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HETEROGENEOUS PRODUCTS OPERATION DECISIONS OF ONLINE DUAL-CHANNEL SUPPLY CHAIN CONSIDERING ONLINE REVIEWS UNDER DIFFERENT FINANCING MODES

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Abstract. This research examines the operation decisions of the online supply chain for heterogeneous products under different financing modes: e-commerce platform financing or bank financing, when manufacturers face funding constraints. Considering the manufacturer's adoption of differentiated channel strategies when providing heterogeneous products is also considered, and combined with the impact of online reviews on consumer utility, an e-commerce platform online dual-channel financing model is constructed. The research findings are as follows: (i) when the effectiveness of online reviews differs within a certain range, the equilibrium solution exists. If the relative interest rates of the e-commerce platforms and bank change within a certain range, the same financing mode can bring mutual benefits to both the manufacturer and e-commerce platform, resulting in a "win–win" situation. (ii) If the interest rates under both financing modes are the same, the e-commerce platform financing mode has a higher wholesale price, but the difference in retail prices of distribution products depends on the costs difference between the two products, and at this point, the manufacturer will select e-commerce platform financing mode. (iii) At the optimal interest rate, when the cost of heterogeneous products is the same, the e-commerce platform consistently offers a more favorable interest rate compared to the bank's optimal rate. when the e-commerce platform's commission and the positive difference in product reviews is large, the manufacturer will choose e-commerce platform financing mode. Under the e-commerce platform financing mode, both the manufacturer and e-commerce platform are willing to provide lower retail prices to attract more consumers.

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1. Introduction

The issue of capital shortage is prevalent in the actual operation of supply chains, especially for manufacturers producing various heterogeneous products, capital constraints can be particularly challenging [11, 20]. Traditionally, manufacturers mainly obtain financing through commercial banks, which, however, requires suf-

Keywords. Heterogeneous products, supply chain, e-commerce platform, online reviews, financing model.

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ficient collateral, slow approval processes, and slow disbursements of funds [2]. Fortunately, due to the rapid development of internet finance, major e-commerce platforms like Amazon in North America, and Suning and JD.com in China, have opened their own "internet commercial banks". These platforms use digital management capabilities to provide financing services to manufacturers in the platform, such as "credit financing" and "order financing", offering advantages such as no need for collateral, low thresholds, and rapid disbursement of funds [4, 21]. Online supply chain finance provides solutions for manufacturers to alleviate capital shortages and promote the performance of supply chain capital flow [25]. Considering operations problems in the online supply chain from perspective of different financing models is crucial, which can provide a solution for financing of manufacturers, accelerate the development of financial market, and improve the economic environment. The e-commerce platform has been widely utilized in digital supply chains. Manufacturers may choose to sell through either self-operated distribution channels or flagship store direct sales channels. Similar to traditional offline and online dual-channel models, these two channels on e-commerce platforms also compete and conflict with each other [14, 18]. In order to mitigate competition between online dual channels, manufacturers can strategically deploy heterogeneous products within each channel. As an illustration, Haier adopts a dual-channel strategy by selling the "Haier EG100MATE2S" through the self-operated distribution channel on JD.com and the "Haier B129W" through its flagship store-based direct sales channel. The heterogeneous products layout with dual online channels may influence the choice of finance model and operation decisions. Considerations of financing and heterogeneous products operations in e-commerce platforms make the decision-making process in supply chains complex [20].

Although the sale of heterogeneous products through online dual channels has gradually become an important means for manufacturers, how to measure the differences between heterogeneous products has always been an important issue that has not been unified. Previous literature distinguishes heterogeneous products based on consumers' preferences [10, 30]. However, in the online dual-channel sales, the diversity of purchasing channels can contribute to an increase in uncertainty during the consumer purchasing process [9, 16]. After purchasing a product online, consumers usually post online reviews about the product. Studies have demonstrated that more than 90.0% of online shoppers browse through product reviews, and 92.2% of consumers report that online reviews have an impact on their purchase decisions [1, 7]. Therefore, Distinguishing the differences in heterogeneous products through online dual channels from the perspective of online reviews and customer preferences is more effective.

On the basis of the above observations, this study considers a supply chain composed of an e-commerce platform and a manufacturer, where the manufacturer can distribute heterogeneous products through online distribution and direct sales on the e-commerce platform, and takes into account the impact of online reviews on consumer utility. Specifically, the operational decisions of a financially constrained manufacturer are investigated under two financing modes: e-commerce platform financing and bank financing. Specifically, the primary goal of this paper is to explore the answers to the following research questions:

(i) When does the e-commerce platform provide financing services?
(ii) Which financing mode should the manufacturer choose under the influence of online dual-channel for heterogeneous products?
(iii) How does differentiation of heterogeneous products affect pricing and profits for manufacturers and e-commerce platforms under different financing modes?
(iv) What is the optimal interest rate under different financing modes?

In this paper, a game model is constructed to address the above questions under two decision-making modes: e-commerce platform financing and bank financing. Firstly, based on the financing interest rate relationship under the two financing modes, the paper examines the prerequisites for e-commerce platform to provide financing services and the financing decisions of the manufacturer. Secondly, under the optimal interest rate, the paper analyzes the financing choices of the manufacturer and the operational decisions of the online dual-channel supply chain.

The contributions of this research are mainly reflected in three aspects. we contribute to the study of supply chain financing by considering the dual role of e-commerce platforms as both price decision-makers and financing providers in the supply chain, which improves practitioners' decision-making. Second, the wholesale and agency modes are both considered for the online dual channels on e-commerce platforms layout of heterogeneous products, which contributes to dual-channel supply chain decision-making. Third, we distinguish the differences in heterogeneous products from a comprehensive view of customer preferences and online reviews, which enrich the literature on heterogeneous products.

2. Literature review

The study pertaining to this paper primarily encompasses three key aspects: supply chain financing, dual-channel supply chain decision-making and heterogeneous products.

2.1. Supply chain financing

Supply chain finance has emerged as an effective solution to address funding constraints, and has garnered significant attention in the literature. Recently, supply chain finance has been investigated from aspects such as financing mode choice, supply chain coordination, contract design [2, 20, 28, 32].

Prior research has primarily focused on examining internal and external financing activities among supply chain members. Existing research has mainly focused on the effects of financing through different channels on supply chain performance. Cao *et al.* [2] investigated a supply chain that includes a supplier and a carbon-dependent manufacturer with funding constraints, where the manufacturer had the choice between internal financing from the supplier and external financing from a bank, and showed that regardless of the consideration of carbon reduction investment, trade credit financing from the supplier was a unique financing equilibrium for the manufacturer. More recently, Sun *et al.* [20] explored pricing, financing, and channel structure in a dual-channel supply chain, where a capital-constrained manufacturer sells two different products *via* a retailer's retail channel and a direct sales channel. And from the manufacturer's perspective, they evaluated the efficacy of three financing strategies, including early payment financing, bank financing, and the combination financing.

The rise of e-commerce has led to an increasing number of e-commerce platforms venturing into the financial sector, making e-commerce and finance integration a major topic of academic research. For instance, Zhen *et al.* [32] analyzed how capital-constrained manufacturers can alleviate their financial pressures through credit financing from third-party platforms, banks and retailers, identifying the optimal financing strategies for manufacturers under three financing modes. Yan *et al.* [28] investigated the impact of the dual roles of online retailers on pricing in different dual-channel competition structures, revealing that financing for online retailers can benefit for all parties involved, with online retailers earning higher profits in vertical competition structures. Ma and Meng [11] studied the financing strategies for manufacturers in a dual-channel closed-loop supply chain that is composed of capital-constrained manufacturers and retailers, and found that retailer credit financing and equity financing are optimal financing strategies for manufacturers when the equity financing ratio satisfies certain conditions. Dong *et al.* [4] studied channel selection and pricing strategies for a capital constrained supplier in the digital supply chain with the consideration of supply chain finance and blockchain. In a study by Chen and Chen [3], the optimal operation strategy of supply chain members was investigated, incorporating four financing modes: internal trade financing, blockchain platform trade financing, external bank financing, and blockchain platform external financing. Considering the competition between national brands and store brands, Wang *et al.* [25] focused on the operation decision-making and financing choice of manufacturers under bank financing and platform financing modes, and found that, despite a higher platform loan interest rate compared to bank loan interest rates, manufacturers will still choose the platform financing model in the cases of elevated production costs and intensified competition.

Based on the above review, there is an increase of research regarding the financing models of e-commerce platforms, which generally consider the role of these platforms as providers of transaction data and financing services in the supply chain system, without participating in price decision-making. While Yan *et al.* [28] and

Dong *et al.* [4] analyzed the impact of e-commerce platform financing on pricing decisions in online channels, Yan *et al.* [28] only focused on the value-added effects of such financing, and Dong *et al.* [4] only studied the price strategy for direct sales channels and platform channels of a single product under the electronic platform financing mode. This study differs from previous research by considering the dual role of e-commerce platforms as both price decision-makers and financing providers in the supply chain, while also exploring and comparing the effects of e-commerce platform financing and bank financing on supply chain operational strategies. This research aims to fill the gap in current literature by examining the role of e-commerce platforms in supply chain financing and their effects on operational strategies.

