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DETERIORATION AND AGE GOVERNED GREENHOUSE GAS EMISSIONS FROM THE PRODUCT ITSELF: AN OPTIMUM INVENTORY CONTROL PROBLEM

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Abstract. Greenhouse gas emissions contribute significantly to global warming. This occurs during various operations, including the production, storage, and transportation of an inventory, but in some cases, the inventory itself becomes the emission source. Emissions from cattle dunk cakes, livestock slurry, manure, crude oil, and gasoline can be considered examples of this type of emissions. This paper proposes a mathematical formulation for a deteriorating inventory model utilizing the inventory as a source of emissions. The model is developed under inflation. Other activities, such as energy consumption for warehousing, are considered to be contributing to greenhouse gas emissions here. The rate of emissions from the product is exponentially governed by the rate of deterioration and the age of the material. The trapezoidal-type demand rate is considered using the Heaviside step function. Shortages are permitted but partially backlogged, and the backlogging rate is supposed to be decreasing exponentially with the increased waiting time. The numerical illustration of the model is provided to illustrate the mathematical expressions, and the effect of parametric variation is reported to give managerial insights. The results reveal that the greenhouse gas emissions are proportional to the variable rate of emissions cost and the deterioration rate. However, the increment in total emissions with respect to variable emissions cost increases with the deterioration rate. If the deterioration rate is negligible, total carbon emissions do not fluctuate much, but it increases rapidly if it is relatively high. The preservation technology is used to reduce deterioration and helps reduce emissions.

Mathematics Subject Classification. 90B05, 90B06.

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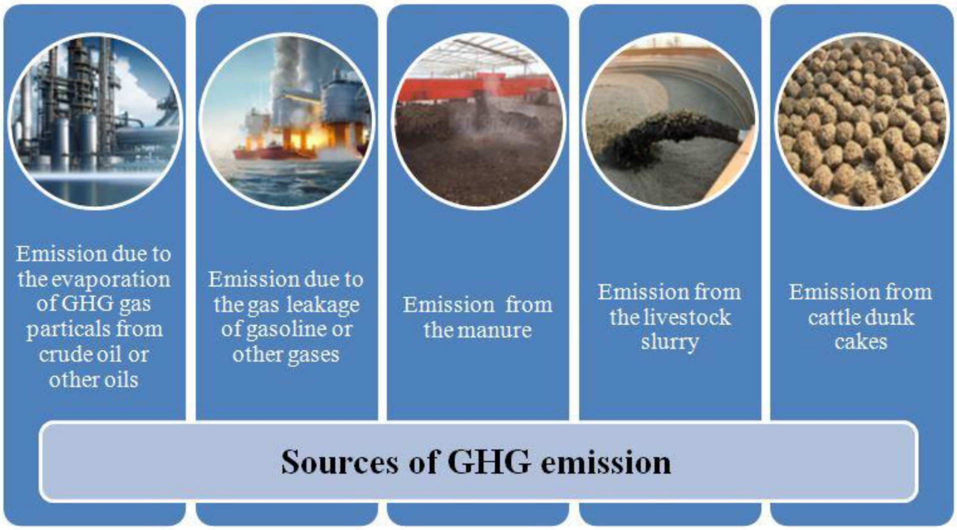

FIGURE 1. Sources of greenhouse gas (GHG) emissions.

1. Introduction

Gasoline, crude oil, cattle dung cake, and slurry are the major sources of energy in many countries, but handling these items turned out to be the most significant source of greenhouse gas (GHG) emissions. Gasoline is a venomous and highly combustible fluid that contains very harmful substances such as nitrogen oxides (NO), carbon monoxide (CO), and sulfur oxides (SO). The evaporation of gasoline generally happens during storage and transportation, and the vapors are given off when gasoline evaporates. Thus, the storage of this kind of product produces emissions of toxic air pollutants [15]. On the other hand, it had been was investigated by Owen and Silver [38] that 20% of non-carbon dioxide (CO_2) GHG emissions, such as methane (CH_4), nitrous oxide (N_2O), and fluorinated compounds, had their roots in animal farming. Livestock manure management is an important reason for GHG and ammonia (NH_3) emissions from agriculture [73, 75], while the storage of slurry is a significant source of emissions for greenhouse gases like CO_2, CH_4, hydrogen sulfide (H_2S), NH_3, and N_2O in livestock farming [28] (Fig. 1). According to Guo *et al.* [22], ponds filled with livestock slurry are were the leading cause of methane emission CH_4 emissions, and around 10% of global methane CH_4 emissions originate from agriculture [71], which can exceed up to 50% depending on the climate changes [38]. Moreover, Davidson [13] stated that livestock manure storage accounts for around 44% of overall N_2O emissions. These statistics emphasize the requirement for an accurate assessment of GHG emissions from manure management [5].

1.1. Literature review

The commercials store the items discussed above for further use, which needs to be managed economically. Therefore, evaluating costs and predicting the optimum replenishment policies are essential for running any business. All the above-discussed articles are agricultural papers and have not discussed inventory control problems. In a supply chain system, transportation is the primary cause of carbon emissions, which must be studied [69]. It was proposed to develop a multilayer deterministic model by Hammami *et al.* [24] that incorporated carbon emissions costs associated with production, storage, and transportation. Many supply chain models were presented with carbon emissions costs [6], such as a vendor-managed inventory (VMI) model, which was investigated by Bai *et al.* [4]. They considered preservation like cold storage and all to deal with decaying products capable of huge carbon emissions. Therefore, a cap-and-trade policy was suggested to control this large amount of emissions. A study by Das *et al.* [10] examined carbon tax and cap policies. An inventory control model with variables associated with carbon emissions was presented by Das *et al.* [12]. Another study by Das *et al.* [11] aimed at investigating the impact of carbon tax policies on sustainable development. A carbon emissions and waste reduction model incorporating green technology and autonomous inspection was investigated by Dey *et al.* [17]. Mridha *et al.* [36] presented a model with quality improvement and emissions reduction from the system. Mishra *et al.* [34] developed an emissions-controlled inventory system for deteriorating products without considering preservation. Sarkar *et al.* [51] discussed a three-echelon supply chain with an emissions-controlled transportation system. They used variable and constant types of emissions costs for emissions reduction. There were several studies that developed inventory control policies that incorporate carbon emission costs. Essentially, all the articles mentioned above consider the emissions due to the facilities provided for the smooth functioning of the system. However, products emit GHG during storage, which was never taken into account based on the literature review.

Zhu *et al.* [74] developed a customer-centric model to eliminate the chances of emissions from waste. They used the carbon cap-trade for controlling emissions from deteriorating products. Regarding remanufacturing and waste management, circular economy [63] is one of the latest tools for sustainable development, which targets emissions reduction. The location of remanufacturing centers and waste collection centers play important roles in carbon emissions reduction [32]. Thus, the carbon market [59] is gradually increasing, and information [72] about green technologies is one of the keys to sustainability. Recently, Mridha *et al.* [37] explored the joint effects of improving the production quality of green products and reducing carbon emissions in a smart sustainable supply chain management context. Eslamipoor [18] presented an optimization model for a green supply chain that considered emissions tax rates for incongruous vehicles. The study aimed to design a more eco-friendly supply chain network by incorporating emission tax rates as a decision variable. By considering emissions taxes, the research aimed to encourage companies to adopt greener transportation options and reduce their carbon footprint in the transportation sector. Alsaedi *et al.* [2] developed a sustainable green supply chain model that accounted for carbon emissions in the context of defective items under a fuzzy environment. The study integrated sustainability considerations and fuzzy logic to address uncertainties related to carbon emissions and defective products. Abbasi and Choukolaei [1] conducted a systematic review of green supply chain network design with a specific focus on carbon policies. The research aimed to provide an overview of existing studies in the field and highlight the role of carbon policies in shaping green supply chain practices.

Overall, these papers contribute to the growing body of literature on green supply chain management and highlight various aspects of sustainability, waste reduction, renewable energy resources [35], carbon emissions reduction, and the integration of green practices in supply chain decision-making. These researches aim to drive more environmentally responsible and efficient supply chain practices for a greener and more sustainable future. On the other hand, in many studies, researchers applied the time-varying demand rate to inventory control problems [16]. The demand rate is assumed to be either increasing or decreasing with time. For example, the demand rate for seasonal products increases with time in the initial phase of its shelf-life. Still, it remains steady after reaching a saturation point without any significant rise or fall. For example, the demand for cattle dung cakes is higher during the winter (used as a source of energy), agricultural farming (used as organic fertilizer), and during the festive seasons and decreases afterward.

Panda *et al.* [40] investigated an inventory model for fashionable products with a time-dependent demand rate. They provided an optimal replenishment policy for decaying products, and the deterioration rate was constant. After that, an economic order quantity (EOQ) model for ramp-type demand was investigated by Skouri *et al.* [56]. They developed the best-suited replenishment policies for allowable partial backlogging. Deterioration was taken into consideration. Furthermore, Kawakatsu [25] investigated the seasonal effect of an inventory model having an infinite replenishment rate. The demand was ramp-type and considered a time and stock level function. In a subsequent study, Cheng *et al.* [7] established a deteriorating inventory model along the same line of research. In that model, a trapezoidal type demand pattern was considered for seasonal products. Shortages were allowed and partially backlogged. In some studies, Wang and Huang [67] and Wu *et al.* [68]

proposed their inventory model assuming the seasonal effect of the deteriorating items. Wang and Huang [67] established their model assuming the price and time-varying demand rate, while Wu *et al.* [68] investigated their model for the trapezoidal-type demand rate with shortages and partial backlog. A supply chain model for innovative products was developed by Sarkar *et al.* [52], while Saha *et al.* [45] investigated a waste reduction model with dynamic investment. An inventory model for vendor-buyer integration with random misplacement was studied by Sarkar and Guchhait [48].

A study for optimal delivery policy along with returnable products was discussed by Sarkar *et al.* [49]. Similarly, another model with advertising policy and uncertain demand was developed by Sarkar *et al.* [50]. On the other hand, Saxena *et al.* [53] provided a reverse logistics model with decaying items considering eco-design. Saranya and Chandrasekaran [47] discussed an inventory management model for products that deteriorate over time, where the demand rate varied with time and influenced by advertising costs. Rukonuzzaman *et al.* [43] investigated the impact of a quantity-based discount program on inventory planning for businesses selling non-immediate perishable items, with a focus on the storage time of the items and changing demand. Khan *et al.* [26] investigated a comprehensive approach to inventory management, pricing decisions, and advance payment strategies in the presence of all-unit discounts. Yang [70] discussed an inventory management model that considers various factors, including fluctuating demand, limited storage capacity, deteriorating items, downstream partial trade credit transactions, and discounted cash-flow considerations. Lin [31] explored a supplier-dominated inventory system where the retailer has the flexibility to order more quantities under the combined influence of inventory-level-dependent demand (ILDD) and trade credit. That study addressed several important issues related to trade-credit, demand stimulation, and capacity utilization.

Inflation is a persistent economic phenomenon affecting various economic sectors, including the industrial sector. Inflation in the industrial sector can have profound consequences, impacting businesses, consumers, and the overall economic landscape. Many researchers explored potential replenishment strategies to mitigate the challenges posed by inflation in the industrial sector. Saha and Sen [44] addressed an inventory management model that dealt with deteriorating items, where both time and price fluctuations influenced demand, while shortages were considered under inflation. In the study by Sundararajan *et al.* [61], the primary focus was on examining how delays in payment, shortages, and inflation influenced an EOQ model with bivariate demand. Udayakumar *et al.* [64] addressed an inventory management model for non-instantaneous deteriorating items. This model considered various factors, including money inflation, time discounting, a permissible delay period offered by the supplier as an alternative to price discounts, and the salvage value associated with deteriorated units. Additionally, it allowed for partially backlogged shortages. Mallick *et al.* [33] focused on developing an EOQ model for deteriorating items. That model considered the joint effects of stock-dependent demand and inflation.

