REVUE FRANÇAISE D'AUTOMATIQUE, D'INFORMATIQUE ET DE RECHERCHE OPÉRATIONNELLE. RECHERCHE OPÉRATIONNELLE

A. K. GOVIL

Stochastic behaviour of a complex system with bulk failure and priority repairs

Revue française d'automatique, d'informatique et de recherche opérationnelle. Recherche opérationnelle, tome 5, nº V1 (1971), p. 51-56.

<http://www.numdam.org/item?id=RO_1971__5_1_51_0>

R.I.R.O.
(5e année, V-1, 1971, p. 51-56)

STOCHASTIC BEHAVIOUR OF A COMPLEX SYSTEM WITH BULK FAILURE AND PRIORITY REPAIRS

By A. K. Govil (1)

Abstract. — *Behaviour of a complex system consisting of two classes of components L_1 and L_2 is considered, where failure in class L_1 results in complete breakdown of the system and that in class L_2 brings the system to a reduced efficiency state. In class L_1 failure follows exponential distribution but unlike the earlier studies* [1, 2, 4], *in class L_2 bulk failure is assumed. In both the classes, repair follows general distribution. Use of Laplace Transforms and Supplementary variable technique have been made to obtain the solution. Asymptotic behaviour of the system has also been examined.*

INTRODUCTION

In many operating systems, it is observed that there are components or parts which when fail, bring the system to a reduced efficiency state. Quite a good number of studies evaluating the behaviour of such systems have been carried out [1, 2, 3, 4]. In all such studies, it has been assumed that in a small interval of time, only one component fails. But in many realistic situations like Telephone exchanges and electronic computers, it is observed that components do fail in bulk.

Keeping this in view, a complex system comprising of two classes of components (denoted hereafter as class L_1 and L_2) has been considered. Class L_1 consists of N components connecting in series where failure of a component brings about the complete breakdown of the system wherein failure and repair of components of this class follow exponential and general distributions, respectively. Class L_2 consists of several components which fail in bulk and the failure form a Poisson process with an average failure rate λ'. The failure of components in class L_2 causes the system to work in reduced efficiency state. For repair purposes, it has been furthur assumed that the failed components of class L_1 join priority class and those of class L_2 join the non-priority class. Behaviour of such a system has been evaluated under | Head of Line |

(1) Directorate of Scientific Evaluation, Ministry of Defence, New Delhi (India).

priority discipline [5]. Use of Laplace Transforms and supplementary variable technique has been made to obtain the solution. Asymptotic behaviour of such a system has also been examined.

NOTATIONS

$\lambda_i \equiv$ constant failure rate of the ith component of class L_1,

$\lambda' \equiv$ average failure rate of components of class L_2,

$S_i(x) \equiv$ the general probability density function of the ith component of class L_1,

$S'(y) \equiv$ the general probability density function of a component of class L_2,

$\mu_i(x)\Delta \equiv$ the first order conditional probability that a failed component of class L_1 will be repaired between $(x, x + \Delta)$, subject to the condition that the repair was not completed upto time x,

$\mu'(y)\Delta \equiv$ the first order conditional probability that a failed component of class L_2 will be repaired between $(y, y + \Delta)$ subject to the condition that repair was not completed upto time y,

$\lambda \equiv \sum_{i=1}^{N} \lambda_i$.

It may be noted that

$$S_i(x) = \mu_i(x) \exp\left[-\int_0^x \mu_i(x)\, dx\right],$$

and

$$S'(y) = \mu'(y) \exp\left[-\int_0^y \mu'(y)\, dy\right].$$

Define,

$P_{0,0}(t) \equiv$ the probability that at time t, all the components of class L_1 and L_2 are working in normal efficiency,

$P_{0,m}(y, t)\Delta \equiv$ the probability that at time t, m components of class L_2 are in the failed state and the component is being repaired with elapsed repair time $(y, y + \Delta)$, $m > 0$,

$q_{li,m}(x, t)\Delta \equiv$ the probability that at time t, m components of class L_2 are already in the failed state and the system is in the failed state due to the failure of ith component of class L_1, is being repaired and the elapsed repair time lies in the interval $(x, x + \Delta)$, $m \geqslant 0$,

$p(r) \equiv$ the probability that non-priority bulk is comprised of r components. Obviously $p(r) = 0$ for $r < 1$,

$K(z) \equiv$ the probability generating function, given by $\sum_{m=1}^{\infty} z^m p(m)$.

