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OPTIMAL AGE REPLACEMENT POLICY OF A *k*-OUT-OF-*n* SYSTEM WITH AGE-DEPENDENT MINIMAL REPAIR (*)

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Abstract. – *A k-out-of-n system is a system with n independent components such that the system operates if and only if at least k of these n components operate successfully. A general age replacement of a k-out-of-n system is introduced which incorporates minimal repair, planned and unplanned replacements, and general random repair costs. The long-run expected cost rate is obtained. The aim of the paper is to find the optimal age replacement time T which minimizes the long-run expected cost per unit time of the policy. Various special cases are considered.*

Keywords: Maintenance, reliability, repair, replacement policy.

Résumé. – *Un système « k-parmi-n » est un système avec n composantes indépendantes fonctionnant si et seulement si au moins k de ces n composantes fonctionnent. Nous exposons une politique générale de renouvellement d'un système k-parmi-n incluant la réparation minimale, les renouvellements planifiés ou non et les coûts de réparation aléatoires. Nous obtenons l'espérance du coût sur le long terme. L'objet de cet article est de trouver le temps optimal T de renouvellement qui minimise cette espérance de coût par l'unité de temps lorsqu'on applique cette politique. Nous considérons divers cas spéciaux.*

Mots clés : Maintenance, fiabilité, réparation, politique de renouvellement.

1. INTRODUCTION

A k-out-of-n system is a system with n independent components such that the system operates if and only if at least k of these n components operate successfully. Such a system occurs quite naturally in many physical and biomedical models. As an example of 2-out-of-4 systems, consider an airplane which can function satisfactorily if and only if at least two of its four engines are functioning. When $k=n$ (or $k=1$), we obtain series (or parallel) system as special cases of k-out-of-n systems.

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A preventive maintenance policy is of great importance in modern complex systems, since such a policy enables us to reduce the operating cost and the risk of a catastrophic breakdown. Preventive maintenance policies with minimal repair have, of late, attracted the attention of several authors. Many contributions to such preventive maintenance policies with minimal repair have been made by Barlow and Hunter [1], Muth [12], Beichelt and Fisher [3], Beichelt [4], Boland and Proschan [10], Boland [9], Berg and Cléroux [5], Tilquin and Cléroux [21], Cléroux, Dubuc and Tilquin [11], Berg, Bienvenu and Cléroux [6], Block, Borges and Savits [8], Nakagawa and Kowada [13], Sheu [16, 17, 18], Tahara and Nishida [20], Sheu, Kuo and Nakagawa [19] and others, Muth [12] suggested minimal repairs for a period of length t and then replacement at the first failure after t. Tahara and Nishida [20] considered a generalization of Muth's policy and suggested the following (t, T) policy. "Replace a system when the first failure after t hours operating or the total operating time reachs T $(0 \leqq t \leqq T)$, whichever occurs first. Each failure in the interval $[0, t]$ is removed by a minimal repair." A modification of the (t, T) policy is investigated by Sheu, Kuo and Nakagawa [19]. It can be seen that the policy considered by Sheu, Kuo and Nakagawa [19] is a generalization on previously known age replacement policies for an one unit system.

Most studies on the optimal age replacement policy concentrate on an one unit system. In realistic application a k-out-of-n system or a multi-unit system occur quite naturally in many physical and biomedical models. In this article optimal age replacement policy of a k-out-of-n system is presented which incorporates minimal repair, planned and unplanned replacement and general random repair costs. The cost of the i-th minimal repair of the component at age y depends on the age-dependent random part $C(y)$ and the deterministic part $c_i(y)$ which depends on the age and the number of the minimal repair. The model is described explicitly at the beginning of the next section. The expected long-run cost per unit time is obtained for this model and optimization results are obtained for the infinite-horizon case. As special cases, various results are obtained from Barlow and Hunter [1], Boland [9], Boland and Proschan [10], Cléroux, Dubuc and Tilquin [11], Tilquin and Cléroux [21], Berg, Bienvenu and Cléroux [6], Block, Borges and Savits [8], and Yasui, Nakagawa and Osaki [22].

