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OPTIMUM ORDERING POLICIES WITH LEAD TIME FOR AN OPERATING UNIT (*)

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Abstract. — *We consider two ordering policies for spare units, where each spare unit can be delivered after a constant lead time. Introducing the cost structure, we discuss the optimum ordering policies minimizing the expected costs per unit of time. We further consider the repair limit policies.*

1. INTRODUCTION

Consider an operating single unit subject to failure. The failed unit is replaced immediately by a spare one if it is available, otherwise the failed unit must wait for replacement until a spare one is delivered. If the spare unit is always available for replacement immediately, i. e., the lead time for the spare units is negligible, the so-called "replacement problems" arise [3]. Many contributions to such replacement problems have been made by Barlow and Proschan [3], Scheaffer [8], Cleroux and Hanscom [4], and so on. However, if we cannot neglect the lead time for replacement, we should consider ordering policies for spare units. In such problems, Allen and D'Esopo [1, 2] considered a model in which some failed units are repaired and the others are scrapped with certain probabilities. Wiggins [9] considered the ordering policy in which the spare unit is ordered at t_0 units of time after installation of the original unit or at failure of the original unit, whichever occurs first. He obtained the optimum ordering time t_0^* minimizing the cost function given by the inventory and shortage costs.

In this paper, we consider two ordering policies for spare units; the first is a case that all the failed units are nonrepairable (i. e., replaced) and the second is a case that some of the failed units are repairable. In the first, we generalize Wiggins' model. Introducing a shortage, an inventory and an ordering cost, we derive the expected cost per unit of time in equilibrium. An optimum ordering

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time t_0^* minimizing the expected cost is given by a unique solution of the equation under suitable conditions. We further discuss a model in which the original unit can be replaced by a spare one as soon as it is delivered, even if the original one can work.

In the second, we adopt two repair limit policies [5, 6] to decide whether we should repair or scrap. If the failed unit is scrapped, the spare unit is ordered immediately. We derive the respective expected costs per unit of time in equilibrium and discuss the respective optimum repair limit policies.

Numerical examples of the optimum ordering policies in this paper are finally presented for illustration.

2. NON-REPAIRABLE UNITS

The original unit has been operating. If the unit fails then it is scrapped and is replaced immediately by a spare unit when it is available. We carry on hand at most one spare unit to serve as a replacement. Then, the following two ordering policies are considered.

(i) Wiggins' model

Wiggins considered the following spare unit policy: The original unit begins to operate at time 0. If the unit has not failed up to a fixed time t_0 $(0 \leqq t_0 \leqq \infty)$ then a spare unit is ordered at time t_0. After a constant lead time L, the spare unit is delivered and begins to operate when the original unit has failed or is put into inventory when the original unit has not failed. If the unit fails before the time t_0 then the expedited order is made immediately and the spare unit begins to operate as soon as it is delivered, after a lead time L. We assume that the failure time for each unit has an arbitrary distribution $F(t)$ with finite mean $1/\lambda$.

Introduce the following costs: If the unit fails when the spare unit is not delivered, then it can not operate until the spare unit is delivered. This incurs a constant cost k_1 per unit of time for shortage. On the other hand, if the spare unit is delivered before it is needed, then it is put into inventory. This incurs a constant cost k_2 per unit of time for inventory. Further, a cost c_1 is suffered for each expedited order of the spare unit when the original unit fails before time t_0. A cost c_2 is suffered for each ordinary order of the spare unit which is ordered at time t_0. We assume that the expedited ordering cost c_1 is greater than c_2. The assumption is plausible in practical fields.

In this paper, we consider an infinite planning horizon. Wiggins derived the optimum policy minimizing the total expected cost of the shortage and the inventory without considering the ordering cost, under the assumptions that the failure time is exponential and the inventory cost increases exponentially. For an infinite planning horizon, it is appropriate to adopt an expected cost per unit of time in equilibrium as an objective function.

