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RESEARCH ON DYNAMIC PRICING AND COORDINATION MODEL OF FRESH PRODUCE SUPPLY CHAIN BASED ON DIFFERENTIAL GAME CONSIDERING TRACEABILITY GOODWILL

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Abstract. This study aims to solve the current pricing and coordination problems in the fresh produce supply chain by proposing a dynamic pricing and coordination differential game model based on traceability goodwill to optimize the operational efficiency and profitability of the supply chain. Freshness and traceability are critical consumer trust and brand value factors incorporated into the dynamic pricing and coordination model in the agricultural supply chain. The research methodology of this paper integrates critical factors such as supply chain cost and market demand to construct a comprehensive differential game model. It also highlights the motivation of dynamic pricing strategy and coordination mechanism based on traceability goodwill. Through traceability goodwill, operating companies can better assess and manage product quality and improve consumer trust in agricultural products, thus enhancing brand value. The dynamic pricing strategy based on traceability goodwill can flexibly adjust prices according to changes in the quality characteristics of agricultural products and market demand, realizing a dynamic balance between supply and demand. On the other hand, the mutual cost-sharing and coordination mechanism promotes the synergy between the various links of the supply chain. It improves overall efficiency through cooperation coo, coordination, and other means. The validity and reliability of the model are verified through numerical simulation experiments. The numerical simulation results show that the dynamic pricing and coordination model based on traceability will significantly improve supply chain efficiency and profitability compared with the traditional model. Specifically, the interests of supply chain participants are balanced, and the overall supply chain operational efficiency is improved. The main contribution of this study is to propose a dynamic pricing and coordination model based on traceability goodwill and verify its superiority through numerical simulation experiments. This study provides a valuable reference for fresh agricultural products supply chain management and offers new ideas for improving supply chain efficiency and profitability.

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1. Introduction

Fresh produce is a necessity of life for the residents of society. It is one of the primary food sources for the residents of the society, with the same important role as water, electricity, gas, etc. However, the frequent occurrence of stagnant fresh produce, coupled with the sudden impact of the new crown epidemic, has brought unprecedented difficulties to fresh produce operators. According to the 21st Century Business Herald, China's annual post-production loss rate of fruits, vegetables, and potatoes is as high as 15% to 25%, with an annual loss of nearly 200 million tons, amounting to 300 billion yuan. In order to solve the problem of agricultural losses and accelerate the movement of agricultural products out of villages and into cities, the Ministry of Agriculture and Rural Affairs proposed for the first time in a report on March 17, 2020, the "first mile" for agricultural products. The "first mile" is the most critical issue for fresh agricultural products: freshness. However, the fresh produce supply chain operators need to improve in freshness preservation.

As people's living standards continue to improve, consumers are particularly concerned about the freshness of their agricultural products when they purchase them. Because freshness is the main factor affecting the quality of fresh agricultural products, quality affects demand, and demand affects the profit of operating companies. Therefore, how operating companies can provide consumers with the freshness of agricultural products is a significant concern for both operating companies and academics. The supply chain of fresh agricultural products is a complex network connecting various links such as agricultural producers, processors, wholesalers, and retailers, and its efficient operation is of great significance in guaranteeing food safety, improving the quality of agricultural products and promoting the sustainable development of agriculture. However, due to the unique nature of fresh agricultural products, such as perishability, seasonality, and locality, their supply chain management faces many challenges. Among them, pricing and coordination issues are core issues in supply chain management. Traditional supply chain pricing models are mainly based on static assumptions, *i.e.*, assuming that factors such as market demand, production costs, and product prices are constant. However, in fresh produce supply chains, these factors tend to change dynamically. For example, market demand is affected by factors such as consumer preferences, income levels, and seasons; production costs are affected by factors such as weather, natural disasters, and labor costs; and product prices are affected by factors such as market competition, government policies, and brand effects. Therefore, traditional pricing models cannot accurately reflect the pricing and coordination problems in the fresh produce supply chain. Cai *et al.* [1] introduced the freshness of products ensured by suppliers and retailers through preservation efforts, thus constructing the freshness function as an increasingly differentiable function of preservation efforts. Piramuthu and Zhou [2] proposed that product freshness is an essential factor affecting quality. Yan *et al.* [3] considered product freshness in three sales periods. They concluded that the freshness of fresh produce decreases continuously with sales time, and freshness is an essential guarantee of product quality. Liu *et al.* [4] pointed out that product freshness is decreasing due to natural decay (quality reduction) and that instantaneous freshness can be enhanced by preservation efforts (quality enhancement), leading to the conclusion that freshness increment is instantaneous freshness with preservation efforts while delaying natural decay freshness. However, little attention has been paid to the application of traceability goodwill to fresh produce, although the effect of freshness on fresh produce quality has been demonstrated some years earlier.

Tamayo *et al.* [5] argued that establishing a comprehensive traceability system for the agricultural products supply chain can effectively monitor the whole process from the production site of agricultural products to the supermarket to the table. Chen *et al.* [6] analyzed consumers' preference for ecological agricultural traceability technology and found that consumers were most willing to pay for agricultural products with traceability information. Banterle and Stranieri [7] concluded that fresh produce traceability systems could benefit agricultural supply chain operating companies. On this basis, Salah *et al.* [8] introduced an agricultural products supply chain traceability system, which not only improves the adequate supply of agricultural products and shortens the circulation time but also reduces the loss in circulation and the value loss caused by the loss. Liu *et al.* [9] introduced traceability goodwill into the food supply chain. They pointed out that the investment in traceability goodwill not only improves food traceability goodwill but also increases the profits of supply chain operating enterprises. Luo *et al.* [10] on the impact of advertising delay on inventory control strategies in fresh produce supply chains. The study found that the advertising delay effect exists and impacts supply chain operations. Therefore, this study aims to develop a just-in-time inventory control strategy for advertising delay effect in the fresh produce supply chain. The optimal equilibrium strategies, product reputation, and optimal inventory trajectories of suppliers and retailers under secondary replenishment decisions and decentralized decisions are comparatively analyzed by establishing a game model based on the Nerlove–Arrow time-delay differential equations and Pontryagin's maximum principle. The results show that the total profit of the supply chain under the secondary replenishment decision is more significant than that under the decentralized decision only when the sales cycle satisfies certain conditions. Although product reputation increases with increasing delay time, achieving this maximum value becomes difficult for longer delay times. These findings provide a theoretical basis for fresh produce supply chain members to choose replenishment and inventory control strategies when using advertising and marketing with different delay levels.

In recent years, as people's attention to food safety and quality has been increasing, the traceability goodwill of fresh agricultural products has become one of the critical factors in consumers' purchasing decisions. However, the price of fresh agricultural products in the current market is highly fluctuating, and the lack of coordination between upstream and downstream enterprises in the supply chain has led to frequent problems such as unreasonable pricing and information asymmetry. Therefore, studying the dynamic pricing and coordination model of fresh produce supply chain based on traceability goodwill is of great theoretical and practical significance.

First, as a trust mechanism, traceability goodwill can provide product traceability and quality assurance, enhancing consumers' trust in products. However, the establishment and maintenance costs of traceability goodwill and its relationship with pricing and coordination strategies have yet to be thoroughly investigated in actual supply chains. Therefore, we need to explore the role of traceability goodwill in supply chains and how to improve its effectiveness through dynamic pricing and coordination strategies.

Second, dynamic pricing is of great significance in fresh produce supply chains. Due to the unique characteristics of fresh agricultural products, their prices are greatly influenced by market demand, seasonal changes, and competition [10]. Therefore, an essential issue for supply chain managers is adjusting the price flexibly according to the changes in market demand to achieve the balance of supply and demand and maximize the supply chain profit.

In addition, coordination among parties in the supply chain is also a key factor. Problems such as information asymmetry and unstable cooperative relationships may lead to conflicts and inefficiencies in the supply chain. Therefore, it is of great practical significance to study how to improve supply chain performance and enhance cooperation and trust among parties through coordination mechanisms.

In summary, from the existing research literature, it is found that some scholars have explored the issue of traceability systems and traceability goodwill inputs in-depth. However, studies have yet to be conducted on the dynamic effects of traceability goodwill inputs for agricultural products. To address the above situation, this chapter discusses the following questions: (1) What optimal pricing, freshness input, and traceability goodwill input strategies are under the centralized and decentralized scenarios? (2) What is the impact of traceability goodwill input on the optimal strategy? (3) What kind of contract can coordinate the supply chain?

To solve the above problem, this paper considers a secondary supply chain of fresh agricultural products considering traceability goodwill in a dynamic environment, in which suppliers determine the wholesale price and freshness inputs of fresh agricultural products, and retailers determine the retail price and traceability goodwill inputs of fresh agricultural products. First, the optimal dynamic pricing, freshness input, and traceability goodwill input strategies are given for the centralized and decentralized scenarios; second, the equilibria under the two scenarios are compared; and finally, mutual cost-sharing contracts are proposed to coordinate the supply chain. The main findings show that the profits in the centralized scenario are higher than those in the decentralized scenario. In contrast, the selling prices in the centralized scenario may be lower or higher than those in the decentralized scenario, which originates from the parameter values. Reciprocal cost-sharing contracts can satisfactorily coordinate the supply chain. The main contributions of this article include: (1) Dynamic pricing and coordination strategies are proposed. The study proposes a dynamic pricing and coordination strategy

Table 1. Symbol description.

Symbols	Meaning	Symbols	Meaning
$u_s(t), u_o(t)$	Freshness and traceability goodwill input	η_1, η_2	Preservation of fresh inputs and traceability goodwill cost factors
β	Traceability of goodwill input effectiveness factor	α	Attenuation factor of retroactive goodwill
φ	Traceability goodwill contribution factor	ϕ	Traceability goodwill contribution margin factor

based on traceability goodwill for the characteristics of fresh produce supply chains. This strategy considers the interests and objectives of different participants in the supply chain and maximizes the benefits of the supply chain through dynamic pricing and coordination. (2) A secondary traceable fresh produce supply chain model is established. In order to study the game relationship in the supply chain, a two-level traceable fresh produce supply chain model, including fresh produce suppliers and retailers, was established. This model can capture the interactions and influences between participants, providing a basis for studying dynamic pricing and coordination strategies. (3) The importance of traceability goodwill is analyzed. The study emphasizes the importance of traceability goodwill in the supply chain. Tracking and recording information about the origin and quality of a product and traceability goodwill can increase consumer trust in the product, thereby increasing sales and market share. The impact of traceability goodwill is also analyzed in depth and included as one of the considerations for dynamic pricing and coordination strategies. (4) Numerical simulation experiments demonstrate the effectiveness and practicality of the strategy. The effectiveness and practicality of the proposed dynamic pricing and coordination strategy are verified through numerical simulation experiments. The experimental results show that the strategy can improve the supply chain's overall efficiency, balance each participant's interests, and enhance traceability goodwill's influence on market share.

This paper is structured as follows: Section 1 describes the problem background; Section 2 model building; Section 3 solves the optimal fresh produce dynamic pricing, freshness input, and traceability goodwill input strategies for operating firms; Section 4 discusses equilibrium comparison under two scenarios; Section 5 mutual cost-sharing fresh produce supply chain coordination; Section 6 numerical analysis; and Section 7 conclusion.

2. Model building

2.1. Model notation

The symbols used are shown in Table 1.

2.2. Model description

This paper considers a secondary supply chain of fresh produce consisting of a supplier s and a retailer r. The fresh produce supplier decides on freshness input $u_s(t)$ and wholesale price $w(t)$, the fresh produce retailer decides on retail price $p(t)$ and traceability goodwill input $u_o(t)$, and $g(t)$ denotes freshness of fresh produce. Fresh food supply chain modeling under traceability goodwill, as shown in Figure 1. Simple differential game flowchart, as shown in Figure 2. Suppliers follow the principle of sustainable development to provide retailers with uncontaminated fresh produce that meets food safety standards and is accompanied by traceability following specific growing methods; in the growing process, traceability is simulated, and clean energy or low-energy growing processes are used to enhance the growth of production in a suitable growing environment. Suppliers (followers) and retailers (leaders) use traceability goodwill to build a traceability system during the production, cultivation, harvesting, transportation, and sales of fresh produce to ensure the safety and reliability of fresh produce and enhance consumer confidence.

