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INSPECTION POLICIES: COMPARISONS AND MODIFICATIONS (*)

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Abstract. — *In this paper, we discuss:* (1) *Comparisons between optimum and nearly optimum inspection policies:* (2) *Inpection policies for modified inspection models. In Section* 2, *we numerically compare the optimum inspection policy by Barlow* et al. *with the nearly optimum ones by Kaio and Osaki, Munford and Shahani, and Nakagawa and Yasui, assuming a gamma lifetime distribution. In Section* 3, *we treat a modified inspection model taking account of two kinds of imperfect inspection probabilities, obtain the structure of the optimum inspection policy, and discuss the optimum policy minimizing the total expected cost up to the detection of the failure. The numerical examples are presented. In Section* 4, *we consider another modified inspection model with imperfect inspection probability and checking time. We obtain the nearly optimum inspection policy which minimizes the nearly total expected cost up to the detection of the failure.*

Keywords : Optimum inspection policy, nearly optimum inspection policy, comparison, modified inspection model, imperfect inspection probability.

Résumé. — *Dans cet article, nous traitons des points suivant :* 1) *comparaisons entre politiques optimales et politiques approximativement optimales;* 2) *politiques d'inspection pour les modèles d'inspection modifiés. Dans la section* 2, *nous comparons numériquement la politique d'inspection optimale de Barlow avec les politiques approxivement optimales de Osaki, Munford et Shahani, d'une part, et de Nakagawa et Yasui, d'autre part, en supposant une loi gamma. Dans la section* 3, *nous traitons un modèle d'inspection modifié prenant en compte deux sortes de probabilité d'inspection imparfaite. Nous obtenons la structure de la politique d'inspection optimale, et discutons de la politique optimale minimisant le coût total moyen jusqu'à la détection de la panne. Des exemples numériques sont présentés. Dans la section* 4, *nous considérons un autre modèle d'inspection modifié avec des probabilités et des temps de vérifications imparfaits. Nous obtenons les coûts moyens approximatifs jusqu'à la détection de la panne.*

1. INTRODUCTION

In a system whose failure can be revealed only by inspection executed at specific time sequences, we must apply the most effective inspection procedure to detect the system failure. If we execute frequent inspections to detect the

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failure as soon as possible, we must suffer much cost for inspection. Conversely, if we make infrequent inspections to decrease cost for inspection, we must suffer much cost for system down because of its longer interval. We must obtain the inspection procedure which balances costs for inspection and system down, i. e., the optimum inspection policy which minimizes the total expected cost composed of costs for inspection and system down. Based on this point of view, several papers for inspection policies have been published [1-10].

In these inspection policies, the inspection policy obtained by Barlow *et al.* [1, 2] is the most famous. They have discussed the optimum inspection policy in the following model: A one-unit system is considered, which obeys an arbitrary lifetime distribution $F(t)$ with a pdf (probability density function) $f(t)$. The system begins operating at time 0, and the planning horizon is infinite. The system failure is detected only by inspection. The system is inspected at prespecified times t_k $(k=1, 2, 3, \ldots)$, where each inspection is executed perfectly and instantaneously and does not cause the deterioration or the failure of the system. The policy continues until the system failure is detected by any inspection. The system is not repaired or replaced by a new unit, i. e. the system is not renewed and the policy terminates, when the system failure is detected. The two costs considered are the inspection cost c_c per each inspection and the cost of system down time k_f per unit time. Then, the total expected cost up to the detection of the failure is;

$$C_B = \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} [c_c (k+1) + k_f (t_{k+1} - t)]\, d F(t). \tag{1.1}$$

Barlow *et al.* [1, 2] have obtained the algorithm to seek the optimum inspection time sequence which minimizes the total expected cost in equation (1.1) by using the recurrence formula;

$$t_{k+1} - t_k = [F(t_k) - F(t_{k-1})]/f(t_k) - c_c/k_f; k=1, 2, 3, \ldots, \tag{1.2}$$

where $f(t)$ is a PF_2 (Pólya frequency function of order 2) with $f(t+\Delta)/f(t)$ strictly decreasing for $t \geqq 0$, $\Delta > 0$ and with $f(t) > 0$ for $t > 0$, and $t_0 = 0$. The alforithm is as follows:

begin

choose t_1 to satisfy $c_c = k_f \int_0^{t_1} F(t)\, dt;$

repeat

compute $t_2, t_3, \ldots$ recursively using equation (1.2);

if any $t_{k+1} - t_k > t_k - t_{k-1}$

```
    then reduce t_1;
    if any t_{k+1} - t_k < 0
    then increase t_1;
  until t_1 < t_2 < ... are determined to the degree of accuracy required
end;
```