Consider one cycle from the beginning of the operating unit to its replacement. Then, the expected cost of one cycle is given by the sum of the following three costs: (i) The expected shortage cost is

$$k_1\left[\int_0^{t_0} L\,dF(t)+\int_{t_0}^{t_0+L}(t_0+L-t)\,dF(t)\right]=k_1\int_{t_0}^{t_0+L}F(t)\,dt, \tag{1}$$

since the shortage cost is proportional to the shortage time. (ii) The expected inventory cost is

$$k_2\int_{t_0+L}^{\infty}(t-t_0-L)\,dF(t)=k_2\int_{t_0+L}^{\infty}\overline{F}(t)\,dt, \tag{2}$$

where $\overline{F}(t)\equiv 1-F(t)$. (iii) The expected ordering cost is

$$c_1 F(t_0)+c_2\overline{F}(t_0). \tag{3}$$

Further, the mean time of one cycle is

$$\int_0^{t_0}(L+t)\,dF(t)+\int_{t_0}^{t_0+L}(t_0+L)\,dF(t)+\int_{t_0+L}^{\infty}t\,dF(t)$$
$$=1/\lambda+\int_{t_0}^{t_0+L}F(t)\,dt. \tag{4}$$

Thus, the total expected cost per one cycle is

$$C_1(t_0)=\frac{k_1\int_{t_0}^{t_0+L}F(t)\,dt+k_2\int_{t_0+L}^{\infty}\overline{F}(t)\,dt+c_1F(t_0)+c_2\overline{F}(t_0)}{1/\lambda+\int_{t_0}^{t_0+L}F(t)\,dt}. \tag{5}$$

The expected cost and the mean time at each cycle are the same, and hence, $C_1(t_0)$ is equal to the expected cost per unit of time in equilibrium (*see* Ross [7], p. 52).

Of our interest is to obtain the optimum ordering time t_0^* minimizing the expected cost $C_1(t_0)$ in (5) under the assumption that $c_1>c_2$. It is assumed that there exists the density $f(t)$ of the failure time distribution $F(t)$. Let $R(t_0)\equiv[F(t_0+L)-F(t_0)]/\overline{F}(t_0)$ and $r(t_0)\equiv f(t_0)/\overline{F}(t_0)$. Then, both $R(t_0)$ and

$r(t_0)$ are called by the failure rates and they have the same properties of the failure rates, i. e., $R(t_0)$ is increasing (decreasing) iff $r(t_0)$ is increasing (decreasing), respectively (*see* Barlow and Proschan [3], p. 23). We restrict ourselves to the case in which the failure rate has a monotone property and continuous. Further, we assume that $C_1(\infty) < k_1$, i. e., $c_1 < k_1/\lambda$, because the expected cost of the system in which the order is made after failure of the original unit would be less than that of the system which remains inoperative forever. Let

$$q_1(t) = R(t)\left[k_1/\lambda + k_2\left(L + \int_0^t \overline{F}(u)\,du\right) - c_1 F(t) - c_2 \overline{F}(t)\right] + [(c_1 - c_2) r(t) - k_2]\left[1/\lambda + \int_t^{t+L} F(u)\,du\right], \tag{6}$$

for simplicity of equations. Then, we have the following theorem.

THEOREM 1: *Assume that* $c_1 < k_1/\lambda$: 1° *suppose that* $r(t)$ *is monotonely increasing. If* $q_1(0) < 0$ *and* $q_1(\infty) > 0$, *then* t_0^* *exists uniquely on* $(0, \infty)$ *as the solution to* $q_1(t_0) = 0$. *Otherwise,* $t_0^* = \infty$ *or* 0 *according as* $q_1(\infty) \leqq 0$ *or* $q_1(0) \geqq 0$, *respectively;*

2° *suppose that* $r(t)$ *is non-increasing. Then,* $t_0^* = \infty$ (0) *if*

$$(1/\lambda + L)\left(k_2 \int_L^\infty \overline{F}(t)\,dt - c_1 + c_2\right) \geqq (<) (k_1/\lambda - c_1) \int_0^L \overline{F}(t)\,dt.$$

Proof: Differentiating $C_1(t_0)$ with respect to t_0 and setting it equal to zero, we have $q_1(t_0) = 0$. Further, from the assumption that $c_1 < k_1/\lambda$, $q_1(t_0)$ is monotonely increasing (non-increasing) if $r(t_0)$ is monotonely increasing (non-increasing), respectively.

First suppose that $r(t_0)$ is monotonely increasing. If $q_1(0) < 0$ and $q_1(\infty) > 0$ then from the monotonicity and the continuity of $q_1(t_0)$, t_0^* exists uniquely on $(0, \infty)$ as the solution to $q_1(t_0) = 0$, which minimizes the expected cost $C_1(t_0)$. Further, it is easily shown that if $q_1(\infty) \leqq 0$ then $t_0^* = \infty$ and if $q_1(0) \geqq 0$ then $t_0^* = 0$.

