RAIRO-Oper. Res. 59 (2025) 3015–3040
https://doi.org/10.1051/ro/2025105

RAIRO Operations Research
www.rairo-ro.org

THE IMPACT OF PARTIAL VERTICAL INTEGRATION ON SUPPLIER ENCROACHMENT

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Abstract. The activity that upstream suppliers introduce online channel to sell products and compete with downstream offline retailers is referred to as "supplier encroachment". Previous studies on this problem mainly focus on centralized and decentralized supply chain structures, while this paper considers partial vertical integrated supply chain. We establish a three-stage Stackelberg game model to analyze the impact of partial forward integration (PFI) and partial backward integration (PBI) on supplier encroachment, and find that in PFI, supplier encroachment always harms the retailer and results in two outcomes: win–lose or lose–lose. Specifically, when the equity and the direct-selling cost are large, the supplier's optimal choice is to not encroach. In PBI, supplier encroachment may benefit the retailer and lead to four outcomes: win–lose, win–win, lose–lose, or lose–win. Here, when the equity is moderate and the direct-selling cost is low, encroachment will benefit both the supplier and the retailer. Furthermore, two extensions have been made to verify the robustness of our findings. We contend that the key results remain valid when considering channel substitution, while a new outcome of win–win in PFI arises when considering sequential quantity decisions.

Mathematics Subject Classification. 90B06.

Received May 21, 2024. Accepted July 20, 2025.

1. Introduction

In business practice, partial vertical integration is very common [44,54]. That is, a firm acquires an ownership stake in another vertically related firm [15, 28]. For example, in the soft drink industry, Coca-Cola holds an 18% share of its downstream independent distributor Coca-Cola European Partners [14]. In 2009, the largest Chinese down wear manufacturer, Bosideng, acquired 1.76% of the shares of the Dashang Group, a downstream department retailer in northeast China. In the same year, Red Dragon, an upstream leading Chinese shoemaker, owned 2.2% of the shares of Dashang Group [9]. Toyota acquired a 29.8% stake in CFAO, a leading distributor of automotive in Africa in 2012, and a 10% share of Purdy Motor S.A., an automotive distributor in Costa Rica in 2021 [36]. The upstream firm owning some of the shares of a downstream firm is called partial forward integration (PFI) [27]. Conversely, the downstream firm holding a portion of shares of an upstream firm is characterized as partial backward integration (PBI) [26,29]. As an illustration, in May 2007, Jinghai Guarantee Co., Ltd., which is composed of several of Gree's downstream retailers, held 10% of the shares in Gree [43]. In October 2021,

Keywords. Game theory, partial backward integration, partial forward integration, supplier encroachment.

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Yili, a dairy manufacturer, acquired a 34.33% stake in its upstream supplier Ausnutria, which provides high-end milk source. In the automotive industry, the automaker Honda possessed a 1% stake in battery supplier CATL, and Volkswagen held a 26% stake in Guoxuan High-Tech. While partial vertical integration affords the holding firm a claim on the target firm's profits, in many cases it does not provide the holding firm with control rights, implying that the latter cannot directly influence the target firm's decisions [13]. In this study, we focus on such a case.

Driven by the rapid development of e-commerce, many suppliers and manufacturers sell their products directly to consumers *via* e-platforms (*e.g.*, Amazon, Taobao, and JD), in addition to indirectly selling through traditional offline retailers [4,6,7,20,59]. For instance, Gree and Haier sell their home appliances through brick-and-mortar retailers and the JD platform [67]. Crocs, a celebrated footwear manufacturer, sells shoes through both the offline store Macy's and the e-platform Zappos.com [11]. There are many other examples, including Dell and Cisco in the computer industry, Coca-Cola and Pepsi-Cola in drinks, and Apple and Samsung in electronics [24]. The activity of an upstream supplier introducing a direct online channel and competing with a downstream offline retailer is defined as "supplier encroachment" [2].

Most of the previous academic research on supplier encroachment has focused on two supply chain structures: centralization and decentralization. The motivation of this paper stems from the need to address the encroachment problem under a new supply chain structure – partial vertical integration. Intuitively, a supplier intends to improve its profitability through encroachment as it provides the supplier with greater pricing and production flexibility. However, supplier encroachment will reduce the market share of the downstream retailer. Therefore, when the upstream supplier holds some of the shares of the downstream retailer (*i.e.*, under PFI), encroachment will cannibalize some of the retailer's profits. As a result, the losses of the retailer's profits will also reduce the supplier's equity dividends. This might reduce the incentive of the supplier to engage in encroachment. In contrast, when the downstream retailer holds shares in the supplier, although supplier encroachment will harm the retailer's interests, the increase in the supplier's direct-selling profits will increase the retailer's equity dividends; thus, the retailer may not be resistant to the supplier's encroachment. To test the preliminary intuition and examine the effects of PFI and PBI on supplier encroachment, we establish a two-level supply chain consisting of a supplier (she) and a brick-and-mortar retailer (he). The supplier who sells her products indirectly through the offline retailer first decides whether or not to encroach on the retail market, then the supplier sets the wholesale price and product quality, and finally, both the supplier and the retailer choose product quantity simultaneously for their respective channels. We hope to answer the following research questions:

(1) How would PFI and PBI affect the supplier's encroachment strategy?
(2) How do the supplier's choice and the two partial vertical integration structures influence the retailer's profits?
(3) What are the impacts of equity in different partial vertical integration structures on firms' optimal decision-making and profits?

The key contributions of this paper can be summarized as follows:

(1) Previous studies on supplier encroachment mainly focus on centralized and decentralized supply chain structures, while this paper investigates the problem from a new structure – partial vertical integrated supply chain. According to the shares holding structure, we categorize the partial vertical integration into PFI and PBI types.
(2) Existing literatures about partial vertical integrated supply chain primarily research the issues of product pricing, order quantity, green R&D and carbon reduction, but have not yet studied the problem of supplier encroachment as we do.
(3) We establish a three-stage Stackelberg game model to analyze the impact of PFI and PBI on supplier encroachment, and find that supplier encroachment will result in win–lose or lose–lose outcomes in PFI, while lead to win–lose, win–win, lose–lose, or lose–win outcomes in PBI. In addition, when considering sequential quantity decisions, there is a new outcome of win–win in PFI.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 provides the model description and hypothesis. Section 4 is the model analysis which analyzes the effects of PFI and PBI on firms' decisions and profits. In Section 5, our basic model is extended by considering channel substitution and sequential quantity decision. In Section 6, conclusions and managerial insights are provided. Finally, proofs are provided in the Appendix A.

2. Literature review

Our research is related to two streams of literature: (1) supplier (or manufacturer) encroachment and (2) partial vertical integration.

2.1. Supplier (or manufacturer) encroachment

The first stream of supplier (or manufacturer) encroachment has been extensively studied in the field of operations management. Xia and Niu [55] investigated whether a supplier should encroach on the market under service spillovers and different channel power. Zheng and Yu [71] explored the interplay between manufacturer encroachment and the manufacturer's equal pricing strategy. Yao *et al.* [60] studied the effect of a retailer's strategic pricing on a supplier's encroachment incentive. Wan *et al.* [52] examined how retail pricing leadership affected a manufacturer's encroachment decision. Shi *et al.* [46] considered that a manufacturer has the flexibility to delegate the decision-making related to opening an online direct channel to its e-commerce division and analyzed the effects of different internal organizational structures on manufacturer encroachment. The mentioned literature examined the issue of supplier (or manufacturer) encroachment by considering price competition, whereas our study considers quantity competition and analyzes firms' optimal quantity decisions. Numerous studies have also considered quantity competition (Cournot competition). For instance, Arya *et al.* [2] investigated the effect of the encroachment cost on supplier encroachment. Yang *et al.* [58] sought out the implications of supplier encroachment on contractual agreements and firms' profitability under imperfect substitutes by considering bargaining power and revenue-sharing contract. Sun *et al.* [48] inspected the impact of quantity-based cost declines on supplier encroachment. Zhang and Zhang [62] examined the strategic interaction between manufacturer's encroachment and retailers' financing choices: trade credit financing or external financing. These studies considered sequential quantity decisions, while some scholars have considered simultaneous quantity decisions. For example, Yoon [61] assumed that the supplier and the retailer decided on their quantity simultaneously, and examined the effect of investment spillover on supplier encroachment. Zhang *et al.* [65] supposed the manufacturer and the retailer make the quantity decision simultaneously and explored the influence of different advertising schemes on manufacturer encroachment. Lu *et al.* [39] analyzed how retailer overconfidence affects supply chain transparency for manufacturers who can encroach on retail channels. Liang *et al.* [37] investigated the effect of the option of switching supplier on supply chain players' strategic interactions under the threat of supplier encroachment. However, none of these studies have considered quality decision. The most strongly related literature to the present work is Ha *et al.* [18], who examined manufacturer encroachment in a decentralized supply chain by allowing quality to be endogenous and discussed both sequential and simultaneous quantity decisions. Differing from that work, we consider a partial vertical integration supply chain and analyze the effect of PFI and PBI on supplier encroachment. Ha *et al.* [18] found that encroachment will lead to win–lose in the sequential quantity decisions, while we obtain win–lose or lose–lose outcomes in PFI. When considering simultaneous quantity decisions, Ha *et al.* [18] obtained win–lose or lose–lose outcomes, while we have the same results in PFI but find win–lose, win–win, lose–lose, or lose–win results in PBI. Moreover, we compare PFI and PBI, and find that the supplier and the retailer have opposite preferences for partial vertical integration. Last but not least, Ha *et al.* [18] assumed that offline and online channels are perfect substitutions, while we consider channel substitution and discover that the primary results fit with the basic model.

2.2. Partial vertical integration

Another stream of partial vertical integration is mainly in the field of economics. Serbera [45] studied the interactions between vertical and horizontal partial ownership giving no control over the target. Fiocco [14] investigated strategic incentives for partial vertical integration between manufacturers and retailers when retailers privately know their costs and engage in price competition with differentiated goods. Levy *et al.* [29] examined the incentive to acquire a partial stake in a vertically related firm and then foreclose on rivals. In recent years, related research has emerged in the field of operations management. For example, Chen *et al.* [9] studied partial vertical integration in push-and-pull supply chains and demonstrated that neither the supply chain nor any member's profit changes with the percentage of the leader's stakes the follower holds. Li *et al.* [33] compared three supply chain structures: complete centralization, complete decentralization, and partial vertical centralization. They demonstrated that partial vertical centralization is the equilibrium channel structure until the substitutability level of competing products exceeds a certain threshold. Xiao *et al.* [57] investigated the influence of backward shareholding on each player's pricing and production decisions and quality cooperation strategies. Xia *et al.* [56] examined how partial vertical integration affects price, carbon emissions reduction, and firms' profits amidst different power structures, namely manufacturer-dominated and retailer-dominated supply chains. Ren *et al.* [43] tested vertical shareholding's impacts on green investment, prices, and profits. He *et al.* [22] studied how tax difference and vertical shareholding ratios affect the contract clauses and financing equilibrium in either bank credit financing or buyer direct financing. Pishchulov *et al.* [42] investigated the effect of partial backward integration on supply chain coordination and performance under asymmetric information. Avinadav and Shamir [3] explored the influence of partial backward ownership on capacity investment and information exchange. These studies primarily focused on the effects of partial vertical integration on firm pricing, production, green investing, information sharing, and profits, but have not yet considered the issue of supplier encroachment as we do. Furthermore, most of the literature considers one partial vertical integration structure. For example, Li *et al.* [33] discussed PFI, while Xiao *et al.* [57], Pishchulov *et al.* [42], and Avinadav and Shamir [3] discussed PBI. In contrast, we consider both types of partial vertical integration, PFI and PBI, and analyze their different effects on firms' decisions and profits.

In short, to the best of our knowledge, we make the first contribution toward combining the two streams and examine the impact of PFI and PBI on supplier encroachment. For the sake of clarity, we illustrate the originality of this paper and the differences compared with previous studies in Table 1.

3. Model description and hypothesis

We construct a two-echelon supply chain consisting of a supplier (she) and a brick-and-mortar retailer (he). The supplier produces products with a quality g and sells them to the retailer at a wholesale price w. The retailer then resells them to consumers at a retail price p. The supplier may also sell directly to consumers through an online channel, which incurs a direct-selling cost due to the need to pay commissions to the platform [19,39]. Without loss of generality, we suppose that the supplier's commission rate is β for each unit sold *via* the platform, while the selling cost of the retailer is normalized to zero [12,21,23,58]. By searching the commission rates of e-platforms, we find the maximum rate is 45% in reality, so we assume $0 < \beta \leq 45\%$. For example, Taobao charges a 7% commission rate for clothing and 30% for home appliances. Amazon lists an 8% commission rate for electronics and 45% for its device accessories. Following the extant encroachment literature [18,40,68], we normalize the supplier's production cost to zero, and the quality investment cost as $C(g) = \frac{1}{2}kg^2$, where k captures the cost coefficient of the quality investment.

A consumer's surplus from purchasing a product is $u = vg - p$, where v represents the consumer's sensitivity to quality and is uniformly distributed on $[0, 1]$ [18,35,66]. A customer will buy a product if $u > 0$ (*i.e.*, $v > \frac{p}{g}$), and the consumer demand is $q = 1 - \frac{p}{g}$. Therefore, the inverse demand function is given as $p = g(1 - q)$, where p is the market clearing price and q is the total quantity of the product for sale [18,63]. We consider two structures of partial vertical integration: PFI, in which the upstream supplier holds θ shares in the downstream retailer;

TABLE 1. Comparison between this study and other relevant studies.

Representative papers	Supplier encroachment	Price (P)/ Quantity(Q) competition	Quality decision	Partial forward integration (PFI)	Partial backward integration (PBI)
Zheng and Yu [71]	√	P			
Yao *et al.* [60]	√	P			
Wan *et al.* [52]	√	P			
Shi *et al.* [46]	√	P			
Arya *et al.* [2]	√	Q			
Yoon [61]	√	Q			
Zhang *et al.* [68]	√	Q			
Liang *et al.* [37]	√	Q			
Ha *et al.* [18]	√	Q	√		
Zhang *et al.*[63]	√	Q	√		
Li *et al.* [33]				√	
Xiao *et al.* [57]					√
Pishchulov *et al.* [42]					√
Avinadav and Shamir [3]					√
Ren *et al.* [43]				√	√
This paper	√	Q	√	√	√

TABLE 2. Notation list.

Notations	Description	Notations	Description
NF	The case of no encroachment in PFI	p	retail price
EF	The case of encroachment in PFI	w	wholesale price
NB	The case of no encroachment in PBI	g	quality level
EB	The case of encroachment inPBI	q_R	retailer's order quantity
β	Supplier's commission rate	q_S	Supplier's direct-selling quantity
k	Cost coefficient of quality investment	π_R	Retailer's profit
θ	The shares of supplier hold in retailer	π_S	Supplier's profit
δ	The shares of retailer hold in supplier		

and PBI, whereby the downstream retailer owns δ shares in the supplier. Referring to the related literature [9,43,56], we presume $\theta, \delta \in (0, 0.5]$. If $\theta > 0.5$ (or $\delta > 0.5$), the supplier (or the retailer) becomes the majority shareholder and has the actual control right of the retailer (or the supplier) in business practice, so all decisions are determined by the supplier (retailer). This case degenerates into a centralized supply chain. If $\theta = 0$ (or $\delta = 0$), the supplier (or the retailer) does not have a share in the retailer (or the supplier), and thus, this case represents a decentralized supply chain. The problem of supplier encroachment has been widely studied in centralized and decentralized supply chains [1,16,17,25,48,51,53,69], while this paper explores the issue in a partial vertical integrated supply chain.