However, this algorithm by Barlow *et al.* is complicated to be executed, because one must apply trial and error to decide the first inspection time t_1 and the assumption to $f(t)$ is really strong. To overcome these difficulties, some improved procedures to obtain the nearly optimum inspection policy have been proposed [3-8]. For example, Keller [3] proposed the nearly optimum inspection policy introducing a smooth density which presents the number of inspections per unit time. Further, Kaio and Osaki [4] have developed Keller's method using the smooth density, which is called inspection density, and obtained the more analytically exact nearly optimum inspection policy. Munford and Shahani [5] have obtained the nearly optimum inspection policy assuming that the probability of the failure occurence between the successive inspections is constant. Nakagawa and Yasui [8] have proposed an improved method based on Barlow *et al.*'s one [1, 2] and obtained the nearly optimum inspection policy in which the successive inspection times are computed backward assuming that an appropriate inspection time is previously given after a large number of inspections.

Several modifications to Barlow *et al.*'s model [1, 2] have been proposed [4, 9, 10]. For example, inspection model where the system may be incapable of detecting its failure due to imperfect inspection has been considered [4, 9]. Kaio and Osaki [4] have considered the inspection model with checking time. Wattanapanom and Shaw [10] have discussed the inspection model in which the system deteriorates by each inspection.

In this paper, we discuss: (1) Comparisons between optimum and nearly optimum inspection policies: (2) Inspection policies for modified inspection models. In Section 2, we numerically compare the optimum inspection policy by Barlow *et al.* [1, 2] with nearly optimum ones by Kaio and Osaki [4], Munford and Shahani [5], and Nakagawa and Yasui [8], assuming a gamma lifetime distribution. In Section 3, we treat a modified inspection model taking account of two kinds of imperfect inspection probabilities, and obtain the structure of the optimum inspection policy, and discuss the optimum policy minimizing the total expected cost up to the detection of the failure. The numerical examples are presented. In section 4, we consider second modified inspection model with imperfect inspection probability and checking time. We obtain the nearly optimum inspection policy which minimizes the nearly total expected cost up to the detection of the failure.

2. COMPARISONS BETWEEN OPTIMUM AND NEARLY OPTIMUM INSPECTION POLICIES

In this section, we numerically compare the optimum inspection policy by Barlow *et al.* [1, 2] with nearly optimum ones by Kaio and Osaki [4], Munford and Shahani [5], and Nakagawa and Yasui [8], assuming a gamma lifetime distribution. We declare that there are not significant differences among the optimum and nearly optimum inspection policies and we should apply a handy inspection policy such as by Kaio and Osaki [4] irrespective of optimum and nearly optimum ones.

2.1. Review on three nearly optimum inspection policies

The nearly optimum inspection policies proposed by Kaio and Osaki [4], Munford and Shahani [5], and Nakagawa and Yasui [8] are reviewed. The inspection model and notation used in each policy follow Barlow *et al.*'s ones in Section 1. For details, see each contribution.

2.1.1. *Nearly optimum inspection policy by Kaio and Osaki* (*K & O policy*)

Let us introduce the inspection density at time t, $n(t)$, which is a smooth function and denotes the approximate number of inspections per unit time at time t. Then, the nearly total expected cost up to the detection of the failure is;

$$C_n(n(t))=c_c\int_0^\infty n(t)\bar{F}(t)\,dt+k_f\int_0^\infty 1/[2n(t)]\,dF(t), \tag{2.1}$$

where $\bar{\Psi}=1-\Psi$, in general. The density $n(t)$ which minimizes the functional $C_n(n(t))$ in equation (2.1) is;

$$n(t)=[k_c r(t)]^{1/2}, \tag{2.2}$$

where $k_c=k_f/(2c_c)$, and $r(t)=f(t)/\bar{F}(t)$; a failure rate.

The inspection times t_k $(k=1, 2, 3, \ldots)$ satisfy;

$$k=\int_0^{t_k} n(t)\,dt; \qquad k=1, 2, 3, \ldots. \tag{2.3}$$

Substituting $n(t)$ in equation (2.2) into equation (2.3) yields the nearly optimum inspection time sequence.

