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DEVELOPMENT AND MARKETING STRATEGIES FOR A CLASS OF R AND D PROJECTS, WITH TIME INDEPENDENT STOCHASTIC RETURNS (*)

by Abraham MEHREZ ([1])

Abstract. — *This paper extends the work of Aldrich and Morton* [1], *on the so-called Lucas* [7] *model that selects a spending function maximizing the expected value of an R and D project. First, a sensitivity analysis of the project's cost and discounted expected value is carried out with regard to infinitesimal changes in the returns. Second, a modified version of the measure of the expected value of perfect information (EVPI) is defined to analyze marketing uncertainties. The value of the measure has been proved to be non-negative. Furthermore, bounds on the measure are determined. Finally, an admissible set of spending policies is defined and identified as the set of the so-called optimal spending policies, and the characteristics of the optimal policies assuming different objective functions, including quadratic functions and a probability of out of loss are established.*

Keywords: R and D project; EVPI; optimal control; loss functions.

Résumé. — *Cet article approfondit le travail d'Aldrich et Morton* [1], *sur le modèle appelé Lucas* [7] *qui sélectionne une fonction de défense amenant au point maximal la valeur moyenne d'un projet R et D. Premièrement, nous avons effectué une analyse de sensibilité sur le coût du projet et la valeur moyenne escomptable par rapport aux changements infinitésimaux dans le gain. Deuxièmement, une version modifiée de la mesure de la valeur moyenne de l'information parfaite (EVPI) est définie pour analyser les incertitudes du marché. La valeur de la mesure est prouvée être non-negative. Puis, sont déterminées les limites sur la mesure. Finalement, un groupe de systèmes de défense valables est défini et puis identifié comme étant un groupe de systèmes de défense optimaux. Enfin, les caractéristiques des systèmes optimaux assumant des fonctions différentes, incluant des fonctions au second degré et une probabilité de non-perte sont établies.*

INTRODUCTION

The central factor in determining the time patterns of R and D project spending is the value at completion. Given actions of competitors and changes in consumer tastes and preferences, this value is uncertain to the planner during the time the project is progressing. Thus, a planner operating in the framework of the Lucas model [7] has to establish both a marketing policy directed to reduce the uncertainties surrounding the project's value at completion and a criterion to establish a development spending policy.

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Empirical works by Mansfield and Wagner [9] and others have shown that expenditures on these policies and their timing might be critical to the project's economic success. The purpose of this paper is to study the characteristics of these policies for the case of time independent stochastic returns. In the next section the Aldrich-Morton version of the Lucas model is reformulated for this case, and some results regarding the temporary properties of the project's expected value and the optimal development spending policy are established as a function of the returns on a completed project. As an example, it is shown that the expenses on development and the project's expected value strictly increase in the returns on the completed project, for any given level of cumulative technical effort devoted to a profitable project.

In Section III, the expected value of perfect information to reduce marketing uncertainties, the timing of these operations and their posterior effects on the development spending are analyzed using a modified version of the measure of the expected value of perfect information. The expected value of this information is shown to be non-negative. Furthermore, the properties of the optimal development policy for the "recourse problem" named by Walkup and Wets [13] are discussed in this context.

In Section IV, the development policy is further investigated and some properties of the admissible spending set of spending policies are identified.

THE MODEL

The basic features of the model we present have appeared in the works of Lucas [7], Kamien and Schwartz [5] and Aldrich and Morton [1].

These authors have treated the time patterns of spending on technical information for a class of R and D projects, with time independent returns. The factors determining the dynamics of spending for this class of projects are the following: the profitability of a completed project, the probability function of technical success, the opportunity cost of a project and the function relating the rate of dollar spending on the project to the rate of change in effort or knowledge devoted to the project.

For given levels of profitability and opportunity cost these authors have shown that the patterns of spending depend on the so-called "conditional completion density function", a function which determines the temporary chances for technically completing a given project. When this function is monotone increasing in the cumulative effort devoted to the project, both functions, the rate of spending and the discounted expected value of the project, increase as the project progresses. When the conditional completion density function is at first strictly monotonically increasing and then strictly monotonically decreasing, both functions possess a unique maximum. Moreover, it is worthwhile in this case to terminate the project's effort above a cutoff level.

