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EVALUATION OF RADIALITY CONSTRAINTS FOR POWER DISTRIBUTION NETWORKS

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Abstract. This work carried out an empirical study with the objective of determining the most efficient strategy in guaranteeing the radiality of the distribution network during the reconfiguration. The reconfiguration problem in a distribution network aims to reduce technical losses by changing the paths that energy takes through the network until it reaches consumers. In this problem, a mandatory constraint is to ensure that the network operates radially. However, the literature does not present a consensus on the best way to guarantee this restriction. Computational experiments were carried out using the two main methodologies in the literature: limiting the number of input edges; and search with removal of cycles in each partial solution. A third and new strategy was proposed as an alternative to the literature using the concept of minimal cuts in graphs. Computational tests showed that the strategy of limiting the number of input edges was the most efficient in ensuring radiality. We also point out a drawback in the applicability of this strategy to networks with high penetrations of distributed generation (DG). For this reason, our experiments also analyze the other strategies that are suitable for DG power flows. For this case, we show that the methodology proposed in this paper provides a better way to tackle modern networks arising from applications of smart grids.

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1. Introduction

The transportation of electrical power, whether in the transmission or distribution stage, inevitably generates technical energy losses, either because of the transformation of electrical energy into thermal energy, which occurs in the conductors, and equipment like transformers, capacitors, switches, and so on [1]. Most distribution networks are radially operated, that is, there is a single energized path between the distribution substation and each consumer unit [2]. Nonetheless, distribution networks present switches in their composition that can assume the state of open or closed, thus changing the paths where power flows, changing the losses in the network [3].

Keywords. Network reconfiguration, radiality, minimum cut.

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Originally, switches were designed as protection devices serving to confine the effects of contingencies. Merlin and Back [3] observed that changing the state of the keys of a network could provide opportunities to reduce technical losses. This process is called network reconfiguration, and several works in the literature have proposed techniques to find optimal solutions to this problem [4]. Cavellucci and Lyra [5] proposed a methodology that merged non-linear flows with search algorithms. Bueno *et al.* [6] considered the reconfiguration problem with variable demands and proposed mathematical models to solve it. A Tabu Search was developed by Guimaraes and Castro [7] using artificial intelligence techniques; Queiroz and Lyra [8] proposed an Adaptive Hybrid Genetic Algorithm applied to the reconfiguration problem with variable demands. Genetic Algorithms were also used by Torres-Jimenez *et al.* [9] but considering only fixed demands. Another populational approach as develop by Reddy and Reddy [10] in their Sine Cosine optimization algorithm.

Due to the nature of the operation of distribution networks, guaranteeing the radiality of the solution is an important step in any optimization model applied to the reconfiguration problem [11]. Therefore, choosing the best strategy to guarantee radiality directly impacts the quality of the solutions obtained and the total execution time.

In other words, saying that a distribution network operates radially is the same as saying that the graph $G = (V, E)$, which represents this network, is a tree. In other words, there is only one path between each consumer node and its supply substation. In the case of graphs with fixed weights on the edges, this is a problem with an efficient solution already known in the literature [12]. However, in the reconfiguration problem this is not true, each tree generated as a solution has its own values of active power (P) and reactive power (Q) passing through the edges. Therefore, the big challenge here is: determining which tree and which weights (P and Q) should be used as a solution.

In networks without Distributed Generation (DG), to impose radiality it is sufficient to limit the number of edges with input flow to one per node [13]. However, in the presence of DG, this restriction cannot be applied. To deal with such networks, Jabr *et al.* [14] impose an edge orientation so that each node has a single parent node. Despite its widespread use, the literature has already shown that this methodology produces disconnected graphs [15]. A model based on dual graphs was suggested by Ahmadi and Martí [15], but it is limited to planar graphs. For a general network, the most common methodology for ensuring radiality is still the elimination of cycles in each partial solution found by the model [16]. Despite the clear importance of guaranteeing radiality in solving the reconfiguration problem, it is observed in the literature a lack of experiments that evaluate the existing strategies and determine which is the most efficient.

This work proposes the application of a new methodology to guarantee radiality using the concept of minimum cuts in graphs. To evaluate the performance of the new proposed methodology, comparisons were made with two well-known strategies to guarantee radiality in the reconfiguration problem: (i) limit the amount of input edges in each vertex [13]; (ii) search and removal of cycles in each partial solution [16]. A performance comparison of the methodologies was conducted with different scenarios, using a benchmark with real and artificial networks. Determining the most efficient strategy is very important in this specific problem, since a considerable part of the problem solution time is used to guarantee radiality. To the best of our knowledge, this is the first work to offer a computational study of this nature.

Another important difference of this work is the set of benchmark networks that was used and is being made available to the community. This is a benchmark with 72 distribution networks, real and artificial, of varying sizes (from 13 to 10 560 nodes) and different topology. With this benchmark, it is possible to more accurately assess the behavior of models compared to real distribution networks, since real networks are considered large from 20 thousand nodes onward. Despite this, current literature only focuses on smaller theoretical networks, up to 205 nodes [17].

