RAIRO-Oper. Res. 58 (2024) 4651–4680
https://doi.org/10.1051/ro/2024177

RAIRO Operations Research
www.rairo-ro.org

INTERVAL EFFICIENCY ESTIMATION USING RELATIONAL DYNAMIC DEA APPROACH: CASE OF INDIAN BANKS

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Abstract. Data envelopment analysis (DEA) facilitates relative performance estimation of homogenous decision-making units (DMUs), whereas dynamic DEA pertains to intertemporal elements like carryovers for measuring DMUs' efficiencies over time. To enhance DMU's performance, there is a need to measure its efficiency with dynamic structure and further to determine input-output target points for making significant improvements in an inefficient DMU. Therefore, the present study proposes a relational dynamic DEA approach that comprises a dynamic structure in which periods are connected through good and bad links/carry-overs along with desirable and undesirable outputs and utilizes interval data. The system and period efficiency intervals are derived using a unique set of weights based on common weights methodology. Moreover, the relationship between the complement of the lower (upper) bound system and period efficiencies is established. Lastly, the input-output target points are suggested to improve DMUs in terms of upper-bound system efficiencies in a dynamic environment. To validate the usefulness of the proposed approach, a case study in Indian banks for the period 2017–2021 is presented. This study investigates the impact of stressed assets (bad link/carryover) and loss due to non-performing assets (bad/undesirable output) on banks' dynamic efficiency. The results are compared with the static structure. The findings, targets, and implications of the study can assist bank experts and policymakers in formulating policies/strategies for further improvements.

Mathematics Subject Classification. 90B50, 90C05, 90C90, 97M40.

Received October 31, 2023. Accepted September 11, 2024.

1. Introduction

Performance evaluation is a prime aspect of controlling and planning the performance of profit and non-profit organizations [19]. Typically, the financial sector is the fundamental part of any nation's monetary framework, and hence assessing the effectiveness of the same assumes an essential part of economic growth and development. To assess efficiency, the existing literature suggests two main approaches, namely, 'ratio approach' and 'frontier approach'. In the ratio approach, a ratio is assessed by focusing on one or more outputs and their corresponding inputs to perform an efficiency analysis of decision-making units (DMUs), whereas the frontier approach deals with the production frontier for classifying DMUs as efficient/inefficient. However, to apply the ratio approach, one requires selecting the ratio and determining the benchmark against which an organization's productivity is to be assessed. Due to this, the frontier approach has been widely used by researchers for which there are five

Keywords. Dynamic DEA, imprecise data, interval efficiencies, system targets, relational model.

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main techniques available in the literature [13]. The three techniques, namely, thick frontier approach (TFA), stochastic frontier analysis (SFA), and distribution-free approach (DFA), require a specified production frontier and come under the category of parametric one. In contrast, the other two techniques, namely, free disposal hull (FDH) and data envelopment analysis (DEA) do not require a predefined production frontier and are known as non-parametric techniques [55]. The development of methods for efficiency evaluation of DMUs and their suggested improvements play a significant part in the development of DEA [50].

DEA is an efficacious technique that assesses the performance of similar DMUs in terms of multiple inputs/outputs [16]. For the past three decades, it has grown considerably in the direction of improvement in modeling to represent the complexities of real systems. This evolution also facilitated a large number of real-world problems in different areas and encouraged researchers and practitioners to conduct research and development. DEA effectively classifies DMUs as efficient and inefficient in both profit and non-profit organizations and investigates factors that cause inefficiency. CCR [16] and BCC [10] models are the basic radial models available in DEA to analyze the performance with constant and variable returns to scale, respectively. Tone [67] presented a non-radial model called slacks-based measure (SBM) to assess the input excesses and output shortfalls. Further, Scheel [62], Jahanshahloo *et al.* [37], and Tone and Tsutsui [68] investigated the efficiency of DMUs with undesirable inputs/outputs. The theoretical and methodological extensions of DEA models are reported in Cooper *et al.* [19], Cook and Seiford [17], Emrouznejad and Yang [23], and Amirteimoori *et al.* [5]. Despite various developments and advantages, there are two significant limitations of DEA which are listed as follows.

(i) DEA assumes crisp data for inputs and outputs; however, the variables like environment pollution and customer satisfaction can only be estimated in imprecise form.
(ii) the conventional DEA investigates a DMU's performance statically for a particular period by ignoring the presence of intertemporal elements like carryovers between consecutive periods that may yield misleading efficiency results/measures.

To deal with the first limitation, Cooper *et al.* [18] introduced imprecise DEA models to measure efficiency in uncertain environments. Despotis and Smirlis [21] presented an alternative DEA model to deal with imprecise data (mixture of exact, interval, and ordinal data) using the unified frontier and provided input targets for inefficient DMUs to make them efficient. Mo *et al.* [52] proposed slacks-based DEA models to evaluate the lower and upper bound efficiencies in the most unfavorable and most favorable situations, respectively, with interval data in the presence of undesirable outputs. Izadikhah *et al.* [39] also presented imprecise DEA models that can handle fuzzy or ordinal data by converting it into interval numbers. Later, Zhang *et al.* [81] addressed the issue of imprecision for DMUs with network structures in DEA and assessed the lower and upper bound efficiencies in pessimistic and optimistic environments along with the targets for efficiency improvements. Some other researchers have also addressed imprecision in DEA as interval numbers, fuzzy numbers, linguistic variables, etc. [1,6,35,56,57,70,74]. The core idea in the above-mentioned studies lies in assessing the efficiency of DMUs in a static environment without considering the intertemporal dependence among periods. To overcome this limitation of DEA, the inter-relations between consecutive periods were incorporated to evaluate the intertemporal efficiency of a DMU in a dynamic environment. Many researchers have developed approaches like the Malmquist DEA index, window analysis, and dynamic DEA to assess efficiency that changes over time [2,15]. Both Malmquist DEA index and window analysis approaches consider the time effect and ignore intertemporal elements like carryovers between two periods. Therefore, these approaches intend to optimize DMU's performance in every single term independently and locally [72]. To investigate the performance of multi-period DMUs in the presence of intertemporal elements, Färe and Grosskopf [25] introduced the concept of dynamic DEA. Emrouznejad and Thanassoulis [22] evaluated the dynamic efficiency by considering the intertemporal dependency of inputs-outputs caused by the capital stock. Tone and Tsutsui [72] introduced a slacks-based model to compute dynamic efficiency when all the periods are linked through carryovers. Kao [40] presented a DEA model considering dynamic factors to evaluate efficiencies for the system as well as the periods with the same weights assigned to the carry-overs, irrespective of (i) the period they belong to, and (ii) whether they act as input or output. Hajiagha *et al.* [33] presented a method to determine the common set of weights in a dynamic DEA for measuring multi-period efficiency using mean-variance criteria. Mariz *et al.* [50] classified intertemporal elements as (i) intermediate inputs/outputs, (ii) storable inputs, (iii) quasi-fixed inputs, (iv) lagged productive effects, (v) adjustment costs, and (vi) carry-overs. Later, Mavi and Mavi [51] presented a common set of weights methodology that not only assesses the dynamic efficiency of DMUs in DEA but also highlights the significance of utilizing common weights to improve the discrimination power of the model in dynamic DEA. Bansal and Mehra [11] evaluated the efficiency of DMUs with negative and interval data in dynamic environments. Soleimani-Chamkhorami and Ghobadi [66] considered the dependency of data in different periods using reserve capital for an assessment window and examined the cost efficiency of DMUs in dynamic framework. Later, Ghobadi *et al.* [29] proposed dynamic inverse DEA models to suggest the least input augmentation and the highest output augmentation to maintain or improve the efficiency of DMUs in dynamic structures with data interdependence. Yekta *et al.* [79] developed a dynamic DEA approach to achieve strictly positive and common weights and further applied it to the Iranian airlines with carryovers linking the consecutive periods.

To deal with data uncertainty in a dynamic environment, many researchers have presented dynamic DEA models with data imprecision of interval form which are reported in studies like Keikha-Javan *et al.* [41], Li *et al.* [46], and Bansal and Mehra [11] etc. The detailed reviews on efficiency in dynamic DEA are presented by Fallah-Fini *et al.* [24] and Mariz *et al.* [50]. Dynamic DEA has been applied to various organizations like electric utilities [53], transmission system operators [73], energy consumption [42], and banking sector [12] etc. In real situations, intertemporal elements like intermediate inputs/outputs and carry-overs in a dynamic environment might be undesirable (bad) [48] and should be reduced. For example, non-performing assets (NPAs) and losses carried forward in a banking sector [11] are two undesirable factors that refrain banks from producing income and need to be reduced. As far as handling undesirable outputs in dynamic DEA is concerned, Lotfi and Poursakhi [48], Tone and Tsutsui [69], Bangarwa and Roy [9], and Bansal and Mehra [11] examined the dynamic efficiency of DMUs in the presence of undesirable resources.

The available literature on dynamic DEA possesses certain gaps that need to be addressed to handle present practical situations. The existing studies lack in (i) considering the initial and terminal links, in conjunction with the undesirable outputs and imprecise data (particularly, interval data) in one framework in the production process of DMUs with dynamic structure, and (ii) providing target points to strengthen system inefficient DMUs in dynamic DEA. However, it is very important to suggest not only what to improve but how to improve. The present study tries to overcome the above-mentioned gaps. The main focus of the present study that distinguishes it from the existing ones is to develop a relational dynamic interval DEA approach that considers a dynamic structure that not only incorporates good/bad links (both initial and terminal) and undesirable outputs but also handles data uncertainty of interval form. It estimates system and period interval efficiencies along with their relationships. Moreover, it provides specific targets to eliminate the system inefficiencies and hence imparts the capability to suggest what to improve and how to improve in order to enhance DMUs' system efficiency. Lastly, to prove the applicability of the proposed relational dynamic interval DEA approach in real-world, it has been implemented to a set of domestic Indian banks. For validation, the proposed approach is compared against the static approach as a special case.

The rest of the paper is designed as follows. Section 2 briefly reviews DEA with undesirable outputs. Section 3 describes the novel relational dynamic interval DEA approach. Section 4 presents target points for upper-bound system efficiency. Section 5 depicts a complete interval dynamic DEA performance decision process. Section 6 presents a case study in Indian banks and a comparison with the static DEA approach. Section 7 concludes the present work along with practical implications and future scope.

2. DEA with undesirable outputs

In a dynamic structure, the consecutive periods can be linked by desirable (good) and undesirable (bad) links along with individual inputs and outputs (both good and bad) [48]. In existing DEA literature, undesirable resources are treated in different ways categorized as direct and indirect approaches [34, 62]. In the indirect approach, the data for bad output $\left(u^b\right)$ is translated using a monotone decreasing function f to transform it into desirable output before incorporating it into the DEA model. The indirect approaches available in the literature are (i) additive inverse (ADD) approach in which u^b is transformed as $f\left(u^b\right)=-u^b$ [43] so that the undesirable output becomes desirable, (ii) translated (TRβ) approach in which u^b is transformed as $f\left(u^b\right)=-u^b+\beta$ [3], where β is a sufficiently large scalar to make $f\left(u^b\right)$ positive and (iii) multiplicative inverse (MLT) approach which transforms u^b as $f\left(u^b\right)=1/u^b$ [30], *i.e.*, incorporating the inverse of an undesirable output into the DEA model. In contrast, the direct approach treats the undesirable outputs directly in the model. The other approach, reviewed by Halkos and Petrou [34], is either ignoring the undesirable output or incorporating it into a non-linear model. Among all the prevailing approaches for dealing with undesirable outputs, the direct approaches have procured more attention from researchers and are widely used in the DEA literature. Korhonen and Luptacik [44] introduced a direct approach to deal with undesirable outputs in DEA models. They have assigned negative weights to the undesirable outputs in the weighted sum of all outputs (desirable and undesirable), which augments the desirable outputs and reduces the undesirable ones simultaneously. Puri and Yadav [56] pointed out that Korhonen and Luptacik's approach [44] may produce negative efficiency while evaluating the cross efficiency, where cross efficiency for DMU_j is the efficiency evaluated using optimal weights obtained for DMU_k. Therefore, Puri and Yadav [56] extended their approach by adding an additional set of constraints stating that the efficiency of every DMU should be greater than or equal to zero. To better understand the direct approach mentioned above, a DEA model suggested by Puri and Yadav [56] is discussed below.

Consider n DMUs wherein each DMU produces s outputs (s_1 desirable + s_2 undesirable) utilizing m inputs. For DMU_j, x_{ij} be the ith input, y^g_{rj} be the rth desirable output, and y^b_{qj} be the qth undesirable output and v_i, u^g_r and u^b_q be their respective weights. Then, the efficiency (E_k) of DMU_k is evaluated as the ratio of virtual output to the virtual input of DMU_k and is defined as $E_k=\left(\sum_{r=1}^{s_1}u^g_r y^g_{rk}-\sum_{q=1}^{s_2}u^b_q y^b_{qk}\right)/\sum_{i=1}^{m}v_i x_{ik}$.