Next, suppose that $r(t_0)$ is non-increasing. Then, $q_1(t_0)$ is also non-increasing. Thus, it is easily seen that $C_1(0)$ or $C_1(\infty)$ is not greater than $C_1(t_0)$ for any t_0. Therefore, we have $t_0^* = \infty$ if $C_1(\infty) \leqq C_1(0)$, i. e.,

$$(1/\lambda + L)\left(k_2 \int_L^\infty \overline{F}(t)\,dt - c_1 + c_2\right) \geqq (k_1/\lambda - c_1) \int_0^L \overline{F}(t)\,dt,$$

and *vice versa*.

Q.E.D.

In case of $q_1\ (0)<0$ and $q_1\ (\infty)>0$ of 1 in theorem 1, the expected cost is given by

$$C_1(t_0^*)=k_1+k_2-\frac{k_2-r(t_0^*)(c_1-c_2)}{R(t_0^*)} \tag{7}$$

Further, the ordering policy when $t_0^*=\infty$ represents that the order of the spare unit is made immediately after failure of the original unit and the policy when $t_0^*=0$ represents that the order of the spare unit is made at the same time of the beginning of the original unit.

In the above theorem, we have assumed that $c_1<k_1/\lambda$. Of course, we can also prove that the above theorem under the weaker condition than $c_1<k_1/\lambda$, for instance, $c_1<k_1/\lambda+k_2 L$. However, in actual situations, any ordering policy might be better than no order, in which the system has been remaining inoperative forever. It would be waste to discuss an optimum policy under the assumption that $c_1\geqq k_1/\lambda$.

In case of $q_1\ (0)<0$ and $q_1\ (\infty)>0$ of 1 in theorem 1, we can obtain the following upper limit of the optimum ordering time t_0^*. This could be applied to compute t_0^* by the successive approximations (*see* numerical examples below).

THEOREM 2: *Suppose that* $c_1<k_1/\lambda$, $q_1\ (0)<0$, $q_1\ (\infty)>0$ *and* $r(t)$ *is monotonely increasing. If* $\bar{t}_0$ *is a solution to* $h_1\ (t_0)=0$ *then* $\bar{t}_0$ *exists uniquely (possibly infinite) and* $t_0^*<\bar{t}_0$, *where*

$$h_1\ (t_0)=R\ (t_0)[k_1/\lambda+k_2 L-c_2]+[(c_1-c_2)r(t_0)-k_2]\left[1/\lambda+\int_0^L F(t)\,dt\right]. \tag{8}$$

Proof: We can easily obtain

$$r(t_0)>F(t_0)\Bigg/\int_0^{t_0}\bar{F}(t)\,dt, \tag{9}$$

$$R(t_0)>\left[\int_{t_0}^{t_0+L}F(t)\,dt-\int_0^L F(t)\,dt\right]\Bigg/\int_0^{t_0}\bar{F}(t)\,dt, \tag{10}$$

since the failure rate is monotonely increasing. Thus, we have $q_1\ (t_0)>h_1\ (t_0)$ for $0<t_0<\infty$. If $\bar{t}_0$ is a solution to $h_1(t_0)=0$ then $\bar{t}_0$ is unique because $h_1\ (t_0)$ is monotonely increasing and $t_0^*<\bar{t}_0$.

Q.E.D.

So far we have assumed that the expedited order has the same lead time L as the ordinary order. In reality, it will be able to be smaller. So, we suppose that the expedited order has a lead time L_1 which might be not greater than L. Then, in a similar way, the expected cost is

$$C_1(t_0) = \frac{\left\{ \begin{array}{l} k_1 \left[\int_{t_0}^{t_0+L} F(t)\,dt - (L-L_1) F(t_0) \right] \\ + k_2 \int_{t_0+L}^{\infty} \overline{F}(t)\,dt + c_1 F(t_0) + c_2 \overline{F}(t_0) \end{array} \right\}}{1/\lambda + \int_{t_0}^{t_0+L} F(t)\,dt - (L-L_1) F(t_0)}. \tag{11}$$

(ii) Modified Wiggins' model

In the Wiggins' model, it has been assumed that the delivered unit is put into inventory if the original unit is operative. Here, the model has the same assumptions as the Wiggins' model except that the original unit is always replaced as soon as the spare unit is delivered, even if it is operating. This model is appropriate in cases such that an inventory task is very difficult or there is no places to put a spare unit in inventory.