In the second section the model is described, then the total expected long-run cost per unit time is found. Theorem 1 gives a general optimization result for the infinite-horizon case. In the third section various special cases are discussed.

2. GENERAL MODEL

We consider a preventive model in which minimal repair or replacement takes place according to the following scheme. A k-out-of-n system is completely replaced whenever it reaches age $T (T > 0)$ at a cost c_0 (planned replacement). The component of the system has two failure type. If it fails at age y, type I failure occurs with probability $q(y)$ and is corrected with minimal repair, whereas type II failure occurs with probability $p(y) = 1 - q(y)$ and a failed component is lying idle. A k-out-of-n system is completely replaced at the occurence of the $(n-k+1)$-th idle component at a cost c_∞ (unplanned replacement at system failure). The cost of the i-th minimal repair of the component at age y is $g(C(y), c_i(y))$, where $C(y)$ is the age-dependent random part, $c_i(y)$ is the deterministic part which depends on the age and the number of the minimal repair, and g is a positive non-decreasing and continuous function. Suppose that the random part $C(y)$ at age y has distribution $L_y(x)$, density function $l_y(x)$ and finite mean $E[C(y)]$. After a complete replacement (*i. e.* an unplanned or planned replacement by a new system), the procedure is repeated. We assume all failures are instantly detected and repaired. We always assume $c_0 > 0$.

Assume that the system consists of n i.i.d. components each with distribution $F(x)$, density function $f(y)$. Then the failure rate (or the hazard rate) is $r(x) = f(x)/\bar{F}(x)$ and the cumulative hazard is $R(x) = \int_0^x r(y)\,dy$, which has a relation $\bar{F}(x) = \exp\{-R(x)\}$, where $\bar{F}(x) = 1 - F(x)$. It is further assumed that the failure rate $r(x)$ is continuous, monotone increasing, and remains undisturbed by minimal repair.

If no planned replacement are considered (*i. e.*, $T = \infty$), the survival distribution of the time until the type II failure for each component is given by

$$\overline{F}_p(y) = \exp\left\{-\int_0^y p(x)\,r(x)\,dx\right\}. \tag{1}$$

Let $Y_1, Y_2, \ldots, Y_n$ be i.i.d. random variables with survival distribution $\bar{F}_p$. Then $Y_{(1)} \leqq Y_{(2)} \leqq \cdots \leqq Y_{(n)}$ the corresponding order value are called the order statistics from $\bar{F}_p$. Note that the order statistics may be interpreted as successive times of type II failure of the components of a system, so that the $(n-k+1)$-th order statistics may be considered as the time of failure of a k-out-of-n system. The order statistics $Y_{(j)}$ have the survival distribution

$$\left.\begin{aligned}\bar{F}_{j,p}(y) = \sum_{i=n-j+1}^{n} \binom{n}{i} (\bar{F}_p(y))^i (F_p(y))^{n-i},\\ \text{for} \quad j = 1, 2, \cdots, n.\end{aligned}\right\} \tag{2}$$

Let $Z_1, Z_2, \ldots$ be i.i.d. random variables with survival distribution $\bar{F}_{n-k+1,p}$ and $Z_i^* = Z_i \wedge T$ [where $a \wedge b = \min(a, b)$] for $i = 1, 2, \ldots$ Let R_i^* denote the operational cost over the renewal interval Z_i^*. Thus $\{(Z_i^*, R_i^*)\}$ constitutes a renewal reward process. If $K(t)$ denotes the expected cost of operating the system over the time interval [0, t], then it is well-known that

$$\lim_{t\to\infty} \frac{K(t)}{t} = \frac{E[R_1^*]}{E[Z_1^*]}, \tag{3}$$

(*see,* e.g., Ross [14], p. 52). We shall denote the right-hand side of (3) by $B(T)$.

The expected cycle length is given by

$$E[Z_1^*] = \int_0^T \bar{F}_{n-k+1,p}(y)\,dy. \tag{4}$$

We need only evaluate $E[R_i^*]$. We now give a derivation of the expression for $E[R_1^*]$. First, however, we must describe in more detail the failure process of the component.