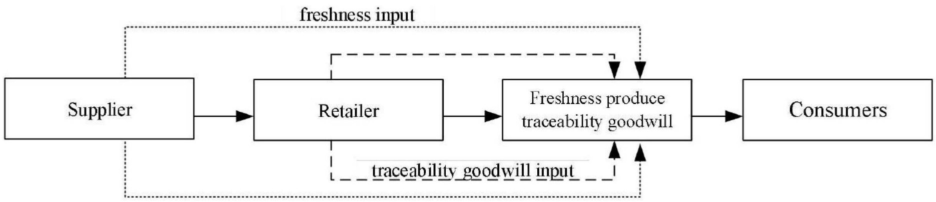

FIGURE 1. Fresh food supply chain modeling under traceability goodwill.

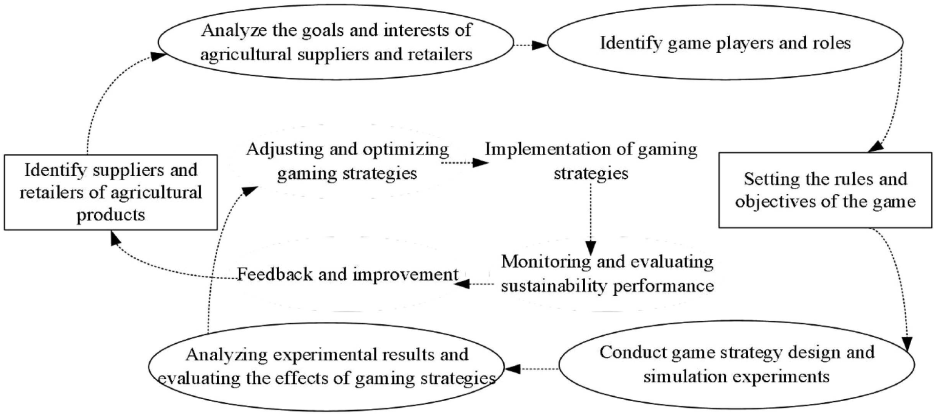

FIGURE 2. Simple differential game flowchart.

Drawing on the ideas of scholars such as El Ouardighi and Pasin [11] and Liu *et al.* [9], this paper proposes that the freshness of fresh produce is a dynamic change function of preservation effort inputs. Since fresh produce suppliers' freshness preservation efforts can increase (relative to the natural decay of fresh produce that is delayed by not taking preservation) the freshness of fresh produce. Therefore, the freshness model is used to portray the dynamic change of freshness of fresh produce with the level of a preservation effort. Thus, the freshness model of fresh produce is:

$$\dot{g}(t) = \lambda u_s(t) - \delta g(t), g(0) = g_0. \tag{1}$$

As a real-time information monitoring system, the traceability system of fresh agricultural products meets the strong demand of consumers for the quality and safety of fresh agricultural products, which is not only beneficial to the expansion of demand for fresh agricultural products but also plays an essential role in promoting the brand image of operating companies, *i.e.*, "traceability goodwill of fresh agricultural products." Traceability goodwill is closely related to the traceability inputs (efforts made by operators to trace the traceability information of fresh agricultural products using traceability goodwill) of operators, such as the use of QR code labels, the establishment of query systems or APPs, etc. Therefore, the traceability goodwill of fresh agricultural products is represented by the Nerlove–Arrow model of Nerlove and Arrow [12] as a dynamic change process:

$$\dot{q}(t) = \beta u_o(t) - \alpha q(t), q(0) = q_0. \tag{2}$$

Traceability goodwill can improve the efficiency of fresh produce traceability systems to meet consumers' needs in a safer, more trustworthy, and time-efficient manner.

Freshness preservation inputs improve the freshness of produce, but they also bring additional costs, such as product management costs [13]. Specifically, the higher the freshness of the produce, the more freshness inputs are required, making it necessary for suppliers to bear high unit production costs. Thus, the variable unit costs for fresh produce suppliers and retailers are:

$$c_s(t) = c_{s0} + c_1 g(t), c_r(t) = c_0 - c_{s0} = c_{r0}. \tag{3}$$

The variable unit cost of the retailer is assumed to be the initial unit cost for calculation convenience, while $c_0 = c_{s0} + c_{r0}$ is the initial unit cost of fresh produce. c_1 is the marginal cost of fresh produce supplier's freshness preservation inputs to improve freshness and is referred to as the operational inefficiency factor of the supplier. Furthermore, Zhang *et al.* [14] used the operational inefficiency coefficient to portray the variable unit cost of green products produced by green manufacturers. However, the existing literature rarely incorporates operational inefficiency to study the variable unit cost of fresh produce. To obtain a positive equilibrium result, c_1 in equation (3) needs to satisfy: $0 < c_1 < \frac{\phi}{b}$; a larger c_1 indicates a more significant inefficiency of the fresh produce preservation process. This linear cost function has been widely used by scholars in industries such as green products and pharmaceuticals, *e.g.*, Zhang *et al.* [15] and Papalexi *et al.* [16]. However, the existing literature is rarely used for the cost of agricultural products. Operational inefficiencies do not exist when the unit cost of raw produce is a fixed constant when $c_1 = 0$.

In the operations management domain, input costs are assumed to be quadratic functions [9, 10, 17], with fresh produce supplier s performing production as well as preservation effort inputs $u_s(t)$ and fresh produce retailer r performing traceability goodwill inputs as $u_o(t)$. The fresh produce supplier's preservation costs and the retailer's traceability goodwill cost functions are:

$$C_s(u_s(t)) = \frac{\eta_1}{2} u_s^2(t), C_o(u_o(t)) = \frac{\eta_2}{2} u_o^2(t) \tag{4}$$

where $C_s(u_s(t)), C_o(u_o(t))$ is the fresh produce supplier's preservation cost and retailer's traceability goodwill cost, respectively, and equation (4) is a quadratic function of the fresh produce supplier's preservation cost and preservation input and retailer's traceability goodwill cost and traceability goodwill input.

Market demand is influenced by the combination of price, freshness, and traceability goodwill of fresh produce, the classical demand function $a - bp$, and the demand functions proposed by scholars such as El Ouardighi and Pasin [11] and Liu *et al.* [9].

$$D(p(t), g(t), q(t)) = a - bp(t) + \theta g(t) + \phi q(t). \tag{5}$$

Considering the non-negativity between the demand and price of fresh produce, *i.e.*, $Q(p(t), G(t), Q(t)) \geq 0$ and $p(t) \geq 0$, the following assumptions are made: $0 \leq p(t) \leq \frac{a}{b}$. Fresh produce operators operate in practice, and most fresh produce is affected by the environment in which they operate. Therefore, suppliers will decide whether to use preservation to reduce the decay rate of fresh produce. However, in general, suppliers and retailers are limited in the number of freshness inputs and traceability goodwill inputs they can adopt due to resource (financial) constraints of the operating companies. Assuming that the maximum freshness input is $u_T(t)$ and the traceability goodwill input is $u_I(t)$, there are the following constraints: $0 \leq u_s(t) \leq u_T(t), 0 \leq u_o(t) \leq u_I(t)$. The relationship between fresh produce's decay rate or coefficient and the maximum freshness input can be expressed as $\delta(u(t)) = \delta_0 - du_T(t)$. The relationship between the decay coefficient of the brand image of traceable fresh produce caused by consumer forgetfulness and other reasons and the maximum traceability goodwill input can be expressed as $\alpha(u_I(t)) = \alpha_0 - ku_I(t)$. In particular, even if the fresh produce suppliers make the maximum freshness input and traceability goodwill input, the quality of perishable fresh produce and traceability goodwill will still decay continuously with time delay. The quality and traceability goodwill of perishable fresh produce decay continuously with time delay, and thus have $d \leq \frac{\delta_0}{u_T(t)}, k \leq \frac{\alpha_0}{u_I(t)}$.

Within the fresh produce life cycle $[0, T]$, the target panoply of fresh produce suppliers and retailers are.

$$J_S = \int_0^T e^{-\rho t}\Big((w(t) - c_0 - c_1 g(t)) D(p(t), g(t), q(t)) - \frac{\eta_1}{2} u_s^2(t)\Big)\,\mathrm{d}t$$

$$J_R = \int_0^T e^{-\rho t}\Big((p(t) - w(t)) D(p(t), g(t), q(t)) - \frac{\eta_2}{2} u_o^2(t)\Big)\,\mathrm{d}t$$

where ρ is the discount rate.

3. Model construction and solution

This section transforms the fresh produce supply chain system strategy under the consideration of traceability goodwill into an optimal control problem. With the help of game theory and Hamilton's equation, the problem is determined from two dimensions of centralized and decentralized decision-making.

3.1. Centralized decision making

Centralized decision-making means that the decision is made in the interest of the fresh produce supply chain as a whole, and the decision is made to maximize the benefit of the whole. However, fresh produce suppliers and retailers can be considered a whole in this decision-making model, so much so that there is no transactional relationship between them. Therefore, the total profit of the fresh produce supply chain in the fresh produce life cycle $[0, T]$ is discounted by the discount factor ρ. The profit objective function of the secondary traceable supply chain of fresh produce is:

$$\max_{p(\cdot),u_s(\cdot),u_o(\cdot)} \int_0^T e^{-\rho t}\Big((p(t)-c_0-c_1 g(t))D(p(t),g(t),q(t)) - \frac{\eta_1}{2}u_s^2(t) - \frac{\eta_2}{2}u_o^2(t)\Big)\,\mathrm{d}t$$
$$\text{s.t.} \begin{cases} \dot{g}(t) = \lambda u_s(t) - \delta g(t), g(0)=g_0, g(T)=0 \\ \dot{q}(t) = \beta u_o(t) - \alpha q(t), q(0) = q_0. \end{cases} \tag{6}$$

There are the following conclusions regarding optimal pricing, freshness input, and traceability goodwill input strategies.

Proposition 1. *Under centralized decision-making, the optimal pricing, freshness inputs, and traceability goodwill inputs of suppliers and retailers in the fresh produce supply chain are:*

$$p^C(t) = \frac{\eta_2\alpha(\rho+\alpha)(\eta_1\delta(\rho+\delta)(a+c_0 b)+\lambda(\theta-bc_1)(ac_1-c_0\theta)) - \eta_1 c_0\delta\phi^2\beta^2(\rho+\delta)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2} \tag{7}$$

$$u_s^c(t) = \frac{\eta_2\delta\alpha\lambda(\rho+\alpha)(a-c_0 b)(\theta-bc_1)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2} \tag{8}$$

$$u_o^c(t) = \frac{\eta_1\alpha\beta\delta\phi(\rho+\delta)(a-bc_0)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}. \tag{9}$$

Proof. To obtain the feedback equilibrium strategy of the fresh produce supply chain members, the optimization problem of (6) is solved using optimal control theory to construct the following Hamiltonian function [18–21]:

$$H^C(g,q,p,\lambda,t) = (p(t)-c_0-c_1 g(t))(a-bp(t)+\theta g(t)+\phi q(t)) - \frac{\eta_1 u_s^2(t)}{2}$$
$$- \frac{\eta_2 u_o^2(t)}{2} + \lambda_1(t)(\lambda u_s(t)-\delta g(t)) + \lambda_2(t)(\beta u_o(t)-\alpha q(t)) \tag{10}$$

where $\lambda_1(t), \lambda_2(t)$ is the shadow price (covariate) concerning the state variables, the freshness of fresh produce, and traceability goodwill, the economics of $\lambda_1(t), \lambda_2(t)$ is interpreted as the incremental increase in future revenue per unit of fresh produce sold. A favorable shadow price indicates that the monopolist can benefit from increased future sales by reducing prices, increasing freshness inputs, or increasing traceability goodwill inputs. □