2.2. Dual-channel supply chain decision-making

In recent times, an expanding body of literature has focused on decision-related challenges within dual-channel supply chains, encompassing aspects such as pricing, distribution, and coordination [6, 26, 27]. For instance, He *et al.* [6] delved into the study of optimal two-stage pricing strategies of manufacturers and retailers, and further analyzed the impacts of channel preference, price competition, and market changes on channel equilibrium. Similarly, Xu and Qiu [26] examined optimal pricing and distribution strategies of the manufacturer in the dual-channel supply chain of online direct sales and offline retail channels, as well as the impact of new distribution strategies on price competition and member's revenue in the dual-channel supply chain. Taking into account the influence of virtual exhibition hall behavior on consumers' purchasing decisions, Xu *et al.* [27] factored in random retailer demand and explored the impact of data-driven marketing, stock shortage substitution rate and product quality sensitivity on the optimal decision and maximum profit of the dual-channel supply chain, and designed a manufacturer's buyback contract to coordinate the supply chain. With the increasing prevalence of online sales models, scholars have also turned their attention to online hybrid channels [12, 15, 22]. Considering the two competing suppliers chooses its distribution strategy and an agency selling contract or a wholesale contract, Matsui [12] examined which of an agency selling or a wholesale contract offered by an e-commerce platform competing suppliers with typical dual-channel supply chains should adopt. Tian *et al.* [22] analyzed the impact of order completion costs and upstream competitive intensity on the sales mode of retailers among three available options: distributor sales, online market sales, and mixed sales. In another study, Ryan *et al.* [15] explored the pricing decisions of retailers and online markets to determine the optimal sales mode for retailers, considering the choice between selling products through their own website or paying a revenue-sharing fee to sell products through an online market.

Although literature has focused on the contract selection and pricing issues of online mixed dual channels on both direct sales and e-commerce platforms [12, 15, 22], there are few studies involving online dual channels on the same platform. In contrast to the above research, this paper focuses on the online dual-channel differential sales model of heterogeneous products to examine the optimal pricing decisions of manufacturers and e-commerce platforms, and further investigate how e-commerce platforms contribute the management of supply chain operations.

2.3. Heterogeneous products

In practice, with the increasing demand for personalized consumption, the strategy of offering heterogeneous product or offering goods that are similar but have different qualities or specifications is becoming increasingly common [10,23,30]. Tian *et al.* [23] investigated how manufacturers can make decisions on channel differentiation strategies when facing free-riding, which involve selling homogeneous or differentiated products through online channels and independent retailers. Their analysis revealed that consumers tend to free-ride more when facing differentiated products than homogeneous ones. While considering that consumers have varying levels of acceptance for different products online, Liu *et al.* [10] addressed the pricing problem of two differentiated products in a dual-channel supply chain involving a dominant manufacturer and a retailer. Zhang *et al.* [30] investigated the optimal distribution strategy for a manufacturer to distribute high-quality and low-quality products through either direct channels (such as their own online channels) or indirect channels (such as traditional retail

channels), or *vice versa*, and their analysis showed that the manufacturer's optimal distribution strategy depends on the type of product. Considering the substitution effect between the two differentiated products, Zhang *et al.* [31] explored how a dominant supplier of differentiated products can determine the optimal online mode under different distribution strategies, and the research suggested that the mode choice is highly dependent on the inherent attributes of the product. In a literature review on heterogeneous products, most studies focused on distinguishing between product differentiation and in consumer preferences and different services. Actually, when consumers buy products or services online, they face uncertainty about quality and value. Online reviews can help consumers identify products that meet their needs [24], distinguish between quality differences among products [17, 19, 24], and more accurately estimate their true utility [5], thereby reducing uncertainty for consumers and influencing their purchasing decisions. Therefore, starting from the actual purchase of consumers, this paper distinguishes the differences in heterogeneous products from a comprehensive of customer preferences and online reviews.

3. Problem description

In the context of commerce, manufacturers often offer different products to meet the diverse needs of consumers. In this study, we consider a scenario where a manufacturer (referred to as M) offers two different products (L and E) for sale through a single e-commerce platform (referred to as R). The e-commerce platform can engage with the manufacturer through wholesale mode (referred to as W) or agency mode (referred to as A). Under the wholesale mode, the e-commerce platform observes the manufacturer's wholesale price w_i and determines the selling price p_i of the product. Under the agency mode, the manufacturer determines the selling price of the product, and the e-commerce platform charges a certain commission rate η for each transaction. Specifically, for each unit of product sold, the manufacturer must pay a proportion η of the revenue to the e-commerce platform and retain $(1-\eta)$ part of the revenue. To align with industry practices, the commission rate is assumed to be exogenous given. We consider products L and E cooperating with the e-commerce platform through wholesale (W) and agency mode (A), respectively.

Consumers' valuations of products are assumed to follow a uniform distribution, denoted as $v \sim U[0,1]$. To account for product differentiation in consumers' valuations, we assume that consumers' valuations of products L and E are v and θv, respectively, where θ represents the preference for the two products due to differences in their content. To ensure that the differentiation between the two products falls within a certain range, we assume that product L is preferred over product E and constrain $\theta \in (1/2, 1)$. This assumption ensures that the two products are targeted at similar consumers.

Due to consumers' inability to accurately perceive a product's quality level before receiving it, online review information can help to reduce this uncertainty. Assume v_i $(i = L,\ E)(-1 < v_i < 1)$ represents the additional utility that online review information brings to consumers, where $1 > v_i > 0\,(-1 < v_i < 0)$ indicates that consumers observe positive (negative) online review information. The additional utility of online review information for consumers regarding product E is taken as the benchmark ($v_E = 0$), and define $\Delta v = v_L - v_E$ is as the difference in additional utility of online review information for consumers regarding the two products. $\Delta v > 0$ indicates that product L receives more positive online review information than product E, so online review information can bring higher additional utility to consumers who choose product L than those who choose product E. Furthermore, the larger the absolute value $|\Delta v|$ of online review information, the weaker the competition between the two products. Therefore, under the influence of online review information, according to Yang *et al.* [29] consumers' valuations of products L and E can be updated as $(1-\rho)v + \rho v_L$ and $(1-\rho)\theta v + \rho v_E$, respectively, where $\rho(0 < \rho < 1)$ represents the weight of online reviews on product evaluation. A larger ρ indicates that consumers are more willing to adjust their quality evaluations based on online review information.

The net utility of a consumer choosing product L and E can be represented as $U_L = (1-\rho)v + \rho v_L - p_L = (1-\rho)v + \rho\Delta v - p_L$ and $U_E = (1-\rho)\theta v + \rho v_E - p_E = (1-\rho)\theta v - p_E$, respectively. A consumer chooses product L if $U_L - U_E \geq 0$ and $U_L \geq 0$. Similarly, a consumer chooses product E if $U_E - U_L \geq 0$ and $U_E \geq 0$. Let

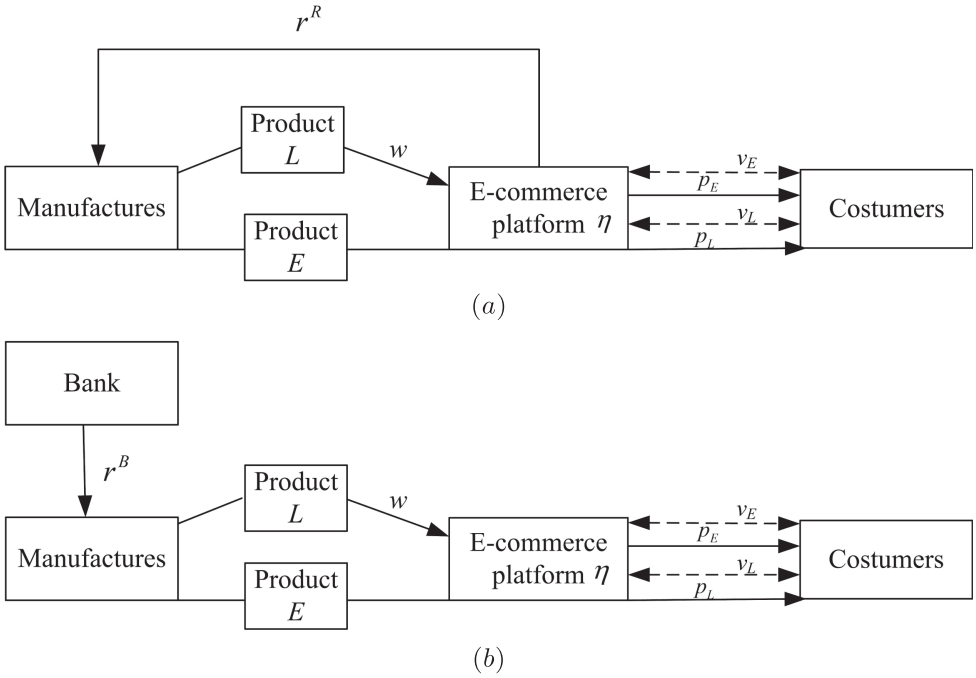

FIGURE 1. (a) E-commerce platform financing. (b) Bank financing.