If we have only minimal repairs at failures of the component and the time for minimal repair is negligible, then the failures occur according to a nonhomogeneous Poisson process with mean valued function $R(x) = -\log \bar{F}(x)$ (*see,* e.g., Nakagawa and Kowada [13]). Consider a nonhomogeneous Poisson process $\{N(t),\ t \geqq 0\}$ with intensity $r(t)$ and successive arrival times $S_1, S_2, \ldots$ At time S_n we flip a coin. We designate the outcome by σ_n which takes the value one (head) with probability $p(S_n)$ and the value zero (tail) with probability $q(S_n)$. Let $L(t) = \sum_{n=1}^{N(t)} \sigma_n$ and $M(t) = N(t) - L(t)$. Then it can be shown that the processes $\{L(t),\ t \geqq 0\}$ and $\{M(t),\ t \geqq 0\}$ are independent nonhomogeneous Poisson processes with respective intensities $p(t)\,r(t)$ and $q(t)\,r(t)$ (*see,* e.g., Savits [15]). This is similar to the classical decomposition of a Poisson process for constant p. For our models we have the identification $Y_1 = \inf\{t \geqq 0 : L(t) = 1\}$ and $M(Y_1)$ counts the number of minimal repairs of the component in [0, Y_1]. Note that Y_1 is independant of $\{M(t),\ t \geqq 0\}$.

We also require the following Lemma from Sheu [18].

LEMMA 1: *Let* $\{M(t),\ t \geqq 0\}$ *be a non-homogeneous Poisson process with intensity* $q(t)\,r(t)$, $t \geqq 0$ *and* $\Lambda(t) = E[M(t)] = \int_0^t q(y)\,r(y)\,dy$. *Denote the successive arrival times by* $S_1, S_2, \ldots$ *Assume that at time* S_i *a cost of* $g(C(S_i), c_i(S_i))$ *is incurred. Suppose that* $C(y)$ *at age y is a random variable with finite mean* $E[C(y)]$ *and g is a positive, non-decreasing continuous function. If* $A(t)$ *is the total cost incurred over* $[0, t)$, *then*

$$E[A(t)] = \int_0^t h(y)\,q(y)\,r(y)\,dy, \tag{5}$$

where $h(y) = E_{M(y)}[E_{C(y)}[g(C(y),\ c^{(y)}_{M(y)+1})]]$ *which is the expectation with respect to random variables* $M(y)$ *and* $C(y)$.

We are now ready to derive the expression for $E[R_1^*]$. First note that

$$\begin{aligned} R_1^* =& I_{\{Y_{(n-k+1)} \leqq T\}} \left\{ c_\infty + \sum_{j=1}^{n-k} \sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\ c_i(S_i)) \right. \\ & \left. + k \sum_{i=1}^{M(Y_{(n-k+1)})} g(C(S_i),\ c_i(S_i)) \right\} \\ & + \sum_{m=0}^{n-k} \left\{ I_{\{Y_{(m)} \leqq T < Y_{(m+1)}\}} \left\{ c_0 + \sum_{j=1}^{m} \sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\ c_i(S_i)) \right. \right. \\ & \left. \left. + (n-m) \sum_{i=1}^{M(T)} g(C(S_i),\ c_i(S_i)) \right\} \right\} \end{aligned} \tag{6}$$

We can write

$$\begin{aligned} E[R_1^*] =& c_\infty F_{n-k+1,\,p}(T) \\ & + E\left[\sum_{j=1}^{n-k} \sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\ c_i(S_i))\, I_{\{Y_{(n-k+1)} \leqq T\}} \right] \\ & + k\,E\left[\sum_{i=1}^{M(Y_{(n-k+1)})} g(C(S_i),\ c_i(S_i))\, I_{\{Y_{(n-k+1)} \leqq T\}} \right] \\ & + c_0 \sum_{m=0}^{n-k} (\overline{F}_{m+1,\,p}(T) - \overline{F}_{m,\,p}(T)) \end{aligned}$$

$$+ E\left[\sum_{m=0}^{n-k}\sum_{j=1}^{m}\sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(m)} \leqq T < Y_{(m+1)}\}}\right]$$

$$+ E\left[\sum_{m=0}^{n-k}(n-m)\sum_{i=1}^{M(T)} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(m)} \leqq T < Y_{(m+1)}\}}\right],$$

where I_A is the indicator function of event A.