This work is organized as follows: initially, Section 1 presents the objective of this work and its motivation, and the evolution of the literature on this topic. Section 2 contains all the concepts necessary for a complete understanding of this article, from the definitions of graphs, networks, flows and cuts to the calculation of technical losses. The mathematical model proposed to solve the reconfiguration problem is exposed in Section 3, together with the three radiality guarantee strategies considered: Cycle Elimination, Edge Limitation, and

Minimum Cut. The computational studies carried out are described and discussed in Section 4. Finally, the Section 5 lists all the conclusions obtained with this work together with future perspectives.

2. Preliminary concepts

This section contains the main theoretical concepts and the mathematical notation used in this work.

2.1. Networks and flows

A graph G is defined by a set of vertices $V(G)$ and a set of edges $E(G)$, in which an edge is an unordered pair $a = \{v_1, v_2\}$, v_1 and $v_2 \in V(G)$. When unambiguous, $V(G)$ and $E(G)$ are denoted by V and E, respectively. An oriented graph $\overrightarrow{G}$ has its edges composed of ordered pairs (v_1, v_2), where the edge represents the direction from v_1 to v_2. An oriented graph $\overrightarrow{G}$, where each edge $(v_1, v_2) \in E(\overrightarrow{G})$ has a capacity value $c(v_1, v_2) \in \mathbb{R}^+$ associated, is called a network. Given a node $v_1 \in V(G)$, the set of all nodes $v_2 \in V(G)$ for which there is an edge $(v_1, v_2) \in E(G)$ is denoted by $\delta(v_1)^-$. Analogously, $\delta(v_1)^+$ is the set of all nodes $v_0 \in V(G)$ for which there is an edge $(v_0, v_1) \in E(G)$.

As defined by Cormen *et al.* [18], a given network $\overrightarrow{G} = (V, E)$ where each edge $(u, v) \in E$ has a capacity $c(u, v) \geq 0$ and two vertices, an origin s and a destination (or sink) t, a flow in G is a real value function $f : V \times V \to R$ satisfying the properties:

- **Capacity constraint** – For all $u, v \in V$, $f(u, v) \leq c(u, v)$.
- **Flow conservation** – For all $u \in V \setminus \{s, t\}$, $\sum_{v \in V} f(u, v) = \sum_{v \in V} f(v, u)$.

The maximum flow problem in networks consists of: given a network, a source vertex s and a destination vertex t, determine a flow assignment to the edges of the network satisfying the two properties and such that the flow from s to t is as large as possible [18].

Given a network $N(E, V, c)$, a subset of vertices $S \subset V$, such that, $s \in S$, $t \notin S$ and $\overline{S} = V \setminus S$. A cut $(S, \overline{S})$ is the set of all edges of the network that have one end at S and another end at $\overline{S}$. The cut capacity $c(S, \overline{S})$ is the sum of the capacities of all the cut edges. The Minimum Cut problem consists of finding the cut with the smallest capacity in the network [19].

According to the maximum flow and minimum cut theorem, the maximum flow of a network is equal to the Minimum Cut capacity of that network [20].

2.2. Power flows and losses

Look at Figure 1, it shows a simplified representation of a distribution network:

Starting from the substations (SE1 and SE2), the energy travels along the wires (edges) to the consumers (vertices). The path it takes in the network is determined by the switching switches, closed switches ((SE, 1), (13, 14) and (6, 7)) allow the passage of electric current, while open switches ((2, 5) and (12, 15)) block the passage. In order to reduce short-circuit current and the complexity of protection and switching systems, distribution networks are organized radially, that is, there is only one energized path between the distribution substation and each consumer [21]. Using the concepts of graph theory, distribution networks are trees, without cycles.

The power distribution network is represented by a three $G = (V, E)$, where each edge $(i, j) \in E$ has a resistance value, $r_{i,j} \in \mathbb{R}^+$. There is a subset of vertices $F \subset V$ that represent the sources or substations of the distribution network. Every vertex $j \in V - F$ has an active power load $L_j^P \in \mathbb{R}^+$ and a reactive power load $L_j^Q \in \mathbb{R}^+$, which must be served by the substations.

Technical losses are calculated based on network power flows. A radial network can have its power flows and losses calculated through the recursive equations of [22]. In order to reduce the complexity of the mathematical models needed to solve the problem, Baran and Wu [22] also presented simplifications for the equations for calculating power flows. As is commonly used in the literature [5, 23–26], in this work, the simplified equations

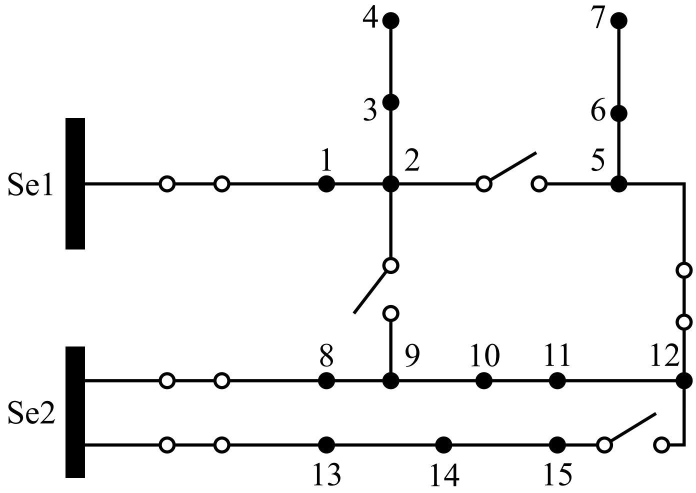

FIGURE 1. Example of a distribution network, containing: substation, consumers, open switches and closed switches.