$U_E = U_L$, consumers with valuation above $\frac{p_L - p_E - \rho\Delta v}{(1-\rho)(1-\theta)}$ will choose product L. Otherwise, setting $U_E = 0$ consumers with valuation between $\frac{p_L - p_E - \rho\Delta v}{(1-\rho)(1-\theta)}$ and $\frac{p_E}{(1-\rho)\theta}$ will choose product E, while the remaining consumers will exit the market. Therefore, the demands of products L and E are given by equations (1) and (2):

$$D_L = 1 - \frac{p_L - p_E - \rho\Delta v}{(1-\rho)(1-\theta)} \tag{1}$$

$$D_E = \frac{p_L - p_E - \rho\Delta v}{(1-\rho)(1-\theta)} - \frac{p_E}{(1-\rho)\theta}. \tag{2}$$

This paper considers a manufacturer with financial constraints using financing to alleviate funding pressure. Specifically, it examines the operational decisions of a heterogeneous product online dual-channel supply chain under online reviews when an e-commerce platform (Fig. 1a) or a bank financing (Fig. 1b) strategy is implemented by the financially constrained e-commerce platform.

Before the start of the selling season, the manufacturer requires a capital amount of $(D_E c_E + D_L c_L)$, where c_E and c_L represent the unit cost of the two products purchased by the manufacturer during the peak season. Assuming the manufacturer has an initial capital of K, the amount of financing for the manufacturer under the financial constraint is $(D_E c_E + D_L c_L) - K$. At the end of the selling season, the manufacturer repays the principal and interest $[(D_E c_E + D_L c_L) - K](1 + r^j)$ based on the interest rate r^j ($j = R$ represents e-commerce platform financing, $j = B$ represents bank financing) set by the e-commerce platform or bank. The online dual-channel supply chain in this paper consists of a manufacturer and an e-commerce platform, where the manufacturer dominates. The notations and definitions are shown in Table 1.

TABLE 1. Notations and definitions.

Notations	Definitions
v	Valuation of product by customer, $v \in (0,1)$
θ	Customer preference coefficient of product E, $\theta \in (1/2,1)$
η	Percentage of the commission received by e-commerce platform from manufacturer, $\eta \in (0,1)$
v_i $(i = L,\ E)$	Additional utility provided to customers by online review information, $v_i \in (0,1)$
Δv	Additional utility difference of customers regarding two products due to online review information, $\Delta v \in (-2,2)$
ρ	Weight of online reviews on product evaluation, $\rho \in (0,1)$
U_i	Net effect of customer purchasing product $i, U_i > 0$
w_L	Wholesale price of product $L, w_L > 0$
p_i	Price of product $i, p_i > 0$
D_i	Demand for product $i, D_i > 0$
π_R	Profit of e-commerce platform, $\pi_R > 0$
π_M	Profit of manufacturer, $\pi_M > 0$
π_B	Profit of bank, $\pi_B > 0$
r^R	Interest rate of e-commerce platform, $r^R \in (0,1)$
r^B	Interest rate of bank, $r^B \in (0,1)$

4. EQUILIBRIUM DECISIONS UNDER DIFFERENT FINANCING SCENARIOS

In this section, we consider a manufacturer facing financial constraints and two financing options: obtaining financing from the e-commerce platform or a bank loan. The game sequence follows a Stackelberg model, where the manufacturer chooses a financing mode (e-commerce platform financing or bank financing), and sets the wholesale price of the distribution channel w_L and the retail price of the direct sales channel p_E. Then, the e-commerce platform determines the retail price p_L of product L based on the manufacturer's pricing and wholesale price.

4.1. E-commerce platform financing ($j = R$)

Considering the manufacturer's financing from the e-commerce platform, according to equations (1) and (2), the profit functions of the manufacturer and the e-commerce platform can be obtained as follows:

$$\pi_M^{WAR} = w_L D_L + [(1-\eta)p_E]D_E - K - [(D_E c_E + D_L c_L) - K]\left(1 + r^R\right) \tag{3}$$

$$\pi_R^{WAR} = (p_L - w_L)D_L + \eta p_E D_E + [(D_E c_E + D_L c_L) - K]\, r^R. \tag{4}$$

Proposition 4.1. *Under the e-commerce platform financing, when* $\Delta v \in [\max(-1, v_1^R), \min(1, v_2^R)]$, *the optimal wholesale and retail price decisions of the manufacturer and the e-commerce platform are:*

$$w_L^{WAR^*} = \frac{1 - \rho - \eta\theta + \eta\rho\theta + \rho\Delta v + c_L(1 + 2r^R)}{2} \tag{5}$$

$$p_L^{WAR^*} = \frac{(1-\eta)(c_L + 3\rho\Delta v) + (3-\theta)(1-\rho)(1-\eta) + c_E(1 + \eta + 2r^R)}{4(1-\eta)} \tag{6}$$

$$p_E^{WAR^*} = \frac{c_E(1 + r^R) + \theta(1-\eta)(1-\rho)}{2(1-\eta)} \tag{7}$$

where, $\Delta v_1^R = \dfrac{(c_L - c_E) - (1-\rho)(1-\theta)}{\rho}$,

$$\Delta v_2^R = \frac{\theta(1-\rho)(1-\theta)(1-\eta) - \theta[c_L(1-\eta) + c_E(1+\eta)] - 2c_E(1+r^R - \theta r^R) + \eta\theta^2(1-\rho)}{\theta\rho(1-\rho)}.$$

The proof of Proposition 4.1 and the other propositions are in Appendix A.

4.2. Bank financing ($j = B$)

In this section, a manufacturer with capital constraints chooses to loan from a bank to alleviate financial pressure. Based on demand equations (1) and (2), the profit functions of the manufacturer, e-commerce platform, and bank are obtained, respectively.

$$\pi_M^{WAB} = w_L D_L + [(1-\eta)p_E]D_E - K - [(D_E c_E + D_L c_L) - K]\left(1 + r^B\right) \tag{8}$$
$$\pi_R^{WAB} = (p_L - w_L)D_L + \eta p_E D_E \tag{9}$$
$$\pi_B^{WAB} = [(D_E c_E + D_L c_L) - K]\, r^B. \tag{10}$$

Proposition 4.2. *Under the bank financing, when* $\Delta v \in [\max(-2, v_1^R), \min(2, v_2^R)]$*, the optimal wholesale and retail price decisions of the manufacturer and the e-commerce platform are:*

$$w_L^{WAB^*} = \frac{1 - \rho - \eta\theta + \eta\rho\theta + \rho\Delta v + c_L(1 + r^B)}{2} \tag{11}$$
$$p_L^{WAB^*} = \frac{(1-\eta)(c_L - c_L r^b + 3\rho\Delta v) + (3-\theta)(1-\rho)(1-\eta) + c_E(1 + \eta + 3r^B - \eta r^B)}{4(1-\eta)} \tag{12}$$
$$p_E^{WAB} = \frac{c_E(1 + r^B) + \theta(1-\eta)(1-\rho)}{2(1-\eta)} \tag{13}$$

where, $\Delta v_1^B = \dfrac{(c_L - c_E)(1 + r^B) - (1-\rho)(1-\theta)}{\rho}$,

$$\Delta v_2^B = \frac{\theta(1-\rho)(1-\theta)(1-\eta) + c_L(1-\eta) + c_E(\theta + \theta\eta - 2) - \theta r^B(1-\eta)(c_L - c_E) - 2c_E r^B(1-\theta)}{\theta\rho(1-\eta)}.$$

The proof of Proposition 4.2 is similar to that of Proposition 4.1.

Corollary 4.3. *The sensitivity of the optimal wholesale, retail price decisions for each key parameter under different financing models are listed in Table 2.*

TABLE 2. Sensitivity of optimal decisions to key parameters in different financing modes.

	r^j	c_E	c_L	Δv	ρ	θ	η
w_L^{WAj*}	↗	–	↗	↗	if $\Delta v > 1 - \eta\theta$, then ↗	↘	↘
p_L^{WAj*}	if $c_L < \frac{3-\eta}{1-\eta}c_E$, then ↗ $(j = B)$; ↗ $(j = R)$	↗	↗	↗	if $\Delta v > \frac{3-\theta}{3}$, then ↗	↘	↘
p_E^{WAj*}	↗	↗	–	–	↘	↗	↗

Notes. ↗ indicates positive correlation, ↘ indicates negative correlation, – indicates irrelevant.

