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THE INTERACTION BETWEEN INTRODUCTION STRATEGY AND COMMISSION FORMAT IN LIVE-STREAMING E-COMMERCE

Qi Kou[1,*] and Yuyan Liao[2]

Abstract. Nowadays, an increasing number of manufacturers and retailers have begun partnering with internet celebrities to boost product sales. Furthermore, two primary commission formats have emerged for collaborations between internet celebrities and firms: the commission rate format and the fixed commission format. This paper examines the conditions under which manufacturers or retailers should cooperate with internet celebrities. Moreover, we also investigate the firms' strategic choices regarding commission format and examine the impacts of internet celebrities on firms' decisions. Based on the introducer of the live-streaming channel and the commission format, we use game theory to establish five different models in a supply chain comprising a manufacturer, a retailer, and an internet celebrity. Comparing the optimal decisions under different scenarios, we obtain the following results: (i) The retail price in the live-streaming channel is not always lower than that in the traditional channel. Correspondingly, the market demand for the live streaming channel increases even despite a higher retail price for the live streaming room. (ii) Both manufacturers and retailers are incentivized to embrace the live-streaming channel, particularly when the switch effect brought by the internet celebrity is small or large. (iii) Retailers have a broader range of conditions under which they are more likely to introduce the live streaming channel compared to manufacturers. There exists a transfer zone where the manufacturer prefers the retailer to cooperate with the internet celebrity under a fixed commission contract. (iv) Finally, the choice of commission format is influenced not only by the fixed commission, but also by the switch effect and consumers' preference for the traditional channel. These findings provide insights into the live transformation of firms in a supply chain.

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1. Introduction

The e-retailing market has experienced significant growth driven by the rapid advancement of electronic commerce. In 2019, sales on Chinese e-commerce platforms reached \$1.935 trillion, accounting for 36.6% of total retail sales, while the e-commerce platforms in the United States generated \$586.9 billion in sales[1]. Live streaming commerce, a novel marketing approach, is occupying the limelight. The unexpected outbreak of the

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[1] Department of Electronic Business, South China University of Technology, Guangzhou 510006, P.R. China.

[2] School of Economics, Nankai University, Tianjin 300071, P.R. China.

*Corresponding author: kq1843100574@163.com

[1]Available at: https://www.huxiu.com/article/482395.html.

COVID-19 pandemic accelerated growth in the live streaming market around the world. By 2022, the market size has surged to \$1.15 trillion in the United States and \$1.78 trillion in China[2].

The rapid growth of live streaming commerce has led to the emergence of numerous internet celebrities who rely on their follower base to promote and sell products. Nowadays, lots of companies are leveraging live streaming commerce as a sales channel. For instance, major brands, like Olay and Philips, have actively partnered with prominent internet celebrities and key opinion leaders (*e.g.*, Viya, Jiaqi Li, Pari Ehsan, and Olivia Culpo) to boost their product sales. Moreover, as competition intensifies, electric vehicle manufacturers, such as Tesla, GreatWall, and Geely, are increasingly turning to cooperate with famous internet celebrities with massive followers for marketing purposes. A representative from Geely stated: "Recruiting internet celebrities is one of the most effective ways to connect with current and next-generation customers". Chengcheng Ji, Dcar's director of content operations, remarked, "New online celebrities can create a sense of comfort and build stronger trust with viewers, ultimately driving more sales[3]". DJI, the world's largest drone manufacturer, builds a superior brand image in overseas markets with the help of social media and internet celebrities (*e.g.*, Casey Neistat, Kara and Nate, and Jordi Koalitic)[4]. These examples demonstrate that both manufacturers and retailers are increasingly adopting live streaming commerce. However, in a supply chain consisting of a manufacturer and a retailer, the manufacturer's introduction of the live-streaming channel may potentially rob the market share of the retailer, while the retailer's introduction of the live streaming may erode the pricing power of the manufacturer. Therefore, it is worth exploring who should introduce live streaming in a supply chain.

Furthermore, two common commission formats are prevalent in the live streaming industry [34]. First, the commission rate format involves manufacturers or retailers paying a commission for each unit of product sold by an internet celebrity through the live-streaming channel. Several studies on live streaming have developed theoretical models based on this commission structure [8, 14, 15]. Second, the fixed commission format entails manufacturers or retailers paying a predetermined commission to the internet celebrity for conducting a live broadcast. For example, several brands, such as Daniel Wellington, Audible and Sperry, use this fixed commission contract to collaborate with small internet celebrities. These varying commission formats can significantly influence the profits of manufacturers and retailers. In light of these dynamics, we investigate the commission selection for the supply chain members introducing the live streaming commerce.

The prior literature mostly focuses on the introduction of the live streaming commerce in the context of commission rate contract, overlooking the role of different introducers and other commission formats [16, 27, 28, 38, 41]. Furthermore, the existing studies characterize the live streaming from various aspects. For example, Niu *et al.* [28] consider the network effect brought by internet celebrities. Yang *et al.* [38] and Zhang *et al.* [42] spotlight on the sales effort made by the famous streamers. Zhou *et al.* [45] discuss the role of danmuku and the social interaction in live streaming platforms. However, they ignore the negative effects of live streaming. To fill these gaps, we consider the bright and dark sides of live streaming to examine the interplays between the introducers and commission formats in a supply chain. Specifically, we explore the following questions: (i) Under what conditions should manufacturers and retailers implement live streaming commerce? (ii) Which commission format is optimal for the party introducing the live streaming as a sales channel? (iii) How does live streaming commerce affect the optimal decisions of manufacturers and retailers?

Nowadays, a variety of methods are being employed to address challenges in operations management. For example, Midya *et al.* [23] develop a deterministic model of the intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in green supply chain network. Modal and Roy [24] incorporate Fermatean fuzzy-Mahalanobis distance into a three-way multi-attribute decision-making model and test the application of the model through two supply chain management cases. Mondal *et al.* [25] use an interval type-2 robust programming to predict demand in the sustainable-resilient-responsive supply chain. There are other methods to address the challenges in operations management [9, 10, 13, 15, 32]. In contrast to these methods, we establish

[2] Available at: https://global.chinadaily.com.cn/a/201811/20/WS5bf40b12a310eff303289eda.html.

[3] Available at: https://new.qq.com/rain/a/20210806A01WDH00.

[4] Available at: https://www.insmarkmcn.com/sys-nd/184.html.

a stylized model using game theory in a supply chain consisting of a manufacturer a retailer and an internet celebrity. Game theory, a widely used tool in supply chain management, has been employed in numerous studies [7,11,12]. Furthermore, our focus is to identify the conditions under which either the manufacturer or the retailer adopts the live streaming channel. Thus, we apply game theory to explore and address the issues presented in this paper.

Using game theory, we characterize two effects that capture the impact of live streaming commerce: the switch effect and the fan effect. The switch effect refers to the loss of customers from the traditional channel due to the competition from the live-streaming channel and the influence of the internet celebrity. The fan effect, on the other hand, describes the efforts made by the internet celebrity to encourage their followers to purchase products.

Then, considering different live-streaming introducers and commission modes, we explore the following five scenarios: (i) scenario N without the live-streaming channel as the baseline, (ii) scenario SM, where the manufacturer introduces the internet celebrity with the commission rate format; (iii) scenario SR, where the retailer adopts the internet celebrity with the commission rate format; (iv) scenario FM, where the manufacturer uses the internet celebrity with the fixed commission format; (v) and scenario FR, where the retailer introduces the internet celebrity with the fixed commission format. To analyze this complex supply chain system, we solve the optimal decisions of supply chain partners under each scenario. Comparing these results to the base model, we investigate the impact of live streaming and identify the conditions under which the live-streaming channel should be adopted. Moreover, we examine the optimal commission contract for the introducer if the live-streaming channel is implemented.

The contributions of this study are summarized as follows.

- Leveraging the bright and dark sides of internet celebrities and characterizing the switch and fan effects.
- Developing a theoretical model in a live streaming supply chain consisting of a manufacturer, a retailer, and an internet celebrity.
- Examining the interplay between the introduction strategies and commission formats.
- Enriching the existing literature and providing valuable insights into business practices on live streaming introduction and cooperation strategies.

The rest of the paper is organized as follows. Section 2 provides an overview of the relevant literature. In Section 3, we describe the problem and construct demand functions. Section 4 formulates the optimal decisions under different scenarios and compares them with those of the basic model. Section 5 explores equilibrium decisions under different scenarios and analyzes the impact of live streaming and optimal commission format selection. Finally, Section 6 concludes the paper and discusses future research directions. All proofs and thresholds are presented in Appendix A.

2. Literature review

Our study contributes to two streams of literature: live streaming commerce and channel management. Below, we outline the contributions to each of these areas in detail.

2.1. Live streaming commerce

There is significant attention in the literature on live streaming commerce. Previous studies have examined consumers' motivations [3,35,36,45] and the influences of live streaming [5,21,31,37] through using empirical approaches. Furthermore, several scholars have developed theoretical models to examine the impact of live streaming on firms' decisions in a supply chain. For example, Zheng *et al.* [44] analyze the optimal sharing ratio of platforms and seek to explain the reasons for the insufficient revenue of live stream platforms based on the principal-agent model. Considering the marketing efforts of internet celebrities, Yu *et al.* [39] develop a supply chain game model under three different power structures and analyze the optimal operational decisions for live streaming e-commerce. Niu *et al.* [28] examine the effect of key opinion leaders in livestream e-commerce and

TABLE 1. Summary of relevant literature.

Authors	Introducer of live streaming		Commission formats		Characteristics of live streaming
	Manufacturer /Supplier	Retailer	Commission rate format	Fixed commission format	
Hao and Yang [16]		✓	✓		Selling effort
Ji *et al.* [17]	✓	✓	✓		Promotion price
Niu *et al.* [28]	✓		✓		Network effect
Wang *et al.* [34]	✓		✓	✓	Sales effort
Yang *et al.* [38]	✓		✓		Sales effort
Zhang and Tang [40]	✓		✓		Number of followers and sales effort
Zhang *et al.* [41]	✓		✓		Service level
This paper	✓	✓	✓	✓	Switch and fan effects

explore the strategic choices of brand owners. The prior literature has mainly studied the introduction of live streaming by one member while ignoring the introduction behavior of the other party. Our study contributes to theoretically analyzing the impacts of different introducers and commission formats of internet celebrities on firms' decision-making in live streaming commerce.

2.2. Channel management

Our paper also contributes to the literature on channel management, which has attracted the attention of numerous scholars. The research in dual-channel supply chains is common [2, 4, 6, 18, 19]. Lou *et al.* [20] explore logistics service choices in a retailer-led supply chain and find that retailers may not always benefit from sharing logistics services. Zhu *et al.* [46] study optimal channel selection and battery capacity allocation strategies, considering the possibility of battery recycling by the manufacturer. Ha *et al.* [14] investigate optimal choices in the presence of increased marketing efforts by online platforms. Ji *et al.* [17] examine the channel selection problem in the context of live-streaming by considering two price schemes. Zhang *et al.* [43] extend this research to the secondary market and analyze optimal collect-and-resell and channel choice strategies. Feng *et al.* [12] combine channel selection and logistics choice in the context of financial constraints. Considering the carbon emissions in a dual channel, Barman *et al.* [1] examine the effects of different carbon policies and find that compared with the carbon tax policy, the manufacturer can benefit from the carbon cap-and-trade policy. Unlike the literature of Barman *et al.* [1], we focus on live streaming commerce and examine the conditions under which the manufacturer and the retailer partner with the internet celebrity.