Based on the literature review provided above, Table 1 presents an author contribution summary to outline the contributions of each author to this line of research.

1.2. Research gap

The literature mentioned above reveals that most of the inventory control papers developed with GHG emissions either considered the emission due to the facilities available during production and storage or the transportation. Many agricultural research papers have explored the emission during storage and stated that farming products are the largest source of GHG emissions. Still, they did not consider the inventory control policies to analyze the corporation's cost. Furthermore, several papers have examined the seasonal effect in inventory control models with deterioration, backlogging, and various demand rates. However, few ramp-type models developed with different assumptions, such as Pal *et al.* [39] established their model under fuzzy and inflation, and Skouri *et al.* [57] proposed an EOQ model with a credit policy for payment. Singh *et al.* [55] derived the optimal approaches for the green items with advertisement policy and reduced environmental impact from the system. No single paper provides an inventory control policy for deteriorating products concerning the emissions from the product itself. In the current scenario, many organizations have concentrated on lessening GHG emissions when legislative controls and customer concerns intensify regarding ecological issues.

TABLE 1. Author contribution table.

Authors	GHG emissions	Demand rate	Deterioration	Inflation	Shortage
Alsaedi *et al.* [2]	Production, storage, and landfill	Fuzzy	–	–	Full backlogging
Baral *et al.* [5]	Manure management	–	–	–	–
Cheng *et al.* [7]	–	Trapezoidal-type demand	Yes	–	Partial backlogging
Davidson [13]	Manure management	–	–	–	–
DeLuchi [15]	Gasoline storage and transportation, emphasizing toxic air pollutants and emissions	–	–	–	–
Eslamipoor [18]	Production and transportation	Constant	–	–	–
Guo *et al.* [22]	Livestock slurry as a source of methane emissions	–	–	–	–
Hammami *et al.* [24]	Production, storage, and transportation	Constant	–	–	–
Kawakatsu [25]	–	Ramp-type	–	–	–
Khan *et al.* [26]	–	Power demand pattern	–	–	–
Kupper *et al.* [28]	CO_2, CH_4, H_2S, NH_3, and N_2O emissions from the storage of slurry in livestock farming	–	–	–	–
Lin [31]	–	Inventory-level-dependent	–	–	–
Mallick *et al.* [33]	–	Stock-dependent	Yes	Yes	–
Mridha *et al.* [37]	Biofuel	–	–	–	–
Owen and Silver [38]	Animal farming to non-CO_2 GHG emissions, specifically CH_4, N_2O, and fluorinated compounds	–	–	–	–
Panda *et al.* [40]	–	Time-dependent	Yes	–	–
Rukonuzzaman *et al.* [43]	–	Time-dependent	Yes	–	Shortage
Saha and Sen [44]	–	Time and price-dependent	–	Yes	Yes
Saranya and Chandrasekaran [47]	–	Advertising cost-dependent	Yes	–	–
Yadav *et al.* [69]	Production and transportation	Crossprice elasticity-dependent	Yes	–	–
Saha *et al.* [45]	–	Ramp-type	Yes	–	–

TABLE 1. continued.

Authors	GHG emissions	Demand rate	Deterioration	Inflation	Shortage
Saxena *et al.* [53]	Transportation, storage, production, landfilling, building construction, power supply, and loading/unloading	–	Yes	–	–
Skouri *et al.* [56]	–	Ramp-type	Yes	–	Partial backlogging
Sundararajan *et al.* [61]	–	Bivariate	–	Yes	Yes
Udayakumar *et al.* [64]	–	Price and advertisement-dependent	Yes	Yes	Yes
Wang and Huang [67]	–	Price and ramp type time-varying	Yes	–	–
Wu *et al.* [68]	–	Trapezoidal-type	Yes	–	Partial backlogging
Yang [70]	–	Fluctuating demand	Yes	–	–
Yusuf *et al.* [71]	Livestock-related	–	–	–	–
Zhu *et al.* [73]	Livestock manure management as a source of GHG and NH_3 emissions in agriculture	–	–	–	–
This study	Agricultural product	Trapezoidal-type demand rate with Heaviside step function	Yes	Yes	Partial backlogging

1.3. Motivation and contribution

The necessity of developing inventory control policies for agricultural products concerning the GHG emissions and the ignorance of the researchers along this side of research motivates to investigate the GHG emissions from these products and to provide the optimum replenishment policies for their business. The study is provided here to build an optimal replenishment policy for a deteriorating inventory model. This article develops the model for the deteriorating product responsible for emitting greenhouse gases GHG. These products emit this GHG in evaporation, deterioration, or spoilage. This emissions depend on the product's age; the decay rate governs emissions. The proposed model investigates the seasonal effect on the EOQ inventory model for deteriorating products. However, the trapezoidal type demand rate is considered by using the Heaviside step function. Shortages are permitted and partly retrieved depending on time. The economic decision under realistic environment costing is analyzed with considering the inflation. The objective of this article is to propose economically wise decisions while taking care of environmental responsibilities. The graphical representation of the problem is depicted in Fig. 2.

1.4. Structure of study

The article is developed for inventory control problem considering the GHG emissions from the inventory itself. The article is divided into four sections. Section 1 elaborates introduction, literature review, research gaps, and contribution of the paper. The next section, Section 2, provides the nomenclature and assumptions for the model. The mathematical modeling for the paper is provided in Section 3, while Section 4 is given to

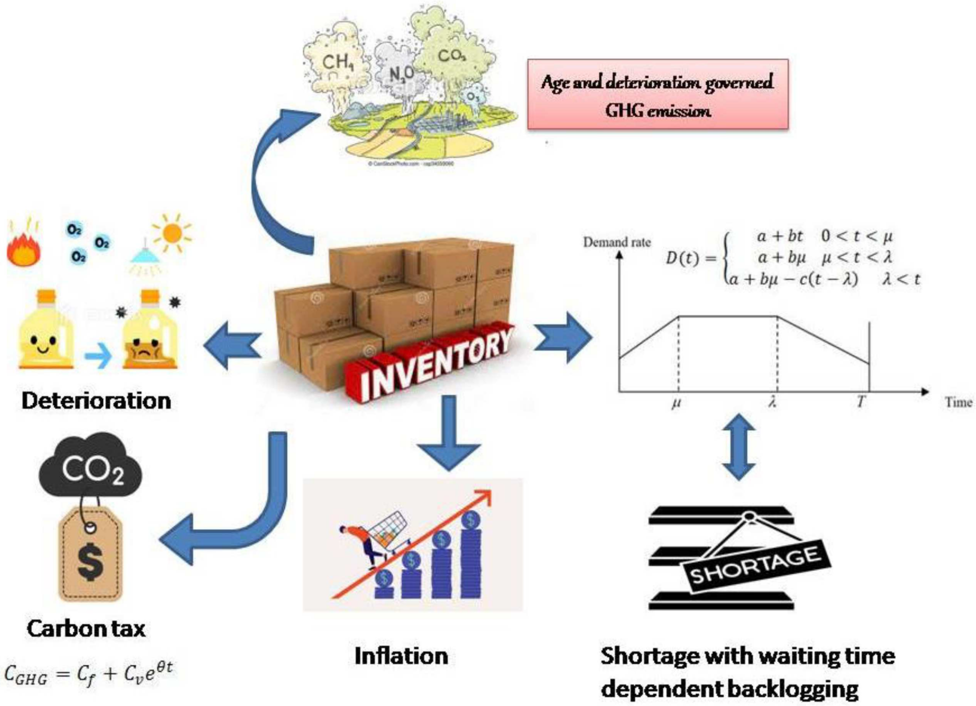

FIGURE 2. Graphical representation of the problem.

verify the mathematical results with the help of numerical examples. Section 5 provides a case study of the paper, whereas Section 6 provides discussions about the results. Section 7 is the conclusions of the paper.

2. Problem description

To derive the rate of the emissions, several papers like Tingey *et al.* [62], Sommer *et al.* [58], and Guo *et al.* [22] are observed. Sommer *et al.* [58] reported a technical paper on the GHG emissions from the storage of livestock slurry. That report stated that the log-transformed emissions are linearly related to the inverse temperature. They provided a function for the emission s(F) as

$$F = A \times \exp(E/RT_m)$$

where A is an Arrhenius parameter, E is the apparent activation energy, R is the gas constant, and T_m is the temperature.

According to Tingey *et al.* [62], the leakage can be measured as $\frac{\mathrm{d}C_{\mathrm{CO}_2}}{\mathrm{d}t} = LC_{\mathrm{CO}_2}$. They calculated the CO_2 leakage as

$$L * V * D * T_c * P * \frac{(C - C_a)}{A_s}$$

where L is the leakage decay constant, V is the volume of the chamber, D is the density of CO_2, T_c is temperature correction, P is the pressure correction, $(C - C_a)$ is the CO_2 concentration gradient, C_a is the ambient CO_2, C is the concentration of CO_2, and A_s is the surface area. After that, a positive relationship between temperature and CH_4 emission was investigated by Guo *et al.* [22]. The results above show that the log-transformed emissions are inversely proportional to the temperature and slurry or cattle dung cakes. The rate for GHG emissions is derived as it is clear that the emissions rate are proportional to the decay constant, or it can be said that it is the deterioration rate. Thus, anyone can derive the rate from Tingey *et al.*'s [62] model. That is

$$\frac{\mathrm{d}E_{\mathrm{GHG}}}{\mathrm{d}t} = \theta E_{\mathrm{GHG}}$$

where E_{GHG} is the emissions rate per unit time, θ is the deterioration rate, and t is the time. After solving the above equation, one can say that

$$\mathrm{Log}\,(E_{\mathrm{GHG}}) = \theta t + \mathrm{Log}\,(C_1)$$
$$E_{\mathrm{GHG}} = C_1 e^{\theta t}$$

where C_1 is the constant parameter. GHG emissions due to the facilities provided in the warehouse are C_2 per unit per unit time. That is, the complete emissions per unit time is

$$E_{\mathrm{GHG}} = C_1 e^{\theta t} + C_2.$$

The penalty cost per unit emissions is assumed to be C_{em}, that is, the actual emissions penalty per unit per unit time is computed as

$$C_{\mathrm{GHG}} = C_{em}\left(C_1 e^{\theta t} + C_2\right) \text{ or } C_{\mathrm{GHG}} = C_f + C_v e^{\theta t}$$

where C_f and C_v are fixed and variable costs of emission per unit per unit time, respectively.

2.1. Nomenclature

The following notation is used throughout the paper.

Decision variables

t_1	Optimum consumption period when inventory becomes zero (time unit)

Dependent variables

S	Initial inventory I(0) (unit per cycle)
Q	Ordered quantity (unit per cycle)

Parameters

$D(t)$	Seasonal demand rate ($ per time unit)
a	Base demand rate (unit time unit)
b, c	Shape parameters for demand rate
θ	Rate of deterioration
i	Rate of inflation
α	Discount rate
r	$(\alpha - i)$, net rate of discount of inflation
A	Ordering cost ($ per order)
c_1	Cost of purchasing ($ per unit item)
h	Holding cost ($ per unit per unit time)
C_f	Fixed cost of emissions ($ per unit per unit time)
C_v	Variable cost of emissions ($ per unit per unit time)
c_2	Cost of backlogging ($ per unit per unit time)
c_3	Opportunity cost due to lost sale ($ per unit item)
δ	Backlogging rate parameter
T	Cycle length (time unit)
$\mathrm{TC}_1(t_1)$	Total cost of Case 1 ($ per cycle)

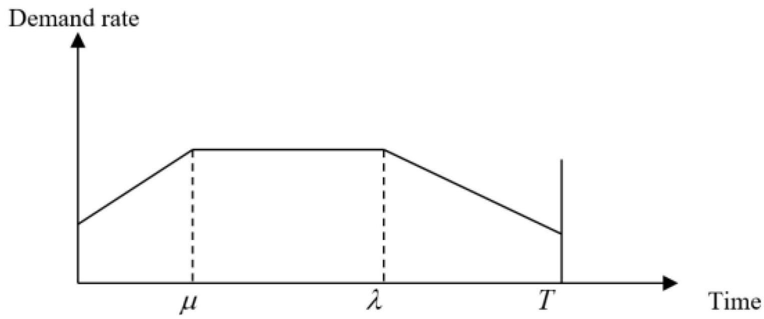

FIGURE 3. Trapezoidal type demand rate.