The conditions for maximizing (10)-style:

$$\frac{\partial H^C}{\partial p(t)} = a - bp(t) + \theta g(t) + \phi q(t) - (p(t) - c_0 - c_1 g(t))b = 0 \tag{11}$$

$$\frac{\partial H^C}{\partial u_s(t)} = \alpha\lambda_1(t) - \eta_1 u_s(t) = 0 \tag{12}$$

$$\frac{\partial H^C}{\partial u_o(t)} = \beta\lambda_2(t) - \eta_2 u_o(t) = 0. \tag{13}$$

Since b, η_1, η_2 are positive parameters, the following inequality always holds for the optimality condition of the optimal control problem (10), *e.g.*:

$$\frac{\partial^2 H^C}{\partial p(t)^2} = -2b < 0 \cdot \frac{\partial^2 H^C}{\partial u_s(t)^2} = -\eta_1 < 0 \cdot \frac{\partial^2 H^C}{\partial u_o(t)^2} = -\eta_2 < 0$$

$$\frac{\partial^2 H^C}{\partial p(t)^2}\frac{\partial^2 H^C}{\partial u_s(t)^2} - \left(\frac{\partial^2 H^C}{\partial p(t)\partial u_s(t)}\right)^2 = 2\eta_1 b > 0 \cdot \frac{\partial^2 H^C}{\partial p(t)^2}\frac{\partial^2 H^C}{\partial u_o(t)^2} - \left(\frac{\partial^2 H^C}{\partial p(t)\partial u_o(t)}\right)^2 = 2\eta_2 b > 0$$

$$\frac{\partial^2 H^C}{\partial u_s(t)^2}\frac{\partial^2 H^C}{\partial u_o(t)^2} - \left(\frac{\partial^2 H^C}{\partial u_s(t)\partial u_o(t)}\right)^2 = \eta_1\eta_2 > 0.$$

We are differentiating $g(t)$ and $q(t)$ concerning H^C, respectively, yields:

$$\frac{\partial H^C}{\partial g(t)} = \theta(p(t) - c_0 - c_1 g(t)) - c_1(a - bp(t) + \theta g(t) + \phi q(t)) - \lambda_1(t)\delta \tag{14}$$

$$\frac{\partial H^C}{\partial q(t)} = \phi(p(t) - c_0 - c_1 g(t)) - \lambda_2(t)\alpha. \tag{15}$$

The shadow prices (covariates) $\lambda_1(t)$ and $\lambda_2(t)$ satisfy the following differential equations:

$$\dot\lambda_1(t) = \rho\lambda_1(t) - \frac{\partial H^C}{\partial g(t)} = (\rho + \delta)\lambda_1(t) - \theta(p(t) - c_0 - c_1 g(t)) + c_1(a - bp(t) + \theta g(t) + \phi q(t)) \tag{16}$$

$$\dot\lambda_2(t) = \rho\lambda_2(t) - \frac{\partial H^C}{\partial q(t)} = (\rho + \alpha)\lambda_2(t) - \phi(p(t) - c_0 - c_1 g(t)). \tag{17}$$

The cross-sectional conditions of equations (16) and (17) are $\lambda_1(T) = 0, \lambda_2(T) = 0$.

$\lambda_1(t)$ and $\lambda_2(t)$ are the shadow prices associated with $g(t)$ and $q(t)$ at moment t, respectively. Thus, the Hamiltonian function (10) can be interpreted as dynamic profitability consisting of three terms:

(1) Net profit: $\pi(t) = (p(t) - c_0 - c_1 g(t))(a - bp(t) + \theta g(t) + \phi q(t)) - C_s(u_s(t)) - C_o(u_o(t))$.
(2) Freshness value: $\lambda_1(t)\dot g(t) = \lambda_1(t)(\lambda u_s(t) - \delta g(t))$.
(3) Retroactive goodwill value: $\lambda_2(t)\dot q(t) = \lambda_2(t)(\beta u_o(t) - \alpha q(t))$.

In addition, equations (16) and (17) show that the marginal opportunity cost $(\rho+\delta)\lambda_1(t)$ of the freshness input is equal to the marginal profit of producing freshness $\frac{\partial\pi(t)}{\partial g(t)} = \theta(p(t) - c_0 - c_1 g(t)) - c_1(a - bp(t) + \theta g(t) + \phi q(t))$ and the capital gain $d\lambda_1(t) = \dot\lambda_1(t)\,\mathrm{d}t$, and; the marginal opportunity cost $(\rho + \alpha)\lambda_1(t)$ of the traceability goodwill input is equal to the marginal profit of produce freshness $\frac{\partial\pi(t)}{\partial q(t)} = \phi(p(t) - c_0 - c_1 g(t))$ and the capital gain $d\lambda_2(t) = \dot\lambda_2(t)\,\mathrm{d}t$.

In addition, equations (16) and (17) show that the marginal opportunity cost $(\rho + \delta)\lambda_1(t)$ of the freshness input is equal to the sum of the marginal profit $\frac{\partial\pi(t)}{\partial g(t)} = \theta(p(t) - c_0 - c_1 g(t)) - c_1(a - bp(t) + \theta g(t) + \phi q(t))$

of produce freshness and the capital gain $d\lambda_1(t) = \dot{\lambda}_1(t)\,\mathrm{d}t$; the marginal opportunity cost $(\rho+\alpha)\lambda_1(t)$ of the traceability goodwill input is equal to the sum of the marginal profit $\frac{\partial \pi(t)}{\partial q(t)} = \phi(p(t) - c_0 - c_1 g(t))$ of produce freshness and the capital gain $d\lambda_2(t) = \dot{\lambda}_2(t)\,\mathrm{d}t$.

From equations (12) and (13), we have:

$$\lambda_1(t) = \frac{\eta_1 u_s(t)}{\lambda}, \lambda_2(t) = \frac{\eta_2 u_o(t)}{\beta}\cdot \tag{18}$$

The pricing strategy of the decision maker of the fresh produce supply chain operator under the centralized scenario can be obtained from the maximization condition (11) equation:

$$p(g,q) = \frac{a + (c_1 g + c_0)b + \theta g + \phi q}{2b}\cdot \tag{19}$$

It is clear from equation (19) of the pricing strategy that the selling price of fresh produce increases with the increase of freshness or traceability goodwill, *i.e.*, A. Next, discussing the law of change of freshness input and traceability goodwill input, the derivatives of the maximization conditions (12) and (13) are found for time, and then the covariance equations (16) and (17) are brought into equation (19), which leads to:

$$\dot{u}_s(t) = (\rho+\delta)u_s(t) - \frac{\alpha\big(\big(c_1c_0 + c_1^2 g(t)\big)b^2 - ((c_0 + 2c_1 g)\theta + c_1(a+\phi q))b + (a + \theta g + \phi q)\theta\big)}{2\eta_1 b} \tag{20}$$

$$\dot{u}_o(t) = (\rho+\alpha)u_o(t) - \frac{\beta\big(q\phi^2 - (a + \theta g + (c_1 g + c_0)b)\phi\big)}{2\eta_2 b}\cdot \tag{21}$$

We can see $\frac{\partial \dot{u}_s(t)}{\partial g(t)} = -\frac{\lambda(\theta - bc_1)^2}{2\eta_1 b} < 0$ and $\frac{\partial \dot{u}_o(t)}{\partial q(t)} = -\frac{\beta\phi^2}{2\eta_2 b} < 0$ from equations (20) and (21).

This inequality shows that: as the freshness or traceability goodwill of agricultural products increases, the rate of freshness input and traceability goodwill input decreases, which means that when the freshness and traceability goodwill of agricultural products reach the optimal level, the freshness input and traceability goodwill input will remain constant, and the corresponding freshness and traceability goodwill are fixed, *i.e.*, there exists $g(t) = g^*, q(t) = q^*$, such that $\frac{\partial \dot{u}_s(t)}{\partial g(t)} = 0$, $\frac{\partial \dot{u}_o(t)}{\partial q(t)} = 0$. At this point, we will give the dynamic control system of decision-makers of fresh produce supply chain operating companies and calculate the equilibrium solution. The dynamic system of the decision maker of the fresh produce supply chain operation enterprise under the centralized scenario can be derived by associating the dynamic equations (1), (20) and (2), (21), which are:

$$\begin{cases} \dot{u}_s(t) = (\rho+\delta)u_s(t) - \frac{\lambda\left(\left(c_1c_0 + c_1^2 g(t)\right)b^2 - (c_0 + 2c_1 g)\theta b + (\theta - bc_1)(a+\phi q) + \theta^2 g\right)}{2\eta_1 b} \\ \dot{g}(t) = \lambda u_s(t) - \delta g(t) \end{cases} \tag{22}$$

$$\begin{cases} \dot{u}_o(t) = (\rho+\alpha)u_o(t) - \frac{\beta\left(q\phi^2 - (a+\theta g + (c_1 g + c_0)b)\phi\right)}{2\eta_2 b} \\ \dot{q}(t) = \beta u_o(t) - \alpha q(t) \end{cases} \tag{23}$$

when the dynamic system is equilibrated, *i.e.*, we have $\dot{u}_s(t) = \dot{g}(t) = 0, \dot{u}_o(t) = \dot{q}(t) = 0$, we can derive the equilibrium solution of the dynamic system (22), (23) equations, *i.e.*, $\{u_s^C, g^C\}, \{u_o^C, q^C\}$. Where:

$$u_s^C = \frac{\eta_2\delta\alpha\lambda(\rho+\alpha)(a - c_0 b)(\theta - bc_1)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta - bc_1)^2}$$

$$g^C = \frac{\eta_2\alpha\lambda^2(\rho+\alpha)(a - bc_0)(\theta - bc_1)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta - bc_1)^2}$$

$$u_o^C = \frac{\eta_1\alpha\beta\delta\phi(\rho+\delta)(a - bc_0)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta - bc_1)^2}$$

$$q^C = \frac{\eta_1\delta\phi\beta^2(\rho+\delta)(a-bc_0)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}$$

where it is assumed that $\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2>0$ and thus $(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)>\frac{\eta_2\alpha\lambda^2(\rho+\alpha)}{\eta_1\delta(\rho+\delta)}(\theta-bc_1)^2>0$, since $\frac{\eta_2\alpha\lambda^2(\rho+\alpha)}{\eta_1\delta(\rho+\delta)}>0$ and thus $(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-(\theta-bc_1)^2>0$ can be obtained, otherwise the equilibrium solution $\{u_s^C, g^C\}, \{u_o^C, q^C\}$ will lose its real economic meaning.

Bringing the equilibrium solutions g^C and q^C into equation (19) yields the equilibrium price, which is:

$$p^C = \frac{\eta_2\alpha(\rho+\alpha)(\eta_1\delta(\rho+\delta)(a+c_0b)+\lambda(\theta-bc_1)(ac_1-\theta c_0))-\eta_1 c_0\delta\phi^2\beta^2(\rho+\delta)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}. \tag{24}$$

Bringing the equilibrium solutions g^c, q^c, p^c, u_s^c, and u_o^c into equation (6) yields the total supply chain profit, which is:

$$J^C = \frac{\eta_1\eta_2(\alpha\delta(a-bc_0))^2\Big(\eta_1(\rho+\delta)^2\Big(\big(2\eta_2 b(\rho+\alpha)^2-\phi^2\beta^2\big)-\lambda^2(\theta-bc_1)^2\Big)\Big)}{2\Big(\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2\Big)^2}. \tag{25}$$

Bringing the equilibrium solutions u_s^c and u_o^c into equation (18) yields the shadow price, which is:

$$\begin{aligned}\lambda_1(t) &= \frac{\eta_1\eta_2\delta\alpha(\rho+\alpha)(a-bc_0)(\theta-bc_1)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}\\ \lambda_2(t) &= \frac{\eta_1\eta_2\lambda\delta\theta(\rho+\delta)(a-bc_0)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}.\end{aligned} \tag{26}$$

At this point, the stability of the equilibrium solution of the dynamic system equations (22) and (23) is proved. Let $H_s^C = \frac{\partial(\dot{u}_s(t),\dot{g}(t))}{\partial(u_s(t),g(t))}, H_o^C = \frac{\partial(\dot{u}_o(t),\dot{q}(t))}{\partial(u_o(t),q(t))}$ be the Jacobi matrices of equations (22) and (23), with $H_s^C = \begin{bmatrix}\rho+\delta-\frac{\lambda(\theta-bc_1)^2}{2\eta_1 b}\\ \lambda-\delta\end{bmatrix}, H_o^C = \begin{bmatrix}\rho+\alpha-\frac{\beta\phi^2}{2\eta_2 b}\\ \beta-\alpha\end{bmatrix}$.