Since

$$E\left[\sum_{m=0}^{n-k}\sum_{j=1}^{m}\sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(m)} \leqq T < Y_{(m+1)}\}}\right]$$

$$= E\left[\sum_{j=1}^{n-k}\sum_{m=j}^{n-k}\sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(m)} \leqq T < Y_{(m+1)}\}}\right]$$

$$= E\left[\sum_{j=1}^{n-k}\sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(j)} \leqq T < Y_{(n-k+1)}\}}\right]$$

$$= E\left[\sum_{j=1}^{n-k}\sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(j)} \leqq T\}}\right]$$

$$- E\left[\sum_{j=1}^{n-k}\sum_{i=1}^{M(Y_{(j)})} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(n-k+1)} \leqq T\}}\right]$$

and

$$E\left[\sum_{m=0}^{n-k}(n-m)\sum_{i=1}^{M(T)} g(C(S_i),\, c_i(S_i))\, I_{\{Y_{(m)} \leqq T < Y_{(m+1)}\}}\right]$$

$$= E\left[\sum_{i=1}^{M(T)} g(C(S_i),\, c_i(S_i))\right]\sum_{m=0}^{n-k}(n-m)(\overline{F}_{m+1,p}(T) - \overline{F}_{m,p}(T))$$

$$= E\left[\sum_{i=1}^{M(T)} g(C(S_i),\, c_i(S_i))\right]\left(\sum_{j=1}^{n-k}\overline{F}_{j,p}(T) + k\,\overline{F}_{n-k+1,p}(T)\right),$$

we obtain

$$
\begin{aligned}
E\left[R_1^*\right] =& c_\infty F_{n-k+1,p}(T) \\
&+ k\,E\left[\sum_{i=1}^{M(Y_{(n-k+1)})} g\left(C(S_i),\ c_i(S_i)\right) I_{\{Y_{(n-k+1)} \leqq T\}}\right] \\
&+ c_0\,\overline{F}_{n-k+1,p}(T) + E\left[\sum_{j=1}^{n-k}\ \sum_{i=1}^{M(Y_{(j)})} g\left(C(S_i),\ c_i(S_i)\right) I_{\{Y_{(j)} \leqq T\}}\right] \\
&+ E\left[\sum_{i=1}^{M(T)} g\left(C(S_i),\ c_i(S_i)\right)\right] \\
&\times \left(\sum_{j=1}^{n-k} \overline{F}_{j,p}(T) + k\,\overline{F}_{n-k+1,p}(T)\right). \qquad (7)
\end{aligned}
$$

Using the Lemma 1, we can write

$$
\begin{aligned}
E\left[R_1^*\right] =& c_\infty F_{n-k+1,p}(T) \\
&+ k \int_0^T \int_0^y h(z)\,q(z)\,r(z)\,dz\,dF_{n-k+1,p}(y) \\
&+ c_0\,\overline{F}_{n-k+1,p}(T) + \sum_{j=1}^{n-k} \int_0^T \int_0^y h(z)\,q(z)\,r(z)\,dz\,dF_{j,p}(y) \\
&+ \int_0^T h(y)\,q(y)\,r(y)\,dy \left(\sum_{j=1}^{n-k} \overline{F}_{j,p}(T) + k\,\overline{F}_{n-k+1,p}(T)\right) \\
=& c_\infty F_{n-k+1,p}(T) + c_0\,\overline{F}_{n-k+1,p}(T) \\
&+ k \int_0^T h(y)\,q(y)\,r(y)\left(\overline{F}_{n-k+1,p}(y) - \overline{F}_{n-k+1,p}(T)\right) dy \\
&+ \sum_{j=1}^{n-k} \int_0^T h(y)\,q(y)\,r(y)\left(\overline{F}_{j,p}(y) - \overline{F}_{j,p}(T)\right) dy \\
&+ \int_0^T h(y)\,q(y)\,r(y)\,dy \left(\sum_{j=1}^{n-k} \overline{F}_{j,p}(T) + k\,\overline{F}_{n-k+1,p}(T)\right)
\end{aligned}
$$