Further, to compute the relative efficiency of DMU_k, E_k is maximized subject to the constraint that the efficiency of every DMU_j (E_j) belongs to $[0,1]$. Mathematically, it is given by the following Model-1:

Model-1

$$
\begin{aligned}
&\max \quad E_k=\frac{\sum_{r=1}^{s_1}u^g_r y^g_{rk}-\sum_{q=1}^{s_2}u^b_q y^b_{qk}}{\sum_{i=1}^{m}v_i x_{ik}}\\
&\text{subject to} \quad 0\le E_j=\frac{\sum_{r=1}^{s_1}u^g_r y^g_{rj}-\sum_{q=1}^{s_2}u^b_q y^b_{qj}}{\sum_{i=1}^{m}v_i x_{ij}}\le 1,\ \forall j=1,2,\ldots,n,\\
&u^g_r\ge\epsilon\ \forall r;\ \ u^b_q\ge\epsilon\ \forall q;\ \ v_i\ge\epsilon\ \forall i;\ \ \epsilon>0,
\end{aligned}
$$

where ϵ is a small positive constant.

The present study analyzes the impact of loss due to NPAs (undesirable output) and stressed assets (undesirable link) on the dynamic efficiency of banks, and the suitable approach for this analysis is the direct approach of incorporating the bad outputs and links into the model with negative weights. Moreover, Model-1 reflects the static efficiency of DMU_k in a specific time period using precisely defined input-output data. It ignores inter-relationships between consecutive periods reflected by carry-over activities like unused assets (good carry-over) and stressed assets (bad carry-over) in the banking sector. Therefore, there is a need to analyze efficiency in a dynamic manner that truly reflects the intertemporal relationships between consecutive periods in the form of carry-overs (links) and evaluates the system and period efficiencies in a dynamic environment using imprecise data, particularly interval data.

3. Proposed relational dynamic DEA approach with interval data

The growth of financial institutions in a country has a direct impact on economic growth. The financial organization often engages in several functions. For example, a bank is a financial entity that transforms deposits into loans and investments, exchanges, brokerage, sale of investments, sale of revaluations, and sale of insurance

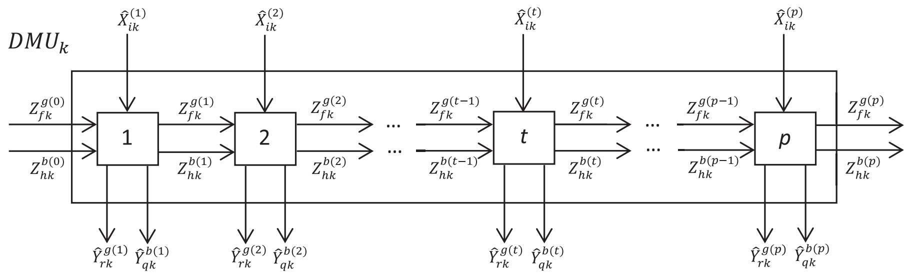


FIGURE 1. Dynamic structure of DMU_k with interval data over p periods.

TABLE 1. Nomenclature.

Notation	Description
n	Number of DMUs.
p	Number of periods.
m	Total number of inputs.
s1	Total number of desirable outputs.
s2	Total number of undesirable outputs.
c1	Total number of good links.
c2	Total number of bad links.
$\hat{X}_{ij}^{(t)}$	ith interval input consumed by DMU_j in period t with value $\left[X_{ij}^{(t)L}, X_{ij}^{(t)U}\right]$, where $X_{ij}^{(t)L}$ and $X_{ij}^{(t)L}$ are the lower and upper bounds of the respective interval input.
$\hat{Y}_{rj}^{g(t)}$	rth interval desirable output produced by DMU_j in period t with value $\left[Y_{rj}^{g(t)L}, Y_{rj}^{g(t)U}\right]$, where $Y_{rj}^{g(t)L}$ and $Y_{rj}^{g(t)U}$ are the lower and upper bounds of the respective interval desirable output.
$\hat{Y}_{qj}^{b(t)}$	qth interval undesirable output produced by DMU_j in period t with value $\left[Y_{qj}^{b(t)L}, Y_{qj}^{b(t)U}\right]$, where $Y_{qj}^{b(t)L}$ and $Y_{qj}^{b(t)U}$ are the lower and upper bounds of the respective interval undesirable output.
$Z_{fj}^{g(t)}$	fth good link produced by DMU_j in period t and consumed in period $t+1$, where $t = 1, \ldots, p$.
$Z_{hj}^{b(t)}$	hth bad link produced by DMU_j in period t and consumed in period $t+1$, where $t = 1, \ldots, p$.
$Z_{fj}^{g(0)}$	fth initial good link consumed by the first period.
$Z_{hj}^{b(0)}$	hth initial bad link consumed by the first period.
$\hat{X}_{ij}$	ith interval input consumed by DMU_j with value $\sum_{t=1}^{p} \hat{X}_{ij}^{(t)}$.
$\hat{Y}_{rj}^{g}$	rth interval desirable output produced by DMU_j with value $\sum_{t=1}^{p} \hat{Y}_{rj}^{g(t)}$.
$\hat{Y}_{qj}^{b}$	qth interval undesirable output produced by DMU_j with value $\sum_{t=1}^{p} \hat{Y}_{qj}^{b(t)}$.

products. The source of a bank's income primarily depends on the interest earned by converting deposits into loans and investments. Therefore, to analyze such a specific and primary function of any DMU like a bank, a simple structure of p periods with interval data and links (good and bad) connecting the periods in a dynamic environment is studied in the present paper as depicted in Figure 1. The nomenclature for the proposed approach is given in Table 1.

The intermediates (links) in the present study represent the interconnection between two consecutive periods, and the links produced by a period act as input for the next period. The present study assumes that the total amount of a link produced by a DMU in any period is fully utilized by that DMU in successive period to maximize the dynamic system efficiencies. Therefore, the intermediates are considered to act as dual-role factors (output of tth period and input to $(t+1)$th period) with precise values in the current study. However to incorporate the crisp intermediates into the proposed interval dynamic DEA model, these are considered as intervals with equal lower and upper bounds. Following the nomenclature given in Table 1, the system efficiency $\left(\hat{E}_k^{(a)}\right)$ of DMU_k is given by

$$\hat{E}_k^{(a)} = \frac{\sum_{r=1}^{s_1} u_r^g \hat{Y}_{rk}^g - \sum_{q=1}^{s_2} u_q^b \hat{Y}_{qk}^b + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)}}{\sum_{i=1}^{m} v_i \hat{X}_{ik} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)}} \tag{1}$$

$$= \frac{\sum_{r=1}^{s_1} u_r^g \left[Y_{rk}^{gL}, Y_{rk}^{gU}\right] - \sum_{q=1}^{s_2} u_q^b \left[Y_{qk}^{bL}, Y_{qk}^{bU}\right] + \sum_{f=1}^{c_1} w_f^g \left[Z_{fk}^{g(p)}, Z_{fk}^{g(p)}\right] - \sum_{h=1}^{c_2} w_h^b \left[Z_{hk}^{b(p)}, Z_{hk}^{b(p)}\right]}{\sum_{i=1}^{m} v_i \left[X_{ik}^L, X_{ik}^U\right] + \sum_{f=1}^{c_1} w_f^g \left[Z_{fk}^{g(0)}, Z_{fk}^{g(0)}\right] - \sum_{h=1}^{c_2} w_h^b \left[Z_{hk}^{b(0)}, Z_{hk}^{b(0)}\right]} \tag{2}$$

$$= \left[\frac{\sum_{r=1}^{s_1} u_r^g Y_{rk}^{gL} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{bU} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)}}{\sum_{i=1}^{m} v_i X_{ik}^U + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)}}, \frac{\sum_{r=1}^{s_1} u_r^g Y_{rk}^{gU} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{bL} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)}}{\sum_{i=1}^{m} v_i X_{ik}^L + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)}}\right] \text{(using interval arithmetic)} \tag{3}$$

$$= \left[E_k^{(a)L}, E_k^{(a)U}\right] \text{(say)} \tag{4}$$

where u_r^g, and w_f^g are the weights corresponding to rth desirable output and fth good link, respectively, and v_i, u_q^b, and w_h^b are the weights corresponding to ith input, qth undesirable output and hth bad link, respectively; $E_k^{(a)L}$ and $E_k^{(a)U}$ are denoted as lower and upper bounds of system interval efficiencies for DMU_k, respectively.

Similarly, interval efficiency of period t for DMU_k, denoted by $\hat{E}_k^{(t)}$, is defined as

$$\hat{E}_k^{(t)} = \frac{\sum_{r=1}^{s_1} u_r^g \hat{Y}_{rk}^{g(t)} - \sum_{q=1}^{s_2} u_q^b \hat{Y}_{qk}^{b(t)} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(t)}}{\sum_{i=1}^{m} v_i \hat{X}_{ik}^{(t)} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(t-1)}} \tag{5}$$

$$= \left[\frac{\sum_{r=1}^{s_1} u_r^g Y_{rk}^{g(t)L} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{b(t)U} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(t)}}{\sum_{i=1}^{m} v_i X_{ik}^{(t)U} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(t-1)}} \frac{\sum_{r=1}^{s_1} u_r^g Y_{rk}^{g(t)U} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{b(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(t)}}{\sum_{i=1}^{m} v_i X_{ik}^{(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(t-1)}}\right] \text{(using interval arithmetic)} \tag{6}$$

$$= \left[E_k^{(t)L}, E_k^{(t)U}\right] \text{(say)} \tag{7}$$

Here, $E_k^{(t)L}$ and $E_k^{(t)U}$ are the lower and upper bound efficiencies of period t for DMU_k, respectively.

Model-1 maximizes the system efficiency of DMU_k under the set of constraints, ensuring that the system efficiency of each DMU is greater than or equal to zero but does not exceed 1. However, it ignores the interrelationships of periods among each other and assumes that precise data are available. However, the production of a period is determined and influenced by the previous period as well. Also, if input-output data are of interval form, the system and period efficiencies $\hat{E}_k^{(a)}$ and $\hat{E}_k^{(t)}$ also need to be interval numbers. Therefore, Model-1 is extended to measure efficiency in a dynamic environment with interval data by maximizing the upper (lower) bound of system interval efficiency subject to the constraints that the system and period interval efficiencies should be the subsets of [0, 1]. Furthermore, let

$$E_k^{(a)L} \geq 0 \text{ and } E_k^{(a)U} \leq 1,\ \forall k,\ \text{then},\ \hat{E}_k^{(a)} = \left[E_k^{(a)L}, E_k^{(a)U}\right] \subseteq [0,1],\ \forall k. \tag{8}$$

$$\text{Similarly, let } E_k^{(t)L} \geq 0 \text{ and } E_k^{(t)U} \leq 1,\ \forall t,k,\ \text{then},\ \hat{E}_k^{(t)} = \left[E_k^{(t)L}, E_k^{(t)U}\right] \subseteq [0,1],\ \forall t,k. \tag{9}$$

By using equations (8) and (9), the following Model-2 can be constructed to measure the upper bound system efficiency $E_k^{(a)U}$ for DMU_k which is of fractional form:
Model-2

$$\max \quad E_k^{(a)U} = \frac{\sum_{r=1}^{s_1} u_r^g Y_{rk}^{gU} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{bL} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)}}{\sum_{i=1}^{m} v_i X_{ik}^{L} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)}}$$

$$\text{subject to} \quad E_j^{(a)U} = \frac{\sum_{r=1}^{s_1} u_r^g Y_{rj}^{gU} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{bL} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(p)}}{\sum_{i=1}^{m} v_i X_{ij}^{L} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(0)}} \leq 1,\ \forall j, \tag{10}$$

$$E_j^{(a)L} = \frac{\sum_{r=1}^{s_1} u_r^g Y_{rj}^{gL} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{bU} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(p)}}{\sum_{i=1}^{m} v_i X_{ij}^{U} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(0)}} \geq 0,\ \forall j, \tag{11}$$

$$E_j^{(t)U} = \frac{\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)U} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)}}{\sum_{i=1}^{m} v_i X_{ij}^{(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t-1)}} \leq 1,\ \forall j,\ \forall t, \tag{12}$$

$$E_j^{(t)L} = \frac{\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)L} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)U} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)}}{\sum_{i=1}^{m} v_i X_{ij}^{(t)U} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t-1)}} \geq 0,\ \forall j,\ \forall t, \tag{13}$$

$$u_r^g \geq \epsilon\ \forall r;\quad u_q^b \geq \epsilon\ \forall q;\quad v_i \geq \epsilon\ \forall i;\quad w_f^g \geq \epsilon\ \forall f;\quad w_h^b \geq \epsilon\ \forall h;\quad \epsilon > 0.$$

The proposed efficiency in Model-2 is developed on a similar concept to the input-oriented CCR models and utilized to evaluate the technical efficiency of DMU_k under constant returns to scale. Model-1 can be considered a special case of Model-2 in the sense that if we ignore the links between periods in Model-2, it will reduce to Model-1 (after omitting the redundant constraints). Assuming that the denominators in all the upper and lower bound efficiencies are greater than zero, the third set of constraints in Model-2 on simplification gives

$$\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)U} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)} - \sum_{i=1}^{m} v_i X_{ij}^{(t)L} - \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t-1)} + \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t-1)} \leq 0,\ \forall j,\ \forall t. \tag{14}$$

$$\Longrightarrow \sum_{t=1}^{p} \left(\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)U} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)} - \sum_{i=1}^{m} v_i X_{ij}^{(t)L} - \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t-1)} + \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t-1)} \right) \leq 0,\ \forall j. \tag{15}$$

$$\implies \sum_{r=1}^{s_1} u_r^g Y_{rk}^{gU} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{bL} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)} - \sum_{i=1}^{m} v_i X_{ik}^L$$
$$- \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} + \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)} \leq 0, \ \forall j, \quad (16)$$
$$i.e., \ E_j^{(a)U} \leq 1, \ \forall j. \quad (17)$$

By using equation (17), the set of system constraints $\left(E_j^{(a)U} \leq 1, \ \forall j\right)$ can be deduced by summing over the third set of constraints corresponding to the period efficiencies $\left(E_j^{(t)U} \leq 1, \ \forall j, \ \forall t\right)$, so the first set of constraints in Model-2 is redundant.