Let E be the second-order unit matrix $r_{si}^c, i=1,2$ and $r_{oj}^C, j=1,2$ be the corresponding real eigenvalues of Jacobi matrices H_s^C and H_o^C. Calculating the eigenequations H_s^C and H_o^C corresponding to $\left|H_s^C-r_{si}^C E\right|=0$ and $\left|H_o^C-r_{oj}^C E\right|=0$, and the corresponding eigenvalues be can as: $r_{s1}^C = \frac{\eta_1 b\rho+\sqrt{\eta_1 b\left(\eta_1 b(\rho+2\delta)^2-2\lambda^2(\theta-bc_1)^2\right)}}{2\eta_1 b}$, $r_{s2}^c = \frac{\eta_1 b\rho-\sqrt{\eta_1 b\left(\eta_1 b(\rho+2\delta)^2-2\lambda^2(\theta-bc_1)^2\right)}}{2\eta_1 b}$, $r_{o1}^c = \frac{\eta_2 b\rho+\sqrt{\eta_2 b(\eta_2 b(\rho+2\alpha)^2-2\phi^2\beta^2)}}{2\eta_2 b}$, $r_{o2}^C = \frac{\eta_2 b\rho-\sqrt{\eta_2 b(\eta_2 b(\rho+2\alpha)^2-2\phi^2\beta^2)}}{2\eta_2 b}$.

Since $\eta_1 b(\rho+2\delta)^2-2\lambda^2(\theta-bc_1)^2>0$ and $\eta_2 b(\rho+2\alpha)^2-2\phi^2\beta^2>0$, we have: $r_{s1}^C>0, r_{s2}^C<0, r_{o1}^C>0, r_{o2}^C<0$.

However, the corresponding eigenvectors of $r_{s1}^C, r_{s2}^C, r_{o1}^C, r_{o2}^C$ are:

$$H_{s1}^C = \left[\frac{\lambda(\theta-bc_1)^2}{\eta_1 b(\rho+2\delta)-\sqrt{\eta_1 b\Big(\eta_1 b(\rho+2\delta)^2-2\lambda^2(\theta-bc_1)^2\Big)}}\right]$$

$$H_{s2}^C = \left[\frac{\lambda(\theta-bc_1)^2}{\eta_1 b(\rho+2\delta)+\sqrt{\eta_1 b\Big(\eta_1 b(\rho+2\delta)^2-2\lambda^2(\theta-bc_1)^2\Big)}}\right]$$

$$H_{o1}^C = \left[\frac{1}{\eta_2 b(\rho+2\alpha)-\sqrt{\eta_2 b(\eta_2 b(\rho+2\alpha)^2-2\phi^2\beta^2)}}\right]$$

$$H_{o2}^C = \left[\frac{\beta\phi^2}{\eta_2 b(\rho+2\alpha) + \sqrt{\eta_2 b(\eta_2 b(\rho+2\alpha)^2 - 2\phi^2\beta^2)}} \right].$$

Referring to Zhang and Liu [22] and Goli *et al.* [23–28],the above analysis is summarized as the equilibrium solution A, B of the dynamic control system (22) and (23) equations of the decision maker of the fresh produce supply chain operator is saddle point equilibrium in the centralized scenario.

Optimal pricing, freshness input, and traceability goodwill input are given in equations (7), (8), and (9), respectively.

Furthermore, the optimal solution of Proposition 1 has the following properties.

Corollary 1. *The optimal pricing strategy for fresh produce depends on the supply chain system parameters: when A, the optimal selling price keeps increasing with time, i.e., the penetration pricing strategy is chosen; when B, the optimal selling price keeps decreasing with time, i.e., the skimming pricing strategy is chosen; when C, the optimal selling price is a constant over the life cycle C of fresh produce. However, no matter how the parameters of the fresh produce supply chain system change, the freshness input continuously increases with time. The shadow price of fresh produce freshness and traceability goodwill is always favorable.*

Proof. According to equation (19), it can be derived that:

$$\dot{p} = \frac{\eta_2\alpha\lambda(\rho+\alpha)(\theta-bc_1)(ac_1-\theta c_0) - \eta_1 c_0\delta\phi^2\beta^2(\rho+\delta)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}.$$

Because $\phi > 0$ can be obtained $\eta_2\alpha\lambda(\rho+\alpha)(\theta-bc_1)(ac_1-\theta c_0) - \eta_1 c_0\delta\phi^2\beta^2(\rho+\delta) > 0$, that is $(\theta - bc_1)(ac_1-\theta c_0) > \frac{\eta_1 c_0\delta(\rho+\delta)}{\eta_2\alpha\lambda(\rho+\alpha)}\phi^2\beta^2$, because $\frac{\eta_1 c_0\delta(\rho+\delta)}{\eta_2\alpha\lambda(\rho+\alpha)} > 0$ can be obtained: $(\theta-bc_1)(ac_1-\theta c_0) > \phi^2\beta^2$, so when $(ac_1 - c_0\phi)(\phi - c_1\kappa) > \beta^2\varphi^2, \dot{p} > 0$; when $(\theta-bc_1)(ac_1-\theta c_0) < \phi^2\beta^2, \dot{p} < 0$.

However, it can be similarly verified within the fresh produce life cycle $[0,T], \dot{g} > 0, \dot{q} > 0$. □

Corollary 1 suggests that the choice of fresh produce pricing strategy depends on the supply chain system parameters: price sensitivity coefficient, the influence coefficient of traceability goodwill input on traceability goodwill, traceability goodwill contribution coefficient, freshness contribution coefficient, and operational inefficiency, *i.e.*, $b, \beta, \phi, \theta, c_1$. When the supply chain system parameters satisfy $(\theta-bc_1)(ac_1-\theta c_0) > \phi^2\beta^2$, the selling price of fresh agricultural products keeps increasing with time, *i.e.*, the operating companies choose the penetration pricing strategy. When θ is more extensive, c_1 is more minor, ϕ is smaller, and β is smaller, the above conditions are easily satisfied, indicating that when the freshness contribution is high, fresh produce operators are willing to adopt a low initial price and then continuously increase the price, which enables operators to occupy a larger market share with a low price at the beginning of the fresh product life cycle $[0,T]$ and maintain a high marginal profit with a high price at the end of the sale. Conversely, when the supply chain system parameters satisfy $(\theta-bc_1)(ac_1-\theta c_0) < \phi^2\beta^2$, the selling price of fresh produce decreases over time, *i.e.*, the operating firm chooses a skimming pricing strategy. This condition is more likely to hold for a minor θ and a larger c_1, indicating that lower freshness contribution and higher operational inefficiencies spur the operator to be the first to earn more current profit at a high price. Then the operator reduces the price to sustain higher market demand.

According to equations (8) and (9), both optimal freshness input and traceability goodwill input are incremental within the fresh produce life cycle $[0,T]$. It can be seen from equation (8) that $\frac{\partial u_s^C(0)}{\partial T} > 0$ can be deduced from initial freshness input $u_s^c(0) = \frac{\eta_2\delta\alpha\lambda(\rho+\alpha)(a-c_0 b)(\theta-bc_1)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}$, indicating that the operating firm is more willing to set a higher freshness input within the fresh produce life cycle $[0,T]$. A more intuitive explanation is that, since the freshness contribution rate θ of produce is relatively higher than that of companies' operational inefficiency c_1, *i.e.*, $\theta > bc_1$, a high initial freshness input will lead to the high freshness of produce to the extent that it attracts more consumer demand, making the benefits from freshness contribution rate dominate over the drawbacks from operational inefficiency during the fresh produce life cycle $[0,T]$. It can be seen from equation (9) that from the initial traceability goodwill input $u_o^C(0) = \frac{\eta_1\alpha\beta\delta\phi(\rho+\delta)(a-bc_0)}{\eta_1\delta(\rho+\delta)(2\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)^2}$, it can be deduced that $\frac{\partial u_o^C(0)}{\partial T} > 0$, indicating that operating companies are more willing to set a higher traceability goodwill input during the fresh produce life cycle $[0,T]$. A high traceability goodwill input will lead to a high traceability goodwill, making the traceability goodwill enhancement help attract consumer demand during the fresh produce life cycle $[0,T]$.

Notice from equation (26) that the shadow price is favorable throughout the life cycle $[0,T]$ of the fresh produce, indicating that the operating firm always benefits from selling one unit of fresh produce outside of the concentration scenario.

3.2. Decentralized decision making

This section investigates the equilibrium sales pricing and freshness preservation input strategies under a decentralized scenario, where the profit objective functions of fresh produce supply chain suppliers and retailers over the life cycle A of fresh produce.

$$\begin{aligned}
&\max_{w(\cdot),u_s(\cdot)} \int_0^T e^{-\rho t}\Big((w(t)-c_{s0}-c_1 g(t))D(p(t),g(t),q(t)) - \frac{\eta_1}{2}u_s^2(t)\Big)\,\mathrm{d}t\\
&\max_{p(\cdot),u_o(\cdot)} \int_0^T e^{-\rho t}\Big((p(t)-w(t)-(c_0-c_{s0}))D(p(t),g(t),q(t)) - \frac{\eta_2}{2}u_o^2(t)\Big)\,\mathrm{d}t\\
&\text{s.t.} \begin{cases} \dot{g}(t) = \lambda u_s(t) - \delta g(t), g(0) = g_0, g(T) = 0\\ \dot{q}(t) = \beta I_o(t) - \alpha q(t), q(0) = q_0. \end{cases}
\end{aligned} \tag{27}$$

This model is a Stackelberg differential game model, in which the fresh produce supplier is the leader of the supply chain, and the retailer is the follower. The game sequence between the supplier and the retailer is as follows: firstly, the supplier decides the fresh produce preservation effort level input and wholesale price, and then, the retailer decides the fresh produce sales price on this basis.

This model is also solved by the inverse induction method in this section. The decision problem of the retailer is solved first, and then the decision problem of the supplier is solved. The equilibrium strategies of suppliers and retailers of fresh produce are concluded as follows.