$$= c_\infty F_{n-k+1,p}(T) + c_0 \overline{F}_{n-k+1,p}(T) + \int_0^T h(y)\, q(y)\, r(y) \left(\sum_{j=1}^{n-k} \overline{F}_{j,p}(y) + k \overline{F}_{n-k+1,p}(y) \right) dy, \quad (8)$$

where $h(y) = E_{M(y)}[E_{C(y)}[g(C(y), c^{(y)}_{M(y)+1})]]$.

For the infinite-horizon case we want to find a T which minimizes $B(T)$, the total expected long-run cost per unit time. Recall that

$$B(T) = \left\{ c_\infty F_{n-k+1,p}(T) + c_0 \overline{F}_{n-k+1,p}(T) + \int_0^T h(y)\, q(y)\, r(y) \left(\sum_{j=1}^{n-k} \overline{F}_{j,p}(y) + k \overline{F}_{n-k+1,p}(y) \right) dy \right\} \Big/ \left\{ \int_0^T \overline{F}_{n-k+1,p}(y)\, dy \right\}. \quad (9)$$

Let $r_{n-k+1,p}(x)$ denote the failure rate function of the distribution $F_{n-k+1,p}(x)$. We now assume that the functions r, p, $r_{n-k+1,p}$, and h are continuous. In this case we can differentiate B with respect to T. We see that $dB/dT=0$ if and only if

$$0 = \int_0^T \left\{ \left[(c_\infty - c_0)\, r_{n-k+1,p}(T) + h(T)\, q(T)\, r(T) \left(\sum_{j=1}^{n-k} \frac{\overline{F}_{j,p}(T)}{\overline{F}_{n-k+1,p}(T)} + k \right) \right] - \left[(c_\infty - c_0)\, r_{n-k+1,p}(y) + h(y)\, q(y)\, r(y) \left(\sum_{j=1}^{n-k} \frac{\overline{F}_{j,p}(y)}{\overline{F}_{n-k+1,p}(y)} + k \right) \right] \right\} \times \overline{F}_{n-k+1,p}(y)\, dy - c_0. \quad (10)$$

THEOREM 1: *Let* $F_{n-k+1,p}$ *have failure rate* $r_{n-k+1,p}$ *and suppose that the function r, p,* $r_{n-k+1,p}$ *and h are continuous. Then, if*

$$(c_\infty - c_0)\, r_{n-k+1,p}(y) + h(y)\, q(y)\, r(y) \left(\sum_{j=1}^{n-k} \frac{\overline{F}_{j,p}(y)}{\overline{F}_{n-k+1,p}(y)} + k \right)$$

increases to ∞, *there exist at least one finite positive* T^* *which minimizes the total expected long-run cost per unit time* $B(T)$. *Furthermore, if*

$$(c_\infty - c_0)\, r_{n-k+1,p}(y) + h(y)\, q(y)\, r(y) \left(\sum_{j=1}^{n-k} \frac{\overline{F}_{j,p}(y)}{\overline{F}_{n-k+1,p}(y)} + k \right)$$

is strictly increasing of y so T^* *is unique and*

$$\begin{aligned} B(T^*) =& (c_\infty - c_0)\, r_{n-k+1,p}(T^*) \\ &+ h(T^*)\, q(T^*)\, r(T^*) \left(\sum_{j=1}^{n-k} \frac{\overline{F}_{j,p}(T^*)}{\overline{F}_{n-k+1,p}(T^*)} + k \right). \end{aligned} \qquad (11)$$

Proof: If the condition of the theorem is satisfied, then the right-hand side of (10) is a continuous increasing function of T which is negative $(-c_0)$ at $T=0$ and tends to $+\infty$ as $T \to \infty$. Hence there is at least on value $0<T^*<\infty$ that satisfies (10). Since $B'(T)$ has the same pattern (–, 0, +), it follows that $B(T)$ has a minimum at T^*. Under the strictly increasing assumption, the right-hand side of (10) is strictly increasing so T^* is unique. If T^* is the solution, then from equations (9) and (10) it is easy to get the expression (11).