$\mathrm{TC}_2(t_1)$	Total cost of Case 2 ($ per cycle)
$\mathrm{TC}_3(t_1)$	Total cost of Case 3 ($ per cycle)
$H(t,\lambda), H(t,\mu)$	Heaviside step function of t
λ, μ	Switch value of Heaviside function
$B(t)$	Backlog (unit per cycle)
$I(t)$	Inventory level at time t
B_1, B_2, B_3	Backlog for Cases 1, 2, and 3, respectively (unit per cycle)

2.2. Assumptions

An advanced inventory model is developed here for seasonal products using the following assumptions.

(1) Seasonal items are perceived to be deteriorating in nature, and the rate of deterioration is constant, i.e., $\theta(0<\theta<1)$ [8].
(2) The amount of carbon emissions depends on the deterioration rate and the product's age. The cost of emissions varies exponentially; the age and deterioration of the unit govern its variation as $C(\theta,t) = C_f + C_v e^{\theta t}$.
(3) The seasonal demand rate $D(t)$ is a trapezoidal type function of time [68] (Fig. 3). The demand rate is $D(t) = a + bt - b(t-\mu)H(t,\mu) - c(t-\lambda)H(t,\lambda)$, where $0<\mu<\lambda, a, b$, and $c>0$, governed by the Heaviside step function

$$H(t,\mu) = \begin{cases} 1, & t \geq \mu \\ 0, & t < \mu \end{cases} \quad \text{and} \quad H(t,\lambda) = \begin{cases} 1, & t \geq \lambda \\ 0, & t < \lambda. \end{cases}$$

By using the above function, one can write the demand function as

$$D(t) = \begin{cases} a + bt & 0 < t < \mu \\ a + b\mu & \mu < t < \lambda \\ a + b\mu - c(t-\lambda) & \lambda < t. \end{cases}$$

(4) Stockout is allowable and partially backlogged whose rate is a decreasing function of the waiting period, that is $B(t) = e^{-\delta t}$, where $\delta \geq 0$, and t is the waiting time.
(5) The model is developed under inflationary conditions where i is the inflation rate, while α is the discount rate. Therefore, the net rate of inflation discount is given by $r = (\alpha - i)$.

3. Mathematical formulation of the problem

An inventory model is developed for the deteriorating products, and the demand rate is trapezoidal type. The replenishment of the items occurs at time $t = 0$. After that, the inventory level reduces due to demand and deterioration and achieved zero at the time t_1. Subsequently, shortages start to occur due to the non-availability of the inventory in the warehouse. The demand during this stockout period (t_1, T) is partially backlogged, which is fulfilled in the next replenishment. Accordingly, the position of inventory at any time t can be expressed by the following differential equations:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -D(t), \qquad I(t_1) = 0, \quad 0 < t < t_1 \tag{1}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -D(t)B(t), \qquad I(t_1) = 0, \quad t_1 < t < T. \tag{2}$$

Based on the trapezoidal type demand rate, three possible cares arise as follows.

3.1. Case 1. $0 < t_1 \leq \mu$

The differential equations for Case 1 depict the stock behavior (Fig. 4).

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -(a + bt), \qquad 0 < t < t_1 I(t_1) = 0 \tag{3}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -(a + bt)B(T - t), \qquad t_1 < t < \mu I(t_1) = 0 \tag{4}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -(a + b\mu)B(T - t), \qquad \mu < t < \lambda I\left(\mu^-\right) = I\left(\mu^+\right) \tag{5}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\{a + b\mu - c(t - \lambda)\}B(T - t), \qquad \lambda < t < T I\left(\lambda^-\right) = I\left(\lambda^+\right). \tag{6}$$

After solving the above equations (3)–(6), one can evaluate the position of the inventory at time t from the following equations (7)–(10).

$$I(t) = \left(\frac{a + bt_1}{\theta} - \frac{b}{\theta^2}\right)e^{\theta(t_1 - t)} - \frac{a + bt}{\theta} + \frac{b}{\theta^2}, \qquad 0 \leq t \leq t_1 \tag{7}$$

$$I(t) = \frac{\left[(a + bt_1)e^{-\delta(T - t_1)} - (a + bt)e^{-\delta(T - t)}\right]}{\delta} + \frac{b\left[e^{-\delta(T - t)} - e^{-\delta(T - t_1)}\right]}{\delta^2}, \qquad t_1 \leq t \leq \mu \tag{8}$$

$$I(t) = \frac{b\left[e^{-\delta(T - \mu)} - e^{-\delta(T - t_1)}\right]}{\delta^2} + \frac{\left[(a + bt_1)e^{-\delta(T - t_1)} - (a + b\mu)e^{-\delta(T - t)}\right]}{\delta}, \qquad \mu \leq t \leq \lambda \tag{9}$$

$$\begin{aligned} I(t) = {} & \frac{be^{-\delta(T - \mu)}}{\delta^2} + \frac{ce^{-\delta(T - \lambda)}}{\delta^2} + \frac{e^{-\delta(T - t_1)}[(a + bt_1)\delta - b]}{\delta^2} \\ & - \frac{e^{-\delta(T - t)}[\{(a + b\mu) - c(t - \lambda)\}\delta + c]}{\delta^2}, \qquad \lambda \leq t \leq T. \end{aligned} \tag{10}$$

The initial stock level can be determined from equation (7) as

$$S = I(0) = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - 1\right) + \frac{bt_1 e^{\theta t_1}}{\theta}\cdot \tag{11}$$

The model is developed under inflationary conditions to predict the system's real-time costing; hence, the present value of each cost is calculated. By evaluating the position of inventory and the initial stock level, one can analyze the costs as follows.

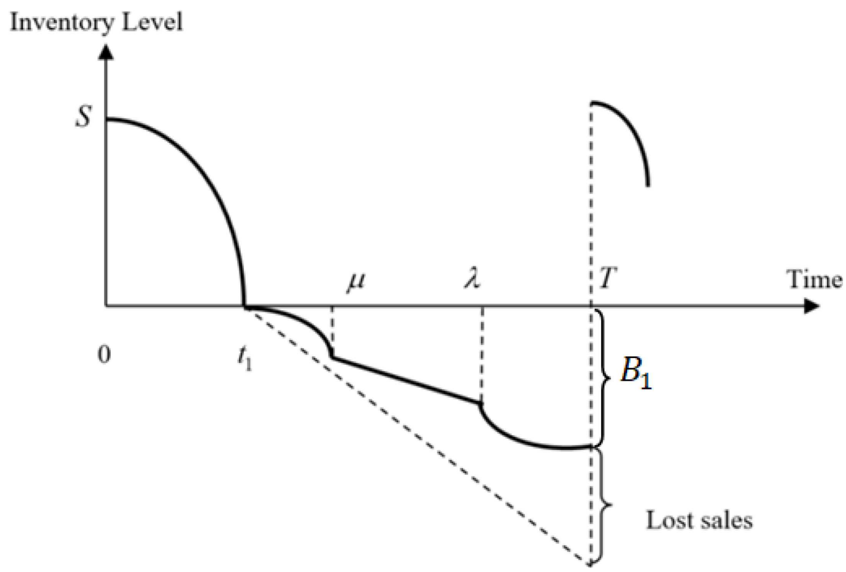

FIGURE 4. The behavior of inventory under the condition $0 < t_1 \leq \mu$ (Case 1).

The total number of products to be ordered for any cycle is the sum of the initial stock level and the backlogged demand of the previous process. Therefore, the present value of purchasing cost can be calculated as follows:

$$\begin{aligned}\mathrm{MC} &= c_1\left\{S + e^{-rT}\left[\int_{t_1}^{\mu}(a+bt)\,\mathrm{d}t + \int_{\mu}^{\lambda}(a+b\mu)\,\mathrm{d}t + \int_{\lambda}^{T}\{(a+b\mu) - c(t-\lambda)\}\,\mathrm{d}t\right]\right\} \\ &= c_1\left[\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - 1\right) + \frac{bt_1e^{\theta t_1}}{\theta} + e^{-rT}\left\{a(\mu - t_1) + \frac{b\left(\mu^2 - t_1^2\right)}{2} + (a+b\mu)(\lambda-\mu)\right.\right. \\ &\quad \left.\left. + \{(a+b\mu) + c\lambda\}(T-\lambda) - \frac{c\left(T^2-\lambda^2\right)}{2}\right\}\right]. \end{aligned} \tag{12}$$

Products are kept in the warehouse to sell in the future, and the cost involved in storage can be computed as follows:

$$\begin{aligned}\mathrm{HC} &= h\int_0^{t_1} I(t)e^{-rt}\,\mathrm{d}t \\ &= h\left\{\frac{\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - e^{-rt_1}\right)}{(r+\theta)} + \frac{(a+bt_1)e^{\theta t_1}}{r\theta} + \frac{(r-\theta)b(1-e^{-rt_1})}{r^2\theta^2} - \frac{a}{\theta}\right\}. \end{aligned} \tag{13}$$

As discussed earlier, carbon is emitted during production and storage, even from that product itself. The government imposes a tax on the carbon they emit to discourage the industrialist's emissions. The amount of carbon emissions depends on the rate of deterioration and the age of the product. The cost of emissions varies exponentially. The cost for emissions due to age and deterioration of the unit govern its variation as $C(\theta,t) = C_f + C_v e^{\theta t}$, where C_f and C_v are fixed and variable costs of emissions per unit per unit time, respectively. The cost of carbon emissions can be calculated as follows.

$$\begin{aligned}\mathrm{CEC} &= \int_0^{t_1} \left(C_f + C_v e^{\theta t}\right) I(t) e^{-rt}\,\mathrm{d}t \\ &= \frac{C_f\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - e^{-rt_1}\right)}{(r+\theta)} + \frac{C_f(a+bt_1)e^{\theta t_1}}{r\theta} + \frac{C_f(r-\theta)b(1-e^{-rt_1})}{r^2\theta^2} \\ &\quad + \frac{C_v\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - e^{(\theta-2r)t_1}\right)}{2r} - \frac{C_v(a+bt_1)e^{(\theta-r)t_1}}{\theta(\theta-r)} + \frac{C_v(2\theta-r)b\left(1-e^{(\theta-r)t_1}\right)}{\theta(\theta-r)^2} \\ &\quad - \frac{a}{\theta}\left(C_f + \frac{C_v}{(r-\theta)}\right). \end{aligned} \tag{14}$$