For details, *see* Kaio and Osaki [4]. Note that any assumption for the pdf $f(t)$ does not exist.

2.1.2. *Nearly optimum inspection policy by Munford and Shahani* (M & S policy)

Put

$$[F(t_k)-F(t_{k-1})]/\bar{F}(t_{k-1})=p; \qquad k=1, 2, 3, \ldots; \qquad 0<p<1. \tag{2.4}$$

Then, the inspection times t_k $(k=1, 2, 3, \ldots)$ *are;*

$$t_k=F^{-1}(1-\bar{p}^k); \qquad k=1, 2, 3, \ldots, \tag{2.5}$$

where the probability p is chosen such that the nearly total expected cost up to the detection of the failure, $C_p(p)$, is minimized;

$$C_p(p)=c_c/p+k_f\left(\sum_{k=1}^{\infty} t_k\bar{p}^{k-1}p-\int_0^{\infty} tf(t)\,dt\right). \tag{2.6}$$

Any assumption for the *pdf* $f(t)$ does not exist. For details, *see* Munford and Shahani [5], and further Munford and Shahani [6] for the case of Weibull distribution and Tadikamalla [7] for the gamma one.

2.1.3. *Nearly optimum inspection policy by Nakagawa and Yasui* (N & Y policy)

This procedure is based on one by Barlow *et al.* [1, 2]. If the pdf $f(t)$ is a PF_2, the following algorithm is obtained:

```
begin
  choose d appropriately for 0<d<c_c/k_f;
    determine t_n after sufficient time has elapsed to
      give the degree of accuracy required;
    compute t_{n-1} to satisfy
      t_n-t_{n-1}-d=[F(t_n)-F(t_{n-1})]/f(t_n)-c_c/k_f;
  repeat
      compute t_{n-2}>t_{n-3}>... recursively using equation (1.2);
  until t_i<0 or t_{i+1}-t_i>t_i
end;
```

For details, *see* Nakagawa and Yasui [8].

2.2. Numerical comparisons and remarks

We numerically compare the optimum inspection policy by Barlow *et al.* [1, 2] (**B policy** in the following) with nearly optimum ones; K & O, M & S and N & Y policies, assuming a gamma lifetime distribution. For the following numerical examples, when $F(t_N) \geqq 99.9\%$ for the first time, the inspection time t_N is the final one. Each total expected cost is obtained from C_B in equation (1.1).

We discuss the case that the lifetime distribution is a gamma one;

$$F(t) = \int_0^t \exp(-\lambda\tau)\,\lambda(\lambda\tau)^{m-1}/\Gamma(m)\,d\tau; \quad \lambda,\ m>0;$$

$$\text{where } \Gamma(m) \text{ is the gamma function.} \qquad (2.7)$$

TABLE I

Optimum and nearly optimum inspection policies, their total expected costs and the sum of relative errors ($F(t)=\int_0^t \exp(-\lambda\tau)\,\lambda(\lambda\tau)^{m-1}/\Gamma(m)\,d\tau$, $c_c=20$, $k_f=1$, $\lambda=0.01$ *and* $m=2$).

t_k	B policy		K & O policy	M & S policy	N & Y policy ($d=10$)
t_1	122.889	122.941	113.923	130.713	126.167
t_2	199.605	199.718	195.393	206.099	202.523
t_3	269.993	270.202	271.101	272.970	272.789
t_4	337.286	337.649	343.966	335.607	340.051
t_5	402.639	403.257	415.095	395.628	405.444
t_6	466.578	467.617	485.050	453.846	469.502
t_7	529.325	531.071	554.143	510.737	532.487
t_8	590.900	593.836	622.576	566.602	594.490
t_9	651.119	656.062	690.489	621.649	655.449
t_{10}	709.529	717.861	757.978	676.026	715.117
t_{11}	765.285	779.321	825.116	729.844	772.983
t_{12}	816.956	840.526	891.958	783.186	828.133
t_{13}	862.282	901.562	958.547	836.119	879.044
t_{14}	898.005	962.535	–	888.695	923.341
t_{15}	920.038	–	–	940.959	–
t_{16}	924.379	–	–	–	–
Total expected cost	95.1056	95.2103	95.5383	95.3855	95.1314
Sum of relative errors; for total expected cost ($\times 10^{-8}$)	0	110 088	454 968	294 304	27 127.7
for inspection time sequence ($\times 10^{-6}$)	0	198 808 (1)	699 366 (2)	476 718 (3)	171 048 (1)

(1) This is the sum of the relative errors from t_1 to t_{14}.
(2) This is the sum of the relative errors from t_1 to t_{13}.
(3) This is the sum of the relative errors from t_1 to t_{15}.