3. SPECIAL CASES

Case 1 ($n=1$, $k=1$): This case reduces to the optimal age replacement of an one unit system with age-dependent minimal repair. It is more general than the model considered by Block, Borges and Savits [8], since the cost of the i-th minimal repair at age y is $g(C(y), c_i(y))$, which depends on the age-dependent random part $C(y)$, and the deterministic part $c_i(y)$, which depends on the age and the number of the minimal repair.

Case 1 (*a*) ($n=1$, $k=1$, $p(y)=1$): This is the Policy I considered by Barlow and Hunter [1].

Case 1 (*b*) ($n=1$, $k=1$, $p(y)=0$, $g(C(y), c_i(y))=c_M$): This is the Policy II considered by Barlow and Hunter [1].

Case 1 (*c*) ($n=1$, $k=1$, $p(y)=0$, $g(C(y), c_i(y))=c(y)$): This is the case considered by Boland [9].

Case 1 (*d*) ($n=1$, $k=1$, $p(y)=0$, $g(C(y), c_i(y))=c_i$): Boland and Proschan [10] investigated this case. In particular they considered the cost structure $c_i=a+ic$.

Case 1 (*e*) ($n=1$, $k=1$, $p(y)=0$, $g(C(y), c_i(y))=c+c_i(y)$): This is the case considered by Tilquin and Cléroux [21].

Case 1 (*f*) ($n=1$, $k=1$, $p(y)=p$, $0<p<1$, $g(C(y), c_i(y))=C$): This is the case considered by Cléroux, Dubuc and Tilquin [11].

Case 1 (*g*) ($n=1$, $k=1$, $g(C(y), c_i(y))=C(y)$): This is the case considered by Berg, Bienvenu and Cléroux [6].

Case 1 (*h*) ($n=1$, $k=1$, $g(C(y), c_i(y))=c_i(y)$): This is the case considered by Block, Borges and Savits [8].

Case 2 ($n=k$): When $n=k$ this case reduces to optimal age replacement policy of a series system. In this case, if we put $n=k$ in (9), we get the total expected long-run cost per unit time is

$$B(T)=\frac{\left\{\begin{array}{c} c_\infty\left(1-(\overline{F}_p(T))^n\right)+c_0(\overline{F}_p(T))^n) \\ +n\displaystyle\int_0^T h(y)\,q(y)\,r(y)\,(\overline{F}_p(y))^n\,dy \end{array}\right\}}{\displaystyle\int_0^T(\overline{F}_p(y))^n\,dy}. \tag{12}$$

Case 3 ($n>1$, $k=1$): When $n>1$, $k=1$ this case reduces to optimal age replacement policy of a parallel system. In this case, if we put $k=1$ in (9), we get the total expected long-run cost per unit time is

$$B(T)=\frac{\left\{\begin{array}{c} c_\infty(F_p(T))^n+c_0(1-(F_p(T))^n) \\ +\displaystyle\int_0^T\left(\sum_{j=1}^n\overline{F}_{j,p}(y)\right)h(y)\,q(y)\,r(y)\;dy \end{array}\right\}}{\displaystyle\int_0^T(1-(F_p(y))^n)\,dy}. \tag{13}$$

If we put $p(y)=1$, $q(y)=0$, $c_\infty=nc_1+c_2$, $c_0=nc_1$ in (13), then we get the following result as Yasui, Nakagawa and Osaki [22] obtained.

$$B(T) = \frac{nc_1 + c_2 (F(T))^n}{\int_0^T (1 - (F(y))^n) \, dy}, \tag{14}$$

where c_1 = acquisition cost of one unit, c_2 = additional cost of replacement for a failed system.

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