac{\partial d_1}{\partial \beta} = \frac{4-a(2-\beta)^2+\beta^2+(4+\beta^2)c_1+4\beta c_2}{(4-\beta^2)^2} > 0,$
$\frac{\partial p_2}{\partial \beta} = \frac{\partial d_2}{\partial \beta} = \frac{a(2-\beta)^2+4\beta+4\beta c_1+(4+\beta^2)c_2}{(4-\beta^2)^2} > 0.$
(ii) $\frac{\partial p_1^u}{\partial \beta} = \frac{80+46\beta^2+37\beta^4-\beta^6-a(1-\beta)^2(80+32\beta+30\beta^2-4\beta^3-\beta^4)+32\beta(1-\beta^2)^2c_2}{2(8-7\beta^2-\beta^4)^2} > 0,$
$\frac{\partial p_2^u}{\partial \beta} = \frac{a(1-\beta)^2(64-8\beta-8\beta^3+\beta^4)+2\beta(68+8\beta^2+5\beta^4)+(1-\beta^2)^2(32+20\beta^2+\beta^4)c_2}{2(8-7\beta^2-\beta^4)^2} > 0;$
$\frac{\partial d_1^u}{\partial \beta} = \frac{(1-a)(16+22\beta^2+\beta^4)+2\beta(16+16\beta^2+\beta^4)c_2}{2(8+\beta^2)^2} > 0,$
$\frac{\partial d_2^u}{\partial \beta} = \frac{12(1-a)\beta+(16+22\beta^2+\beta^4)c_2}{(8+\beta^2)^2} > 0;$
$\frac{\partial \phi_1^u}{\partial \beta} = \frac{2\beta(56+32\beta^2-7\beta^4)+a(1-\beta)^2(64+16\beta+48\beta^2+16\beta^3+\beta^4)-\beta^2(1-\beta^2)^2(24+\beta^2)c_2}{2(8-7\beta^2-\beta^4)^2} > 0.$
(iii) $\frac{\partial p_1^c}{\partial \beta} = \frac{6(8+7\beta^2+3\beta^4-a(1-\beta)^2(8+2\beta+3\beta^2))}{(8-7\beta^2-\beta^4)^2} > 0;$
$\frac{\partial p_2^c}{\partial \beta} = \frac{6(2\beta(7+2\beta^2)+a(1-\beta)^2(8+2\beta+3\beta^2))}{(8-7\beta^2-\beta^4)^2} > 0; \frac{\partial d_1^c}{\partial \beta} = \frac{12a\beta}{(8+\beta^2)^2} > 0; \frac{\partial d_2^c}{\partial \beta} = \frac{12(1-a)\beta}{(8+\beta^2)^2} > 0;$
$\frac{\partial \phi_1^c}{\partial \beta} = \frac{2\beta(20+8\beta^2-\beta^4)+a(1-\beta)^2(16-8\beta+6\beta^2+4\beta^3+\beta^4)}{(8-7\beta^2-\beta^4)^2} > 0;$
$\frac{\partial \phi_2^c}{\partial \beta} = \frac{16+38\beta^2-\beta^4+\beta^6-a(1-\beta)^2(16-8\beta+6\beta^2+4\beta^3+\beta^4)}{(8-7\beta^2-\beta^4)^2} > 0.$

Proof of Corollary 4.1.

$p_1^u - p_1 = \frac{\beta[3(8+\beta^4+a(-8+8\beta+\beta^3-\beta^4))+(\beta^2-1)(2(8+\beta^2)c_1+3\beta^3c_2)]}{2(-4+\beta^2)(-8+7\beta^2+\beta^4)},$
$p_2^u - p_2 = \frac{4(-8+7\beta^2+\beta^4)c_1+(2+\beta^2)(8+\beta^4+a(-8+8\beta+\beta^3-\beta^4)+\beta^3(\beta^2-1)c_2)}{2(-4+\beta^2)(-8+7\beta^2+\beta^4)}$, we denote $c_1^0 = \frac{3(8+\beta^4+a(-8+8\beta+\beta^3-\beta^4))+(\beta^2-1)3\beta^3c_2}{(1-\beta^2)(2(8+\beta^2))} > 0, c_1^1 = \frac{(2+\beta^2)(8+\beta^4+a(-8+8\beta+\beta^3-\beta^4)+\beta^3(\beta^2-1)c_2)}{4(8-7\beta^2-\beta^4)} > 0,$ and we have $c_1^1 - c_1^0 < 0$.

When $0 < c_1 < c_1^1$, we have $p_1^u > p_1$ and $p_2^u > p_2$. When $c_1^1 < c_1 < c_1^0$, we have $p_1^u > p_1$ and $p_2^u < p_2$. When $c_1^0 < c_1$, we have $p_1^u < p_1$ and $p_2^u < p_2$.