In practical situations, backlogging is unavoidable. Therefore, the stockout is considered here, and the present value of the backlogging cost is calculated as

$$\begin{aligned}\mathrm{SC} &= -\int_{t_1}^{\mu} c_2 I(t) e^{-rt}\,\mathrm{d}t - \int_{\mu}^{\lambda} c_2 I(t) e^{-rt}\,\mathrm{d}t - \int_{\lambda}^{T} c_2 I(t) e^{-rt}\,\mathrm{d}t \\ &= \frac{c_2 e^{-\delta T}}{\delta^2}\left[\frac{\delta^2 e^{-(r-\delta)t_1}[(a+bt_1)(r-\delta)+b]}{r(r-\delta)^2} + \frac{\left(e^{-\delta T} - e^{-r\lambda}\right)\left(ce^{r\lambda} - be^{r\mu}\right)}{r} + \frac{e^{-\delta(T-t_1)}((a+bt_1)\delta - b)}{r} \right. \\ &\quad \left. + \frac{c(2\delta - r)\left(e^{-(r-\delta)T} - e^{-(r-\delta)\lambda}\right)}{(r-\delta)^2} - \frac{\delta\{(a+b\mu) - c(T-\lambda)\}e^{-(r-\delta)T}}{(r-\delta)}\right]. \end{aligned} \tag{15}$$

The present value of the opportunity cost for lost sale during the cycle is

$$\begin{aligned}\mathrm{OPC} &= c_3\left[\int_{t_1}^{\mu} e^{-rt}(1-B(t))(a+bt)\,\mathrm{d}t + \int_{\mu}^{\lambda} e^{-rt}(1-B(t))(a+b\mu)\,\mathrm{d}t \right. \\ &\quad \left. + \int_{\lambda}^{T} e^{-rt}(1-B(t))\{(a+b\mu) - c(t-\lambda)\}\,\mathrm{d}t\right] \\ &= c_3\left[\frac{e^{-rt_1}\left(\frac{b}{r} - (a+bt_1)\right)}{r} - \frac{e^{-\delta T}e^{-(r-\delta)t_1}\left((a+bt_1) + \frac{b}{(r-\delta)}\right)}{(r-\delta)} - \frac{\left(be^{-r\mu} + ce^{-r\lambda}\right)}{r^2} \right. \\ &\quad \left. + \frac{e^{-\delta T}\left(be^{-(r-\delta)\mu} + ce^{-(r-\delta)\lambda}\right)}{(r-\delta)^2} + \frac{e^{-\delta T}\left(\{(a+b\mu) - c(T-\lambda)\} - \frac{(2r-\delta)c}{r(r-\delta)}\right)}{r(r-\delta)}\right]. \end{aligned} \tag{16}$$

By using the above cost components, one can compute the total cost per unit of time under the condition $(t_1 \leq \mu)$ is given by

$$\mathrm{TC}_1(t_1) = \frac{1}{T}[A + \mathrm{MC} + \mathrm{HC} + \mathrm{CEC} + \mathrm{SC} + \mathrm{OPC}] \tag{17}$$

$$\begin{aligned} &= \frac{1}{T}\left[A + c_1\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - 1\right) + \frac{bt_1 e^{\theta t_1}}{\theta} + e^{-rT}\left\{a(\mu - t_1) + \frac{b\left(\mu^2 - t_1^2\right)}{2} + (a+b\mu)(\lambda-\mu)\right.\right.\right. \\ &\quad \left.\left. + \{(a+b\mu) + c\lambda\}(T-\lambda) - \frac{c\left(T^2 - \lambda^2\right)}{2}\right\}\right\} + h\left\{\frac{\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - e^{-rt_1}\right)}{(r+\theta)} + \frac{(a+bt_1)e^{\theta t_1}}{r\theta}\right. \\ &\quad \left. + \frac{(r-\theta)b(1-e^{-rt_1})}{r^2\theta^2} - \frac{a}{\theta}\right\} + \frac{C_f\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - e^{-rt_1}\right)}{(r+\theta)} + \frac{C_f(a+bt_1)e^{\theta t_1}}{r\theta} \\ &\quad + \frac{C_f(r-\theta)b(1-e^{-rt_1})}{r^2\theta^2} + \frac{C_v\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)\left(e^{\theta t_1} - e^{(\theta-2r)t_1}\right)}{2r} - \frac{C_v(a+bt_1)e^{(\theta-r)t_1}}{\theta(\theta-r)} \end{aligned}$$

$$+\frac{C_v(2\theta-r)b\left(1-e^{(\theta-r)t_1}\right)}{\theta(\theta-r)^2}-\frac{a}{\theta}\left(C_f+\frac{C_v}{(r-\theta)}\right)+\frac{c_2e^{-\delta T}}{\delta^2}\left\{\frac{\delta^2e^{-(r-\delta)t_1}[(a+bt_1)(r-\delta)+b]}{r(r-\delta)^2}\right.$$
$$+\frac{\left(e^{-\delta T}-e^{-r\lambda}\right)\left(ce^{r\lambda}-be^{r\mu}\right)}{r}+\frac{e^{-\delta(T-t_1)}((a+bt_1)\delta-b)}{r}+\frac{c(2\delta-r)\left(e^{-(r-\delta)T}-e^{-(r-\delta)\lambda}\right)}{(r-\delta)^2}$$
$$\left.-\frac{\delta\{(a+b\mu)-c(T-\lambda)\}e^{-(r-\delta)T}}{(r-\delta)}\right\}+c_3\left\{\frac{e^{-rt_1}\left(\frac{b}{r}-(a+bt_1)\right)}{r}-\frac{e^{-\delta T}e^{-(r-\delta)t_1}\left((a+bt_1)+\frac{b}{(r-\delta)}\right)}{(r-\delta)}\right.$$
$$\left.\left.-\frac{\left(be^{-r\mu}+ce^{-r\lambda}\right)}{r^2}+\frac{e^{-\delta T}\left(be^{-(r-\delta)\mu}+ce^{-(r-\delta)\lambda}\right)}{(r-\delta)^2}+\frac{e^{-\delta T}\left(\{(a+b\mu)-c(T-\lambda)\}-\frac{(2r-\delta)c}{r(r-\delta)}\right)}{r(r-\delta)}\right\}\right].$$

Now, after differentiating the total average cost $\mathrm{TC}_1(t_1)$ with respect to t_1, one can have

$$\frac{\mathrm{d}\,\mathrm{TC}_1(t_1)}{\mathrm{d}t_1}=\frac{(C_f+h)(a+bt_1)\left(e^{\theta t_1}-e^{-rt_1}\right)}{(r+\theta)}-\frac{C_ve^{(\theta-2r)t_1}\left((a+bt_1)(\theta-2r)+\frac{2rb}{\theta}\right)}{2r\theta}$$
$$+\frac{C_v(a+bt_1)e^{\theta t_1}}{2r}-\frac{C_ve^{(\theta-r)t_1}\left((a+bt_1)+\frac{b(2\theta-r+1)}{(\theta-r)}\right)}{\theta}-(a+bt_1)c_1e^{-rT}$$
$$+\frac{c_2e^{-\delta T}(a+bt_1)\left(e^{-\delta(T-t_1)}-\delta e^{-(r-\delta)t_1}\right)}{\delta r}+c_3\left[(a+bt_1)\left(e^{-rt_1}+e^{-\delta T}\right)-\frac{2be^{-rt_1}}{r}\right]. \tag{18}$$

The necessary condition for $\mathrm{TC}_1(t_1)$ in (17) to be minimized is $\dfrac{\mathrm{d}\,\mathrm{TC}_1(t_1)}{\mathrm{d}t_1}=0.$ (19)

The root $t_1=t_1^*$ of equation (19) will be the optimal solution for $t_1^*\in(0,\mu)$ if

$$\frac{\mathrm{d}^2\mathrm{TC}_1(t_1)}{\mathrm{d}t_1^2}>0 \text{ at } t_1=t_1^*$$

where

$$\frac{\mathrm{d}^2\mathrm{TC}_1(t_1)}{\mathrm{d}t_1^2}=-be^{-rT}c_1-\frac{be^{\delta(-2T+t_1)}\left(-1+e^{T\delta-rt_1}\delta\right)c_2}{r\delta}+be^{-T\delta}c_3+\frac{b\left(-e^{-rt_1}+e^{\theta t_1}\right)(h+C_f)}{r+\theta}$$
$$+\frac{e^{\delta(-2T+t_1)}\left(1+e^{T\delta-rt_1}(r-\delta)\right)c_2(a+bt_1)}{r}+\frac{\left(e^{-rt_1}r+e^{\theta t_1}\theta\right)(h+C_f)(a+bt_1)}{r+\theta}$$
$$-e^{-rt_1}c_3(-3b+ar+brt_1)+\frac{1}{2r\theta^2}e^{(-2r+\theta)t_1}C_v\left(2e^{rt_1}r\theta(b(-2+r-2\theta)+a(r-\theta)+b(r-\theta)t_1)\right.$$
$$\left.+e^{2rt_1}\theta^2(b+a\theta+b\theta t_1)-(2r-\theta)(a(2r-\theta)\theta-b(2r+\theta)+b(2r-\theta)\theta t_1)\right). \tag{20}$$

The total amount of backlogged items

$$B_1=a(\mu-t_1^*)+\frac{b\left(\mu^2-t_1^{*2}\right)}{2}+(a+b\mu)(\lambda-\mu)+(a+b\mu+c\lambda)(T-\lambda)-\frac{c\left(T^2-\lambda^2\right)}{2}.$$

Therefore, the optimal ordered quantity becomes $Q^*=S^*+B_1$, where S^* denotes the optimal value of S.

3.2. Case 2. $\mu\le t_1\le\lambda$

The differential equations for the case depict the stock behavior (Fig. 5).

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t}+\theta I(t)=-(a+bt),\qquad 0<t<\mu \tag{21}$$

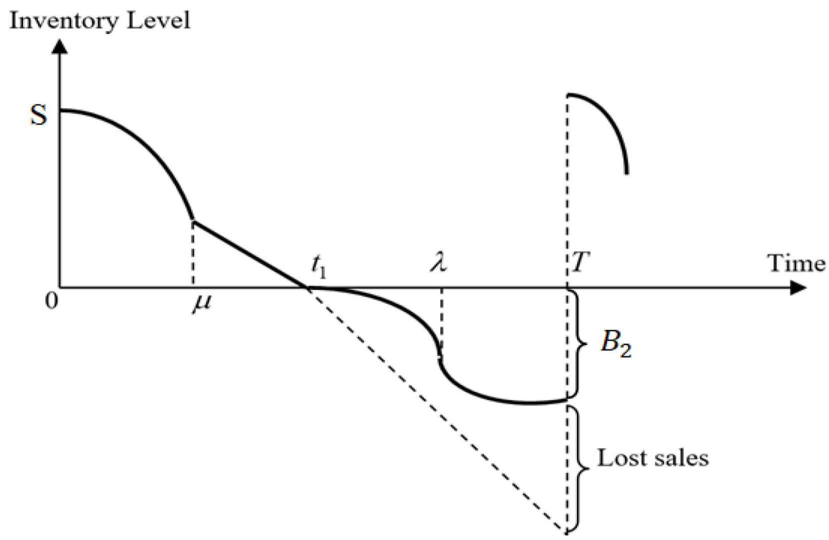

FIGURE 5. The level of inventory for Case 2.