The timeline is as follows: First, the supplier chooses whether to encroach on the retail market. Second, the supplier establishes the quality level g and the wholesale price w. Third, the retailer decides on his order quantity q_R, and simultaneously, the supplier determines her direct-selling quantity q_S if she encroaches. Finally, the market clearing price is realized and the chain members obtain their profits [41,61,65].

Throughout this paper, we use backward induction to derive the equilibrium outcomes. For clarity, the model parameters used in this paper are listed in Table 2.

4. Model analysis

In this section, the supplier encroachment in PFI and PBI is analyzed. Thus, these four scenarios require discussion as follows: (1) No encroachment in PFI, (2) Encroachment in PFI, (3) No encroachment in PBI, and (4) Encroachment in PBI.

4.1. Partial forward integration (PFI)

4.1.1. No encroachment in PFI

First, we examine the case of no encroachment in PFI. The profits of the supplier and the retailer are as follows:

$$\pi_S^{\mathrm{NF}} = wq_R - \frac{1}{2}kg^2 + \theta(g(1-q_R)-w)q_R \tag{1}$$

$$\pi_R^{\mathrm{NF}} = (g(1-q_R)-w)q_R - \theta(g(1-q_R)-w)q_R. \tag{2}$$

Where the first to third items in equation (1) represent the wholesale revenue, the quality investment cost, and the equity dividends from the retailer, respectively; the first to second items in equation (2) represent the retail revenue, and the losses of equity dividends, respectively.

Using backward induction, we obtain Lemma 1.

Lemma 1. *With no encroachment in PFI, the equilibrium wholesale price, quality level, retailer's order quantity, and the retailer and supplier profits are respectively:*

$$w^{\mathrm{NF}^*} = \frac{1-\theta}{4k(2-\theta)^2}, \quad g^{\mathrm{NF}^*} = \frac{1}{8k-4k\theta}, \quad q_R^{\mathrm{NF}^*} = \frac{1}{4-2\theta},$$
$$\pi_R^{\mathrm{NF}^*} = \frac{1-\theta}{16k(2-\theta)^3}, \; \pi_S^{\mathrm{NF}^*} = \frac{1}{32k(2-\theta)^2}.$$

Next, we test the impact of θ on the equilibrium results as stated in Lemma 1.

Proposition 1. $\frac{\partial w^{\mathrm{NF}^*}}{\partial\theta} < 0$, $\frac{\partial g^{\mathrm{NF}^*}}{\partial\theta} > 0$, $\frac{\partial q_R^{\mathrm{NF}^*}}{\partial\theta} > 0$, $\frac{\partial \pi_R^{\mathrm{NF}^*}}{\partial\theta} > 0$, $\frac{\partial \pi_S^{\mathrm{NF}^*}}{\partial\theta} > 0$.

As shown in Proposition 1, the optimal wholesale price decreases in θ, while the optimal quality level and the retailer's quantity increase in θ. With the increases of θ, the supplier can receive more dividends, so she has an incentive to increase the retailer's profits by lowering the wholesale price and improving the product quality, which in turn leads to the retailer ordering more.

Proposition 1 also reveals a counterintuitive but very interesting phenomenon in which the retailer's profits increase with θ. Intuitively, the larger the equity share (θ), the more profits the retailer must share with the supplier. Therefore, the retailer's profits should decrease with θ. However, Proposition 1 makes the opposite conclusion. This is primarily because the equity shares increment leads to the sales quantity increment and the decrement in wholesale price (or the increment in marginal profit), which in turn leads to increased market profitability. Thus, the gain from the increased profits offsets the losses of dividends. These results suggest that PFI will benefit both firms, so the supplier should acquire as many shares of the retailer as possible under the no encroachment case.

4.1.2. Encroachment in PFI

The following section analyzes the case of encroachment in PFI. The profits of the supplier and the retailer are as follows:

$$\pi_S^{\mathrm{EF}} = wq_R - \frac{1}{2}kg^2 + (1-\beta)g(1-q_R-q_S)q_S + \theta(g(1-q_R-q_S)-w)q_R \tag{3}$$

$$\pi_R^{\mathrm{EF}} = (g(1-q_R-q_S)-w)q_R - \theta(g(1-q_R-q_S)-w)q_R. \tag{4}$$

Where the first to third items in equation (3) represent the wholesale revenue, the quality investment cost, direct-selling profits, and the equity dividends from the retailer, respectively; the first to second items in equation (4) represent the retail revenue, and the losses of equity dividends, respectively.

Similar to Lemma 1, we can acquire Lemma 2 by backward induction.

Lemma 2. *With encroachment in PFI, the equilibrium wholesale price, quality level, retailer's order quantity, supplier's direct-selling quantity, and the retailer and supplier profits are respectively:*

$$w^{\mathrm{EF}^*} = \frac{(1-\beta)(1-\theta)(5-4\beta-\theta)[5+\beta(2\beta-7)-6\theta+5\beta\theta+\theta^2]}{4k[5-\beta(4+\beta)-6\theta+4\beta\theta+\theta^2]^2},$$

$$g^{\mathrm{EF}^*} = \frac{(1-\beta)(1-\theta)(5-4\beta-\theta)}{4k[5-\beta(4+\beta)-6\theta+4\beta\theta+\theta^2]},$$

$$q_R^{\mathrm{EF}^*} = \frac{(1-\beta)\beta}{5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2}, \quad q_S^{\mathrm{EF}^*} = \frac{5-5\beta-6\theta+3\beta\theta+\theta^2}{2[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]},$$

$$\pi_R^{\mathrm{EF}^*} = \frac{(1-\beta)^3\beta^2(1-\theta)^2(5-4\beta-\theta)}{4k[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^3}, \quad \pi_S^{\mathrm{EF}^*} = \frac{(1-\beta)^2(1-\theta)^2(5-4\beta-\theta)^2}{32k[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^2}.$$

Checking the impact of θ on the above equilibriums, we get Proposition 2.

Proposition 2. $\frac{\partial w^{\mathrm{EF}^*}}{\partial\theta} < 0,\ \frac{\partial g^{\mathrm{EF}^*}}{\partial\theta} > 0,\ \frac{\partial q_R^{\mathrm{EF}^*}}{\partial\theta} > 0,\ \frac{\partial q_S^{\mathrm{EF}^*}}{\partial\theta} < 0,\ \frac{\partial \pi_R^{\mathrm{EF}^*}}{\partial\theta} > 0,\ \frac{\partial \pi_S^{\mathrm{EF}^*}}{\partial\theta} > 0.$

Notably, most of the results in Proposition 2 are the same as those in Proposition 1. The wholesale price decreases with θ, while the quality level, the sales quantity, and the supplier and the retailer profits increase with θ. Additionally, as shown in Proposition 2, the optimal direct-selling quantity for the supplier decreases in θ. This is because with increases in θ, the supplier can gain more bonuses from the retailer, and will also downscale direct sales to alleviate channel conflicts. These results imply that in PFI, supplier encroachment does not change the impact mechanism of θ.

4.1.3. Comparison of no encroachment and encroachment in PFI

In this subsection, we compare the equilibrium outcomes of no-encroachment and encroachment and explore the effect of PFI on firms' decisions and profits.

Proposition 3. (1) $q_R^{\mathrm{EF}^*} < q_R^{\mathrm{NF}^*}$*;*
(2) $w^{\mathrm{EF}^*} > w^{\mathrm{NF}^*}$ *when* $0 < \beta \le \beta_1$ *or* $\beta_1 < \beta < \beta_2$ *and* $0 < \theta < \theta_1$, $w^{\mathrm{EF}^*} \le w^{\mathrm{NF}^*}$ *when* $\beta_1 < \beta < \beta_2$ *and* $\theta_1 \le \theta \le 0.5$ *or* $\beta_2 \le \beta \le 0.45$*;*
(3) $g^{\mathrm{EF}^*} > g^{\mathrm{NF}^*}$ *when* $0 < \beta \le \beta_3$ *or* $\beta_3 < \beta \le 0.45$ *and* $0 < \theta < \theta_2$, $g^{\mathrm{EF}^*} \le g^{\mathrm{NF}^*}$ *when* $\beta_3 < \beta \le 0.45$ *and* $\theta_2 \le \theta \le 0.5$.

Where $\beta_1 \approx 0.36$ is the root of function $H(\beta) = 405 - 2259\beta + 4082\beta^2 - 2776\beta^3 + 512\beta^4, \beta_2 = \frac{7-\sqrt{34}}{3} \approx 0.39, \beta_3 = \frac{35-\sqrt{73}}{64} \approx 0.41$, and θ_1, θ_2 are described in proof.

Proposition 3(1) indicates that in PFI, the retailer's order quantity with encroachment is always less than that with no encroachment (see Fig. 1) because the supplier encroachment takes the retailer's market share.

Proposition 3(2) shows that when the commission rate is low or the commission rate is moderate and the equity θ is low, the wholesale price with encroachment is greater than that with no encroachment, and *vice versa* (see Fig. 2). This can be explained as when the commission rate is low ($0 < \beta \le \beta_1$), that is, the cost of the direct channel is low, the supplier will prefer the online channel to sell her products. Therefore, she will increase the wholesale price to reduce offline sales. As the commission rate increases, the supplier would like to shift more sales to the offline channel, and when the equity owned is high ($\theta_1 \le \theta \le 0.5$), the supplier will receive more dividends, so she will be motivated to lower the wholesale price to increase the retailer's profits. This result suggests that when the direct online channel is costlier and the supplier holds more shares in the retailer, encroachment can mitigate the double marginal effect.

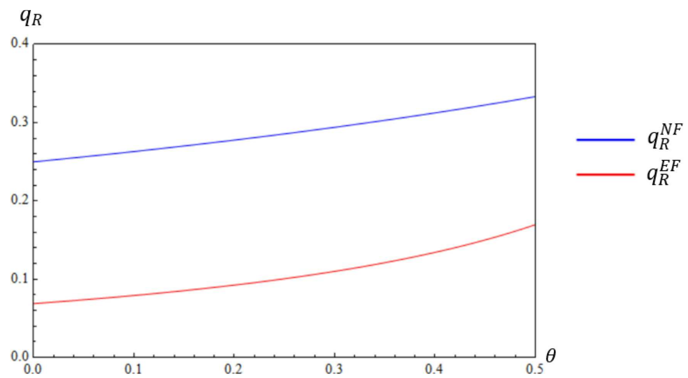

FIGURE 1. Comparison of the retailer's order quantity in PFI ($k = 1, \beta = 0.37$).

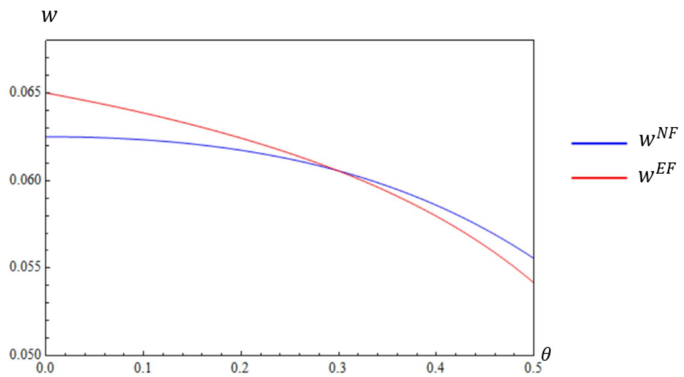

FIGURE 2. Comparison of the wholesale price in PFI ($k = 1, \beta = 0.37$).

Proposition 3(3) demonstrates that the quality level with encroachment is greater than that with no encroachment when the commission rate is low or the commission rate is high but the equity θ is low, and *vice versa* (see Fig. 3). The reason is that a higher quality level will increase the quality investment cost, but it can also expand the market and increase the direct-selling profits, so the supplier distorts quality upward ($g^{\mathrm{EF}^*} > g^{\mathrm{NF}^*}$) when the commission rate is low. However, as the commission rate increases, the supplier has less incentive to improve quality ($\frac{\partial g^{\mathrm{EF}^*}}{\partial \beta} < 0$), and when the equity is high ($\theta_2 \leq \theta \leq 0.5$), the supplier will earn lower wholesale profits due to a decreased wholesale price and a reduced order quantity. Thus, the supplier will lower the quality level. This finding suggests that when the cost of the direct online channel is cheaper or the direct channel is costlier but the supplier holds fewer shares in the retailer, encroachment can promote quality improvement.

Proposition 4. (1) $\pi_R^{\mathrm{EF}^*} < \pi_R^{\mathrm{NF}^*}$;
(2) $\pi_S^{\mathrm{EF}^*} > \pi_S^{\mathrm{NF}^*}$ *when* $0 < \beta \leq \beta_3$ *or* $\beta_3 < \beta \leq 0.45$ *and* $0 < \theta < \theta_2$, $\pi_S^{\mathrm{EF}^*} \leq \pi_S^{\mathrm{NF}^*}$ *when* $\beta_3 < \beta \leq 0.45$ *and* $\theta_2 \leq \theta \leq 0.5$.

Proposition 4 reveals that the retailer's profits with encroachment are less than that with no encroachment; thus, the retailer is always harmed by encroachment in PFI. This finding differs from previous studies which have declared that supplier encroachment can also benefit the retailer since the supplier will reduce the wholesale price to relieve channel conflicts [2,8,10,50]. The primary reason for the outcome difference is that in this paper we consider PFI in which the supplier not only divides the retailer's market but also shares his profits, both of which result in the retailer always losses.

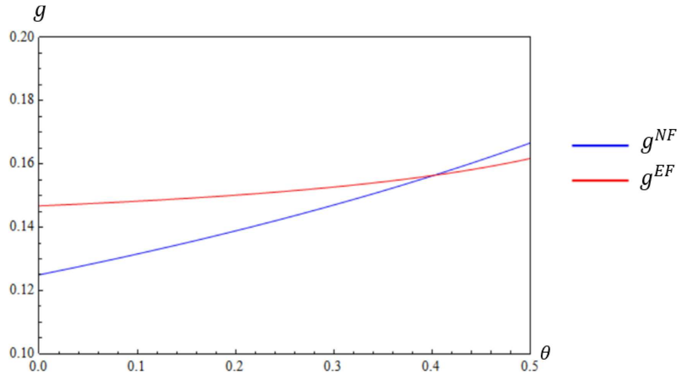

FIGURE 3. Comparison of the quality level in PFI ($k = 1, \beta = 0.43$).

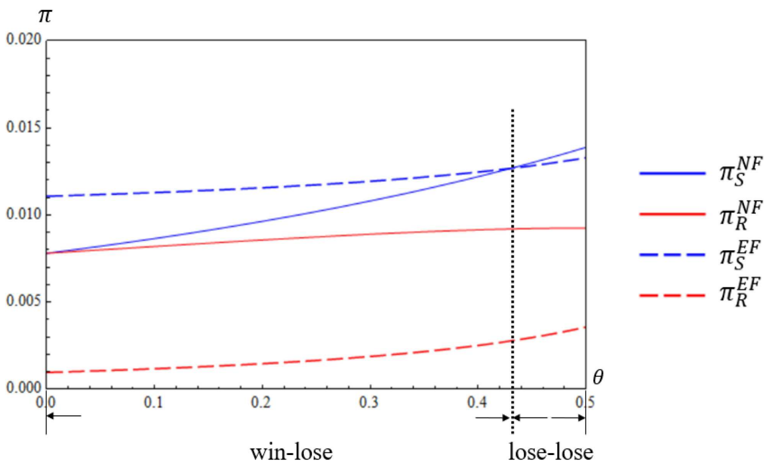

FIGURE 4. Comparison of the firms' profits in PFI ($k = 1, \beta = 0.43$).