In this Section we study the time varying properties of the project's discounted expected value, and the optimal development spending policy as a function of completed project profitability.

Assume that the profitability of a completed project, defined by R_0 on (Ω, $\mathfrak{A}$, p) a probability space and T, a time index set, is a continuous, real-valued, bounded, time independent stochastic process, i. e. $R_0(\omega, t) = R_0(\omega)$ and $R_0(\omega) \in [a_0, b_0]$, $\forall \delta \in \mathfrak{A}$ and $t \in T$(1). Following Aldrich and Morton, let $m(t)$ = the rate of dollar spending at time t, $z(t)$ = the cumulative effort devoted to the project by time t, $F(z)$ = the probability that the project will be completed by the time the cumulative effort is z. Define by $h(z) = F'(z)/(1-F)(z)$, the conditional completion density function and let $dz/dt = g(m(t))$ be the function relating the rate of change in effort to the rate of dollar spending at time t.

Furthermore, we assume that

$$g(0)=0, g'(m)>0, g''(m)<0, \lim_{m \to \infty} g'(m)=0,$$

that $F(0)=0$, $F'(z)>0$, $\lim_{z \to \infty} F(z)=1$, that the $\inf_z h(z)$ exists and is nonzero and that the $\sup_z h(z)$ exists and is finite.

Aldrich and Morton [th. 1, p. 453], imply the existence of the value:

$$V(z, \omega) \approx \operatorname*{Max}_{m \geqq 0} [-m\Delta t + h(z) g(m) R_0(\omega) \Delta t + (1 - r\Delta t - h(z) g(m) \Delta t))(z + g(m) \Delta t, \omega)] \quad (1)$$

where:

$$r = \text{interest rate}$$

and:

$$V(z, \omega) \equiv V(z, R_0(\omega))$$

is the discounted expected value of the project, given that effort already expended is z, that potential profit will be $R_0(\omega)$, and that an optimal spending path is followed over the remaining infinite horizon.

(1) The profitability of a completed project is defined for the purposes of sections II and III abstractly on the probability space (Ω, Ω, p). In fact, the functional relationship between R_0 and (Ω, Ω, p) are very complex and attempts to capture this relation were made by Sherrer [12], Kamien and Schwartz [5] and others. The restrictive assumption of time independent returns does not seem to affect the generality of the results to be shown here.

The explanation for the formulation is the following: If the firm spends at rate m, its spending in an interval of length Δt is $m\Delta t$. This will lead to completion in the current period with an asymptotic probability of $h(z) g(m) \Delta t$ and with current expected profit $h(z) g(m) R_0(\omega) \Delta t$. If completion does not occur, the firm must go on with an effort that will increase to $z+g(m)\Delta t$. The firm's maximum return at this point is $V(z+g(m)\Delta t, \omega)$ which it earns with probability $(1-h(z) g(m))\Delta t$, all of which must be discounted. (1) is the infinite horizon formulation of the discounted expected value of the project. $V(t, z, \omega, s)$, the expected value of the project at time t, given that the effort already expended is z, ω is the profit realization, and an optimal spending path is followed from time s to the end of the horizon. In their basic theorem Aldrich and Morton have shown that $V(t, z, \omega, s)$ converges uniformly as $s \to \infty$ to a unique limit given by $V(z, \omega)$.

For the convenience of the reader we will give the summary of Aldrich and Morton's results, quoted from their work (pp. 494-495).

"1. An increasing (decreasing) success rate is associated with increasing (decreasing) spending and return. This is an extension of the Kamien and Schwartz theorem on increasing success rates.

2. An asymptotically constant rate is associated with asymptotically constant spending and return. Moreover, theorem 4 implies that optimal spending and return will approach their asymptotic values from the same direction as the success rate.

3. By theorem 4 and its first corollary, a unimodal success rate is associated with spending and return functions having at most one mode. For decreasing success rates, once a project is stopped, it should not be resumed.

