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J. H. Kuiper

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LOCATION PATTERNS AND DISTANCES IN TINBERGEN-BOS SYSTEMS (*)

by J. H. KUIPER (1)

Abstract. — *In this paper a general mathematical programme is introduced, for the computation of optimal location patterns for a member of industrial sectors in well-defined areas. The economy is organized according to Tinbergen-Bos systems. The principles introduced in this paper are the starting point of a number of new investigations leading to optimal location patterns in Manhattan circles.*

Keywords : Location pattern; Tinbergen-Bos system; computation of transportation costs; networks.

1. INTRODUCTION

In Tinbergen-Bos systems for general location analysis the spatial dispersion of economic activities is studied; it includes many elements: multiple sectors, multiple production centres and multiple market areas. These models lead to an optimal location pattern of production centres based on the criterion of minimum transportation cost.

A production centre is a spatial point, where one or more industrial sectors are produced; a system of centres is a set of centres where all defined industrial sectors are located. Using the equilibrium conditions concerning the balance of trade, income and production equations of those centres, it is possible to express the total value of transported products between the centres in an exogenous income variable of the system (Y).

An important problem is the computation of the transportation costs determining the optimal system; in order to be able to compute the transportation costs, the distance between centres should be determined and in order

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(1) Erasmus University Rotterdam, Faculty of Economics, Department of Theoretical Spatial Economics, The Netherlands.

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to be able to compute the distances, the locations should be known. This paper will focus on the computation of the transportation costs and therefore on the distances and the location patterns. The production centres are located in a well-defined bordered area. The value of product streams between the centres will depend on the distance between the centres and also on the internal organisation of each centre; a centre may include many different industrial sectors (a concentration) or just one sector (a deconcentration).

So, in order to be able to compute the transportation costs, a number of hypothesis concerning the area (its shape and its site), the potential locations and the way distances are computed, will be necessary.

First a survey of compositions of optimal systems will be presented, including advantages and disadvantages of these methods; characteristic for each method will be the computation of distances between centres. Finally a general mathematical programme is presented leading to an optimal location pattern in a Tinbergen-Bos system.

2. DISTANCES IN TINBERGEN-BOS SYSTEMS

In the first models (Paelinck and Nijkamp, 1975) the transportation costs were computed without defining or computing a location pattern of centres. Transportation costs are determined by

$$T=\sum_i \Theta_i E_i \tag{2.1}$$

(E_i being the transported quantity of sector i, Θ_i is a constant value depending on sector i).

So the transportation costs are found by a weighted sum of product values; this means that in this area the discrete distance measure is used:

$$d(x, y)=\begin{cases} 0 & x=y \\ 1 & x\neq y \end{cases}$$

(x and y are centre locations).

By expressing all values of E_i in an exogenous income variable Y, the transportation costs can be computed in a simple way; an optimal location pattern of the centres cannot be found in this way, In fact, space is supposed to consist of a very limited number of points, without knowing the exact locations.

Only the optimal number of centres and their type (concentrated or not) can be found. Optimal solutions found, in this way, showed concentrated centres (Kuiper, Paelinck, 1984).

A second way, dealing with transportation costs, without using a location pattern, is to assume that these costs depend on average distances. Transportation costs of a given volume of commodities from one centre to a sufficiently large number of other centres are approximated by the costs of transporting the same volume from this centre over a surrounding continuous circular market with a constant densitiy of demand using Euclidean distances (Paelinck and Nijkamp, 1975) or assuming a squared market area with rectangular distances (Kuiper, 1988).

In this way it is not possible to compute minimum transportation costs; an optimal solution will depend on the location pattern of economic activities. Space in these models only consists of one or more continuous areas (circles or squares). In case of circles the space is not covered completely by market areas.

In order to compute optimal location patterns it was necessary to define locations in a differentiated space; the computation of transportation costs between two centres is possible if, both the location of the centres and the volume of the product stream is known. In Kuiper (1989 *a*, 1989 *b*) a square is used of known dimensions where all activities take place; the production centres are supposed to be located on a rectangular network. Like in all other models the agricultural sector is distributed in a continuous way over the area. In constructing a spatial pattern some extra hypotheses are used: each firm i is connected to an equal part of the bounded area; this area is a square so the total area is divided in a number of squares related to each defined sector.

This assumption is based on the idea that sectors of high rank need a sufficiently large area, both to be able to find enough inputs and enough other centres and consumers to sell their products. Low ranked sectors need just a small area because they are less specialized so need relatively few inputs and products are sold directly to the customers. Centres of equal rank are related to equal parts of the area, they have equal market areas.