Furthermore, we find that encroachment is no longer beneficial to the supplier. Specifically, when the commission rate and the equity θ are large, encroachment will harm her. This can be explained as follows. When the supplier encroaches, she can gain additional direct-selling profits, but it will decrease with increasing commission rates and equity. Yet, she will also receive diminished equity dividends as the retailer's profits shrink (Prop. 4(1)) and will lose more wholesale profits as the retailer's order quantity and wholesale price decrease (Prop. 3(1) and (2)). When the commission rate and the equity are large, the latter effect dominates the former. Thus, the supplier's profits with encroachment are less than that with no encroachment when the commission rate and the equity θ are large.

Briefly, supplier encroachment will result in two outcomes in PFI: win–lose or lose–lose (see Fig. 4). Particularly, the consequence of lose–lose suggests that when the supplier holds larger equity in the retailer and the direct online channel is costlier, she should not encroach.

4.2. Partial backward integration (PBI)

4.2.1. No encroachment in PBI

In this section, we analyze partial backward integration (PBI). First, we examine the case of no encroachment. The profits of the supplier and the retailer are as follows:

$$\pi_S^{\mathrm{NB}} = wq_R - \frac{1}{2}kg^2 - \delta\left(wq_R - \frac{1}{2}kg^2\right) \tag{5}$$

$$\pi_R^{\mathrm{NB}} = (g(1-q_R) - w)q_R + \delta\left(wq_R - \frac{1}{2}kg^2\right). \tag{6}$$

Where the first to third items in equation (5) represent the wholesale revenue, the quality investment cost, and the losses of equity dividends, respectively; the first to second items in equation (6) represent the retail revenue, and the equity dividends from the supplier, respectively.

Similar to Lemma 1, we establish Lemma 3.

Lemma 3. *With no encroachment in PBI, the equilibrium wholesale price, quality level, retailer's order quantity, and profits for the two members are, respectively:*

$$w^{\mathrm{NB}^*} = \frac{1}{16k(2-\delta)^2},\quad g^{\mathrm{NB}^*} = \frac{1}{8k-8k\delta},\quad q_R^{\mathrm{NB}^*} = \frac{1}{4},$$
$$\pi_R^{\mathrm{NB}^*} = \frac{1-2\delta}{128k(2-\delta)^2},\ \pi_S^{\mathrm{NB}^*} = \frac{1}{128k(1-\delta)}.$$

Next, we test the impact of δ on the equilibrium results as stated in Lemma 3.

Proposition 5. $\frac{\partial w^{\mathrm{NB}^*}}{\partial \delta} > 0,\ \frac{\partial g^{\mathrm{NB}^*}}{\partial \delta} > 0,\ \frac{\partial q_R^{\mathrm{NB}^*}}{\partial \delta} = 0,\ \frac{\partial \pi_R^{\mathrm{NB}^*}}{\partial \delta} < 0,\ \frac{\partial \pi_S^{\mathrm{NB}^*}}{\partial \delta} > 0.$

As shown in Proposition 5, the optimal wholesale price and quality level increase in δ. When the equity δ increases, the supplier should provide more dividends to the retailer; therefore, to offset profit losses, the supplier will be incentivized to increase the wholesale price and product quality to increase her profits. Moreover, the optimal retailer's quantity is independent of δ. This is because when δ increases, on the one hand, the wholesale price increase will prompt the retailer to reduce order quantity, but on the other hand, the product quality increases and the retailer can receive more dividends, both of which promote him to increase order quantity. The negative and positive effects counteract and ultimately result in the optimal retailer's quantity not being influenced by δ.

Proposition 5 reveals an interesting result that the retailer's profits decrease with δ, while the supplier's profits increase with δ. The larger the δ, the more profits the supplier must share with the retailer. Therefore, intuitively, the retailer's profits should increase with δ while the supplier's profits should decrease with δ. However, Proposition 5 yields the opposite result. The reason is that with δ increases, the retailer will gain more dividends, but an increase in wholesale price will reduce sales margin, which makes his profits decrease. These results demonstrate that differing from PFI, PBI shows the opposite effect on retailer and supplier profits. Thus, the retailer should not engage in PBI when there is no encroachment.

4.2.2. Encroachment in PBI

The following section analyzes the case of encroachment in PBI. The profits of the supplier and the retailer are as follows:

$$\pi_S^{\mathrm{EB}} = (1-\delta)\left[wq_R - \frac{1}{2}kg^2 + (1-\beta)g(1-q_R-q_S)q_S\right] \tag{7}$$

$$\pi_R^{\mathrm{EB}} = (g(1-q_R-q_S) - w)q_R + \delta\left[wq_R - \frac{1}{2}kg^2 + (1-\beta)g(1-q_R-q_S)q_S\right]. \tag{8}$$

The definition of equations (7) and (8) are similar to equations (3) and (4), respectively.

Lemma 4 is acquired in a manner similar to Lemma 2.

Lemma 4. *With encroachment in PBI, the equilibrium wholesale price, quality level, retailer's order quantity, supplier's direct-selling quantity, profits of the two members are respectively:*

$$\begin{aligned}
w^{\mathrm{EB}^*} &= \frac{\left[(3-\delta+\beta\delta)^2-4(1+\beta)\right]\left[(3-\delta+\beta\delta)^2-2(2+\beta)\right]}{8k(1-\delta)^2(5+\beta-\delta+\beta\delta)^2},\\
g^{\mathrm{EB}^*} &= \frac{(3-\delta+\beta\delta)^2-4(1+\beta)}{4k(1-\delta)(5+\beta-\delta+\beta\delta)},\\
q_R^{\mathrm{EB}^*} &= \frac{\beta}{5+\beta-\delta+\beta\delta},\ q_S^{\mathrm{EB}^*} = \frac{5-\delta+\beta\delta}{2(5+\beta-\delta+\beta\delta)},\\
\pi_R^{\mathrm{EB}^*} &= \frac{[(3-\delta+\beta\delta)^2-4(1+\beta)]Z}{32k(1-\delta)^2(5+\beta-\delta+\beta\delta)^3},\ \pi_S^{\mathrm{EB}^*} = \frac{[(3-\delta+\beta\delta)^2-4(1+\beta)]^2}{32k(1-\delta)(5+\beta-\delta+\beta\delta)^2}.
\end{aligned}$$

Where $Z=(5-\delta)^2(1-\delta)\delta-3\beta(5-\delta)(1-\delta)^2\delta-\beta^3\delta^2(4-\delta-\delta^2)+\beta^2[8-3\delta(2-\delta)(4+\delta-\delta^2)]$.

Checking the impact of δ on the above equilibriums, we get Proposition 6.

Proposition 6. $\frac{\partial w^{\mathrm{EB}^*}}{\partial\delta}>0,\ \frac{\partial g^{\mathrm{EB}^*}}{\partial\delta}>0,\ \frac{\partial q_R^{\mathrm{EB}^*}}{\partial\delta}>0,\ \frac{\partial q_S^{\mathrm{EB}^*}}{\partial\delta}<0,\ \frac{\partial \pi_R^{\mathrm{EB}^*}}{\partial\delta}>0,\ \frac{\partial \pi_S^{\mathrm{EB}^*}}{\partial\delta}<0.$

Differing from Proposition 5, the retailer's order quantity under the encroachment case increases with δ. This is primarily because, on the one hand, the retailer wants to increase order quantity to increase market share, and on the other hand, increasing order quantity can also generate more profit sharing from the supplier. In contrast, the sales quantity in the direct channel decreases with δ. This is because the supplier will reduce the direct-selling quantity to reduce competition as δ increases. Additionally, an increment in δ will benefit the retailer but harm the supplier, which indicates that the impacts of δ on the retailer and the supplier profits under encroachment differ from those under no encroachment.

4.2.3. Comparison of no encroachment and encroachment in PBI

In this subsection, the equilibrium outcomes of no-encroachment and encroachment scenarios are compared, and the effects of PBI on supplier encroachment are examined.

Proposition 7. (1) $q_R^{\mathrm{EB}^*}<q_R^{\mathrm{NB}^*}$;

(2) $w^{\mathrm{EB}^*}>w^{\mathrm{NB}^*}$ *when* $0<\beta<\beta_2$ *and* $0<\delta<\delta_1$, $w^{\mathrm{EB}^*}\le w^{\mathrm{NB}^*}$ *when* $0<\beta<\beta_2$ *and* $\delta_1\le\delta\le 0.5$ *or* $\beta_2\le\beta\le 0.45$;

(3) $g^{\mathrm{EB}^*}>g^{\mathrm{NB}^*}$ *when* $0<\delta<\delta_2$, $g^{\mathrm{EB}^*}\le g^{\mathrm{NB}^*}$ *when* $\delta_2\le\delta\le 0.5$.

Where $\beta_2=\frac{7-\sqrt{34}}{3}\approx 0.39$, and δ_1,δ_2 are described in proof.

Proposition 7(1) demonstrates that in PBI, the retailer's sales quantity with encroachment is always less than that with no encroachment, which is the same as the PFI (see Fig. 5). This suggests that no matter what type of partial vertical integration, supplier encroachment will always reduce the retailer's sales quantity.

Moreover, Proposition 7(2) demonstrates that, with encroachment, the wholesale price is greater than that with no encroachment when both the commission rate and the equity δ are low, and *vice versa* (see Fig. 6). As the commission rate is low, the supplier will prefer the online channel to sell products, so she would like to increase the wholesale price to reduce offline sales. But as the equity δ increases, the supplier will share more profits from the additional direct channel with the retailer, so she will favor the offline channel to distribute products and lower the wholesale price. This result suggests that when the selling costs of the direct channel are lower and the retailer holds fewer shares in the supplier, encroachment will aggravate the double marginalization.

Additionally, Proposition 7(3) reveals that the quality level with encroachment is greater than that with no encroachment when the equity δ is low, and *vice versa* (see Fig. 7). Higher quality will improve market demand

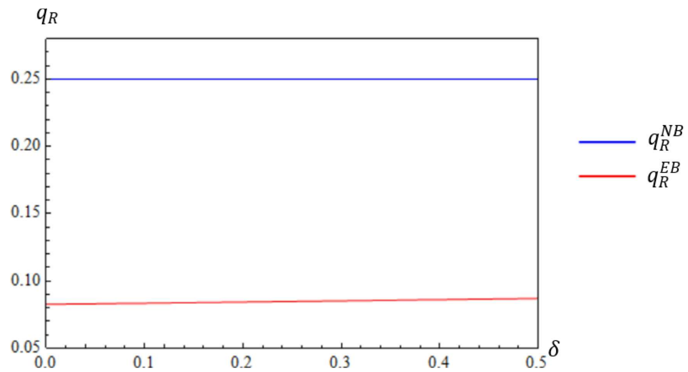

FIGURE 5. Comparison of the retailer's order quantity in PBI ($k = 1, \beta = 0.37$).

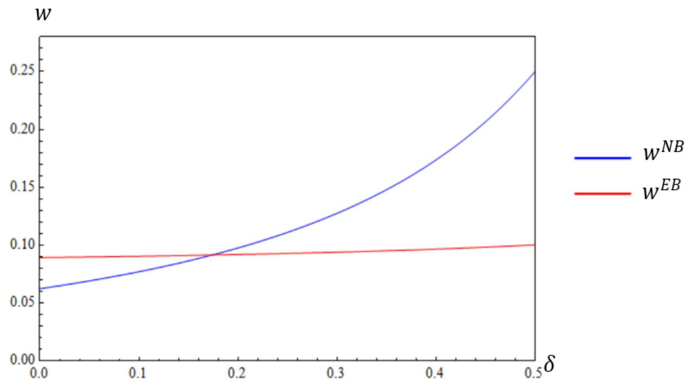

FIGURE 6. Comparison of the wholesale price in PBI ($k = 1, \beta = 0.2$).

and profitability, but as the equity δ increases, the supplier will share more profits with the retailer, and the benefits of increasing quality are transferred to the retailer; therefore, the supplier will distort quality downward ($g^{\mathrm{EB}^*} \leq g^{\mathrm{NB}^*}$) when the equity δ is high. This finding suggests that when the retailer holds fewer shares in the supplier, encroachment can facilitate quality development.

Proposition 8. (1) $\pi_R^{\mathrm{EB}^*} < \pi_R^{\mathrm{NB}^*}$ *when* $0 < \delta < \delta_3$, $\pi_R^{\mathrm{EB}^*} \geq \pi_R^{\mathrm{NB}^*}$ *when* $\delta_3 \leq \delta \leq 0.5$;
(2) $\pi_S^{\mathrm{EB}^*} > \pi_S^{\mathrm{NB}^*}$ *when* $0 < \delta < \delta_2$, $\pi_S^{\mathrm{EB}^*} \leq \pi_S^{\mathrm{NB}^*}$ *when* $\delta_2 \leq \delta \leq 0.5$.

Where $\delta_2 > \delta_3$ when $0 < \beta < \frac{5\sqrt{85}-19}{98} \approx 0.28$, otherwise $\delta_2 \leq \delta_3$. The parameters δ_2 and δ_3 are described in proof.

Differing from PFI, Proposition 8 demonstrates that the retailer can benefit from encroachment when he holds large amounts of equity in the supplier. This is because supplier encroachment will take some profits from the retailer, but the retailer can receive more dividends when he owns large amounts of equity, which can cover the losses. Therefore, the retailer could acquire as large a stake as possible to cope with supplier encroachment. Similar to PFI, we also discover that supplier encroachment may harm herself when the equity δ is large because the supplier must share more profits with the retailer.

In conclusion, supplier encroachment will lead to four outcomes in PBI: win–lose, win–win, lose–lose, or lose–win. Peculiarly, when the commission rate is low and the equity is moderate ($0 < \beta < 0.28$ and $\delta_3 < \delta < \delta_2$, see Fig. 8), encroachment will benefit both the supplier and the retailer. Therefore, when the direct channel cost is low, the supplier can appropriately sacrifice her equity to achieve a win–win situation with the retailer.

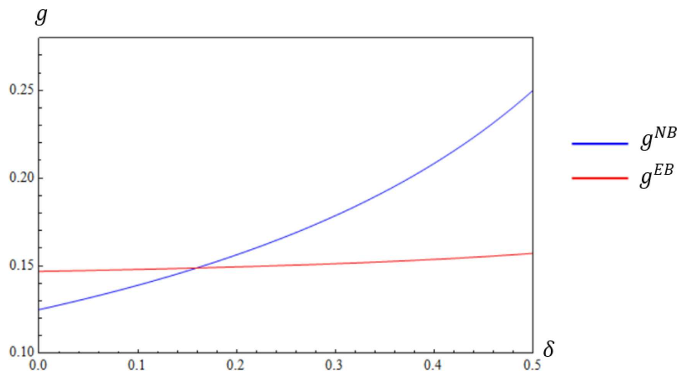

FIGURE 7. Comparison of the quality level in PBI ($k = 1, \beta = 0.43$).

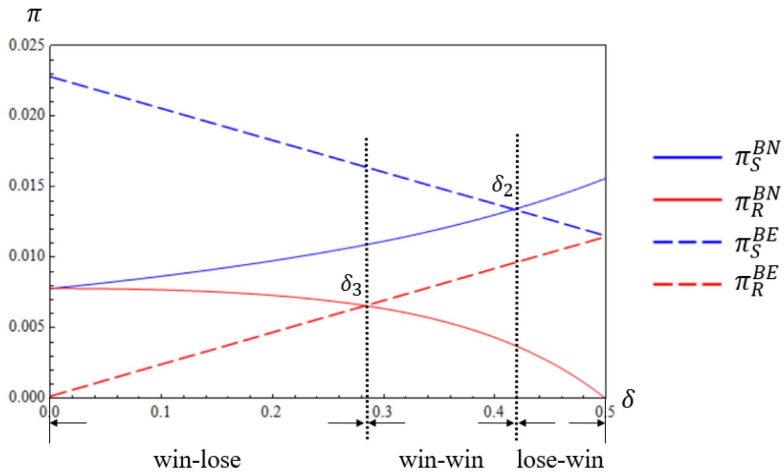

FIGURE 8. Comparison of the firms' profits in PBI ($k = 1, \beta = 0.15$).