Difference-Differential Equations Governing the Behaviour of the System

$$\left(\frac{\partial}{\partial t} + \lambda + \lambda'\right)P_{0,0}(t) = \sum_{i=1}^{N} \int_0^\infty q_{li,0}(x, t)\mu_i(x)\,\mathrm{d}x + \int_0^\infty P_{0,1}(y, t)\mu'(y)\,\mathrm{d}y, \quad (1)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_i(x)\right] q_{li,m}(x, t) = 0, \quad (2)$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \lambda + \lambda' + \mu'(y)\right] P_{0,m}(y, t) = \sum_{r=1}^{m-1} P_{0,r}(y, t)\lambda' p(m - r). \quad (3)$$

The above equations are to be solved under the following boundary conditions :

$$q_{li,m}(0, t) = \lambda_i P_{0,m}(y, t), \quad (4)$$

$$P_{0,m}(0, t) = \sum_{i=1}^{N} \int_0^\infty q_{li,m}(x, t)\mu_i(x)\,\mathrm{d}x + \int_0^\infty P_{0,m+1}(y, t)\mu'(y)\,\mathrm{d}y + \lambda' P_{0,0}(t)p(m). \quad (5)$$

It has been assumed that initially all the components are working in normal efficiency i.e. $P_{0.0}(0) = 1$.

Taking Laplace Transforms of equations (1) through (5) and using initial condition, we get

$$[s + \lambda + \lambda']\bar{P}_{0,0}(s) = 1 + \sum_{i=1}^{N} \int_0^\infty \bar{q}_{li,0}(x, s)\mu_i(x)\,\mathrm{d}x + \int_0^\infty \bar{P}_{0,1}(y, s)\mu'(y)\,\mathrm{d}y, \quad (6)$$

$$\left[\frac{\partial}{\partial x} + s + \mu_i(x)\right] \bar{q}_{li,m}(x, s) = 0, \quad (7)$$

$$\left[\frac{\partial}{\partial y} + s + \lambda + \lambda' + \mu'(y)\right] \bar{P}_{0,m}(y, s) = \sum_{r=1}^{m-1} \bar{P}_{0,r}(y, s)\lambda' p(m - r), \quad (8)$$

$$\bar{q}_{li,m}(0, s) = \lambda_i \bar{P}_{0,m}(y, s), \quad (9)$$

$$\bar{P}_{0,m}(0, s) = \sum_{i=1}^{N} \int_0^\infty \bar{P}_{li,m}(x, s)\mu_i(x)\,\mathrm{d}x + \int_0^\infty \bar{P}_{0,m+1}(y, s)\mu'(y)\,\mathrm{d}y + \lambda' \bar{P}_{0,0}(s)p(m). \quad (10)$$

The following generating functions are introduced to solve the equations :

$$\bar{F}_{li}(x, s, \alpha) = \sum_{m=0}^{\infty} \alpha^m \bar{q}_{li,m}(x, s), \quad (11)$$

$$\bar{H}_0(y, s, \alpha) = \sum_{m=1}^{\infty} \alpha^m \bar{\rho}_{0,m}(y, s). \quad (12)$$

Using (11) and (12) in (7) through (10), we get

$$\left[\frac{\partial}{\partial x} + s + \mu_i(x)\right]\bar{F}_{li}(x, s, \alpha) = 0, \tag{13}$$

$$\left[\frac{\partial}{\partial y} + s + \lambda + \lambda'\{1 - K(\alpha)\} + \mu'(y)\right]\bar{H}_0(y, s, \alpha) = 0. \tag{14}$$

$$\bar{F}_{li}(0, s, \alpha) = \lambda_i \bar{P}_{0,0}(s), \tag{15}$$

$$\bar{H}_0(0, s, \alpha) = \sum_{i=1}^{N}\int_0^\infty \bar{F}_{li}(x, s, \alpha)\mu_i(x)\,dx - \sum_{i=1}^{N}\int_0^\infty \bar{q}_{li,0}(x, s)\mu_i(x)\,dx$$
$$+ \frac{1}{\alpha}\int_0^\infty \bar{H}_0(y, s, \alpha)\mu'(y)\,dy - \int_0^\infty \bar{P}_{0,1}(y, s)\mu'(y)\,dy$$
$$+ \lambda'\bar{P}_{0,0}(s)K(\alpha). \tag{16}$$