Proof of Proposition 4.2.

$$\pi_1^u - \pi_1 = \frac{\begin{array}{l}4\beta^2c_1^2(8-7\beta^2-\beta^4)+8\beta c_1(8-7\beta^2-\beta^4)(2a-a\beta+\beta-(2-\beta^2)c_2)-\\(8+\beta^4+a(8\beta-8+\beta^3-\beta^4)+(\beta^5-\beta^3)c_2)^2\end{array}}{4(-4+\beta^2)^2(7\beta^2+\beta^4-8)}.$$

We denote $c_1^2 = \frac{2\beta(7\beta^2-8+\beta^4)(2a-a\beta+\beta+(\beta^2-2)c_2)+\sqrt{\beta^2(7\beta^2-8+\beta^4)(4(7\beta^2-8+\beta^4)(2a-a\beta+\beta+(\beta^2-2)c_2)^2-(8+\beta^4+a(8\beta-8+\beta^3-\beta^4)+\beta^3(\beta^2-1)c_2)^2)}}{2\beta^2(-8+7\beta^2+\beta^4)} > 0$. When $0 < c_1 < c_1^2$, we have $\pi_1^u > \pi_1$. When $c_1 > c_1^2$, we have $\pi_1^u < \pi_1$.

$\pi_2^u - \pi_2 = \frac{1}{(-32+4\beta^2+\beta^4)^2}(a(1-\beta)(\beta^3-8)+8+\beta^4+(\beta^2-2)(8+\beta^2)c_1+\beta^3(\beta^2-1)c_2)(a(2-\beta)(12+2\beta+\beta^3+3\beta^2)-24-4\beta^2+\beta^4+(2-\beta^2)(8+\beta^2)c_1-\beta(16+3\beta^2-\beta^4)c_2)$ We denote $c_1^3 = \frac{a(8-8\beta-\beta^3+\beta^4)-8-\beta^4+(\beta^3-\beta^5)c_2}{-16+6\beta^2+\beta^4}$ and $c_1^4 = \frac{a(2-\beta)(12+2\beta+\beta^3+3\beta^2)-24-4\beta^2+\beta^4+\beta(\beta^4-16-3\beta^2)c_2}{(-2+\beta^2)(8+\beta^2)}$, and we have $0 < c_1^3 < c_1^4$. When $c_1^3 < c_1 < c_1^4$, we have $\pi_2^u > \pi_2$. when $c_1 \in (0, c_1^3) \cup (c_1^4, a)$, we have $\pi_2^u < \pi_2$.

Proof of Corollary 4.3.

$p_1^c - p_1^u = \frac{\beta(2+\beta^2)+a(4-2\beta-\beta^2-\beta^3)-(8-5\beta^2-3\beta^4)c_2}{-2(-8+7\beta^2+\beta^4)}$, $p_2^c - p_2^u = \frac{\beta(2\beta+\beta^3+a(4-2\beta-\beta^2-\beta^3)+(-4+3\beta^2+\beta^4)c_2)}{-2(-8+7\beta^2+\beta^4)}$, $\phi_1^c - \phi_2^u = \frac{\beta\left(\beta(2+\beta^2)-a(2\beta+\beta^2+\beta^3-4)+\beta^2(\beta^2-1)c_2\right)}{2(-8+7\beta^2+\beta^4)} < 0$.

We denote $c_2^0 = \frac{2\beta+\beta^3+a(4-2\beta-\beta^2-\beta^3)}{8-5\beta^2-3\beta^4}$ and $c_2^1 = \frac{2\beta+\beta^3+a(4-2\beta-\beta^2-\beta^3)}{4-3\beta^2-\beta^4} > c_2^0$. When $0 < c_2 < c_2^0$ we have $p_1^c > p_1^u$ and $p_2^c > p_2^u$. When $c_2^0 < c_2 < c_2^1$, we have $p_1^c < p_1^u$ and $p_2^c > p_2^u$. When $c_2 > c_2^1$, we have $p_1^c < p_1^u$ and $p_2^c < p_2^u$.

Proof of Proposition 4.4.

$\pi_1^c - \pi_1^u = \frac{(a(\beta-1)(12-2\beta+\beta^2)-14\beta-\beta^3)(-2\beta-\beta^3-a(4-2\beta-\beta^2-\beta^3))+(7\beta^2-8+\beta^4)c_2(8\beta+2\beta^3+2a(8-4\beta+\beta^2-\beta^3)+(3\beta^2-8+\beta^4)c_2)}{4(-1+\beta^2)(8+\beta^2)^2}$. We denote $a_0 = \frac{\sqrt{(\beta-1)\beta^2(1+\beta)(2+\beta^2)^2(4+\beta^2)(8+\beta^2)(3\beta^2+\beta^4-8)}+(\beta^2-\beta)(8\beta-2\beta^2+21\beta^3+5\beta^4+4\beta^5+\beta^6-16)}{(-1+\beta)(-32+\beta(-64+\beta(4+\beta(-1+2\beta)(16+\beta+3\beta^2))))}$,

$c_2^2 = \sqrt{\frac{4(7\beta^2-8+\beta^4)(4a^2(1-\beta)(4\beta^2-64\beta-32+\beta^3(2\beta-1)(16+\beta+3\beta^2)-4\beta^2(\beta^4+\beta^6-24-23\beta^2))+8a(\beta^2-1)(8\beta-16-2\beta^2+21\beta^3+5\beta^4+4\beta^5+\beta^6))}{2(7\beta^2-8+\beta^4)}} - \frac{4\beta+\beta^3+a(8-4\beta+\beta^2-\beta^3)}{3\beta^2-8+\beta^4}$

and $c_2^3 = \sqrt{\frac{4(7\beta^2-8+\beta^4)(4a^2(1-\beta)(4\beta^2-64\beta-32+\beta^3(2\beta-1)(16+\beta+3\beta^2)-4\beta^2(\beta^4+\beta^6-24-23\beta^2))+8a(\beta^2-1)(8\beta-16-2\beta^2+21\beta^3+5\beta^4+4\beta^5+\beta^6))}{2(7\beta^2-8+\beta^4)(3\beta^2-8+\beta^4)}} + \frac{4\beta+\beta^3+a(8-4\beta+\beta^2-\beta^3)}{3\beta^2-8+\beta^4}$. When $0 < \beta < 0.735, 0 < a < a_0(\beta > 0.735), c_2^2 < c_2 < c_2^3$, we have $\pi_1^c > \pi_1^u$. When $c_2 \in (0, c_2^2) \cup (c_2^3, a), \pi_1^c < \pi_1^u$. When $0 < \beta < 0.735$, $a > a_0$, we have $\pi_1^c < \pi_1^u$.