Additionally, when the commission rate is high and the equity is moderate ($0.28 < \beta \leq 0.45$ and $\delta_2 < \delta < \delta_3$, see Fig. 9), encroachment will harm both, while when equity level is large ($\max\{\delta_2, \delta_3\} < \delta \leq 0.5$), encroachment will harm the supplier but benefit the retailer; therefore, it is better for the supplier to not encroach in these two scenarios.

Comparing Propositions 4 and 8, we find that supplier encroachment will lead to win–lose or lose–lose in PFI, but win–lose, win–win, lose–lose, or lose–win in PBI. The main reasons for the differences between PFI and PBI are as follows: in PFI, the supplier not only has a first-mover advantage in price and quality decision-makings but also owns an equity advantage, so the supplier's encroachment is always detrimental to the retailer. In contrast, in PBI, the retailer possesses the equity advantage, which forces the supplier to consider the consequences of equity losses when making encroachment decision. These results indicate that when the equity holder changes, the influence of encroachment on the two firms' profits will also change greatly.

4.3. Comparison of PFI and PBI

In this section, PFI and PBI are compared, and the impacts of the two partial vertical integrations on the equilibrium results are analyzed. To ensure the results are comparable, we assume equity $\delta = \theta$.

First, we compare PFI and PBI with no encroachment, generating Proposition 9.

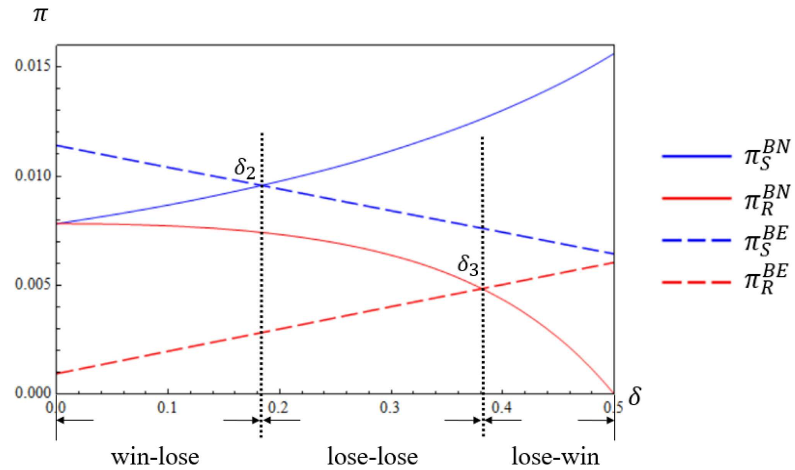

FIGURE 9. Comparison of the firms' profits in PBI ($k = 1, \beta = 0.43$).

Proposition 9. $w^{\mathrm{NF}^*} < w^{\mathrm{NB}^*}$, $g^{\mathrm{NF}^*} < g^{\mathrm{NB}^*}$, $q_R^{\mathrm{NF}^*} > q_R^{\mathrm{NB}^*}$, $\pi_R^{\mathrm{NF}^*} > \pi_R^{\mathrm{NB}^*}$, $\pi_S^{\mathrm{NF}^*} < \pi_S^{\mathrm{NB}^*}$.

Proposition 9 demonstrates that when the supplier does not encroach, the optimal wholesale price and quality level in PFI are less than that in PBI, while the optimal retailer's order quantity in PFI is greater than that in PBI. These can be explained as follows. In PFI, the supplier receives equity dividends from the retailer, so she is motivated to reduce the wholesale price to lessen the retailer's wholesale cost and is incentivized less to improve quality. In contrast, in PBI, the supplier provides equity dividends to the retailer, which motivates the supplier to increase the wholesale price and quality to increase her profits, thereby recovering the equity loss. Although the lower quality will decrease the retailer's order quantity, the lower wholesale price will encourage the retailer to order more; therefore, the retailer's order quantity in PFI is larger than that in PBI.

Intuitively, the retailer's profits should be lower in PFI than that in PBI because he must share his profits in PFI while receiving equity dividends in PBI. However, we observe the opposite result. The primary reason for this is that in PFI, the lower wholesale price and larger order quantity allow the retailer to generate more profits, which can offset the losses of equity dividends. Inversely, the supplier's profit in PFI is lower than that in PBI.

Next, PFI and PBI with supplier encroachment are compared, generating Proposition 10.

Proposition 10. $w^{\mathrm{EF}^*} < w^{\mathrm{EB}^*}$, $g^{\mathrm{EF}^*} > g^{\mathrm{EB}^*}$, $q_S^{\mathrm{EF}^*} < q_S^{\mathrm{EB}^*}$, $q_R^{\mathrm{EF}^*} > q_R^{\mathrm{EB}^*}$, $\pi_R^{\mathrm{EF}^*} < \pi_R^{\mathrm{EB}^*}$, $\pi_S^{\mathrm{EF}^*} > \pi_S^{\mathrm{EB}^*}$.

Differing from Proposition 9, when the supplier encroaches, the optimal quality level is higher than that in PBI, while the supplier's direct-selling quantity is lower in PFI than that in PBI. As the supplier encroaches, she will improve the quality level to increase direct-selling profits, but in PBI, the benefit of quality improvement will be shared with the retailer, so she has less motivation to develop quality. Since the supplier receives dividends from the retailer in PFI, she is more willing to downsize the direct-selling quantity to reduce channel conflicts.

Meanwhile, the retailer's profits in PFI are lower than that in PBI. Similar to when there is no encroachment, the retailer generates lower sales profits through PBI, but differing from it, the retailer will receive more dividends because of the additional direct-selling profits which can compensate for the lower sales profits. Conversely, the supplier's profits in PFI are higher than that in PBI.

From Propositions 9 and 10, we observe that the retailer and the supplier have opposite preferences relative to partial vertical integration, with the retailer preferring PFI when the supplier does not encroach while preferring PBI when the supplier encroaches.

5. Extensions

In this section, two extensions of the basic model are considered with certain assumptions altered.

5.1. Channel substitution

In the basic model, we suppose that products sold through the retailer and the direct channel are perfectly substitutable. This assumption has been adopted in several studies, including [16, 18, 63]. However, the online channel typically provides consumers with only a virtual description of the products and lacks the touch and feel of physical inspection; thereby potentially leading to mistaken evaluations. Therefore, we use the parameter γ $(0 < \gamma < 1)$ to represent the channel substitution, that is, the degree to which consumers accept the online channel as a substitute for shopping at an offline channel [25, 66, 68]. A consumer's surplus from purchasing a product through an offline channel is $u_R = vg - p_R$, and through an online channel is $u_S = v\gamma g - p_S$. The offline consumer demand is $q_R = 1 - \frac{p_R - p_S}{g(1-\gamma)}$ when $u_R > u_S$ (*i.e.*, $v > \frac{p_R - p_S}{g(1-\gamma)}$), while the online consumer demand is $q_S = \frac{p_R - p_S}{g(1-\gamma)} - \frac{p_S}{\gamma g}$ when $u_S > u_R$ and $u_S > 0$ (*i.e.*, $\frac{p_R - p_S}{g(1-\gamma)} > v > \frac{p_S}{\gamma g}$). By calculation, the inverse demand functions are $p_R = g(1 - \gamma q_S - q_R)$ and $p_S = g\gamma(1 - q_S - q_R)$, respectively.

When the supplier encroaches, the profits of the supplier and the retailer in PFI are as follows:

$$\breve{\pi}_S^{\mathrm{EF}^*} = wq_R - \frac{1}{2}kg^2 + (1-\beta)p_S q_S + \theta(p_R - w)q_R \tag{9}$$

$$\breve{\pi}_R^{\mathrm{EF}} = (1-\theta)(g(1-\gamma q_S - q_R) - w)q_R. \tag{10}$$

Lemma 5 is obtained employing the same methodology as in Section 4.1.2.

Lemma 5. *When considering channel substitution and encroachment in PFI, the equilibrium wholesale price, quality level, retailer's order quantity, supplier's direct-selling quantity, and profits of the two members are respectively:*

$$\breve{w}^{\mathrm{EF}^*} = \frac{(1-\beta)((2\beta-\gamma+\gamma\theta)^2 + 4(1-\beta^2-\gamma\theta))\breve{A}}{8k((-1+\beta)(8+(-3+\beta)\gamma) + 2(2-2\beta+\gamma)\theta - \gamma\theta^2)^2},$$

$$\breve{g}^{\mathrm{EF}^*} = \frac{(-1+\beta)(4+\gamma(4\beta(-1+\theta)+\gamma(-1+\theta)^2 - 4\theta))}{4k((-1+\beta)(8+(-3+\beta)\gamma) + 2(2-2\beta+\gamma)\theta - \gamma\theta^2)},$$

$$\breve{q}_R^{\mathrm{EF}^*} = \frac{(-1+\beta)(2+(-2+\beta)\gamma)}{\beta^2\gamma + 4(-2+\theta) - 4\beta(-2+\gamma+\theta) + \gamma(3+2\theta-\theta^2)},$$

$$\breve{q}_S^{\mathrm{EF}^*} = \frac{\beta(6+\gamma(-1+\theta)-4\theta) - (-1+\theta)(-6+\gamma+\gamma\theta)}{2(\beta^2\gamma + 4(-2+\theta) - 4\beta(-2+\gamma+\theta) + \gamma(3+2\theta-\theta^2))},$$

$$\breve{\pi}_R^{\mathrm{EF}^*} = \frac{(-1+\beta)^3(2+(-2+\beta)\gamma)^2(4+\gamma(4\beta(-1+\theta)+\gamma(-1+\theta)^2-4\theta))(1-\theta)}{4k((-1+\beta)(8+(-3+\beta)\gamma) + 2(2-2\beta+\gamma)\theta - \gamma\theta^2)^3},$$

$$\breve{\pi}_S^{\mathrm{EF}^*} = \frac{(-1+\beta)^2(4+\gamma(4\beta(-1+\theta)+\gamma(-1+\theta)^2-4\theta))^2}{32k((-1+\beta)(8+(-3+\beta)\gamma) + 2(2-2\beta+\gamma)\theta - \gamma\theta^2)^2}.$$

Where $\breve{A} = 8(1-\beta)(1-\theta) + 2\gamma((\beta-\theta)^2 - (2-\beta-\theta)) + \gamma^2(1-\beta+\theta)(1-\theta)$.

When the supplier does not encroach, the equilibrium results are the same as Lemma 1. Comparing no-encroachment and encroachment in PFI by considering channel substitution, we obtain Proposition 11.

Proposition 11. (1) *For the retailer:* $\breve{q}_R^{\mathrm{EF}^*} < \breve{q}_R^{\mathrm{NF}^*}$, $\breve{\pi}_R^{\mathrm{EF}^*} < \breve{\pi}_R^{\mathrm{NF}^*}$;

(2) *For the supplier: (i)* $\breve{w}^{\mathrm{EF}^*} > \breve{w}^{\mathrm{NF}^*}$ *when* $0 < \beta < \beta_4$, *otherwise* $\breve{w}^{\mathrm{EF}^*} \leq \breve{w}^{\mathrm{NF}^*}$; *(ii)* $\breve{g}^{\mathrm{EF}^*} < \breve{g}^{\mathrm{NF}^*}$ *and* $\breve{\pi}_S^{\mathrm{EF}^*} < \breve{\pi}_S^{\mathrm{NF}^*}$ *when* $0 < \theta < \theta_3$, $0 < \gamma < \frac{3(7+2\theta)(11-14\theta)}{220(2-\theta)(1-\theta)^2}$ *and* $\beta_5 < \beta \leq 0.45$ *or* $\theta_3 < \theta \leq 0.5$, $0 < \gamma < 1$ *and* $\beta_5 < \beta \leq 0.45$, *otherwise* $\breve{g}^{\mathrm{EF}^*} \geq \breve{g}^{\mathrm{NF}^*}$ *and* $\breve{\pi}_S^{\mathrm{EF}^*} \geq \breve{\pi}_S^{\mathrm{NF}^*}$.

The results of Proposition 11 are consistent with the base model (see Props. 3 and 4). In PFI, supplier encroachment also always harms the retailer and results in win–lose or lose–lose outcomes.

Next, we examine the PBI. The profits of the supplier and the retailer with encroachment are as follows:

$$\breve{\pi}_S^{\mathrm{EB}} = (1-\delta)\left[wq_R - \frac{1}{2}kg^2 + (1-\beta)g\gamma(1-q_R-q_S)q_S\right] \tag{11}$$

$$\breve{\pi}_R^{\mathrm{EB}} = (p_S - w)q_R + \delta\left[wq_R - \frac{1}{2}kg^2 + (1-\beta)g\gamma(1-q_R-q_S)q_S\right]. \tag{12}$$

Lemma 6 is obtained by employing the same methodology as in Section 4.2.2.

Lemma 6. *With encroachment in PBI, the equilibrium wholesale price, quality level, retailer's order quantity, supplier's direct-selling quantity, profits of the two members are respectively:*

$$\breve{w}^{\mathrm{EB}^*} = \frac{(8-2(2+\beta)\gamma - 8(1-\beta)\gamma\delta + \gamma^2(1+\delta-\beta\delta)^2)\breve{B}}{8k(1-\delta)^2(8+\gamma(-3+\beta+(-1+\beta)\delta))^2},$$
$$\breve{g}^{\mathrm{EB}^*} = \frac{4-4\beta\gamma-8(1-\beta)\gamma\delta+\gamma^2(1+\delta-\beta\delta)^2}{4k(1-\delta)(8+\gamma(-3+\beta+(-1+\beta)\delta))},$$
$$\breve{q}_R^{\mathrm{EB}^*} = \frac{2+(-2+\beta)\gamma}{8+\gamma(-3+\beta-\delta+\beta\delta)},\ \breve{q}_S^{\mathrm{EB}^*} = \frac{6+\gamma(-1+(-1+\beta)\delta)}{2(8+\gamma(-3+\beta-\delta+\beta\delta))},$$
$$\breve{\pi}_R^{\mathrm{EB}^*} = \frac{(4+\gamma(\gamma-4\beta-2(1-\beta)(4-\gamma)\delta+(1-\beta)^2\gamma\delta^2))\breve{Z}}{32k(1-\delta)^2(8+\gamma(-3+\beta+(-1+\beta)\delta))^3},$$
$$\breve{\pi}_S^{\mathrm{EB}^*} = \frac{(4-4\beta\gamma-8(1-\beta)\gamma\delta+\gamma^2(1+\delta-\beta\delta)^2)^2}{32k(1-\delta)(8+\gamma(-3+\beta+(-1+\beta)\delta))^2}.$$

Where $\breve{B} = 4+\gamma(\gamma-4\beta-2(1-\beta)(4-\gamma)\delta+(1-\beta)^2\gamma\delta^2)$, $\breve{Z} = 8(2+(-2+\beta)\gamma)^2+(-64+\gamma(196-124\beta-4(-3+\beta)(-10+7\beta)\gamma+(13+\beta(-15+4\beta))\gamma^2))\delta+(1-\beta)\gamma(-52+\gamma(8+9\gamma+2\beta(6+(-7+2\beta)\gamma)))\delta^2+(-1+\beta)^2\gamma^2(16+(-5+\beta)\gamma)\delta^3+(-1+\beta)^3\gamma^3\delta^4$.