In Table I, we present the numerical results with $c_c=20$, $k_f=1$, $\lambda=0.01$ and $m=2$, including B, K & O, M & S and N & Y policies. In B policy, Barlow *et al.*'s algorithm can give several optimum inspection time sequences since t_N is the final inspection time when $F(t_N)\geqq 99.9\%$ for the first time. We present the two results with the smallest t_1 and the largest one, where we regard the former policy as the optimum one since its total expected cost is the smallest among several numerical results. From Table I, we conclude that N & Y policy is relatively better than K & O and M & S ones, but there are not significant differences among the optimum and nearly optimum inspection policies, and thus we should apply a handy inspection policy such as by Kaio and Osaki [4] irrespective of optimum and nearly optimum ones.

There are the following merits when we use K & O policy:

(i) We can obtain the nearly optimum inspection policy uniquely, immediately and easily from equations (2.2) and (2.3) for any distributions, whereas B and N & Y policies cannot treat non-PF_2 distributions.

(ii) We can analyze the more complicated models and easily obtain their nearly optimum inspection policies (e. g., *see* Kaio and Osaki [4] and Section 4).

3. INSPECTION POLICY WITH TWO KINDS OF IMPERFECT INSPECTION PROBABILITIES

In this section, we discuss a modified inspection model with the following two kinds of imperfect inspection probabilities: Consider a system whose failure can be detected only by inspection. However, two kinds of imperfect inspections are possible; (i) the system may be regarded as failure even if it is normally operating due to imperfect inspection (error of the first kind); (ii) the system may be incapable of detecting its failure due to imperfect inspection (error of the second kind). We assume that the two kinds of imperfect inspection probabilities above are given. For this model, we obtain the structure of the optimum inspection policy, and discuss the optimum policy minimizing the total expected cost up to the detection of the failure. We present the numerical examples.

3.1. Model and assumptions

A one-unit system is considered. The new system begins operating at time 0, and the planning horizon is infinite. The failure time for the system itself obeys an exponential distribution with the cumulative distribution function;

$$F(v)=1-\exp(-\lambda v); \qquad v\geqq 0, \qquad \lambda>0. \tag{3.1}$$

The system failure is revealed only by inspection made at inspection times $t_k (k=1, 2, 3, \ldots)$, and each inspection is made instantaneously. Since each inspection is made imperfectly; (i) the system may be regarded as failure even if it is normally operating, with probability a; (ii) the system may be incapable of detecting its failure due to imperfect inspection, with probability b; where $a+b<1$ since these probabilities are relatively small. The policy ends when the inspection indicates the system is failed irrespective of the actual state of the system, and the system does not make any repair or replacement.

The costs considered here are; a cost k_r per unit time that is suffered for residual lifetime when the system is indicated to be failed whereas the system is normally operating; and costs c_c and k_f, those are as same as ones in Barlow *et al.*'s model in Section 1.

3.2. Structure of optimum inspection policy

We define the following:

time t — the time when the inspection is executed;

time $t-$ — the time immediately before the inspection is executed;

time $t+$ — the time immediately after the inspection is executed;

$p(v)$ — the conditional probability that the system is in the failure state at time v, given that the failure has not been revealed between the time interval $[0, v]$.

Then, we clearly have the following relationships among $p(0)$, $p(t-)$ and $p(t+)$.

THEOREM (3.2): *We have the following relationships among the conditional probability $p(v)$;*

$$p(0)=0; \tag{3.3}$$

$$p(0)<p(t+); \tag{3.4}$$

and

$$p(t-)>p(t+). \tag{3.5}$$

Thus, we can obtain the structure of the optimum inspection policy, from the properties of the conditional probability $p(v)$:

Structure of optimum inspection policy: Optimum inspection times are the following;

$$t_1=x; \tag{3.6}$$

$$t_{i+1}=x+i\cdot y; \qquad i=1, 2, 3, \ldots ; \tag{3.7}$$

where

$$x>y. \tag{3.8}$$

That is, the first inspection is executed after the time interval x, and after that the inspections are executed with time interval y periodically.