$\pi_2^c - \pi_2^u = \frac{(2+\beta^2)(4a^2+6(1-a)a\beta+(2-4a+a^2)\beta^2+(a-1)^2\beta^4+(\beta^3+\beta^5-2\beta)((2-2a)c_2+\beta c_2^2))}{(1-\beta^2)(8+\beta^2)^2}$. when $0 < c_2 < a$, we have $\pi_2^c > \pi_2^u$.

Proof of Corollary 4.5.

$p_1^c - p_1 = \frac{16\beta+\beta^3+\beta^5+a(\beta-1)^2(8-\beta^3)+(7\beta^2+\beta^4-8)(2c_2+\beta c_1)}{(-4+\beta^2)(-8+7\beta^2+\beta^4)}$. When $2c_2 + \beta c_1 < \frac{16\beta+\beta^3+\beta^5+a(\beta-1)^2(8-\beta^3)}{8-7\beta^2-\beta^4}$,we have $p_1^c > p_1$; otherwise $p_1^c < p_1$.

$p_2^c - p_2 = \frac{2(2+\beta^2)^2+a(\beta-1)^2(\beta^3-8)+(7\beta^2+\beta^4-8)(2c_1+\beta c_2)}{(-4+\beta^2)(-8+7\beta^2+\beta^4)}$. When $2c_1 + \beta c_2 < \frac{2(2+\beta^2)^2+a(\beta-1)^2(8-\beta^3)}{8-7\beta^2-\beta^4}$, we have $p_2^c > p_1$; otherwise $p_2^c < p_1$.

Proof of Proposition 4.6.

$$\pi_1^c - \pi_1 = \frac{24\beta^4 + 25\beta^6 + a^2(\beta^2-3\beta+2)^2(\beta^2-6\beta-4)(4+\beta^2+2\beta) + 2a(\beta^2-3\beta+2)(5\beta^5-64-112\beta-88\beta^2-28\beta^3-10\beta^4) + 128 - 96\beta^2 + (\beta^2-1)(8+\beta^2)^2(\beta^2 c_1^2 + \beta c_1(4a+2\beta-2a\beta+(\beta^2-2)c_2)) + (\beta^2-2)c_2(4a+2\beta-2a\beta+(\beta^2-2)c_2)}{(-4+\beta^2)^2(1-\beta^2)(8+\beta^2)^2}.$$

We denote $c_1^5 = \frac{-a(-2+\beta)(-1+\beta)\beta(1+\beta)(8+\beta^2)^2 + \beta(-1+\beta^2)(8+\beta^2)^2(\beta+(-2+\beta^2)c_2) + \sqrt{(-2+\beta)^2\beta^2(2+\beta)^2(8+\beta^2)^2(-2+\beta^2+\beta^4)(-4+6a^2(-1+\beta)+\beta^2-2a(-1+\beta)(4+\beta))}}{\beta^2(1-\beta^2)(8+\beta^2)^2}$. When $0 < c_2 < a$, $0 < c_1 < c_1^5$, we have $\pi_1^c > \pi_1$. When $0 < c_2 < a, c_1^5 < c_1 < 1-a$, we have $\pi_1^c < \pi_2$.

$$\pi_2^c - \pi_2 = \frac{48\beta^4 + \beta^8 + a^2(\beta^2-3\beta+2)^2(\beta^2-6\beta-4)(4+\beta^2+2\beta) - 2a(-192+32\beta+224\beta^2-96\beta^3+48\beta^4-12\beta^5-5\beta^7+\beta^8) - 192 + 224\beta^2 + (\beta^2-1)(8+\beta^2)^2((\beta^2-2)^2c_1^2 + (\beta^2-2)(4+2a(\beta-2)c_1+2\beta c_2) + \beta c_2(4+2a(\beta-2)+\beta c_2))}{(1-\beta^2)(-32+4\beta^2+\beta^4)^2}.$$

We denote $c_1^6 = \min$

$(1-a, c_1^5, \frac{36\beta^4+26\beta^6+2\beta^8+a(8+\beta^2)^2(2\beta+6\beta^2-3\beta^3-2\beta^4+\beta^5-4)+\beta(8+\beta^2)^2(2-3\beta^2+\beta^4)c_2+256-320\beta^2+\sqrt{(\beta^2+\beta^4-2)(64-40\beta^2+2\beta^4+\beta^6)^2(6a^2(\beta-1)-\beta^2-2+2a(2-3\beta+\beta^2))}}{(1-\beta^2)(6\beta^2+\beta^4-16)^2}$

and $c_1^7 = \max$

$(0, \frac{36\beta^4+26\beta^6+2\beta^8+a(8+\beta^2)^2(-4+2\beta+6\beta^2-3\beta^3-2\beta^4+\beta^5)+\beta(8+\beta^2)^2(2-3\beta^2+\beta^4)c_2+256-320\beta^2-\sqrt{(-2+\beta^2+\beta^4)(64-40\beta^2+2\beta^4+\beta^6)^2(-2+6a^2(-1+\beta)-\beta^2+2a(2-3\beta+\beta^2))}}{(1-\beta^2)(-16+6\beta^2+\beta^4)^2})$.
We have $c_1^7 < c_1 < c_1^6$, we have $\pi_2^c > \pi_2$; otherwise $\pi_2^c < \pi_2$.