The equilibrium results are the same as Lemma 3 when the supplier does not encroach. Comparing the no-encroachment and encroachment scenarios in PBI by considering channel substitution, we obtain Proposition 12.

Proposition 12. (1) *For the retailer: (i)* $\breve{q}_R^{\mathrm{EB}^*} < \breve{q}_R^{\mathrm{NB}^*}$*; (ii)* $\breve{\pi}_R^{\mathrm{EB}^*} < \breve{\pi}_R^{\mathrm{NB}^*}$ *when* $0<\delta<\delta_4$*, otherwise* $\breve{\pi}_R^{\mathrm{EB}^*} \geq \breve{\pi}_R^{\mathrm{NB}^*}$*;*

(2) *For the supplier: (i)* $\breve{w}^{\mathrm{EB}^*} > \breve{w}^{\mathrm{NB}^*}$ *when* $0<\delta<\delta_5$ *and* $0<\beta<\beta_6$*, otherwise* $\breve{w}^{\mathrm{EB}^*} \leq \breve{w}^{\mathrm{NB}^*}$*; (ii)* $\breve{g}^{\mathrm{EB}^*} > \breve{g}^{\mathrm{NB}^*}$ *and* $\breve{\pi}_S^{\mathrm{EB}^*} > \breve{\pi}_S^{\mathrm{NB}^*}$ *when* $0<\delta<\delta_6$, $0<\gamma<\frac{21}{40}$ *and* $0<\beta<\frac{3+2\gamma}{9}$ *or* $0<\delta<\delta_6$, $\frac{21}{40}<\gamma<1$ *and* $0<\beta\leq 0.45$*, otherwise* $\breve{g}^{\mathrm{EB}^*} \leq \breve{g}^{\mathrm{NB}^*}$ *and* $\breve{\pi}_S^{\mathrm{EB}^*} \leq \breve{\pi}_S^{\mathrm{NB}^*}$.

Analogous to the base model, supplier encroachment will also result in win–lose, win–win, lose–lose, or lose–win outcomes in PBI (see Props. 7 and 8).

In short, the primary results concur with the basic model when considering channel substitution.

5.2. Sequential quantity decision

In the basic model, we assume that the retailer and the supplier simultaneously determine their quantities in their respective channels. This simultaneous movement has been adopted in several studies [25,34,39]. However, there could be sequential quantity decisions that the supplier makes related to her direct-selling quantity after receiving the retailer's order. As encroachment intensifies channel conflict, the desire to use both channels may force the supplier to make concessions on quantity to appease the retailer. For example, IBM accepted orders for PCs over the Internet, but it gave priority to its distributors' sales in an attempt to alleviate channel conflict

[50]. Another example is that when the Air Jordan 2011 shoe was launched, it was only available at retail stores for several months before Nike began selling it on their official website [5]. Notably, the supplier can usually observe the retailer's order decision, whereas the supplier's quantity decision is usually unknown to the retailer. Thus, the supplier cannot credibly commit to not changing her quantity after receiving the retailer's order. This assumption concurs with numerous studies, such as Ha *et al.* [18], Yang *et al.* [58], Sun *et al.* [48], and Zhang *et al.* [68].

Based on the above description, we consider the following sequence of events: First, the supplier decides whether to encroach. Second, the supplier establishes the quality level g and the wholesale price w. Third, the retailer decides upon his order quantity q_R. Fourth, the supplier determines her direct-selling quantity q_M if she will be encroaching. Finally, the market clearing price is realized and the chain members obtain their profits.

Since the equilibrium results of no encroachment and the basic model are identical, we analyze the case of encroachment. Lemma 7 is obtained by employing the same methodology as in Section 4.1.2.

Lemma 7. *When considering sequential quantity decisions and encroachment in PFI, the equilibrium wholesale price, quality level, retailer's order quantity, supplier's direct-selling quantity, and profits of the two members are, respectively:*

$$\hat{w}^{\mathrm{EF}^*} = \frac{(1-\beta)(1-\theta)(3-3\theta-\beta)(3-3\theta-2\beta)}{8k(3+\beta-3\theta)^2(1-\beta-\theta)}, \quad \hat{g}^{\mathrm{EF}^*} = \frac{(1-\beta)(1-\theta)(3-2\beta-3\theta)}{4k(3+\beta-3\theta)(1-\beta-\theta)},$$
$$\hat{q}_R^{\mathrm{EF}^*} = \frac{(1-\beta)\beta}{(3+\beta-3\theta)(1-\beta-\theta)}, \quad \hat{q}_S^{\mathrm{EF}^*} = \frac{3(1-\theta)^2-\beta(3-\theta)}{2(3+\beta-3\theta)(1-\beta-\theta)},$$
$$\hat{\pi}_R^{\mathrm{EF}^*} = \frac{(1-\beta)^2\beta^2(1-\theta)^2(3-2\beta-3\theta)}{8k(3+\beta-3\theta)^3(1-\beta-\theta)^2}, \quad \hat{\pi}_S^{\mathrm{EF}^*} = \frac{(1-\beta)^2(1-\theta)^2(3-2\beta-3\theta)^2}{32k(3+\beta-3\theta)^2(1-\beta-\theta)^2}.$$

Comparing no-encroachment and encroachment in PFI by considering sequential quantity decisions, we obtain Proposition 13.

Proposition 13. (1) *For the retailer: (i)* $\hat{q}_R^{\mathrm{EF}^*} > \hat{q}_R^{\mathrm{NF}^*}$ *when* $\frac{4-\sqrt{10}}{4} \approx 0.21 < \beta \le 0.45$ *and* $\theta_4 < \theta \le 0.5$*, otherwise* $\hat{q}_R^{\mathrm{EF}^*} \le \hat{q}_R^{\mathrm{NF}^*}$*; (ii)* $\hat{\pi}_R^{\mathrm{EF}^*} > \hat{\pi}_R^{\mathrm{NF}^*}$ *when* $\beta_7 < \beta \le 0.45$ *and* $\theta_5 < \theta \le 0.5$*, otherwise* $\hat{\pi}_R^{\mathrm{EF}^*} \le \hat{\pi}_R^{\mathrm{NF}^*}$*;*
(2) *For the supplier: (i)* $\hat{w}^{\mathrm{EF}^*} < \hat{w}^{\mathrm{NF}^*}$ *when* $\beta_8 < \beta \le 0.45$ *and* $0 < \theta < \theta_6$*, otherwise* $\hat{w}^{\mathrm{EF}^*} \ge \hat{w}^{\mathrm{NF}^*}$*; (ii)* $\hat{g}^{\mathrm{EF}^*} > \hat{g}^{\mathrm{NF}^*}$ *and* $\hat{\pi}_S^{\mathrm{EF}^*} > \hat{\pi}_S^{\mathrm{NF}^*}$*.*

Differing from the base model, we find that in PFI, encroachment will result in win–win or win–lose outcomes when the supplier later determines her direct-selling quantity. Notably, the new outcome of win–win replaces the previous lose–lose outcome when the commission rate and the equity θ are large. The reason is that the sequential game diminishes competition in the downstream market. Therefore, when the supplier holds more equity in the retailer and the direct channel is costlier, she can make concessions on the quantity decision which will benefit both the supplier and the retailer.

Next, the PBI is examined. Lemma 8 is obtained by employing the same methodology as in Section 4.2.2.

Lemma 8. *With encroachment in PBI, the equilibrium wholesale price, quality level, retailer's order quantity, supplier's direct-selling quantity, profits of the two members are respectively:*

$$\hat{w}^{\mathrm{EB}^*} = \frac{[(2-\delta+\beta\delta)^2-(1+2\beta)][(2-\delta+\beta\delta)^2-(1+\beta)]}{8k(1-\delta)^2(3+\beta-\delta+\beta\delta)^2},$$
$$\hat{g}^{\mathrm{EB}^*} = \frac{(2-\delta+\beta\delta)^2-(1+2\beta)}{4k(1-\delta)(3+\beta-\delta+\beta\delta)},$$
$$\hat{q}_R^{\mathrm{EB}^*} = \frac{\beta}{3+\beta-\delta+\beta\delta}, \quad \hat{q}_S^{\mathrm{EB}^*} = \frac{3-\delta+\beta\delta}{2(3+\beta-\delta+\beta\delta)},$$
$$\hat{\pi}_R^{\mathrm{EB}^*} = \frac{[(2-\delta+\beta\delta)^2-(1+2\beta)]\check{C}}{32k(1-\delta)^2(3+\beta-\delta+\beta\delta)^3}, \quad \hat{\pi}_S^{\mathrm{EB}^*} = \frac{[(2-\delta+\beta\delta)^2-(1+2\beta)]^2}{32k(1-\delta)(3+\beta-\delta+\beta\delta)^2}.$$

Where $\check{C} = 4\beta^2 + (9 - 3\beta + 14\beta^2)\delta - (1-\beta)(15 + 2\beta - 4\beta^2)\delta^2 + \delta^3(1-\beta)^2(7 + \beta - \delta + \beta\delta)$.

Comparing no-encroachment and encroachment in PBI by considering sequential quantity decisions, we obtain Proposition 14.

Proposition 14. (1) *For the retailer: (i)* $\hat{q}_R^{\mathrm{EB}^*} < \hat{q}_R^{\mathrm{NB}^*}$*; (ii)* $\hat{\pi}_R^{\mathrm{EB}^*} < \hat{\pi}_R^{\mathrm{NB}^*}$ *when* $0 < \delta < \delta_7$*, otherwise* $\hat{\pi}_R^{\mathrm{EB}^*} \geq \hat{\pi}_R^{\mathrm{NB}^*}$*;*

(2) *For the supplier: (i)* $\hat{w}^{\mathrm{EB}^*} > \hat{w}^{\mathrm{NB}^*}$ *when* $0 < \beta < 4 - \sqrt{13} \approx 0.39$ *and* $0 < \delta < \delta_8$*, otherwise* $\hat{w}^{\mathrm{EB}^*} \leq \hat{w}^{\mathrm{NB}^*}$*; (ii)* $\hat{g}^{\mathrm{EB}^*} > \hat{g}^{\mathrm{NB}^*}$ *and* $\hat{\pi}_S^{\mathrm{EB}^*} > \hat{\pi}_S^{\mathrm{NB}^*}$ *when* $0 < \delta < \delta_9$*, otherwise* $\hat{g}^{\mathrm{EB}^*} \leq \hat{g}^{\mathrm{NB}^*}$ *and* $\hat{\pi}_S^{\mathrm{EB}^*} \leq \hat{\pi}_S^{\mathrm{NB}^*}$*.*

Consistent with the base model, supplier encroachment will also lead to win–lose, win–win, lose–lose, or lose–win outcomes in PBI (see Props. 7 and 8).

In short, considering sequential quantity decisions does not change the results in PBI, while a new outcome of win–win is created in PFI.

6. Conclusion

In this paper, we examine the impact of two structures of partial vertical integration (PFI and PBI) on supplier encroachment. Our analysis yields the following primary results:

First, in PFI, supplier encroachment will always harm the retailer and result in two outcomes: win–lose or lose–lose. Particularly, when the commission rate and the equity θ are large, encroachment will harm both, so the supplier should not encroach. However, in PBI, supplier encroachment may benefit the retailer and lead to four outcomes: win–lose, win–win, lose–lose, or lose–win. Peculiarly, when the commission rate is low and the equity δ is moderate, encroachment will benefit both. Therefore, when the direct channel cost is low, the supplier can sacrifice her equity appropriately to achieve a win–win outcome with the retailer. Additionally, when the commission rate is high and the equity is moderate, encroachment will harm both, while when the equity level is large, encroachment will harm the supplier but benefit the retailer; therefore, it is better for the supplier to not encroach in these two scenarios.

Second, in PFI, the equity θ has the same impact mechanisms and will improve both firms' profits regardless of whether the supplier encroaches or not. Nevertheless, in PBI, the equity δ has the opposite impact. In this case, when the supplier does not encroach, the retailer's profits decrease in δ, while the supplier's profits increase in association with it. When the supplier encroaches, the retailer's profits increase in association with δ, while the supplier's profits decrease with it.

Third, when the supplier equity holding (θ) and the retailer equity holding (δ) are equal or relatively close, the supplier will prefer PBI when no encroachment occurs, but she will prefer PFI when encroachment occurs. The preference of the retailer is the reverse.

Additionally, two extensions have been made to verify the robustness of the findings. We contend that the key results remain valid when considering channel substitution, while a new outcome of win–win is created in PFI when considering sequential quantity decisions.

6.1. Managerial insights

These findings are also of practical interest and can enable firms to make better decisions.

For the supplier, she should be cautious about encroaching because it may harm herself. If she intends to open an online channel to expand her business, she must carefully inspect the equity structure and direct-selling costs. To create a win–win scenario with a downstream retailer, the supplier could make concessions on quantity decisions in PFI, while she could sacrifice appropriate equity in PBI.

For the retailer, supplier encroachment will harm him in most cases, but it may also benefit him. To address the potential threat of encroachment, a well-funded retailer should acquire as much of the supplier's equity as possible.

6.2. Limitations and future research

There are some limitations to our study. First, we consider a monopolist supply chain in this paper; thus, introducing upstream or downstream competition in the model would be an interesting research direction. Second, we only consider the supplier's option to encroach on the retail market. It would be worthwhile to examine a retailer's defense strategies, such as providing in-store services or a store brand product to deter manufacturer encroachment [32, 60, 64]. Finally, a major limitation of this paper is that we assume perfect information in the game-theory models. To overcome this limitation, future research can explore imperfect information (*e.g.*, demand information imperfect) by method of signaling game [25, 49, 70].

Funding

This work is supported by National Natural Science Foundation of China (Grant 72371180 and 72071137) and Philosophy and Social Sciences Project of Jiangsu Province Universities (Grant 2024SJYB1079).

Data availability statement

The research data associated with this article are included in the article.

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Appendix A.