Since the denominator in the objective function of Model-2 is assumed to be positive for every DMU, and the objective function in the given model is a fractional number invariant under multiplication of both numerator and denominator by the same nonzero quantity, one can easily apply the Charnes-Cooper transformation on Model-2 [19]. Thus, after omitting the redundant constraints and using Charnes-Cooper transformation [19], Model-2 reduces to the following linear program:

Model-3

$$\max \ E_k^{(a)U} = \sum_{r=1}^{s_1} u_r^g Y_{rk}^{gU} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{bL} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)}$$
$$\text{subject to} \quad \sum_{i=1}^{m} v_i X_{ik}^L + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)} = 1, \quad (18)$$
$$\sum_{r=1}^{s_1} u_r^g Y_{rj}^{gL} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{bU} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(p)} \geq 0, \ \forall j, \quad (19)$$
$$\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)U} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)} - \sum_{i=1}^{m} v_i X_{ij}^{(t)L}$$
$$- \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t-1)} + \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t-1)} \leq 0, \ \forall j, \ \forall t, \quad (20)$$
$$\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)L} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)U} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)} \geq 0, \ \forall j, \ \forall t, \quad (21)$$
$$u_r^g \geq \epsilon \ \forall r; \ \ u_q^b \geq \epsilon \ \forall q; \ \ v_i \geq \epsilon \ \forall i; \ \ w_f^g \geq \epsilon \ \forall f; \ \ w_h^b \geq \epsilon \ \forall h; \ \ \epsilon > 0.$$

The best possible system efficiency of DMU_k in a dynamic environment is the optimal objective function value $E_k^{(a)U^*}$ obtained from Model-3 when every DMU in every period is in the best production activity state. Now, one approach to compute the lower bound system efficiency $E_k^{(a)L^*}$ of DMU_k is by utilizing the weights obtained in Model-3. However, the optimal weights of Model-3 might not be unique, and the resultant efficiency may not be computed as a valid interval. The existing literature suggests the common set of weights methodology as a popular and reliable approach to enhance the discrimination power of dynamic DEA models [20,33,51]. Further, the same methodology is applied by various researchers to evaluate and optimize each DMU's efficiency at the same time [4,54, 58]. Owing to the importance of a common weights methodology, it has been utilized in the present paper along with unified production frontier [74] to estimate the lower and upper bounds of the system and period interval efficiencies in the presence of interval data and to achieve the complete ranking of units possessing interval efficiencies. In the estimation of lower bound efficiency $E_k^{(a)L^*}$ of DMU_k, an additional constraint has been considered by retaining the upper bound system efficiency of DMU_k equal to $E_k^{(a)U^*}$ which results into the following Model-4.

Model-4

$$\max \quad E_k^{(a)L} = \sum_{r=1}^{s_1} u_r^g Y_{rk}^{gL} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{bU} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)}$$

$$\text{subject to} \quad \sum_{i=1}^{m} v_i X_{ik}^{U} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)} = 1, \tag{22}$$

$$\sum_{r=1}^{s_1} u_r^g Y_{rk}^{gU} - \sum_{q=1}^{s_2} u_q^b Y_{qk}^{bL} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(p)} - \left(E_k^{(a)U^*}\right) * \left(\sum_{i=1}^{m} v_i X_{ik}^{L} + \sum_{f=1}^{c_1} w_f^g Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^b Z_{hk}^{b(0)}\right) = 0, \tag{23}$$

$$\sum_{r=1}^{s_1} u_r^g Y_{rj}^{gL} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{bU} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(p)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(p)} \geq 0, \ \forall j, \tag{24}$$

$$\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)U} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)L} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)} - \sum_{i=1}^{m} v_i X_{ij}^{(t)L} - \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t-1)} + \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t-1)} \leq 0, \ \forall j, \ \forall t, \tag{25}$$

$$\sum_{r=1}^{s_1} u_r^g Y_{rj}^{g(t)L} - \sum_{q=1}^{s_2} u_q^b Y_{qj}^{b(t)U} + \sum_{f=1}^{c_1} w_f^g Z_{fj}^{g(t)} - \sum_{h=1}^{c_2} w_h^b Z_{hj}^{b(t)} \geq 0, \ \forall j, \ \forall t, \tag{26}$$

$$u_r^g \geq \epsilon \, \forall r; \ \ u_q^b \geq \epsilon \, \forall q; \ \ v_i \geq \epsilon \, \forall i; \ \ w_f^g \geq \epsilon \, \forall f; \ \ w_h^b \geq \epsilon \, \forall h; \ \ \epsilon > 0,$$

where ϵ is a non-archimedean infinitesimal.

Model-4 measures the lower bound of the system efficiency of DMU_k, which is the best possible relative efficiency in a dynamic environment obtained as the optimal objective function value $E_k^{(a)L^*}$. Note that the constraint set of Model-3 determines the production frontier. Further, Model-4 utilizes that frontier as a benchmark to find common weights for measuring the system efficiency intervals $\left[E_k^{(a)L^*}, E_k^{(a)U^*}\right]$ for every DMU_k. Therefore, $\left[E_k^{(a)L^*}, E_k^{(a)U^*}\right]$ is termed as the best possible relative system efficiency interval of DMU_k in a dynamic environment. Further, the interval efficiencies for the p periods are measured in the following manner.

Definition 3.1. The lower and upper bound period efficiencies $\left(E_k^{(t)L^*} \text{ and } E_k^{(t)U^*}\right)$ for DMU_k by using common weights $\left(u_r^{g^*}, r = 1, 2, \ldots, s_1;\ u_q^{b^*}, q = 1, 2, \ldots, s_2;\ v_i^*, i = 1, 2, \ldots, m;\ w_f^{g^*}, f = 1, 2, \ldots, c_1;\ w_h^{b^*},\right.$ $h = 1, 2, \ldots, c_2)$ obtained from Model-4 are defined as follows:

$$E_k^{(t)L^*} = \frac{\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{g(t)L} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{b(t)U} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(t)U} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}}, \ \forall t = 1, 2, \ldots, p, \tag{27}$$

$$\text{and } E_k^{(t)U^*} = \frac{\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{g(t)U} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{b(t)L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(t)L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}}, \ \forall t = 1, 2, \ldots, p. \tag{28}$$

Remark 3.2. The proposed approach is applicable if data is in ordinal form. Model-3 and Model-4 can easily handle data in the form of ordinal preference relation after converting it into the interval form using Wang *et al.* [74] approach.

Remark 3.3. The proposed approach is also applicable for fuzzy data since Model-3 and Model-4 can handle fuzzy data after its conversion into interval form utilizing the α-cut approach.

Theorem 3.4. *Model-4 is always feasible.*

Proof. The feasibility of Model-4 has been proved using its dual (Model-4D) and primal-dual relationships. The dual of Model-4 is given as follows:
Model-4D

$$\min \quad \Phi = \psi - \epsilon\left(\sum_{i=1}^{m} s_{ik}^{'-} + \sum_{r=1}^{s_1} s_{rk}^{g'+} + \sum_{q=1}^{s_2} s_{qk}^{b'-} + \sum_{f=1}^{c_1} S_{fk}^{g'+} + \sum_{h=1}^{c_2} S_{hk}^{b'-}\right)$$

$$\text{subject to } \sum_{j=1}^{n}\sum_{t=1}^{p} \beta_j^{(t)} X_{ij}^{(t)L} + \xi E_k^{(a)U*} X_{ik}^{L} + s_{ik}^{'} = \psi X_{ik}^{U},\ \forall i, \tag{29}$$

$$-\sum_{j=1}^{n}\alpha_j Y_{rj}^{gL} - \sum_{j=1}^{n}\sum_{t=1}^{p}\gamma_j^{(t)} Y_{rj}^{g(t)L} + \sum_{j=1}^{n}\sum_{t=1}^{p}\beta_j^{(t)} Y_{rj}^{g(t)U} + \xi Y_{rk}^{gU} - s_{rk}^{g'+} = Y_{rk}^{gL},\ \forall r, \tag{30}$$

$$-\sum_{j=1}^{n}\alpha_j Y_{qj}^{bU} - \sum_{j=1}^{n}\sum_{t=1}^{p}\gamma_j^{(t)} Y_{qj}^{b(t)U} + \sum_{j=1}^{n}\sum_{t=1}^{p}\beta_j^{(t)} Y_{qj}^{b(t)U} + \xi Y_{qk}^{bL} + s_{qk}^{b'-} = Y_{qk}^{bL},\ \forall q, \tag{31}$$

$$\begin{aligned} -\sum_{j=1}^{n}\alpha_j Z_{fj}^{g(p)} - \sum_{j=1}^{n}\sum_{t=1}^{p}\gamma_j^{(t)} Z_{fj}^{g(t)} + \sum_{j=1}^{n}\sum_{t=1}^{p}\beta_j^{(t)}\left(Z_{fj}^{g(t)} - Z_{fj}^{g(t-1)}\right) + \psi Z_{fk}^{g(0)} \\ + \xi\left(Z_{fk}^{g(p)} - E_k^{(a)U*} Z_{fk}^{g(0)}\right) - S_{fk}^{g'+} = Z_{fk}^{g(p)}, \forall f, \end{aligned} \tag{32}$$

$$\begin{aligned} -\sum_{j=1}^{n}\alpha_j Z_{hj}^{b(p)} - \sum_{j=1}^{n}\sum_{t=1}^{p}\gamma_j^{(t)} Z_{hj}^{b(t)} + \sum_{j=1}^{n}\sum_{t=1}^{p}\beta_j^{(t)}\left(Z_{hj}^{b(t)} - Z_{hj}^{b(t-1)}\right) + \psi Z_{hk}^{b(0)} \\ + \xi\left(Z_{hk}^{b(p)} - E_k^{(a)U*} Z_{hk}^{b(0)}\right) + S_{hk}^{b'-} = Z_{hk}^{b(p)}, \forall h, \end{aligned} \tag{33}$$

$$\alpha_j \geq 0,\ \ \gamma_j^{(t)} \geq 0,\ \ \beta_j^{(t)} \geq 0,\ \ \forall j,\ \forall t;\ \ \psi \text{ and } \xi \text{ are unrestricted},$$
$$s_{rk}^{g'+} \geq 0\ \forall r;\ \ s_{qk}^{b'-} \geq 0\ \forall q;\ \ s_{ik}^{'-} \geq 0\ \forall i;\ \ S_{fk}^{g'+} \geq 0\ \forall f;\ \ S_{hk}^{b'-} \geq 0\ \forall h,$$

where ψ, ξ, α_j $(\forall j)$, $\beta_j^{(t)}$ $(\forall j,t)$ and $\gamma_j^{(t)}$ $(\forall j,t)$ are the dual variables corresponding to the set of constraints given in (22)–(26), respectively. Model-4D has a feasible solution $\psi = 1,\ \xi = 0,\ \alpha_j = 0\ \forall j,\ \gamma_j^{(t)} = 0\ \forall j,t,\ \beta_k^{(t)} = 1\ \forall t,\ \beta_j^{(t)} = 0\ \forall j \neq k,\ \forall t,\ s_{rk}^{g'+} = Y_{rk}^{gL} - Y_{rk}^{gU}\ \forall r,\ s_{qk}^{b'-} = Y_{qk}^{bU} - Y_{qk}^{bL}\ \forall q,\ s_{ik}^{'-} = X_{ik}^{U} - X_{ik}^{L}\ \forall i,\ S_{fk}^{g'+} = 0\ \forall f,\ S_{hk}^{b'-} = 0\ \forall h$. Using the non-negativity of data and constraints in (29) and (30), we get $\psi > 0$ for all the feasible solutions of Model-4D. Furthermore, $\epsilon > 0$ is a non-archimedean infinitesimal, which implies that there exists no positive real number which is smaller than $\epsilon > 0$, and the same holds for the $K\epsilon$, regardless of how large $K > 0$ [19]. It implies that $\Phi^* > 0$, where Φ^* is the optimal objective function value of Model-4D. Considering that the objective function is bounded from below for all the solutions within feasible region and Model-4D is a minimization problem, the optimality of model is guaranteed, further ensuring the feasibility of Model-4 (primal) using the primal-dual relationships. □

Theorem 3.5. *For the optimal weights* $\left(u_r^{g^*}\ \forall r;\ u_q^{b^*}\ \forall q;\ v_i^*\ \forall i;\ w_f^{g^*}\ \forall f;\ w_h^{b^*}\ \forall h\right)$, *obtained from Model-4, the complement of the upper bound system efficiency* $\left(E_k^{(a)U^*}\right)$ *can be expressed as a convex linear combination of the complement of the upper bound period efficiencies* $\left(E_k^{(t)U^*}, \forall t = 1, 2, \ldots, p\right)$, *provided* $\Delta_U = 0$, *where*

$$\Delta_U = \left(\sum_{t=2}^{p}\left(\sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}\right) \Big/ \left(\sum_{i=1}^{m} v_i^* X_{ik}^L + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}\right)\right).$$