Proof of Consumer surplus analysis

We denote the consumer surplus for competition and unilateral licensing and cross-licensing scenarios as CS, CS^uandCS^c.

$CS = \frac{1}{2(4-\beta^2)^2}((2+a(\beta-2)+(\beta^2-2)c_1+\beta c_2)^2+2\beta(2+a(\beta-2)+(\beta^2-2)c_1+\beta c_2)(2a+\beta-a\beta+\beta c_1+(\beta^2-2)c_2)+(2a+\beta-a\beta+\beta c_1+(\beta^2-2)c_2)^2)$,

$CS^u = \frac{1}{8(8+\beta^2)^2}(4(2+\beta^2)^2(1-a+\beta c_2)^2+4\beta(2+\beta^2)(1-a+\beta c_2)(2\beta+\beta^3-a(\beta-2)(4+\beta+\beta^2)+(\beta^2-8+\beta^4)c_2)+(2\beta+\beta^3-a(\beta-2)(4+\beta+\beta^2)+(\beta^2+\beta^4-8)c_2)^2)$,

$CS^c = \frac{(1-2a(1-a)(1-\beta))(2+\beta^2)^2}{2(8+\beta^2)^2}$

We denote $g_1(c_1,c_2) = CS - CS^u, g_2(c_1,c_2) = CS - CS^c, g_3(c_2) = CS^u - CS^c$.

$g_1(c_1,c_2) = \frac{1}{8(32-4\beta^2-\beta^4)^2}(4(8+\beta^2)^2(4-7\beta^2+3\beta^4)c_1^2+8(\beta^2-1)(8+\beta^2)^2c_1(4-a(\beta-2)^2+\beta^2+\beta^3c_2)-(96+136\beta^2+16\beta^4-5\beta^6+a(2-\beta)(1-\beta)(40\beta-48+16\beta^2+14\beta^3+5\beta^4)+\beta c_2(\beta^2-1)(4+\beta^2)(32-5\beta^2)(8a-8-8a\beta-a\beta^3-(1-a)\beta^4+(\beta^3-\beta^5)c_2)))$;

$g_2(c_1,c_2) = \frac{1}{2(32-4\beta^2-\beta^4)^2}(192+352\beta^2+96\beta^4+9\beta^6-\beta^8-2a(1-a)(2-\beta)^2(1-\beta)^2(4+2\beta+\beta^2)(12+2\beta+3\beta^2+\beta^3)+(8+\beta^2)^2((4-7\beta^2+3\beta^4)c_1^2+2(\beta^2-1)c_1(4-a(2-\beta)^2+\beta^2+\beta^3c_2)+c_2(\beta^2-1)(2a(2-\beta)^2+8\beta+(3\beta^2-4)c_2)))$;

$g_3(c_2) = \frac{1}{8(8+\beta^2)^2}((5\beta(2+\beta^2)+a(1-\beta)(12+2\beta+5\beta^2))(2\beta+\beta^3+a(4-2\beta-\beta^3-\beta^2))+2(\beta^2-1)(\beta(2+\beta^2)(16+5\beta^2)+a(64-32\beta+32\beta^2-26\beta^3+3\beta^4-5\beta^5))c_2+(1-\beta^2)(64-23\beta^4-5\beta^6)c_2^2)$

We have $\lim_{c_1\to 0, c_2\to 0} g_1(c_1,c_2) > 0, \quad \lim_{c_1\to 0, c_2\to 0} g_2(c_1,c_2) > 0, \quad g_3(c_2=0) > 0$

From $\lim_{c_1\to 0, c_2\to 0} g_1(c_1,c_2) > 0, \lim_{c_1\to 0, c_2\to 0} g_2(c_1,c_2) > 0$, we can conclude that When $c_1 and c_2$ are smaller, the competition scenario achieves more consumer surplus than unilateral licensing and cross-licensing scenario.

From $(1-\beta^2)(64-23\beta^4-5\beta^6) > 0$ and $g_3(c_2=0) > 0$, we can conclude that with the increase of c_2, firstly, the unilateral licensing scenario achieves more consumer surplus than the cross-licensing scenario, and then the cross-licensing scenario achieves more consumer surplus than the unilateral licensing scenario.

Due to the complexity of the two functions $(g_1(c_1,c_2), g_2(c_1,c_2))$, it is difficult to determine the positive and negative properties of the functions. So, we use numerical analysis to explain the comparison results of consumer surplus in three scenarios. The results in Figure 7 further support the existing proof results.

Appendix B.

The demand functions of products are given by $d_1 = a + \varepsilon_1 - p_1 + \beta p_2$and $d_2 = 1 - a + \varepsilon_2 - p_2 + \beta p_1$. We assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $\varepsilon \sim U(0,b)$. The expectations of the demands are $E(d_1) = a - p_1 + \beta p_2 + b/2$and $E(d_2) = 1 - a - p_2 + \beta p_1 + b/2$.