(Eqs. (1)–(3)) will be used both for the calculation of power flows and for the calculation of the total losses of the network.

$$P_{ij} = P_{Lj} + \sum_{k \in \delta(j)^-} P_{jk} \tag{1}$$

$$Q_{ij} = Q_{Lj} + \sum_{k \in \delta(j)^-} Q_{jk} \tag{2}$$

$$T_L = \sum_{(i,j) \in E(G)} r_{ij} \left({P_{ij}}^2 + {Q_{ij}}^2 \right) \tag{3}$$

where:

P_{ij} is the active power flow on the edge $(i, j) \in E(G)$;
Q_{ij} is the reactive power flow on the edge $(i, j) \in E(G)$;
T_L is the network total loss;
P_{Lj} is the active power of the load at node j;
Q_{Lj} is the reactive power of the load at node j;
r_{ij} is the resistance on the edge $(i, j) \in E(G)$.

As these are recursive equations, the active power (1) and reactive power (2) must be calculated from the sheets in direction to the roots of the network. All quantities expressed in the equations are normalized using a concept from Electrical Engineering, called P.U. [27].

3. Mathematical model

The objective of the problem is to reduce the value of technical losses in the distribution network. To this end, the following Mixed Integer Linear Programming Model (MILP) is proposed, which will determine the best set of edges to be used, resulting in the lowest possible technical loss, while respecting all the operating restrictions of the distribution network. Binary variables $X_{ij} \in \{0, 1\}, \forall\ (i, j) \in E$, also called decision variables, are defined, which will tell for each edge of the network whether it should be used in the final solution $(X_{ij} = 1)$

or not ($X_{ij} = 0$). Note that the MILP itself already calculates the power flows in each edge (P_{ij} and Q_{ij}) while choosing which edges to use.

$$
\begin{aligned}
&\text{(Base Model)} \\
&\text{Min} \sum_{(i,j)\in E(G)} r_{i,j}\left({P_{ij}}^2 + {Q_{ij}}^2\right) && (4)\\
&\text{s.a.} \\
&\quad \sum_{i\in\delta(j)^+} P_{ij} \geq L_j^P + \sum_{k\in\delta(j)^-} P_{jk}, && \forall\, j \in V(G) && (5)\\
&\quad \sum_{i\in\delta(j)^+} Q_{ij} \geq L_j^Q + \sum_{k\in\delta(j)^-} Q_{jk}, && \forall\, j \in V(G) && (6)\\
&\quad P_{ij} \leq X_{ij} M^P, && \forall\, (i,j) \in E(G) && (7)\\
&\quad Q_{ij} \leq X_{ij} M^Q, && \forall\, (i,j) \in E(G) && (8)\\
&\quad \sum_{(i,j)\in E} X_{ij} = |V| - |F| && X_{ij} \in \{0,1\} && (9)\\
&\quad P_{ij}, Q_{ij} \in \mathbb{R}^+ && && (10)\\
&\quad M^P = \sum_{j\in V} L_j^P && && (11)\\
&\quad M^Q = \sum_{j\in V} L_j^Q && && (12)\\
&\quad G \text{ it's a tree.} && && (13)
\end{aligned}
$$

The objective function (4) minimizes the total losses of the resulting optimal configuration. Constraints (5) and (6) are responsible for the load balance of the network. They guarantee that the flows of active and reactive power on each edge are enough to meet all the demands of the consumers.

The active and reactive power flows of each edge are bounded by constraints (7) and (8). The power flow of an edge can only assumes a positive value if the edge is present in the solution. Valid upper bounds for the active and reactive power flows are given by parameters M^P and M^Q, respectively, and their values are set as the sum of all loads.

Since the distribution networks considered here operate radially, constraint (13) states that the network must be a tree. To ensure that a graph $G = (V, E)$ is a tree if is sufficient that the network: (i) has exactly $|V| - 1$ edges; (ii) is connected [28]. In the case of distribution networks, the solution can actually be formed by multiple trees (forest), where each tree starts at a distribution substation. In this case the number of edges in the solution is actually $|V| - |F|$.

Next, in Sections 3.2, 3.1 and 3.3, three different strategies will be presented to guarantee that the solution is radial.