In recent years, scholars have focused on channel selection in live-streaming e-commerce. Wang *et al.* [34] investigate whether suppliers should choose between reselling and agency selling in the context of live streaming. Zhang *et al.* [42] discuss channel choices from the perspective of multinational firms. Yang *et al.* [38] compare e-commerce platform mode, transferring mode, and live streaming platform mode, and find that the best sales mode depends on net sales volume and channel rate. Hao and Yang [16] consider the impact of consumer returns on channel selection. Niu *et al.* [27] explore when global brands should select an AI live stream over the key opinion leaders in one. In summary, channel selection in the context of live streaming is a relatively new topic. Compared to existing research, our study considers the possibility of the other members of the supply chain adopting the live streaming commerce and characterizes different commission formats adopted by internet celebrities.

Table 1 shows the difference between the key contributions of the work and the contributions of the prior literature.

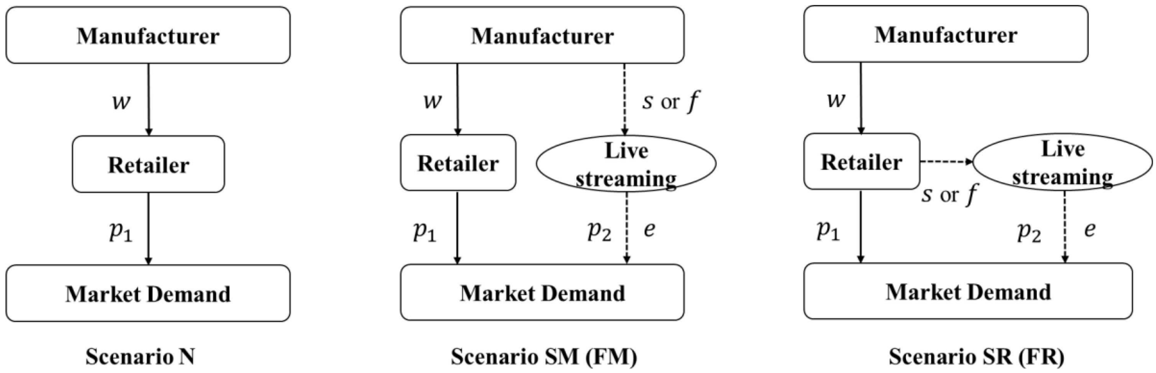

FIGURE 1. Supply chain constructs in different scenarios.

3. MODEL

In this section, we describe the model and characterize the live streaming.

3.1. Model description

We develop a supply chain consisting of a manufacturer (denoted as he), a retailer (denoted as she), and an internet celebrity (denoted as it). The manufacturer and retailer must determine whether to adopt live streaming. If neither adopts live streaming, there is only the traditional channel in which the retailer purchases products from the manufacturer at a wholesale price w, and sells them to consumers at a retail price p_1. If the manufacturer (retailer) introduces live-streaming commerce, in addition to the traditional channel, there is also a live-streaming channel in which the internet celebrity sells the product at a retail price p_2 in the live-streaming room and charges a commission fee from the manufacturer (retailer) in return.

Moreover, there are two commission formats for the live-streaming channel. One is the commission rate format where the internet celebrity charges a commission rate s for each product sold through the live-streaming channel. The total commission fee is the commission rate s multiplied by the demand for the live-streaming channel. The other is the fixed commission format where the manufacturer (retailer) pays a fixed commission f regardless of the number of products sold by the internet celebrity. We use $X \in \{N, M, R\}$ to denote different cases: "N" represents no live-streaming channel, "M" denotes the manufacturer adopting the live-streaming channel, and "R" represents the retailer introducing the live-streaming channel. Additionally, $Y \in \{S, F\}$ denotes different commission formats: "S" stands for the commission rate format, and "F" represents the fixed commission format. The supply chain structures are illustrated in Figure 1.

All notations used in this paper, along with their meanings, are listed in Table 2.

3.2. Demand formulation

When the manufacturer or the retailer adopts the live-streaming channel, following Wang *et al.* [34], the demand functions of the traditional channel and the live-streaming channel are shown as follows:

$$D_1 = a_1 - p_1 + bp_2 - c_1 e,$$
$$D_2 = a_2 - p_2 + bp_1 + c_2 e,$$

where $a_1 = \beta a$ and $a_2 = (1 - \beta)a$ are the potential market sizes of the traditional channel and live streaming channel; c_1 and c_2 are the switch effect coefficient and fan effect coefficient of the live streaming channel, respectively; e represents the effort level of the internet celebrity.

TABLE 2. Summary of notations.

Notation	Description
a	Potential demand size
β	Preference coefficient of the traditional channel
$a_i(i=1,2)$	Overall demand from different channels
b	Cross-price coefficient
w	The wholesale price
$p_i(i=1,2)$	The retail price in different channels
$c_i(i=1,2)$	The influence coefficient of internet celebrity
e	The effort level of the internet celebrity
k	Effort cost coefficient of internet celebrity
s	The commission rate
f	The fixed commission
$D_i(i=N,1,2)$	Demand from the channel
$\pi_j(j=M,R,I)$	Profit of different parties

If the manufacturer and retailer do not introduce an internet celebrity, there is only one channel in the supply chain. In this case, the demand function of the traditional channel is $D_N = a - p_1$.

3.3. Cost function of the live streaming

Following the prior studies [19,25,29,33], we assume that the cost of live-streaming is $C(e) = \frac{k}{2}e^2$. It is obvious that a larger e not only brings a greater demand for the live-streaming channel, but also a higher cost for the internet celebrity (*i.e.*, $\frac{\mathrm{d}C(e)}{\mathrm{d}e} > 0$). In addition, as the effort level increases, the marginal cost of increasing the effort level also rises. That is, $\frac{\mathrm{d}^2C(e)}{\mathrm{d}e^2} > 0$.

3.4. Sequence of events

In Stage 1, the manufacturer or retailer determines whether to adopt an internet celebrity for the live-streaming channel. If the manufacturer introduces an internet celebrity, the manufacturer first sets the wholesale price w and retail price p_2 in the live-streaming channel in Stage 2. The retailer then determines the retail price p_1 for the traditional channel in Stage 3. Finally, in Stage 4, the internet celebrity decides on its effort level e to maximize its profit. If the retailer adopts an internet celebrity, the manufacturer determines the wholesale price w for the traditional channel in Stage 2. Next, the retailer decides on retail prices p_1 and p_2 for the traditional and live-streaming channels in Stage 3. Finally, the internet celebrity determines its effort level e in Stage 4. The sequence of events is shown in Figure 2.

4. OPTIMAL OUTCOMES

In this section, we derive the optimal outcomes under different cases and examine the impacts of fan and switch effects.

4.1. Benchmark case (Scenario N)

We consider scenario N as a benchmark model, in which neither the manufacturer nor the retailer adopts the live-streaming channel. The profit functions of the manufacturer and retailer are $\max_w \pi_M(w) = wD_N$ and $\max_{p_1} \pi_R(p_1) = (p_1 - w)D_N$, respectively.

Using the backward induction approach, we obtain the optimal decisions under scenario N as $w^{\mathrm{N}} = \frac{a}{2}$ and $p_1^{\mathrm{N}} = \frac{3a}{4}$.

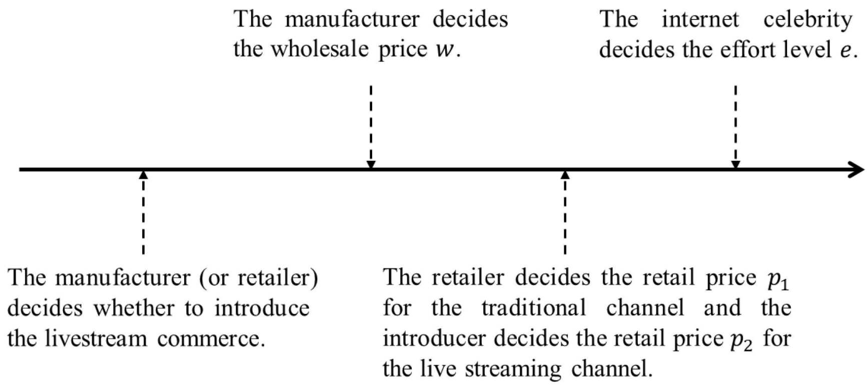

FIGURE 2. Sequence of events.

4.2. Equilibrium under the commission rate format

In this section, we examine the commission rate case in which the introducer (*i.e.*, the manufacturer or retailer) pays a commission fee s to the internet celebrity for each unit of product sold through the live-streaming channel. We then explore the equilibrium results under different cases and compare them to those under scenario N.

4.2.1. Scenario SM

Under scenario SM where the manufacturer introduces the internet celebrity in the commission rate format, the profit functions of the manufacturer, retailer and internet celebrity are presented as follows:

$$\max_{w,p_2} \pi_M^{\mathrm{SM}}(w,p_2) = wD_1 + (p_2 - s)D_2, \tag{1}$$

$$\max_{p_1} \pi_R^{\mathrm{SM}}(p_1) = (p_1 - w)D_1, \tag{2}$$

$$\max_{e} \pi_I^{\mathrm{SM}}(e) = sD_2 - ke^2/2. \tag{3}$$

Lemma 1. *The optimal decisions under scenario SM are:* $w^{\mathrm{SM}} = \frac{k(a_1+ba_2)+sc_2(bc_2-c_1)}{2k(1-b^2)}$, $p_1^{\mathrm{SM}} = \frac{k\left[2ba_2+\left(3-b^2\right)a_1+sb\left(1-b^2\right)\right]-sc_2[3c_1-b(bc_1+2c_2)]}{4k(1-b^2)}$, $p_2^{\mathrm{SM}} = \frac{k[a_2+s+b(a_1-bs)]+sc_2(c_2-bc_1)}{2k(1-b^2)}$, *and* $e^{\mathrm{SM}} = \frac{s}{k}c_2$.

Lemma 1 demonstrates that the optimal effort level depends on parameters k, s, and c_2. In particular, the internet celebrity is inclined to increase its effort level as the commission rate s and the fan effect of the internet celebrity c_2 rise. However, a higher cost coefficient of effort level k discourages the internet celebrity from exerting a larger effort. The finding is intuitive, indicating that the internet celebrity weighs the commission fee against the effort cost.

Then, by analyzing the impact of the live-streaming channel on optimal prices, we derive Corollary 1.