We can get the profit function and decision problem of competition scenario, unilateral licensing and cross-licensing scenario (See Tab. B.1).

According to the characteristics of the Nash and Stackelberg game (same as in the demand determination scenario), the optimal patent license fee, product price and sales volumes in the competition, unilateral and cross-licensing scenarios are obtained, as shown in Table B.2.

Based on the optimal equilibrium results in various scenarios, we adopt numerical analysis to compare the unilateral licensing scenario with the competition scenario (see Fig. B.1), the cross-licensing scenario with the unilateral licensing scenario (see Fig. B.2), and the cross-licensing scenario with the competition scenario (see Fig. B.3). Finally, we analyze the optimal licensing scenario (see Fig. B.4). Additionally, while examining the comparative analysis of different scenarios, we categorize uncertainty into two situations to observe its impact on the comparative analysis results across scenarios ($b = 0.05$ and $b = 0.25$).

Based on the comparison between Figures 3 and B.1, Figures 4 and B.2, Figures 5 and B.3, and Figures 6 and B.4, it can be observed that the uncertainty of demand does not significantly affect the main

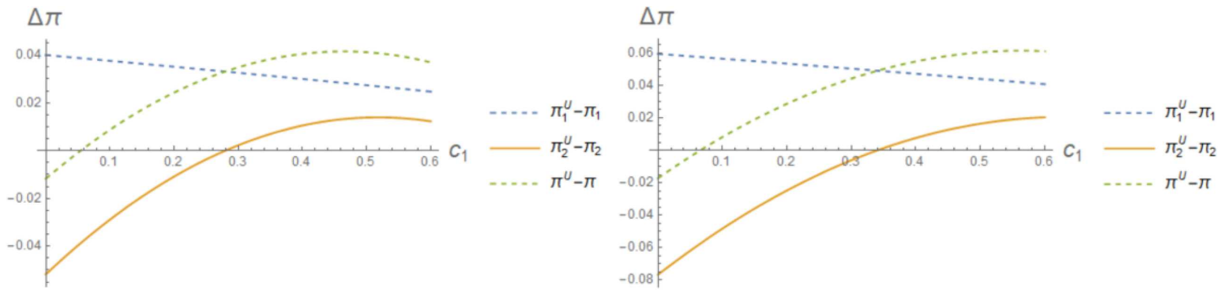
TABLE B.1. The profit function and decision problem under the three licensing scenarios.

Scenarios	Profit function and decision problem
Competition	$\max \pi_1(p_1) = (p_1 - c_2)(a - p_1 + \beta p_2 + \frac{b}{2})$ $\max \pi_2(p_2) = (p_2 - c_1)(1 - a - p_2 + \beta p_1 + \frac{b}{2})\cdot$
Unilateral licensing	$\max \pi_1^u(p_1, \varphi_1) = (p_1 - c_2)(a - p_1 + \beta p_2 + \frac{b}{2}) + \varphi_1(1 - a - p_2 + \beta p_1 + \frac{b}{2})$ $\max \pi_2^u(p_2) = (p_2 - \varphi_1)(1 - a - p_2 + \beta p_1 + \frac{b}{2})\cdot$
Cross-licensing	$\max \pi_1^c(p_1, \varphi_1) = (p_1 - \varphi_2)(a - p_1 + \beta p_2 + \frac{b}{2}) + \varphi_1(1 - a - p_2 + \beta p_1 + \frac{b}{2})$ $\max \pi_2^c(p_2, \varphi_2) = (p_2 - \varphi_1)(1 - a - p_2 + \beta p_1 + \frac{b}{2}) + \varphi_2(a - p_1 + \beta p_2 + \frac{b}{2})\cdot$

TABLE B.2. Equilibrium results for three licensing scenarios under demand uncertainty.

