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*Revue française d'automatique, d'informatique et de recherche
opérationnelle. Recherche opérationnelle*, tome 17, n° 4 (1983),
p. 385-390.

http://www.numdam.org/item?id=RO_1983__17_4_385_0

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CALCULATION OF OPTIMAL CHECKOUT INTERVALS WHEN FAILURE RATE FIGURE INCREASES WITH TIME (*)

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Abstract. — Given the shortcomings of the Poisson's model in describing the behaviour of systems and components subject to wear, the Weibull model has been considered and general formulae have been developed for optimal checkout intervals. Exact and approximate solutions are derived and sensitivity analysis is presented for populations of components for which the failure rate increases linearly in time.

Résumé. — Devant les défauts du modèle de Poisson pour la description du comportement des systèmes et composants sujets à l'usure, on a considéré le modèle de Weibull et on a développé des formules générales pour les intervalles optimaux de révision. On calcule des solutions exactes et approchées et on présente l'analyse de sensibilité pour des composants dont le taux de panne s'accroît linéairement avec le temps.

INTRODUCTION

In the electronic field, the Poisson model is widely used and appreciated. This model consists of the assumptions that the failure rate of a certain population of components is constant in time, that the failures are statistically independent of each other, and that the probability of more than one failure in a very small time interval dt is zero.

In spite of the criticism made of this model, it is universally adopted and is in fact the standard basis for the optimal checkout interval calculations of electronic systems; this is justified by the ease of calculation, which at times is an indispensable factor for the calculation performance itself ([1, 2]).

Among the cases in which the above simple approximation is insufficient to find the solution to specific problems, the example of systems with wear phenomena which cause an increase of the failure rate in time is particularly interesting.

An example of such a problem is the optimal checkout interval calculation of a component which has been subject to wear during the acceptance tests and the

(*) Received May 1982.

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periodic preventive maintenance tests performed on it at regular time intervals during its storage in the arsenal.

The simplest hypothesis which can be formulated to take into account the consequences of wear is to assume an instantaneous failure rate linearly increasing in time:

$$(1.1) \quad z(t) = kt.$$

As is well known, (1.1) is a particular case ($m = 1$) of the more general

$$(1.2) \quad z(t) = kt^m$$

which represents the behaviour in time of the instantaneous failure rate of a population of components whose life duration is given by the probability density function:

$$(1.3) \quad f(t) = kt^m \left[\exp - \frac{k}{m+1} t^{(m+1)} \right],$$

known as the "Weibull distribution".

In this case the general formula for reliability:

$$(1.4) \quad v(t = t_f - t_i) = \exp \left[- \int_{t_i}^{t_f} z(\tau) d\tau \right],$$

where t_i is mission start time and t_f mission end time, becomes:

$$(1.5) \quad v(t) = \exp \left(- \frac{K}{m+1} t^{m+1} \right),$$

when the mission start coincides with the component life start, i.e. $t_i = 0$ and $t_f = t$.

The evaluation of the optimal checkout interval in the case of (1.5) can be analytically carried out only in the case $m = 1$ (Rayleigh model), which is widely known in the literature [3].

In the mechanical and electromechanical field it has the same importance of the Poisson's model in the electronic field, as (1.1) is suitable to represent the behaviour of many mechanical and electromechanical components and systems ([4, 5]).

For example a ball bearing is a very simple component, the failure rate of which can be assumed to depend only on wear. Its behaviour is therefore representable by formula (1.1).

Other examples are the brushes of an electric machine, a pump system for the lifting of liquid, etc.

Therefore after having presented a general model for the calculation of optimal checkout intervals, this note extends Kamin's evaluation model [1] to calculate optimal checkout interval for systems subject to random failures with following survival probability:

$$(1.6) \quad v(t) = \exp(-pt^2)$$

with $p = K/2$.

The analysis is derived following the outline and symbology suggested by Goldmann [2].

2. THE MODEL

In the case of the general formula (1.5), the integrated value of operational ineffectiveness during a cycle is [2]:

$$(2.1) \quad A(T) = \int_R^T [1 - v(t)] dt + (1 - q) v(T) \int_0^R [1 - v(t)] dt + qRv(T) + [1 - v(T)] R.$$

where: T , checkout interval; q , checkout probability of failure; R , checkout operation time.

Therefore average operational readiness is [2]:

$$(2.2) \quad G(T) = 1 - \frac{A(T)}{T}$$

with p , q , and R as input parameters.

By deriving previous equation, necessary condition for optimal T results:

$$(2.3) \quad TA'(T) - A(T) = 0.$$

If Rayleigh's model (1.6) is assumed to be valid, this expression can be analitically worked out.