Proposition 2. *The equilibrium selling prices, wholesale prices, freshness inputs, and traceability goodwill inputs of suppliers and retailers in the fresh produce supply chain under the dispersion scenario are:*

$$p^D(t) = \frac{\eta_1\delta(\rho+\delta)\big(\eta_2\alpha(\rho+\alpha)(3a+bc_0) - c_0\phi^2\beta^2\big) + \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)} \tag{28}$$

$$\begin{aligned}
w^D(t) = &\frac{\eta_2\alpha(\rho+\alpha)\big(bc_1c_{r0}\lambda^2(\theta-bc_1) + 2c_{s0}\theta^2\lambda^2 + b(c_0-2c_{s0})\big(2\eta_1\delta(\rho+\delta)+c_1\theta\lambda^2\big)\big)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}\\
&- \frac{2\eta_1 a\delta(\rho+\delta) + ac_1\lambda^2(2\theta-bc_1) - \eta_1\delta c_{s0}\phi^2\beta^2}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}.
\end{aligned} \tag{29}$$

$$u_s^D(t) = \frac{\eta_2\delta\lambda\alpha(\rho+\alpha)(a-bc_0)(2\theta-bc_1)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)} \tag{30}$$

$$u_o^D(t) = \frac{\eta_1\alpha\beta\delta\phi(\rho+\delta)(a-bc_0)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}. \tag{31}$$

Proof. Because the supplier is the master of the fresh produce supply chain, he must guess the market selling price of the product in advance before fixing the preservation input and wholesale price. Therefore, the selling price of agricultural products should be solved first. However, first, solve the retailer's sales pricing decision problem:

$$\max_{p(\cdot),u_o(\cdot)} \Pi_r = \int_0^T e^{-\rho t}\Big((p(t)-w(t)-(c_0-c_{s0}))D(p(t),g(t),q(t)) - \frac{\eta_2}{2}u_o^2(t)\Big)\,\mathrm{d}t$$
$$\text{s.t. } \dot{q}(t) = \beta I_o(t) - \alpha q(t), q(0)=q_0.$$

□

Hamiltonian functions for retailers:

$$H_r^D(g,q,p,\lambda_r,t) = (p(t)-w(t)-(c_0-c_{s0}))(a-bp(t)+\theta g(t)+\phi q(t)) - \frac{\eta_2}{2}u_o^2(t) + \lambda_s(t)(\lambda u_s(t)-\delta g(t)) + \lambda_r(t)(\beta u_o(t)-\alpha q(t))$$

where $\lambda_r(t)$ is the shadow price (covariate) and satisfies the following differential equation:

$$\dot{\lambda}_r(t) = \rho\lambda_r(t) - \frac{\partial H_r^D}{\partial q(t)} = (\rho+\alpha)\lambda_r(t) - \phi(p(t)-w(t)-(c_0-c_{s0})). \tag{32}$$

Its maximization conditions.

$$\frac{\partial H_r^D}{\partial p(t)} = a - bp(t) + \theta g(t) + \phi q(t) - (p(t)-w(t)-(c_0-c_{s0}))b = 0 \tag{33}$$

$$\frac{\partial H_r^D}{\partial u_o(t)} = \beta\lambda_r(t) - \eta_2 u_o(t) = 0. \tag{34}$$

According to equation (33), we can solve for:

$$p(g,q,w) = \frac{a+(w-(c_0-c_{s0}))b+\theta g+\phi q}{2b}. \tag{35}$$

Equation (35) yields $\frac{\partial p(g,q,w)}{\partial g} = \frac{\theta}{2b} > 0$, $\frac{\partial p(g,q,w)}{\partial q} = \frac{\phi}{2b} > 0$, $\frac{\partial p(g,q,w)}{\partial w} = \frac{1}{2} > 0$. The above three inequalities show that the selling price of fresh produce increases with the product's freshness or traceability goodwill or wholesale price. Next, the optimization problem of the supplier is discussed, giving the objective function of the fresh produce supplier, while the supplier's objective is to find the optimal freshness input and wholesale price to maximize its profit over the life cycle $[0,T]$ of the fresh produce.

Therefore, based on the optimal response function of the fresh produce retailer, the optimization problem of the supplier, *i.e.*:

$$\max_{w(\cdot),u_s(\cdot)} \Pi_s = \int_0^T e^{-\rho t}\Big((w(t)-c_{s0}-c_1 g(t))D(p(t),g(t),q(t)) - \frac{\eta_1}{2}u_s^2(t)\Big)\,\mathrm{d}t$$
$$\text{s.t. } \dot{g}(t) = \lambda u_s(t) - \delta g(t), g(0)=g_0, g(T)=0.$$

Hamiltonian functions for suppliers:

$$H_s^D(g,q,p,\lambda_s,t) = (w(t)-c_{s0}-c_1 g(t))(a-bp(t)+\theta g(t)+\phi q(t)) - \frac{\eta_1 u_s^2(t)}{2} + \lambda_s(t)(\lambda u_s(t)-\delta g(t)) + \lambda_s(t)(\beta u_o(t)-\alpha q(t)).$$

$\lambda_s(t)$ is the shadow price (covariate) and satisfies the following differential equation.

$$\dot{\lambda}_s(t) = \rho\lambda_s(t) - \frac{\partial H_s^D}{\partial g(t)} = (\rho+\delta)\lambda_s(t) - \theta(p(t)-c_{s0}-c_1 g(t)) + c_1(a-bp(t)+\theta g(t)+\phi q(t)). \tag{36}$$

Its maximization conditions:

$$\frac{\partial H_s^D}{\partial w(t)} = \frac{a + \theta g(t) + \phi q(t) + (c_1 g(t) + 2c_{s0} - c_0 - 2w(t))b}{2} = 0 \tag{37}$$

$$\frac{\partial H_s^D}{\partial u_s(t)} = \lambda\lambda_s(t) - \eta_1 u_s(t) = 0. \tag{38}$$

From equation (37), we can obtain the wholesale price of fresh produce at birth as:

$$w(g,q) = \frac{a + \theta g(t) + \phi q(t) + (c_1 g(t) + 2c_{s0} - c_0)b}{2b}. \tag{39}$$

It can be seen from equation (39) that $\frac{\partial w(g,q)}{\partial g} = \frac{\theta + bc_1}{2b} > 0, \frac{\partial w(g,q)}{\partial q} = \frac{\phi}{2b} > 0$. The above two inequalities show that the wholesale price of fresh produce increases with freshness or traceability goodwill. This is because the higher freshness of products implies that suppliers have more freshness inputs or retailers have more traceability goodwill inputs. Therefore, suppliers should charge higher wholesale prices for fresh produce to maximize profits. Next, the variation law of freshness input and traceability goodwill input is discussed, and the time derivatives are found for the maximization conditions (34) and (38). Then the covariance equations (32) and (36) are brought into (35) and (39), which leads to:

$$\dot{u}_s(t) = (\rho + \delta)u_s(t) - \frac{\lambda(2\theta - bc_1)(a + \theta g + \phi q - (c_0 + c_1 g)b)}{4\eta_1 b} \tag{40}$$

$$\dot{u}_o(t) = (\rho + \theta)u_o(t) - \frac{\beta((a + \theta g + \phi q - (c_0 + c_1 g)b)\phi)}{4\eta_2 b}. \tag{41}$$

From equations (40) and (41), it can be seen that $\frac{\partial \dot{u}_s(t)}{\partial g(t)} = -\frac{\eta_1(\theta - bc_1)(2\theta - bc_1)}{4b\lambda} < 0, \frac{\partial \dot{u}_o(t)}{\partial q(t)} = -\frac{\beta\phi}{4\eta_2 b} < 0$, this inequality shows that the freshness input and traceability goodwill input rates are decreasing as the freshness and traceability goodwill of the produce increases in the decentralized scenario. The dynamic system of fresh produce suppliers under the decentralized scenario can be derived by associating the dynamic equations (1), (40) and (2), (41), which are:

$$\begin{cases} \dot{u}_s(t) = (\rho + \delta)u_s(t) - \frac{\lambda(2\theta - bc_1)(a + \theta g + \phi q - (c_0 + c_1 g)b)}{4\eta_1 b} \\ \dot{g}(t) = \lambda u_s(t) - \delta g(t) \end{cases} \tag{42}$$

$$\begin{cases} \dot{u}_o(t) = (\rho + \theta)u_o(t) - \frac{\beta((a + \theta g + \phi q - (c_0 + c_1 g)b)\phi)}{4\eta_2 b} \\ \dot{q}(t) = \beta u_o(t) - \alpha q(t). \end{cases} \tag{43}$$

When the dynamic system is equilibrated, *i.e.*, we have $\dot{u}_s(t) = \dot{g}(t) = 0, \dot{u}_o(t) = \dot{q}(t) = 0$, and can derive the equilibrium solution of the dynamic system (42), (43) equations, *i.e.*, $\{u_s^D, g^D\}, \{u_o^D, q^D\}$. Where:

$$u_s^D = \frac{\eta_2\delta\lambda\alpha(\rho + \alpha)(a - bc_0)(2\theta - bc_1)}{\eta_1\delta(\rho + \delta)(4\eta_2 b\alpha(\rho + \alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho + \alpha)(\theta - bc_1)(2\theta - bc_1)}$$

$$g^D = \frac{\eta_2\alpha\lambda^2(\rho + \alpha)(a - bc_0)(2\theta - bc_1)}{\eta_1\delta(\rho + \delta)(4\eta_2 b\alpha(\rho + \alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho + \alpha)(\theta - bc_1)(2\theta - bc_1)}$$

$$u_o^D = \frac{\eta_1\alpha\beta\delta\phi(\rho + \delta)(a - bc_0)}{\eta_1\delta(\rho + \delta)(4\eta_2 b\alpha(\rho + \alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho + \alpha)(\theta - bc_1)(2\theta - bc_1)}$$

$$q^D = \frac{\eta_1\delta\phi\beta^2(\rho + \delta)(a - bc_0)}{\eta_1\delta(\rho + \delta)(4\eta_2 b\alpha(\rho + \alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho + \alpha)(\theta - bc_1)(2\theta - bc_1)}.$$

It is assumed that $\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)>0$ and thus $(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)>\frac{\eta_2\alpha\lambda^2(\rho+\alpha)}{\eta_1\delta(\rho+\delta)}(\theta-bc_1)(2\theta-bc_1)$, since $\frac{\eta_2\alpha\lambda^2(\rho+\alpha)}{\eta_1\delta(\rho+\delta)}>0$ and thus $(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-(\theta-bc_1)(2\theta-bc_1)>0$ can be obtained. Otherwise, the equilibrium solution $\{u_s^D, g^D\}, \{u_o^D, q^D\}$ will lose its real economic meaning.

Bringing the equilibrium solutions g^D, q^D into equation (19) yields the equilibrium selling price and the wholesale price, which are:

$$p^D = \frac{\eta_1\delta(\rho+\delta)\big(\eta_2\alpha(\rho+\alpha)(3a+bc_0)-c_0\phi^2\beta^2\big)+\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)} \tag{44}$$

$$\begin{aligned} w^D &= \frac{\eta_2\alpha(\rho+\alpha)\big(bc_1c_{r0}\lambda^2(\theta-bc_1)+2c_{s0}\theta^2\lambda^2+b(c_0-2c_{s0})\big(2\eta_1\delta(\rho+\delta)+c_1\theta\lambda^2\big)\big)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)} \\ &\quad - \frac{2\eta_1 a\delta(\rho+\delta)+ac_1\lambda^2(2\theta-bc_1)-\eta_1\delta c_{s0}\phi^2\beta^2}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}. \end{aligned} \tag{45}$$

Bringing the equilibrium solution $g^D, q^D, p^D, w^D, u_s^D, u_o^D$ into equation (27) yields the total fresh produce supplier J_S^D and retailer J_R^D and the total supply chain profit J^D, which is:

$$J_S^D = \frac{\eta_1(\eta_2\delta\alpha(\rho+\alpha)(a-bc_0))^2\Big(4\eta_1(\rho+\delta)^2-\lambda^2(\theta-bc_1)^2\Big)}{\big(\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)\big)^2} \tag{46}$$

$$J_R^D = \frac{\eta_2(\eta_1\delta\alpha(\rho+\delta)(a-bc_0))^2\big(\eta_2 b(\rho+\alpha)^2-\phi^2\beta^2\big)}{\big(\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)\big)^2} \tag{47}$$

$$J^D = \frac{\eta_1\eta_2(\alpha\delta(a-bc_0))^2\Big(\eta_1(\rho+\delta)^2\big(6\eta_2 b(\rho+\varepsilon)^2-\phi^2\beta^2\big)-\eta_2(\lambda(\rho+\alpha)(2\theta-bc_1))^2\Big)}{\big(\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)\big)^2}. \tag{48}$$

Bringing the equilibrium solutions u_s^D, u_o^D into equations (34) and (38) yields the shadow price, which is:

$$\begin{aligned} \lambda_r(t) &= \frac{\eta_1\eta_2\alpha\delta\phi(\rho+\delta)(a-bc_0)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)} \\ \lambda_s(t) &= \frac{\eta_1\eta_2\delta\alpha(\rho+\alpha)(a-bc_0)(2\theta-bc_1)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha)-\phi^2\beta^2)-\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}. \end{aligned} \tag{49}$$

At this point, the stability of the equilibrium solution of the dynamic system equations (42) and (43) is proved. Let $H_s^D = \frac{\partial(\dot{u}_s(t),\dot{g}(t))}{\partial(u_s(t),g(t))}, H_o^D = \frac{\partial(\dot{u}_o(t),\dot{q}(t))}{\partial(u_o(t),q(t))}$ be the Jacobi matrix of equations (42) and (43) with $H_s^D = \begin{bmatrix} \rho+\delta-\frac{\lambda(\theta-bc_1)(2\theta-bc_1)}{2\eta_1 b} \\ \lambda \end{bmatrix}, H_o^C = \begin{bmatrix} \rho+\alpha-\frac{\beta\phi(\theta-bc_1)}{4\eta_2\kappa} \\ \beta-\alpha \end{bmatrix}$.