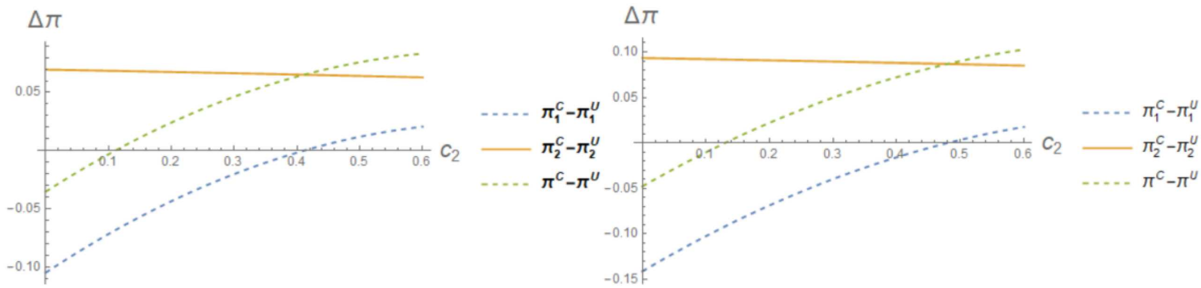
Scenarios	Equilibrium results
Competition	$p_1 = \frac{4a+2b+2\beta-2a\beta+b\beta+2\beta c_1+4c_2}{2(4-\beta^2)}, p_2 = \frac{4-4a+2b+2a\beta+b\beta+4c_1+2\beta c_2}{2(4-\beta^2)}$, $d_1 = \frac{4a+2b+2\beta-2a\beta+b\beta+2\beta c_1-2c_2(2-\beta^2)}{2(4-\beta^2)}, d_2 = \frac{4-4a+2b+2a\beta+b\beta+2\beta c_2-2c_1(2-\beta^2)}{2(4-\beta^2)}$, $\pi_1 = \frac{(4a+2b+2\beta-2a\beta+b\beta+2\beta c_1-2c_2(2-\beta^2))^2}{4(4-\beta^2)^2}, \pi_2 = \frac{(4-4a+2b+2a\beta+b\beta+2\beta c_2-2c_1(2-\beta^2))^2}{4(4-\beta^2)^2}$.
Unilateral licensing	$p_1^u = \frac{8b+20\beta+10b\beta+b\beta^2-2\beta^3-b\beta^3+2a(8-10\beta+\beta^2+\beta^3)+2c_2(8-5\beta^2-3\beta^4)}{4(8-7\beta^2-\beta^4)}$ $p_2^u = \frac{24+12b+b\beta(8-2\beta+\beta^2-\beta^3)-4\beta^2-2\beta^4-2a(12-8\beta-2\beta^2-\beta^3-\beta^4)+2\beta c_2(4-3\beta^2-\beta^4)}{4(8-7\beta^2-\beta^4)}$ $d_1^u = \frac{8b+4\beta+2b\beta+b\beta^2+2\beta^3+b\beta^3+2a(8-2\beta+\beta^2-\beta^3)-2(8-\beta^2-\beta^4)c_2}{4(8+\beta^2)}$ $d_2^u = \frac{(2+\beta^2)(2-2a+b+2\beta c_2)}{2(8+\beta^2)}$ $\varphi_1^u = \frac{(8+\beta^4)(2-2a+b)+(2a+b)(8\beta+\beta^3)-2c_2\beta^3(1-\beta^2)}{4(8-7\beta^2-\beta^4)}$ $\pi_1^u = \frac{1}{16(8-7\beta^2-\beta^4)} 16(1+b+\beta^2) + 12b^2 + b\beta(32+16b+16\beta$ $+5b\beta + 4\beta^2 + 2b\beta^2 + 4\beta^3 + b\beta^3) + 4\beta^4$ $+4a^2(12-16\beta+5\beta^2-2\beta^3+\beta^4) + 4a(1-\beta)(2(4\beta-4+\beta^3)$ $+b(4+4\beta+\beta^2+\beta^3)) - 4c_2(1-\beta^2)(2\beta(4+\beta^2)$ $+2a(8-4\beta+\beta^2-\beta^3) + b(8+4\beta+\beta^2+\beta^3))$ $+4c_2^2(8-11\beta^2+2\beta^4+\beta^6), \pi_2^u = \frac{(2+\beta^2)^2(2-2a+b+2\beta c_2)^2}{4(8+\beta^2)^2}$.
Cross-licensing	$p_1^c = \frac{6a(1-\beta)+3(b+2\beta+b\beta)}{8-7\beta^2-\beta^4}, p_2^c = \frac{3(2+b-2a(1-\beta)+b\beta)}{8-7\beta^2-\beta^4}, d_1^c = \frac{(2a+b)(2+\beta^2)}{2(8+\beta^2)}$, $d_2^c = \frac{(2-2a+b)(2+\beta^2)}{2(8+\beta^2)}, \varphi_1^c = \frac{8-2\beta^2+b(1+\beta)(4+2\beta-\beta^2)+2a(\beta(2+\beta+\beta^2)-4)}{2(8-7\beta^2-\beta^4)}$ $\varphi_2^c = \frac{b(1+\beta)(4-2\beta+\beta^2)+2\beta(2+\beta^2)+2a(4-\beta(2+\beta+\beta^2))}{2(8-7\beta^2-\beta^4)}$, $\pi_1^c = \frac{(2+\beta^2)(8+b(8+3b)+12a^2(1-\beta)+3b\beta(2+b)-2\beta^2(1+b)-4a(1-\beta)(4+b+\beta+b\beta)}{2(1-\beta^2)(8+\beta^2)^2}$ $\pi_2^c = \frac{(2+\beta^2)((1-\beta)(12a^2-4a(2-b-\beta-b\beta))+(1+\beta)(3b^2+2b(2+\beta))+2(2+\beta^2))}{2(1-\beta^2)(8+\beta^2)^2}$.

conclusions of the article. Furthermore, both relatively small and relatively large demand uncertainties do not substantially alter the main conclusions of the article.



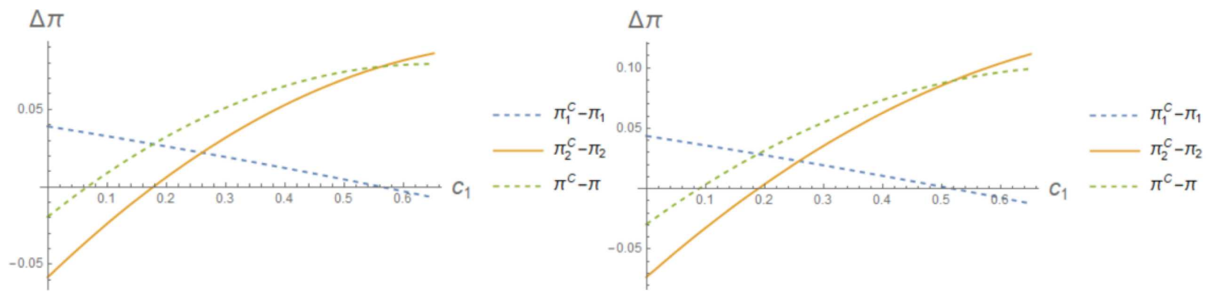
(**A**) $b = 0.05$ (**B**) $b = 0.25$

FIGURE B.1. Unilateral licensing and competition scenarios.