Proof. Let $\left(u_r^{g^*}\ \forall r;\ u_q^{b^*}\ \forall q;\ v_i^*\ \forall i;\ w_f^{g^*}\ \forall f;\ w_h^{b^*}\ \forall h\right)$ be the optimal weights obtained from Model-4. Since,

$$\begin{aligned}
&\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{gU} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{bL} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(p)} - \sum_{i=1}^{m} v_i^* X_{ik}^L - \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} + \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)} \\
&= \sum_{t=1}^{p}\left(\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{g(t)U} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{b(t)L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t)} - \sum_{i=1}^{m} v_i^* X_{ik}^{(t)L}\right. \\
&\qquad\left. - \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} + \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}\right)
\end{aligned} \tag{34}$$

Dividing both sides by $\left(\sum_{i=1}^m v_i^* X_{ik}^L + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}\right)$, we get

$$E_k^{(a)U^*} - 1 = \sum_{t=1}^{p}\left(E_k^{(t)U^*} - 1\right) \frac{\sum_{i=1}^m v_i^* X_{ik}^{(t)L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}}{\sum_{i=1}^m v_i^* X_{ik}^L + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}} = \sum_{t=1}^{p}\left(E_k^{(t)U^*} - 1\right) w^{(t)U} \tag{35}$$

$$\text{where} \quad w^{(t)U} = \frac{\sum_{i=1}^m v_i^* X_{ik}^{(t)L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}}{\sum_{i=1}^m v_i^* X_{ik}^L + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}}. \tag{36}$$

Now,

$$\begin{aligned}
\sum_{t=1}^{p} w^{(t)U} &= \frac{\sum_{i=1}^m v_i^* X_{ik}^L + \left(\sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}\right) + \sum_{t=2}^{p}\left(\sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}\right)}{\sum_{i=1}^m v_i^* X_{ik}^L + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}} \\
&= 1 + \frac{\sum_{t=2}^{p}\left(\sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}\right)}{\sum_{i=1}^m v_i^* X_{ik}^L + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}} = 1 + \Delta_U \text{ (say)}
\end{aligned} \tag{37}$$

From equation (36), we have $w^{(t)U} \geq 0$ as we always try to minimize undesirable links while maximizing efficiency. Also, by the definitions of $E_k^{(a)U}$ and $E_k^{(t)U}$, it can easily be observed that $w^{(t)U}$ is non-zero, *i.e.*,

$$w^{(t)U} > 0,\ \forall t. \tag{38}$$

Further, if $\Delta_U = 0$, then by using equations (35)–(38), $E_k^{(a)U^*} - 1 = \sum_{t=1}^{p}\left(E_k^{(t)U^*} - 1\right) w^{(t)U}$, where $\sum_{t=1}^{p} w^{(t)U} = 1$ and $w^{(t)U} > 0\ \forall t$,

Hence proved. □

Remark 3.6. If $\Delta_U \neq 0$, then the complement of the upper bound system efficiency $\left(E_k^{(a)U^*}\right)$ can be written as a linear combination of the complement of the upper bound period efficiencies $\left(E_k^{(t)U^*}, \forall t = 1, 2, \ldots, p\right)$.

Theorem 3.7. *For the optimal weights* $\left(u_r^{g^*}\ \forall r;\ u_q^{b^*}\ \forall q;\ v_i^*\ \forall i;\ w_f^{g^*}\ \forall f;\ w_h^{b^*}\ \forall h\right)$*, obtained from Model-4, the complement of lower bound of system efficiency* $\left(E_k^{(a)L^*}\right)$ *can be expressed as a convex linear combination of the complement of the lower bound period efficiencies* $\left(E_k^{(t)L^*}, \forall t = 1, 2, \ldots, p\right)$*, provided* $\Delta_L = 0$*, where*

$$\Delta_L = \left(\sum_{t=2}^{p}\left(\sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}\right) \Big/ \left(\sum_{i=1}^{m} v_i^* X_{ik}^U + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}\right)\right).$$

Proof. The Proof is similar to Theorem 3.5. □

Remark 3.8. If $\Delta_L \neq 0$, then the complement of the lower bound system efficiency $\left(E_k^{(a)L^*}\right)$ can be written as a linear combination of the complement of the lower bound period efficiencies $\left(E_k^{(t)L^*}, \forall t = 1, 2, \ldots, p\right)$.

Proposition 3.9. If $\Delta_U = 0$, then $Q_k \leq E_k^{(a)U^*} \leq R_k$ where $Q_k = \min_t \left\{E_k^{(t)U^*}\right\} = E_k^{(e)U^*}$ and $R_k = \max_t \left\{E_k^{(t)U^*}\right\} = E_k^{(f)U^*}$ for any $e, f \in \{1, 2, \ldots, p\}$.

Proof. From equation (35), we have

$$\begin{aligned} E_k^{(a)U^*} - 1 &= \sum_{t=1}^{p}\left(E_k^{(t)U^*} - 1\right) w^{(t)U} = \sum_{t=1}^{p}\left(E_k^{(t)U^*} * w^{(t)U}\right) - \sum_{t=1}^{p} w^{(t)U} \\ &\geq \sum_{t=1}^{p}\left(E_k^{(e)U^*} * w^{(t)U}\right) - \sum_{t=1}^{p} w^{(t)U} = \left(\sum_{t=1}^{p} w^{(t)U}\right)\left(E_k^{(e)U^*} - 1\right) = (1+\Delta_U)\left(E_k^{(e)U^*} - 1\right). \end{aligned} \tag{39}$$

$$\text{If } \Delta_U = 0 \text{ in equation (39), then } \left(E_k^{(a)U^*} - 1\right) \geq \left(E_k^{(e)U^*} - 1\right) \tag{40}$$

$$\text{which implies } E_k^{(a)U^*} \geq E_k^{(e)U^*} = Q_k. \tag{41}$$

$$\begin{aligned} \text{Now, } E_k^{(a)U^*} - 1 &= \sum_{t=1}^{p}\left(E_k^{(t)U^*} - 1\right) w^{(t)U} = \sum_{t=1}^{p}\left(E_k^{(t)U^*} * w^{(t)U}\right) - \sum_{t=1}^{p} w^{(t)U} \\ &\leq \sum_{t=1}^{p}\left(E_k^{(f)U^*} * w^{(t)U}\right) - \sum_{t=1}^{p} w^{(t)U} = \left(\sum_{t=1}^{p} w^{(t)U}\right)\left(E_k^{(f)U^*} - 1\right) = (1+\Delta_U)\left(E_k^{(f)U^*} - 1\right). \end{aligned} \tag{42}$$

$$\text{If } \Delta_U = 0 \text{ in equation (42), then } \left(E_k^{(a)U^*} - 1\right) \leq \left(E_k^{(f)U^*} - 1\right) \tag{43}$$

$$\text{which implies } E_k^{(a)U^*} \leq E_k^{(f)U^*} = R_k. \tag{44}$$

Hence, $Q_k \leq E_k^{(a)U^*} \leq R_k$ □

Proposition 3.10. If $\Delta_L = 0$, then $M_k \leq E_k^{(a)L^*} \leq N_k$ where $M_k = \min_t \left\{E_k^{(t)L^*}\right\} = E_k^{(m)L^*}$ and $N_k = \max_t \left\{E_k^{(t)L^*}\right\} = E_k^{(n)L^*}$ for any $m, n \in \{1, 2, \ldots, p\}$.

Proof. The Proof is similar to Proposition 3.9. □

Theorem 3.11. *For an optimal solution to Model-4, prove that the efficiencies* $\left[E_k^{(a)L^*}, E_k^{(a)U^*}\right]$ *and* $\left[E_k^{(t)L^*}, E_k^{(t)U^*}\right]$, $\forall t = 1, 2, \ldots, p$, *constitute valid intervals for* DMU_k.

Proof. Let $\left(u_r^{g^*}\ \forall r;\ u_q^{b^*}\ \forall q;\ v_i^*\ \forall i;\ w_f^{g^*}\ \forall f;\ w_h^{b^*}\ \forall h\right)$ be the optimal solution to Model-4. Then

$$E_k^{(a)L^*} = \frac{\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{gL} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{bU} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{U} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}} \tag{45}$$

$$\leq \frac{\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{gU} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{bL} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(p)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(0)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(0)}} = E_k^{(a)U^*} \tag{46}$$

$$E_k^{(t)L^*} = \frac{\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{g(t)L} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{b(t)U} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(t)U} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}} \tag{47}$$

$$\leq \frac{\sum_{r=1}^{s_1} u_r^{g^*} Y_{rk}^{g(t)U} - \sum_{q=1}^{s_2} u_q^{b^*} Y_{qk}^{b(t)L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(t)L} + \sum_{f=1}^{c_1} w_f^{g^*} Z_{fk}^{g(t-1)} - \sum_{h=1}^{c_2} w_h^{b^*} Z_{hk}^{b(t-1)}} = E_k^{(t)U^*}. \tag{48}$$

Equations (46) and (48) imply that $E_k^{(a)L^*} \leq E_k^{(a)U^*}$ and $E_k^{(t)L^*} \leq E_k^{(t)U^*}$, $\forall t = 1, 2, \ldots, p$. Therefore, $\left[E_k^{(a)L^*}, E_k^{(a)U^*}\right]$ and $\left[E_k^{(t)L^*}, E_k^{(t)U^*}\right]$, $\forall t = 1, 2, \ldots, p$, constitute valid intervals for DMU_k. □

Theorem 3.12. *For* DMU_k, *let* $\left[E_k^{(a)L^*}, E_k^{(a)U^*}\right]$ *be the system efficiency interval and* $\left[E_k^{(t)L^*}, E_k^{(t)U^*}\right]$ *be the efficiency interval for period* t. *Then,*

(i) $E_k^{(a)U^*} = 1 \iff E_k^{(t)U^*} = 1,\ \forall t$ *and (ii)* $E_k^{(a)L^*} = 1 \iff E_k^{(t)L^*} = 1,\ \forall t$.

Proof. (i) From equation (35), we have $E_k^{(a)U^*} - 1 = \sum_{t=1}^{p}\left(E_k^{(t)U^*} - 1\right) w^{(t)U}$

Now, $E_k^{(a)U^*} = 1 \iff \sum_{t=1}^{p}\left(E_k^{(t)U^*} - 1\right) w^{(t)U} = 0 \iff E_k^{(t)U^*} - 1 = 0\ \ \forall t$ as $w^{(t)U} > 0$ (See Eq. (38)) $\iff E_k^{(t)U^*} = 1\ \ \forall t$.

(ii) The Proof is similar to (i).

□

The theoretical contribution of the present study has been outlined in the given theorems. Theorem 3.4 utilizes the duality theory to validate the feasibility of Model-3. Theorems 3.5 (3.7) outlines the prerequisite for expressing the system's upper (lower) bound efficiency as a convex linear combination of upper (lower) bound period efficiencies. This allows for a more comprehensive study of how the efficiencies of the periods impact the overall efficiency of the system. It further leads to the result in Propositions 3.9 (3.10) that the upper (lower) bound system efficiency for a DMU is bounded by the minimum and maximum of the upper (lower) bound period efficiencies of that DMU under the same condition. The validity of the efficiencies as intervals is proved in Theorem 3.11. Furthermore, Theorem 3.12 deduces that for a DMU to possess a lower (upper) bound system efficiency of 1, it must attain a lower (upper) bound efficiency of 1 in each period.

3.1. Classification of DMUs in a dynamic environment with interval data

Let $\left[E_k^{(a)L^*}, E_k^{(a)U^*}\right]$ be the system efficiency interval evaluated using the proposed relational dynamic DEA approach. Then, Zhang *et al.* [81]'s classification approach has been used to categorize DMUs system-wise, which is as follows:

(i) Fully efficient DMUs: all DMUs that belong to E^{++} are in the category of fully efficient DMUs (E^{++}), where

$$E^{++} = \left\{ DMU_k | E_k^{(a)L^*} \geq \max_{j \neq k} E_j^{(a)U^*} \right\}.$$

(ii) Efficient DMUs: all DMUs belonging to E^{+} are in the category of efficient DMUs (E^{+}), where

$$E^{+} = \left\{ DMU_k | E_k^{(a)L^*} = \max_{j} E_j^{(a)L^*} \text{ or } E_k^{(a)U^*} = \max_{j} E_j^{(a)U^*} \right\}.$$

(iii) Inefficient DMUs: all DMUs belonging to E^{-} are in the category of inefficient DMUs (E^{-}), where

$$E^{-} = \left\{ DMU_k | E_k^{(a)L^*} < \max_{j \neq k} E_j^{(a)L^*} \text{ and } E_k^{(a)U^*} < \max_{j \neq k} E_j^{(a)U^*} \right\}.$$

(iv) Fully inefficient DMUs: all DMUs belonging to E^{--} are in the category of fully inefficient DMUs (E^{--}), where

$$E^{--} = \left\{ DMU_k | E_k^{(a)U^*} < \min_{j \neq k} E_j^{(a)L^*} \right\}.$$

A similar type of classification can be utilized to categorize DMUs period-wise using the interval period efficiencies $\left[E_k^{(t)L^*}, E_k^{(t)U^*}\right]$.