Let E be a unit matrix of order 2, and $r_{si}^D, i=1,2$ and $r_{oj}^D, j=1,2$ be the real eigenvalues corresponding to the Jacobi matrices ${H_s}_s^D$ and ${H_o}^D$. Calculating the eigenequations ${H_s}^D$ and H_o^D corresponding to $\left|H_s^D - r_{si}^D E\right| = 0$ and $\left|H_o^D - r_{oj}^D E\right| = 0$, the corresponding eigenvalues can be derived as:

$$\begin{aligned} r_{s1}^D &= \frac{\eta_1 b\rho+\sqrt{\eta_1 b(\eta_1 b(\rho+2\delta)^2-\lambda^2(\theta-bc_1)(2\theta-bc_1))}}{2\eta_1 b} \\ r_{s2}^D &= \frac{\eta_1 b\rho-\sqrt{\eta_1 b(\eta_1 b(\rho+2\delta)^2-\lambda^2(\theta-bc_1)(2\theta-bc_1))}}{2\eta_1 b} \\ r_{o1}^D &= \frac{\eta_2 b\rho+\sqrt{\eta_2 b(b(\eta_2(\rho+2\alpha)^2+c_1\phi\beta^2)-\theta\phi\beta^2)}}{2\eta_2 b} \\ r_{o2}^D &= \frac{\eta_2 b\rho-\sqrt{\eta_2 b(b(\eta_2(\rho+2\alpha)^2+c_1\phi\beta^2)-\theta\phi\beta^2)}}{2\eta_2 b}. \end{aligned}$$

Since $\eta_1 b(\rho+2\delta)^2 - \lambda^2(\theta - bc_1)(2\theta - bc_1) > 0$ and $b(\eta_2(\rho+2\alpha)^2 + c_1\phi\beta^2) - \theta\phi\beta^2 > 0$, we have $r_{s1}^D > 0, r_{s2}^D < 0, r_{o1}^D > 0, r_{o2}^D < 0$.

However, $r_{s1}^D, r_{s2}^D, r_{o1}^D, r_{o2}^D$, the corresponding eigenvector, are:

$$H_{s1}^D = \left[\frac{\lambda(\theta - bc_1)(2\theta - bc_1)}{2\eta_1 b(\rho+2\delta) - 2\sqrt{\eta_1 b(\eta_1 b(\rho+2\delta)^2 - \lambda^2(\theta - bc_1)(2\theta - bc_1))}}\right]$$
$$H_{s2}^D = \left[\frac{\lambda(\theta - bc_1)(2\theta - bc_1)}{2\eta_1 b(\rho+2\delta) + 2\sqrt{\eta_1 b(\eta_1 b(\rho+2\delta)^2 - \lambda^2(\theta - bc_1)(2\theta - bc_1))}}\right]$$
$$H_{r1}^D = \left[\frac{1}{2\eta_2 b(\rho+2\alpha) - 2\sqrt{\eta_2 b(b(\eta_2(\rho+2\theta)^2 + c_1\phi\beta^2) - \theta\phi\beta^2)}}\right]$$
$$H_{r2}^D = \left[\frac{1}{2\eta_2 b(\rho+2\alpha) + 2\sqrt{\eta_2 b(b(\eta_2(\rho+2\theta)^2 + c_1\phi\beta^2) - \theta\phi\beta^2)}}\right].$$

Summarizing the above analysis, the equilibrium solution $\{u_s^D, g^D\}, \{u_o^D, q^D\}$ of equations (42) and (43) of the dynamic control system of the decision maker of the fresh produce supply chain operator is a saddle point equilibrium in the decentralized scenario.

The optimal sales pricing, wholesale price, freshness input, and traceability goodwill input are given in equations (28), (29), (30), and (31), respectively.

Furthermore, the optimal solution of Proposition 2 has the following properties.

Corollary 2. *The optimal sales pricing strategy for fresh produce depends on the supply chain system parameters: when $\phi^2\beta^2 < (\theta - bc_1)(2\theta - bc_1)$, the optimal sales price keeps increasing with time, i.e., the penetration pricing strategy is chosen; when $\phi^2\beta^2 > (\theta - bc_1)(2\theta - bc_1)$, the optimal sales price keeps decreasing with time, i.e., the skimming pricing strategy is chosen; when $\phi^2\beta^2 = (\theta - bc_1)(2\theta - bc_1)$, the optimal sales price is a constant over the life cycle $[0, T]$ of fresh produce. The optimal wholesale price strategy also depends on the above supply chain system parameters and is similar to the optimal selling price. However, no matter how the fresh produce supply chain system parameters change, the freshness input continuously increases with time. The shadow price of fresh produce freshness and traceability goodwill is always favorable.*

Proof. According to equation (44), it can be derived that: $\dot{p} = \frac{\eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1) - \eta_1 c_0\delta\phi^2\beta^2(\rho+\delta)}{\eta_1\delta(\rho+\delta)(4\eta_2 b\alpha(\rho+\alpha) - \phi^2\beta^2) - \eta_2\alpha\lambda^2(\rho+\alpha)(\theta-bc_1)(2\theta-bc_1)}$. Because $\phi > 0$, we is, $(\theta - bc_1)(2\theta - bc_1) > \frac{\eta_1 c_0\delta(\rho+\delta)}{\eta_2\alpha\lambda^2(\rho+\alpha)}\phi^2\beta^2$ because $\frac{\eta_1 c_0\delta(\rho+\delta)}{\eta_2\alpha\lambda^2(\rho+\alpha)} > 0$, we can get: $(\theta - bc_1)(2\theta - bc_1) > \phi^2\beta^2$, so when $(\theta - bc_1)(2\theta - bc_1) > \phi^2\beta^2$, $\dot{p} > 0$; when $(\theta - bc_1)(2\theta - bc_1) < \phi^2\beta^2, \dot{p} < 0$.

However, it can be similarly verified that within the fresh produce life cycle $[0, T], \dot{w} > 0, \dot{g} > 0, \dot{q} > 0$. □

Corollary 2 shows that the choice of fresh produce sales pricing or wholesale price strategy depends on the supply chain system parameters: price sensitivity coefficient, coefficient of traceability goodwill input on traceability goodwill, coefficient of traceability goodwill contribution, coefficient of freshness contribution, and operational inefficiency, *i.e.*, $b, \beta, \phi, \theta, c_1$.

When the supply chain system parameters satisfy $(\theta - bc_1)(2\theta - bc_1) > \phi^2\beta^2$, the fresh produce sales price or wholesale price keeps increasing over time, *i.e.*, the operating firm chooses the penetration pricing strategy. Conversely, when the supply chain system parameters satisfy $(\theta - bc_1)(2\theta - bc_1) < \phi^2\beta^2$, the fresh produce selling price or wholesale price decreases continuously over time, *i.e.*, the operating firm chooses the skimming pricing strategy.

According to equations (30) and (31), the optimal freshness input and traceability goodwill input are increasing in the fresh produce life cycle $[0, T]$. It is clear from equations (30) and (31) that $\frac{\partial u_s^D(0)}{\partial T} > 0$ and $\frac{\partial u_o^D(0)}{\partial T} > 0$

can be deduced from the initial freshness input $u_s^D(0)$ and the initial traceability goodwill input $u_o^D(0)$, indicating that the operating companies prefer to set a higher freshness input and traceability goodwill input in the fresh produce life cycle E to the extent that it brings more benefits to the operating firm.

From equation (49), shadow prices are favorable throughout the fresh produce life cycle $[0, T]$.

4. Equilibrium comparison under the two scenarios

This section compares fresh produce selling prices, freshness inputs, traceability goodwill inputs, freshness, and channel profits under the two scenarios and draws the following conclusions.

Corollary 3. *The selling prices of fresh produce under the two scenarios are related as follows.*

(1) *When* $(3bc_1 - \theta) > \phi^2\beta^2$, *then* $p^D(t) > p^C(t)$.
(2) *When* $(3bc_1 - \theta) = \phi^2\beta^2$, *then* $p^D(t) = p^C(t)$.
(3) *When* $(3bc_1 - \theta) < \phi^2\beta^2$, *then* $p^D(t) < p^C(t)$.

The following conclusions can be drawn by comparing the freshness input, traceability goodwill input, freshness, and channel profit in the two scenarios.

Corollary 3 shows that when $(3bc_1 - \theta) > \phi^2\beta^2$, the optimal selling price of fresh produce under the centralized scenario is lower than the selling price under the decentralized scenario. On the contrary, when $(3bc_1 - \theta) < \phi^2\beta^2$, the selling price of fresh produce under the centralized scenario is higher than the selling price under the decentralized scenario.

The following conclusions can be drawn by comparing the freshness input, traceability goodwill input, freshness, and channel profit in the two scenarios.

Corollary 4. *The fresh produce freshness input, traceability goodwill input, freshness, traceability goodwill, and channel profit have the following relationships under the two scenarios:*

$$u_s^C(t) > u_s^D(t), u_o^C(t) > u_o^D(t), g^C > g^D, q^C > q^D, J^C > J^D.$$

It is clear from Corollary 4 that fresh produce freshness input, traceability goodwill input, freshness, traceability goodwill, and channel profit are higher in the centralized scenario than the corresponding equilibrium in the decentralized scenario.

The optimal freshness and traceability goodwill inputs in the centralized scenario are higher than those in the decentralized scenario. The reason is that the fresh produce suppliers' freshness input and retailers' traceability goodwill input decisions depend on their expected benefits and costs. In the decentralized scenario, it is clear from $\frac{\partial u_s^D}{\partial g} > 0$, $\frac{\partial u_o^D}{\partial q} > 0$ that the difficulty of developing freshness and traceability technology will increase for each unit of freshness and traceability goodwill. However, the corresponding cost will also increase or increase operational inefficiency. Therefore, suppliers and retailers will reduce their R&D investment in freshness and traceability technologies to save costs. In a centralized scenario, fresh produce suppliers and retailers work closely together to improve freshness and traceability technologies.

5. Mutual cost-sharing contract fresh produce supply chain coordination

Based on the equilibrium comparison between the above two scenarios, it is found that the decentralized scenario leads to low overall supply chain profits. The centralized scenario requires the decision-makers of the fresh produce supply chain to make a unified strategy. However, the operating companies are all independent economic entities, so making a unified decision takes time and effort. Therefore, a coordination model needs to be established among the operating companies. In this model, the objective of the operating companies is to maximize the total profit of the channel and ensure that their profit is greater than the profit in the decentralized scenario so that the operating companies have the incentive to participate in the coordination model. Therefore, this section proposes mutual cost-sharing contracts for supply chain coordination based on the decentralized scenario.