In this case, through (1.6) and (2.1), eq. (2.3) becomes:

$$(2.4) \quad \varphi(\sqrt{p}T) - \varphi(\sqrt{p}R) + (1 - q)(1 + 2pT^2)e^{-pT^2} \varphi(\sqrt{p}R) - 2\sqrt{\frac{p}{\pi}} Te^{-pT^2} = 0,$$

where $\varphi(x)$ is Kramp's function, related to Laplace's integral $F(x)$ by [6]:

$$(2.5) \quad \varphi(x) = 2F(\sqrt{2}x) - 1.$$

Eq. (2.4) can be solved by means of numerical search techniques. As example, we consider following cases:

$$p=0.1, \quad q=0.2, \quad R=1 \text{ day}$$

and:

$$p=0.1, \quad q=0.8, \quad R=2.3 \text{ day}$$

for which eq. (2.4) yields:

$$T^* \cong 1.1; \quad T^* \cong 3.6,$$

to be respectively compared with Kamin's approximate solution $T_{op} = (2qR/p)^{1/2}$:

$$T_{op} \cong 2; \quad T_{op} \cong 6.1.$$

3. APPROXIMATE SOLUTION

As know, $F(x)$ distribution function is almost linear near to the origin of coordinates, i. e., it can be described with a sufficient accuracy by the first two terms of series development [6]:

$$(3.1) \quad F(x) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^{2k-1}}{2^{k-1}(2k-1)(k-1)!}.$$

Linear approximation can also be used for exponential in eq. (2.4) i. e. $e^x \cong 1+x$ ([1, 2]).

In this way eq. (2.4) becomes through (2.5):

$$(3.2) \quad f(T, p, q, R) = T^3 + R(1-2q)T^2 - qR/p = 0.$$

Simple cubic eq. (3.2) yields a good approximation (5%) if R and resulting T are $< 1/\sqrt{2p}$.

As example in the wrong case: $p=0.1, q=0.8, R=2.3$, approximate solution is $T_a=3.2$ against $T^*=3.6$ with a relative error percent of 11%.

4. MODEL SENSITIVITY

In order to explore the sensitivity of the model, differential analysis of eq. (3.2) can be very useful to see the effect on the optimal checkout interval by varying the input parameters.

In this study it is assumed that $q < 0.5$ and the input parameters were varied, one at a time, over a range of values in order to obtain a set of solutions for optimal inspection interval.

Therefore, by means of (3.2), we have:

$$(4.1) \quad \left(\frac{\partial T}{\partial p} \right)_{R, q} = - \frac{f_p}{f_T} = - \frac{qR}{p^2 T [3T + 2R(1-2q)]}$$

Eq. (3.2) can be solved with respect to p :

$$(4.2) \quad p = \frac{qR}{T[T^2 + R(1-2q)T]}$$

Therefore eq. (4.1) becomes through (4.2):

$$(4.3) \quad \left(\frac{\partial T}{\partial p} \right)_{R, q} = - \frac{T^3 [T + R(1-2q)]^2}{qR [3T + 2R(1-2q)]}$$

An analysis of eq. (4.2), (4.3) shows that $T = T(p)$ curve has a negative sensitivity in the region of interest and validity of eq. (3.2), $0 < T < T_L = 1/\sqrt{2p}$; $0 < p < 1$; (i. e. when p increases along p axis, T asymptotically decreases to zero, while sensitivity (4.3) decreases from ∞ to zero).

With reference to R and q parameters, going on in the same way of eqs. (4.2) (4.3), we respectively obtain:

$$(4.4) \quad R = \frac{pT^3}{p - (1-2q)pT^2},$$

$$(4.5) \quad \left(\frac{\partial T}{\partial R} \right)_{p, q} = \frac{[q - (1-2q)pT^2]^2}{pT^2 [3q - (1-2q)pT^2]},$$

and:

$$(4.6) \quad q = \frac{(T+R)pT^2}{R(1+2pT^2)},$$

$$(4.7) \quad \left(\frac{\partial T}{\partial q} \right)_{p, R} = \frac{R(1+2pT^2)^2}{pT(2pT^3 + 3T + 2R)}$$

Eqs. (4.4) (4.5) demonstrate that $T = T(R)$ curve has a positive sensitivity in the

region of validity of eq. (3.2), i. e.: $0 < T < 1/\sqrt{2p}$ and $0 < R < 1/\sqrt{2p}$ i. e. as R increases along R -axis, T also increases asymptotically from zero to $T_0 = \{q[p(1-2q)]\}^{1/2}$ while derivative (4.5) decreases from ∞ to zero.

With reference to positive derivative (4.7), as q and T increase along q - T -axis, it decreases from ∞ to $2R$.

The previous analysis demonstrates that the checkout optimal interval is sensitive to the changes of input parameters, according to the above said functional relationships.

It suggests that a good accuracy of input parameters would be necessary.

CONCLUSIONS

Given the shortenings of Poisson's model in describing the behaviour of systems and components subject to wear, the obtained results are an original contribution to complete and improve optimal checkout interval evaluations of analogous models.

Since Weibull's model with $m = 1$ can be analytically studied and is suitable to represent the behaviour of many mechanical and electromechanical components, this paper is devoted to populations of components for which the failure rate increases linearly in time.

The author derives exact and approximate solutions and presents sensitivity analysis.

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