3.2. Ranking of system and period interval efficiencies

In order to address the issue of ranking interval numbers, many researchers have contributed in this field and suggested different procedures to rank interval numbers based on (i) measure of proximity [45], (ii) minimax-regret based approach [74], (iii) probabilistic approach [64], (iv) Monte Carlo method [38], (v) fuzzy preference ordering [63], (vi) preference index [36], (vii) possibility degree formula [47], and (viii) symmetry axis compensation factor in risk appetite index [49]. Among all suggested approaches, the widely used ranking approach in DEA is found to be the minimax regret-based approach which is appropriate for ranking interval efficiencies of the homogeneous DMUs, and it can be seen in reports like Farzipoor Saen [26,27], Aydin and Zortuk [7], Puri *et al.* [59], Azizi *et al.* [8] and Toloo *et al.* [71], etc. Therefore, the present study utilizes a minimax regret-based approach for ranking system and period interval efficiencies in a dynamic environment. The detailed description of the minimax regret-based approach can be seen in Wang *et al.* [74] and also discussed in Appendix A of *supplemental file*.

4. System targets for upper bound efficiency

The majority of the approaches suggested in the existing literature generally estimate the efficiencies/inefficiencies in dynamic DEA but remain silent on the areas that require improvement to make the DMUs efficient. However, in the present study, the classification of DMUs in Subsection 3.1 suggests that the efficiency of a DMU in a dynamic environment with interval data primarily relies on the upper bound efficiency of that DMU. So it can be inferred that inefficient DMUs can perform better by achieving higher upper-bound efficiency. Here, Model-3 assesses the upper bound system efficiency, and to improve inefficient DMUs, system targets for each inefficient DMU can be evaluated by using the dual of Model-3. Here, Model-3 is a linear optimization model as the objective function and all its constraints are linear functions of the decision variables

u_r^g, u_q^b, v_i, w_f^g, and w_h^b. Therefore by considering θ, $\lambda_j(\forall j)$, $\rho_j^{(t)}(\forall j,t)$, and $\mu_j^{(t)}(\forall j,t)$, respectively, as the dual variables corresponding to the set of constraints given in equations (18)–(21), the dual of Model-3 has been achieved and is given in Model-5.

Model-5

$$\min \quad \phi = \quad \theta - \epsilon\left(\sum_{i=1}^{m} s_{ik}^{-} + \sum_{r=1}^{s_1} s_{rk}^{g+} + \sum_{q=1}^{s_2} s_{qk}^{b-} + \sum_{f=1}^{c_1} S_{fk}^{g+} + \sum_{h=1}^{c_2} S_{hk}^{b-}\right)$$

$$\text{subject to} \quad \sum_{j=1}^{n}\sum_{t=1}^{p} \rho_j^{(t)} X_{ij}^{(t)L} + s_{ik}^{-} = \theta X_{ik}^{L},\ \forall i, \tag{49}$$

$$-\sum_{j=1}^{n} \lambda_j Y_{rj}^{gL} - \sum_{j=1}^{n}\sum_{t=1}^{p} \mu_j^{(t)} Y_{rj}^{g(t)L} + \sum_{j=1}^{n}\sum_{t=1}^{p} \rho_j^{(t)} Y_{rj}^{g(t)U} - s_{rk}^{g+} = Y_{rk}^{gU},\ \forall r, \tag{50}$$

$$-\sum_{j=1}^{n} \lambda_j Y_{qj}^{bU} - \sum_{j=1}^{n}\sum_{t=1}^{p} \mu_j^{(t)} Y_{qj}^{b(t)U} + \sum_{j=1}^{n}\sum_{t=1}^{p} \rho_j^{(t)} Y_{qj}^{b(t)L} + s_{qk}^{b-} = Y_{qk}^{bL},\ \forall q, \tag{51}$$

$$-\sum_{j=1}^{n} \lambda_j Z_{fj}^{g(p)} - \sum_{j=1}^{n}\sum_{t=1}^{p} \mu_j^{(t)} Z_{fj}^{g(t)} + \sum_{j=1}^{n}\sum_{t=1}^{p} \rho_j^{(t)}\left(Z_{fj}^{g(t)} - Z_{fj}^{g(t-1)}\right) + \theta Z_{fk}^{g(0)} - S_{fk}^{g+} = Z_{fk}^{g(p)}, \forall f, \tag{52}$$

$$-\sum_{j=1}^{n} \lambda_j Z_{hj}^{b(p)} - \sum_{j=1}^{n}\sum_{t=1}^{p} \mu_j^{(t)} Z_{hj}^{b(t)} + \sum_{j=1}^{n}\sum_{t=1}^{p} \rho_j^{(t)}\left(Z_{hj}^{b(t)} - Z_{hj}^{b(t-1)}\right) + \theta Z_{hk}^{b(0)} + S_{hk}^{b-} = Z_{hk}^{b(p)}, \forall h, \tag{53}$$

$$\lambda_j \geq 0,\ \ \mu_j^{(t)} \geq 0,\ \ \rho_j^{(t)} \geq 0,\ \ \forall j,\ \forall t;\ \ \theta \text{ is unrestricted},$$
$$s_{rk}^{g+} \geq 0\ \forall r;\ \ s_{qk}^{b-} \geq 0\ \forall q;\ \ s_{ik}^{-} \geq 0\ \forall i;\ \ S_{fk}^{g+} \geq 0\ \forall f;\ \ S_{hk}^{b-} \geq 0\ \forall h,$$

where s_{rk}^{g+}, s_{qk}^{b-}, s_{ik}^{-}, S_{fk}^{g+}, S_{hk}^{b-} be the slacks in the rth desirable output, qth undesirable output, ith input, fth good link, and hth bad link, respectively, and θ is a scalar defined as the (proportional) reduction to all system inputs.

Model-5 has a feasible solution $\theta = 1,\ \lambda_j = 0\ \forall j,\ \mu_j^{(t)} = 0\ \forall j,t,\ \rho_k^{(t)} = 1\ \forall t,\ \rho_j^{(t)} = 0\ \forall j \neq k,\ \forall t,\ s_{rk}^{g+} = 0 \forall r,\ s_{qk}^{b-} = 0\ \forall q,\ s_{ik}^{-} = 0\ \forall i,\ S_{fk}^{g+} = 0\ \forall f,\ S_{hk}^{b-} = 0\ \forall h$. The non-negativity of data and the constraints given in (49)–(53) imply that $\phi > 0$. Therefore, if ϕ^* denotes the optimal value of Model-5, then we have $\phi^* > 0$. The non-zero slacks and $\theta \leq 1$ constitute the amount of upper bound system inefficiency that may exist in the DMU_k. Let the optimal solution of Model-5 be $\left(\theta^*, \lambda_j^* \forall j, \mu_j^{(t)^*} \forall j,t, \rho_j^{(t)^*} \forall j,t, s_{rk}^{g+^*} \forall r, s_{qk}^{b-^*} \forall q, s_{ik}^{-^*} \forall i, S_{fk}^{g+^*} \forall f, S_{hk}^{b-^*} \forall h\right)$. To improve the inefficient DMU_k, using the optimal solution of Model-5, the following can be suggested.

(i) the inputs are deduced radially by the ratio θ and surplus input recorded in $s_{ik}^{-^*}$ $\forall i$ are eliminated in each ith system input.
(ii) undesirable output slacks recorded in $s_{qk}^{b-^*}$ $\forall q$ are eliminated in each qth system undesirable output.
(iii) desirable output shortfalls recorded in $s_{rk}^{g+^*}$ $\forall r$ are augmented in each rth system desirable output.

Thus, the targets for improvement are given by

$$\bar{X}_{ik}^{L} = \theta^* X_{ik}^{L} - s_{ik}^{-^*},\ \bar{Y}_{qk}^{bL} = Y_{qk}^{bL} - s_{qk}^{b-^*} \text{ and } \bar{Y}_{rk}^{gU} = Y_{rk}^{gU} + s_{rk}^{g+^*}, \tag{54}$$

where $X_{ik}^{L} = \sum_{t=1}^{p} X_{ik}^{(t)L}$ and $Y_{qk}^{bL} = \sum_{t=1}^{p} Y_{qk}^{b(t)L}$ are respectively the lower bounds of the ith system input and qth undesirable system output, and $Y_{rk}^{gU} = \sum_{t=1}^{p} Y_{rk}^{g(t)U}$ is the upper bound of rth system desirable output of DMU_k. Further, Model-3 is input-oriented, so the targets imply a certain amount of primary input reduction for each input of a DMU. These projections identify the most sensitive variables (inputs/outputs) that need

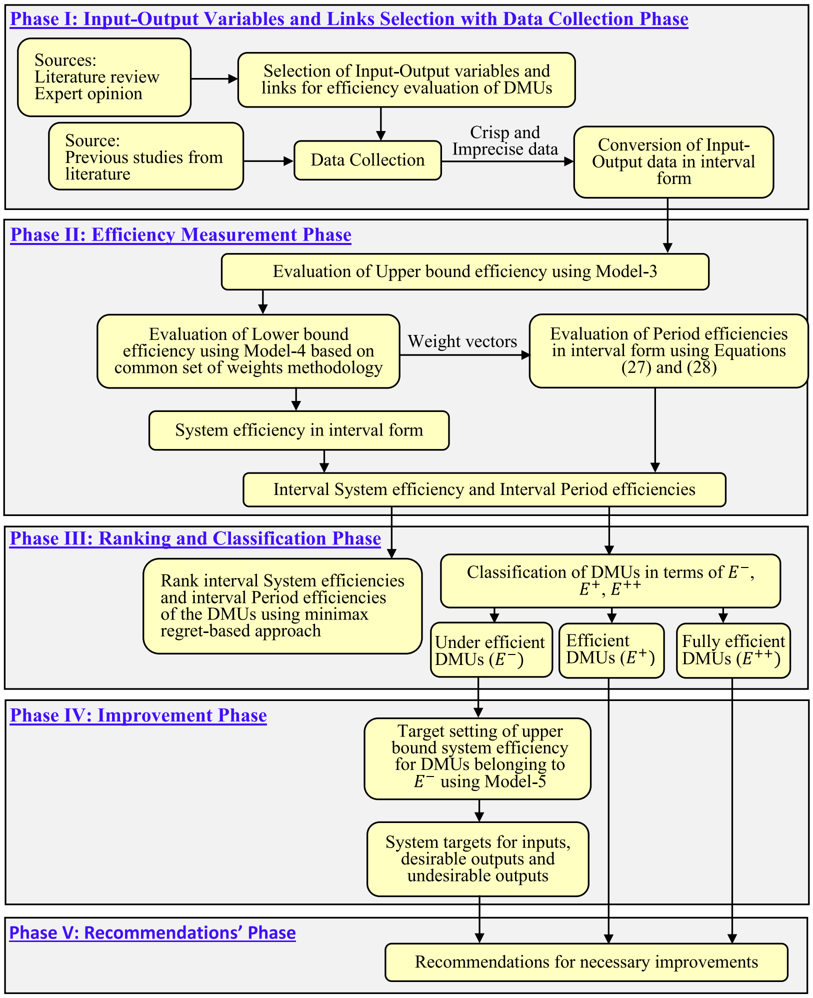


FIGURE 2. Schematic view of the proposed dynamic DEA approach.

improvements for the efficiency enhancement of inefficient DMUs. Specifically, the suggested target points in the proposed approach are helpful in improving the system efficiency of underperformed DMUs in a way that these points assist decision-makers in directing their attention toward variables that are more susceptible to efficiency enhancements. However, the targets suggested in the proposed study may not explicitly make the system efficiency score equal to 1; rather, the decision-maker can utilize the information from the target points to formulate policies for strengthening the system-inefficient units. Thus, this section presents an important managerial implication of the proposed approach for improving inefficient organizations.

5. Complete Interval dynamic DEA performance decision process

The complete interval dynamic DEA performance decision process involves five phases (1) Input-Output variables and links selection with data collection phase, (2) Efficiency measurement phase, (3) Ranking and classification phase, (4) Improvement phase, and (5) Recommendations' phase. The schematic view of the complete interval dynamic DEA performance decision process is depicted in Figure 2.

TABLE 2. List of banks included in case study.

BC	PuSBs	BC	PrSBs
BoB	Bank of Baroda	AXIB	Axis Bank Ltd.
BoI	Bank of India	BDN	Bandhan Bank Ltd.
BoM	Bank of Maharashtra	CUB	City Union Bank Ltd.
CaB	Canara Bank	CSB	CSB Bank Ltd.
CBoI	Central Bank of India	DCB	DCB Bank Ltd.
INB	Indian Bank	FB	Federal Bank Ltd.
IOB	Indian Overseas bank	HDFC	HDFC Bank Ltd.
PSB	Punjab and Sind Bank	ICICI	ICICI Bank Ltd.
PNB	Punjab National Bank	IIB	Indusind Bank Ltd.
SBI	State Bank of India	JKB	Jammu & Kashmir Bank Ltd.
UCOB	UCO Bank	KB	Karnataka Bank Ltd.
UBoI	Union Bank of India	KVB	Karur Vysya Bank Ltd.
		KMB	Kotak Mahindra Bank Ltd.
		NB	Nainital Bank Ltd.
		RBL	RBL Bank Ltd.
		SIB	South Indian bank Ltd.
		TMB	Tamilnad Mercantile Bank Ltd.
		DLB	The Dhanalakshmi Bank Ltd.
		YBL	Yes Bank Ltd.

Notes. BC: Bank Codes, PrSBs: Private Sector Banks, PuSBs: Public Sector Banks.