Under this contract, the retailer bears part of the freshness input cost for the supplier, and the supplier bears the traceability goodwill input cost for the retailer. It is assumed that γ_1 is the proportion of freshness input costs borne by the retailer, and γ_2 is the proportion of traceability goodwill input costs borne by the retailer. The coordination contract $(p, w, \gamma_1, \gamma_2)$ is determined as follows: firstly, the proportion of freshness input costs borne by the supplier and the retailer is determined through negotiation between the supplier and the retailer, and secondly, the supplier and the retailer determine the implementation contract $(p, w, \gamma_1, \gamma_2)$. The target profit functions of the fresh produce supply chain operating companies under the mutual cost-sharing contract are:

$$\begin{aligned}
&\max_{w,u_s} \int_0^T e^{-\rho t}\left((w(t)-c_{s0}-c_1 g(t))D(p(t),g(t),q(t)) - \frac{\eta_1(1-\gamma_1)u_s^2(t)}{2} - \frac{\eta_2\gamma_2 u_o^2(t)}{2}\right)\mathrm{d}t\\
&\max_{p,u_o} \int_0^T e^{-\rho t}\left((p(t)-w(t)-(c_0-c_{s0}))D(p(t),g(t),q(t)) - \frac{\eta_1\gamma_1 u_s^2(t)}{2} - \frac{\eta_2(1-\gamma_2)u_o^2(t)}{2}\right)\mathrm{d}t\\
&\text{s.t.}\begin{cases}\dot{g} = \lambda I_s(t) - \delta g(t), g(0) = g_0, g(T) = 0\\ \dot{q} = \beta I_o(t) - \alpha q(t), q(0) = q_0.\end{cases}
\end{aligned} \tag{50}$$

Proposition 3. *Fresh produce suppliers and retailers coordinate their supply chains using mutual cost-sharing contracts so that the profit under the mutual cost-sharing contract is greater than the profit under the fragmentation scenario.*

Proof. $J_S^T > J_S^D, J_R^T > J_R^D, J^T > J^D$. □

The computational procedure is similar to that of the decentralized scenario and will not be repeated for the sake of space.

Proposition 3 shows that the mutual cost-sharing contract can increase the profits of fresh produce suppliers and retailers and improve the efficiency of the fresh produce supply chain system compared with the decentralized scenario.

From Propositions 2 and 3, it is found that when γ_1, γ_2 can make $J_s^T > J_s^D, J_R^T > J_R^D$, the coordination of fresh produce operating firms can be achieved under the mutual cost-sharing contract, and Corollary 5 can be obtained by calculation.

Corollary 5. *Under the mutual cost-sharing contract, the supply chain can be coordinated when the proportion of costs borne by fresh produce suppliers satisfies* $\gamma_1^* = \frac{\Delta_1+\Delta_2-\Delta_3}{\eta_1(\rho+\delta)(\Delta_4-\Delta_5-\Delta_6)}, \gamma_2^* = \frac{8\eta_2\lambda^4\theta^4(\rho+\theta)^2+\Delta_7+\Delta_8+\Delta_9}{\eta_2(\rho+\theta)\Delta_{10}\Delta_{11}}$. *Where:*

$$\begin{aligned}
\Delta_1 &= 4\eta_1\eta_2 b\alpha\big(\alpha(2\rho+\alpha)(\rho+\delta)(\rho c_1 b + \theta(\rho-\delta)) + \rho^2\big(\delta(\theta\delta+\rho b c_1) - \rho^2(\theta - bc_1)\big)\big)\\
\Delta_2 &= \eta_1\phi^2\beta^2(\alpha(2\rho+\delta)(2\rho\theta - bc_1(\rho+\delta)) - \rho\delta(\rho+\delta)(2\theta - bc_1))\\
\Delta_3 &= \eta_2\alpha\lambda^2(\theta - bc_1)\Big((\theta-bc_1)^2 + \theta(3\theta - 2bc_1)\Big)\big((\rho+\alpha)^2 + 4\theta^3\big)\\
\Delta_4 &= \phi^2\beta^2(\alpha(2\theta(\rho+2\delta)) - bc_1(2\rho+3\delta) + \rho\delta(2\theta - bc_1))\\
\Delta_5 &= 4\eta_2 b\alpha^2(2\rho+\alpha)(\rho(\theta-bc_1) + \delta(3\theta - 2bc_1))\\
\Delta_6 &= \eta_2 b\alpha\rho^2(\rho(\theta+3\delta) - bc_1(\rho+2\delta))\\
\Delta_7 &= \eta_2 c_1^2\lambda^2 b^3(\rho+\alpha)^2\big(4\eta_1(\rho+\delta) + c_1\lambda^2(c_1 - 7\theta)\big)\\
\Delta_8 &= 2c_1\theta\lambda^2\big(9c_1\theta\lambda^2 - 4\eta_1\rho(6\rho+\delta)\big) - 16\delta\eta_2^2\rho^3 - 48\rho\eta_1^2\delta^2(\rho+\delta) + 4\eta_1\delta^2\big(c_1\theta\lambda^2 - 4\eta_1\delta^2\big)\\
\Delta_9 &= 8\delta\eta_1^2\phi^2\beta^2(\rho+\alpha)^3 - \eta_2\lambda^2\theta^2(\rho+\alpha)^2\big(20c_1\theta\lambda^2 - 8\eta_1\big(\rho^2-\delta^2\big)\big)\\
\Delta_{10} &= \lambda^2(\theta - 2bc_1)^2 + 4\eta_1 b(\rho+\delta)^2\\
\Delta_{11} &= \lambda^2(\theta - bc_1)(2\theta - bc_1) - 4\eta_1 b\delta(\rho+\delta).
\end{aligned}$$

Table 2. Parameter values.

Parameters	a	b	θ	λ	ρ	δ	c_{s0}	c_{r0}	c_1	η_1, η_2	ϕ
Value	20	2	1	1	0.1	0.4	0.6	0.4	0.4	2	0.8

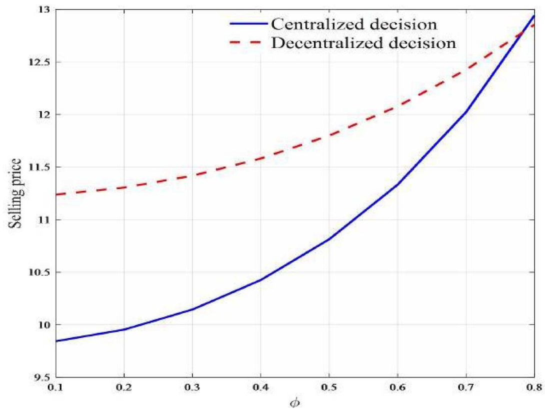

Figure 3. Effect of ϕ on price.

Corollary 5 shows that when $0 < \gamma_1^* < 1, 0 < \gamma_2^* < 1$, under the mutual cost-sharing contract scenario, to induce supply chain coordination, the retailer needs to increase its cost-sharing to the supplier for freshness. In contrast, the supplier likewise needs to share the cost of the retailer's traceability goodwill inputs.

6. Numerical analysis

In this section, numerical simulations are conducted using Matlab software to verify the validity of the theoretical results presented in this paper. The impact of the coefficient of the contribution of traceability goodwill on the equilibrium results and supply chain coordination is mainly investigated. The baseline parameter assignment in the model is mainly borrowed from Yang *et al.* [29], Liu *et al.* [4], Luo *et al.* [10], Jiang *et al.* [21], and the specific values are shown in Table 2.

6.1. Impact of traceability goodwill contribution factor on equilibrium results

This subsection investigates the impact of the traceability goodwill contribution coefficient on the equilibrium outcome, *i.e.*, the impact of traceability goodwill contribution coefficient ϕ on the equilibrium outcome of the fresh produce supply chain under the centralized and decentralized scenarios within the life cycle $[0, T]$ of fresh produce. First, the values of parameters such as effective coefficient of freshness input α, effective coefficient of traceability goodwill input β, cost coefficient of freshness and traceability goodwill input η_1, η_2, price elasticity coefficient b, and operational inefficiency coefficient c_1 are kept constant. Second, the traceability goodwill contribution coefficient ϕ is adjusted to increase from 0.1 to 0.8 at an incremental rate of 0.1, as shown in Figure 3. In order to prove the validity and reliability of the model and method, we take the value of ϕ as 0.1–1.4 for numerical simulation, as shown in Figure 4. Finally, the effects of traceability goodwill contribution coefficient ϕ on equilibrium price, traceability goodwill input, and channel profit are plotted. As shown in Figures 3–7 supplement.

It is clear from Figures 3, 5–7 that the equilibrium in both the centralized and decentralized scenarios increases when the traceability goodwill contribution rate coefficient ϕ increases, thus indicating that both

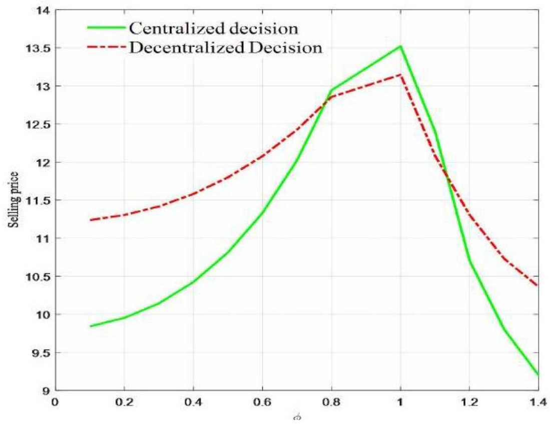

FIGURE 4. Effect of ϕ on price.

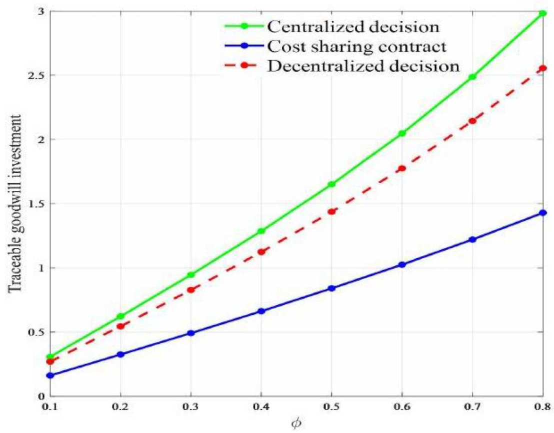

FIGURE 5. Effect of ϕ on the input of traceability goodwill.

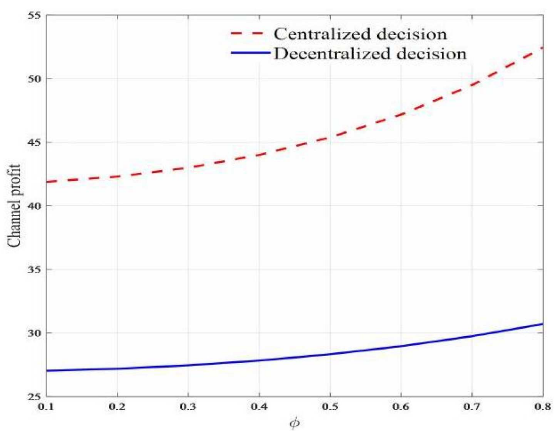

FIGURE 6. Impact of ϕ on channel.

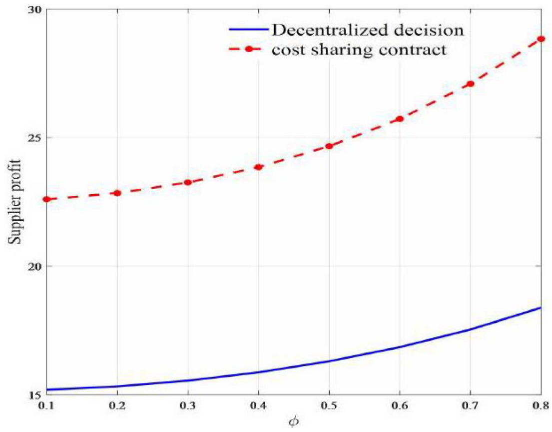

FIGURE 7. Impact of ϕ on supplier.

suppliers and retailers, regardless of the centralized or decentralized channels, will increase their traceability goodwill investment when they are faced with a higher traceability goodwill contribution rate, thus increasing the traceability goodwill of agricultural products and setting higher sales prices to obtain more profits.