Corollary 1. *Under scenario SM, we have:*

(1) $\frac{\partial w^{\mathrm{SM}}}{\partial c_1} < 0$, $\frac{\partial p_1^{\mathrm{SM}}}{\partial c_1} < 0$, *and* $\frac{\partial p_2^{\mathrm{SM}}}{\partial c_1} < 0$;

(2) $\frac{\partial w^{\mathrm{SM}}}{\partial c_2} > 0$ *if* $c_1 < 2bc_2$, $\frac{\partial p_1^{\mathrm{SM}}}{\partial c_2} > 0$ *if* $c_1 < \frac{4bc_2}{3-b^2}$, *and* $\frac{\partial p_2^{\mathrm{SM}}}{\partial c_2} > 0$ *if* $c_1 < \frac{2c_2}{b}$.

Corollary 1 illustrates the impacts of the switch effect and fan effect of the internet celebrity on price equilibrium decisions in scenario SM. Corollary 1(1) shows that with an increase in the switch effect (*i.e.*, a higher c_1), the wholesale and retail prices for different channels decrease. This occurs because a stronger switch effect reduces market demand, prompting the manufacturer and retailer to lower retail prices. Corollary 1(2) reveals

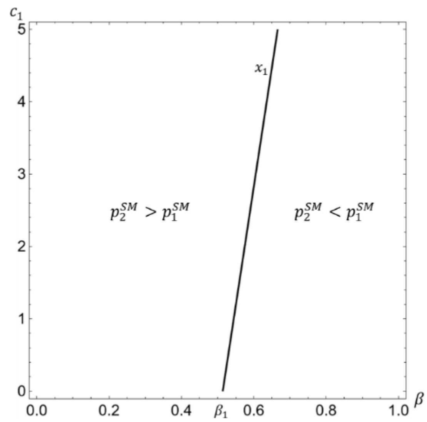

FIGURE 3. Retail price under different channels in scenario SM.

that if the switch effect is small, the wholesale and retail prices increase as the fan effect of the internet celebrity strengthens. However, a strong switch effect forces the firms to decrease retail prices for different channels to shift consumers away from the traditional channel. In this case, an increase in the fan effect (*i.e.*, a higher c_2) leads the manufacturer and retailer to decrease retail prices since the profits from the increased demand dominate the losses caused by the lower prices.

Corollary 2. *Comparing the retail price and the demand under different channels, we have:*

(1) *If* $c_1 > x_1$ *and* $\beta > \beta_1$ *or* $\beta < \beta_1$, $p_2^{\mathrm{SM}} > p_1^{\mathrm{SM}}$, *otherwise,* $p_2^{\mathrm{SM}} < p_1^{\mathrm{SM}}$;
(2) *If* $c_1 > x_2$ *and* $\beta > \beta_2$ *or* $\beta < \beta_2$, $D_2^{\mathrm{SM}} > D_1^{\mathrm{SM}}$, *otherwise,* $D_2^{\mathrm{SM}} < D_1^{\mathrm{SM}}$,

where x_1, x_2, β_1, *and* β_2 *are in the Appendix A.*

It is commonly believed that the retail price for the live-streaming channel is lower because the internet celebrity often adopts a low-price strategy to attract more consumers. Corollary 2(1) shows that the retail price in the live-streaming channel may be higher than that in the traditional channel. Specifically, if the consumers' preference for the traditional channel and the switch effect are high or the consumers' preference for the traditional channel is low, the retail price for the live streaming channel is higher. This phenomenon can be explained from two main aspects. On the other hand, a small β indicates that consumers prefer the live-streaming channel, enhancing the competitiveness of the internet celebrity. On the other hand, although β is large, a significant number of consumers will shift from the traditional channel to the live streaming channel if switch effect (*i.e.*, c_1) is substantial. As a result, the introducer is incentivized to raise the retail price of the live streaming room, as illustrated in Figure 3.

In practice, an infant facial cream, priced at RMB 99 in the live streaming room, had a purchase price of RMB 17.54, indicating that the retail price in the live-streaming channel can be significantly higher[5]. Another example is that Jiaqi Li sold Shiseido's products at high prices in his live broadcast[6]. By analyzing the product data on Taobao, Pan *et al.* [29] find that 14.49% of products are more expensive in the live streaming room. Our study further extends the findings of Pan *et al.* [29] in the environment where the retailer adopts the live-streaming channel.

Chen *et al.* [5] reveal that the swift guanxi affects product sales in the live-streaming channel. Their model, however, differs from ours in that it does not consider switch and fan effects, nor does it examine the different introducers of the live streaming commerce. We find that compared with the traditional channel, the market

[5]Available at: https://www.thepaper.cn/newsDetail_forward_22198662.
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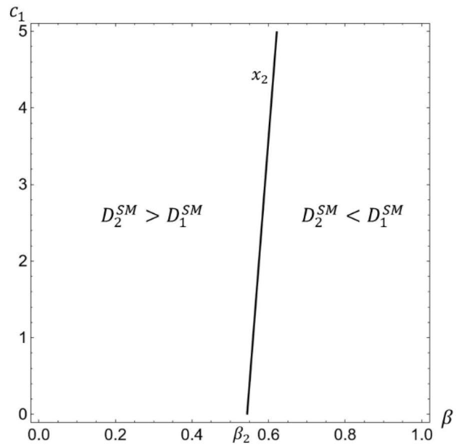

FIGURE 4. Demand under different channels in scenario SM.

demand for the live-streaming channel increases even despite a higher retail price for the live streaming room, which is primarily contingent on the interaction between the two effects brought by the internet celebrity and the price effect. Specifically, the impacts of the fan and switch effects on the demand dominate the impacts of the price effect on the demand, resulting in increased demand for the live streaming channel and decreased demand for the traditional channel. Figure 4 illustrates the demand changes across different channels.

Next, we compare the optimal decisions under scenarios SM and N, as presented in Proposition 1.

Proposition 1. *Comparing scenario SM to scenario N, we have:*

(1) *If* $\beta \le \beta_3$, $w^{\mathrm{N}} \ge w^{\mathrm{SM}}$, *otherwise,* $w^{\mathrm{N}} < w^{\mathrm{SM}}$;
(2) *If* $\beta \le \beta_4$, $p_1^{\mathrm{N}} \ge p_1^{\mathrm{SM}}$, *otherwise,* $p_1^{\mathrm{N}} < p_1^{\mathrm{SM}}$,

where β_3 *and* β_4 *are in the Appendix A.*

Proposition 1 shows that, compared with the results under scenario N, the wholesale price and retail prices under scenario SM are lower when consumers do not prefer the traditional channel. This is because the introduction of the live-streaming channel generates a switch effect. To maintain sales in the traditional channel, the retailer must reduce the retail price, which in turn prompts the manufacturer to cut down the wholesale price.

4.2.2. Scenario SR

In this section, we investigate the optimal decisions under scenario SR where the retailer adopts an internet celebrity in the commission rate format. The profit functions of all members are characterized as follows:

$$\max_{w} \pi_M^{\mathrm{SR}}(w) = w(D_1 + D_2), \tag{4}$$

$$\max_{p_1, p_2} \pi_R^{\mathrm{SR}}(p_1, p_2) = (p_1 - w)D_1 + (p_2 - w - s)D_2, \tag{5}$$

$$\max_{e} \pi_I^{\mathrm{SR}}(e) = sD_2 - ke^2/2. \tag{6}$$

Lemma 2. *The optimal decisions of the manufacturer, the retailer, and the internet celebrity are:* $w^{\mathrm{SR}} = \frac{k[a_1+a_2-(1-b)s]+sc_2(c_2-c_1)}{4k(1-b)}$, $p_1^{\mathrm{SR}} = \frac{k\left[(5+b)a_1+(1+5b)a_2-\left(1-b^2\right)s\right]+sc_2[(1+5b)c_2-(5+b)c_1]}{8k(1-b^2)}$, $p_2^{\mathrm{SR}} = \frac{k\left[(5+b)a_2+(1+5b)a_1+3s\left(1-b^2\right)\right]+sc_2[(5+b)c_2-(1+5b)c_1]}{8k(1-b^2)}$, *and* $e^{\mathrm{SR}} = \frac{s}{k}c_2$.

Similar to Lemma 1, Lemma 2 shows that the optimal effort level depends on parameters k, s, and c_2. Based on Lemmas 1 and 2, we find that a change in the introducer does not affect the optimal effort level of the internet celebrity. We then explore the impact of the live-streaming channel under scenario SR, which is summarized in Corollaries 3 and 4.

Corollary 3. *Under scenario SR, we obtain:*

(1) $\frac{\partial w^{\mathrm{SR}}}{\partial c_1} < 0$, $\frac{\partial p_1^{\mathrm{SR}}}{\partial c_1} < 0$, *and* $\frac{\partial p_2^{\mathrm{SR}}}{\partial c_1} < 0$;
(2) $\frac{\partial w^{\mathrm{SR}}}{\partial c_2} > 0$ *if* $c_1 < 2c_2$, $\frac{\partial p_1^{\mathrm{SR}}}{\partial c_2} > 0$ *if* $c_1 < \frac{2c_2(1+5b)}{5+b}$, *and* $\frac{\partial p_2^{\mathrm{SR}}}{\partial c_2} > 0$ *if* $c_1 < \frac{2c_2(5+b)}{1+5b}$.

Corollary 4. *Comparing the retail price and the demand under different channels, we have:*

(1) *If* $c_1 > x_3$ *and* $\beta > \beta_5$ *or* $\beta < \beta_5$, $p_2^{\mathrm{SR}} > p_1^{\mathrm{SR}}$, *otherwise,* $p_2^{\mathrm{SR}} < p_1^{\mathrm{SR}}$.
(2) *If* $c_1 > x_4$ *and* $\beta > \beta_6$ *or* $\beta < \beta_6$, $D_2^{\mathrm{SR}} > D_1^{\mathrm{SR}}$, *otherwise,* $D_2^{\mathrm{SR}} < D_1^{\mathrm{SR}}$,

where x_3, x_4, β_5, *and* β_6 *are in the Appendix A.*

Corollary 3 illustrates the impacts of the switch and fan effects of the internet celebrity on optimal price decisions under scenario SR. The results mirror those in Corollary 1. Therefore, we do not discuss them here for convenience. Similar to Corollary 2, the retail price and demand for the traditional channel are not always higher than that for the live-streaming channel, as presented in Corollary 4. Specifically, compared with the traditional channel, both the retail price and the demand for the live-streaming channel under scenario SR increase if the consumers' preference for the traditional channel and the switch effect are high or the consumers' preference for the traditional channel is low. Irrespective of who cooperates with the internet celebrity, the impacts of the live streaming channel on firms' optimal decisions are similar.