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -(a+b\mu), \qquad \mu < t < t_1 \tag{22}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -(a+b\mu)B(T-t), \qquad t_1 < t < \lambda \tag{23}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\{(a+b\mu) - c(t-\lambda)\}B(T-t), \qquad \lambda < t < T \tag{24}$$

Solving the differential equations (21)–(24) with $I(t_1) = 0$, one can have

$$I(t) = \left[\frac{(a+b\mu)e^{\theta t_1}}{\theta} - \frac{be^{\theta\mu}}{\theta^2}\right]e^{-\theta t_1} + \frac{b}{\theta^2} - \frac{(a+bt)}{\theta}, \qquad 0 \le t \le \mu \tag{25}$$

$$I(t) = \frac{(a+b\mu)}{\theta}\left(e^{\theta(t_1-t)} - 1\right), \qquad \mu \le t \le t_1 \tag{26}$$

$$I(t) = -\frac{(a+b\mu)}{\delta}\left[e^{-\delta(T-t)} - e^{-\delta(T-t_1)}\right], \qquad t_1 \le t \le \lambda \tag{27}$$

$$I(t) = \frac{ce^{-\delta(T-\mu)}}{\delta^2} + \frac{(a+b\mu)e^{-\delta(T-t_1)}}{\delta} - \frac{e^{-\delta(T-t)}}{\delta^2}[\{(a+b\mu) - c(t-\lambda)\}\delta + c], \qquad \lambda \le t \le T. \tag{28}$$

The initial level of inventory can be determined as

$$S = I(0) = \frac{(a+b\mu)e^{\theta t_1}}{\theta} - \frac{be^{\theta\mu}}{\theta^2} + \frac{b}{\theta^2} - \frac{a}{\theta}. \tag{29}$$

Thus, the present value of material purchasing cost can be computed as

$$\begin{aligned}\mathrm{MC} &= c_1S + c_1e^{-rT}\left[\int_{t_1}^{\lambda}(a+b\mu)\,\mathrm{d}t + \int_{\lambda}^{T}\{(a+b\mu) - c(t-\lambda)\}\,\mathrm{d}t\right]\\ &= c_1S + c_1e^{-rT}\left[(a+b\mu)(\lambda-t_1) + (a+b\mu+c\lambda)(T-\lambda) - \frac{c\left(T^2-\lambda^2\right)}{2}\right].\end{aligned} \tag{30}$$

Total carbon emissions cost due to the energy consumption during storage and from the product is evaluated as

$$\begin{aligned}\mathrm{CEC} &= \int_0^{\mu} \big(C_f + C_v e^{\theta t}\big) I(t) e^{-rt}\,\mathrm{d}t + \int_{\mu}^{t_1} \big(C_f + C_v e^{\theta t}\big) I(t) e^{-rt}\,\mathrm{d}t \\ &= \frac{C_f(a+b\mu)\big(re^{\theta t_1} + \theta e^{-rt_1}\big)}{r\theta(r+\theta)} + \frac{C_f b\big(e^{-r\mu} - e^{\theta\mu}\big)}{\theta^2(r+\theta)} - \frac{C_f a}{r\theta} + \frac{C_f b(r-\theta)(1-e^{-r\mu})}{r^2\theta^2} + \frac{C_v(a+b\mu)e^{\theta t_1}}{r\theta} \\ &\quad - \frac{C_v b e^{\theta\mu}}{r\theta^2} + \frac{C_v b e^{-(r-\theta)\mu}}{r\theta^2} + \frac{C_v b(r-2\theta)\big(1-e^{-(r-\theta)\mu}\big)}{\theta^2(r-\theta)^2} - \frac{aC_v}{\theta(r-\theta)} + \frac{C_v(a+b\mu)e^{-(r-\theta)t_1}}{r(r-\theta)}.\end{aligned}$$

The present value of holding cost per cycle is

$$\begin{aligned}\mathrm{HC} &= h\bigg\{\int_0^{\mu} I(t)e^{-rt}\,\mathrm{d}t + \int_{\mu}^{t_1} I(t)e^{-rt}\,\mathrm{d}t\bigg\} \\ &= h\Bigg\{\frac{(a+b\mu)\big(re^{\theta t_1} + \theta e^{-rt_1}\big)}{r\theta(r+\theta)} + \frac{b\big(e^{-r\mu} - e^{\theta\mu}\big)}{\theta^2(r+\theta)} - \frac{a}{r\theta} + \frac{b(r-\theta)(1-e^{-r\mu})}{r^2\theta^2}\Bigg\}. \end{aligned} \tag{31}$$

The present value of shortage cost during the given cycle is

$$\begin{aligned}\mathrm{SC} &= -\int_{t_1}^{\lambda} c_2 I(t)e^{-rt}\,\mathrm{d}t - \int_{\lambda}^{T} c_2 I(t)e^{-rt}\,\mathrm{d}t \\ &= c_2 e^{-\delta T}\Bigg[\frac{(a+b\mu)e^{-\delta t_1}e^{-rT}}{r\delta} - \frac{ce^{\delta\lambda}\big(-e^{-rT} + e^{-r\lambda}\big)}{r} - \frac{e^{-(r-\delta)T}[\{(a+b\mu) - c(T-\lambda)\}\delta(r-\delta) + rc - 2\delta c]}{\delta^2(r-\delta)^2} \\ &\quad - e^{-(r-\delta)\lambda}\bigg[\frac{c}{r} - \frac{[cr + (a+b\mu)\delta(r-\delta)]}{(r-\delta)\delta^2}\bigg] + \frac{(a+b\mu)e^{-\delta T}\big(\frac{\delta}{r}e^{-(r-\delta)t_1} - e^{-(r-\delta)\lambda}\big)}{\delta(r-\delta)^2}\Bigg]. \end{aligned} \tag{32}$$

The opportunity cost in a cycle due to lost sales can be computed as

$$\begin{aligned}\mathrm{OPC} &= c_3\bigg[\int_{t_1}^{\lambda} e^{-rt}(1-B(t))(a+b\mu)\,\mathrm{d}t + \int_{\lambda}^{T} e^{-rt}(1-B(t))\{(a+b\mu) - c(t-\lambda)\}\,\mathrm{d}t\bigg] \\ &= c_3\Bigg[\frac{ce^{-\delta T}\{(a+b\mu) - c(T-\lambda)\}}{r(r-\delta)} - \frac{(2r-\delta)\delta ce^{-\delta T}}{r^2(r-\delta)^2} - \frac{ce^{-r\lambda}}{r^2} + \frac{ce^{-\delta T}e^{-(r-\delta)\lambda}}{(r-\delta)^2} \\ &\quad + \frac{(a+b\mu)e^{-rt_1}}{r} + \frac{(a+b\mu)e^{-\delta T}e^{-(r-\delta)t_1}}{(r-\delta)}\Bigg]. \end{aligned} \tag{33}$$

Then, the average total cost of this EOQ model under the condition ($\mu \le t_1 \le \lambda$) can be written as

$$\mathrm{TC}_2(t_1) = \frac{1}{T}[A + \mathrm{MC} + \mathrm{HC} + \mathrm{CEC} + \mathrm{SC} + \mathrm{OPC}]. \tag{34}$$

The first-order differentiation of the average cost function $\mathrm{TC}_2(t_1)$ with respect to t_1 can be written as follows.

$$\begin{aligned}\frac{\mathrm{d}\,\mathrm{TC}_2(t_1)}{\mathrm{d}t} &= \frac{(C_f + h)\big(e^{-\theta t_1} - e^{-rt_1}\big)}{(r+\theta)} + \frac{C_v e^{\theta t_1}}{r} - \frac{C_v e^{-(r-\theta)t_1}}{r} - \big(c_1 e^{-rT} + c_3 e^{-rt_1}\big) \\ &\quad - e^{-\delta T}e^{-(r-\delta)t_1}\bigg(\frac{c_2}{r(r-\delta)} + c_3\bigg). \end{aligned} \tag{35}$$

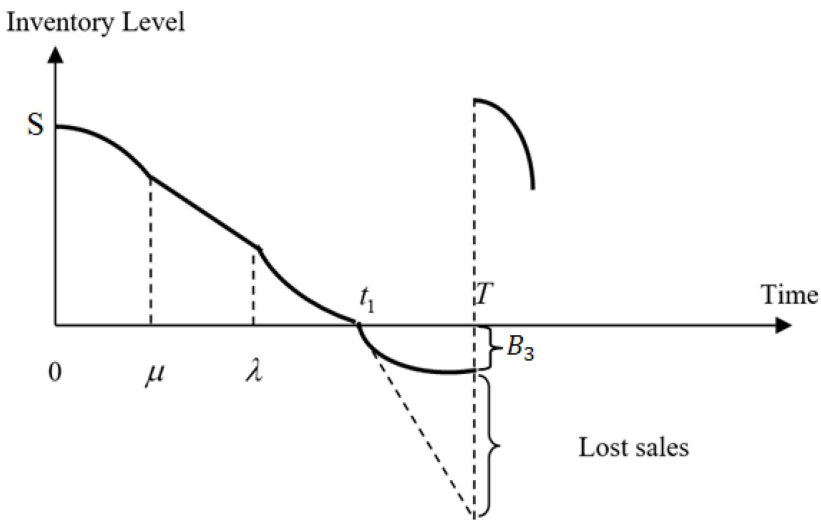

FIGURE 6. The level of inventory for Case 3.

The necessary condition for the average cost function $\mathrm{TC}_2(t_1)$ to have a stationary point is

$$\frac{\mathrm{d}\,\mathrm{TC}_2(t_1)}{\mathrm{d}t_1} = 0. \tag{36}$$

From equation (36), one can find the optimal value of t_1^* if $\frac{\mathrm{d}^2\mathrm{TC}_2(t_1)}{\mathrm{d}t_1^2} > 0$ where

$$\frac{\mathrm{d}^2\mathrm{TC}_2(t_1)}{\mathrm{d}t_1^2} = e^{-rt_1} r c_3 + \frac{e^{-T\delta+(-r+\delta)t_1}(c_2 + r(r-\delta)c_3)}{r} + \frac{\left(e^{-rt_1} r + e^{\theta t_1}\theta\right)(h + C_f)}{r+\theta} + \frac{e^{(-r+\theta)t_1}(r-\theta)C_v}{r} + \frac{e^{\theta t_1}\theta C_v}{r}\cdot \tag{37}$$

The total backorder quantity at the end of the cycle is

$$B_2 = (a + b\mu)(\lambda - t_1^*) + (a + b\mu + c\lambda)(T - \lambda) - \frac{c\left(T^2 - \lambda^2\right)}{2}\cdot$$

Therefore, from equations (29) and (37), one can find the optimal ordered quantity as, $Q^* = S^* + B_2$, where S^* denotes the optimal value of S.

3.3. Case 3. $\lambda \leq t_1 < T$

The behavior of stock for Case 3 is (Fig. 6) given as follows:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -(a + bt), \qquad 0 < t < \mu \tag{38}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -(a + b\mu), \qquad \mu < t < \lambda \tag{39}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \theta I(t) = -\{(a + b\mu) - c(t - \lambda)\}, \qquad \lambda < t < t_1 \tag{40}$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\{(a + b\mu) - c(t - \lambda)\}B(T - t), \qquad t_1 < t < T. \tag{41}$$

Solving the differential equations (38)–(41) with $I(t_1) = 0$, one can have

$$I(t) = \left[\frac{\theta\{(a+b\mu) - c(t_1-\lambda)\} + c}{\theta^2} e^{\theta t_1} - \frac{be^{\theta\mu}}{\theta^2} - \frac{ce^{\theta\lambda}}{\theta^2}\right] e^{-\theta t} + \frac{b}{\theta^2} - \frac{(a+bt)}{\theta}, \qquad 0 \le t \le \mu \tag{42}$$

$$I(t) = \frac{(a+b\mu)}{\theta}\left(e^{\theta(\lambda-t)} - 1\right) + \left(\frac{\{(a+b\mu) - c(t_1-\lambda)\}}{\theta} + \frac{c}{\theta^2}\right) e^{\theta(t_1-t)} - \left(\frac{(a+b\mu)}{\theta} + \frac{c}{\theta^2}\right) e^{\theta(\lambda-t)}, \qquad \mu < t < \lambda \tag{43}$$

$$I(t) = \left(\frac{\{(a+b\mu) - c(t_1-\lambda)\}}{\theta} - \frac{c}{\theta^2}\right) e^{\theta(t_1-t)} - \frac{\{(a+b\mu) - c(t_1-\lambda)\}}{\theta} - \frac{c}{\theta^2}, \qquad \lambda \le t \le t_1 \tag{44}$$

$$I(t) = \frac{e^{-\delta(T-t_1)}[\{(a+b\mu) - c(t_1-\lambda)\}\delta + c]}{\delta^2} - \frac{e^{-\delta(T-t)}[\{(a+b\mu) - c(t-\lambda)\}\delta + c]}{\delta^2}, \qquad t_1 \le t \le T. \tag{45}$$