The solution of the equations (13) and (14), is given by

$$\bar{F}_{li}(x, s, \alpha) = \lambda_i\,\bar{P}_{0,0}(s)\exp\left[-sx - \int_0^x \mu_i(x)\,dx\right], \tag{17}$$

and

$$H_0(y, s, \alpha) = \bar{H}_0(0, s, \alpha)\left[\exp\left\{-\int_0^y \mu'(y)\,dy\right.\right.$$
$$\left.\left. - y(s + \lambda + \lambda'\{1 - k(\alpha)\})\right\}\right] \tag{18}$$

Making use of (6), (17) and (18) in relation (16), we obtain

$$\bar{H}_0(0, s, \alpha) = \frac{1 - \left[s + \sum_{i=1}^{N}\lambda_i\{1 - \bar{S}_i(s)\} + \lambda'\{1 - K(\alpha)\}\right]}{1 - \frac{1}{\alpha}\bar{S}'[s + \lambda + \lambda'\{1 - K(\alpha)\}]} \tag{19}$$

Let α_s be the root of the equation, then

$$\alpha_s = \bar{S}'[s + \lambda + \lambda'\{1 - K(\alpha_s)\}]$$

which lies inside the unit circle $|\alpha| = 1$.

Hence, $\bar{H}_0(0, s, \alpha)$ is a regular function in α for $|\alpha| \leqslant 1$ and $Re(s) > 0$ and the denominator of (19) vanishes for $\alpha = \alpha_s$ and $\lambda = 0$, the numerator of (19) vanishes for $\alpha = \alpha_s$, thus

$$\bar{P}_{0,0}(s) = \frac{1}{\left[s + \sum_{i=1}^{N}\lambda_i\{1 - \bar{S}_i(s)\} + \lambda'\{1 - K(\alpha)\}\right]} \tag{20}$$

Relations (17) and (18), give

$$\bar{F}_{l_i}(s, \alpha) = \frac{\left[\sum_{i=1}^{N} \lambda_i \{ 1 - \bar{S}_i(s) \}\right]}{s\left[s + \sum_{i=1}^{N} \lambda_i \{ 1 - \bar{S}_i(s) \} + \lambda' \{ 1 - K(\alpha) \}\right]} \tag{21}$$

and

$$\bar{H}_0(s, \alpha) = \frac{\bar{P}_{0,0}(s)\left[1 - S'[s + \lambda + \lambda' \{ 1 - K(\alpha) \}]\right]\left[1 - \left\{s + \sum_{i=1}^{N} \lambda_i(1 - \bar{S}_i(s)) + \lambda'(1 - K(\alpha))\right\}\right]}{[s + \lambda + \lambda' \{ 1 - K(\alpha) \}]\left[1 - \frac{1}{\alpha}\bar{S}' \{ s + \lambda + \lambda'(1 - K(\alpha)) \}\right]} \tag{22}$$

Where $\bar{P}_{0,0}(s)$ is given by (20).

Asymptotic Behaviour of the System

Using Abel's Corollary in relation (20), we get

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\bar{f}(s)$$

(if the limit on the right exists)

$$P_{0,0} = \frac{1}{\left[1 + \sum_{i=1}^{N} \lambda_i m_i + \lambda' m\right]}$$

where M is the mean service time of the priority components and m is the mean number of components in bulk.

ACKNOWLEDGEMENTS

The author is extremely grateful to Dr. S. S. Srivastava, Director, Scientific Evaluation, and to Dr. R. C. Garg, Senior Scientific Officer, Gde I, for their keen interest and guidance in the preparation of this paper.

REFERENCES

[1] A. Cobham, « Priority Assignment in waiting line problem », *Opns. Res.*, 1954, 2, 70-76.

[2] R. C. Garg, « Behaviour of a system with regard to Dependability », *Recherche Opérationnelle*, France, 1963, 26, 83-87.

[3] R. C. Garg, « Dependability of a complex system having two types of components », *IEEE Transactions on Reliability*, vol. R-12, n° 3.

[4] A. K. GOVIL, « Dependability of a complex system with priority repair resume », *ZAMM*, Germany, vol. 50, n° 8, 1970.

[5] A. K. GOVIL, « Stochastic behaviour of a complex system under priority repair », *Computing*, Germany, 6, 1970, 200-213.

[6] C. R. HEATHCOTE, « On priority queues », *Biometrika*, 1960, 48.

[7] H. WHITE and L. S. CHRISTIE, « Queuing with preemptive priorities of breakdown », *Opns. Res.*, 1958, 6, 76-96.

[8] D. V. WIDDER, *The Laplace Transforms*, Princeton Univ. Press, Princeton, N. J., 1947.