Furthermore, by comparing the optimal decisions under scenarios SR and N, we obtain Proposition 2 as follows:

Proposition 2. *Comparing scenario SR to scenario N, we have:*

(1) *If* $c_1 \geq x_5$, $w^{\mathrm{N}} \geq w^{\mathrm{SR}}$, *otherwise,* $w^{\mathrm{N}} < w^{\mathrm{SR}}$;
(2) *If* $c_1 \geq x_6$, $p_1^{\mathrm{N}} \geq p_1^{\mathrm{SR}}$, *otherwise,* $p_1^{\mathrm{N}} < p_1^{\mathrm{SR}}$,

where x_5 *and* x_6 *are in the Appendix A.*

From Proposition 2(1), we find that compared with the equilibrium price decisions under scenario N, when the switch effect is high, the wholesale price under scenario SR is lower. The reasons for this are as follows. A large c_1 implies that more consumers from the traditional channel switch to the live streaming channel, dampening the retailer's sales. Therefore, the manufacturer decreases the wholesale price in order to stimulate the retailer to order more products. Furthermore, Proposition 2(2) indicates that the retail price for the traditional channel under scenario N is higher than that under scenario SR if the switch effect is relatively large. The main reason is that under scenario SR, the retailer has the incentive to decrease the retail price for the traditional channel due to a lower wholesale price.

4.3. Equilibrium under the fixed commission format

In this section, we consider the fixed commission case in which the introducer (*i.e.*, the manufacturer or the retailer) pays a fixed commission f to the internet celebrity, regardless of the number of products sold through the live-streaming channel. We first derive the optimal results for different cases and then compare them with those in the benchmark case.

4.3.1. Scenario FM

Under scenario FM, a fixed commission format contract is established between the internet celebrity and the manufacturer. Similar to scenario SM, the profit functions of supply chain members are expressed as follows:

$$\max_{w,p_2} \pi_M^{\mathrm{FM}}(w,p_2) = wD_1 + p_2 D_2 - f, \tag{7}$$

$$\max_{p_1} \pi_R^{\mathrm{FM}}(p_1) = (p_1 - w)D_1, \tag{8}$$

$$\max_{e} \pi_I^{\mathrm{FM}}(e) = f - ke^2/2. \tag{9}$$

Lemma 3. *The optimal decisions of the manufacturer, the retailer and the internet celebrity are:* $w^{\mathrm{FM}} = \frac{a_1+ba_2}{2(1-b^2)}$, $p_1^{\mathrm{FM}} = \frac{(3-b^2)a_1+2ba_2}{4(1-b^2)}$, $p_2^{\mathrm{FM}} = \frac{a_2+ba_1}{2(1-b^2)}$, *and* $e^{\mathrm{FM}} = 0$.

From Lemma 3, we find that the internet celebrity has no incentive to exert positive effort (*i.e.*, $e^{\mathrm{FM}} = 0$) when the fixed commission format is implemented, which aligns with the findings of Wang *et al.* [34]. The reason is that regardless of the number of products sold by the internet celebrity, the effort level does not improve its income, but instead increases the costs. In the following, we examine the impacts of the live-streaming channel and consumers' preferences on optimal prices, as summarized in Corollary 5.

Corollary 5. *Under scenario FM, we have:*

(1) $\frac{\partial w^{\mathrm{FM}}}{\partial c_1} = \frac{\partial p_1^{\mathrm{FM}}}{\partial c_1} = \frac{\partial p_2^{\mathrm{FM}}}{\partial c_1} = 0$ *and* $\frac{\partial w^{\mathrm{FM}}}{\partial c_2} = \frac{\partial p_1^{\mathrm{FM}}}{\partial c_2} = \frac{\partial p_2^{\mathrm{FM}}}{\partial c_2} = 0$;
(2) $\frac{\partial w^{\mathrm{FM}}}{\partial \beta} \geq 0$, $\frac{\partial p_1^{\mathrm{FM}}}{\partial \beta} \geq 0$, *and* $\frac{\partial p_2^{\mathrm{FM}}}{\partial \beta} \leq 0$.

Corollary 5 demonstrates that the switch and fan effects of the celebrity have no influence on the optimal prices under scenario FM. The main reason is that internet celebrities have no incentive to make the effort. Therefore, the switch and fan effects do not affect the sales of the traditional and live-streaming channels, nor do they affect the wholesale and retail prices. However, optimal prices are mainly influenced by consumers' preference for the traditional channel. With an increase in consumers' preference for the traditional channel, the wholesale price and retail price for the traditional channel increase, but the retail price for the live-streaming channel decreases.

Proposition 3. *Comparing scenario FM to scenario N, we have:*

(1) *If* $\beta \leq \beta_7$, $w^{\mathrm{N}} \geq w^{\mathrm{FM}}$, *otherwise,* $w^{\mathrm{N}} < w^{\mathrm{FM}}$;
(2) *If* $\beta \leq \beta_8$, $p_1^{\mathrm{N}} \geq p_1^{\mathrm{FM}}$, *otherwise,* $p_1^{\mathrm{N}} < p_1^{\mathrm{FM}}$,

where β_7 *and* β_8 *are in the Appendix A.*

Similar to Proposition 1, we find that the wholesale and retail prices for the traditional channel under scenario FM are both lower than those under scenario N if consumers have a low preference for the traditional channel. Conversely, compared with scenario N, the wholesale price and the retail price for the traditional channel are higher if β is large. The adoption of the live streaming channel robs heavily the demand of the retailer in the traditional channel, intensifying the competition between the manufacturer and the retailer and increasing the manufacturer's competitive advantage. Therefore, as β increases, the manufacturer has the incentive to raise the wholesale price and the retailer correspondingly increases the retail price.

4.3.2. Scenario FR

Similar to scenario SR, the profit functions of the manufacturer, retailer and internet celebrity under scenario FR are formulated as follows:

$$\max_{w} \pi_M^{\mathrm{FR}}(w) = w(D_1 + D_2), \tag{10}$$

$$\max_{p_1,p_2} \pi_R^{\mathrm{FR}}(p_1,p_2) = (p_1 - w)D_1 + (p_2 - w)D_2 - f, \tag{11}$$

$$\max_{e} \pi_I^{\mathrm{FR}}(e) = f - ke^2/2. \tag{12}$$

Lemma 4. *The optimal decisions of the manufacturer, the retailer, and the internet celebrity are:* $w^{\mathrm{FR}} = \frac{a_1+a_2}{4(1-b)}$, $p_1^{\mathrm{FR}} = \frac{(5+b)a_1+(1+5b)a_2}{8(1-b^2)}$, $p_2^{\mathrm{FR}} = \frac{(1+5b)a_1+(5+b)a_2}{8(1-b^2)}$, *and* $e^{\mathrm{FR}} = 0$.

Based on Lemma 4, we can also derive the impacts of the live-streaming channel on optimal prices under scenario FR, as shown in Corollary 6.

Corollary 6. *Under scenario FR, we have:*

(1) $\frac{\partial w^{\mathrm{FR}}}{\partial c_1} = \frac{\partial p_1^{\mathrm{FR}}}{\partial c_1} = \frac{\partial p_2^{\mathrm{FR}}}{\partial c_1} = 0$ *and* $\frac{\partial w^{\mathrm{FR}}}{\partial c_2} = \frac{\partial p_1^{\mathrm{FR}}}{\partial c_2} = \frac{\partial p_2^{\mathrm{FR}}}{\partial c_2} = 0$;
(2) $\frac{\partial w^{\mathrm{FR}}}{\partial \beta} \geq 0$, $\frac{\partial p_1^{\mathrm{FR}}}{\partial \beta} \geq 0$, *and* $\frac{\partial p_2^{\mathrm{FR}}}{\partial \beta} \leq 0$.

The results of Corollary 6 are the same as those of Corollary 5. This shows that the impacts of the switch effect, fan effect, and consumers' preference for the traditional channel on the optimal decisions remain unchanged, although the introducer has changed. We further compare the optimal prices under scenario FR with those under scenario N, which are summarized in Proposition 4.

Proposition 4. *Comparing scenario FR to scenario N, we have:*

(1) *If* $b \leq \frac{1}{2}$, $w^{\mathrm{N}} \geq w^{\mathrm{FR}}$, *otherwise*, $w^{\mathrm{N}} < w^{\mathrm{FR}}$;
(2) *If* $\beta \leq \beta_9$, $p_1^{\mathrm{N}} \geq p_1^{\mathrm{FR}}$, *otherwise*, $p_1^{\mathrm{N}} < p_1^{\mathrm{FR}}$,

where β_9 *is in the Appendix A.*

Compared with scenario N, Proposition 4 shows that the wholesale price under scenario FR is smaller if consumers' price coefficient is low, and the retail price for the traditional channel under scenario FR is higher if consumers' preference for the traditional channel is low.

5. Analyses

In this section, we investigate the impacts of different introducers and commission formats by comparing the equilibrium results in different scenarios.

5.1. The role of different introducers

5.1.1. Changes in optimal prices

We first analyze the impact of different introducers on optimal decisions when the commission format is the same, as shown in Proposition 5.

Proposition 5. *Comparing scenarios SM, SR, FM, and FR, we have the following results.*

(1) *If* $\beta > \beta_{10}$ *and* $c_1 < x_7$, $w^{\mathrm{SM}} > w^{\mathrm{SR}}$*; if* $\beta > \frac{1}{2}$, $w^{\mathrm{FM}} > w^{\mathrm{FR}}$;
(2) *If* $\beta > \beta_{11}$, $p_1^{\mathrm{SM}} > p_1^{\mathrm{SR}}$*; if* $\beta > \frac{1}{2(1-b)}$, $p_1^{\mathrm{FM}} > p_1^{\mathrm{FR}}$;
(3) *If* $c_1 > x_8$, $p_2^{\mathrm{SM}} > p_2^{\mathrm{SR}}$; $p_2^{\mathrm{FM}} < p_2^{\mathrm{FR}}$;
(4) $e^{\mathrm{SM}} = e^{\mathrm{SR}}$, *and* $e^{\mathrm{FM}} = e^{\mathrm{FR}}$,

where x_7, x_8, β_{10}, *and* β_{11} *are in the Appendix A.*

Based on Proposition 5(1), we observe that, compared to scenario SR, the wholesale price under scenario SM is higher when the consumers' preference for the traditional channel is large and the switch effect is small. Proposition 5(2) indicates that the retail price for the traditional channel under scenario SM is higher when the consumers' preference for the traditional channel is large. A larger β results in higher demand for the traditional channel, increasing the competitive advantage of the retailer. Therefore, the retailer under scenario SR has more power to negotiate a lower wholesale price with the manufacturer. Inspired by a reduced wholesale price, the retailer is also willing to decrease the retail price under scenario SR. Observing the changes in wholesale price and the retail price for the traditional channel between scenarios SM and SR, the introducer also modifies the retail price for the live streaming channel. Specifically, compared with the scenario SR, the introducer increases the retail price if the switch effect is sufficiently large.

Turning attention to the internet celebrity with the fixed commission, we find an interesting insight. Proposition 5(3) indicates that the retail price for the live-streaming channel in scenario FM is always lower. This is because the manufacturer, when cooperating with the internet celebrity needs to reduce the retail price of the live-streaming channel to capture the market. However, the changes in wholesale price and the retail price for the traditional channel between scenarios FM and FR are similar to those between scenarios SM and SR. The wholesale and retail prices for the traditional channel under scenario FR are lower than those under scenario FM if consumers prefer the traditional channel because of the increased bargaining power of the retailer over the wholesale price under scenario FR.

Finally, Proposition 5(4) reveals that if the commission format remains the same, the effort level of the internet celebrity will remain unchanged regardless of the introducer of the live-streaming channel.