3.1. Indegree bounding

The energy distributed in the network is injected by distribution substations or by a DG source. It means that to supply a customer represented by a node $j \in V$, it is necessary to have a single energized path from a power source to j. However, constraints (5) and (6) do not guarantee that there will be a unique energized path for every node [29]. The bounding indegree strategy ensures radiality by making incoming edge [13]. This rule can be asserted through the following constraint:

$$\sum_{i\in\delta(j)^+} X_{i,j} = 1 \qquad \forall\, j \in V - F. \tag{14}$$

The indegree bounding has a polynomial number of constraints which makes the model lighter and could reduce the computational effort [13].

A drawback of the indegree bounding is that it is not suitable for network with DG. It should be noticed that there are two possibilities for a DG node to be part of a radial network: (i) the node is part of a substation's tree; (ii) the node is the root of its own tree [11]. Forcing every node to have an incoming edge prevents case (ii), thus making the model an approximation of the reconfiguration problem.

3.2. Cycle elimination

The cycle elimination strategy is the strategy most used in literature and it assumes that if a graph G has $|V|-1$ edges, but it is not a tree, them G has at least one cycle and it is not connected [28].

The cycle elimination is represented by:

$$\sum_{(i,j)\in E(S)} X_{i,j} \leq |S|-1 \qquad \forall\ S \subseteq V(G). \tag{15}$$

Constraint (15) limits the number of edges in any subgraph of the network. However, the number of subgraphs in the network grows exponentially with the size of the network. For this reason, we first check if our solution contains cycles, and that is the case, then we insert enough constraints (15) to remove the cycles. More formally, if a cycle C is found in the solution, a new constraint

$$\sum_{(i,j)\in E(C)} X_{i,j} \leq |C|-1$$

is included to avoid the formation of this cycle.

3.3. Minimum cut

The minimum cut strategy proposed in this work guarantees radiality by making use of the concepts presented in Section 2.1. The underlying principle is that any subset of customers with positive demands must be connected to at least one power source. This is modeled by constraints (16) that enforce at least one edge connecting any such subset of costumers:

$$\sum_{(i,j)\in \delta(S,\overline{S})} X_{i,j} \geq 1 \qquad \forall (S,\overline{S}) \subseteq V-\{F\cup V_0\}. \tag{16}$$

As the cycle elimination strategy (Sect. 3.2), once again we have an exponential number of constraints. To decide which constraints to consider, the first step is to build a graph G_f, induced by a fractional solution of the Base Model, such that capacity $c(v_1,v_2)=X_{v_1v_2}$ is added to each edge (v_1,v_2). Add a new node s to the graph G_f, denominated as source, and add a new edge (s,v_2) for each substation $v_i\in F$ capacity $c(s,v_2)=\infty$. Also add a new vertex t in G, denominated as target, and add an edge (v_1,t) for each non-source vertex $v_i\in V-F$, with capability $c(v_1,t)=\infty$. With G_f we find a minimum cut $\delta(S,\overline{S})$ that separates s from t. If $\delta(S,\overline{S})<1$ then a constraint (16) is violated, thus we include it in the Base Model.

4. Computational experiments

A performance comparison among the tree radiality strategies were carried out for the problem of reconfiguration of distribution networks. A benchmark of seven IEEE classical networks was adopted (bus_13, bus_32, bus_83, bus_135, bus_201, bus_873, bus_10476). We have also created new instances extracted from random subgraphs of the bus_10476 network: bus_1036, bus_1579, bus_2011, bus_2572, and bus_3115.

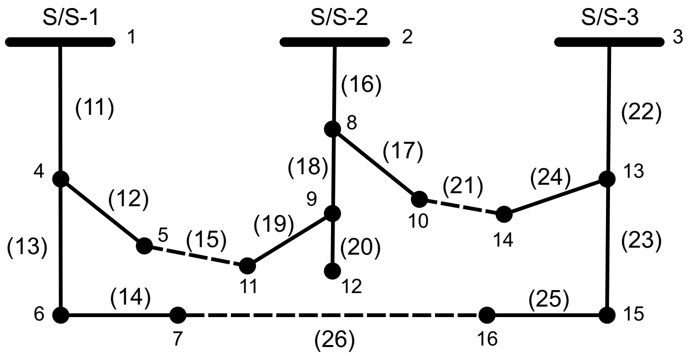

Figure 2. IEEE bus_13 network before reconfiguration.

To evaluate the performance of the methodologies for different networks densities, we have included 5 variations for each of the 12 inputs networks with respect to the number of open switches. The variations were created with 10%, 20%, 30%, 40%, and 50% of additional edges with open switches.

In total, the tests involved 72 different distribution networks. All networks used as input can be accessed *via* the link `https://github.com/jonatastbelotti/reconfiguration-networks`. The objective of this set of inputs was to provide a performance comparison closer to the execution of the models in real city networks, which are significantly larger than those commonly used in the literature. While the literature focuses on networks of up to 205 vertices [17], in practice, real networks are considered large from 20 thousand nodes onward.

For illustration purposes, Figure 2 shows the IEEE bus_13 input network before the reconfiguration process. It is possible to visualize its 3 substations, 16 consumer vertices and 16 switches (every edge is a switch), 3 of which are open and 13 are closed.