4. Local maxima (minima) of spending and return functions occur prior to local maxima (minima) of the success rate. Again, this is an extension of a Kamien and Schwartz theorem for unimodal success rates."

The results achieved by the Aldrich and Morton formulation are similar to those obtained by Lucas [7] and Kamien and Schwartz [5]. Lucas assumed that the effort invested in the project is related to the probability of technical success *via* a special distribution relation expressed by the gamma distribution. He also assumed that the function relating efforts to monetary expenditure is piecewise linear. This model was further investigated by Kamien and Schwartz to include general types of effort-information distribution functions. Aldrich and Morton have suggested their model formulation to avoid some of the difficulties arising from the finite horizon optimal control formulation of Kamien and Schwartz. They argued that "Kamien and Schwartz investigated the model further, but were unable to provide constructive solutions."

A sensitivity analysis is carried out using the Aldrich and Morton formulation of the Lucas Model. A summary of the results of this study is provided at the end of this Section.

The following lemma is useful in prooving lemma 2 and theorem 1.

LEMMA 1:

$$V(z, \omega^1) \leqq V(z, \omega^2) \qquad \textit{if} \quad R_0(\omega^1) \leqq R_0(\omega^2).$$

Proof (see Mehrez [11]).

In the following the argument ω is dropped for notational simplicity and V is observed *via* R_0.

LEMMA 2: *If $V(z, R_0)$ is continuous in R_0, then $A = \{ R_0 \mid V(z, R_0) = 0 \}$ is either a closed bounded interval or an empty set.*

Proof :

(i) A is closed, since V is continuous.

(ii) A is bounded by definition.

(iii) Lemma 1 implies that A is a connected set. By (i)-(iii) and the theorem for a closed bounded set, A is either a closed bounded interval or an empty set.

THEOREM 1: *$m(z, R_0)$ and $V(z, R_0)$ are strictly monotone increasing in R_0 on $A^c = \{ R_0 \mid V(z, R_0) \neq 0 \}$* (²).

Proof: Aldrich and Morton [pp. 493-494] imply that:

$$r V(z, R_0) = \operatorname*{Max}_{m>0} \left\{ -m + g(m) \left[h(z)\,(R_0 - V(z, R_0)) + \frac{dV(z, R_0)}{dz} \right] \right\} \qquad (2)$$

and:

$$\Phi(m, R_0) = -m + g(m) \left[h(z)\,(R - V(z, R_0)) + \frac{dV(z, R_0)}{dz} \right] \qquad (3)$$

and that a necessary condition for m to be optimal for a given R_0 is that $d\Phi/dm = 0$. Furthermore, inspection of $d^2\Phi/dm^2$, shows that for R_0 on A^c:

$$U(z, R_0) = h(z)\,(R_0 - V(z, R_0)) + \frac{dV(z, R_0}{dz} > 0, \qquad (4)$$

(²) The differentiability property is supposed to hold whenever required in the body of the proof.

thus, the optimal spending is determined by $d\Phi/d^m = 0$, a parametric equation in m and R_0.

Now by differentiating $d\Phi/dm$ with respect to m and R_0, and rearranging terms we get by (3) and (4) that:

$$\frac{dm}{dR_0} = \left(-g'(m)\frac{du}{dR_0}\right)\Big/(m) \,.\, U(R^0)) \tag{5}$$

where the argument z is dropped for notational simplicity.

Furthermore, (4) and (5) imply that the sign of dm/dR_0 is determined by the sign of du/dm. To conclude on the first part of the theorem we exclude the possibility of the following cases:

(*a*) $R_0^1 < R_0^2 \to U(R_0^2) < U(R_0^1),$

(*b*) $R_0^1 < R_0^2 \to U(R_0^2) = U(R_0^1).$

Suppose (*a*) holds, then combining (2) and (4) results in:

$$V(z, R_0^2) < V(z, R_0^1)$$

which is a contradiction to lemma 1.