Corollary 4.3 demonstrates that the optimal wholesale, retail price decisions for each key parameter under different financing models are influenced by the combined effects of cost, interest rate, online review, percentage of the commission. The specific impacts are as follows:

(i) As the interest rate of the e-commerce platform increases, the manufacturer may increase the wholesale price of product L and the retail price of product E through its direct sales channel to reduce the financing cost. This increase in the wholesale price may also result in the e-commerce platform raising the retail price of product L.

(ii) The price of product L is not only related to its own production cost, but also to the cost of product E. As c_E increases, the manufacturer may also increase the retail price of product E. In such cases, the e-commerce platform will also adopt a similar strategy of raising the retail price of product L, but according to $\frac{\partial p_E^{WAj*}}{\partial c_E} > \frac{\partial p_L^{WAj*}}{\partial c_E}$, the increase will not be as large as that of the manufacturer.

(iii) As the difference in online reviews between product L and product E increases, the manufacturer will increase the wholesale price of product L, and the e-commerce platform will also increase the retail price of product L. When there is a significant difference in online reviews between the two products, the price of product L tends to rise due to weight of online reviews on product evaluation. This is because online reviews can reduce consumers' uncertainty about product valuation and, to some extent, offset the negative impact of high prices on consumer purchasing behavior. This also implies that consumers may need to pay a premium for the product at a higher level of online review effectiveness.

(iv) The wholesale and retail prices of product L decrease as the preference coefficient of product E increases. This is because the increase in preference for product E intensifies the competition between the two products in the market, and the e-commerce platform can only increase sales by lowering prices.

(v) The price of product E rises as the percentage of commission received by e-commerce platform increases. This is because the sales cost of product E increases with the percentage of commission of the e-commerce platform, and the manufacturer will increase the price of product E to transfer the cost to consumers. At the same time, the manufacturer will lower the wholesale price of product L to sell more of it and make up for the loss of sales costs. However, the e-commerce platform will adopt the same strategy of raising prices as product E to gain more profit.

Proposition 4.4. (i) *When* $\frac{r^B}{r^R} > \frac{4A_3(r^R+2)-8A_4}{A_1(r^B)+A_2-2\theta(1+\eta)^2{c_E}^2}$, *then* $\pi_R^{WAR^*} > \pi_R^{WAB^*}$; *When* $\frac{r^B}{r^R} < \frac{4A_3(r^R+2)-8A_4}{A_1(r^B)+A_2-2\theta(1+\eta)^2{c_E}^2}$, *then* $\pi_R^{WAR^*} < \pi_R^{WAB^*}$.

(ii) *When* $\frac{r^B}{r^R} < \frac{2{c_E}^2(1-\theta)(r^R+2)+4B_1}{B_2(r^B)+4B_1+4{c_E}^2(1-\theta)}$, *then* $\pi_M^{WAR^*} > \pi_M^{WAB^*}$; *When* $\frac{r^B}{r^R} > \frac{2{c_E}^2(1-\theta)(r^R+2)+4B_1}{B_2(r^B)+4B_1+4{c_E}^2(1-\theta)}$, *then* $\pi_M^{WAR^*} < \pi_M^{WAB^*}$.

Where, $A_1 = {c_E}^2(4\eta+3\theta+3\eta^2\theta-10\eta\theta)+3c_L\theta(1-\eta)^2(c_L-2c_E)$,
$A_2 = 2\theta(1-\eta)^2\left[2c_Ec_L+8\eta{c_E}^2+(1-\rho)(1-\theta)(c_L-c_E)+\rho v(c_L-c_E)\right]$,
$A_3 = {c_E}^2(2-\eta)(1-\theta)$, $A_4 = (1-\eta)(1-\theta)\left[(c_E\theta-2K\theta)(1-\rho)(1-\eta)+{c_E}^2\right]$,
$B_1 = \theta(2K-c_E)(1-\rho)(1-\theta)(1-\eta)$, $B_2 = 2{c_E}^2(1-\theta)+\theta(1-\eta)(c_L-c_E)^2$.

Proposition 4.4(i) shows that if the e-commerce platform is willing to provide financing services, its profit under the e-commerce platform financing model will be higher than that under the bank financing model. Otherwise, the e-commerce platform is not willing to provide financing services.

(ii) If the manufacturer chooses the e-commerce platform financing mode, then its profit under the e-commerce platform financing mode is higher than that under the bank financing mode. Otherwise, the manufacturer chooses the bank financing mode.

Corollary 4.5. (i) *if* $\dfrac{4A_3(r^R+2)-8A_4}{A_1(r^B)+A_2-2\theta(1+\eta)^2{c_E}^2} < \dfrac{2{c_E}^2(1-\theta)(r^R+2)+4B_1}{B_2(r^B)+4B_1+4{c_E}^2(1-\theta)}$ *and* $\dfrac{4A_3(r^R+2)-8A_4}{A_1(r^B)+A_2-2\theta(1+\eta)^2{c_E}^2} < \dfrac{r^B}{r^R} < \dfrac{2{c_E}^2(1-\theta)(r^R+2)+4B_1}{B_2(r^B)+4B_1+4{c_E}^2(1-\theta)}$, *then* $\pi_R^{WAR^*} > \pi_R^{WAB^*}$ *and* $\pi_M^{WAR^*} > \pi_M^{WAB^*}$.

(ii) *if* $\dfrac{2{c_E}^2(1-\theta)(r^R+2)+4B_1}{B_2(r^B)+4B_1+4{c_E}^2(1-\theta)} < \dfrac{4A_3(r^R+2)-8A_4}{A_1(r^B)+A_2-2\theta(1+\eta)^2{c_E}^2}$ *and* $\dfrac{2{c_E}^2(1-\theta)(r^R+2)+4B_1}{B_2(r^B)+4B_1+4{c_E}^2(1-\theta)} < \dfrac{r^B}{r^R} < \dfrac{4A_3(r^R+2)-8A_4}{A_1(r^B)+A_2-2\theta(1+\eta)^2{c_E}^2}$, *then* $\pi_R^{WAR^*} < \pi_R^{WAB^*}$ *and* $\pi_M^{WAR^*} < \pi_M^{WAB^*}$.

Corollary 4.5 indicates that when the interest rates of e-commerce platforms and banks satisfy the condition (i), the manufacturer's profit is higher under the e-commerce platform financing mode than under the bank financing mode. At the same time, offering financing services can increase the e-commerce platform's profit, which is beneficial for both the e-commerce platform and the manufacturer, resulting in a win–win situation. Similarly, under condition (ii), the e-commerce platform is unwilling to provide financing services, and the manufacturer can obtain more profits under the bank financing mode. Therefore, bank financing is beneficial for both the manufacturer and the e-commerce platform, resulting in a win–win situation.

Proposition 4.6. *If the interest rates in the two financing modes are the same, that is,* $r^R = r^B = r$ *are then* $p_E^{WAR^*} = p_E^{WAB^*}$, $w_L^{WAR^*} > w_L^{WAB^*}$ *and*

(i) *if* $c_L > c_E$, *then* $p_L^{WAR^*} > p_L^{WAB^*}$, $D_L^{WAR^*} < D_L^{WAB^*}$, $D_E^{WAR^*} > D_E^{WAB^*}$;

(ii) *if* $c_L < c_E$, *then* $p_L^{WAR^*} < p_L^{WAB^*}$, $D_L^{WAR^*} > D_L^{WAB^*}$, $D_E^{WAR^*} < D_E^{WAB^*}$;

(iii) $\pi_M^{WAR^*} \geq \pi_M^{WAB^*}$.

Proposition 4.6 indicates that if the interest rate of the bank is equal to that of the e-commerce platform, then the retail price of product E through direct sales is the same under both financing modes. This suggests that the difference in direct sales prices under different financing modes is only determined by the interest rates. Additionally, the wholesale price of product L is higher under the e-commerce platform financing mode than under the bank financing mode. (i) When the cost of product L is high ($c_L > c_E$), the retail price of the distribution channel is higher in the e-commerce platform financing mode than in the bank financing mode. This indicates that the increase in wholesale prices leads to an increase in the cost of the e-commerce platform. To maintain its profits, the e-commerce platform will further increase the retail price. (ii) When the cost of product L is low, the retail price of the distribution channel is lower under the e-commerce platform financing mode than under the bank financing mode. This is because when the manufacturer chooses the e-commerce platform financing, it will increase the wholesale price to reduce the financing cost. At this time, the profit of the e-commerce platform consists of two parts: sales profit and financing profit. In order to increase the financing profit, the e-commerce platform will choose to offer lower retail prices to promote the increase in market demand of the distribution channel. (iii) When the loan interest rates are the same, the manufacturer always prefers to choose financing through the e-commerce platform.