5.1.2. Introduction strategy

In this section, we further examine whether and when the manufacturer or retailer introduces the live-streaming channel. Specifically, we first study the motivation of the manufacturer or retailer to introduce the live-streaming channel when the internet celebrity charges the commission rate.

Proposition 6. *When the internet celebrity adopts the commission rate format, the optimal profits of the manufacturer and the retailer are satisfied:*

(1) *If $x_9 < c_1 < x_{10}$, the manufacturer does not introduce the live streaming channel (i.e., $\max\{\pi_M^{\mathrm{SM}}, \pi_M^{\mathrm{SR}}\} < \pi_M^{\mathrm{N}}$); otherwise, the manufacturer adopts the live streaming channel (i.e., $\pi_M^{\mathrm{SM}} \geq \max\{\pi_M^{\mathrm{N}}, \pi_M^{\mathrm{SR}}\}$);*
(2) *If $x_{11} < c_1 < x_{12}$, the retailer does not implement the live streaming channel (i.e., $\pi_R^{\mathrm{N}} > \max\{\pi_R^{\mathrm{SR}}, \pi_R^{\mathrm{SM}}\}$); otherwise, the retailer embraces the live streaming channel (i.e., $\pi_R^{\mathrm{SR}} \geq \max\{\pi_R^{\mathrm{N}}, \pi_R^{\mathrm{SM}}\}$),*

where x_9, x_{10}, x_{11} and x_{12} are in the Appendix A.

Proposition 6 suggests that the benefit to the introducer from introducing and operating the live streaming channel with the commission rate format is contingent on the switch effect. If the switch effect is moderate, the introducer (*i.e.*, the manufacturer or the retailer) fails to earn a higher profit by adopting the live-streaming channel. Only if the switch effect is low or high, the introducer has the incentive to cooperate with the internet celebrity with the commission rate to operate a live streaming channel. The rationale behind this phenomenon arises from two aspects. On the one hand, a large switch effect (*i.e.*, c_1) implies that more consumers move from the traditional channel to the live streaming channel. The profit obtained from the increased demand in the live streaming channel dominates the loss caused by the decreased demand in the traditional channel. On the other hand, a small switch effect (*i.e.*, c_1) indicates that the influence of the internet celebrity on the traditional channel is insignificant. The profit earned from the live streaming channel outweighs the cost paid to the internet celebrity. Therefore, the introducer has the incentive to open the live streaming if c_1 is small or large. Otherwise, the opening of the live streaming channel damages the introducer if c_1 is within a moderate interval. This implies that the live streaming is a double-edged sword, which requires the introducer to consider the effects brought by the internet celebrity before deciding whether to introduce the live streaming channel or not.

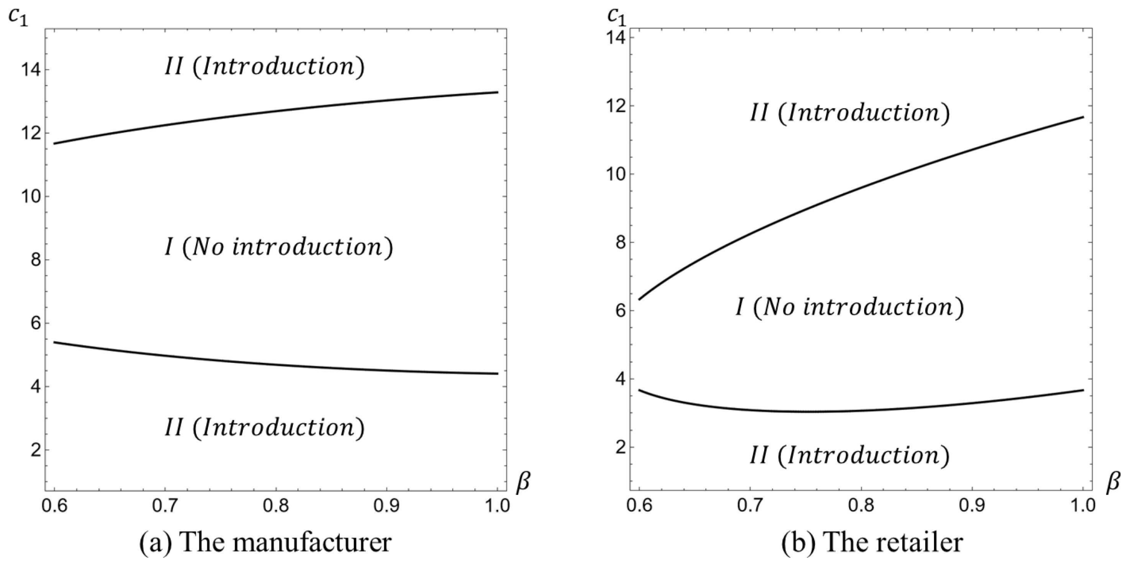

FIGURE 5. The area group of the different introducers. (a) The manufacturer. (b) The retailer.

Intuitively, Figure 5 illustrates that the manufacturer or the retailer introduces the live-streaming channel if the switch effect is large or small (region II in Figs. 5a and 5b). By observing Figure 5, we discover that there is a larger area for the retailer to introduce and operate the live streaming channel. On the one hand, the retailer is forced to participate in the live streaming to expand the market demand as well as to keep up with selling trends. On the other hand, the manufacturer can hitch a ride when the downstream retailer introduces the live streaming channel, which leads to less incentive to partner with the internet celebrity. This observation also explains why a lot of brands (or retailers) cooperate with internet celebrities and relatively few manufacturers introduce the live streaming channel.

Furthermore, we investigate the conditions under which the manufacturer or retailer collaborates with an internet celebrity charging a fixed commission. The conclusions are presented in Proposition 7.

Proposition 7. *When the internet celebrity charges the fixed commission, the optimal profits of the manufacturer and the retailer are satisfied:*

(1) *The manufacturer adopts the live streaming channel (i.e.,* $\pi_M^{\mathrm{FM}} > \max\{\pi_M^{\mathrm{N}}, \pi_M^{\mathrm{FR}}\}$*) only if* $f < \min\{f_1, f_2\}$*, where* $f_1 > 0$ *only if* $\beta \le \frac{1}{3}$ *or* $\beta > \frac{1}{3}$ *and* $b > b_1$*; otherwise, the manufacturer does not open the live streaming channel (i.e.,* $\pi_M^{\mathrm{FM}} < \min\{\pi_M^{\mathrm{N}}, \pi_M^{\mathrm{FR}}\}$*);*

(2) *The retailer implements the live streaming channel (i.e.,* $\pi_R^{\mathrm{FR}} > \max\{\pi_R^{\mathrm{N}}, \pi_R^{\mathrm{FM}}\}$*) only if* $f < \min\{f_3, f_4\}$*, where* $f_3 > 0$ *only if* $\beta \le \frac{1}{4}$ *or* $\beta \ge \frac{3}{4}$ *or* $\frac{1}{4} < \beta < \frac{3}{4}$ *and* $b > b_2$*; otherwise, the retailer does not introduce the live streaming channel (i.e.,* $\pi_R^{\mathrm{FR}} < \min\{\pi_R^{\mathrm{N}}, \pi_M^{\mathrm{FM}}\}$*),*

where f_1*,* f_2*,* f_3*,* f_4*,* b_1*, and* b_2 *are in the Appendix A.*

When an internet celebrity charges a fixed commission, Proposition 7 reveals that the introduction strategy of the live-streaming channel is determined by the fixed commission. The manufacturer or the retailer prefers to cooperate with the internet celebrity and operate the live-streaming channel only if the fixed commission is small. An important consideration here is the impact of the consumers' preference for the traditional channel and the price competition intensity on the values of two thresholds on f. Specifically, if β is large and b is small, the threshold f_1 is always less than zero and thus the manufacturer has no incentive to cooperate with the internet celebrity who charges a fixed commission in return. The main reason is that a small market share for the live streaming channel and a weak price competition diminish the competitiveness of the internet celebrity. The small consumers' preference for the live streaming channel and the weak price competition erect barriers for the manufacturer to adopt the internet celebrity on a fixed commission format. In contrast, when β is moderate and b is small, the threshold f_3 is always below zero and the retailer is not willing to introduce the internet celebrity. This implies the trade-off between the gains from the traditional channel and the additional costs of introducing the live streaming channel.

Interestingly, we find that the manufacturer prefers the retailer to cooperate with the internet celebrity when the fixed commission is moderate (*i.e.*, $f_2 < f < f_1$) although he can benefit from the introduction of the live-streaming channel. The main reason is that the manufacturer can indirectly enjoy the market demand brought about by the internet celebrity through the retailer's orders. The gains from the increased order quantity dominate the costs caused by the introduction of the live-streaming channel. However, the retailer never stands to benefit from the introduction of the internet celebrity by the manufacturer since the price competition will intensify and the demand for the retailer will decrease.

5.2. The role of the commission format

5.2.1. Changes in optimal prices

In this section, we explore the impact of the commission format on the optimal decisions when the introducer of the live-streaming channel remains unchanged, as presented in Proposition 8.

Proposition 8. *Comparing scenarios SM, SR, FM, and FR, we obtain the following results.*

(1) *If* $c_1 < bc_2$, $w^{\mathrm{SM}} > w^{\mathrm{FM}}$; *if* $c_1 < x_{13}$, $w^{\mathrm{SR}} > w^{\mathrm{FR}}$;
(2) *If* $c_1 < x_{14}$, $p_1^{\mathrm{SM}} > p_1^{\mathrm{FM}}$; *if* $c_1 < x_{15}$, $p_1^{\mathrm{SR}} > p_1^{\mathrm{FR}}$;
(3) $p_2^{\mathrm{SM}} > p_2^{\mathrm{FM}}$, *and* $p_2^{\mathrm{SR}} > p_2^{\mathrm{FR}}$;
(4) $e^{\mathrm{SM}} > e^{\mathrm{FM}}$, *and* $e^{\mathrm{SR}} > e^{\mathrm{FR}}$,

where x_{13}, x_{14}, *and* x_{15} *are in the Appendix A.*

Based on Propositions 8(1) and 8(2), we observe that regardless of the introducer of the live-streaming channel, the wholesale and retail prices for the traditional channel are lower under the fixed commission format only if the switch effect is small. This is because the influence of the internet celebrity on the traditional channel is minimal when the switch effect is low. The introducer of the live-streaming channel anticipates that the cost of using the commission rate format may exceed that of the fixed commission format. Consequently, the manufacturer or retailer increases prices to compensate for the potential loss caused by the live-streaming channel and generate higher profits.

Propositions 8(3) and 8(4) indicate that compared to the fixed commission format, the retail price for the live-streaming channel is higher, and the internet celebrity also exerts a higher effort level in the commission rate format. The primary reason is that the internet celebrity under the commission rate format, sells more products through the live-streaming channel and earns higher profits as the effort level increases. However, under a fixed commission contract, the internet celebrity's profit remains the same, regardless of the number of products sold, leading to a lack of incentive to exert effort. Consequently, the internet celebrity in the commission rate format charges a higher commission, prompting the introducer to increase the retail price of the live-streaming channel to mitigate the costs associated with introducing the live-streaming channel.