Suppose (*b*) holds, then (2) implies that $m(R_0^1) = m(R_0^2)$, and by lemma 1; and (*a*) $V(R_0)$ and $U(R_0)$ are constants on $\{R_0^1, R_0^2\}$. Now, the constancy of V implies the constancy of dV/dz in R_0 on A^c, and from (4), $dU/dR_0 = h(z) > 0$. Thus, the constancy of U is contradicted by assumption and $R_0^1 < R_0^2 \to U(R_0^2) > U(R_0^1)$ which implies further that $dm/dR_0 > 0$. This result and (2) imply directly the second part of the theorem.

THEOREM 2: *V is a convex in R_0 on $[a_0, b_0]$.*

Proof: Let $E[f^\infty(R_0)]$ = the expected discounted probability and $E[m^\infty(R_0)]$ = the expected discounted cost of completing the project by the infinite horizon, given that it has not been completed when the effort already expanded is z and $m^\infty(R_0)$, the optimal infinite spending policy corresponding to R_0 is employed.

By the implicit definition of V used by Aldrich and Morton (p. 492):

$$V(R_0^2) = R_0^2 E(f^\infty(R_0^2)) - E(m^\infty(R_0^2)) \tag{7}$$

and:

$$V(R_0^2) \geqq R_0^2 E(f^\infty(R_0^1)) - E(m^\infty(R_0^1)) \tag{8}$$

Now, the convexity of V follows immediately from the definition of convexity, (7) and (8).

THEOREM 3:

(*a*)
$$V'(z, R_0) = E(f^{\infty}(R_0))$$

and:

(*b*)
$$R_0 E'(f^{\infty}(R_0)) = E'(m^{\infty}(R_0)) \text{ (}^3\text{)}.$$

Proof: Let $\Phi(z, R_0^2, m^{\infty}(R_0^1))$ be defined by:

$$\Phi(z, R_0^2, m^{\infty}(R_0^1)) = R_0^2 E(f^{\infty}(R_0^1)) - E(m^{\infty}(R_0^1)). \quad (9)$$

Now, (7) and (9) imply that:

$$\Phi(z, R_0^2, m^{\infty}(R_0^1)) = V(z, R_0^1) + E(f^{\infty}(R_0^1))(R_0^2 - R_0^1) \quad (10)$$

and:

$$\Phi(z, R_0^2, m^{\infty}(R_0^1)) \leqq V(z, R_0^2), \qquad \forall (R_0^1, R_0^2) \in [a_0, b_0]. \quad (11)$$

Furthermore, by (10) and (11) it follows that $\forall R_0^2 \geqq R_0^1$:

$$E(f^{\infty}(R_0^1)) \leqq V(z, R_0^2) - V(z, R_0^1)/(R_0^2 - R_0^1) \quad (12)$$

and thus, at the limit:

$$E(f(R_0^1)) \leqq V'(z, R_0^1). \quad (13)$$

By a similar argument, it follows that $\forall R_0^2 \leqq R_0^1$:

$$E(f^{\infty}(R_0^1)) \geqq V'(z, R_0^1), \quad (14)$$

combining (13) and (14) results in (*a*).

Now, differentiating (7) and substituting (*a*) into it gives (*b*).

For the convenience of the reader the results of this section are summarized below.

(*a*) In case a research project is economically feasible, a higher level of expected return conditioned on completion will carry the firm to a more costly policy of spending to increase the discounted expected value of the project (th. 1). This result provides some insight into the subject of R and D spending

(3) A dot (.) denotes the first derivative with respect to R_0.

policy. It indicates that if changes due to actions of competitors and to changes in consumers' tastes and preferences take place in the course of the development of a project, a more or less offensive policy of spending should be taken by the firm.

(*b*) An increase in the discounted expected value of the project due to an increase in the returns on a completed project is measured by the rate of change of V in R_0.

This rate is non-negative on $[a_0, b_0]$ (lemma 2 and th. 1). [th. 3, part (*a*)]. In case, the project is economically feasible it is stricly monotone increasing (th. 1). This result is intuitively obvious.

The rate of change of V is equal to the expected discounted probability for completing the project [th. 3, part (*a*)]. The practical meaning of this result is the following: If the level of returns on a completed project is expected to increase by one dollar by the time the cumulative effort is Z, the value of the project shows approximately an increase of less than one dollar, precisely it is the expected discounted probability for completing the project; a measure that determines the amount of technical uncertainties or risk inherent to the project or risk inherent to the project.