The initial level of inventory can be calculated as

$$S = I(0) = \frac{\theta\{(a+b\mu) - c(t_1-\lambda)\} + c}{\theta^2} e^{\theta t_1} - \frac{be^{\theta\mu}}{\theta^2} - \frac{ce^{\theta\lambda}}{\theta^2} + \frac{b - a\theta}{\theta^2}. \tag{46}$$

The purchasing cost of items is computed as $c_1 Q$ where Q is the purchased inventory.

$$\begin{aligned} \mathrm{MC} &= c_1 S + c_1 e^{-rT}\left[\int_\lambda^T \{(a+b\mu) - c(t-\lambda)\}\,\mathrm{d}t\right] \\ &= c_1 S + c_1 e^{-rT}\left[\{(a+b\mu) + c\lambda\}(T-\lambda) - \frac{c\left(T^2 - \lambda^2\right)}{2}\right]. \end{aligned} \tag{47}$$

The present value of the holding cost per cycle is

$$\begin{aligned} \mathrm{HC} &= h\left\{\int_0^\mu I(t)e^{-rt}\,\mathrm{d}t + \int_\mu^\lambda I(t)e^{-rt}\,\mathrm{d}t + \int_\lambda^T I(t)e^{-rt}\,\mathrm{d}t\right\} \\ &= \left[\frac{\theta\{(a+b\mu) - c(t_1-\lambda)\} + c}{\theta^2} e^{\theta t_1} - \frac{be^{\theta\mu}}{\theta^2} - \frac{ce^{\theta\lambda}}{\theta^2}\right] \frac{h\left(1 - e^{-(r+\theta)\mu}\right)}{(r+\theta)} + \frac{hb(1 - e^{-r\mu})\left(\frac{1}{r\theta} - 1\right)}{\theta} \\ &+ \frac{h\left(e^{-(r+\theta)\mu} - e^{-(r+\theta)\lambda}\right)}{\theta(r+\theta)}\left[\left(\{(a+b\mu) - c(t_1-\lambda)\} + \frac{c}{\theta}\right)e^{\theta t_1} - \frac{e^{\theta\lambda}c}{\theta}\right] - \frac{ha}{r\theta} \\ &+ \left[\frac{\{(a+b\mu) - c(t_1-\lambda)\}}{\theta} + \frac{c}{\theta^2}\right] \frac{he^{\theta t_1}\left(e^{-(r+\theta)\lambda} - e^{-(r+\theta)t_1}\right)}{(r+\theta)} + \frac{hc(r-\theta)\left(e^{-rt_1} - e^{-r\lambda}\right)}{(r\theta)^2} \\ &+ \frac{h\{(a+b\mu) - c(t_1-\lambda)\}e^{\theta t_1}}{\theta r}. \end{aligned}$$

The present value of the carbon emissions cost per cycle is

$$\begin{aligned} \mathrm{CEC} &= \int_0^\mu \left(C_f + C_v e^{\theta t}\right) I(t)e^{-rt}\,\mathrm{d}t + \int_\mu^\lambda \left(C_f + C_v e^{\theta t}\right) I(t)e^{-rt}\,\mathrm{d}t + \int_\lambda^{t_1} \left(C_f + C_v e^{\theta t}\right) I(t)e^{-rt}\,\mathrm{d}t \\ &= \left[\frac{\theta\{(a+b\mu) - c(t_1-\lambda)\} + c}{\theta^2} e^{\theta t_1} - \frac{be^{\theta\mu}}{\theta^2} - \frac{ce^{\theta\lambda}}{\theta^2}\right] \frac{C_f\left(1 - e^{-(r+\theta)\mu}\right)}{(r+\theta)} + \frac{C_f b(1 - e^{-r\mu})\left(\frac{1}{r\theta} - 1\right)}{\theta} \\ &+ \frac{C_f\left(e^{-(r+\theta)\mu} - e^{-(r+\theta)\lambda}\right)}{\theta(r+\theta)}\left[\left(\{(a+b\mu) - c(t_1-\lambda)\} + \frac{c}{\theta}\right)e^{\theta t_1} - \frac{e^{\theta\lambda}c}{\theta}\right] - \frac{C_f a}{r\theta} \end{aligned}$$

$$+\left[\frac{\{(a+b\mu)-c(t_1-\lambda)\}}{\theta}+\frac{c}{\theta^2}\right]\frac{C_f e^{\theta t_1}\left(e^{-(r+\theta)\lambda}-e^{-(r+\theta)t_1}\right)}{(r+\theta)}+\frac{C_f c(r-\theta)\left(e^{-rt_1}-e^{-r\lambda}\right)}{(r\theta)^2}$$
$$+\frac{C_f\{(a+b\mu)-c(t_1-\lambda)\}e^{\theta t_1}}{\theta r}+\left[\frac{\theta\{(a+b\mu)-c(t_1-\lambda)\}+c}{\theta^2}e^{\theta t_1}-\frac{be^{\theta\mu}}{\theta^2}-\frac{ce^{\theta\lambda}}{\theta^2}\right]\frac{C_v(1-e^{-r\mu})}{r}$$
$$+\frac{C_v b\left(1-e^{-(r-\theta)\mu}\right)\left(\frac{1}{(r-\theta)\theta}-1\right)}{\theta}+\frac{C_v\left(e^{-r\mu}-e^{-r\lambda}\right)}{\theta^2}\left[\left(\{(a+b\mu)-c(t_1-\lambda)\}+\frac{c}{\theta}\right)e^{\theta t_1}-\frac{e^{\theta\lambda}c}{\theta}\right]$$
$$-\frac{C_v a}{(r-\theta)\theta}+\left[\frac{\{(a+b\mu)-c(t_1-\lambda)\}}{\theta}+\frac{c}{\theta^2}\right]\frac{C_v e^{\theta t_1}\left(e^{-r\lambda}-e^{-rt_1}\right)}{r}+\frac{C_v c(r-2\theta)\left(e^{-(r-\theta)t_1}-e^{-(r-\theta)\lambda}\right)}{\theta^2(r-\theta)^2}$$
$$+\frac{C_v\{(a+b\mu)-c(t_1-\lambda)\}e^{\theta t_1}}{\theta(r-\theta)}.$$

The present value of the shortage cost during the given cycle is

$$\begin{aligned}\mathrm{SC}&=-\int_{t_1}^{T}c_2 I(t)e^{-rt}\,\mathrm{d}t\\&=\frac{c_2e^{-\delta T}}{\delta}\left[\frac{e^{-rT}e^{-\delta t_1}\{\{(a+b\mu)-c(t_1-\lambda)\}+c\}}{r}+\frac{\delta e^{-(r-\delta)t_1}[\delta(r-\delta)\{(a+b\mu)-c(t_1-\lambda)\}-c(2r-\delta)]}{r(r-\delta)^2}\right.\\&\left.-\frac{e^{-(r-\delta)T}}{(r-\delta)}\left[\{(a+b\mu)-c(T-\lambda)\}\delta+\frac{c(r-2\delta)}{(r-\delta)}\right]\right].\end{aligned}\tag{48}$$

The present value of opportunity cost due to lost sales in a cycle is

$$\begin{aligned}\mathrm{OPC}&=c_3\left[\int_{t_1}^{T}e^{-rt}(1-B(t))\{(a+b\mu)-c(t-\lambda)\}\,\mathrm{d}t\right]\\&=c_3\left[\frac{ce^{-\delta T}\{(a+b\mu)-c(T-\lambda)\}}{r(r-\delta)}-\frac{(2r-\delta)\delta ce^{-\delta T}}{r^2(r-\delta)^2}-\frac{ce^{-rt_1}}{r^2}+\frac{ce^{-\delta T}e^{-(r-\delta)t_1}}{(r-\delta)^2}\right.\\&\left.+\frac{\{(a+b\mu)-c(t_1-\lambda)\}e^{-rt_1}}{r}-\frac{\{(a+b\mu)-c(t_1-\lambda)\}e^{-\delta T}e^{-(r-\delta)t_1}}{(r-\delta)}\right].\end{aligned}\tag{49}$$

By adding up the costs mentioned above, one can find the total cost of the given EOQ model per unit of time under the condition ($\lambda\le t_1\le T$) as

$$\mathrm{TC}_3(t_1)=\frac{1}{T}[A+\mathrm{MC}+\mathrm{HC}+\mathrm{CEC}+\mathrm{SC}+\mathrm{OPC}].\tag{50}$$

The first-order total derivative of $\mathrm{TC}_3(t_1)$ with respect to t_1 is as follows:

$$\frac{\mathrm{d}\,\mathrm{TC}_3(t_1)}{\mathrm{d}t}=\frac{-(C_f+h)c\left(1-e^{-(r+\theta)\mu}\right)}{\theta^2(r+\theta)}+\frac{(C_f+h)\left(e^{-(r+\theta)\mu}-e^{-(r+\theta)\lambda}\right)\{(a+b\mu)-c(t_1-\lambda)\}e^{\theta t_1}}{\theta(r+\theta)}$$
$$+\frac{(C_f+h)\{(a+b\mu)-c(t_1-\lambda)\}e^{\theta t_1}\left(e^{-(r+\theta)\lambda}-e^{-(r+\theta)t_1}\right)}{(r+\theta)}$$
$$-(C_f+h)\left[\frac{\{(a+b\mu)-c(t_1-\lambda)\}}{\theta}+\frac{c}{\theta^2}\right]e^{\theta t_1}e^{-(r+\theta)t_1}-\frac{C_v c(1-e^{-r\mu})}{\theta^2 r}$$
$$+\frac{C_v\left(e^{-r\mu}-e^{-r\lambda}\right)\{(a+b\mu)-c(t_1-\lambda)\}e^{\theta t_1}}{r}+\frac{C_v\{(a+b\mu)-c(t_1-\lambda)\}e^{\theta t_1}\left(e^{-r\lambda}-e^{-rt_1}\right)}{r}$$

$$- C_v\left[\frac{\{(a+b\mu)-c(t_1-\lambda)\}}{\theta}+\frac{c}{\theta^2}\right]e^{-(r-\theta)t_1}+\frac{c_2e^{-\delta T}}{r}\left\{e^{-rT}e^{-\delta t_1}[\{(a+b\mu)-c(t_1-\lambda)\}\delta+2c]\right.$$
$$\left.-\frac{e^{-(r-\delta)t_1}}{(r-\delta)}[\delta(r-\delta)\{(a+b\mu)-c(T-\lambda)\}-2rc]\right\}. \tag{51}$$