6. Application to the Indian Banking sector

In this section, the proposed approach is applied to the Indian banking sector to demonstrate its effectiveness. The banking sector in India dominates the Indian financial system by holding approximately 64% of the total assets and is regulated by the central bank of India, known as the Reserve Bank of India (RBI). For the economic growth of the country, it is required to analyze the performance of banks regularly. There exists comprehensive literature on the performance analysis of the banking sector in India using DEA and its extensions like Puri and Yadav [56], Puri and Yadav [58], Gulati and Kumar [31], Gulati and Kumar [32], Bansal and Mehra [11] and Wanke *et al.* [78]. However, it is evident to measure the bank efficiency and analyze the impact of stressed assets on banks in a dynamic environment. Therefore, in the present study, a total number of 31 scheduled banks which include 12 public sector banks and 19 private sector banks, are selected as DMUs to analyze the banks' performance over five consecutive periods (2017–2021) and are listed in Table 2. In 2015–16, RBI conducted a special inspection named asset quality review (AQR), exposing that many banks were hiding bad loans or NPAs under regulatory forbearance. It was found that NPAs increased by 80% after AQR. A large number of bad loans resulted from restructured standard advances in these banks, which after adding to the gross NPAs, gives the realistic figure of total stressed assets in banks. Owing to the role of total stressed assets, these must be considered as a bad link in analyzing bank performance in a dynamic environment. Many authors have applied dynamic DEA approaches to evaluate the efficiency of banks in a dynamic environment, which are summarized in Table 3. The existing literature on dynamic bank efficiency has not considered the variables like stressed assets as bad carry-overs and loss due to NPAs as undesirable output.

6.1. Selection of data variables

There exist several approaches in the literature, namely, the 'assets approach', 'production approach', 'user-cost approach', 'intermediation approach', 'income-based approach', and 'value-added approach' to select input/output variables, out of which the 'production', and the 'intermediation' approaches have been widely used in performance analysis of banking sector [57]. In the production approach, banks utilize physical inputs

TABLE 3. Review of research articles on banking efficiency in dynamic environment.

Paper	Area of study	Methodology	Variables	Periods	DMUs
Shafiee *et al.* [65]	Iran	SBM in dynamic DEA	Inputs: Average monthly salaries, operating expenses. Outputs: Total value of loans. Links: Net profit (good link), loans losses (bad link).	3	10
Wanke *et al.* [76]	Brazil	Dynamic SBM model	Inputs: Non-interest expenses, no. of branches (control variable). Outputs: Total equity, Liquid assets, total assets. Links: Net income (good link), loan losses reserves (bad link).	16	40
Zha *et al.* [80]	China	Dynamic two stage, SBM model	Inputs: interest cost, operating cost. Outputs: Interest income, operation income. Intermediates: Deposits. Links: Non-performing loans (NPLs).	5	25
Zhou *et al.* [82]	Ghana	Dynamic network SBM model	Inputs: Personnel expenses, interest expenses. Outputs: Loans, interest, non-interest income (desirable), NPLs (undesirable). Intermediates: Deposits. Links: Previous loans, previous NPLs.	6	27
Zhou *et al.* [83]	China	Multi period three-stage DEA model	Inputs: Employees' salaries, fixed assets (shared inputs), interest payments. Outputs: Net interest income, NPLs. Intermediates: Deposits, due from banks, total loans. Link: Unused assets.	3	16
Wang *et al.* [75]	All over the world	Dynamic SBM model	Inputs: Assets, capitalization, liabilities. Outputs: Revenue. Links: Net interest income (good link).	5	18
Wanke *et al.* [77]	OECD banks	Dynamic network DEA and SFA models	Inputs: Net loans, personnel expenses, total earning assets, fixed assets, loan loss reserves, costs. Outputs: Net interest margin (shared), total equity, net income. Intermediates: Net interest margin, total equity, total assets (shared), cash, and due from banks. Links: Total assets, net income (shared), net interest income.	10	124
Bansal *et al.* [12]	India	Dynamic productivity index	Inputs: Labor, fixed assets, loan loss provisions, inter-bank borrowings (desirable), NPLS (undesirable), equity (fixed input). Outputs: Net interest income, non-interest income (desirable outputs), NPLs (undesirable output). Intermediates: Deposits, performing loans, investments. Links: Net profit, unused assets.	4	60
Razipour-GhalehJough *et al.* [60]	Iran	Dynamic SBM model	Inputs: Staff privileges, profits paid. Outputs: Facilities, profits received, bank charges received (desirable), deferred claims (undesirable). Links: Other sources, total deposits (depended inputs).	12	36
Wanke *et al.* [78]	India	Dynamic SBM model	Inputs: Interest expense, operating expense. Outputs: Deposits, loans, investments. Links: Net profits (good link), NPLs (bad link).	13	58

like capital, number of employees, and number of branches to produce outputs like the number of deposit accounts and loan accounts. On the contrary, in the intermediation approach, a bank consumes employees' cost, interest cost, and other operating costs to produce outputs like the value of advances, total income, and value of investments. So intermediation approach considers the monetary values of labor and funds and the monetary values of assets as inputs and outputs, respectively. Berger and Humphrey [14] suggested that the production approach can be a better approach for measuring the efficiencies of bank branches, whereas the intermediation approach is better for evaluating efficiency at the bank level. So, the latter approach is followed in this study to select input/output variables, keeping in mind the data availability.

In the present study, the dynamic structure of the Indian banks for the periods 2016–17 to 2020–21 has been shown in Figure 3, in which every bank uses three inputs (Employees, Fixed assets, and Loanable funds) to produce three desirable outputs (Advances, Investments, and Net profit), along with one undesirable output, namely, Loss due to NPAs. Stressed assets are considered a bad link and calculated as the sum of NPAs, write-offs, and restructured loans during the financial year. Due to the data availability, it is not possible to distinguish the links in the form of assets that come from loans and assets that come from securities [28]. Therefore, the carry-over assets that come from loans and securities together are named as unused assets, and it is considered as a good link [11,28]. Table 4 presents a brief description of all the inputs/outputs and links used for the present

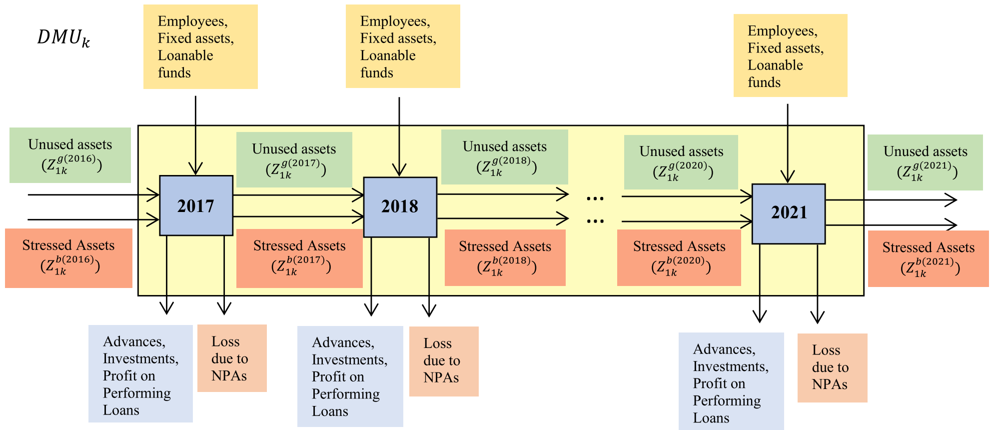


FIGURE 3. Dynamic structure of the kth bank for periods 2017–2021.

TABLE 4. Input-Output variables and links for banks.

Inputs	Employees (Emp.)	These include officers, clerks, and sub-staff members working with the bank.
	Fixed assets (FA)	These include premises, fixed assets under construction, and other fixed assets.
	Loanable Funds (LF)	It is the amount of deposits and borrowings which is available with the bank for lending loans.
Desirable Outputs	Advances (Adv.)	These include cash credits, bills purchased and discounted, term loans, and overdrafts and loans.
	Investments (Inv.)	These include shares, government securities, other approved securities, debentures and bonds, subsidiaries and joint ventures, and others.
	Net profit (NP)	It is calculated by subtracting the interest expenditure from total earned interests.
Undesirable Output	Loss due to NPAs (LNPAs)	It is the amount that a bank holds as a provision for NPAs where NPAs stand for the non-performing assets.
Good Link	Unused assets (UA)	Unused assets are calculated by subtracting required reserves, loans, security investments, and fixed capital from total assets.
Bad Link	Stressed assets (SA)	Stressed assets of a financial year are the sum of NPAs, write-offs, and restructured loans during the year.

case study. The data for inputs/outputs and links have been collected from RBI [61] and the descriptive statistics of data for input-output variables and links are listed in Appendix B (Tables B.1 and B.2) of *supplemental file*, in which all the variables except employees are measured in Rs. Crores. Later, for the normalization, the data for all inputs/outputs/links are divided by the respective number of branches of each bank [12].

6.2. Imprecision in inputs and outputs

The data collected for input-output variables are available in crisp form. However, some imprecision always exists in the data, which can be well represented in the interval or fuzzy or ordinal form. Also, the amount and category of imprecision may vary for different variables in the banking sector. In the present study, we are taking the imprecision in the form of interval numbers. As the transfer of deposits into loans and investments

TABLE 5. Interval efficiencies of the banks from the proposed approach.

DMU	System	P1	P2	P3	P4	P5
BoB	[0.5453, 0.6787]	[0.4939, 0.6091]	[0.5268, 0.6603]	[0.5568, 0.6931]	[0.5733, 0.7204]	[0.5689, 0.7027]
BoI	[0.5648, 0.6724]	[0.4995, 0.5952]	[0.5305, 0.6390]	[0.5702, 0.6889]	[0.5814, 0.6908]	[0.6331, 0.7371]
BoM	[0.7438, 0.8641]	[0.5594, 0.6621]	[0.6323, 0.7304]	[0.8182, 0.9767]	[0.8116, 0.9293]	[0.8816, **1.0000**]
CaB	[0.5784, 0.6414]	[0.6115, 0.6842]	[0.5920, 0.6712]	[0.5631, 0.6307]	[0.5822, 0.6443]	[0.5515, 0.5927]
CBoI	[0.8412, 0.8658]	[0.7289, 0.7603]	[0.7101, 0.7273]	[0.8698, 0.8897]	[0.9605, 0.9842]	[0.9772, 0.9993]
INB	[0.6547, 0.7719]	[0.5939, 0.7115]	[0.6140, 0.7446]	[0.6479, 0.7564]	[0.6857, 0.8019]	[0.7214, 0.8313]
IOB	[0.7162, 0.7552]	[0.6849, 0.7646]	[0.6489, 0.6898]	[0.6372, 0.6635]	[0.7357, 0.7623]	[0.8666, 0.8873]
PSB	[0.6211, 0.7075]	[0.6345, 0.7301]	[0.6669, 0.7603]	[0.5641, 0.6373]	[0.5575, 0.6383]	[0.7088, 0.8050]
PNB	[0.6665, 0.7631]	[0.6096, 0.7043]	[0.6162, 0.7197]	[0.6559, 0.7468]	[0.7008, 0.8042]	[0.7424, 0.8304]
SBI	[0.6987, 0.8681]	[0.6505, 0.8205]	[0.7193, 0.9060]	[0.6452, 0.8135]	[0.6801, 0.8254]	[0.7848, 0.9603]
UCOB	[0.8396, 0.9178]	[0.7678, 0.8194]	[0.7923, 0.8682]	[0.8717, 0.9226]	[0.9010, **1.0000**]	[0.9035, 0.9970]
UBoI	[0.6410, 0.7663]	[0.5999, 0.7167]	[0.6117, 0.7504]	[0.6070, 0.7263]	[0.6470, 0.7733]	[0.7274, 0.8502]
AXIB	[0.5937, 0.8360]	[0.5895, 0.8260]	[0.5693, 0.8592]	[0.5765, 0.8292]	[0.6012, 0.8259]	[0.6314, 0.8420]
BDN	[0.7645, 0.9729]	[0.8279, 0.8990]	[0.9637, 0.9816]	[0.9075, 0.9300]	[0.6443, **1.0000**]	[0.7242, **1.0000**]
CUB	[0.7101, 0.7832]	[0.6946, 0.7547]	[0.7067, 0.8019]	[0.7492, 0.8047]	[0.6820, 0.7666]	[0.7173, 0.7868]
CSB	[0.7513, 0.7849]	[0.8625, 0.8664]	[0.6258, 0.6285]	[0.6763, 0.6763]	[0.8401, 0.9164]	[0.7521, 0.8492]
DCB	[0.6451, 0.7623]	[0.6611, 0.7431]	[0.6640, 0.7623]	[0.6551, 0.7671]	[0.6266, 0.7500]	[0.6248, 0.7872]
FB	[0.6409, 0.7719]	[0.4611, 0.5530]	[0.5867, 0.7062]	[0.6737, 0.8127]	[0.7212, 0.8702]	[0.7520, 0.9064]
HDFC	[0.8120, **1.0000**]	[0.8084, **1.0000**]	[0.7916, **1.0000**]	[0.8244, **1.0000**]	[0.8202, **1.0000**]	[0.8459, **1.0000**]
ICICI	[0.6210, 0.8485]	[0.5936, 0.8717]	[0.5991, 0.9052]	[0.5996, 0.8381]	[0.6280, 0.8359]	[0.7075, 0.8319]
IIB	[0.6615, 0.8981]	[0.6535, 0.8332]	[0.6610, 0.8934]	[0.6212, 0.8344]	[0.7066, 0.9917]	[0.7497, 0.9753]
JKB	[0.6117, 0.6538]	[0.6345, 0.6782]	[0.5832, 0.6233]	[0.6307, 0.6808]	[0.5882, 0.6273]	[0.6229, 0.6615]
KB	[0.6190, 0.6495]	[0.7936, 0.8122]	[0.5845, 0.5962]	[0.5547, 0.5968]	[0.5506, 0.5983]	[0.6398, 0.6630]
KVB	[0.6966, 0.7663]	[0.6923, 0.7611]	[0.7155, 0.7978]	[0.7608, 0.8307]	[0.6234, 0.6769]	[0.6907, 0.7653]
KMB	[0.7503, 0.9304]	[0.7206, 0.8822]	[0.7502, 0.9176]	[0.7288, 0.9025]	[0.7587, 0.9404]	[0.8566, **1.0000**]
NB	[0.6082, 0.6082]	[0.7238, 0.7239]	[0.5739, 0.5739]	[0.5288, 0.5288]	[0.5145, 0.5145]	[0.7202, 0.7203]
RBL	[0.6573, 0.8809]	[0.6037, 0.8195]	[0.6435, 0.8595]	[0.6223, 0.8193]	[0.6941, **1.0000**]	[0.7276, 0.9169]
SIB	[0.5934, 0.6516]	[0.5951, 0.6301]	[0.6054, 0.6645]	[0.6151, 0.6788]	[0.5914, 0.6720]	[0.5613, 0.6101]
TMB	[0.7538, 0.7595]	[0.8350, 0.8351]	[0.7631, 0.7792]	[0.6703, 0.6703]	[0.6703, 0.6785]	[0.7596, 0.7597]
DLB	[0.8408, 0.8659]	[0.8251, 0.8410]	[0.8736, 0.9225]	[0.8659, 0.8912]	[0.7898, 0.8096]	[0.8647, 0.8814]
YBL	[0.5531, 0.9513]	[0.6064, 0.8748]	[0.6066, 0.9775]	[0.5725, **1.0000**]	[0.4276, **1.0000**]	[0.5601, 0.8957]