(*c*) The possibility of adjusting the spending for an increase in R_0 guarantees that V is strictly convex in R_0 (th. 2). Theorem 2 is intuitively appealling. If the level of returns on a completed project is expected to increase by the time the cumulative effort is Z and no change is being made in the rate of spending, then the discounted expected value of the project increases linearlly. Thus, an optimal adjustment of the spending policy will guarantee that V is stricly convex.

(*d*) Theorem 3(*b*) is intuitively appealing; it gives a simple marginal cost-benefit rule to adjust the optimal spending policy to changes in R_0. The meaning of this rule is the following: An increase in returns conditioned on completion will induce the firm to a more costly policy of spending. At the optimum, the marginal discounted expected cost due to such a policy is offset by a marginal increase in the discounted expected benefits due to multiplicative effects of returns and a marginal decrease in technical risk.

The results achived in the previous section suggest that a planner should consider in the project's life the economic feasibility of marketing operations to reduce the uncertainties regarding R_0 ([4]). A dynamic version of the measure

([4]) Without loss of generality, R_0 could be defined as the expected returns on a completed project, whereas the reduction of uncertainties is done on the basis of learning some of the factors to affect R_0.

of the expected value of perfect information is derived below to identify the expected benefits arising from performing such operations.

Let EVPI (z), the expected value of perfect information given that a level z of effort is already expanded or:

$$\mathrm{EVPI}(z)=\mathrm{WS}(z)-RP(z) \quad (^5), \tag{15}$$

where:

$$\mathrm{WS}(z)=E[V(z, R_0)] \tag{16}$$

and

$$\mathrm{RP}(z)=\underset{m^\infty \in \mathrm{M}}{\mathrm{Max}}\, E[\Phi(z, R_0, m^\infty)], \tag{17}$$

where m^∞ belongs to M the set of all differentiable non-negative spending policy (6).

Combining (15)-(17) we note that E VPI (z) is defined as the difference between the solutions of two problems: WS, or the "Wait and See" problem defined by Madansky [8] to compute the expected value of the project given that first R_0 is observed and then an optimal spending policy is conducted. The RP, of the Recourse Problem, defined by Walkup and Wets [13] to compute the expected value of the project in case where the optimal spending policy is calculated without the actual knowledge of R_0. E VPI (z) as defined by (15) reflects on an extreme situation where the option of purchasing information is given once and forever at level z. Thus (15) exhibits the "worst" situation the planner may be faced with. The advantage of dealing with (15) is that it provides an upper bound on the expected value of perfect information derived for other situations. A lower bound on E VPI (z) is given by theorem 4.

THEOREM 4:

$$\mathrm{E\,VPI}(z) \geqq 0.$$

Proof: By (7) and (8):

$$RP(z)=V(z, E(R)) \tag{18}$$

Thus:

$$\mathrm{EVPI}(z)=E(V(z, R_0))-V(z, E(R)). \tag{19}$$

Now, by theorem 2, V is convex in R, and, by Jensen's inequality derived in Ferguson [2], p. 76 and (19):

$$\mathrm{E\,VPI}(z) \geqq 0.$$

(5) (15) competes with the assumption that the manager is risk neutral and the benefits of reducing uncertainties are measured in monetary terms.

Clearly, by (19), $m^{\infty}(E(R))$ is the optimal policy of spending, up to the time information is purchased. Furthermore, the timing of purchasing information depends on z, or $h(z)$. By and large this dependency is not easily characterized. However, some properties of the dependency are known by the results of Kamien and Schwartz, and verified by Morton and Aldrich. As an example, if $h(z)$ is constant, the timing is independent of z, and the decision to purchase information is a static one determined by the magnitude of E VPI (z) and the costs to purchase information. In case that $\lim_{z \to \infty} h(z) = \infty$ and $h'(z) > 0$, $\lim_{z \to \infty} \text{E VPI}(z) = 0$ and it is not worthwhile to purchase information above a cutoff level. The same conclusions hold in case that $h(z) < 0$ for $z \leqq z^x$ and $\lim_{z \to \infty} h(z) = 0$.