5.2.2. Selection strategy

In this section, we further explore the selection strategy of the commission format from the perspective of the introducer. By comparing the optimal profits between scenarios SM (SR) and FM (FR), we derive Proposition 9.

Proposition 9. *Given the live streaming channel is adopted, the selection strategy of the commission format satisfies:*

(1) *The manufacturer chooses the internet celebrity with the fixed commission only if* $f < f_5$, *where* $f_5 > 0$ *only if* $c_1 < x_{16}$ *and* $\beta > \beta_{12}$; *Otherwise, the manufacturer chooses the internet celebrity with the commission rate.*

(2) *The retailer chooses the internet celebrity with the fixed commission only if* $f < f_6$, *where* $f_6 > 0$ *only if* $c_1 < x_{17}$ *and* $\beta > \beta_{13}$; *Otherwise, the retailer chooses the internet celebrity with the commission rate,*

where x_{16}, x_{17}, β_{12}, β_{13}, f_5 *and* f_6 *are in the Appendix A.*

Proposition 9 suggests that the selection strategy for the commission format mainly depends on the fixed commission. Given the live streaming channel is adopted, the introducer (*i.e.*, the manufacturer or the retailer) cooperates with the internet celebrity with a fixed commission if f is relatively small. An important consideration here is the impact of the consumers' preference for the traditional channel and the switch effect on the values of two thresholds on f. Specifically, if β is small and c_1 is large, the thresholds (*i.e.*, f_5 and f_6) are always less than zero and thus the introducer has the incentive to choose the internet celebrity with the commission rate format. On the one hand, if c_1 is relatively large, the influence of the internet celebrity on the traditional channel is significant so that more market demand will shift away from the traditional channel. On the other hand, a small β implies that consumers prefer the products from the live streaming channel. In such cases, to motivate the internet celebrity to exert more effort in selling products, the introducer also prefers the commission rate contract, bundling the internet celebrity and sales performance. This implies that the introducer of the live-streaming channel should establish an appropriate contract with the internet celebrity, taking into account the influence of the internet celebrity, including the fan effect, switch effect, and consumers' preference for the live-streaming channel.

6. Conclusions

6.1. Main findings

In this study, we analyze a multi-echelon supply chain system consisting of a manufacturer, a retailer, and an internet celebrity where either the manufacturer or the retailer can introduce the live-streaming channel by collaborating with the internet celebrity under two different types of commission. We examine the firms' optimal decisions and introduction strategies concerning the live-streaming channel, considering both commission formats. Additionally, we explore the selection strategy of the commission format from the perspective of the introducer.

Based on our analyses, we summarize the main conclusions as follows. First, under a commission rate contract, the manufacturer and retailer prefer to operate the live-streaming channel when the switch effect is small or large. Under a fixed commission contract, the manufacturer and retailer are willing to cooperate with the internet celebrity if the fixed commission is relatively small. Furthermore, under a fixed commission contract, there exists a transfer zone where the manufacturer prefers the retailer to cooperate with the internet celebrity although he can benefit from the introduction of the live-streaming channel. Second, the selection strategy of the commission format is not only dependent on the fixed commission, but also on the switch effect and consumers' preference for the traditional channel. Finally, the retail price under the live-streaming channel is not always lower than that under the traditional channel. Notably, under a commission rate contract, the firms' decisions are positively influenced by the fan effect only if the switch effect is small.

6.2. Managerial insights

Our study generates some management implications in the context of supply chain members hesitating to cooperate with internet celebrities to promote product sales. First, the manufacturer and the retailer should investigate the influence of the internet celebrity, such as his followers, effort level and degree of consumer preference, when they determine to introduce the live-streaming channel. If the influence of the internet celebrity and the commission fee do not match, the manufacturer and the retailer have no incentives to open the live-streaming channel. Second, the manufacturer and the retailer should choose the appropriate internet celebrity based on the merchandise type. If products have a large market share in the traditional channel (*e.g.*, household goods), the manufacturer and the retailer should opt to pay the internet celebrity a low fixed commission fee. If consumers prefer to purchase goods in a live streaming room (*e.g.*, beauty products, apparel), the manufacturer and the retailer should sign a commission rate contract with the internet celebrity. Finally, the firms adopting the live-streaming channel should make appropriate pricing decisions for products sold through different channels. If the consumers' preference for the traditional channel and the switch effect are high or the consumers' preference for the traditional channel is low, the firms have incentives to set a higher retail price for products sold in the live room.

6.3. Limitations and future research

Although we consider two effects (*i.e.*, switch effect and fan effect) and two different contracts of live streaming commerce to examine the conditions under which the manufacturer or the retailer adopts the live-streaming channel, there are some limitations in our model that can be addressed in future research. First, we only consider the introduction of the live-streaming channel by either the manufacturer or the retailer, without exploring the case where the manufacturer and the retailer may introduce the live streaming simultaneously. Thus, it is interesting to explore the impact of the live streaming when two supply chain members cooperate with the internet celebrity simultaneously. Secondly, incorporating the aspect of product returns, which is common in live streaming due to potential mismatches between advertised and actual products, would further enhance the analyses. Finally, investigating scenarios in which the internet celebrity is either a third party or is established by the brand, would provide additional insights into the dynamics of live-streaming commerce.

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Data availability statement

No new data/codes were created or analyzed in this study.

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Appendix A.

Proof of Lemma 1. From equation (3), it is easy to find that $\partial^2\pi_I^{\mathrm{SM}}/\partial(e)^2=-k<0$. Let $\frac{\partial\pi_I^{\mathrm{SM}}}{\partial e}=0$, we can obtain the optimal effort level $e^{\mathrm{SM}}=\frac{s}{k}c_2$. By substituting e^{SM} into equation (2) and computing the second derivative of π_R^{SM} with respect to p_1, $\partial^2\pi_R^{\mathrm{SM}}/\partial(p_1)^2=-2<0$. Let $\frac{\partial\pi_R^{\mathrm{SM}}}{\partial p_1}=0$ and we formulate the optimal price for the traditional channel, $p_1^{\mathrm{SM}}=\frac{a_1+bp_2+w-c_1e^{\mathrm{SM}}}{2}$. Finally, by substituting p_1^{SM} and e^{SM} into the equation (1), it is easy to find that $\frac{\partial^2\pi_M^{\mathrm{SM}}}{\partial(w)^2}=-1$, $\frac{\partial^2\pi_M^{\mathrm{SM}}}{\partial(p_2)^2}=b^2-2$, $\frac{\partial^2\pi_M^{\mathrm{SM}}}{\partial w\partial p_2}=\frac{\partial^2\pi_M^{\mathrm{SM}}}{\partial p_2\partial w}=b$. The Hessian matrix of the manufacturer is $H_{\mathrm{SM}}=\begin{bmatrix}-1 & b\\ b & b^2-2\end{bmatrix}$, which is clearly negative definite since $0<b<1$. Combining $\frac{\partial\pi_M^{\mathrm{SM}}}{\partial w}=0$ and $\frac{\partial\pi_M^{\mathrm{SM}}}{\partial p_2}=0$, we solve $w^{\mathrm{SM}}=\frac{k(a_1+ba_2)+sc_2(bc_2-c_1)}{2k(1-b^2)}$ and $p_2^{\mathrm{SM}}=\frac{k[a_2+s+b(a_1-bs)]+sc_2(c_2-bc_1)}{2k(1-b^2)}$. By substituting the manufacturer's optimal decisions into p_1^{SM} and e^{SM}, we have $p_1^{\mathrm{SM}}=\frac{k\left[2ba_2+\left(3-b^2\right)a_1+sb\left(1-b^2\right)\right]-sc_2[3c_1-b(bc_1+2c_2)]}{4k(1-b^2)}$ and $e^{\mathrm{SM}}=\frac{s}{k}c_2$. Thus, we have equilibrium outcomes in Lemma 1. □

Proof of Lemma 2. From equation (6), we can know that $\partial^2\pi_I^{\mathrm{SR}}/\partial(e)^2=-k<0$. Let $\frac{\partial\pi_I^{\mathrm{SR}}}{\partial e}=0$, we can obtain the optimal effort level $e^{\mathrm{SR}}=\frac{s}{k}c_2$. By substituting e^{SR} into equation (5), we calculate that $\frac{\partial^2\pi_R^{\mathrm{SR}}}{\partial(p_1)^2}=-2$, $\frac{\partial^2\pi_R^{\mathrm{SR}}}{\partial(p_2)^2}=-2$, $\frac{\partial^2\pi_R^{\mathrm{SR}}}{\partial p_1\partial p_2}=\frac{\partial^2\pi_R^{\mathrm{SR}}}{\partial p_2\partial p_1}=2b$ and obtain the Hessian matrix of the retailer $H_{\mathrm{SR}}=\begin{bmatrix}-2 & 2b\\ 2b & -2\end{bmatrix}$, which is obviously negative definite since $0<b<1$. Let $\frac{\partial\pi_R^{\mathrm{SR}}}{\partial p_1}=0$ and $\frac{\partial\pi_R^{\mathrm{SR}}}{\partial p_2}=0$, then we get $p_1^{\mathrm{SR}}=\frac{k[a_1+w+b(a_2-bw)]+sc_2(bc_2-c_1)}{2k(1-b^2)}$ and

$p_2^{\text{SR}} = \frac{k[a_2+s+w+b(a_1-bs-bw)]+sc_2(c_2-bc_1)}{2k(1-b^2)}$. By substituting the optimal decisions of the retailer and internet celebrity into equation (4), it is easy to find that $\partial^2\pi_M^{\text{SR}}/\partial(w)^2 = -2(1-b) < 0$. Through solving $\frac{\partial\pi_M^{\text{SR}}}{\partial w} = 0$, we get $w^{\text{SR}} = \frac{k[a_1+a_2-(1-b)s]+sc_2(c_2-c_1)}{4k(1-b)}$. Finally, by substituting w^{SR} into p_1^{SR}, p_2^{SR} and e^{SR}, we can obtain the optimal decisions, which is summarized in Lemma 2. □

Proof of Lemma 3. The proof of Lemma 3 is similar to the proof of Lemma 1. For simplicity, we omit the proof. □

Proof of Lemma 4. The proof of Lemma 4 is similar to the proof of Lemma 2. For simplicity, we omit the proof. □