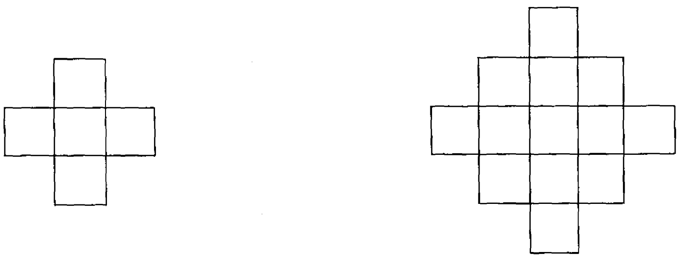
In Kuiper, Mares (1990) a number of general conditions are derived for an optimal location pattern using these assumptions.

Introducing a differentiated space leads to a number of useful conclusions. Firms prefer a deconcentrated location if transported volumes or tarifs of their products to the agricultural sector are important. Sectors tend to concentrate if many goods have to be transported between them and if tarifs are high.

Resuming, a location area should be defined in a more general way, offering enough possibilities to locate an arbitrary number of firms; a programme should be constructed that allows the computation of a general optimal spatial location pattern; the optimal solution has to indicate the number of centres of each type, including the number of firms of each sector located in it and its location; also all transported quantities between each pair of centres has to be a part of the optimal solution.

3. A GENERAL MATHEMATICAL PROGRAMME

A first step to a more general spatial structure (in Kuiper, Paelinck, Rosing, 1989) was the introduction of "Manhattan circles" where all sectors are produced. The area is organised around a rectangular network with unit mazes. The "radius" of the Manhattan circles is integer; the area is structured by unit Manhattan circles (squares) centered on the network nodes.



$R=1$ $R=2$

Figure 3.1. – Manhattan circles.

The number of potential locations, depending on the radius R of the circle, equals: $2R(R+1)+1$.

A mathematical programme will be derived, leading to optimal Tinbergen-Bos systems.

Industrial production in Tinbergen-Bos models takes place in a hierarchically organised production structure. There are I industrial sectors and for each sector production is performed in firms of optimal size, $\bar{Y}_i$; the number of firms needed to produce the known total demand of product i (Y_i) is n_i (n_i

is integer). It is assumed that

$$n_1 \geqq n_2 \geqq \ldots \geqq n_I = 1.$$

Industry I showing the highest rank; the production centres are characterised by the industry of highest rank located in the centre. According to a classical assumption, trade between centres of the same rank is excluded.

Next exogenous variables will be used in the programme.

Y_i total income of sector i

n_i number of firms of sector i,

a_i propensity of consume products of sector i

d_{sr} distance between location s and r; $d_{sr}=0$ if $s=r$,

t^i_{sr} transport tarif of transporting product i from centre s to r.

Next endogenous variables are used in the programme:

x_{is} number of firms of sector i located in s (x_{is} is integer); (agriculture $i=0$ is distributed homogeneously, so $x_{0s}=1$, $\forall s$).

X^i_{sr} transported volume of i from s to r, $X^i_{sr} \geqq 0$.

Each firm of type i has to deliver an equal share to all firms of type i'; all firms i' require $a_i Y'_i$ of i, so from each firm i, $a_i Y'_i/n_i$ is transported to firms i'.

The total exported volume of i is $a_i \sum_{i'=0}^{I} Y_{i'} = a_i (Y - Y_i)$; so each firm exports $(a_i/n_i)(Y - Y_i)$.

In each location s demand for i of firms i' is:

$$x_{i's} \frac{a_i Y'_i}{n_{i'}};$$

total demand in s for products i is:

$$\sum_{\substack{i'=0 \\ i' \neq i}} x_{i's} \frac{a_i Y_{i'}}{n_{i'}}$$

Each firm has to be located somewhere inside the Manhattan circle, so

$$\sum_s x_{is} = n_i, \qquad \forall i, \quad i \neq 0. \tag{3.1}$$

The rank of a sector is higher according as the number of firms is lower. Regions are also ranked; the rank of a region is determined by the rank of the highest ranked sector located in the region. According to a classical hypothesis of Tinbergen-Bos systems, only 1 firm of the highest rank is located in the region. This can be achieved by

$$\left(\sum_{i=0}^{I} x_{is}-1\right)\left(\prod_{i=1}^{I}(x_{is}-1)\right)=0, \qquad \forall s \tag{3.2}$$

Looking at the supply side, X^i_{sr} is defined as the value of i delivered by the centres located in s to the centres located in r. Let $X^i_{s.}$ be the total supply of i from s, so

$$X^i_{s.} \stackrel{\Delta}{=} \sum_r X^i_{sr}$$

The total supply of i in the system should equal the total demand so

$$\sum_s X^i_{s.} = a_i Y, \qquad \forall i \tag{3.3}$$

In s, there are x_{is} firms producing i, so

$$X^i_{s.} = \frac{x_{is}}{n_i} Y_i, \qquad \forall s, \quad \forall i \tag{3.4}$$