The necessary condition for $\mathrm{TC}_3(t_1)$ in (50) to be minimized is

$$\frac{\mathrm{d}\,\mathrm{TC}_3(t_1)}{\mathrm{d}t_1}=0. \tag{52}$$

The root of the equation (52) will be the optimal solution $t_1^*\in(0,\mu)$ if at $t_1=t_1^*$

$$\frac{\mathrm{d}^2\mathrm{TC}_3(t_1)}{\mathrm{d}t_1^2}>0$$

where

$$\frac{\mathrm{d}^2\mathrm{TC}_3(t_1)}{\mathrm{d}t_1^2}=\frac{1}{T}\Bigg(-\Big(ce^{-rt_1}-ce^{-T\delta+(-r+\delta)t_1}-e^{-rt_1}r(a+c(-T+\lambda)+b\mu)$$
$$-e^{-T\delta+(-r+\delta)t_1}(-r+\delta)(a+c(-T+\lambda)+b\mu)\Big)c_3$$
$$+\frac{1}{r\theta^2(r+\theta)}e^{-rt_1}(h+C_f)\Big(c\Big(r^4-r^2\theta(2+\theta)+e^{(r+\theta)t_1}\theta^3(-2+\theta\lambda)-r^3(1+\theta+\theta\lambda)$$
$$+r\theta^2\Big(\theta+e^{(r+\theta)t_1}(-3+2\theta\lambda)\Big)\Big)+\theta\Big(-r^3+2e^{(r+\theta)t_1}r\theta^2+e^{(r+\theta)t_1}\theta^3\Big)(a+b\mu)$$
$$+c\theta\Big(r^3-2e^{(r+\theta)t_1}r\theta^2-e^{(r+\theta)t_1}\theta^3\Big)t_1\Big)+C_v\Bigg(-\frac{2ce^{\theta t_1}\left(e^{-r\lambda}-e^{-rt_1}\right)}{r}+\frac{ce^{(-r+\theta)t_1}(r-2\theta)}{\theta^2}$$
$$-\frac{2ce^{(-r+\theta)t_1}}{\theta}+\frac{2ce^{\theta t_1}}{-r+\theta}-\frac{e^{\theta t_1}\theta(a+c\lambda+b\mu-ct_1)}{-r+\theta}$$
$$+\frac{e^{\theta t_1}\left(e^{-r\lambda}-e^{-rt_1}\right)(c+c\theta\lambda+\theta(a+b\mu)-c\theta t_1)}{r}-\frac{e^{(-r+\theta)t_1}(r-\theta)(c+c\theta\lambda+\theta(a+b\mu)-c\theta t_1)}{\theta^2}$$
$$+\frac{e^{(-r+\theta)t_1}(c+c\theta\lambda+\theta(a+b\mu)-c\theta t_1)}{\theta}+\frac{e^{-r\mu+\theta t_1}(-1+e^{r\mu})(c(-1+\theta\lambda)+\theta(a+b\mu)-c\theta t_1)}{r}$$
$$+\frac{e^{-r(\lambda+\mu)+\theta t_1}\left(e^{r\lambda}-e^{r\mu}\right)(c(-1+\theta\lambda)+\theta(a+b\mu)-c\theta t_1)}{\theta}\Bigg)+\frac{1}{r}\Big\{e^{-T\delta}c_2\Big(2ce^{-rT-\delta t_1}$$
$$+2ce^{(-r+\delta)t_1}\delta+e^{-rT-\delta t_1}\delta(a+c+c\lambda+b\mu-ct_1)$$
$$+e^{(-r+\delta)t_1}(c(-2r+\delta)+(r-\delta)\delta(a+c\lambda+b\mu-ct_1))\Big)\Big\}\Bigg).$$

One knows that the total backorder amount at the end of the cycle is

$$B_3=(a+b\mu+c\lambda)(T-\lambda)-\frac{c\left(T^2-\lambda^2\right)}{2}\cdot \tag{53}$$

Therefore, the optimal order quantity, denoted by Q^*, is $Q^*=S^*+B_3$, where S^* denotes the optimal value of S.

4. Numerical illustration

The established model gives three different numerical examples for a single type of product for the above-mentioned situations. Data is obtained from the literature in the required units for the numerical analysis.

Example 1. The analytical results of the problem for Case 1 are verified by the numerical example [54] having the following input parameters in their appropriate units for Case 1.

$A = \$1000$ per order, $T = 5$ months, $\mu = 4$ months, $\lambda = 4.5$ months, $a = 500$ products per month, $b = 10$, $c = 8$, $C_v = \$0.75$ per unit per month, $C_f = \$0.25$ per unit per month, $\theta = 0.25$, $h = \$1.5$ per unit per month, $c_2 = \$2.5$ per unit per month, $c_3 = \$10$ per unit, $c_1 = \$30$ per unit, $r = 0.1$ and $\delta = 0.25$.

The optimal results are calculated as the optimum time period $(t_1^*) = 3.65$ months, optimum initial inventory $(S^*) = 31\,00.99$ unit per month, optimum ordered quantity $(Q^*) = 3543.44$ unit per month, and $\mathrm{TC}_1(t_1^*) = \$14\,663.7$ per month.

In this example, it is noticed that the number of backlogged items is much higher than in the other two cases. This is because in the first case, $0 < t_1 \leq \mu < \lambda$, and the demand is low and constant during the inventory depletion period; afterward, it increases and is at its peak during the shortages. That is why the backlogged items are higher in the Case 1.

Example 2. The inventory system is verified for Case 2 with the described assumptions above. The input data of parameters are same as Example 1.

Therefore, the optimal policies for Case 2 $(0 < \mu < t_1 \leq \lambda)$ are $t_1^* = 2.99$ months, $S^* = 2294.33$ unit per month, $Q^* = 2815.33$ unit per month, and $\mathrm{TC}_2(t_1^*) = \$30\,602.76$ per month in their appropriate units.

Example 3. For Case 3, the inventory system is verified with the described assumptions above. The input data of parameters are similar as previous. Changed data is given below.

$T = 8$ months, $\mu = 2$ months, $\lambda = 3.5$ months, $r = 0.2$ and $\delta = 0.1$.

Therefore, the optimal policies for Case 3 $(0 < \mu < \lambda \leq t_1)$ are $t_1^* = 7.38$ months, $S^* = 6845.95$ unit per month, $Q^* = 2259$ unit per month, and $\mathrm{TC}_3(t_1^*) = \$165\,244.55$ per month in their appropriate unit.

4.1. Sensitivity analysis

This section provides the changes of the total cost with respect to the cost parameters of the system. Changes for Case 1 are listed in Tables 2–7. Increasing the value of a increases the total cost of the system. A similar phenomenon happens for the parameter b. Meanwhile, the increasing value of fixed carbon emissions cost C_f decreases the total cost of the system, but the total cost increases for increasing the value of variable emissions cost C_v. Similarly, Tables 8–13 provide the sensitivity for Case 2 and Tables 14–19 for Case 3, respectively.

4.2. Observations and managerial insights

(1) Tables 2 and 3 show that the increasing demand is responsible for the increment in the ordering quantity and total cost. It is noticed that although the increment in demand parameter b resulted in an increase in the value of t_1, the increase in parameter a can lead to a shorter consumption period.
(2) From Table 4, one can observe that an increment in the carbon emissions cost due to the storage resulted in a reduction in the optimum consumption period and, therefore, the lower ordering quantity reduces the system's total cost.
(3) It is observed from Table 7 that the carbon emitted from the inventory is highly positively sensitive to the changes in both demand parameters and deterioration rate. The increment in the carbon emissions due to the increment in demand rate increases when the deterioration rate is higher.

TABLE 2. Effect of variation of the base demand a for Case 1.

a (unit per month)	t_1^* (month)	S (unit per month)	Q (unit per month)	TC_1 ($ per month)
400	3.79823	2674.45	2994.44	12 808.9
450	3.71515	2887.75	3268.75	13 737.9
500	3.64609	3100.99	3543.44	14 663.7
550	3.58777	3314.18	3818.43	15 586.9
600	3.53789	3527.34	4093.64	16 508.2

TABLE 3. Effect of variation of the scaling parameter b for Case 1.

b	t_1^* (month)	S (unit per month)	Q (unit per month)	TC_1 ($ per month)
5	3.30356	2616.01	3149.72	11 819.3
7.5	3.48163	2858.63	3345.51	13 250.1
10	3.64609	3100.99	3543.44	14 663.7
12.5	3.79823	3343.06	3743.2	16 061.7
15	3.93923	3584.83	3944.5	17 445.7

TABLE 4. Effect of variation of the fixed carbon emissions cost C_f for Case 1.

C_f ($ per unit per month)	t_1^*	S (unit per cycle)	Q (unit per month)	TC_1 ($ per cycle)
0.15	3.73149	3216.3	3630.95	14 858.8
0.20	3.68832	3157.68	3586.39	14 756.5
0.25	3.64609	3100.99	3543.44	14 663.7
0.30	3.60476	3046.13	3502.02	14 579.9
0.35	3.5643	2993.01	3462.06	14 504.8

TABLE 5. Total carbon emissions cost due to the facilities in the warehouse for the storage of products for Case 1.

C_f ($ per unit per month)	Total carbon emissions cost
0.15	670.37
0.20	893.83
0.25	1117.28
0.30	1340.74
0.35	1564.20

(4) For Cases 3 and 4 (Tables 8, 9, 14, and 15), it is advisable for the practitioners to increase the replenishment period and the ordering quantity if demand increases.
(5) It is noticed from the observations (Tables 5, 11, and 17) that the total carbon emissions cost is proportional to the fixed cost of carbon emissions. The total carbon emissions increases in the same proportion of C_f.
(6) Figures 7–9 show that the carbon emissions cost increases with the increment in the variable carbon emissions cost and the deterioration rate. However, the increment in the total carbon emissions with respect to variable emissions cost increases with the deterioration rate. That is, if the deterioration rate is negligible, total carbon emissions do not vary much with C_v but it increases rapidly if the deterioration rate is relatively high.

TABLE 6. Effect of variation of the variable emissions cost C_v for Case 1.

C_v ($ per unit per cycle)	t_1^*	S (unit per month)	Q (unit per month)	TC_1 ($ per month)
0.25	3.85697	3390.61	3764.31	11 619.1
0.50	3.74603	3236.2	3646.15	12 986.6
0.75	3.64609	3100.99	3543.44	14 663.7
1.00	3.56454	2995.45	3467.18	16 703.8
1.25	3.45458	2887.85	3397.41	18 841.2

TABLE 7. Effect of deterioration and demand on the total carbon emissions from the product itself for Case 2.

		θ		
		0.2	0.25	0.3
a (unit per month)	250	604.55	688.86	786.15
	500	1152.73	1312.74	1497.33
	750	1700.91	1936.63	2208.52

TABLE 8. Effect of variation of the base demand rate a for Case 2.

a (unit per month)	t_1^*	S (unit per month)	Q (unit per month)	TC_2 ($ per month)
400	2.99	1849.90	2270.90	24 767.41
450	2.99	2072.12	2543.12	27 685.08
500	2.99	2294.33	2815.33	30 602.76
550	2.99	2516.54	3087.54	33 520.45
600	2.99	2738.76	3359.76	36 438.10

TABLE 9. Effect of variation of demand rate parameter b for Case 2.

b	t_1^*	S (unit per month)	Q (unit per month)	TC_2 ($ per month)
5	2.99	2258.24	2766.74	29 865.2
7.5	2.99	2276.28	2791.03	30 234.1
10	2.99	2294.33	2815.33	30 602.7
12.5	2.99	2312.38	2839.63	30 971.5
15	2.99	2330.42	2863.92	31 340.3

TABLE 10. Effect of variation of the fixed carbon emissions cost C_f for Case 2.

C_f ($ per unit per month)	t_1^*	S (unit per month)	Q (unit per month)	TC_2 ($ per month)
0.15	2.99	2293.3	2814.3	30 543.1
0.20	2.99	2293.8	2814.8	30 572.9
0.25	2.99	2294.33	2815.33	30 602.7
0.30	2.99	2294.8	2815.8	30 632.6
0.35	2.99	2295.3	2816.3	30 662.4

TABLE 11. Total carbon emissions cost due to the facilities provided in the warehouse for the storage of the product for Case 2.

C_f ($ per unit per month)	Total carbon emissions cost
0.15	80.32
0.20	107.12
0.25	133.93
0.30	160.75
0.35	187.58

TABLE 12. Effect of variation of the variable carbon emissions cost C_v for Case 2.