In real life, the e-commerce platform and bank typically set different interest rates. This is because bank loans are only affected by the loan amount, while e-commerce platform is both a participant in product distribution and a borrower in manufacturer financing, making its interest rate influenced by both the manufacturer's business strategy and the loan amount. As a result, e-commerce platform and bank may set different interest rates. In this paper, we assume that both e-commerce platform and bank are rational decision-makers and will adjust their interest rates to maximize their own profits. Therefore, the following proposition is obtained.

Proposition 4.7. *When the manufacturer is in financing, the optimal loan interest rate is as follows:*

(i) *The optimal interest rate of the e-commerce platform is:*

$$r^{R^*} = \frac{\theta(1-\rho)(c_E-2K)(1-\eta)^2-{c_E}^2}{{c_E}^2(2-\eta)}. \tag{14}$$

(ii) *When* $\theta(1-\eta)(c_L-c_E)^2 < 2{c_E}^2(1-\theta)$, *the optimal interest rate of bank is:*

$$r^{B^*} = \frac{\theta(1-\theta)(1-\eta)(1-\rho)(c_L+c_E-4K) - \rho\theta\Delta v(1-\eta)(c_L-c_E) + \theta(1-\eta)(c_L-c_E)^2}{2\left[2{c_E}^2(1-\theta) - \theta(1-\eta)(c_L-c_E)^2\right]} - \frac{1}{2}. \tag{15}$$

Proposition 4.7 suggests that the optimal interest rates for both the e-commerce platform and the bank are influenced by market factors, such as consumer preferences for products and the accuracy of online reviews. These factors are determined by consumers and are difficult for businesses to change. Therefore, in practice, e-commerce platforms can focus on factors that can be modified, such as the revenue-sharing ratio and the production costs of both products L and E.

Corollary 4.8. $\dfrac{\partial r^{R^*}}{\partial c_E} < 0,\ \dfrac{\partial r^{R^*}}{\partial \theta} > 0,\ \dfrac{\partial r^{R^*}}{\partial \eta} < 0,\ \dfrac{\partial r^{j*}}{\partial K} < 0.$

Corollary 4.8 shows that the optimal interest rate of the e-commerce platform is only related to c_E, and the optimal interest rate decreases as c_E increases. This is because the e-commerce platform can control its profit factors by setting the price of product L. For product E, the optimal interest rate is the only factor that can be controlled to achieve profitability. When the manufacturer's cost increases, the retail price increases, leading to a decrease in demand for product E. At this point, the e-commerce platform will lower the interest rate to increase the manufacturer's financing amount. As the preference for product E increases and demand rises, the e-commerce platform will increase its interest rate to obtain higher financing income. Moreover, as the percentage of commission increases, the e-commerce platform will lower its interest rate because in price due to the higher commission rate leads to decrease in demand, and the decrease in interest rate can promote financing. Furthermore, as the initial capital of the manufacturer increases, both the e-commerce platform and the bank will lower the interest rate to encourage the manufacturer's willingness to participate in financing.

Proposition 4.9. *When* $c_L = c_E = c$, $r^{B^*} > r^{R^*}$.

Proposition 4.9 indicates that when the production costs of products L and E are equal for the manufacturer, the optimal interest rate of the bank is always higher than that of the e-commerce platform. The manufacturer can further control the interest rate and affect the choice of financing mode and the cost of the financing by adjusting the product cost.

5. Numerical analysis

The previous section has provided a detailed analysis of the equilibrium pricing and optimal interest rates under the two financing modes. In the following section, a numerical example using MATLAB is conducted to examine the impact of important parameters on the manufacturer's financing decisions and supply chain operations. Numerical analysis involves the use of computer technology for numerical simulation, widely applied in fields such as economics and management. While simple linear game models yield relatively easy analytical solutions, complex linear or nonlinear game models often require numerical analysis due to the complexity of analytical formulas. Numerical analysis proves useful in obtaining the impact of parameters on the behavior of game participants.

In line with Ling and Wang [8], who set the replacement rate of products from two manufacturers is 0.8 in the numerical simulation, we adopt the same, setting $\theta = 0.8$. Following the insights from Yang *et al.* [29], where the weight of online reviews on product evaluation is defined within the range $[0, 0.5]$, our numerical analysis use $\rho = 0.4$. Consistent with Parlaktürk [13] and Tang *et al.* [21], we let $c_L = 0.2$, $c_E = 0.25$, $\eta = 0.15$, $K = 0$. To ensure that the demand is greater than zero, we assume $\Delta v = -0.1$. Unless explicitly stated otherwise, these parameter values are assumed for all instances.

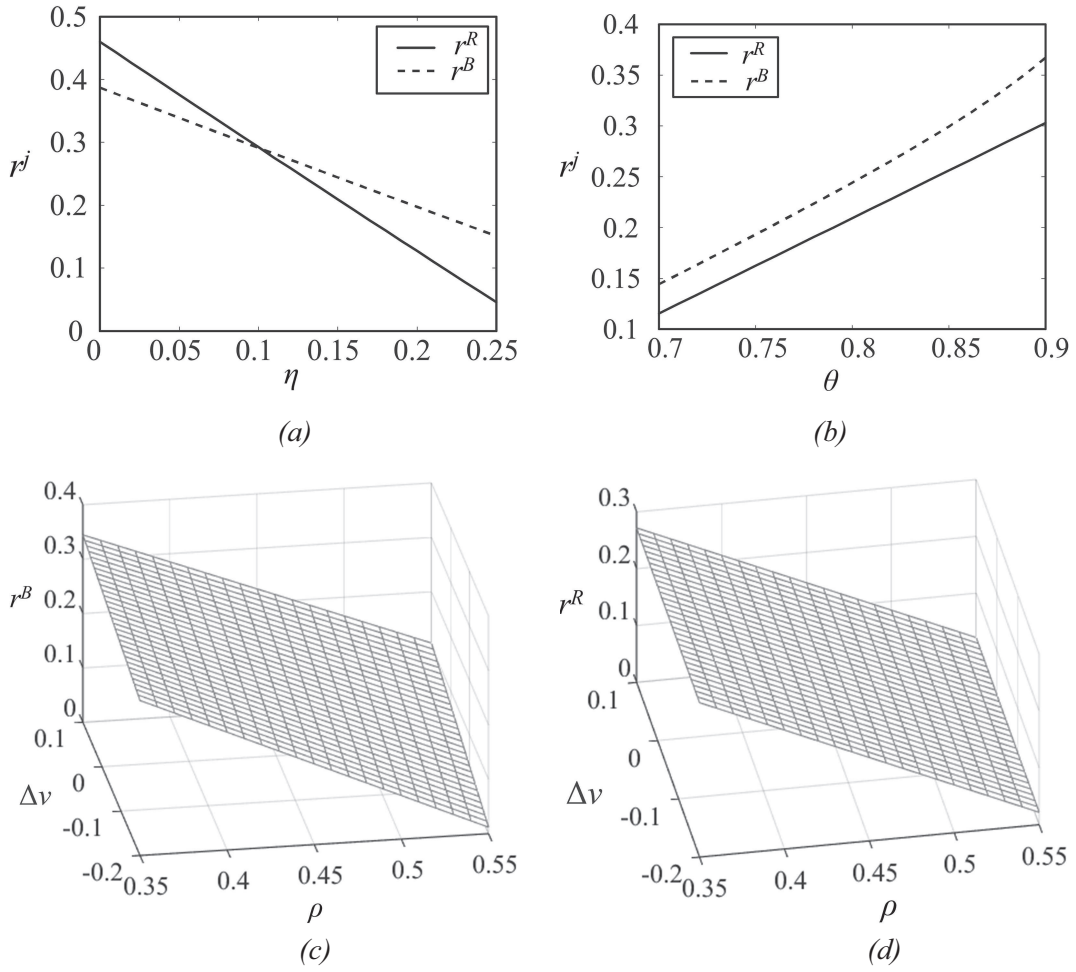

FIGURE 2. (a) Impacts of η on r^R and r^B. (b) Impacts of θ on r^R and r^B. (c) Impacts of ρ on Δv and r^B. (d) Impacts of ρ on Δv and r^R.

5.1. Interest rate analysis under different financing modes

This section compares the interest rates under two financing modes and analyzes the impacts of several key factors, including the percentage of the commission received by e-commerce platform (η), customer preference coefficient of product E (θ), online review differences (Δv), and weight of online reviews on product evaluation (ρ), on interest rate decisions.

(i) As shown in Figure 2a, when the percentage of commission received by e-commerce platform is low, the interest rate under the e-commerce financing mode is higher than that under the bank financing mode. And as he percentage of the commission received by e-commerce platform increases, the interest rates under both financing modes show a decreasing trend. However, the e-commerce platform's interest rate decreases at a faster rate, indicating that a higher percentage of the commission received by e-commerce platform would incentive it to lower its interest rate, thereby increasing the number of loans.