Table 1 presents the main characteristics of the input networks, containing the number of nodes in each network, the number of edges, the number of substations (S/E), the total active load (Total P), the total reactive load (Total Q) and the initial network loss. The number at the end of each network name indicates the percentage of open switches present on the network.

The methodologies were implemented in the C++ programming language using the GCC 8.4.0 compiler making use of the 03 optimization level. As the MLP solver, Gurobi 9.5.2 was addressed with the following execution parameters: one thread (Threads=1), RAM memory usage limit of 7 GB (MemLimit=7), maximum execution time of one hour (TimeLimit=3600), and all Gurobi cuts have been disabled (Cuts=0). The experiments were performed on a computer with 32 GB of RAM, a CPU with 6 cores of 1.6 GHz each, and Ubuntu 18.04.6 LTS 64-bits operational system.

New constraints were added to the model using lazy constraints during Gurobi's callback. In the cycle elimination strategy, each fractional solution was converted into an integer solution, in which every edge with a value greater than or equal to 0.2 was included (to choose this value, tests were performed with 0.1, 0.2, 0.3, 0.4 and 0.5). To find all cycles in the solution, the algorithm of [30] implemented by Boost C++ Libraries. For the minimum cut strategy, the Goldberg's preflow algorithm [31] from the Lemon library were used.

4.1. Case studies

Table 2 contains the results achieved by the three strategies (Indegree bounding, Cycle elimination, Minimum cut). In the columns are depicted the execution time in seconds, and the final loss obtained by each model.

The Table 2 shows that all strategies reached the optimal solution for the bus_13 base networks. These results were expected, since this is the smallest network, with only 13 nodes, offering no difficulty for any model applied. In such small input networks, the chosen radiality guarantee strategy does not have direct implications on the

TABLE 1. Distribution networks data.

Network	Nodes	Edges	S/E	P (kW)	Q (VAr)	Loss (kW)
bus_13	16	16	3	28 700.00	17 300.00	613.25
bus_13_10	16	16	3	28 700.00	17 300.00	613.25
bus_13_20	16	17	3	28 700.00	17 300.00	613.25
bus_13_30	16	19	3	28 700.00	17 300.00	613.25
bus_13_40	16	22	3	28 700.00	17 300.00	613.25
bus_13_50	16	26	3	28 700.00	17 300.00	613.25
bus_32	33	37	1	3715.00	2300.00	182.19
bus_32_10	33	37	1	3715.00	2300.00	182.19
bus_32_20	33	40	1	3715.00	2300.00	182.19
bus_32_30	33	46	1	3715.00	2300.00	182.19
bus_32_40	33	54	1	3715.00	2300.00	182.19
bus_32_50	33	64	1	3715.00	2300.00	182.19
bus_83	94	96	11	28 350.00	20 700.00	488.64
bus_83_10	94	96	11	28 350.00	20 700.00	488.64
bus_83_20	94	104	11	28 350.00	20 700.00	488.64
bus_83_30	94	119	11	28 350.00	20 700.00	488.64
bus_83_40	94	139	11	28 350.00	20 700.00	488.64
bus_83_50	94	166	11	28 350.00	20 700.00	488.64
bus_135	143	156	8	18 312.81	7930.26	297.21
bus_135_10	143	156	8	18 312.81	7930.26	297.21
bus_135_20	143	169	8	18 312.81	7930.26	297.21
bus_135_30	143	193	8	18 312.81	7930.26	297.21
bus_135_40	143	225	8	18 312.81	7930.26	297.21
bus_135_50	143	270	8	18 312.81	7930.26	297.21
bus_201	204	216	3	27 571.56	17 084.54	511.15
bus_201_10	204	224	3	27 571.56	17 084.54	511.15
bus_201_20	204	252	3	27 571.56	17 084.54	511.15
bus_201_30	204	288	3	27 571.56	17 084.54	511.15
bus_201_40	204	335	3	27 571.56	17 084.54	511.15
bus_201_50	204	402	3	27 571.56	17 084.54	511.15
bus_873	880	900	7	124 871.61	74 362.22	1413.59
bus_873_10	880	970	7	124 871.61	74 362.22	1413.59
bus_873_20	880	1092	7	124 871.61	74 362.22	1413.59
bus_873_30	880	1248	7	124 871.61	74 362.22	1413.59
bus_873_40	880	1455	7	124 871.61	74 362.22	1413.59
bus_873_50	880	1746	7	124 871.61	74 362.22	1413.59
bus_1036	1043	1178	7	146 740.47	86 021.23	1979.37
bus_1036_10	1043	1178	7	146 740.47	86 021.23	1979.37
bus_1036_20	1043	1295	7	146 740.47	86 021.23	1979.37
bus_1036_30	1043	1480	7	146 740.47	86 021.23	1979.37
bus_1036_40	1043	1727	7	146 740.47	86 021.23	1979.37
bus_1036_50	1043	2072	7	146 740.47	86 021.23	1979.37
bus_1579	1592	1798	13	225 653.26	135 844.39	2467.04
bus_1579_10	1592	1798	13	225 653.26	135 844.39	2467.04
bus_1579_20	1592	1974	13	225 653.26	135 844.39	2467.04
bus_1579_30	1592	2256	13	225 653.26	135 844.39	2467.04
bus_1579_40	1592	2632	13	225 653.26	135 844.39	2467.04
bus_1579_50	1592	3158	13	225 653.26	135 844.39	2467.04
bus_2011	2026	2289	15	287 005.24	172 137.81	3240.20
bus_2011_10	2026	2289	15	287 005.24	172 137.81	3240.20
bus_2011_20	2026	2514	15	287 005.24	172 137.81	3240.20
bus_2011_30	2026	2873	15	287 005.24	172 137.81	3240.20
bus_2011_40	2026	3352	15	287 005.24	172 137.81	3240.20
bus_2011_50	2026	4022	15	287 005.24	172 137.81	3240.20
bus_2572	2594	2931	22	366 397.71	216 527.30	3599.20
bus_2572_10	2594	2931	22	366 397.71	216 527.30	3599.20
bus_2572_20	2594	3215	22	366 397.71	216 527.30	3599.20
bus_2572_30	2594	3675	22	366 397.71	216 527.30	3599.20
bus_2572_40	2594	4287	22	366 397.71	216 527.30	3599.20
bus_2572_50	2594	5144	22	366 397.71	216 527.30	3599.20
bus_3115	3139	3546	24	447 321.05	261 237.30	5567.09
bus_3115_10	3139	3546	24	447 321.05	261 237.30	5567.09
bus_3115_20	3139	3894	24	447 321.05	261 237.30	5567.09
bus_3115_30	3139	4450	24	447 321.05	261 237.30	5567.09
bus_3115_40	3139	4287	24	447 321.05	261 237.30	5567.09
bus_3115_50	3139	5144	24	447 321.05	261 237.30	5567.09
bus_10476	10 560	10 728	84	1 490 720.00	886 739.00	16 585.20
bus_10476_10	10 560	11 640	84	1 490 720.00	886 739.00	16 585.20
bus_10476_20	10 560	13 095	84	1 490 720.00	886 739.00	16 585.20
bus_10476_30	10 560	14 966	84	1 490 720.00	886 739.00	16 585.20
bus_10476_40	10 560	17 460	84	1 490 720.00	886 739.00	16 585.20
bus_10476_50	10 560	20 952	84	1 490 720.00	886 739.00	16 585.20