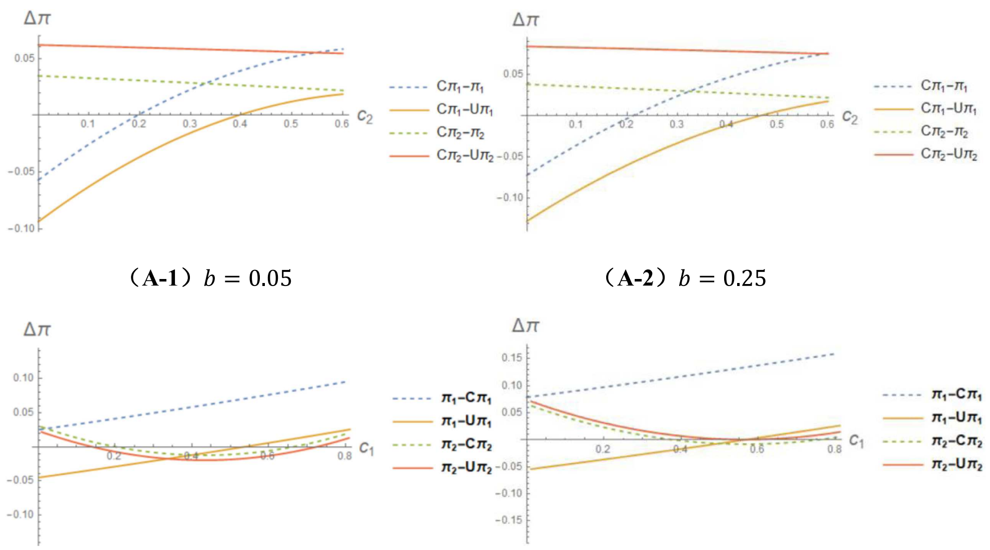
(**A**) $b = 0.05$ (**B**) $b = 0.25$

FIGURE B.2. Cross and unilateral licensing scenarios.



(**A**) $b = 0.05$ (**B**) $b = 0.25$

FIGURE B.3. Cross-licensing and competition scenarios.



(**A-1**) $b = 0.05$ (**A-2**) $b = 0.25$

(**B-1**) $b = 0.05$ (**B-2**) $b = 0.25$

FIGURE B.4. Competition, unilateral licensing and cross-licensing scenarios.

APPENDIX C.

We refer to the demand function forms used in existing literatures, which are mainly divided into the following forms: (1) Exponential form: $d_1 = e^a - e(p_1 - \beta p_2 - \varepsilon_1) and d_2 = e(1-a) - e(p_2 - \beta p_1 - \varepsilon_2)$; (2) Cobb-Douglas form: $d_1 = \frac{(a+\varepsilon_1)p_2}{p_1^2}$ and $d_2 = \frac{(1-a+\varepsilon_2)p_1}{p_2^2}$; (3) Power form: $d_1 = a + \varepsilon_1 - p_1^2 + \beta p_2^2$ and $d_2 = 1 - a + \varepsilon_2 - p_2^2 + \beta p_1^2$ (see, [3, 4, 12, 18, 27]).

(1) The demand functions of products are given by $d_1 = e^a - e^{(p_1 - \beta p_2 - \varepsilon_1)}$ and $d_2 = e^{(1-a)} - e^{(p_2 - \beta p_1 - \varepsilon_2)}$The demand functions satisfy some conditions, such as $\frac{\partial d_1}{\partial p_1} < 0, \frac{\partial d_1}{\partial p_2} > 0, a = p_1 - \beta p_2 - \varepsilon_1 (d_1 = 0)$.We assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $\varepsilon \sim U(0, b)$. The expectations of the demands are $E(d_1) = e^a - \frac{e^b - 1}{be^b} \cdot e^{(p_1 - \beta p_2)}$ and $E(d_2) = e^{1-a} - \frac{e^b - 1}{be^b} \cdot e^{p_2 - \beta p_1}$.
We can get the profit function and decision problem of competition scenario, unilateral licensing and cross-licensing scenario (See Tab. C.1).
(2) The demand functions of products are given by $d_1 = \frac{(a+\varepsilon_1)p_2}{p_1^2}$ and $d_2 = \frac{(1-a+\varepsilon_2)p_1}{p_2^2}$. The demand functions satisfy some conditions, such as $\frac{\partial d_1}{\partial p_1} < 0, \frac{\partial d_1}{\partial p_2} > 0$. We assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $\varepsilon \sim U(0, b)$. The expectations of the demands are $E(d_1) = \frac{(a+\frac{b}{2})p_2}{p_1^2}$ and $E(d_2) = \frac{(1-a+\frac{b}{2})p_1}{p_2^2}$.
We can get the profit function and decision problem of competition scenario, unilateral licensing and cross-licensing scenario (See Tab. C.2)
(3) The demand functions of products are given by $d_1 = a + \varepsilon_1 - p_1^2 + \beta p_2^2$ and $d_2 = 1 - a + \varepsilon_2 - p_2^2 + \beta p_1^2$. The demand functions satisfy some conditions, such as $\frac{\partial d_1}{\partial p_1} < 0, \frac{\partial d_1}{\partial p_2} > 0$. We assume $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $\varepsilon \sim U(0, b)$. The expectations of the demands are $E(d_1) = a - p_1^2 + \beta p_2^2 + \frac{b}{2}$ and $E(d_2) = 1 - a - p_2^2 + \beta p_1^2 + \frac{b}{2}$.