(so if $x_{is}=0$, $X^i_{s.}=0$).
In each location s, supply equals demand for i, so

$$\sum_s X^i_{sr} = \sum_{i'} x_{i'r} \frac{a_i Y^{i'}_i}{n_{i'}}, \qquad \forall r, \quad \forall i. \tag{3.5}$$

For each centre import equals export, so

$$\sum_i \sum_{\substack{r \\ r \neq s}} X^i_{sr} = \sum_i \sum_{\substack{r \\ r \neq s}} X^i_{rs}, \qquad \forall s \tag{3.6}$$

The system is optimised by minimisation of the total transportation costs. The transportation costs related to the transportation of i between s and r are:

$$T^i_{sr} = d_{sr} X^i_{sr} t^i_{sr} \tag{3.7}$$

The total transportation costs between s and r are:

$$T_{sr}=\sum_{i} d_{sr} X^{i}_{sr} t^{i}_{sr}$$

The total transportation costs T, that should be minimised, are

$$T=\sum_{s} \sum_{r} \mathrm{T}_{sr}. \qquad (3.8)$$

To sum up next mathematical programme has to be solved:

$$\min T=\sum_{s} \sum_{r} \sum_{i} \mathrm{d}_{sr} X^{i}_{sr} t^{i}_{sr}$$

s. t.

$$\sum_{s} x_{is}=n_{i}, \qquad \forall i, \quad i \neq 0$$

$$\left(\sum_{i=0}^{I} x_{is}-1\right)\left(\prod_{i=0}^{I} (x_{is}-1)\right)=0, \qquad \forall s$$

$$X^{i}_{s.} \overset{\Delta}{=} \sum_{r} X^{i}_{sr}$$

$$X^{i}_{s.}=\frac{x_{is}}{n_{i}} Y_{i}, \qquad \forall i, \quad \forall s$$

$$\sum_{s} X^{i}_{sr}=\sum_{i'} x_{i'r} a_{i} \frac{Y_{i'}}{n_{i'}}, \qquad \forall r, \quad \forall i$$

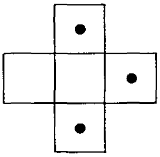
$$\sum_{i} \sum_{\substack{r \\ r \neq s}} X^{i}_{sr}=\sum_{i} \sum_{\substack{r \\ r \neq s}} X^{i}_{rs}, \qquad \forall s$$

$$X^{i}_{sr} \geqq 0$$

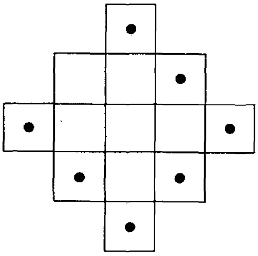
$$x_{os}=1, \qquad \forall s$$

$$x_{is} \text{ is integer}, \qquad \forall i, \quad \forall s$$

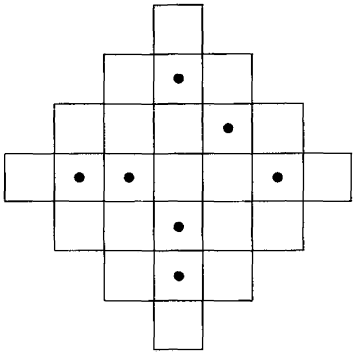
In Kuiper, Paelinck, Rosing (1989) this model was solved for one sector, using Manhattan circles $R=1$, 2 and 3. Both the optimal location pattern and the (relative) transportation costs were computed. The location pattern of mono-industrial centres shows a dominant pattern of dispersed location; of course one has to be careful in generalising this result. Introducing more sectors will probably lead to a pattern with concentrations, because of intersectoral production streams. Some examples of optimal location patterns from the previous paper are:



number of firms: 3
transport. costs: 3.0



number of firms: 7
transport. costs: 8.0



number of firms: 7
transport. costs: 19.1

4. CONCLUSIONS

In general, Tinbergen-Bos systems are optimised by minimising total transportation costs. The computation of transportation costs is directly related to the compulation of distances between locations, so it is necessary to define locations of production centres. In the first models, the locations of firms were not defined individual, but in a collective way. This implies that only a special metric can be used, the discrete metric. Another solution is to define mean distances (euclidean or rectangular); in this way it is not possible to find an optimal system of different types of location centres; that is only possible if each firm is located on a well-defined optimal spot.

Defining networks, including a number of different potential location possibilities, it is possible to find a "richer" solution including location pattern indicating exactly the location of each firm.

In Kuiper, Paelinck, Rosing (1989) Manhattan circles were introduced and a mathematical programme was solved, showing one industrial sector and the agricultural sector.

One of their conclusions was the need for further strategic extensions concerning the raise of the number of industrial sectors to 2 or more. Here a programme is presented including I different industrial sectors. Although the programme is not solved, yet, it will be the start for a number of investigations leading to general solutions on location problems.

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