Table 2. Case studies results.

		Ind. Bound.		Cyc. Elim.		Min. Cut	
Network	Loss	Time	Loss	Time	Loss	Time	Loss
bus_13_3	613	0.01330	570	0.09534	570	0.09693	570
bus_13_3_10	613	0.01427	570	0.05415	570	0.09699	570
bus_13_3_20	613	0.01708	534	0.08568	534	0.14893	534
bus_13_3_30	613	0.02233	467	0.34331	467	0.66888	467
bus_13_3_40	613	0.04994	560	0.94132	560	1.64615	560
bus_13_3_50	613	0.06589	382	3.91968	382	4.46747	382
bus_32_1	182	0.06640	127	2.62123	127	2.29046	127
bus_32_1_10	182	0.08513	127	2.61203	127	3.59915	127
bus_32_1_20	182	0.10871	98	12	98	18	98
bus_32_1_30	182	0.16297	36	1324	36	1644	36
bus_32_1_40	182	0.56927	36	3600	36	3600	36
bus_32_1_50	182	2.95108	31	3600	31	3600	31
bus_83_11	489	0.15322	438	253	438	192	438
bus_83_11_10	489	0.16090	438	253	438	188	438
bus_83_11_20	489	0.38310	349	3600	349	3600	349
bus_83_11_30	489	1.64262	211	3600	211	3600	211
bus_83_11_40	489	51	111	3600	111	3600	111
bus_83_11_50	489	3600	136	3600	120	3600	120
bus_135_8	297	0.45114	265	3.00383	265	601	265
bus_135_8_10	297	0.27655	265	2.81654	265	498	265
bus_135_8_20	297	53	159	3600	159	3600	159
bus_135_8_30	297	3600	117	3600	113	3600	111
bus_135_8_40	297	3600	58	3600	63	3600	48
bus_135_8_50	297	3600	41	3600	34	3600	34
bus_201_3	511	1.3109	479	7.57554	479	3600	479
bus_201_3_10	511	3.41594	396	3600	396	3600	396
bus_201_3_20	511	3600	66	3600	64	3600	63
bus_201_3_30	511	3600	13	3600	12	3600	12
bus_201_3_40	511	3600	10	3600	10	3600	9
bus_201_3_50	511	3600	15	79	15	3600	15
bus_873_7	1414	60	452	3600	452	3600	452
bus_873_7_10	1414	3600	206	3600	206	3600	206
bus_873_7_20	1414	3600	104	3600	106	3600	105
bus_873_7_30	1414	3600	105	3600	106	3600	106
bus_873_7_40	1414	3600	38	3600	40	3600	40
bus_873_7_50	1414	3600	28	3600	30	3600	29
bus_1036_7	1979	3600	339	3600	339	3600	341
bus_1036_7_10	1979	3600	339	3600	342	3600	342
bus_1036_7_20	1979	3600	144	3600	145	3600	146
bus_1036_7_30	1979	3600	167	3600	169	3600	171
bus_1036_7_40	1979	3600	48	3600	92	3600	50
bus_1036_7_50	1979	3600	57	3600	61	3600	59
bus_1579_13	2467	3600	331	3600	333	3600	337
bus_1579_13_10	2467	3600	331	3600	333	3600	335
bus_1579_13_20	2467	3600	209	3600	220	3600	219
bus_1579_13_30	2467	3600	158	3600	221	3600	202
bus_1579_13_40	2467	3600	110	3600	130	3600	193
bus_1579_13_50	2467	3600	45	3600	287	3600	208
bus_2011_15	3240	3600	584	3600	617	3600	593
bus_2011_15_10	3240	3600	583	3600	617	3600	596
bus_2011_15_20	3240	3602	334	3600	340	3600	344
bus_2011_15_30	3240	3603	212	3600	1046	3600	786
bus_2011_15_40	3240	3600	130	3600	1713	3600	1238
bus_2011_15_50	3240	3600	97	3600	465	3600	245
bus_2572_22	3599	3600	477	3600	697	3600	503
bus_2572_22_10	3599	3600	477	3600	715	3600	507
bus_2572_22_20	3599	3600	332	3600	502	3600	559
bus_2572_22_30	3599	3600	220	3600	1842	3600	500
bus_2572_22_40	3599	3600	130	3600	3103	3600	2547
bus_2572_22_50	3599	3600	304	3600	3236	3600	2916
bus_3115_24	5567	3600	659	3600	748	3600	672
bus_3115_24_10	5567	3600	659	3600	735	3600	672
bus_3115_24_20	5567	3600	446	3600	693	3600	593
bus_3115_24_30	5567	3600	260	3600	4518	3600	344
bus_3115_24_40	5567	3600	214	3600	2757	3600	1964
bus_3115_24_50	5567	3600	259	3600	3154	3600	1002
bus_10476_84	16 585	3600	8701	3600	9807	3600	9088
bus_10476_84_10	16 585	3600	13 946	3603	16 585	3600	13 351
bus_10476_84_20	16 585	3600	16 585	3600	16 584	3600	16 551