So far, we assumed that the planner is a risk neutral. Thus, the optimal spending policy $m^{\infty}(E(R_0))$, and, $m^{\infty}(E(R_0))$ is shown to be the solution of:

$$\text{Min}\, P_1 = E[V(z, R_0) - \Phi(z, R_0, m)] \quad \text{s. t. } m^{\infty} \in \text{M}, \tag{20}$$

where $(V(z, R_0) - \Phi(z, R_0, m^{\infty}))$ establishes the loss from deriving a policy m^{∞} for a given R_0, due to the lack of perfect information on R_0.

To introduce risk elements into the decision problem (7), we consider the solution of the following problem measuring risk *via* the loss of efficiency:

$$\text{Min}\, P_2 = E[V(z, R_0) - \Phi(z, R_0, m)]^2 \quad \text{s. t. } m^{\infty} \in M. \tag{21}$$

Alternatively, P_2 can be rewritten as follows:

$$P_2 = \text{Var}[\Phi(z, R_0, m^{\infty})] + [E(\Phi(z, R_0, m^{\infty})]^2 \tag{22}$$

where the deviation of P_2 from 0 is due to the variance of Φ, and the squared distance of $E(V)$ from $E(\Phi)$. Furthermore, we use (9) to develop:

$$\text{Var}[\Phi(z, R_0, m^{\infty})] = E[f(z, m^{\infty})]^2 \, \text{Var}(R_0). \tag{23}$$

and by (23) we observe that P_2 is increasing in $\text{Var}(R_0)$. Now, combining theorem 3 (*a*), (20), (22), and (23), it follows, finally, that:

$$\text{Min}\, P_2 \leqq E[V(z, R_0) - \Phi(z, R_0, m^{\infty} E(R_0))]^2$$
$$\leqq E[V(z, R_0) - \Phi(z, R_0, m^{\infty}(R_0))]^2, \qquad \forall R_0 \geqq E(R_0).$$

(6) Different dynamic structures dealing with the problem of purchasing information are studied by [4].

(7) Elements of Risk Analysis have been introduced into the general problem of R and D project selection by [3].

Thus a planner operating under criterion P_2 will spend, as it is intuitively appealing, at most $m^{\infty}(E(R_0))$.

So far, it is shown that the determination of a spending policy depends on the criterion to be selected. Furthermore, the solution for a given criterion is not necessarily unique, as illustrated for:

$$\text{Min}\, P_3 = P(\Phi(z, R_0, m^{\infty}) \leqq 0) \quad \text{s. t. } m^{\infty} \in M \tag{24}$$

and the case requiring that $V(z, R_0) \geqq 0$:

$$\forall R_0 \in [a_0, b_0].$$

This observation indicates that a test is required to examine the admissibility of a solution derived for a given criterion. To derive this test we first note the definition of admissibility.

DEFINITION 1: A policy $m^{\infty}_{(1)} \in M$ is an admissible policy if there is no other policy $m^{\infty}_{(2)}$ such that $\Phi(z, R_0, m^{\infty}_{(2)}) \geqq \Phi(z, R_0, m^{\infty}_{(1)})$, $\forall R_0 \in [a_0, b_0]$ and a strict inequality holds for a non-trivial subset of $[a_0, b_0]$.

A strong test to identify an admissible policy is given by theorem 5.

THEOREM 5: *$m^{\infty} \in M$ is admissible if it is optimal for some $R_0 \in [a_0, b_0]$.*

Proof (see Mehrez [11]).

We note that if $A \{ R_0 \mid V(z, R_0) = 0 \} = \Phi$, then a stronger result could be stated by theorem 6.