(ii) As shown in Figure 2b, the interest rate under the bank financing mode is always higher than that under the e-commerce financing mode. As the product preference θ of product E increases, the interest rates under

both financing modes show an increasing trend. This suggests that when customer preference coefficient of product E increases, the increase in interest rates can enable the e-commerce platform or bank to obtain more loan income.

(iii) As observed from Figures 2c and 2d, as the weight of online reviews on product evaluation (ρ) increases, the interest rates under different financing modes show a downward trend. When product L receives more positive online reviews compared to product E (Δv), the bank interest rate tends to increase, while the e-commerce platform interest rate remains unchanged. Additionally, it can be observed that the weight of online reviews on product evaluation has a significant impact on the changes in interest rates. Therefore, both banks and e-commerce platforms should focus on the changes of the weight of online reviews on product evaluation to adjust their interest rates.

5.2. Analysis of wholesale prices and retail prices under different financing modes

This section compares wholesale prices and retail prices under two financing modes and analyzes the impacts of several key factors, including the percentage of the commission received by e-commerce platform (η), customer preference coefficient of product E (θ), online review differences (Δv), and weight of online reviews on product evaluation (ρ) on wholesale prices and retail prices.

(i) As shown in Figure 3a, under the financing mode of the e-commerce platform, the manufacturer tends to offer lower wholesale prices for product L. Moreover, when the e-commerce platform has a higher percentage of commission, the prices of product L and E under the e-commerce platform financing mode are lower than those under the bank financing mode. This indicates that when the manufacturer chooses e-commerce platform financing and the percentage of commission received by e-commerce platform is high enough, both the manufacturer and the e-commerce platform are willing to offer lower retail prices to promote consumer purchases.

(ii) As shown in Figure 3b, in the e-commerce financing mode, both the wholesale prices and the prices for heterogeneous products are lower. And as customer preference coefficient of product E increases, the retail and wholesale prices of both product L and product E are lower in the e-commerce financing mode. This indicates that when product preference competition reaches a certain level, the income generated by the increased demand is sufficient to offset the losses caused by the decrease in retail prices. The Manufacturer tends to offer lower retail prices to promote the increase in sales of product E, and at this point, product L will also adopt the same low-price strategy (according to Prop. 4.4).

(iii) As shown in Figures 3c–3e, (A) when product L receives more positive online review information than product E, the wholesale and retail prices of product L under the e-commerce platform financing mode are lower than under the bank financing mode. This indicates that under the e-commerce platform financing mode, the higher the effectiveness of online reviews for product L, the more likely the manufacturer is to offer a lower wholesale price, and the e-commerce platform is more likely to offer a lower retail price. At the same time, the manufacturer will adopt the same pricing strategy for product E, promoting market demand increase. (B) As the weight of online reviews on product evaluation increases, the wholesale and retail prices for product L tend to increase in the same direction. When product L receives more positive online reviews than product E, as the weight of online reviews on product evaluation increases, the manufacturer will accelerate the reduction of product E's price under the e-commerce financing model to cope with the adverse effects of the increasing effectiveness of product L's online reviews. (C) Taking into account the dual effects of online review effectiveness and the weight of online reviews on product evaluation, when the negative difference of product L and E in online review effectiveness is greater and the weight of online reviews on product evaluation is higher, the wholesale and retail prices under the bank financing mode are both higher.

5.3. Profit analysis under different financing models

In this section, whether the e-commerce platform is willing to provide financing services and the manufacturer's strategic choices are analyzed by comparing the profit functions under two financing modes. Firstly, the

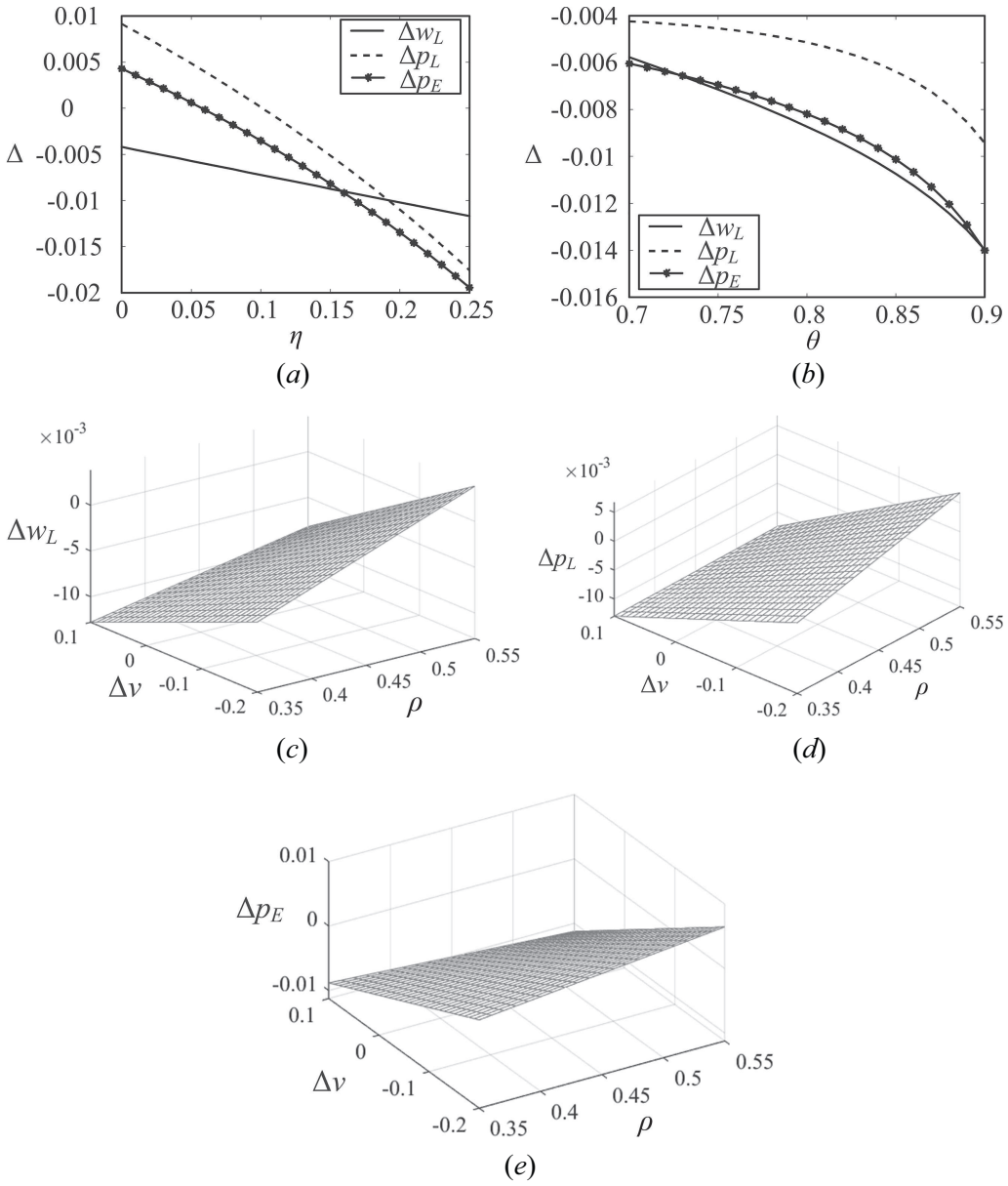

FIGURE 3. (a) Impacts of η on Δw_L, Δp_L and Δp_E. (b) Impacts of θ on Δw_L, Δp_L and Δp_E. (c) Impacts of ρ and Δv on Δw_L. (d) Impacts of ρ and Δv on Δp_L. (e) Impacts of ρ and Δv on Δp_E.

impact of the interest rates of the e-commerce platform and the bank on the manufacturer's financing decision is investigated. Secondly, under the optimal interest rate, the impacts of several key factors, including the percentage of the commission received by e-commerce platform (η), customer preference coefficient of product E (θ), online review differences (Δv), and weight of online reviews on product evaluation (ρ) on profits of e-commerce platform and manufacturer.