bus_10476_84_30	16 585	3600	16 519	3600	16 585	3600	16 572
bus_10476_84_40	16 585	3603	16 585	3637	16 585	3600	16 585
bus_10476_84_50	16 585	3600	16 585	3600	16 585	3600	16 585

Table 3. Pooled results for instances with known optimals.

	Best solution	Best time	Mean deviation
Indegree bounding	24	23	0.0%
Cycle elimination	24	1	0.0%
Minimum cut	24	0	0.0%

Table 4. Pooled results for instances with suboptimal results.

	Best solution	Best time	Mean deviation
Indegree bounding	39	48	1.85%
Cycle elimination	7	48	228.46%
Minimum cut	12	48	122.58%

final quality of the solution found by the model. The same situation is repeated in all bus_32 base networks, and in bus_83 bases up to bus_83_11_40.

On the other hand, the data in Table 2 show that none of the tested models managed to find the optimal solution for any network with at least 1036 nodes. Although the models found solutions that provided significant reductions in energy losses, these networks presented great difficulty for the models to prove their optimality within the execution time limit. Therefore, based on this input size, choosing the most efficient radiality guarantee strategy has a direct impact on the quality of the final solution found.

Table 3 brings a performance comparison among the radiality strategies considering only the networks in which at least one of the three models was able to find the optimal solution. The results show that for 23 of 24 networks the indegree bounding presented the best execution time. Unlike the other strategies that have the overhead of callbacks, the indegree bounding seems to benefit from being a light model. Observing the number of times each strategy reached the best solution, all of them reached optimality for 24 networks.

Table 4 shows the results for a group of 48 networks in which none of the three strategies was able to prove the optimal solution. The number of times the best solution was attained by the indegree bounding was substantially higher than the other strategies, achieving 39 of the 48 (81%) networks. With respect to the deviation from the best solution, the degree bounding outperformed the others strategies, reaching a deviation of 1.85%. We highlight the deviations obtained by the other two strategies: 228.46% and 122.58%. These values can be explained using callbacks that drastically reduced the convergence of the models for large networks.

As previously mentioned, in practice, distribution networks in real cities must have at least 20 thousand nodes to be considered large. With the aim of providing a better understanding of the behavior of the methodologies tested in real networks, Table 5 presents a summary of the results obtained for the bus_10476_84 base networks. The first point to note is that, despite the high number of nodes and edges present in the input, the three models were able to provide at least one valid solution with lower loss than the original network. Once again, the fact of not using callbacks benefited the indegree bounding strategy, having an average deviation of 0.77%. Despite the use of callbacks, the minimum cut strategy came in second place, with an average deviation of 0.79%, a difference of just 0.02% in relation to first place. Again, in last place was Cycle Elimination, with 6.25% average deviation from the best solution found.