is a major source of a bank's income, so most of the employees of a bank are involved in this activity [58]. To represent the realistic picture, it is considered that 60–80% of the total employees are involved in this activity of transforming deposits into loans and investments, which is well represented in interval numbers, where lower and upper bounds of the interval for employees (Emp.) are taken as 60% and 80% of total employees, respectively. This imprecision may vary at the branch and bank levels. The other input variable, named loanable funds, also constitutes some imprecision in data. Loanable funds are the funds used to lend loans to customers. From the definition of the loanable funds, these mainly depend on the performing deposits, which are calculated as performing deposits = Total deposits − ((CRR+CAR+SLR)% of total deposits) where CRR, CAR, and SLR stand for the cash reserve ratio, capital adequacy ratio, and statutory liquidity ratio, respectively. Therefore, the lower bound of the loanable funds can be taken as performing deposits. Further, the upper bound of the loanable funds is calculated as the sum of performing deposits and borrowings since the banks sometimes take borrowings to meet the increasing demand for short-term loans. So, the data for loanable funds are incorporated as interval numbers for efficiency measurement in the present study. The conventional dynamic DEA models can not handle interval data, but the proposed approach overcomes this limitation of the existing models. To the best of our knowledge, this is the first study that measures the efficiency of banks in India with interval data in a dynamic environment in the presence of stressed assets as bad link.

6.3. Empirical results and discussion

System and Period interval efficiencies for 31 banks are evaluated by using the proposed approach and are listed in Table 5, where P1, P2, P3, P4, P5 denote the period 1 (2016–17), period 2 (2017–18), period 3 (2018–19), period 4 (2019–20), and period 5 (2020–21), respectively. Table 5 shows that HDFC is the only bank that has

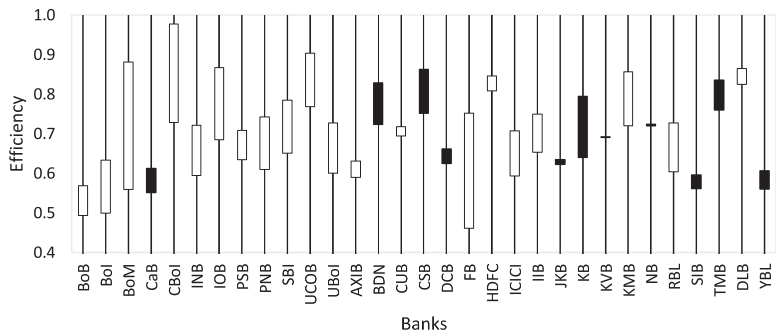
TABLE 6. Ranking of system and period interval efficiencies.

System	***CBoI*** ≻ DLB ≻ UCoB ≻ HDFC ≻ BDN ≻ TMB ≻ CSB ≻ KMB ≻ BoM ≻ IOB ≻ CUB ≻ SBI ≻ KVB ≻ PNB ≻ IIB ≻ RBL ≻ YBL ≻ INB ≻ DCB ≻ UBoI ≻ FB ≻ ICICI ≻ AXIB ≻ PSB ≻ KB ≻ JKB ≻ NB ≻ SIB ≻ CaB ≻ BoI ≻ **BoB**.
P1	HDFC ≻ CSB ≻ BDN≻ TMB ≻ DLB ≻ KB ≻ UCOB ≻ ***CBoI*** ≻ KMB ≻ NB ≻ CUB ≻ KVB ≻ IOB ≻ DCB≻ IIB ≻ SBI ≻ JKB ≻ PSB ≻ CaB ≻ PNB ≻ YBL ≻ ICICI ≻ RBL ≻ AXIB ≻ UBoI ≻ INB ≻ SIB ≻ BoM ≻ **BoB** ≻ BoI ≻ FB.
P2	BDN ≻ DLB ≻ HDFC ≻ UCOB ≻ TMB ≻ KMB ≻ SBI ≻ KVB ≻ ***CBoI*** ≻ CUB ≻ YBL ≻ PSB ≻ DCB ≻ IIB ≻ IOB ≻ ICICI ≻ RBL ≻ AXIB ≻ BoM ≻ CSB ≻ UBoI ≻ INB ≻ PNB ≻ FB ≻ SIB ≻ CaB ≻ KB ≻ JKB ≻ NB ≻ **BoB** ≻ BoI.
P3	BDN ≻ UCOB ≻ ***CBoI*** ≻ DLB ≻ HDFC ≻ BoM ≻ KVB ≻ CUB ≻ KMB ≻ YBL ≻ CSB ≻ FB ≻ TMB ≻ PNB ≻ DCB ≻ INB ≻ SBI ≻ IOB ≻ JKB ≻ RBL ≻ IIB ≻ SIB ≻ ICICI ≻ AXIB ≻ UBoI ≻ BoI ≻ **BoB** ≻ PSB ≻ CaB ≻ KB ≻ NB.
P4	***CBoI*** ≻ UCOB ≻ CSB ≻ HDFC ≻ BoM ≻ DLB ≻ KMB ≻ IOB ≻ FB ≻ IIB ≻ PNB ≻ RBL ≻ INB ≻ CUB ≻ SBI ≻ TMB ≻ UBoI ≻ BDN ≻ ICICI ≻ DCB ≻ KVB ≻ YBL ≻ AXIB ≻ **BoB** ≻ BoI ≻ SIB ≻ CaB ≻ JKB ≻ PSB ≻ KB ≻ NB.
P5	***CBoI*** ≻ UCOB ≻ BoM ≻ IOB ≻ DLB ≻ KMB ≻ HDFC ≻ SBI ≻ TMB ≻ CSB ≻ FB ≻ IIB ≻ BDN ≻ RBL ≻ PNB ≻ UBoI ≻ INB ≻ NB ≻ CUB ≻ PSB ≻ ICICI ≻ KVB ≻ KB ≻ BoI ≻ AXIB ≻ YBL ≻ DCB ≻ JKB ≻ **BoB** ≻ SIB ≻ CaB.

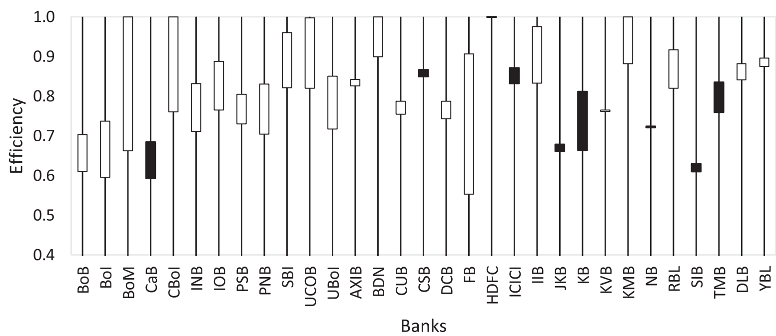
attained an upper bound system and period efficiencies equal to one. CBoI and HDFC have respectively attained the highest lower and upper bound efficiencies, therefore these two banks are the efficient banks (E^+). All other banks except CBoI and HDFC are in the category of inefficient banks. No bank belongs to the category of fully efficient or fully inefficient banks. In the period-wise study, P4 constitutes the maximum number of efficient banks (namely CBoI, UCoB, BDN, HDFC, RBL, and YBL) followed by P5 with five efficient banks, namely, CBoI, BoM, BDN, HDFC, KMB. Further, the ranking of the system and period interval efficiencies are obtained using a minimax regret-based approach (See Appendix A of *supplemental file*) and is given in Table 6. Ranking of the system interval efficiencies indicates that CBoI is the most efficient and BoB is the least efficient among all the banks, with ranks 1 and 31, respectively. The minimax-regret approach enables the decision maker to rank the equi-centred but different-width intervals. In this approach, the DMU with the smallest maximum loss of efficiency is regarded as the more efficient than others, where the maximum loss of efficiency for a DMU is calculated as the difference between the highest upper bound efficiency (among all DMUs) and the lower bound efficiency of that DMU for which the maximum loss of efficiency is being evaluated. Therefore, the ranking of banks is affected not only by the upper bound efficiencies but by the width of the interval efficiencies, which makes CBoI attain a higher system rank than DLB instead of DLB having upper bound system efficiency greater than CBoI.

6.3.1. Performance trend of lower and upper bound efficiencies from P1 to P5

The performance trend of lower and upper bound period efficiencies from P1 to P5 has been depicted in Figure 4. The upper starting point and lower ending point of a black box in Figure 4 represent the lower (upper) bound efficiencies of P1 and P5, respectively. In contrast, a white box's lower ending point and upper starting point represent the lower (upper) bound efficiencies of P1 and P5, respectively. Figures 4a and 4b depict that there is an increasing trend of lower and upper bound efficiencies from P1 to P5 for most of the banks. The lower bound efficiency of 20 banks and the upper bound efficiency of 22 banks have significantly increased from P1 to P5. However, the upper bound efficiency of the HDFC bank remains the same in both periods. Figure 4a suggests that maximum and minimum increases in lower bound efficiencies from P1 to P5 can be seen in BOM and CUB, respectively. Similarly, Figure 4b depicts that a maximum and minimum increase in upper bound efficiencies for the same is reported in FB and AXIB, respectively. Further, the banks that have



(A)



(B)

FIGURE 4. Performance trend of interval efficiencies from P1 (2016–17) to P5 (2020–21) in dynamic environment. (A) Performance trend of lower bound efficiency from P1 (2016–17) to P5 (2020–21). (B) Performance trend of upper bound efficiency from P1 (2016–17) to P5 (2020–21).

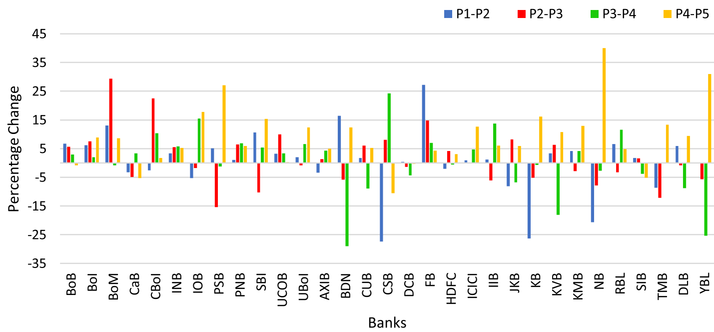
shown a decreasing trend in lower and upper bound efficiencies are CaB, CSB, JKB, KB, NB, SIB, and TMB, out of which maximum and minimum decrease in both lower and upper bound efficiencies has occurred in KB and NB, respectively.