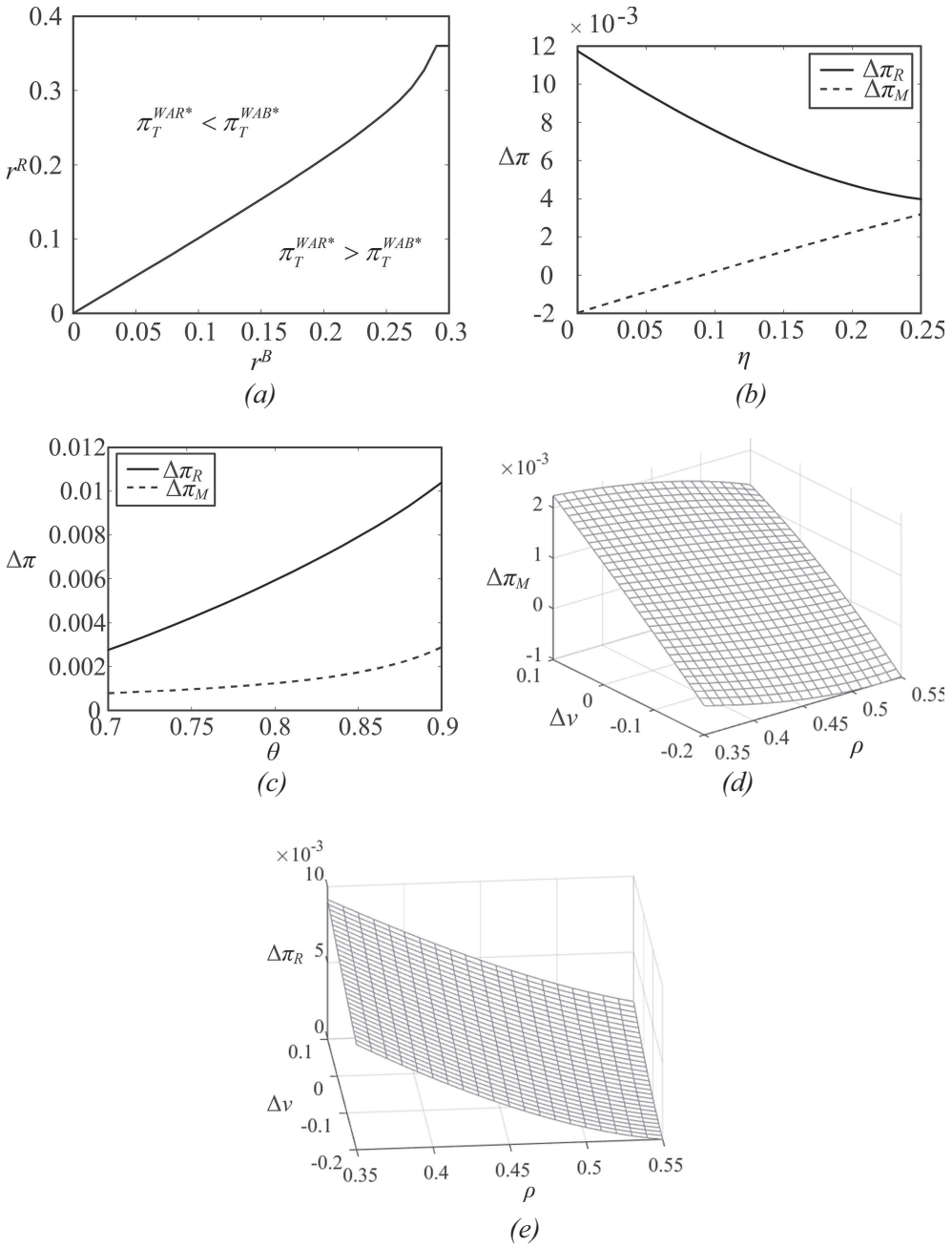

FIGURE 4. (a) Impacts of r^R and r^B on $\Delta\pi_M$. (b) Impacts of η on $\Delta\pi_M$ and $\Delta\pi_R$. (c) Impacts of θ on $\Delta\pi_M$ and $\Delta\pi_R$. (d) Impacts of ρ and Δv on $\Delta\pi_M$. (e) Impacts of ρ and Δv on $\Delta\pi_R$.

(i) When e-commerce platforms are willing to provide financing services, financially constrained manufacturer has two options. As shown in Figure 4a, when the bank interest rate is higher than the e-commerce platform interest rate, the manufacturer chooses e-commerce platform financing, which is consistent with reality. Even when the e-commerce platform interest rate is slightly higher than the bank interest rate, the manufacturer still tends to choose e-commerce platform financing because e-commerce platforms have a dual role in the supply chain, which indicates that e-commerce platforms can influence the manufacturer's financing mode by adjusting the interest rate.

(ii) As shown in Figure 4b, (A) when the percentage of the commission received by e-commerce platform is low, the manufacturer chooses bank financing; when the percentage of the commission received by e-commerce platform is high, the manufacturer chooses e-commerce platform financing. Therefore, the e-commerce platform can influence the financing strategy of the manufacturer by setting the percentage of the commission received by e-commerce platform. (B) When the manufacturer chooses e-commerce platform financing, the e-commerce platform can obtain more profit, which is independent of the percentage of the commission. Therefore, the e-commerce platform is always willing to provide financing services. (C) As the percentage of the commission received by e-commerce platform increases, the profit difference of manufacturer between the two modes gradually increases, while the profit difference of the e-commerce platform shows the opposite trend. Therefore, the e-commerce platform can set a reasonable percentage of the commission based on the profit of both parties.

(iii) As shown in Figure 4c, as the consumer preference coefficient of product E increases, the profit difference between the two financing modes for the e-commerce platform and the manufacturer shows an upward trend, and the profit of the e-commerce platform increases more rapidly. This suggests that as the consumer preference coefficient of product E increases, the competition in the markets for products L and E becomes more intense, which is more advantageous for both the e-commerce platform and the manufacturer.

(iv) As shown in Figures 4d and 4e, (A) When product L receives more positive online review information compared to product E, the manufacturer prefers e-commerce financing. When product L receives more negative reviews than product E, the manufacturer gradually shifts to bank financing. (B) The e-commerce platform is always willing to provide financing services, regardless of online review information. (C) When product L receives higher positive online review information compared to product E, and when weight of online reviews on product evaluation is low, the e-commerce financing model can generate greater profits for the e-commerce platform and the manufacturer. Therefore, the greater the preference for product L in terms of positive online review information, the more beneficial it is for both the e-commerce platform and the manufacturer. The manufacturer should increase the effectiveness of online reviews for product L. (D) As the weight of online reviews on product evaluation increases, the profit difference between the two financing models gradually decreases. The e-commerce platform experiences a faster decrease in profit difference between the two financing models. Combining Figures 3d and 3e, the increase in price under the e-commerce financing model gradually reduces profits, leading to a smaller advantage in providing financing services to the e-commerce platform.

6. Conclusion

6.1. Main conclusion

Although dual-channel supply chain and supply chain finance are popular research topics, there has been limited attention paid to the interaction between financial constraints and dual-channel operations in the context of supply chain management. Previous studies on dual-channel supply chain have mainly focused on single-product online and offline channels, while this study examines a heterogeneous dual-channel based on an e-commerce platform. In this setting, the manufacturer wholesales product L to the platform for distribution and sells product E directly by joining the e-commerce platform. By considering the impact of online reviews on consumer utility, the pricing strategies of the manufacturer and e-commerce platform is analyzed by constructing a supply chain financing model. Additionally, the financing strategies adopted by the financially constrained

manufacturer are investigated. Furthermore, the main contributions and conclusions of the paper are summarized below.

When the additional utility difference between two products due to online reviews falls within a certain range, an equilibrium solution exists. The study analyzes the feasibility conditions for e-commerce platforms to provide financing services and the financing preferences of financially constrained manufacturers, based on the equilibrium pricing decision under e-commerce platform financing and bank financing. When the relative interest rates of the e-commerce platform and bank fall within a certain range, both the manufacturer and the e-commerce platform can achieve a "win–win" situation under the same financing model.

When the interest rates of the e-commerce platform and the bank are the same, the manufacturer tends to offer a higher wholesale price under the e-commerce platform financing mode. However, the retail price for distribution depends on the cost of selling the products directly. In this case, the manufacturer prefers to choose the e-commerce platform financing mode. At the optimal interest rate, when the costs of heterogeneous products are equal, the optimal interest rate of the bank is always higher than that of the e-commerce platform. The e-commerce platform is always willing to provide financing services, but the manufacturer will only choose e-commerce platform financing when the percentage of the commission received by e-commerce platform is high and there is a significant positive difference in online reviews between product L and product E. Meanwhile, under the e-commerce platform financing mode, both the e-commerce platform and the manufacturer are willing to offer lower retail prices to attract more consumers.

6.2. Managerial insights

This paper provides valuable insights into the financing strategies and decision-making processes of supply chain members in an online dual-channel supply chain dominated by the manufacturer. Some managerial insights can be summarized as follows.

No matter what financing mode is adopted, key parameters related to heterogeneous products (online review, customer preference, cost) significantly impact the decisions of supply chain members. Manufactures, when making pricing and financing model decisions, should pay more attention the comprehensive influence of heterogeneous product on the operational decision of the supply chain. For manufacturers, there are no optimal financing model. The relative interest rates of e-commerce platforms and banks play a crucial role in the loan provision of e-commerce platforms and the financing mode choices of manufacturers. In practice, manufacturers should fully understand the interest rate situation to make informed decisions and achieve mutually beneficial results. For e-commerce platform, the percentage of the commission received by e-commerce platform and the differences in the effectiveness of heterogeneous product online reviews affect the manufacturers' financing model selection decisions. In order to guide manufacturers to choose e-commerce platform financing models, the e-commerce platforms should negotiate for a higher commission during discussions with manufacturers and encourage positive product reviews on the direct channel by providing high-quality service.