Proof of Corollary 1. Taking the first derivative of the optimal decisions with respect to c_1 under the scenario SM, we have: $\frac{\partial w^{\text{SM}}}{\partial c_1} = -\frac{sc_2}{2k(1-b^2)}$, $\frac{\partial p_1^{\text{SM}}}{\partial c_1} = -\frac{sc_2(3-b^2)}{4k(1-b^2)}$, and $\frac{\partial p_2^{\text{SM}}}{\partial c_1} = -\frac{bsc_2}{2k(1-b^2)}$. It is easy to find that $\frac{\partial w^{\text{SM}}}{\partial c_1} < 0$, $\frac{\partial p_1^{\text{SM}}}{\partial c_1} < 0$, and $\frac{\partial p_2^{\text{SM}}}{\partial c_1} < 0$. Similarly, we obtain that $\frac{\partial w^{\text{SM}}}{\partial c_2} = \frac{s(2bc_2-c_1)}{2k(1-b^2)}$, $\frac{\partial p_1^{\text{SM}}}{\partial c_2} = \frac{s(4bc_2-(3-b^2)c_1)}{4k(1-b^2)}$, and $\frac{\partial p_2^{\text{SM}}}{\partial c_2} = \frac{s(2c_2-bc_1)}{2k(1-b^2)}$. Therefore, if $c_1 < 2bc_2$, then $\frac{\partial w^{\text{SM}}}{\partial c_2} > 0$. $\frac{\partial p_1^{\text{SM}}}{\partial c_2} > 0$ holds if $c_1 < \frac{4bc_2}{3-b^2}$, and $\frac{\partial p_2^{\text{SM}}}{\partial c_2} > 0$ if $c_1 < \frac{2c_2}{b}$. □

Proof of Corollary 2. We can calculate the first derivative of the optimal decision in Lemma 1 with respect to c_1 and c_2, respectively. It is easy to find the outcomes in Corollaries 2(1) and 2(2). Then, $p_2^{\text{SM}} - p_1^{\text{SM}} = \frac{c_2s[(3+b)c_1+2c_2]+ks(2-b)(1+b)-ak[(5+b)\beta-2]}{4k(1+b)}$. If $c_2s[(3+b)c_1+2c_2]+ks(2-b)(1+b)-ak[(5+b)\beta-2] > 0$, we have $c_1 > x_1$. Notably, when $x_1 > 0$, $\beta > \beta_1$ holds. When $\beta < \beta_1$, $x_1 < 0$. Therefore, when $c_1 > x_1$ and $\beta > \beta_1$ or $\beta < \beta_1$, $p_2^{\text{SM}} > p_1^{\text{SM}}$. Otherwise, $p_2^{\text{SM}} < p_1^{\text{SM}}$. Using the same approach, we can obtain that: if $c_1 > x_2$ and $\beta > \beta_2$ or $\beta < \beta_2$, $D_2^{\text{SM}} > D_1^{\text{SM}}$. Here, $x_1 = \frac{ak[(5+b)\beta-2]-2sc_2^2-(2-b)(1+b)ks}{(3+b)sc_2}$, $x_2 = \frac{[(3-b)\beta-2]ak-2sc_2^2+(2+b-b^2)ks}{(1-b)sc_2}$, $\beta_1 = \frac{2ak+2sc_2^2+(2+b-b^2)ks}{(5+b)ak}$, and $\beta_2 = \frac{2ak+2sc_2^2-(2+b-b^2)ks}{(3-b)ak}$. □

Proofs of Corollaries 3, 5 and 6. Proofs of Corollaries 3, 5 and 6 are similar to the proof of Corollary 1. For simplicity, we omit these proofs. □

Proof of Corollary 4. Proof of Corollary 4 is similar to the proof of Corollary 2. For simplicity, we omit this proof. Note that $x_3 = \frac{(2\beta-1)ak-sc_2^2-(1+b)ks}{sc_2}$, $x_4 = \frac{ak(2\beta-1)+(k+bk-c_2^2)s}{sc_2}$, $\beta_5 = \frac{ak+sc_2^2+(1+b)ks}{2ak}$, and $\beta_6 = \frac{ak+(c_2^2-k-bk)s}{2ak}$. □

Proof of Proposition 1. Based on $w^{\text{N}} - w^{\text{SM}} = \frac{ak\left(1-b-b^2\right)+sc_2(bc_2-c_1)-ak\beta(1-b)}{2k\left(1-b^2\right)}$, it is easy to find that if $\beta \le \beta_3$, $w^{\text{N}} - w^{\text{SM}} \ge 0$, that is $w^{\text{N}} \ge w^{\text{SM}}$. Similarly, according to $p_1^{\text{N}} - p_1^{\text{SM}} = \frac{ak\left(3-2b-3b^2\right)+sc_2\left(3c_1-b^2c_1-2bc_2\right)+bks\left(b^2-1\right)-ak\beta(3-2b-b^2)}{4k\left(1-b^2\right)}$, we can obtain that if $\beta \le \beta_4 \frac{ak\left(3-2b-3b^2\right)+sc_2\left(3c_1-b^2c_1-2bc_2\right)+bks(b^2-1)}{ak(3-2b-b^2)}$, then $p_1^{\text{N}} \ge p_1^{\text{SM}}$. Here, $\beta_3 = \frac{ak\left(1-b-b^2\right)+sc_2(bc_2-c_1)}{ak(1-b)}$ and $\beta_4 = \frac{ak\left(3-2b-3b^2\right)+sc_2\left(3c_1-b^2c_1-2bc_2\right)+bks(b^2-1)}{ak(3-2b-b^2)}$. □

Proofs of Propositions 2, 3, 4, 5 and 8. Proofs of Propositions 2–6 are similar to the proof of Proposition 1. We ignore these proofs for convenience. Note that $x_5 = \frac{ak(2b-1)+s(c_2^2-k+bk)}{sc_2}$, $x_6 = \frac{(1+5b)sc_2^2-(1-b^2)ks+ak(b(5+6b-4\beta)+4\beta-5)}{(5+b)sc_2}$, $x_7 = \frac{(k+bk-c_2^2)s+ak(2\beta-1)}{sc_2}$, $x_8 = \frac{ak+sc_2^2-(1-b)ks}{sc_2}$, $x_{13} = \frac{c_2^2-(1-b)k}{c_2}$, $x_{14} = \frac{b(2c_2^2+k-b^2k)}{(3-b^2)c_2}$, $x_{15} = \frac{(1+5b)c_2^2-(1-b^2)k}{(5+b)c_2}$, $\beta_7 = \frac{1-b-b^2}{1-b}$, $\beta_8 = \frac{3-2b-3b^2}{3-2b+b^2}$, $\beta_9 = \frac{5-5b-6b^2}{4(1-b)}$, $\beta_{10} = \frac{ak+sc_2^2-(1+b)ks}{2ak}$, and $\beta_{11} = \frac{ak+(c_1-2bc_1+c_2)sc_2-(1-b)(1+2b)ks}{2ak(1-b)}$. □

Proof of Proposition 6. We first prove Proposition 6(1). Taking the difference of the manufacturer's profits between scenarios SM and N, we have:

$$\begin{aligned}\pi_M^{\text{SM}} - \pi_M^{\text{N}} = \frac{1}{8(1-b^2)k^2}\Big(&\left(c_2^2\left((1+b^2)c_1^2 - 4bc_1c_2 + 2c_2^2\right) + 2(1-b^2)(bc_1-2c_2)c_2k + (2-3b^2+b^4)k^2\right)s^2 \\ &+ 2aks\left(2\left(-bc_1c_2 + c_2^2 - k + b^2k\right) - (1-b)(c_2(c_1-bc_1+2c_2) - (2-b)(1+b)k)\beta\right) \\ &+ a^2k^2\left(1+b^2-4\beta(1-b)+(3-b)(1-b)\beta^2\right)\Big).\end{aligned}$$

Let $G_1 = (c_2^2((1+b^2)c_1^2 - 4bc_1c_2 + 2c_2^2) + 2(1-b^2)(bc_1 - 2c_2)c_2k + (2 - 3b^2 + b^4)k^2)s^2 + 2aks(2(-bc_1c_2 + c_2^2 - k + b^2k) - (1-b)(c_2(c_1 - bc_1 + 2c_2) - (2-b)(1+b)k)\beta) + a^2k^2(1 + b^2 + 4(-1+b)\beta + (-3+b)(-1+b)\beta^2)$. Taking the first- and second-order derivatives of G_1 with respect to c_1, we have that $\frac{\partial G_1}{\partial c_1} = sc_2(2sc_2(c_1 + b^2c_1 - 2bc_2) + 2bks(1 - b^2) - 4abk - 2ak\beta(1-b)^2) < 0$ since a is enough large and $\frac{\partial^2 G_1}{\partial c_1^2} = 2s^2c_2^2(1+b^2) > 0$. Therefore, G_1 firstly increases and then decreases as c_1 increases. Let $G_1 = 0$, we can obtain x_9 and x_{10}. If $x_9 < c_1 < x_{10}$, $\pi_M^{\mathrm{SM}} < \pi_M^{\mathrm{N}}$. If $c_1 < x_9$ and $x_{10} < c_1$, $\pi_M^{\mathrm{SM}} > \pi_M^{\mathrm{N}}$. Here, $x_9 = \frac{sc_2(bs(2c_2^2 - k + kb^2) + ak[2b + \beta(1-b)^2]) - \sqrt{M_1}}{(1+b^2)s^2c_2^2}$, $x_{10} = \frac{sc_2(bs(2c_2^2 - k + kb^2) + ak[2b + \beta(1-b)^2]) + \sqrt{M_1}}{(1+b^2)s^2c_2^2}$, and $M_1 = s^2c_2^2(1-b^2)[2s^2(c_2^2 - k)^2 + 4aks(c_2^2 - k)(1-\beta) + a^2k^2(1 - b^2 - 4\beta + 2\beta^2)]$.

Furthermore, we take the difference of the manufacturer's profits between scenarios SM and SR and obtain the equation as follows:

$$\begin{aligned}\pi_M^{\mathrm{SM}} - \pi_M^{\mathrm{SR}} = &-\frac{1}{16k^2(1-b^2)}\Big(s^2\Big(c_2^2\big(c_1^2\big(b - 1 - 2b^2\big) - 2c_1c_2(1-3b) - c_2^2(3-b)\big)\\&+ 2kc_2(1-b)(1+b)(c_1 - 2bc_1 + 3c_2) - k^2(1-b)(1+b)^2(3-2b)\Big) + a^2k^2\big(8\beta(1-b)\\&- 2\beta^2(3-b)(1-b) - 3 + b\big) + 2aks(k(1-b)(1+b)(3 - 4\beta + 2b\beta) + c_1c_2(2\beta(1-b)^2 - 1 + 3b)\\&+ c_2^2(4\beta - 4b\beta - 3 + b))\Big).\end{aligned}$$

Let $G_2 = s^2(c_2^2(c_1^2(b - 1 - 2b^2) - 2c_1c_2(1-3b) - c_2^2(3-b)) + 2kc_2(1-b)(1+b)(c_1 - 2bc_1 + 3c_2) - k^2(1-b)(1+b)^2(3-2b)) + a^2k^2(8\beta(1-b) - 2\beta^2(3-b)(1-b) - 3 + b) + 2aks(k(1-b)(1+b)(3 - 4\beta + 2b\beta) + c_1c_2(2\beta(1-b)^2 - 1 + 3b) + c_2^2(4\beta - 4b\beta - 3 + b))$. Because $\frac{\partial^2 G_2}{\partial c_1^2} = -2s^2c_2^2(1 - b + 2b^2) < 0$, $\frac{\partial G_2}{\partial c_1}$ decreases with c_1. There are two possible scenarios. On the one hand, $\frac{\partial G_2}{\partial c_1} < 0$ always holds, which implies that G_2 decreases with c_1. On the other hand, $\frac{\partial G_2}{\partial c_1}|_{c_1=0}$ is greater than zero, which means that G_2 first increases and then decreases as c_1 increases. When $c_1 = 0$, $G_2|_{c_1=0} = s^2(-c_2^4(3-b) + 6kc_2^2(1-b)(1+b) - k^2(1-b)(1+b)^2(3-2b)) + a^2k^2(8\beta(1-b) - 2\beta^2(3-b)(1-b) - 3 + b) + 2aks(k(1-b)(1+b)(3 - 4\beta + 2b\beta) + c_2^2(4\beta - 4b\beta - 3 + b))$. When $c_1 = c_2$, $G_2|_{c_1=c_2} = s^2(1-b)(4kc_2^2(2 + b - b^2) - 2c_2^4(3-b) - (1+b)^2(3-2b)k^2) + a^2k^2(8\beta(1-b) - 2\beta^2(3-b)(1-b) - 3 + b) + 2aks(1-b)(k(1+b)(3 - 2\beta(2-b)) - 2c_2^2(2 - 3\beta + b\beta))$. It is simple to find that $G_2|_{c_1=0} < 0$ and $G_2|_{c_1=c_2} < 0$ since a is enough large and $8\beta(1-b) - 2\beta^2(3-b)(1-b) - 3 + b < 0$. Therefore, $\pi_M^{\mathrm{SM}} > \pi_M^{\mathrm{SR}}$ always holds.