Proof of Lemma 1. Given w and g, π_R^{NF} is a concave downward function of q_R as $\frac{\partial^2\pi_R^{\mathrm{NF}}}{\partial {q_R}^2} = -2g(1-\theta) < 0$. It is easy to obtain the retailer's optimal order quantity $q_R(g,w) = \frac{g-w}{2g}$ by $\frac{\partial\pi_R^{\mathrm{NF}}}{\partial q_R} = 0$. Substituting $q_R(g,w)$ into π_S^{NF}, we have $\pi_S^{\mathrm{NF}}(g,w) = \frac{-2g^3k+w^2(-2+\theta)-2gw(-1+\theta)+g^2\theta}{4g}$. As $\frac{\partial^2\pi_S^{\mathrm{NF}}}{\partial w^2} = -\frac{2-\theta}{2g} < 0$, π_S^{NF} is a concave downward function of w. Let $\frac{\partial\pi_S^{\mathrm{NF}}}{\partial w} = 0$, we get the supplier's optimal wholesale price $w(g) = \frac{g(1-\theta)}{2-\theta}$. Then, substituting $w(g)$ into π_S^{NF}, we have $\pi_S^{\mathrm{NF}}(g) = \frac{g[1-2gk(2-\theta)]}{4(2-\theta)}$ and $\frac{\partial^2\pi_S^{\mathrm{NF}}}{\partial g^2} = -k < 0$. By $\frac{\partial\pi_S^{\mathrm{NF}}}{\partial g} = 0$, we get the optimal product quality $g^{(\mathrm{NF}^*)} = \frac{1}{8k-4k\theta}$. Last, substituting the equilibrium $g^{(\mathrm{NF}^*)}$ to the responding functions, we get Lemma 1. □

Proof of Proposition 1. $\frac{\partial w^{\mathrm{NF}^*}}{\partial\theta} = -\frac{\theta}{4k(2-\theta)^3} < 0$, $\frac{\partial g^{\mathrm{NF}^*}}{\partial\theta} = \frac{1}{4k(2-\theta)^2} > 0$, $\frac{\partial q_R^{\mathrm{NF}^*}}{\partial\theta} = \frac{1}{2(2-\theta)^2} > 0$, $\frac{\partial\pi_R^{\mathrm{NF}^*}}{\partial\theta} = \frac{1-2\theta}{16k(2-\theta)^4} > 0$, $\frac{\partial\pi_S^{\mathrm{NF}^*}}{\partial\theta} = \frac{1}{16k(2-\theta)^3} > 0$. □

Proof of Lemma 2. Given w and g, π_R^{EF} and π_S^{EF} are concave downward functions of q_R and q_S respectively since $\frac{\partial^2\pi_R^{\mathrm{EF}}}{\partial {q_R}^2} = -2g(1-\theta) < 0$ and $\frac{\partial^2\pi_S^{\mathrm{EF}}}{\partial {q_S}^2} = -2g(1-\beta) < 0$. We can obtain the retailer's optimal order quantity $q_R(g,w) = \frac{(g-2w)(1-\beta)}{g(3-3\beta-\theta)}$ and the supplier's direct-selling quantity $q_S(g,w) = \frac{w(1-\beta+\theta)-g(1-\beta-\theta)}{g(3-3\beta-\theta)}$ by $\frac{\partial\pi_R^{\mathrm{EF}}}{\partial q_R} = 0$ and $\frac{\partial\pi_S^{\mathrm{EF}}}{\partial q_S} = 0$, respectively. Substituting $q_R(g,w)$ and $q_S(g,w)$ into π_S^{EF}, we have $\pi_S^{\mathrm{EF}}(g,w) = -\frac{1}{2g(-3+3\beta+\theta)^2}(2g^2(-1+\beta)^3 + g^3k(-3+3\beta+\theta)^2 + 2w^2(-1+\beta)(-5+\beta^2-4\beta(-1+\theta)+6\theta-\theta^2) + 2gw(-1+\beta)(5+2\beta^2-6\theta+\theta^2+\beta(-7+5\theta)))$. As $\frac{\partial^2\pi_S^{\mathrm{EF}}}{\partial w^2} = -\frac{2(-1+\beta)(-5+\beta^2-4\beta(-1+\theta)+6\theta-\theta^2)}{g(-3+3\beta+\theta)^2} < 0$, π_S^{EF} is a concave downward function of w. Let $\frac{\partial\pi_S^{\mathrm{EF}}}{\partial w} = 0$, we get the supplier's optimal wholesale price $w(g) = \frac{g(5-7\beta+2\beta^2-6\theta+5\beta\theta+\theta^2)}{2(5-4\beta-\beta^2-6\theta+4\beta\theta+\theta^2)}$. Then, substituting $w(g)$ into π_S^{EF}, we have $\pi_S^{\mathrm{EF}}(g) = -\frac{g(-(-1+\beta)(-1+\theta)(-5+4\beta+\theta)+2gk(-5+\beta^2-4\beta(-1+\theta)+6\theta-\theta^2))}{4(-5+\beta^2-4\beta(-1+\theta)+6\theta-\theta^2)}$ and $\frac{\partial^2\pi_S^{\mathrm{EF}}}{\partial g^2} = -k < 0$. By $\frac{\partial\pi_S^{\mathrm{EF}}}{\partial g} = 0$, we get the optimal product quality $g^{\mathrm{EF}^*} = \frac{(1-\beta)(1-\theta)(5-4\beta-\theta)}{4k[5-\beta(4+\beta)-6\theta+4\beta\theta+\theta^2]}$. Last, substituting the equilibrium g^{EF^*} to the responding functions, we get Lemma 2. □

Proof of Proposition 2. $\frac{\partial w^{\mathrm{EF}^*}}{\partial\theta} = -\frac{\beta(1-\beta)A}{8k[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^3}$, where $A = 65+8\beta^4-108\theta+54\theta^2-12\theta^3+\theta^4+\beta^3(-92+76\theta)+\beta^2(231-274\theta+51\theta^2)+4\beta(-53+77\theta-27\theta^2+3\theta^3)$. By the conditions of $0<\theta\le 50\%$ and $0<\beta\le 45\%$, we have $A>0$, then we get $\frac{\partial w^{\mathrm{EF}^*}}{\partial\theta} < 0$. $\frac{\partial g^{\mathrm{EF}^*}}{\partial\theta} = \frac{\beta^2(1-\beta)(3-2\beta-\theta)}{2k[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^2} > 0$, $\frac{\partial q_R^{\mathrm{EF}^*}}{\partial\theta} = \frac{2\beta(1-\beta)(3-2\beta-\theta)}{[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^2} > 0$, $\frac{\partial q_S^{\mathrm{EF}^*}}{\partial\theta} = -\frac{\beta[11+3\beta^2-2\beta(7-\theta)-2\theta-\theta^2]}{2[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^2} < 0$. $\frac{\partial\pi_R^{\mathrm{EF}^*}}{\partial\theta} = \frac{(1-\beta)^3\beta^2(1-\theta)B}{4k[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^4}$, where $B = 35-8\beta^3-57\theta+25\theta^2-3\theta^3+\beta^2(27-19\theta)-16\beta(3-4\theta+\theta^2)$. By the conditions of $0<\theta\le 50\%$ and $0<\beta\le 45\%$, we have $B>0$, then we get $\frac{\partial\pi_R^{\mathrm{EF}^*}}{\partial\theta} > 0$. $\frac{\partial\pi_S^{\mathrm{EF}^*}}{\partial\theta} = \frac{(1-\beta)^2\beta^2(1-\theta)[15+8\beta^2-8\theta+\theta^2+\beta(6\theta-22)]}{8k[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^3} > 0$. □

Proof of Proposition 3. (1) $q_R^{\mathrm{EF}^*} - q_R^{\mathrm{NF}^*} = -\frac{5-8\beta+3\beta^2-6\theta+6\beta\theta-2\beta^2\theta+\theta^2}{2(2-\theta)(5-4\beta-\beta^2-6\theta+4\beta\theta+\theta^2)} < 0$ since $0<\theta\le 50\%$ and $0<\beta\le 45\%$.

(2) $w^{\mathrm{EF}^*} - w^{\mathrm{NF}^*} = \frac{(1-\theta)C}{8k}$, where $C = \frac{(1-\beta)[(5-\theta)^2(1-\theta)-8\beta^3+\beta^2(38-22\theta)-\beta(55-56\theta+9\theta^2)]}{[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^2} - \frac{2}{(2-\theta)^2}$. By the conditions of $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $w^{\mathrm{EF}^*} > w^{\mathrm{NF}^*}$ (*i.e.*, $C > 0$) when $0 < \beta \leq \beta_1$ or $\beta_1 < \beta < \beta_2$ and $0 < \theta < \theta_1$, $w^{\mathrm{EF}^*} \leq w^{\mathrm{NF}^*}$ (*i.e.*, $C \leq 0$) when $\beta_1 < \beta < \beta_2$ and $\theta_1 \leq \theta \leq 0.5$ or $\beta_2 \leq \beta \leq 0.45$, where θ_1 is the root of function $F(\theta) = 50 - 240\beta + 360\beta^2 - 200\beta^3 + 30\beta^4 + (-120 + 508\beta - 644\beta^2 + 288\beta^3 - 32\beta^4)\theta + (117 - 412\beta + 413\beta^2 - 134\beta^3 + 8\beta^4)\theta^2 + (-59 + 159\beta - 114\beta^2 + 22\beta^3)\theta^3 + (13 - 24\beta + 9\beta^2)\theta^4 + (-1+\beta)\theta^5$, $\beta_1 \approx 0.36$ is the root of function $H(\beta) = 405 - 2259\beta + 4082\beta^2 - 2776\beta^3 + 512\beta^4$, $\beta_2 = \frac{7-\sqrt{34}}{3} \approx 0.39$.

(3) $g^{\mathrm{EF}^*} - g^{\mathrm{NF}^*} = \frac{D}{4k(2-\theta)[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]}$, where $D = \beta^2(3-2\theta)^2 + (5-\theta)(1-\theta)^2 - \beta(14-25\theta+12\theta^2-\theta^3)$. By the conditions of $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $g^{\mathrm{EF}^*} > g^{\mathrm{NF}^*}$ (*i.e.*, $D > 0$) when $0 < \beta \leq \beta_3$ or $\beta_3 < \beta \leq 0.45$ and $0 < \theta < \theta_2$, $g^{\mathrm{EF}^*} \leq g^{\mathrm{NF}^*}$ (*i.e.*, $D \leq 0$) when $\beta_3 < \beta \leq 0.45$ and $\theta_2 \leq \theta \leq 0.5$, where θ_2 is the root of function $G(\theta) = 5 - 14\beta + 9\beta^2 + (-11 + 25\beta - 12\beta^2)\theta + (7 - 12\beta + 4\beta^2)\theta^2 + (-1+\beta)\theta^3$, $\beta_3 = \frac{35-\sqrt{73}}{64} \approx 0.41$. □

Proof of Proposition 4. (1) $\pi_R^{\mathrm{EF}^*} - \pi_R^{\mathrm{NF}^*} = -\frac{(1-\theta)E}{16k}$, where $E = \frac{1}{(2-\theta)^3} - \frac{4(1-\beta)^3\beta^2(1-\theta)(5-4\beta-\theta)}{[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^3}$. By the conditions of $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $E > 0$, *i.e.*, $\pi_R^{\mathrm{EF}^*} < \pi_R^{\mathrm{NF}^*}$.

(2) $\pi_S^{\mathrm{EF}^*} - \pi_S^{\mathrm{NF}^*} = \frac{F}{32k}$, where $F = \frac{(1-\beta)^2(1-\theta)^2(5-4\beta-\theta)^2}{[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^2} - \frac{1}{(2-\theta)^2}$. By the conditions of $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $\pi_S^{\mathrm{EF}^*} > \pi_S^{\mathrm{NF}^*}$ (*i.e.*, $F > 0$) when $0 < \beta \leq \beta_3$ or $\beta_3 < \beta \leq 0.45$ and $0 < \theta < \theta_2$, $\pi_S^{\mathrm{EF}^*} \leq \pi_S^{\mathrm{NF}^*}$ when $\beta_3 < \beta \leq 0.45$ and $\theta_2 \leq \theta \leq 0.5$. □

Proof of Lemma 3. The proof is similar to Lemma 1, we omit it here. □

Proof of Proposition 5. $\frac{\partial w^{\mathrm{NB}^*}}{\partial \delta} = \frac{1}{8k(1-\delta)^3} > 0$, $\frac{\partial g^{\mathrm{NB}^*}}{\partial \delta} = \frac{1}{8k(1-\delta)^2} > 0$, $\frac{\partial q_R^{\mathrm{NB}^*}}{\partial \delta} = 0$, $\frac{\partial \pi_R^{\mathrm{NB}^*}}{\partial \delta} = -\frac{\delta}{64k(1-\delta)^3} < 0$, $\frac{\partial \pi_S^{\mathrm{NB}^*}}{\partial \delta} = \frac{1}{128k(1-\delta)^2} > 0$. □

Proof of Lemma 4. The proof is similar to Lemma 2, we omit it here. □

Proof of Proposition 6. $\frac{\partial w^{\mathrm{EB}^*}}{\partial \delta} = \frac{\beta(3-\delta+\beta\delta)[(1+\beta)(3-\delta+\beta\delta)^2-(3\beta+2)(\beta+2)]}{2k(1-\delta)^3(5+\beta-\delta+\beta\delta)^3}$, by the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $(1+\beta)(3-\delta+\beta\delta)^2 - (3\beta+2)(\beta+2) > 0$, then we get $\frac{\partial w^{\mathrm{EB}^*}}{\partial \delta} > 0$. $\frac{\partial g^{\mathrm{EB}^*}}{\partial \delta} = \frac{\beta^2(3-\delta+\beta\delta)}{2k(1-\delta)^2(5+\beta-\delta+\beta\delta)^2} > 0$, $\frac{\partial q_R^{\mathrm{EB}^*}}{\partial \delta} = \frac{(1-\beta)\beta}{(5+\beta-\delta+\beta\delta)^2} > 0$, $\frac{\partial q_S^{\mathrm{EB}^*}}{\partial \delta} = -\frac{(1-\beta)\beta}{2(5+\beta-\delta+\beta\delta)^2} < 0$. $\frac{\partial \pi_R^{\mathrm{EB}^*}}{\partial \delta} = \frac{L}{32k(-1+\delta)^3(5+\beta-\delta+\beta\delta)^4}$, where $L = (-5+\delta)^4(-1+\delta)^3 - 6\beta(-5+\delta)^3(-1+\delta)^4 + \beta^6\delta^3(16-9\delta-\delta^2+\delta^3+\delta^4) - 4\beta^3(-1+\delta)^3(-39+44\delta+68\delta^2-40\delta^3+5\delta^4) + \beta^2(-1+\delta)^3(-195-676\delta+668\delta^2-180\delta^3+15\delta^4) - 2\beta^5\delta(24-90\delta+54\delta^2+7\delta^3-5\delta^4-9\delta^5+3\delta^6) + \beta^4(-176+472\delta-210\delta^2-162\delta^3+15\delta^4+183\delta^5-105\delta^6+15\delta^7)$. By the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $L > 0$, so $\frac{\partial \pi_R^{\mathrm{EB}^*}}{\partial \delta} > 0$. $\frac{\partial \pi_S^{\mathrm{EB}^*}}{\partial \delta} = -\frac{M}{32k(1-\delta)^2(5+\beta-\delta+\beta\delta)^3}$, where $M = (5-4\beta+6(-1+\beta)\delta+(-1+\beta)^2\delta^2)(25-\beta(15+16\beta)-35\delta+\beta(33+6\beta-4\beta^2)\delta+(-1+\beta)^2(11+\beta)\delta^2+(-1+\beta)^3\delta^3)$. By the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $M > 0$, so $\frac{\partial \pi_S^{\mathrm{EB}^*}}{\partial \delta} < 0$. □

Proof of Proposition 7. (1) $q_R^{\mathrm{EB}^*} - q_R^{\mathrm{NB}^*} = -\frac{5-3\beta-\delta+\beta\delta}{4(5+\beta-\delta+\beta\delta)} < 0$ since $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$.

(2) $w^{\mathrm{EB}^*} - w^{\mathrm{NB}^*} = \frac{R}{16k(1-\delta)^2(5+\beta-\delta+\beta\delta)^2}$, where $R = 2[(3-\delta+\beta\delta)^2 - 4(1+\beta)][(3-\delta+\beta\delta)^2 - 2(2+\beta)] - (5+\beta-\delta+\beta\delta)^2$. By the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $w^{\mathrm{EB}^*} > w^{\mathrm{NB}^*}$ (*i.e.*, $R > 0$) when $0 < \beta < \beta_2 = \frac{7-\sqrt{34}}{3} \approx 0.39$ and $0 < \delta < \delta_1$, $w^{\mathrm{EB}^*} \leq w^{\mathrm{NB}^*}$ (*i.e.*, $R \leq 0$) when $0 < \beta < \beta_2$ and $\delta_1 \leq \delta \leq 0.5$ or $\beta_2 \leq \beta \leq 0.45$, where δ_1 is the root of function $O(\delta) = 25 - 70\beta + 15\beta^2 - (110 - 184\beta + 74\beta^2)\delta + (91 - 194\beta + 115\beta^2 - 12\beta^3)\delta^2 - (24 - 72\beta + 72\beta^2 - 24\beta^3)\delta^3 + (2 - 8\beta + 12\beta^2 - 8\beta^3 + 2\beta^4)\delta^4$.