3.3. Optimum inspection time intervals x and y

We obtain the optimum inspection time intervals x and y in the structure of optimum inspection policy. We apply, as a criterion of optimality, the total expected cost from the beginning of the operation at time 0 to the detection of the failure, and obtain the optimum intervals x and y which minimize this total expected cost.

The following three expected costs are obtained: (i) The expected inspection cost, from the beginning of the operation to the detection of the failure;

$$c_c[e^{-\lambda x}/(1-\bar{a}e^{-\lambda y})+1/\bar{b}]. \tag{3.9}$$

(ii) The expected loss cost, suffered for the residual lifetime when the system is indicated to be failed whereas the system is normally operating;

$$k_r ae^{-\lambda x}[1/\lambda-x-a\bar{a}e^{-\lambda(x+y)}y/(1-\bar{a}e^{-\lambda y})^2]/(1-\bar{a}e^{-\lambda y}). \tag{3.10}$$

(iii) The expected shortage cost, suffered for the system down from the system failure to its detection;

$$\begin{aligned}k_f[1-e^{-\lambda x}+\bar{a}e^{-\lambda x}(1-e^{-\lambda y})/(1-\bar{a}e^{-\lambda y})]\\ \times[x+\{\bar{a}e^{-\lambda x}(1-e^{\lambda y})/(1-\bar{a}e^{-\lambda y})^2+b/\bar{b}\}y-1/\lambda].\end{aligned} \tag{3.11}$$

Thus, the total expected cost from the beginning of the operation at time 0 to the detection of the failure is as follows;

$$\begin{aligned}C(x; y)=c_c[e^{-\lambda x}/(1-\bar{a}e^{-\lambda y})+1/\bar{b}]\\ +k_r ae^{-\lambda x}[1/\lambda-x-a\bar{a}e^{-\lambda(x+y)}y/(1-\bar{a}e^{-\lambda y})^2]\\ /(1-\bar{a}e^{-\lambda y})\\ +k_f[1-e^{-\lambda x}+\bar{a}e^{-\lambda x}(1-e^{-\lambda y})/(1-\bar{a}e^{-\lambda y})]\\ \times[x+\{\bar{a}e^{-\lambda x}(1-e^{-\lambda y})/(1-\bar{a}e^{-\lambda y})^2\\ +b/\bar{b}\}y-1/\lambda].\end{aligned} \tag{3.12}$$

The optimum inspection time intervals x and y which minimize the total expected cost in equation (3.12) are obtained as the solutions of the following simultaneous equations;

$$\begin{cases} -c_c\lambda e^{-\lambda x}/(1-\bar{a}e^{-\lambda y})+k_r a e^{-\lambda x}[2\{-1+\lambda a\bar{a}e^{-\lambda(x+y)}y \\ \qquad /(1-\bar{a}e^{-\lambda y})^2\}+\lambda x]/(1-\bar{a}e^{-\lambda y}) \\ \qquad +k_f[ae^{-\lambda x}\{\lambda x+\lambda(2\bar{a}e^{-\lambda x}(1-e^{-\lambda y})/(1-\bar{a}e^{-\lambda y})^2 \\ \qquad +b/\bar{b})y-2\}/(1-\bar{a}e^{-\lambda y})+1-\lambda\bar{a}e^{-\lambda x}(1-e^{-\lambda y})y \\ \qquad /(1-\bar{a}e^{-\lambda y})^2]=0; & (3.13) \\ k_r a\bar{a}e^{-\lambda(x+y)}[1+ae^{-\lambda x}\{(\bar{a}e^{-\lambda y}/(1-\bar{a}e^{-\lambda y})+1)\lambda y \\ \qquad -1\}/(1-\bar{a}e^{-\lambda y})]/(1-\bar{a}e^{-\lambda y})^2 \\ \qquad +k_f[1-e^{-\lambda x}+\bar{a}e^{-\lambda x}(1-e^{-\lambda y})/(1-\bar{a}e^{-\lambda y})] \\ \qquad \times[\bar{a}e^{-\lambda y}\{-1+\lambda e^{-\lambda x}(1-\bar{a}(1-e^{-\lambda y})/(1-\bar{a}e^{-\lambda y}))y \\ \qquad /(1-\bar{a}e^{-\lambda y})\}/(1-\bar{a}e^{-\lambda y})+\bar{a}e^{-\lambda x}(1-e^{-\lambda y}) \\ \qquad /(1-\bar{a}e^{-\lambda y})^2+b/\bar{b}]=0. & (3.14) \end{cases}$$

3.4. Remarks

For the probabilities a and b, it is stated that it may be that there is a choice among available inspection methods, so that the probabilities a and b could be controllable parameters (*see* Shahani and Crease [11]), where the case that $a \geqq b$ even occurs. Also, in this model, if we put $a=0$, then this model becomes equivalent to the model discussed by Sengupta [9].