In sum, if $x_9 < c_1 < x_{10}$, $\pi_M^{\mathrm{N}} > \max\{\pi_M^{\mathrm{SM}}, \pi_M^{\mathrm{SR}}\}$. If $c_1 < x_9$ and $x_{10} < c_1$, $\pi_M^{\mathrm{SM}} > \max\{\pi_M^{\mathrm{N}}, \pi_M^{\mathrm{SR}}\}$.

Using a similar approach, we can obtain that $\pi_R^{\mathrm{N}} > \max\{\pi_R^{\mathrm{SM}}, \pi_R^{\mathrm{SR}}\}$ if $x_{11} < c_1 < x_{12}$, and $\pi_R^{\mathrm{SR}} \geq \max\{\pi_R^{\mathrm{N}}, \pi_R^{\mathrm{SM}}\}$ if $c_1 < x_{11}$ and $c_1 > x_{12}$. Note that $x_{11} = \frac{(5b-3)s^2c_2^3 + ksc_2[3s(1-b^2) + a(5b - 3 + 8\beta - 8b\beta)] - \sqrt{M_2}}{(5-3b)s^2c_2^2}$, $x_{12} = \frac{(5b-3)s^2c_2^3 + ksc_2[3s(1-b^2) + a(5b - 3 + 8\beta - 8b\beta)] + \sqrt{M_2}}{(5-3b)s^2c_2^2}$, and $M_2 = 2s^2c_2^2(1-b^2)[a^2k^2(16\beta - 8\beta^2 - 3 - 3b) + 16aks(k - c_2^2)(1-\beta) - 8s^2(c_2^2 - k)^2]$. □

Proof of Proposition 7. We first prove Proposition 7(1). Taking the difference of the manufacturer's profits between scenarios FM and N, we have: $\pi_M^{\mathrm{FM}} - \pi_M^{\mathrm{N}} = \frac{a^2(1 + b^2 - 4(1-b)\beta + (3-b)(1-b)\beta^2)}{8(1-b^2)} - f$. Let $f_1 = \frac{a^2(1 + b^2 - 4(1-b)\beta + (3-b)(1-b)\beta^2)}{8(1-b^2)}$. When $f_1 > 0$, we obtain that $\beta \leq \frac{1}{3}$ or $\beta > \frac{1}{3}$ and $b > b_1$, where $b_1 = \frac{\sqrt{\beta^4 - 4\beta^3 + 4\beta - 1} - 2\beta(1-\beta)}{1+\beta^2}$. It is easy to obtain that $\pi_M^{\mathrm{FM}^*} > \pi_M^{\mathrm{N}^*}$ if $f < f_1$ and $\pi_M^{\mathrm{FM}} < \pi_M^{\mathrm{N}}$ if $f > f_1$. Similarly, we take the difference of the manufacturer's profits between scenarios FM and FR and obtain the equation as follows: $\pi_M^{\mathrm{FM}} - \pi_M^{\mathrm{FR}} = \frac{a^2(3 - b - 8(1-b)\beta + 2(3-b)(1-b)\beta^2)}{16(1-b^2)} - f$. We denote $f_2 = \frac{a^2(3 - b - 8\beta(1-b) + 2(3-b)(1-b)\beta^2)}{16(1-b^2)}$ and $f_2 > 0$ always holds. Therefore, $\pi_M^{\mathrm{FM}} > \pi_M^{\mathrm{FR}}$ if $f < f_2$ and $\pi_M^{\mathrm{FM}} < \pi_M^{\mathrm{FR}}$ if $f > f_2$. In brief, $\pi_M^{\mathrm{FM}} > \max\{\pi_M^{\mathrm{N}}, \pi_M^{\mathrm{FR}}\}$ holds if $f < \min\{f_1, f_2\}$ and $\pi_M^{\mathrm{FM}} < \min\{\pi_M^{\mathrm{N}}, \pi_M^{\mathrm{FR}}\}$ holds if $f > \min\{f_1, f_2\}$.

Then, we prove Proposition 7(2). Taking the difference of the retailer's profits between scenarios FM and N, we have: $\pi_R^{\mathrm{FR}} - \pi_R^{\mathrm{N}} = \frac{a^2(3 - 3b + 2b^2 - 16\beta(1-b) + 16\beta^2(1-b))}{32(1-b^2)} - f$. We denote $f_3 = \frac{a^2(3 - 3b + 2b^2 - 16\beta(1-b) + 16\beta^2(1-b))}{32(1-b^2)}$. It is easy to obtain that $\pi_R^{\mathrm{FM}} > \pi_R^{\mathrm{N}}$ if $f < f_3$ and $\pi_R^{\mathrm{FM}} < \pi_R^{\mathrm{N}}$ if $f > f_3$. Note that $f_3 > 0$ holds if $\beta \leq \frac{1}{4}$ or $\beta \geq \frac{3}{4}$ or $\frac{1}{4} < \beta < \frac{3}{4}$ and $b > b_2$, where $b_2 = \frac{1}{4}(3 - 16\beta(1-\beta) + \sqrt{(5 - 4\beta)(3 - 4\beta)(4\beta - 1)(1 + 4\beta)})$. Similarly, we take the difference of the retailer's profits between scenarios FM and FR and obtain the equation as follows: $\pi_R^{\mathrm{FR}} - \pi_R^{\mathrm{FM}} = \frac{a^2(5 - 3b - 16\beta(1-b) + 2\beta^2(7-b)(1-b))}{32(1-b^2)} - f$. Furthermore, we obtain that $\pi_R^{\mathrm{FM}} > \pi_R^{\mathrm{FR}}$ if $f < f_4$ and $\pi_R^{\mathrm{FM}} < \pi_R^{\mathrm{FR}}$ if $f > f_4$, where $f_4 = \frac{a^2(5 - 3b - 16\beta(1-b) + 2\beta^2(7-b)(1-b))}{32(1-b^2)}$. In sum, $\pi_R^{\mathrm{FR}} > \max\{\pi_R^{\mathrm{N}}, \pi_R^{\mathrm{FM}}\}$ holds if $f < \min\{f_3, f_4\}$, while $\pi_R^{\mathrm{FR}} < \min\{\pi_R^{\mathrm{N}}, \pi_M^{\mathrm{FM}}\}$ holds if $f > \min\{f_3, f_4\}$. □

Proof of Proposition 9. Taking the difference of the manufacturer's profits between scenarios FM and SM, we obtain the following equation: $\pi_M^{\mathrm{SM}} - \pi_M^{\mathrm{FM}} = f - f_5$, where $f_5 = \frac{2aksA_1 + A_2 s^2}{8k^2(1-b^2)}$, $A_1 = \beta(1-b)(c_2(c_1 - bc_1 + 2c_2) + (-2+b)(1+b)k) - 2(c_2^2 - bc_1c_2 - k + b^2k)$, and $A_2 = 2kc_2(1-b^2)(2c_2 - bc_1) - c_2^2((1+b^2)c_1^2 - 4bc_1c_2 + 2c_2^2) - (2 - 3b^2 + b^4)k^2$. We obtain that $\pi_M^{\mathrm{SM}} > \pi_M^{\mathrm{FM}}$ if $f > f_5$ and $\pi_M^{\mathrm{SM}} < \pi_M^{\mathrm{FM}}$ if $f < f_5$. However, we need to note that $f_5 > 0$ holds if $c_1 < x_{16}$ and $\beta > \beta_{12}$, where $x_{16} = \frac{2k(1+b-c_2^2) - b^2k}{(1-b)c_2}$ and $\beta_{12} = \frac{4ak(c_2^2 - bc_1c_2 + b^2k - k) + s(c_1^2c_2^2(1+b^2) + 2c_2^4 - 4kc_2^2(1-b^2) + k^2(2-3b^2+b^4) - 2bc_1c_2(2c_2^2 - k + b^2k))}{2ak(1-b)(2c_2^2 + c_1c_2(1-b) - k(2+b-b^2))}$.

Similarly, we take the difference of the retailer's profits between scenarios FM and SM, and obtain the following equation: $\pi_R^{\mathrm{SR}} - \pi_R^{\mathrm{FR}} = f - f_6$, where $f_6 = \frac{2aksA_3 + A_4 s^2}{32k^2(1-b^2)}$, $A_3 = c_2^2(8\beta(1-b) - 5 + 3b) + c_1c_2(8\beta(1-b) - 3 + 5b) + k(1-b^2)(8\beta - 5)$, and $A_4 = 2kc_2(1-b^2)(3c_1 + 5c_2) - c_2^2((5-3b)c_1^2 + 2c_1c_2(3-5b) + (5-3b)c_2^2) - k^2(1-b)(1+b)(5+3b)$. It is easy to find that $\pi_R^{\mathrm{SM}} > \pi_R^{\mathrm{FM}}$ if $f > f_6$ and $\pi_R^{\mathrm{SM}} < \pi_R^{\mathrm{FM}}$ if $f < f_6$. However, f_6 is not always greater than zero, which is contingent on c_1 and β. $f_6 > 0$ holds if $c_1 < x_{17}$ and $\beta > \beta_{13}$, where $x_{17} = \frac{k(1+b) - c_2^2}{c_2}$, $\beta_{13} = \frac{2ak((5-3b)c_2^2 + (3-5b)c_1c_2 + 5b^2k - 5k) + A_5 s}{16ak(1-b)(c_2^2 + c_1c_2 - k - bk)}$ and $A_5 = (5-3b)c_2^2(c_1^2 + c_2^2) + 2c_1c_2^3(3-5b) - 6kc_1c_2(1-b^2) - 10kc_2^2(1-b^2) + k^2(5 + 3b - 5b^2 - 3b^3)$. □