(3) $g^{\mathrm{EB}^*} - g^{\mathrm{NB}^*} = \frac{S}{8k(1-\delta)(5+\beta-\delta+\beta\delta)}$, where $S = 2(1-\beta)^2\delta^2 - 11(1-\beta)\delta + 5 - 9\beta$. By the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $g^{\mathrm{EB}^*} > g^{\mathrm{NB}^*}$ (*i.e.*, $S > 0$) when $0 < \delta < \delta_2$, $g^{\mathrm{EB}^*} \leq g^{\mathrm{NB}^*}$ (*i.e.*, $S \leq 0$) when $\delta_2 \leq \delta \leq 0.5$, where $\delta_2 = \frac{11-3\sqrt{9+8\beta}}{4(1-\beta)}$. □

Proof of Proposition 8. (1) $\pi_R^{\mathrm{EB}^*} - \pi_R^{\mathrm{NB}^*} = \frac{T}{128k(1-\delta)^2(5+\beta-\delta+\beta\delta)^3}$, where $T = \beta(-75 + (145 - 129\beta)\beta) + \beta(-595 + \beta(-429 + 575\beta))\delta + (-1+\beta)(1465 + \beta(-832 - 717\beta + 96\beta^2))\delta^2 - (-1+\beta)^2(-1191 + \beta(163 + 208\beta))\delta^3 - 2(-1+\beta)^3(-213 + 8(-1+\beta)\beta)\delta^4 + 4(-1+\beta)^4(17+\beta)\delta^5 + 4(-1+\beta)^5\delta^6 + 25(-5+33\delta)$. By the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $\pi_R^{\mathrm{EB}^*} < \pi_R^{\mathrm{NB}^*}$ (*i.e.*, $T < 0$) when $0 < \delta < \delta_3$, $\pi_R^{\mathrm{EB}^*} \geq \pi_R^{\mathrm{NB}^*}$ (*i.e.*, $T \geq 0$) when $\delta_3 \leq \delta \leq 0.5$, where δ_3 is the root of function $P(\delta) = -125 - 75\beta + 145\beta^2 - 129\beta^3 + (825 - 595\beta - 429\beta^2 + 575\beta^3)\delta + (-1465 + 2297\beta - 115\beta^2 - 813\beta^3 + 96\beta^4)\delta^2 + (1191 - 2545\beta + 1309\beta^2 + 253\beta^3 - 208\beta^4)\delta^3 + (-426 + 1262\beta - 1214\beta^2 + 330\beta^3 + 64\beta^4 - 16\beta^5)\delta^4 + (68 - 268\beta + 392\beta^2 - 248\beta^3 + 52\beta^4 + 4\beta^5)\delta^5 + (-4 + 20\beta - 40\beta^2 + 40\beta^3 - 20\beta^4 + 4\beta^5)\delta^6$.

(2) $\pi_S^{\mathrm{EB}^*} - \pi_S^{\mathrm{NB}^*} = \frac{U}{128k(1-\delta)(5+\beta-\delta+\beta\delta)^2}$, where $U = 4[(3-\delta+\beta\delta)^2 - 4(1+\beta)]^2 - (5+\beta-\delta+\beta\delta)^2$. By the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $\pi_S^{\mathrm{EB}^*} > \pi_S^{\mathrm{NB}^*}$ (*i.e.*, $U > 0$) when $0 < \delta < \delta_2$, $\pi_S^{\mathrm{EB}^*} \leq \pi_S^{\mathrm{NB}^*}$ (*i.e.*, $U \leq 0$) when $\delta_2 \leq \delta \leq 0.5$, where $\delta_2 = \frac{11-3\sqrt{9+8\beta}}{4(1-\beta)}$.

Moreover, $\frac{\partial P(\delta)}{\partial \delta} = 825 - 595\beta - 429\beta^2 + 575\beta^3 + 2(-1465 + 2297\beta - 115\beta^2 - 813\beta^3 + 96\beta^4)\delta + 3(1191 - 2545\beta + 1309\beta^2 + 253\beta^3 - 208\beta^4)\delta^2 + 4(-426 + 1262\beta - 1214\beta^2 + 330\beta^3 + 64\beta^4 - 16\beta^5)\delta^3 + 5(68 - 268\beta + 392\beta^2 - 248\beta^3 + 52\beta^4 + 4\beta^5)\delta^4 + 6(-4 + 20\beta - 40\beta^2 + 40\beta^3 - 20\beta^4 + 4\beta^5)\delta^5$, by the conditions of $0 < \delta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $\frac{\partial P(\delta)}{\partial \delta} > 0$, *i.e.*, $P(\delta)$ is a monotonically increasing function with respect to δ. Let $P(\delta_2) = 0$, we can get $\beta = \frac{5\sqrt{85}-19}{98} \approx 0.28$, so according to the monotonicity of $P(\delta)$, we have $\delta_2 > \delta_3$ when $0 < \beta < \frac{5\sqrt{85}-19}{98} \approx 0.28$, otherwise $\delta_2 \leq \delta_3$. □

Proof of Proposition 9. $w^{\mathrm{NF}^*} - w^{\mathrm{NB}^*} = -\frac{\theta(8-11\theta+4\theta^2)}{16k(2-\theta)^2(1-\theta)^2} < 0$, $g^{\mathrm{NF}^*} - g^{\mathrm{NB}^*} = -\frac{\theta}{8k(2-3\theta+\theta^2)} < 0$, $q_R^{\mathrm{NF}^*} - q_R^{\mathrm{NB}^*} = \frac{\theta}{4(2-\theta)} > 0$, $\pi_R^{\mathrm{NF}^*} - \pi_R^{\mathrm{NB}^*} = \frac{\theta(4-6\theta+5\theta^2-2\theta^3)}{128k(2-\theta)^3(1-\theta)^2} > 0$, $\pi_S^{\mathrm{NF}^*} - \pi_S^{\mathrm{NB}^*} = -\frac{\theta^2}{128k(2-\theta)^2(1-\theta)} < 0$. □

Proof of Proposition 10. $w^{\mathrm{EF}^*} - w^{\mathrm{EB}^*} = -\frac{I}{8k(1-\theta)^2}$, where $I = \frac{(1-\beta)(1-\theta)^3(5-4\beta-\theta)[5+\beta(2\beta-7)-6\theta+5\beta\theta+\theta^2]}{[5-\beta(4+\beta)-6\theta+4\beta\theta+\theta^2]^2} + \frac{[(3-\theta+\beta\theta)^2-4(1+\beta)][(3-\theta+\beta\theta)^2-2(2+\beta)]}{(5+\beta-\theta+\beta\theta)^2}$. By the conditions of $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $I > 0$, *i.e.*, $w^{\mathrm{EF}^*} < w^{\mathrm{EB}^*}$;

$$g^{\mathrm{EF}^*} - g^{\mathrm{EB}^*} = \frac{\beta^3\theta(2-2\theta+\beta\theta)}{4k(1-\theta)(5+\beta-\theta+\beta\theta)[5-\beta(4+\beta)-6\theta+4\beta\theta+\theta^2]} > 0;$$

$$q_S^{\mathrm{EF}^*} - q_S^{\mathrm{EB}^*} = -\frac{\beta\theta(10+\beta\theta-\beta-\beta^2-2\theta)}{2(5+\beta-\theta+\beta\theta)[5-\beta(4+\beta)-6\theta+4\beta\theta+\theta^2]} < 0;$$

$$q_R^{\mathrm{EF}^*} - q_R^{\mathrm{EB}^*} = \frac{\beta\theta(5-2\beta-\beta^2-\theta)}{(5+\beta-\theta+\beta\theta)[5-\beta(4+\beta)-6\theta+4\beta\theta+\theta^2]} > 0;$$

$$\pi_R^{\mathrm{EF}^*} - \pi_R^{\mathrm{EB}^*} = \frac{(1-\beta)^3\beta^2(1-\theta)^2(5-4\beta-\theta)}{4k[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^3} - \frac{[(3-\theta+\beta\theta)^2-4(1+\beta)]Z}{32k(1-\theta)^2(5+\beta-\theta+\beta\theta)^3},$$

where $Z = (5-\theta)^2(1-\theta)\theta - 3\beta(5-\theta)(1-\theta)^2\theta - \beta^3\theta^2(4-\theta-\theta^2) + \beta^2[8-3\theta(2-\theta)(4+\theta-\theta^2)]$. By the conditions of $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $\pi_R^{\mathrm{EF}^*} - \pi_R^{\mathrm{EB}^*} < 0$, *i.e.*, $\pi_R^{\mathrm{EF}^*} < \pi_R^{\mathrm{EB}^*}$;

$\pi_S^{\mathrm{EF}^*} - \pi_S^{\mathrm{EB}^*} = \frac{J}{32k(1-\theta)}$, where $J = \frac{(1-\beta)^2(1-\theta)^3(5-4\beta-\theta)^2}{[5-\beta^2-4\beta(1-\theta)-6\theta+\theta^2]^2} - \frac{[(3-\theta+\beta\theta)^2-4(1+\beta)]^2}{(5+\beta-\theta+\beta\theta)^2}$. By the conditions of $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $J > 0$, *i.e.*, $\pi_S^{\mathrm{EF}^*} > \pi_S^{\mathrm{EB}^*}$. □

Proof of Lemma 5. Using the same process developed in the proof of Lemma 2, we can easily show that Lemma 5 holds. □

Proof of Proposition 11. (1) $\breve{q}_R^{\mathrm{EF}^*} - \breve{q}_R^{\mathrm{NF}^*} = \frac{(-1+\beta)(2+(-2+\beta)\gamma)}{\beta^2\gamma+4(-2+\theta)-4\beta(-2+\gamma+\theta)+\gamma(3+2\theta-\theta^2)} - \frac{1}{4-2\theta}$, $\breve{\pi}_R^{\mathrm{EF}^*} - \breve{\pi}_R^{\mathrm{NF}^*} = \frac{(\beta-1)^3(2-2\gamma+\beta\gamma)^2(4+4\gamma(\beta\theta-\theta-\beta)+\gamma^2(1-\theta)^2)(1-\theta)}{4k((\beta-1)(8+(-3+\beta)\gamma)+2(2-2\beta+\gamma)\theta-\gamma\theta^2)^3} - \frac{1-\theta}{16k(2-\theta)^3}$. By the conditions of $0 < \gamma < 1$, $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $\breve{q}_R^{\mathrm{EF}^*} - \breve{q}_R^{\mathrm{NF}^*} < 0$ and $\breve{\pi}_R^{\mathrm{EF}^*} - \breve{\pi}_R^{\mathrm{NF}^*} < 0$, *i.e.*, $\breve{q}_R^{\mathrm{EF}^*} < \breve{q}_R^{\mathrm{NF}^*}$ and $\breve{\pi}_R^{\mathrm{EF}^*} < \breve{\pi}_R^{\mathrm{NF}^*}$.

(2) $\breve{w}^{\mathrm{EF}^*} - \breve{w}^{\mathrm{NF}^*} = \frac{(1-\beta)((2\beta-\gamma+\gamma\theta)^2+4(1-\beta^2-\gamma\theta))(8(1-\beta)(1-\theta)+2\gamma((\beta-\theta)^2-(2-\beta-\theta))+\gamma^2(1-\beta+\theta)(1-\theta))}{8k((-1+\beta)(8+(-3+\beta)\gamma)+2(2-2\beta+\gamma)\theta-\gamma\theta^2)^2} - \frac{1-\theta}{4k(2-\theta)^2}$. By the conditions of $0 < \gamma < 1$, $0 < \theta \leq 50\%$ and $0 < \beta \leq 45\%$, we have $\breve{w}^{\mathrm{EF}^*} > \breve{w}^{\mathrm{NF}^*}$ when $0 < \beta < \beta_4$, otherwise $\breve{w}^{\mathrm{EF}^*} \leq \breve{w}^{\mathrm{NF}^*}$, where β_4 is the root of function $K(\beta) = 2\beta^4\gamma(-1+\theta)(-5+2\theta)(-3+2\theta) + (-1+\theta)(\gamma^3(1+\theta)(2-3\theta+\theta^2)^2 - 8(-2+\theta)(2+\theta(-4+3\theta)) - 2\gamma^2(-2+\theta)^2(2+\theta(-1+\theta(2+\theta))) + 2\gamma(-1+\theta)(-15+\theta(13+\theta(-19+7\theta)))) + 2\beta^3(3\gamma^2(2-3\theta+\theta^2)^2 + 4(-2+\theta)(-3+2\theta)(4+\theta(-5+2\theta)) - 4\gamma(2+\theta(-6+\theta(12+\theta(-9+2\theta))))) + \beta^2(\gamma^3(-2+\theta)^2(-1+\theta)^3 - 4\gamma^2(-2+\theta)^2(-1+\theta)(-2+\theta+2\theta^2) - 16(-2+\theta)(-13+3\theta(9+\theta(-7+2\theta))) + 4\gamma(23+\theta(-41+2\theta(14+\theta(2+\theta(-7+2\theta)))))) - \beta(\gamma^3(-2+\theta)^2(-1+\theta)^3(2+\theta) - 8(-2+\theta)(-16+\theta(37+11(-3+\theta)\theta)) - 2\gamma^2(-2+\theta)^2(-1+\theta)(1+\theta(-2+\theta(6+\theta))) + 4\gamma(4+\theta(8+\theta(-38+\theta(57+\theta(-37+8\theta))))))$.