In the model, we do not need to consider the inventory cost because of the assumption. The shortage cost is equal to (1) and hence, the total expected cost per one cycle is

$$C_2(t_0) = \frac{k_1 \int_{t_0}^{t_0+L} F(t)\,dt + c_1 F(t_0) + c_2 \overline{F}(t_0)}{L + \int_0^{t_0} \overline{F}(t)\,dt}. \tag{12}$$

Of our interest is to obtain the optimum t_0^* minimizing the expected cost $C_2(t_0)$ in (12) under the assumption that $c_1 > c_2$. Let

$$q_2(t) = [r(t) + b_1 R(t)] \left[L + \int_0^t \overline{F}(u)\,du \right] - F(t) - b_1 \int_t^{t+L} F(u)\,du, \tag{13}$$

where $b_1 = k_1/(c_1 - c_2)$ and $b_2 = c_2/(c_1 - c_2)$. Then, from the discussions similar to the previous theorems, we obtain the following theorems without proving.

THEOREM 3: $1°$ *suppose that $r(t)$ is monotonely increasing. If $q_2(0) < b_2$ and $q_2(\infty) > b_2$, then t_0^* exists uniquely on $(0, \infty)$ as the solution to $q_2(t_0) = b_2$. Otherwise, $t_0^* = \infty$ or 0 according as $q_2(\infty) \leqq b_2$ or $q_2(0) \geqq b_2$, respectively;* $2°$ *suppose that $r(t)$ is non-increasing. Then, $t_0^* = \infty$ (0) if*

$$(1/\lambda + L)\left(k_1 \int_0^L \overline{F}(t)\,dt - c_1 \right) \leqq (>) (k_1/\lambda - c_1) L.$$

THEOREM 4: *Suppose that* $q_2(0) < b_2$, $q_2(\infty) > b_2$ *and* $r(t)$ *is monotonely increasing. If* $\bar{t}_0$ *is a solution to* $h_2(t_0) = 0$ *then* $\bar{t}_0$ *exists uniquely (possibly infinite) and* $t_0^* < \bar{t}_0$, *where*

$$h_2(t_0) = r(t_0) + b_1 R(t_0) - \left[b_1 \int_0^L F(t)\,dt + b_2 \right] \Big/ L.$$

In case of $q_2(0) < b_2$ and $q_2(\infty) > b_2$ of 1 in theorem 3, the expected cost is

$$C_2(t_0^*) = k_1 R(t_0^*) + (c_1 - c_2)\, r(t_0^*).$$

3. REPAIRABLE UNITS

Suppose that the failed unit can be repairable. However, it might be sometimes better to scrap than to repair in actual situations. From this point of view, the so-called "repair limit policies" have been considered by [5, 6]. In this section, we consider the two repair limit policies which are given by a repair time limit and a repair cost limit. If the failed unit is decided to scrap by a repair limit policy, then the spare unit is ordered immediately and is delivered after a lead time L. In the two models, we derive the expected costs per unit of time in equilibrium and discuss the optimum policies minimizing the expected costs.

(i) Repair time limit

When the original unit fails, the repair time is estimated. If the repair is estimated to be completed up to time t_0, then the failed unit undergoes repair immediately. It is assumed that the estimation time is negligible. If the estimated repair time is greater than t_0, then the failed unit is scrapped without repair and the spare unit is ordered immediately. The ordering policy depends on the repair time limit t_0. It is assumed that the estimated repair time for each unit has an arbitrary distribution $G(t)$ with finite mean $1/\mu$. The repair incurs a constant cost k_0 per unit of time. The other notations are the same as the previous models.