TABLE II

Optimum inspection time intervals x and y, and their minimal expected cost $C(x;\, y)$ ($\lambda=0.02$, $c_c=10$, $k_r=3$, $k_f=4$ and $b=0.1$).

a	x	y	$C(x;\, y)$
0.0	15.11	13.37	77.48
0.1	20.04	7.70	13.68
0.2	24.53	9.65	5.17
0.3	27.78	11.13	3.89
0.4	30.42	12.03	4.32
0.5	32.67	12.28	5.27
0.6	34.67	11.69	6.33
0.7	36.48	9.76	7.28
0.8	38.19	5.14	7.85

In Table II, we present the numerical examples of the optimum inspection time intervals x and y, and their minimal expected cost $C(x;\, y)$, as the dependence of the probability a where all the other parameters are fixed. We have computed the numerical examples by implementing the software for personal computer **GINO/PC** (General **IN**teractive **O**ptimizer) to solve the non-linear equations.

4. INSPECTION POLICY WITH IMPERFECT INSPECTION PROBABILITY AND CHECKING TIME

In this section, we discuss the inspection policy for the modified inspection model with; (1) the imperfect inspection probability that the system may be incapable of detecting its failure due to imperfect inspection (*see* Section 3); (2) the checking time, i. e., the time for an inspection. We obtain the nearly optimum inspection policy which minimizes the nearly total expected cost up to the detection of the failure. Examples are presented for illustration.

4.1. Model and assumptions

The system may be incapable of detecting its failure due to imperfect inspection, with probability b, i. e., the failure may be detected with probability $\bar{b}$, where $0 \leqq b < 1$. Each inspection has the constant checking time T_c. Other assumptions and notation used in this policy follow Barlow *et al.*'s ones in Section 1 and Kaio and Osaki's ones in Section 2.

4.2. Analysis and result

The following two expected costs are obtained: (i) The nearly expected inspection cost up to the detection of the failure;

$$c_c\left[\int_0^\infty n(t)\,\bar{F}(t-T_c)\,dt+b/\bar{b}\right]. \tag{4.1}$$

(ii) The nearly expected cost suffered for the system down from the failure to its detection;

$$k_f\left[(1+b)/(2\,\bar{b})\int_0^\infty 1/n(t)\,dF(t)+T_c\right]. \tag{4.2}$$

Thus, the nearly total expected cost from the beginning of the operation at time 0 to the detection of the failure is;

$$C_m(n(t)) = c_c \int_0^\infty n(t)\,\bar{F}(t-T_c)\,dt + k_f(1+b)/(2\bar{b}) \int_0^\infty 1/n(t)\,dF(t) + c_c b/\bar{b} + k_f T_c. \quad (4.3)$$

We obtain the inspection density $n(t)$ which minimizes the functional $C_m(n(t))$. This is a problem of calculus of variations in which $n(t)$ is the unknown function. We obtain Euler's equation;

$$c_c \bar{F}(t-T_c) - k_f(1+b) f(t)/[2\bar{b}n^2(t)] = 0. \quad (4.4)$$

We solve equation (4.4) with respect to $n(t)$;

$$n(t) = K_b[\bar{F}(T_c \mid t-T_c)\, r(t)]^{1/2}, \quad (4.5)$$

where $K_b = [(1+b)k_f/(2\bar{b}c_c)]^{1/2}$. The resultant $n(t)$ in equation (4.5) also satisfies the sufficient condition.

Substituting $n(t)$ in euqation (4.5) into equation (2.3) yields the nearly optimum inspection time sequence in a similar fashion of the procedure by Kaio and Osaki in Section 2. We can easily use this result for practical numerical computation.