While many studies have explored the strategy of offering heterogeneous products in dual-channel supply chains and the choice of financing modes in capital-constrained supply chains, limited attention has been given to the interaction between heterogeneous products in online dual-channel supply chains and the financing behavior of capital-constrained manufacturers. However, in the online dual-channel supply chain, many manufacturers encounter capital shortage issues. Thus, by expressing differences in heterogeneous products through consumer preferences and online reviews, this paper discusses heterogeneous product's pricing of enterprises in the online dual-channel supply chain under two financing modes, and analyzes the financing mode selection behavior of manufacturers. This research not only broadens the research perspective on dual supply chains but also contributes to the development of sustainable supply chain financing. It has practical implications for the operation and management of supply chains facing funding constraints. While the study considers the percentage of the commission received by e-commerce platform as an exogenous variable, in reality, the platform may adjust this percentage of the commission to maximize profits. Therefore, in the future research, percentage of the commission received by e-commerce platform could be considered as an endogenous variable that affects supply chain operations. In addition, while the study focuses on the impact of online reviews and product preferences

on demand, other factors such as channel preference may also influence demand and warrant further exploration in future research.

6.3. Limitations and future research

In the future, this paper can be extended from the following aspects. Firstly, in this paper, during the model construction process, we assume that the percentage of the commission is an exogenous variable. In reality, the e-commerce platform may adjust the percentage of the commission to impact manufactures' decision-making. Thus, this study can be extended by considering the case where percentage of the commission received by e-commerce platform could be considered as an endogenous variable that affects online dual-supply chain operations. Secondly, this study focuses on the impact of online reviews and product preferences on demand. Different online channels can have an impact on consumer purchasing behavior, such as, compared to merchants opening stores on the JD platform, consumers are willing to purchase products from JD flagship stores. Therefore, it will be interesting to explore how consumer preferences for online dual channels would affect the results of this study. Thirdly, in the online dual channel cooperation model of heterogeneous products, this paper only considers the case where product L adopts wholesale mode and product E adopts agency mode. In the future, other cooperation models for online dual channel sales of heterogeneous products can be further explored, and the impact of different cooperation models on financing mode selection can be compared. Finally, this paper considers that supply chain members are all rational decision makers, but in fact, members will be affected by irrational behavior in decision-making, such as risk aversion, fairness concern, altruistic preference, etc., and the role of irrational behavior in supply chain financing decision-making can be further explored in the future.

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Appendix A.

Proof of Proposition 4.1. The backward induction is used to solve the problem. Firstly, we take the derivative of equation (4) with respect to p_L and set $\frac{\partial \pi_R^{WAR}}{\partial p_L} = 0$, then $p_L = \frac{w_L + (c_E - c_L)r^R + (1+\eta)p_E + \rho\Delta v + (1-\rho)(1-\theta)}{2}$ is obtained. Substituting p_L into equation (3), according to $\frac{\partial^2 \pi_M^{WAR}}{\partial {w_E}^2} = -\frac{1}{(1-\theta)(1-\rho)} < 0$, $\frac{\partial^2 \pi_M^{WAR}}{\partial {p_L}^2} = -\frac{(1-\eta)(2-\theta-\theta\eta)}{\theta(1-\theta)(1-\rho)} < 0$, and $\frac{\partial^2 \pi_M^{WAR}}{\partial {w_L}^2}\frac{\partial^2 \pi_M^{WAR}}{\partial {p_E}^2} - \left(\frac{\partial^2 \pi_M^{WAR}}{\partial p_E \partial w_L}\right)^2 = \frac{2(1-\eta)}{\theta(1-\theta)(1-\rho)^2} > 0$, π_M^{WAt} is a jointly concave function with respect to w_L and p_E. By solving the system of equations $\frac{\partial \pi_M^{WAR}}{\partial w_L} = 0$, and $\frac{\partial \pi_M^{WAR}}{\partial p_E} = 0$, (5) and (6) are obtained. Substituting equations (5) and (6) into p_L, equation (7) is gotten. Since demand is greater than 0, the constraint of Δv is obtained. Thus, Proposition 4.1 is proved. □

Proof of Proposition 4.4. Let the equilibrium solution substituted into the profit function of the two different financing modes, it is obtained as

$$\Delta\pi_R = \pi_R^{WAR^*} - \pi_R^{WAB^*} = \frac{r^B\left[A_1(r^B) + A_2 - 2\theta(1+\eta)^2{c_E}^2\right] - r^R\left[4A_3(r^R+2) - 8A_4\right]}{16\theta(1-\rho)(1-\theta)(1-\eta)^2}.$$

Set $\Delta\pi_R > 0$ or $\Delta\pi_R < 0$, Proposition 4.4(i) can be obtained. In addition, $\Delta\pi_M = \pi_M^{WAR^*} - \pi_M^{WAB^*} = \frac{r^R\left[2{c_E}^2(1-\theta)(r^R+2)+4B_1\right]-r^B\left[B_2(r^B)+4B_1+4{c_E}^2(1-\theta)\right]}{8\theta(1-\rho)(1-\theta)(1-\eta)}$.

Set $\Delta\pi_M > 0$ or $\Delta\pi_M < 0$, Proposition 4.4(ii) can be obtained. □

Proof of Proposition 4.6. If $r^R = r^B = r$ then,

$$\Delta w_L = w_L^{WAR^*} - w_L^{WAB^*} = \frac{c_E(2r^R - r^B)}{2} = \frac{c_E r}{2} > 0$$

$$\Delta p_E = p_E^{WAR^*} - p_E^{WAB^*} = \frac{c_E(r^R - r^B)}{2(1-\eta)} = 0$$

$$\Delta\pi_M = \pi_M^{WAR^*} - \pi_M^{WAB^*} = \frac{(c_L - c_E)^2 r^2}{8(1-\rho)(1-\theta)} \geq 0$$

$$\Delta p_L = p_L^{WAR^*} - p_L^{WAB^*} = \frac{c_L r^B(1-\eta) + c_E(2r^R - 3r^B + \eta r^B)}{4(1-\eta)} = \frac{(c_L - c_E)r}{4}$$

$$\Delta D_L = D_L^{WAR^*} - D_L^{WAB^*} = \frac{(c_E - c_L)r^B}{4(1-\rho)(1-\theta)} = \frac{(c_E - c_L)r}{4(1-\rho)(1-\theta)}$$

$$\Delta D_E = D_E^{WAR^*} - D_E^{WAB^*} = \frac{c_L\theta(1-\eta)r^B - 2c_E(r^R - r^B) + 2\theta c_E r^R - c_E\theta r^B(3-\eta)}{4\theta(1-\rho)(1-\theta)(1-\eta)} = \frac{(c_L - c_E)r}{4(1-\rho)(1-\theta)}.$$

When $c_L > c_E$, then $\Delta p_L > 0$, $\Delta D_L < 0$, $\Delta D_E > 0$.

When $c_L < c_E$, then $\Delta p_L < 0$, $\Delta D_L > 0$, $\Delta D_E < 0$. □

Proof of Proposition 4.7. Substitute the conclusion of Proposition 4.1 into equation (4). According to $\frac{\partial^2\pi_R^{WAR^*}}{\partial(r^R)^2} = -\frac{{c_E}^2(2-\eta)}{2(1-\eta)^2(1-\rho)} < 0$, $\pi_O^{WAR^*}$ is a concave function with respect to r^R. Let $\frac{\partial\pi_R^{WAR^*}}{\partial r^R} = 0$, Proposition 4.7(i) can be obtained. Substitute the conclusion of Proposition 4.2 into equation (9). According to $\frac{\partial^2\pi_B^{WAB^*}}{\partial(r^B)^2} = \frac{\theta(1-\eta)(c_L - c_E)^2 - 2{c_E}^2(1-\theta)}{2\theta(1-\rho)(1-\theta)(1-\eta)}$, when $\theta(1-\eta)(c_L - c_E)^2 < 2{c_E}^2(1-\theta)$, $\pi_B^{WAB^*}$ is a concave function with respect to r^B. Let $\frac{\partial\pi_B^{WAB^*}}{\partial r^B} = 0$, Proposition 4.7(ii) can be obtained. □

Proof of Proposition 4.9. Substitute $c_L = c_E = c$ into equations (14) and (15), $r^{B^*} - r^{R^*} = \frac{\eta\left[\theta(1-\rho)(c-2K)(1-\eta)+c^2\right]}{c^2(2-\eta)}$ is obtained. According to $r^{R^*} = \frac{\theta(1-\rho)(c_E-2K)(1-\eta)^2 - {c_E}^2}{{c_E}^2(2-\eta)} > 0$, $c_E > 2K$ can be obtained. Then, we can get $r^{B^*} > r^{R^*}$. □

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