Consider one cycle from the beginning of the operating unit to the next operation. If the failed unit is repaired, then the expected cost is $(k_0 + k_1) \int_0^{t_0} t\,dG(t)$ which is the sum of the repair cost and the shortage cost. If the failed unit is scrapped, then the expected cost is $(k_1 L + c_1) \bar{G}(t_0)$ which is the sum of the ordering cost and the shortage cost. Thus, the total expected cost per one cycle is easily given by

$$C_3(t_0) = \frac{(k_0 + k_1) \int_0^{t_0} t\,dG(t) + (k_1 L + c_1) \bar{G}(t_0)}{1/\lambda + \int_0^{t_0} t\,dG(t) + L\bar{G}(t_0)}. \tag{16}$$

Of our interest is to obtain the optimum t_0^* minimizing the expected cost $C_3\ (t_0)$ in (16). Let

$$q_3\ (t_0)=[(k_0+k_1)/\lambda]\, t_0+(k_0\, L-c_1)\int_0^{t_0}\overline{G}\ (t)\, dt. \tag{17}$$

Then, we have the following:

THEOREM 5: *The* t_0^* *exists uniquely on* $(0, \infty)$ *as the solution to* $q_3\ (t_0)=(k_1\, L+c_1)/\lambda$.

Proof: Differentiating $C_3\ (t_0)$ with respect to t_0 and setting it equal to zero, we have $q_3\ (t_0)=(k_1\, L+c_1)/\lambda$. It is evident that $q_3\ (0)=0$ and $q_3\ (\infty)=\infty$.

If $(k_0+k_1)/\lambda \geqq c_1-k_0\, L$, then $q_3\ (t_0)$ is monotonely increasing, otherwise $q_3\ (t_0)$ is a convex function. In either case, t_0^* exists uniquely on $(0, \infty)$ as the solution to $q_3\ (t_0)=(k_1\, L+c_1)/\lambda$.

Q.E.D.

In this case, the expected cost is

$$C_3\ (t_0^*)=\begin{cases} k_1+(k_0\, t_0^*-c_1)/(t_0^*-L) & \text{if}\quad k_0\, L\neq c_1, \\ (k_0+k_1)\Big/\left[1+1\Big/\left(\lambda\displaystyle\int_0^L\overline{G}\ (t)\, dt\right)\right] & \text{if}\quad k_0\, L=c_1. \end{cases} \tag{18}$$

It is further shown that $t_0^*\leqq L$ iff $k_0\, L\geqq c_1$.

(ii) Repair cost limit

When the original unit fails, we estimate the repair cost of the failed unit. If the estimated repair cost is less than a fixed cost c_0, then the failed unit begins to repair. It is assumed that the estimated repair cost of the failed unit has an arbitrary distribution $H\ (x)$. On the other hand, if the estimated repair cost is not less than the cost c_0, the failed unit is scrapped and the spare unit is ordered immediately.

In a similar way of obtaining (16), the expected cost per unit of time in equilibrium is

$$C_4\,(c_0)=\frac{\displaystyle\int_0^{c_0} x dH\,(x)+(k_1/\mu)\, H\,(c_0)+(k_1\, L+c_1)\,\overline{H}\,(c_0)}{1/\lambda+H\,(c_0)/\mu+\overline{H}\,(c_0)\, L}. \tag{19}$$

Let

$$q_4\,(c)=(1/\lambda+1/\mu)\,(c-c_1)+(L-1/\mu)\left[\int_0^c\overline{H}\,(x)\, dx-k_1/\lambda\right]. \tag{20}$$

Then, we have the following theorem without proving:

THEOREM 6: 1° *suppose that* $L \neq 1/\mu$. *If* $q_4(0)<0$ *then* c_0^* *exists uniquely on* $(0, \infty)$ *as the solution to* $q_4(c_0)=0$, *otherwise* $c_0^*=0$.

2° *suppose that* $L=1/\mu$. *Then,* $c_0^*=c_1$.

In case of 1 in theorem 6, the expected cost is

$$C_4(c_0^*)=k_1+(c_1-c_0^*)/(L-1/\mu).$$

In the above two models, we have assumed that the order of the spare unit is made after the failed unit is decided to scrap by the repair limit policy. However, it would be reasonable to consider an advanced ordering policy in which the order might be made before failure, taking account of the length of service of an operating unit. The problem of such a policy will be able to be solved by connection with the ordering policy discussed in section 2 and the repair limit policy. However, it might be too difficult to do so analytically.

4. NUMERICAL EXAMPLES

We have discussed the optimum ordering policies, where we have adopted, as the criteria of optimality, the expected costs per unit of time in equilibrium. In this section, we show numerical examples of the optimum ordering time t_0^* and their associated values.