C_v ($ per unit per month)	t_1^*	S (unit per month)	Q (unit per month)	TC_2 ($ per month)
0.25	2.95	2253.4	2774.4	30 125.7
0.50	2.97	2273.8	2794.8	30 362.5
0.75	2.99	2294.33	2815.33	30 602.7
1.00	3.01	2314.8	2835.8	30 846.5
1.25	3.03	2335.4	2856.5	31 093.6

TABLE 13. Effect of parameter C_v on the total carbon emissions cost due to the emissions from the product itself for Case 2.

		θ		
		0.2	0.25	0.3
C_v ($ per unit per month)	0.50	203.63	230.26	261.40
	0.75	307.93	349.89	399.38
	1.00	413.88	472.55	542.35

TABLE 14. Effect of variation of the base demand rate a for Case 3.

a (unit per month)	t_1^*	S (unit per month)	Q (unit per month)	TC_3 ($ per month)
400	7.34	5447.54	1809	131 140
450	7.36	6146.5	2034	148 190
500	7.38	6845.95	2259	165 244.55
550	7.39	7545.6	2484	182 303
600	7.41	8245.5	2709	199 365

TABLE 15. Effect of variation of the demand rate parameter b for Case 3.

b	t_1^*	S (unit per month)	Q (unit per month)	TC_3 ($ per month)
5	7.37	6717.12	2214	161 701
7.5	7.37	6779.02	2236.5	163 453
10	7.38	6845.95	2259	165 244.55
12.5	7.38	6902.8	2281.5	166 959
15	7.38	6964.73	2304	168 712

Table 16. Effect of variation of the fixed carbon emissions cost C_f for Case 3.

C_f ($ per unit per month)	t_1^*	S (unit per month)	Q (unit per month)	TC_3 ($ per month)
0.15	7.47	6979.19	2259	165 496
0.20	7.42	6910.6	2259	165 359
0.25	7.38	6845.95	2259	165 244.55
0.30	7.34	6784.7	2259	165 151
0.35	7.29	6726.7	2259	165 076

Table 17. Total carbon emissions cost due to the facilities provided in the warehouse for the storage of the product for Case 3.

C_f ($ per unit per month)	Total carbon emissions cost
0.15	1147.18
0.20	1515.31
0.25	1894.14
0.30	2272.97
0.35	2651.80

Table 18. Effect of variation of the variable carbon emissions cost C_v for Case 3.

C_v ($ per unit per month)	t_1^*	S (unit per month)	Q (unit per month)	TC_3 ($ per month)
0.25	7.14	6504.7	2259	152 143
0.50	7.26	6675.13	2259	158 565
0.75	7.38	6845.95	2259	165 244
1.00	7.49	7017.37	2259	172 185
1.25	7.61	7189.59	2259	179 389

Table 19. Effect of parameter C_v on the total carbon emissions cost due to the emissions from the product itself for Case 3.

		θ		
		0.12	0.15	0.18
C_v ($ per unit per month)	0.50	8404	10 753	21 931
	0.75	12 894	16 514	33 733
	1.00	17 583	22 533	46 088

5. Case study

A case study is presented to justify the study and provide a comparison with existing literature based on numerical findings. The input data of parameters are taken from a neighboring cattle dung compost organic fertilizer supplier in India.

$A = \$1250$ per cycle, $T = 6$ months, $\mu = 3$ months, $\lambda = 2$ months, $a = 50$ metric ton per month, $b = 5$, $c = 3$, $C_v = \$0.05$ per metric ton per month, $C_f = \$0.10$ per metric ton per month, $\theta = 0.1$, $h = \$1$ per metric ton per month, $c_2 = \$1.5$ per metric ton per month, $c_3 = \$5$ per metric ton, $c_1 = \$35$ per metric ton, $r = 0.1$ and $\delta = 0.12$.

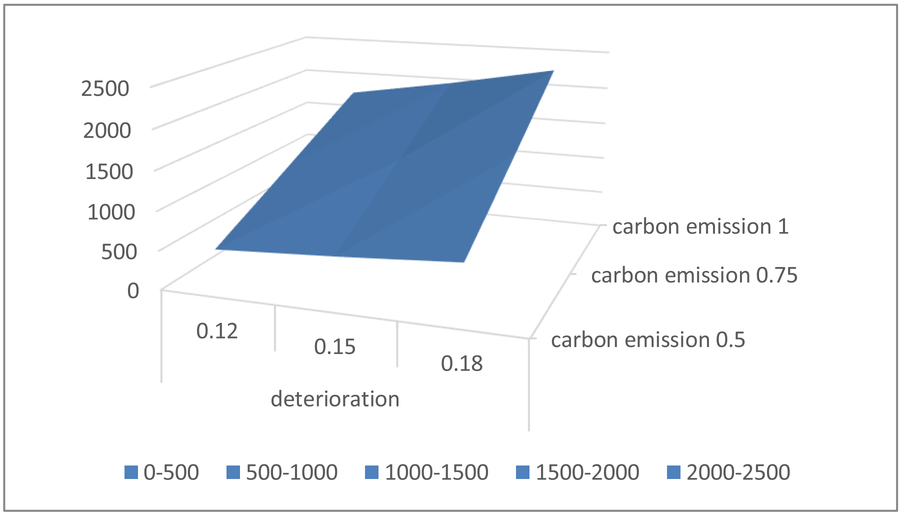

FIGURE 7. Behavior of the total carbon emissions with respect to varying deterioration rate and carbon emissions cost for Case 1.

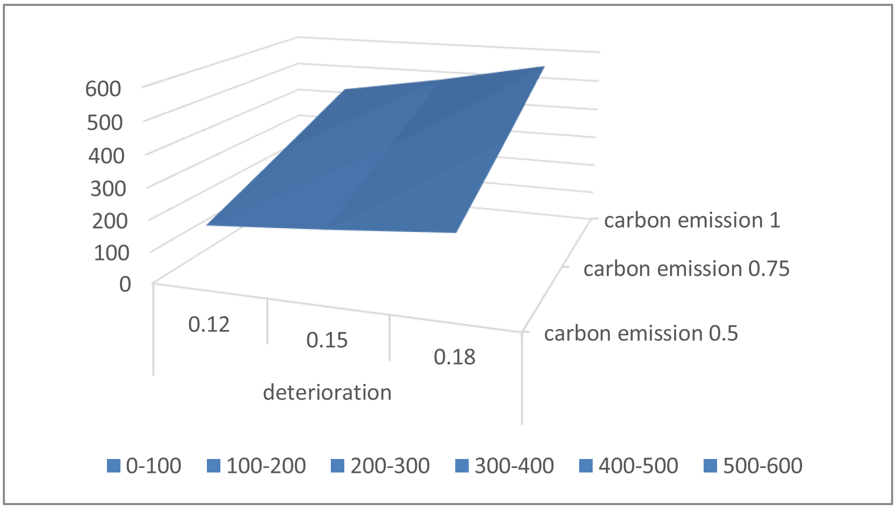

FIGURE 8. Behavior of the total carbon emissions with respect to varying deterioration rate and carbon emissions cost for Case 2.

The optimal policies under assumed conditions are $t_1^* = 2.236$ months, $S^* = 3100.99$ metric ton per month, $Q^* = 3543.4$ metric ton per month and $\mathrm{TC}_1(t_1^*) = \$36\,143.6$ per month.

6. Managerial insights

This article offers valuable managerial insights for supply chain professionals and businesses seekers to optimize inventory management while addressing environmental responsibilities [19]. Notably, it highlights the impact of increasing demand rates on ordering quantity and total costs, emphasizing the need for adaptive inventory strategies as demand fluctuates. Moreover, it underscores the significance of carbon emissions costs

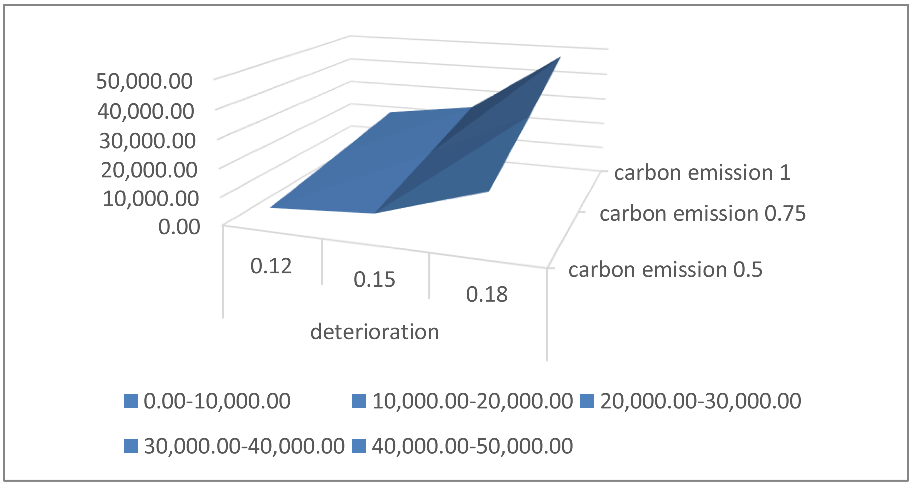

FIGURE 9. Behavior of the total carbon emissions with respect to varying deterioration rate and carbon emissions cost for Case 3.

associated with storage, advocating for a reduction in ordering quantity when these costs rise. The study's sensitivity analysis reveals that carbon emissions are highly responsive to changes in both demand parameters and deterioration rates, particularly when deterioration rates are high, emphasizing the importance of preservation techniques for mitigating emissions. Practical recommendations, backed by numerical examples, provide actionable guidance for real-world applications, while the article's focus on seasonal demand patterns adds relevance for businesses in various industries. Overall, the insights, offered here, bridge the gap between economic considerations and environmental responsibility [65], empowering supply chain managers [66] to make informed decisions that benefit both their bottom line and the planet.

7. Conclusions

Some products such as crude oil, gasoline, dung cake, or livestock slurry were responsible for the huge emissions of GHG in storage due to evaporation or decay. The model for the deteriorating products that were responsible for releasing greenhouse gases was studied in this article. The demand for the product was seasonal, increasing over time. Three cases were defined based on the trapezoidal interval and the shortage point. The effects of inflation throughout the cycle were taken into account. Shortages were allowed, and a partial backlogging strategy was adopted to meet the overall back-ordered demand when the next replenishment was received. The partial backlog rate was the decreasing waiting time function for the next refill. This article proposed economically sensible decisions while taking responsibility for the environment. In this study, numerical examples were provided to illustrate the assumptions. The sensitivity analysis for each case was provided to show the correct picture of the proposed model.

This study shed light on the intricate relationship between inventory management, environmental costs, and carbon emissions in deteriorating product production systems. It highlighted the significance of considering factors such as demand rates, carbon emissions costs, and deterioration rates in making informed decisions.

Practitioners are encouraged to adapt their replenishment strategies to meet changing demand effectively. Moreover, recognizing the impact of carbon emissions on total costs, this study emphasizes the importance of environmentally conscious inventory management [41,46]. Sensitivity analysis revealed that carbon emissions are highly responsive to variations in both demand and deterioration rates, emphasizing the need for preservation techniques when deterioration was a concern.

Lastly, the study underscored that both variable emissions costs and deterioration rates influenced carbon emissions costs. Businesses must balance minimizing costs and reducing their carbon footprint [14]. These findings offer a holistic perspective, guiding companies toward sustainable and responsible inventory management practices in the face of dynamic market conditions and environmental concerns.

This study contains several limitations which must be further examined in future studies. This model can be extended by considering preservation technology and a customer-centric approach in the future version of the current study [9, 20]. Besides, the best supplier selection for the products can be an interesting extension of this study for international supply chain [21]. The present study can be extended by including the permissible assumption of delay for process inventory management and its routing network [3, 42] further research. More environmental issues, such as reverse logistics [60], green design [30], and green production [29], can be mentioned in future studies. The used energy for the production system can be considered in further research, which can be renewable energy [23, 27] for reducing carbon footprint.

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