We can get the profit function and decision problem of competition scenario, unilateral licensing and cross-licensing scenario (See Tab. C.3)

TABLE C.1. The profit function and decision problem under the three licensing scenarios.

Scenarios	Profit function and decision problem
Competition	$\max \pi_1(p_1) = (p_1 - c_2)\left(e^a - \frac{e^b-1}{be^b} \cdot e^{(p_1-\beta p_2)}\right)$ $\max \pi_2(p_2) = (p_2 - c_1)(e^{1-a} - \frac{e^b-1}{be^b})e^{(p_2-\beta p_1)}$
Unilateral licensing	$\max \pi_1^u(p_1,\varphi_1) = (p_1 - c_2)(e^a - \frac{e^b-1}{be^b})e^{(p_1-\beta p_2)}$ $+\varphi_1\left(e^{1-a} - \frac{e^b-1}{be^b}\right)e^{(p_2-\beta p_1)}$ $\max \pi_2^u(p_2) = (p_2 - \varphi_1)(e^{1-a} - \frac{e^b-1}{be^b})e^{(p_2-\beta p_1)}$
Cross-licensing	$\max \pi_1^c(p_1,\varphi_1) = (p_1 - \varphi_2)(e^a - \frac{e^b-1}{be^b})e^{(p_1-\beta p_2)}$ $+\varphi_1\left(e^{1-a} - \frac{e^b-1}{be^b}\right)e^{(p_2-\beta p_1)}$ $\max \pi_1^c(p_2,\varphi_2) = (p_2 - \varphi_1)(e^{1-a} - \frac{e^b-1}{be^b})e^{(p_2-\beta p_1)}$ $+\varphi_1\left(e^{1-a} - \frac{e^b-1}{be^b}\right)e^{(p_1-\beta p_2)}$

TABLE C.2. The profit function and decision problem under the three licensing scenarios.

Scenarios	Profit function and decision problem
Competition	$\max \pi_1(p_1) = (p_1 - c_2)(\frac{(a+\frac{b}{2})p_2}{p_1^2})$ $\max \pi_2(p_2) = (p_2 - c_1)(\frac{(1-a+\frac{b}{2})p_1}{p_2^2})$
Unilateral licensing	$\max \pi_1^u(p_1,\varphi_1) = (p_1 - c_2)(\frac{(a+\frac{b}{2})p_2}{p_1^2}) + \varphi_1(\frac{(1-a+\frac{b}{2})p_1}{p_2^2})$ $\max \pi_2^u(p_2) = (p_2 - \varphi_1)(\frac{(1-a+\frac{b}{2})p_1}{p_2^2})$
Cross-licensing	$\max \pi_1^c(p_1,\varphi_1) = (p_1 - \varphi_2)(\frac{(a+\frac{b}{2})p_2}{p_1^2}) + \varphi_1(\frac{(1-a+\frac{b}{2})p_1}{p_2^2})$ $\max \pi_2^c(p_2,\varphi_2) = (p_2 - \varphi_1)(\frac{(1-a+\frac{b}{2})p_1}{p_2^2}) + \varphi_2(\frac{(a+\frac{b}{2})p_2}{p_2^1})$

TABLE C.3. The profit function and decision problem under the three licensing scenarios.

Scenarios	Profit function and decision problem
Competition	$\max \pi_1(p_1) = (p_1 - c_2)(a - p_1^2 + \beta p_2^2 + \frac{b}{2})$ $\max \pi_2(p_2) = (p_2 - c_1)(1 - a - p_2^2 + \beta p_1^2 + \frac{b}{2})$
Unilateral licensing	$\max \pi_1^u(p_1,\varphi_1) = (p_1 - c_2)(a - p_1^2 + \beta p_2^2 + \frac{b}{2})$ $+\varphi_1(1 - a - p_2^2 + \beta p_1^2 + \frac{b}{2})$ $\max \pi_2^u(p_2) = (p_2 - \varphi_1)(1 - a - p_2^2 + \beta p_1^2 + \frac{b}{2})$
Cross-licensing	$\max \pi_1^c(p_1,\varphi_1) = (p_1 - \varphi_2)(a - p_1^2 + \beta p_2^2 + \frac{b}{2})$ $+\varphi_1(1 - a - p_2^2 + \beta p_1^2 + \frac{b}{2})$ $\max \pi_2^c(p_2,\varphi_2) = (p_2 - \varphi_1)(1 - a - p_2^2 + \beta p_1^2 + \frac{b}{2})$ $+\varphi_2(a - p_1^2 + \beta p_2^2 + \frac{b}{2})$

The decision-making process in this section is the same as that in Appendix A. Due to the nature of the demand function and the high index of the decision variable, we are unable to obtain the optimal analytical solution.

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