The results in Table 5 show the performance obtained by the minimum cut strategy proposed by this work. Despite making use of callbacks, it does so in a more intelligent way than the literature standard, which is the cycle elimination, thus obtaining a significant performance improvement and achieving results very close to the most efficient strategy, especially in large network scenarios.

Table 5. Pooled results for base bus_10476_84 instances.

	Best solution	Best time	Mean deviation
Indegree bounding	4	6	0.77%
Cycle elimination	2	6	6.25%
Minimum cut	4	6	0.79%

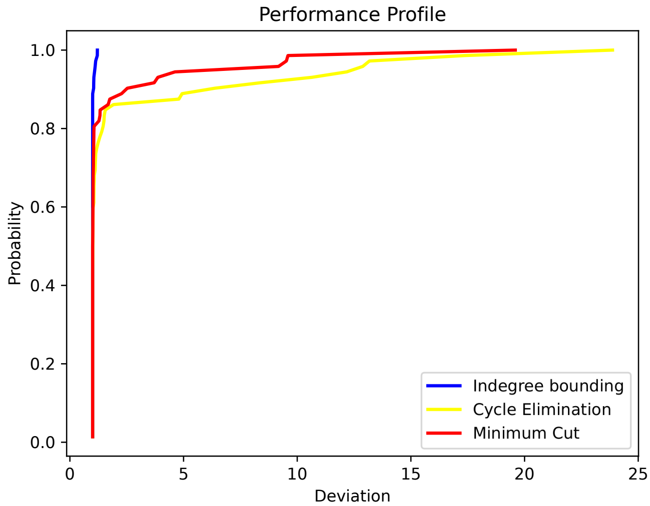

Figure 3. Performance Profile Chart for the three connectivity strategies against the tested instances.

Performance Profile is a performance metric based on a cumulative distribution function, it allows you to see how much a profile has deviated from its reference [32]. In this work we use the relationship between the solution found by a methodology and the best solution found by all methodologies, this way it is possible to check how far the solution of a methodology is from the best solution found by all methodologies. Figure 3 presents a compilation of the results, considering all the 72 networks. The ordinate axis represents the rate of instances that the algorithm was able to solve within the deviation from the best solution indicated in the abscissa axis.

The performance profiles confirm the dominance of the indegree bounding strategy with respect to the number of best solutions and deviation from the best solution, just check that the blue curve was the one with the smallest deviation (smallest change in x) along the entire y axis. Similarly, the next curves with smaller deviations along the y axis are red and green, confirming the minimum cut and cycle elimination strategies as second and third places.

The results showed that the strategy with the best performance in guaranteeing the radiality of the distribution network was the indegree bounding. Therefore, it is the most suitable to the reconfiguration problem. Despite its good general performance, we discussed in Section 3.1 that this methodology can prevent the optimal solution from being found, transforming the exact model into an approximate model for networks with DG. In order to provide better decision making in this case, the Table 6 contains a comparison of the results obtained only between the cycle elimination strategy, which is the literature standard, with the minimum cut methodology proposed by this work.

TABLE 6. Pooled results for Cycle elimination *vs.* Minimum cut.

	Best solution	Best time	Mean deviation
Cycle elimination	39	70	15.6%
Minimum cut	64	58	1.59%

The data in Table 6 leave no doubt about the superior performance of the proposed minimum cut methodology, when compared to cycle elimination. The minimum cut strategy achieved a greater reduction in energy loss by 64% more instances than cycle elimination, with an average deviation from the best solution found of just 1.59%. This shows that a more intelligent use of solver callbacks generated a significant gain in the efficiency of the mathematical model. Therefore, for distribution networks with DG, we recommend the minimum cut strategy, that obtained substantially better results that the cycle elimination strategy.

5. Conclusion

This work addressed three different strategies to guarantee the radiality of the networks: limiting the number of incoming edges (Sect. 3.1), elimination of cycles (Sect. 3.2), and minimum cut (Sect. 3.3). An empirical evaluations was carried out to determine which of the three strategies is the most effective. To this end, we develop a benchmark of 72 instances.

The results showed that for distribution networks without DG, the strategy that should be chosen is bounding the number of incoming edges. On the other hand, in the presence of DG, bounding the number of incoming edges should not be applied as it can prevent the optimal solution from being found. Therefore, in this case, the best strategy to guarantee the radiality of the network is the minimum cut.

A natural continuation of this work is the verification of whether the results found here remain the same when we add more complexity to the mathematical model, such as sources capacity (substations and DG) and volt/var control. Another interesting path would be to verify how the strategies presented here behave in a stochastic scenario, that is, when there is uncertainty in the input data.

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Data availability statement

No new data/codes were created or analyzed in this study.

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