As shown in Figure 3, when the traceability goodwill contribution rate is lower than threshold $\bar{\phi}$, the price under the centralized scenario is lower than that under the decentralized scenario. When the traceability goodwill contribution rate coefficient is higher than this threshold, the price under the centralized scenario is higher than the price under the decentralized scenario. This conclusion verifies Corollary 3, which shows that the centralized scenario will not necessarily be a low selling price because, under the centralized channel, high traceability goodwill input will lead to high traceability goodwill so that the operating companies can set higher prices.

It is clear from the scientific research in Figure 4 that when $\phi = 1$, the influence of the contribution ratio coefficient of traceability goodwill on the price of fresh agricultural products, reaches the maximum under the centralized and decentralized example scenarios, and when ϕ takes a value of more than 1, the sales price decreases with the increase of the value of ϕ (This implies that traceability goodwill hurts the selling price of fresh produce, *i.e.*, the higher the traceability goodwill, the lower the selling price. This phenomenon may be because high traceability goodwill can provide more information and assurance, and consumers have higher trust in the quality and safety of fresh produce. As a result, operating companies in the supply chain may lower the selling price to attract more consumers and increase sales. In this way, operating companies can gain market share and competitive advantage through traceability goodwill). This conclusion is in line with the actual operation situation. Therefore, the model and method proposed in this paper are valid and reliable.

Compared with the decentralized scenario, both the traceability goodwill input and profit are higher under the centralized scenario (as shown in Figs. 5–7), which thoroughly verifies Corollary 4. Moreover, as the traceability goodwill contribution increases, the gap between traceability goodwill input and channel profit under both scenarios becomes larger, indicating that the equilibrium increases faster under the centralized scenario when the traceability goodwill contribution is more considerable.

6.2. Impact of traceability goodwill contribution factor on supply chain coordination

As shown in Figures 8 and 9, Figures 8 and 9 portray the impact of the traceability goodwill contribution factor ϕ on the profits of suppliers and retailers. As the contribution rate of traceability goodwill increases, the difference between the profits of suppliers and retailers under the cost-sharing contract and the decentralized scenario becomes larger, indicating that the profits of operating companies under the cost-sharing contract

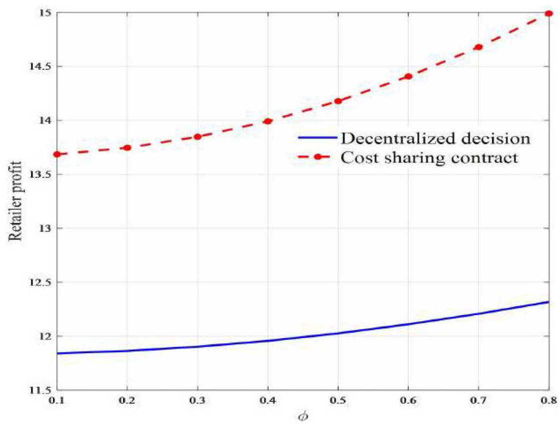

FIGURE 8. The impact of ϕ on retailers' profits.

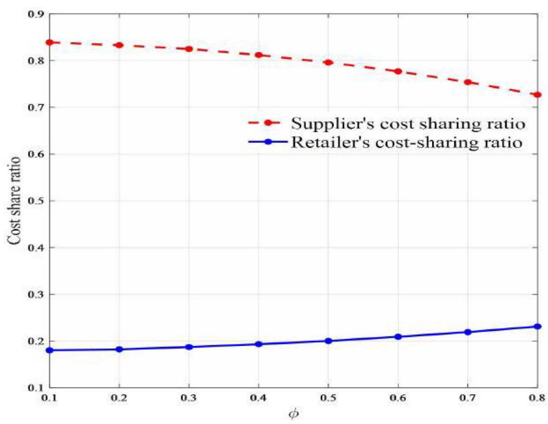

FIGURE 9. Effect of ϕ on cost-sharing ratio.

increase faster when the contribution rate of traceability goodwill is more significant. Therefore, the cost-sharing contract can successfully coordinate the supply chain to benefit both operating companies.

Figure 9 depicts the effect of the traceability goodwill contribution factor ϕ on the cost-sharing ratio between suppliers and retailers. This means that a high traceability goodwill contribution factor makes retailers increase their share of freshness inputs and drives suppliers to increase their freshness input enthusiasm to ensure high freshness of produce, which can. However, as demand continues to rise, suppliers will reduce retailers' traceability goodwill input costs. Therefore, suppliers and retailers negotiate cost-sharing contracts to improve the agility of the operating companies in coordinating the supply chain.

7. Discussion

In this paper, we propose a dynamic pricing and coordination differential game model for the fresh produce supply chain based on traceability goodwill, which considers the impact of traceability goodwill on fresh produce's pricing and supply chain coordination. This model has critical managerial discussions and practical implications for agricultural supply chain managers and policymakers.

First, the model proposes the coefficient of contribution of traceability goodwill as an indicator to measure the degree of influence of traceability goodwill on selling price. Managers can utilize this indicator to assess the impact of traceability goodwill on pricing and adjust pricing strategies according to the actual situation. When the coefficient of the contribution of traceability goodwill is $0.1-0.8$, managers can consider continuously increasing the sales price to maximize revenue.

Second, this paper proposes the mutual cost-sharing contract model, which emphasizes the coordination problem in the supply chain. By considering mutual cost sharing among supply chain parties to enhance cooperation, managers can promote supply chain coordination by establishing and enhancing traceability goodwill. Traceability goodwill can enhance consumers' trust in fresh produce and reduce information asymmetry and risk, thus promoting cooperation and coordination in the supply chain.

In practical application, the model and methodology of this study can help fresh produce supply chain managers develop more effective pricing and supply chain coordination strategies. By reasonably assessing the impact and importance of traceability goodwill, managers can adjust product pricing according to market demand and competition and conduct practical cooperation and coordination with all parties in the supply chain.

In addition, the study can provide a reference for government regulators. Based on the conclusions and methods of this study, the government can formulate relevant policies and standards to promote the construction and supervision of traceability systems for agricultural products and to safeguard the rights and interests of consumers and the quality and safety of products.

In summary, this paper's managerial discussion and practical implications include helping managers assess the impact of traceability goodwill on pricing, facilitating supply chain coordination and cooperation, guiding pricing strategies and supply chain decisions, and providing references for government regulation. These practical applications can help improve the efficiency and competitiveness of fresh produce supply chains while enhancing consumer trust and satisfaction with the products.

8. Conclusion

Consumers pay special attention to the freshness of agricultural products when purchasing them, and the Ministry of Agriculture and Rural Development proposes solving the problem of "first mile" agricultural products. For fresh produce operations, the level of freshness effort is the way to solve the freshness problem. Thus, fresh produce suppliers should follow the principle of sustainable development and provide retailers with uncontaminated fresh produce that meets food safety standards and is accompanied by traceability goodwill according to specific growing methods to ensure the safety and reliability of fresh produce and enhance consumer confidence. In this paper, we construct a dynamic model of freshness and traceability goodwill and a demand model in which fresh produce price, freshness, and traceability goodwill jointly influence each other and discuss the dynamic pricing, freshness input, and traceability goodwill input decisions of fresh produce supply chain operators under centralized and decentralized scenarios, and compare the decision results under the two scenarios. Mutual cost-sharing contracts are designed to coordinate the supply chain. The main conclusions are as follows: traceability goodwill contribution rate and freshness effectiveness have the same effect on the price, *i.e.*, the price in the centralized scenario is not necessarily lower than that in the decentralized scenario; when the traceability goodwill contribution rate exceeds a threshold value, *i.e.*, A, the price in the centralized scenario is higher than that in the decentralized scenario; when suppliers and retailers negotiate a cost-sharing contract, B, it can satisfactorily coordinate the supply chain; retailers The proportion of suppliers' share of freshness input costs increases with the increasing contribution of traceability goodwill, while the proportion of suppliers' share of retailers' traceability goodwill input costs decreases in the opposite direction, *i.e.*, with the increasing contribution of traceability goodwill. The management insights of this paper are as follows: First, it provides reference or guidance for fresh produce operating companies to formulate sales price strategies. When the operational inefficiencies of freshness effectiveness and traceability goodwill contribution rate are high and low, operating companies can gain more profits by increasing sales prices, and vice versa Second, it provides theoretical support or a basis for negotiating cost-sharing contracts between fresh produce suppliers and retailers. By designing a cost-sharing contract, the operating companies can have their respective profits in the cost-sharing contract scenario exceed the profits in the decentralized scenario and the overall profits.

In this paper, only fresh produce suppliers for freshness inputs and retailers for traceability goodwill inputs are considered, and the introduction of joint inputs from suppliers and retailers can be considered. In addition, competition among suppliers or retailers is not considered, and studying dynamic pricing and coordination of fresh produce supply chains under market competition will be more realistic and applicable.

While this paper considers the impact of freshness and traceability goodwill on the market demand for fresh produce, there are limitations to the research and directions for future work.

The study limitations are relevant: First, the model of the study is based on certain assumptions and simplified conditions, such as rational decision-making. However, supply chains often face complex, irrational behavior, information asymmetry, and cost fluctuations. Therefore, the study's conclusions may be affected by these real-world factors. Future research can further consider these real-world factors to improve the model's accuracy and approximate the real-world situation.

Secondly, the study's data validation and numerical simulation experiments were conducted based on a specific dataset. The sample size is relatively small and covers only specific fresh produce and supply chain scenarios. Therefore, the study's conclusions may be affected by the limitations of the sample and the particular situation. Future studies could expand the sample size (selecting multiple data sets for simulation) to cover more produce and supply chain contexts to verify the generalizability and reliability of the model.

In addition, the study needed to consider changes in consumer behavior and market demand adequately. Various factors, such as seasonal variations, market trends, and competitive pressures, may influence consumer demand and preference for fresh produce. Future research can further investigate the impact of traceability goodwill on consumer purchasing decisions and market responses by considering the dynamic changes in consumer behavior and market demand.

Finally, the empirical analysis of this study is mainly based on the numerical simulation of the model and needs more validation of field research and actual cases. Field research and case studies can provide a deeper understanding of the actual situation and problems in the fresh produce supply chain to validate and improve the feasibility and practicality of the model. Future research can combine field research and case studies to validate further and generalize the study results.

In summary, the study's limitations in this paper include more accurate modeling of the actual situation, expanding the sample size, considering changes in consumer behavior and market demand, and validation in conjunction with field research and case studies. These limitations provide directions for future research to improve further and expand theory and practice in this area.

Relevant elements for the direction of future work: First, future research can further improve the traceability of goodwill measurement and assessment methods. In the current study, the traceability goodwill contribution rate coefficient is introduced as a measure of the degree of influence of traceability goodwill on pricing. However, it remains a challenge to calculate and assess the contribution of traceability goodwill accurately. Future research could explore more precise metrics incorporating consumer behavior and market reaction data to accurately measure the traceability of goodwill's impact on pricing.

Second, future research could further consider the issue of risk management and flexibility in the supply chain. Fresh produce supply chains face various risks, such as weather disasters and disease outbreaks. How to consider risk management and flexibility in dynamic pricing and coordination to cope with uncertainty and change is an important research direction. Future research can explore the impacts of risk management and flexibility mechanisms on supply chain performance and benefits and propose corresponding strategies and methods.

In addition, future research could further investigate the traceability of goodwill's impact on consumer behavior and market demand. Consumers are increasingly concerned about the trust and safety of products, and traceability goodwill, as one of the essential attributes of products, significantly impacts consumers' purchasing decisions and behaviors. Future research can combine the dynamic changes in consumer behavior and market demand to study the impact mechanism of traceability goodwill on consumer purchase intention, brand loyalty, and other aspects.

Finally, future research can expand the scope to cover more fresh produce and supply chain situations. The current study mainly focuses on a specific fresh produce or a specific supply chain segment, and future studies can expand the sample size to cover more products and supply chain contexts to verify the applicability and universality of the model.

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