$\breve{g}^{\mathrm{EF}^*} - \breve{g}^{\mathrm{NF}^*} = \frac{(-1+\beta)(4+\gamma(4\beta(-1+\theta)+\gamma(-1+\theta)^2-4\theta))}{4k((-1+\beta)(8+(-3+\beta)\gamma)+2(2-2\beta+\gamma)\theta-\gamma\theta^2)} - \frac{1}{4k(2-\theta)}$, $\breve{\pi}_S^{\mathrm{EF}^*} - \breve{\pi}_S^{\mathrm{NF}^*} = \frac{(-1+\beta)^2(4+\gamma(4\beta(-1+\theta)+\gamma(-1+\theta)^2-4\theta))^2}{32k((-1+\beta)(8+(-3+\beta)\gamma)+2(2-2\beta+\gamma)\theta-\gamma\theta^2)^2} - \frac{1}{32k(2-\theta)^2}$. By the conditions of $0 < \gamma < 1$, $0 < \theta \leq 50\%$ and

$0 < \beta \le 45\%$, we have $\breve{g}^{\mathrm{EF}^*} < \breve{g}^{\mathrm{NF}^*}$ and $\breve{\pi}_S^{\mathrm{EF}^*} < \breve{\pi}_S^{\mathrm{NF}^*}$ when $0 < \theta < \theta_3$, $0 < \gamma < \frac{3(7+2\theta)(11-14\theta)}{220(2-\theta)(1-\theta)^2}$ and $\beta_5 < \beta \le 0.45$ or $\theta_3 < \theta \le 0.5$, $0 < \gamma < 1$ and $\beta_5 < \beta \le 0.45$, otherwise $\breve{g}^{\mathrm{EF}^*} \ge \breve{g}^{\mathrm{NF}^*}$ and $\breve{\pi}_S^{\mathrm{EF}^*} \ge \breve{\pi}_S^{\mathrm{NF}^*}$, where $\theta_3 \approx 0.40$ is the root of function $H(\theta) = -209 + 872\theta - 964\theta^2 + 220\theta^3$, $\beta_5 = \frac{8+\gamma}{8} + \frac{\gamma}{16(3-2\theta)^2} - \frac{\gamma\theta}{8} - \frac{16+\gamma}{16(3-2\theta)} - \frac{(1-\theta)\sqrt{(6-4\theta-2\gamma+\gamma\theta)^2-\theta\gamma^2(2-\theta)^3}}{2(3-2\theta)^2}$. □

Proof of Proposition 12. (1) $\breve{q}_R^{\mathrm{EB}^*} - \breve{q}_R^{\mathrm{NB}^*} = \frac{2+(-2+\beta)\gamma}{8+\gamma(-3+\beta-\delta+\beta\delta)} - \frac{1}{4}$. By the conditions of $0 < \gamma < 1$, $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $\breve{q}_R^{\mathrm{EB}^*} < \breve{q}_R^{\mathrm{NB}^*}$;

$\breve{\pi}_R^{\mathrm{EB}^*} - \breve{\pi}_R^{\mathrm{NB}^*} = \frac{(4+\gamma(\gamma-4\beta-2(1-\beta)(4-\gamma)\delta+(1-\beta)^2\gamma\delta^2))\breve{Z}}{32k(1-\delta)^2(8+\gamma(-3+\beta+(-1+\beta)\delta))^3} - \frac{1-2\delta}{128k(-1+\delta)^2}$, where $\breve{Z} = 8(2 + (-2+\beta)\gamma)^2 + (-64 + \gamma(196 - 124\beta - 4(-3+\beta)(-10+7\beta)\gamma + (13 + \beta(-15+4\beta))\gamma^2))\delta + (1-\beta)\gamma(-52 + \gamma(8 + 9\gamma + 2\beta(6 + (-7+2\beta)\gamma)))\delta^2 + (-1+\beta)^2\gamma^2(16 + (-5+\beta)\gamma)\delta^3 + (-1+\beta)^3\gamma^3\delta^4$. Let $V = \breve{\pi}_R^{\mathrm{EB}^*} - \breve{\pi}_R^{\mathrm{NB}^*}$, we have $\frac{dV}{d\delta} > 0$ by the conditions of $0 < \gamma < 1$, $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, *i.e.*, V increases monotonically with respect to δ. Since $0 < \gamma < 1$, $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $V(\delta = 0) = \frac{32(2+(-2+\beta)\gamma)^2(4-4\beta\gamma+\gamma^2)-(8+(-3+\beta)\gamma)^3}{128k(8+(-3+\beta)\gamma)^3} < 0$, $V(\delta = \frac{1}{2}) = \frac{\gamma(16-16\gamma+(3-\beta)^2\gamma^2)(336-272\beta-16(24-27\beta+7\beta^2)\gamma+(3-\beta)(43-48\beta+13\beta^2)\gamma^2)}{64k(16-7\gamma+3\beta\gamma)^3} > 0$. Then according to the monotonicity of the function V, there exists a certain value δ_4 such that $V < 0$ (*i.e.*, $\breve{\pi}_R^{\mathrm{EB}^*} < \breve{\pi}_R^{\mathrm{NB}^*}$) when $0 < \delta < \delta_4$ and $V \ge 0$ (*i.e*, $\breve{\pi}_R^{\mathrm{EB}^*} \ge \breve{\pi}_R^{\mathrm{NB}^*}$) when $\delta_4 \le \delta \le 0.5$, where δ_4 is the root of function V.

(2) $\breve{w}^{\mathrm{EB}^*} - \breve{w}^{\mathrm{NB}^*} = \frac{X}{16k(1-\delta)^2(8+\gamma(-3+\beta+(-1+\beta)\delta))^2}$, where $X = 2(8 - 2(2+\beta)\gamma + 8(-1+\beta)\gamma\delta + \gamma^2(1+\delta-\beta\delta)^2)(4 + \gamma(-4\beta + \gamma - 2(-1+\beta)(-4+\gamma)\delta + (-1+\beta)^2\gamma\delta^2)) - (8 + \gamma(-3+\beta+(-1+\beta)\delta))^2$. By the conditions of $0 < \gamma < 1$, $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $X > 0$ (*i.e.*, $\breve{w}^{\mathrm{EB}^*} > \breve{w}^{\mathrm{NB}^*}$) when $0 < \delta < \delta_5$ and $0 < \beta < \beta_6$, otherwise $X \le 0$ (*i.e.*, $\breve{w}^{\mathrm{EB}^*} \le \breve{w}^{\mathrm{NB}^*}$), where δ_5 is the root of function $x(\delta) = 16 + 15\gamma - 8\gamma^2 + 2\gamma^3 + (-176 + 106\gamma - 48\gamma^2 + 8\gamma^3)\delta + (151\gamma - 72\gamma^2 + 12\gamma^3)\delta^2 + (-32\gamma^2 + 8\gamma^3)\delta^3 + 2\gamma^3\delta^4$, β_6 is the root of function $y(\beta) = 16 - 96\beta + 15\gamma + 38\beta\gamma + 15\beta^2\gamma - 8\gamma^2 - 12\beta\gamma^2 + 2\gamma^3 + 2(-1+\beta)(88 + \gamma(-53 - 4(-6+\gamma)\gamma + \beta(-49+12\gamma)))\delta + (-1+\beta)^2\gamma(151 + 12\gamma(-6 - \beta + \gamma))\delta^2 - 8(-1+\beta)^3(-4+\gamma)\gamma^2\delta^3 + 2(-1+\beta)^4\gamma^3\delta^4$.

$\breve{g}^{\mathrm{EB}^*} - \breve{g}^{\mathrm{NB}^*} = \frac{4-4\beta\gamma-8(1-\beta)\gamma\delta+\gamma^2(1+\delta-\beta\delta)^2}{4k(1-\delta)(8+\gamma(-3+\beta+(-1+\beta)\delta))} - \frac{1}{8k-8k\delta}$, $\breve{\pi}_S^{\mathrm{EB}^*} - \breve{\pi}_S^{\mathrm{NB}^*} = \frac{(4-4\beta\gamma-8(1-\beta)\gamma\delta+\gamma^2(1+\delta-\beta\delta)^2)^2}{32k(1-\delta)(8+\gamma(-3+\beta+(-1+\beta)\delta))^2} - \frac{1}{128k-128k\delta}$. By the conditions of $0 < \gamma < 1$, $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $\breve{g}^{\mathrm{EB}^*} > \breve{g}^{\mathrm{NB}^*}$ and $\breve{\pi}_S^{\mathrm{EB}^*} > \breve{\pi}_S^{\mathrm{NB}^*}$ when $0 < \delta < \delta_6$, $0 < \gamma < \frac{21}{40}$ and $0 < \beta < \frac{3+2\gamma}{9}$ or $0 < \delta < \delta_6$, $\frac{21}{40} < \gamma < 1$ and $0 < \beta \le 0.45$, otherwise $\breve{g}^{\mathrm{EB}^*} \le \breve{g}^{\mathrm{NB}^*}$ and $\breve{\pi}_S^{\mathrm{EB}^*} \le \breve{\pi}_S^{\mathrm{NB}^*}$, where $\delta_6 = \frac{15-4\gamma-3\sqrt{25-16\gamma+8\beta\gamma}}{4(1-\beta)\gamma}$.

□

Proof of Proposition 13. (1) $\hat{q}_R^{\mathrm{EF}^*} - \hat{q}_R^{\mathrm{NF}^*} = \frac{(1-\beta)\beta}{(3+\beta-3\theta)(1-\beta-\theta)} - \frac{1}{4-2\theta}$. By the conditions of $0 < \theta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{q}_R^{\mathrm{EF}^*} > \hat{q}_R^{\mathrm{NF}^*}$ when $\frac{4-\sqrt{10}}{4} \approx 0.21 < \beta \le 0.45$ and $\theta_4 < \theta \le 0.5$, otherwise $\hat{q}_R^{\mathrm{EF}^*} \le \hat{q}_R^{\mathrm{NF}^*}$, where $\theta_4 = \frac{(3-2\beta+\beta^2)-\sqrt{(3-\beta)(2-\beta)\beta(1+\beta)}}{3}$.

$\hat{\pi}_R^{\mathrm{EF}^*} - \hat{\pi}_R^{\mathrm{NF}^*} = \frac{(1-\beta)^2\beta^2(1-\theta)^2(3-2\beta-3\theta)}{8k(3+\beta-3\theta)^3(1-\beta-\theta)^2} - \frac{1-\theta}{16k(2-\theta)^3}$. By the conditions of $0 < \theta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{\pi}_R^{\mathrm{EF}^*} > \hat{\pi}_R^{\mathrm{NF}^*}$ when $\beta_7 < \beta \le 0.45$ and $\theta_5 < \theta \le 0.5$, otherwise $\hat{\pi}_R^{\mathrm{EF}^*} \le \hat{\pi}_R^{\mathrm{NF}^*}$, where $\beta_7 \approx 0.35$ is the root of function $a(\beta) = 27 - 54\beta - 234\beta^2 + 620\beta^3 - 482\beta^4 + 248\beta^5$, θ_5 is the root of function $b(\theta) = -27\beta(-1+\theta)^4 - 27(-1+\theta)^5 + 6\beta^2(-1+\theta)^2(-11 + \theta(15 + (-6+\theta)\theta)) - 2\beta^3(-3+\theta)(-1+\theta)(-23 + 2\theta(21 + \theta(-13+3\theta))) + \beta^5(33 + 4\theta(-20 + \theta(18 + (-7+\theta)\theta))) + \beta^4(-1+\theta)(105 + 2\theta(-108 + \theta(78 + \theta(-25+3\theta))))$.

(2) $\hat{w}^{\mathrm{EF}^*} - \hat{w}^{\mathrm{NF}^*} = \frac{(1-\beta)(1-\theta)(3-3\theta-\beta)(3-3\theta-2\beta)}{8k(3+\beta-3\theta)^2(1-\beta-\theta)} - \frac{1-\theta}{4k(2-\theta)^2}$. By the conditions of $0 < \theta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{w}^{\mathrm{EF}^*} < \hat{w}^{\mathrm{NF}^*}$ when $\beta_8 < \beta \le 0.45$ and $0 < \theta < \theta_6$, otherwise $\hat{w}^{\mathrm{EF}^*} \ge \hat{w}^{\mathrm{NF}^*}$, where $\theta_6 \approx 0.33$ is the root of function $d(\theta) = -583 + 1824\theta + 949\theta^2 - 4210\theta^3 + 2200\theta^4$, β_8 is the root of function $e(\beta) = 2\beta^3(3 - 4\theta + \theta^2) - 9(1-\theta)^2(2 - 2\theta + \theta^2) + \beta^2(-54 + 90\theta - 47\theta^2 + 9\theta^3) + 3\beta(22 - 56\theta + 52\theta^2 - 21\theta^3 + 3\theta^4)$.

$\hat{g}^{\mathrm{EF}^*} - \hat{g}^{\mathrm{NF}^*} = \frac{(1-\beta)(1-\theta)(3-2\beta-3\theta)}{4k(3+\beta-3\theta)(1-\beta-\theta)} - \frac{1}{8k-4k\theta}$, $\hat{\pi}_S^{\mathrm{EF}^*} - \hat{\pi}_S^{\mathrm{NF}^*} = \frac{(1-\beta)^2(1-\theta)^2(3-2\beta-3\theta)^2}{32k(3+\beta-3\theta)^2(1-\beta-\theta)^2} - \frac{1}{32k(2-\theta)^2}$. By the conditions of $0 < \theta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{g}^{\mathrm{EF}^*} > \hat{g}^{\mathrm{NF}^*}$ and $\hat{\pi}_S^{\mathrm{EF}^*} > \hat{\pi}_S^{\mathrm{NF}^*}$.

□

Proof of Proposition 14. (1) $\hat{q}_R^{\mathrm{EB}^*} - \hat{q}_R^{\mathrm{NB}^*} = \frac{\beta}{3+\beta-\delta+\beta\delta} - \frac{1}{4}$. By the conditions of $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{q}_R^{\mathrm{EB}^*} < \hat{q}_R^{\mathrm{NB}^*}$.

$\hat{\pi}_R^{\mathrm{EB}^*} - \hat{\pi}_R^{\mathrm{NB}^*} = \frac{[(2-\delta+\beta\delta)^2-(1+2\beta)][4\beta^2+(9-3\beta+14\beta^2)\delta-(1-\beta)(15+2\beta-4\beta^2)\delta^2+\delta^3(1-\beta)^2(7+\beta-\delta+\beta\delta)]}{32k(1-\delta)^2(3+\beta-\delta+\beta\delta)^3} - \frac{1-2\delta}{128k(1-\delta)^2}$. By the conditions of $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{\pi}_R^{\mathrm{EB}^*} < \hat{\pi}_R^{\mathrm{NB}^*}$ when $0 < \delta < \delta_7$, otherwise $\hat{\pi}_R^{\mathrm{EB}^*} \ge \hat{\pi}_R^{\mathrm{NB}^*}$, where

δ_7 is the root of function $f(\delta) = 39\beta^2 - 27 - 27\beta - 33\beta^3 + (189 - 63\beta - 205\beta^2 + 175\beta^3)\delta - (387 - 501\beta - 187\beta^2 + 349\beta^3 - 48\beta^4)\delta^2 + (1-\beta)^2(379 - 19\beta - 128\beta^2)\delta^3 - 2(1-\beta)^3(93 + 8\beta - 8\beta^2)\delta^4 + 4(1-\beta)^4(11+\beta)\delta^5 - 4(1-\beta)^5\delta^6$.

(2) $\hat{w}^{\mathrm{EB}^*} - \hat{w}^{\mathrm{NB}^*} = \frac{[(2-\delta+\beta\delta)^2-(1+2\beta)][(2-\delta+\beta\delta)^2-(1+\beta)]}{8k(1-\delta)^2(3+\beta-\delta+\beta\delta)^2} - \frac{1}{16k(1-\delta)^2}$. By the conditions of $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{w}^{\mathrm{EB}^*} > \hat{w}^{\mathrm{NB}^*}$ when $0 < \beta < 4 - \sqrt{13} \approx 0.39$ and $0 < \delta < \delta_8$, otherwise $\hat{w}^{\mathrm{EB}^*} \le \hat{w}^{\mathrm{NB}^*}$, where δ_8 is the root of function $h(\delta) = 9 - 24\beta + 3\beta^2 - (42 - 68\beta + 26\beta^2)\delta + (1-\beta)^2(43 - 6\beta)\delta^2 - 16(1-\beta)^3\delta^3 + 2(1-\beta)^4\delta^4$.

$\hat{g}^{\mathrm{EB}^*} - \hat{g}^{\mathrm{NB}^*} = \frac{(2-\delta+\beta\delta)^2-(1+2\beta)}{4k(1-\delta)(3+\beta-\delta+\beta\delta)} - \frac{1}{8k-8k\delta}$, $\hat{\pi}_S^{\mathrm{EB}^*} - \hat{\pi}_S^{\mathrm{NB}^*} = \frac{[(2-\delta+\beta\delta)^2-(1+2\beta)]^2}{32k(1-\delta)(3+\beta-\delta+\beta\delta)^2} - \frac{1}{128k-128k\delta}$. By the conditions of $0 < \delta \le 50\%$ and $0 < \beta \le 45\%$, we have $\hat{g}^{\mathrm{EB}^*} > \hat{g}^{\mathrm{NB}^*}$ and $\hat{\pi}_S^{\mathrm{EB}^*} > \hat{\pi}_S^{\mathrm{NB}^*}$ when $0 < \delta < \delta_9$, otherwise $\hat{g}^{\mathrm{EB}^*} \le \hat{g}^{\mathrm{NB}^*}$ and $\hat{\pi}_S^{\mathrm{EB}^*} \le \hat{\pi}_S^{\mathrm{NB}^*}$, where $\delta_9 = \frac{7-\sqrt{25+40\beta}}{4(1-\beta)}$.

□