4.3. Examples

4.3.1. *Case with an exponential distribution*

If the lifetime obeys the exponential distribution in equation (3.1), then we obtain

$$n(t) = K_b[\lambda \exp\{\lambda \max(0, t-T_c) - \lambda t\}]^{1/2} \quad (4.6)$$

$$= \begin{cases} K_b[\lambda \exp(-\lambda t)]^{1/2}; & t \leqq T_c \\ K_b[\lambda \exp(-\lambda T_c)]^{1/2}; & t \geqq T_c. \end{cases}$$

Thus, if $t \geqq T_c$, then

$$t_k - t_{k-1} = 1/[K_b\{\lambda \exp(-\lambda T_c)\}^{1/2}]. \quad (4.7)$$

That is, the interval between the inspections increases initially and is constant after that.

4.3.2. *Case with a Weibull distribution*

If the lifetime obeys the Weibull distribution;

$$F(t)=1-\exp(-t^{y}); \qquad t \geqq 0, \quad y>0, \tag{4.8}$$

then,

$$n(t)=K_b [y t^{y-1} \exp\{(\max(0, t-T_c))^{y}-t^{y}\}]^{1/2}. \tag{4.9}$$

Thus, from equation (2.3),

$$k=K_b \int_0^{t_k} [y t^{y-1} \exp\{(\max(0, t-T_c))^{y}-t^{y}\}]^{1/2} dt; \qquad k=1, 2, 3, \ldots, \tag{4.10}$$

from which we can obtain the nearly optimum inspection policy. Case with the exponential distribution is a special one of this example.

4.4. Remarks

The function $n(t)$ in equation (4.5) gives the local minimum, strictly speaking. However, that $n(t)$ gives a minimum, perhaps.

If we specify $b=0$ and/or $T_c=0$, then this model becomes equivalent to the model discussed by Kaio and Osaki [4], respectively.

5. CONCLUSIONS

In this paper, we have summarized the inspection policies/models, i. e., we have discussed: (1) Comparisons between optimum and nearly optimum inspection policies: (2) Inspection policies for modified inspection models. In Section 2, we have numerically compared the optimum inspection policy by Barlow *et al.* [1, 2] with nearly optimum ones by Kaio and Osaki [4], Munford and Shahani [5], and Nakagawa and Yasui [8], and we have concluded that there are not significant differences among the optimum and nearly optimum inspection policies and we should apply a handy inspection policy such as by Kaio and Osaki [4]. In Section 3, we have discussed the modified inspection model taking account of the errors of the first and second kinds, and obtained the optimum inspection policy minimizing the total expected cost up to the detection of the failure. In Section 4, we have treated the second modified inspection model taking account of the error of the second kind and the checking time, and obtained the nearly optimum inspection policy minimizing the nearly total expected cost up to the detection of the failure.

The inspection policy is one of the most important policies applicable in the practical systems. Thus, to widen state of this art is powerfully wished in theoretical and/or practical aspects.

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REFERENCES

1. R. E. Barlow, L. C. Hunter and F. Proschan, *Optimum checking procedures*, J. Soc. Indust. Appl. Math., Vol. 11, 1963, pp. 1078-1095.
2. R. E. Barlow and F. Proschan, *Mathematical theory of reliability*, Wiley, New York, 1965.
3. J. B. Keller, *Optimum checking schedules for systems subject to random failure*, Management Science, Vol. 21, 1974, pp. 256-260.
4. N. Kaio and S. Osaki, *Some remarks on optimum inspection policies*, I.E.E.E. Trans. Reliab., Vol. R-33, 1984, pp. 277-279.
5. A. G. Munford and A. K. Shahani, *A nearly optimal inspection policy*, Opl Res. Q., Vol. 23, 1972, pp. 373-379.
6. A. G. Munford and A. K. Shahani, *An inspection policy for the weibull case*, Opl Res. Q., Vol. 24, 1973, pp. 453-458.
7. P. R. Tadikamalla, *An inspection policy for the gamma failure Distributions*, J. Opl Res. Soc., Vol. 30, 1979, pp. 77-80.
8. T. Nakagawa and K. Yasui, *Approximate calculation of optimal inspection times*, J. Opl Res. Soc., Vol. 31, 1980, pp. 851-853.
9. B. Sengupta, *An exponential riddle*, J. Appl. Prob., Vol. 19, 1982, pp. 737-740.
10. N. Wattanapanom and L. Shaw, *Optimal inspection schedules for failure detection in a model where tests hasten failures*, Oper. Res., Vol. 27, 1979, pp. 303-317.
11. A. K. Shahani and D. M. Crease, *Towards Models of screening for early detection of disease*, Adv. Appl. Prob., Vol. 9, 1977, pp. 665-680.