We assume that $dF(t)=\alpha(\alpha t)\exp(-\alpha t)dt\,(\alpha>0)$. Then,

$$\bar{F}(t)=(1+\alpha t)\exp(-\alpha t),$$
$$r(t)=\alpha^2 t/(1+\alpha t),$$
$$R(t)=1-e^{-\alpha L}-\alpha L e^{-\alpha L}/(1+\alpha t),$$
$$1/\lambda=2/\alpha.$$

It is noted that the failure time distribution is a gamma distribution with a shape parameter 2, which has a monotone increasing failure rate with $r(0)=0$ and $r(\infty)=\alpha$.

As an example, we consider the modified Wiggins' model. From theorem 3, if

$$(L+2/\alpha)[\alpha+b_1(1-e^{-\alpha L})]>1+b_1 L+b_2$$

and

$$b_1[2/\alpha-(2/\alpha+2L+\alpha L^2)e^{-\alpha L}]<b_2,$$

then t_0^* exists uniquely on $(0, \infty)$ as the solution to

$$A(1+\alpha t)+B e^{-\alpha t}=C, \tag{22}$$

where

$$A \equiv (L+2/\alpha)[\alpha+b_1(1-e^{-\alpha L})]-(1+b_1 L+b_2),$$
$$B \equiv 1+b_1 L e^{-\alpha L},$$
$$C \equiv \alpha(L+2/\alpha)(1+b_1 L e^{-\alpha L}).$$

In this case, the expected cost is

$$C_2(t_0^*)=k_1(1-e^{-\alpha L})+[(c_1-c_2)\alpha^2 t_0^*-k_1 \alpha L e^{-\alpha L}]/(1+\alpha t_0^*). \tag{23}$$

Moreover, from the inequality that $t_0^* < \bar{t}_0$ in theorem 4, we have

$$\alpha t_0^* < \frac{b_2-b_1[2/\alpha-(2/\alpha+2L+\alpha L^2)e^{-\alpha L}]}{(L+2/\alpha)[\alpha+b_1(1-e^{-\alpha L})]-2-b_1 L(1+e^{-\alpha L})-b_2}, \tag{24}$$

if the right-hand side is positive.

TABLE I

Dependence of the mean failure time $1/\lambda$ in the optimum ordering time t_0^* and its associated expected cost $C_2(t_0^*)$ for modified Wiggins' model

($1/\lambda=2/\alpha$, $dF(t)=\alpha(\alpha t)\exp(-\alpha t)\,dt$, $c_1=3$, $c_2=1$, $k_1=0.01$, $L=5$)

$1/\lambda$	t_0^*	$\bar{t}_0$	$C_2(t_0^*)$
15	7.01	21.03	0.1319
16	7.89	29.42	0.1271
17	8.80	43.09	0.1225
18	9.74	68.75	0.1182
19	10.71	132.81	0.1142
20	11.70	552.70	0.1104
21	12.72	∞	0.1068
22	13.75	∞	0.1033
23	14.80	∞	0.1001
24	15.87	∞	0.0971
25	16.95	∞	0.0942

TABLE II

Dependence of the lead time L in the optimum ordering time t_0^* and its associated expected cost $C_2(t_0^*)$ for modified Wiggins' model

($1/\lambda=2/\alpha=20$, $dF(t)=\alpha(\alpha t)\exp(-\alpha t)\,dt$, $c_1=3$, $c_2=1$, $k_1=0.01$)

L	t_0^*	$\bar{t}_0$	$C_2(t_0^*)$
1	23.24	∞	0.1405
2	19.15	∞	0.1326
3	16.05	∞	0.1249
4	13.63	∞	0.1175
5	11.70	552.70	0.1104
6	10.15	44.36	0.1036
7	8.88	23.03	0.0973
8	7.84	15.52	0.0914
9	6.97	11.68	0.0859
10	6.25	9.35	0.0809

Therefore, the optimum ordering time t_0^* can be obtained from (22) by estimating the initial value from (24) (or the mean failure time $1/\lambda$) and adopting the successive approximations. Table I and II show the numerical examples of the modified Wiggins' model of the dependence of the mean failure time $1/\lambda$ and the lead time L, respectively in the optimum ordering times t_0^* and their associated quantities.

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