THEOREM 6: *Suppose $\Phi(z, R_0, m^{\infty}) > 0$, $\forall R_0 \in [a_0, b_0]$.*

Proof: Let $m^{\infty} = m^{\infty}(R_0)$ for an $R_0 \in [a_0, b_0]$. Then $\Phi(z, R_0, m^{\infty}) = V(z, R_0)$ and by theorem 1 it is the unique policy that obtains $V(z, R_0)$. Thus by definition 1 m^{∞} is an admissible policy. To show that an admissible m^{∞} policy is optimal for some R_0, we consider three cases:

(*a*) $\forall R_0 \in [a_0, b_0]$, $E(f^{\infty}(m^{\infty}(R_0))) < E(f^{\infty}(m^{\infty}))$;

(*b*) $\forall R_0 \in [a_0, b_0]$, $E(f(m^{\infty}(R_0))) > E(f^{\infty}(m^{\infty}(R_0)))$;

(*c*) $\forall R_0 \in [a_0, b_0]$, $E(f(m^{\infty}(R_0))) = E(f^{\infty}(m^{\infty}(R_0)))$.

Now, suppose that m^{∞} satisfies the condition of case (*a*). Then either $E(m^{\infty}) \leqq E(m^{\infty}(R_0))$ for some $R_0 \in [a_0, b_0]$, a contradiction to the optimality of $m^{\infty}(R_0)$, or $E(m^{\infty}) > E(m^{\infty}(R_0))$, which is a contradiction to the admissibility of m^{∞}. Hence case (*a*) is excluded and we will consider cases (*b*) and (*c*) further.

By simular argument, we can show that case (*b*) is impossible. We claim that $\exists R_0 \in [a_0, b_0]$ such that $E(f^{\infty}(m^{\infty})) = E(f^{\infty}(m^{\infty}(R_0)))$ and furthermore,

$E(m^{\infty})=E(m^{\infty}(R_0))$. Suppose that equality does not hold between the expected discounted costs of m^{∞} and $m^{\infty}(R_0)$, then we have established a contradiction to either the admissibility of m^{∞} on the optimality of R_0. Three remarks follows from theorems 5 and 6.

(1) If for some $R_0 \in [a_0, b_0]$ there exists a null optimal solution, then the uniqueness of $m^{\infty} R'_0)$ cannot be established and, thus, the results of theorem 6 can not be guaranteed.

(2) One can easily show that if

$$\forall (R_0^1, R_0^2, R_0^3) \in [a_0, b_0] \quad \text{and} \quad R_0^1 < R_0^2 < R_0^3,$$

then the following is true:

$$\Phi(z, R_1, m^{\infty}(R_0^2)) \geqq \Phi(z, R_1, m^{\infty}(R_0^3)).$$

This statement means that if the decision maker conducts a policy of spending that is optimal to R_0^{∞}, but in the end the observed profit is R_0^1, then he could do better by developing a policy which is optimal to $R_0^2 < R_0^3$.

(3) Under the conditions that are satisfied by theorem 6, a null policy is an inferior (nonadmissible) solution.

CONCLUSIONS

The element of stochastic returns is added to the dynamic programming formulation of the Lucas model to focus on risky R and D/Marketing decision problems.

We have shown that:

(*a*) An increase in the profit will induce the D.M. ([8]), under the set of realizations of profit corresponding to non-null policies and the criterion of maximizing the discounted expected value of the project, to compress spending and, consequently, to marginally increase the discounted expected value of the project.

(*b*) Up to some level of profit, null policy is optimal and consequently the discounted expected value of the project is zero.

(*c*) The economic interpretation of V' has been studied in terms of the costs and the probability characteristics of the R and D project.

(*d*) The value of purchasing marketing information, the timing of these operations and the characteristics of the optimal spending policy have been analyzed with a modified version of the measure of perfect information. The

([8]) Decision marker (D.M.).

value of the measure has been proved to be non-negative. Furthermore, $m^{\infty}(\varepsilon(R_0))$ has been shown to be the spending policy to conduct under the problem defined by the measure. Other properties of the policy have also been shown.

(*e*) Alternative R and D decision problems have been formulated and analyzed with regard to the problem of selecting a spending policy. The objectives that have been defined and analyzed include, in addition to the linear objective, a quadratic and zero-one expected discounted loss function.

(*f*) The non-inferior (admissible) set of spending policies were identified. Empirical research aimed at observing the actual decisions taken by the R and D and Marketing planners, and the extension of this model to allow time dependent returns are two directions to investigate the appropriateness of those rules.

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