6.3.2. Percentage change in lower and upper bound period efficiencies in consecutive periods

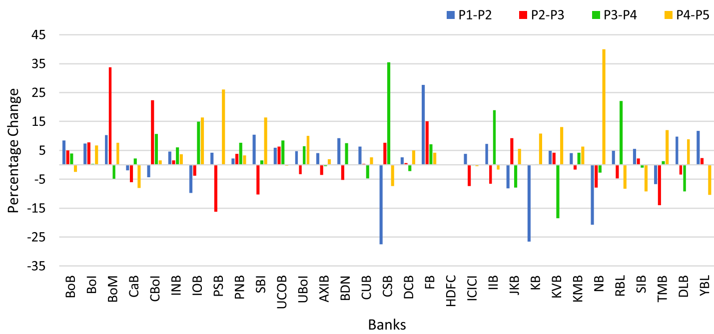
The percentage change has also been analyzed in consecutive periods and is presented in Figure 5, which depicts that FB, BoM, CSB, and NB have shown the highest percentage increase of efficiencies in P1-P2, P2-P3, P3-P4, and P4-P5, respectively. However, a few banks have shown a percentage decrease in lower and upper-bound efficiencies from P1 to P5. Only four banks (BoB, CaB, CSB, and SIB) and nine banks (BoB, CaB, UCOB, CSB, ICICI, IIB, RBL, SIB, and YBL) have shown a percentage decrease in P4-P5 for lower and upper bound efficiencies, respectively. The efficiency of BoI, INB, PNB, UCOB, and FB has increased in each period to some extent.

6.4. Comparison with a static approach

The static DEA models ignore the links connecting the periods and are based on the assumption that a DMU's efficiency solely depends on the inputs and outputs of a particular period. The proposed approach has taken into account the good and bad links to measure performance in a dynamic environment. A comparative analysis has been done between our proposed approach and the static approach. The static interval efficiencies of



(A)



(B)

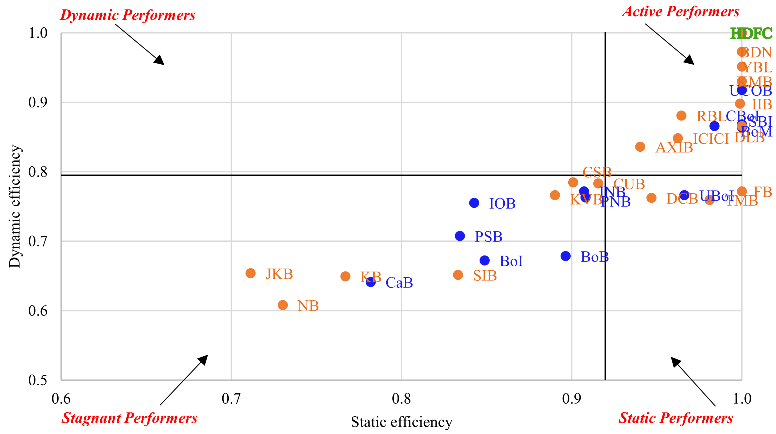
FIGURE 5. Percentage change in efficiency of banks during each period from P1 to P5 in dynamic environment. (A) Percentage change in lower bound efficiency of banks during consecutive periods. (B) Percentage change in upper bound efficiency of banks during consecutive periods.

31 banks are evaluated by ignoring the good and bad links and compared with the dynamic interval efficiencies obtained from the proposed approach.

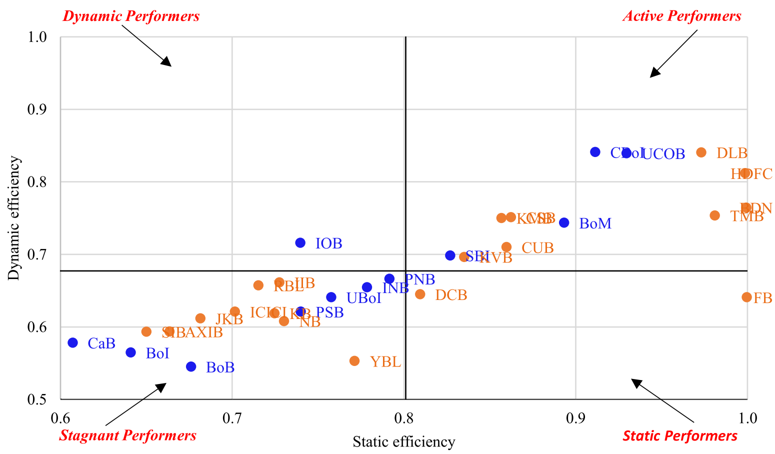
The comparison asserts that each bank has shown a reduction in lower and upper bound system efficiencies, respectively, in a dynamic environment as compared to a static environment. Moreover, nine banks (BoM, SBI, UCOB, BDN, FB, HDFC, KMB, DLB, and YBL) have upper bound efficiencies equal to 1 in the static model. However, using the proposed dynamic approach, HDFC is the only bank that has attained the upper bound efficiency equal to 1. FB is the most efficient bank with lower and upper bound efficiencies of 1 when examined in a static environment, whereas it ranked 21 in a dynamic environment. Thus, the proposed relational dynamic DEA approach depicts the effect of carryovers (Stressed assets and unused assets) on the efficiencies of Indian banks for the selected financial years. Therefore, it is apparent to use the proposed dynamic DEA approach with common weights and imprecise data for performance evaluation.

6.4.1. Comparison of banks using static-dynamic system efficiency matrix

Figures 6a and 6b depict a comparison of static and dynamic efficiencies for all the selected banks using the static-dynamic efficiency matrices in which the horizontal and vertical axes represent the lower bound static and dynamic efficiencies, respectively. In Figure 6a, the static-dynamic efficiency matrix is divided into four quadrants using the mean values of lower bound static efficiency and dynamic efficiency, represented by the horizontal and vertical lines in the matrix, respectively. The four quadrants are designated as follows:



(A)



(B)

FIGURE 6. Static-dynamic efficiency matrices. (A) Lower bound static-dynamic system efficiency matrix. (B) Upper bound static-dynamic system efficiency matrix.

(i) **Dynamic performers**: banks with lower bound static efficiency below the mean value of lower bound static efficiencies and lower bound dynamic efficiency above the sample mean value of lower bound dynamic efficiencies.

(ii) **Active performers**: banks having their lower bound static and dynamic efficiencies greater than the mean values of respective lower bound efficiencies.

(iii) **Stagnant performers**: banks having their lower bound static and dynamic efficiencies below the mean values of the corresponding lower bound efficiencies.

(iv) **Static performers**: banks with lower bound static efficiency above the sample mean value of lower bound static efficiencies and lower bound dynamic efficiency below the sample mean value of lower bound dynamic efficiencies.

A similar criterion is used to classify banks in the upper bound static-dynamic efficiency matrix (Fig. 6b). In Figures 6a and 6b, the banks in blue and orange colors, respectively, denote the 12 PuSBs and 19 PrSBs.

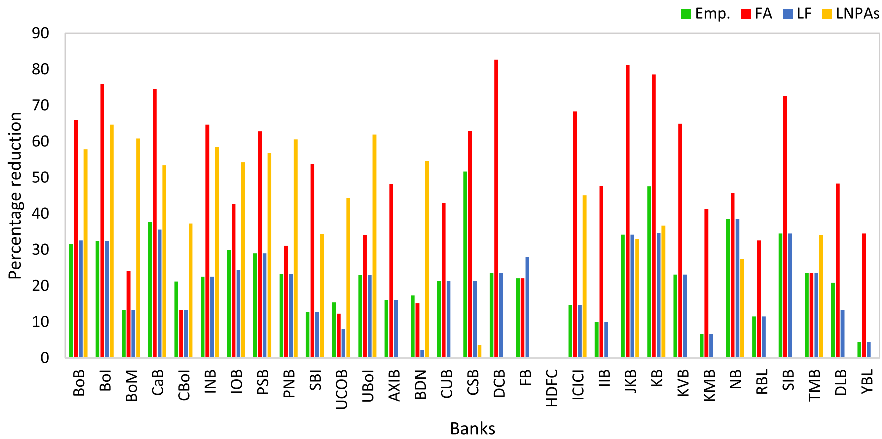


FIGURE 7. Percentage reduction in inputs and outputs for Indian banks.

Figures 6a and 6b reveal that in lower (upper) bound static-dynamic system efficiency matrix, (i) 8.33% (0%) PusBs and 0% (0%) PrSBs are dynamic performers, (ii) 33.33% (33.33%) PuSBs and 42.10% (47.36%) PrSBs are active performers, (iii) 58.33 % (58.33%) PuSBs and 47.37% (36.84%) PrSBs are stagnant performers, and (iv) 0% (8.33%) PuSBs and 10.53% (15.78%) PrSBs are static performers.

6.5. System targets for inefficient banks

Since the improvement in inefficient banks relies mainly on the improvements in their upper bound efficiencies, system targets for inefficient banks are evaluated using Model-5 and equation (54). Figure 7 presents the required percentage reduction in inputs and undesirable outputs of inefficient banks to achieve improvements and indicates that the highest percentage reduction has been found in FA (input), followed by the reduction in LNPAs (undesirable output) and Emp. (input). The targets identify the most sensitive variables (inputs/outputs) that need improvements for the efficiency enhancement of system inefficient banks. Furthermore, the least percentage reduction is found in LF (input). The percentage reduction ranges from (i) 12% to 82% in FA, (ii) 3% to 64% in LNPAs for 19 banks out of 30 inefficient banks, (iii) 4% to 51% in Emp., and (iv) 2% to 38% in LF.

7. CONCLUSION

In this study, a relational dynamic DEA approach is developed that consists of a dynamic structure having p periods, which are connected through good and bad links along with desirable and undesirable outputs in the presence of imprecise data. A common set of weights methodology is applied to develop lower and upper-bound models that preserve the linearity of DEA. The proposed approach evaluates system and period efficiencies in the form of interval numbers and theoretically validates (i) the relationship between the complement of the lower (upper) bounds of system and period efficiencies in the form of convex linear combination, (ii) stronger discrimination power of an approach to achieve the complete ranking of the DMUs in terms of system and period interval efficiencies, and (iii) estimation of inputs/outputs targets for inefficient DMUs to improve upper bound system efficiencies.

Moreover, to validate the practical applicability of the proposed approach in real life, it has been applied to 31 banks in the Indian banking sector consisting of five periods that summarizes the following practical implications:

(i) the present study exhibits valuable results for the policymakers and bank experts. The findings conclude that CBoI and HDFC are the efficient banks. All other banks can treat them as a benchmark for their efficiency improvement,

(ii) the number of efficient banks is more in P4 and P5 as compared to other periods; therefore, among all the periods, banks have performed better in these periods in a dynamic environment,

(iii) a large number of banks have shown an increasing trend in lower and upper bound efficiencies from P1 to P5. The maximum increase of 57.6% in the lower bound efficiency of BoM and 63.91% in the upper bound efficiency of FB is seen from P1 to P5. On the other hand, KB has shown the highest percentage decrease of 19% and 18.38% in lower and upper bound period efficiencies, respectively, and its rank has dropped from 6 in P1 to 23 in P5. The banks which have shown decreasing trend in terms of efficiencies are recommended to improve their efficiency by following the benchmark banks,

(iv) the percentage change in lower and upper bound period efficiencies in consecutive periods infer that among all the banks, almost 30%, 50%, and 31% of the banks have shown a percentage decrease in all the consecutive periods P1-P2, P2-P3, and P3-P4, respectively; however almost 80% banks have shown percentage increase in consecutive periods P4-P5, in particular, NB is the only bank that has firstly shown percentage decrease in efficiency during periods P1-P2, P2-P3, and P3-P4; however, it has shown tremendous increase in period P4-P5. Therefore, period P4-P5 outperforms all other periods in lower and upper bound efficiencies,

(v) the comparative analysis of the dynamic proposed approach and static approach depicts a fall in the efficiencies from static to the dynamic environment. It implies that the bad link (stressed assets) has shown an adverse effect on system efficiency in a dynamic environment as compared to the static one. Therefore, it should be considered while measuring the dynamic efficiency of the banking sector,

(vi) almost 29% banks (25% PuSBs and 30% PrSBs) are evaluated as efficient in a static environment; however, only one bank (HDFC) is assessed as a benchmark in a dynamic environment. It clearly indicates the impact of links (good and/or bad) on the system efficiency of the banks. Therefore, policymakers are suggested to utilize efficiency results evaluated in a dynamic manner for policy implications,

(vii) the classification of the static and dynamic efficiency results in the form of a static-dynamic system efficiency matrix exhibits that IOB is the only bank that is evaluated as a dynamic performer in terms of lower bound system efficiency. The highest percentage of PuSBs comes under the section of stagnant performers, whereas the highest percentage of PrSBs comes under the section of active performers, and

(viii) the system targets for inefficient banks are also evaluated, which indicates that the highest percentage reduction has been found in FA, followed by LNPAs and Emp., whereas the least percentage reduction is seen in LF.

The present study can handle the fuzzy/ordinal data by converting it to interval numbers and thus enables the decision-makers to assess the technical efficiency of DMUs in uncertain environments. Moreover, it considers the undesirable outputs and bad links along with initial and terminal links, which makes it more advantageous. The targets evaluated using the proposed approach can further assist decision-makers in refining their strategies for efficiency enhancements.

However, the present study is limited to its implementation on entities having dynamic structures without networks. But in practical situations, the functioning of several organizations, such as insurance, health care, power plants, etc., is well represented by complex network dynamic structures. Moreover, the proposed models in the present study exhibit constant returns to scale. Therefore, we plan to extend the current approach to a dynamic network structure that can also handle real-time data sets with different types of data uncertainties and incorporate variable returns to scale. It leads to the technique applicable for more practical applications.

Acknowledgements

The authors are grateful to the anonymous reviewers for their constructive comments and suggestions that helped us improve the paper significantly. The first author is also thankful to the FIST grant (SR/FST/MS-I/2017/13) for providing support facilities at Thapar Institute of Engineering and Technology, Patiala, that helped us in the smooth conduct of the present work.

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