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AN ECONOMICAL BACKUP WARNING STRATEGY FOR A HARD DISK

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Abstract. – *A hard disk has, in recent years, become an essential component for a personal computer as well as a work station. A hard disk stores a variety of files, which are daily updated. The files in a hard disk are, however, lost because of human errors and a failure of the hard disk. This is called a* hard disk failure. *In order to protect us from such a serious loss, it is important to make a backup copy of files on a magnetic tape or on several floppy disks. Frequent backup operations require much cost in backup operations themselves, while rare backup operations would make the loss at a hard disk failure very large. This indicates that the backup timing should adequately be determined considering the above two factors.*

This study discusses an economical backup warning stragegy, which prescribes the time to give us a warning for a backup operation at age T, where age refers to the elapsed time since the previous backup operation or a recovery operation whichever occurred most recently. When a job accessing the hard disk is being processed at the warning time, the backup operation is conducted after the process of the job is finished. The expected cost per unit time under this strategy is formulated as an objective function. The existence of an economical warning time that minimizes the expected cost is shown. Numerical examples are also presented to illustrate the theoretical underpinnings of the economical backup warning strategy formulation.

Keywords: Hard disk; backup warning; expected cost; economical strategy.

Résumé. – *Le disque dur est devenu, dans les récentes années, un composant essentiel d'un PC aussi bien que d'une station de travail. Divers fichiers sont enregistrés sur un disque dur, et sont quotidiennement mis à jour. Ces fichiers sont malheureusement perdus, à cause d'erreurs humaines ou d'une panne du disque dur. C'est ce qu'on appelle* une panne de disque dur. *La méthode la plus simple de nous protéger d'une perte si sévère est de faire périodiquement une copie de secours des fichiers sur une bande magnétique ou sur plusieurs disquettes. De fréquentes opérations de copie seraient coûteuses à cause des opérations elles-mêmes, tandis que de rares opérations de copie rendraient plus grande la perte due à une panne de disque dur. Cela indique que le choix de l'instant d'une opération de copie de secours devrait être déterminé en utilisant ces deux facteurs.*

Nous exposons une stratégie économique d'avertissement, qui prescrit l'instant d'avertissement d'une copie de sauvegarde à l'âge T, où l'âge est le temps couru depuis la dernière opération de sauvegarde, ou depuis la dernière opération de récupération. Lorsqu'un travail accédant au

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disque dur est en train d'être exécuté à l'instant d'avertissement, l'opération de copie de secours est effectué après la fin de l'exécution de ce travail. Nous formulons le coût moyen par unité de temps, correspondant à cette stratégie, comme une fonction-objectif. Nous montrons l'existence d'un temps économique d'avertissement qui minimise le coût moyen. Nous présentons aussi des exemples numériques illustrant intuitivement la théorie.

Mots clés : Disque dur, avertissement de copie de secours, coût moyen, stratégie économique.

1. INTRODUCTION

A hard disk has, in recent years, a large-size memory and can be obtained at lower prices. A variety of software that requires hard disk are also, in recent years, being developed for a personal computer. For these reasons, a hard disk has become an essential component for a personal computer as well as a work station.

Various files are stored on a hard disk and are daily updated. These files are, however, lost because of human errors and a failure of the hard disk. This is called *a hard disk failure*. The simplest method for protecting us from such a serious loss is to make a backup copy of files on a magnetic tape or on several floppy disks (backup disks in short) periodically. In the case of a hard disk failure, the hard disk can partially be recovered using the backup disks. The recovery would be partial since the backup disks only preserve the files at the last backup time.

Frequent backup operations would require much cost in operations themselves, while rare backup operations would make the loss at a hard disk failure very large. This signifies that the adequate timing of a backup operation should be determined considering these two factors.

Similar problems can be observed in the main internal memory of a main frame computer, where data stored in the main memory are occasionally lost. For these systems, several studies on rollback and recovery strategies have been reported [1-5], which suggest to periodically make a backup copy of data in the main internal memory on hard disks.

These strategies were originally devised for fear of a failure of an online banking system. In the case of a failure of an online banking system, all the data in the main internal memory at a failure must be recovered. For this reason, all the log files are also stored on magnetic tapes. With both the data backed up on the hard disk and the log files on magnetic tapes, the system can perfectly be recovered although it will spend much time.

If the system can perfectly be recovered up to the state at its failure, a formulation based on the renewal reward process [6] is possible. Furthermore, the underlying idea in the formulation is identical of that of replacement policies for a system where the cost structure depends on the age of the failed unit at its failure [7-9].

For protecting files on the hard disks of personal computers and engineering work stations, however, it should be reminded that the recovery from a hard disk failure is partial, *i.e.*, the hard disk can only be recovered up to the state at the last backup operation. For such a problem, Sandoh, Kaio and Kawai [10] have proposed an optimal backup strategy where a backup operation is conducted at age T. *Age* refers to the elapsed time since the previous backup operation or a recovery operation from a hard disk failure, whichever occurred most recently, and this term was originally used in reliability theory [11]. Sandoh and Kawai [12] have proposed another optimal backup strategy considering jobs under which a backup operation is conducted when N jobs updating the hard disk are processed. Sandoh and Kawai [13] have also proposed a backup strategy peculiar to floppy disks, which are usually used to preserve personal document files. Under this strategy, a backup operation is carried out when $1/N$ of the total memory size of a floppy disk is consumed.

This study extends the above two backup strategies for a hard disk [10, 12] and proposes an economical backup warning strategy, which provides us with a warning to backup files at age T. In case a job accessing the hard disk is being processed at age T, the backup operation is conducted after the process of the job is completed. Such a job is called *a warned job.* The expected cost per unit time is formulated under the proposed backup warning strategy, which focuses on the two kinds of costs; the cost for the troublesomeness of a backup operation and the cost for losing files because of a hard disk failure. It is shown that there always exists an economical warning time minimizing the expected cost. Numerical examples are also presented to illustrate the theoretical underpinnings of the backup warning strategy formulation.

2. MODEL FORMULATION

2.1. Assumptions and notation

Consider a backup warning strategy where a warning to backup files is given at age T and the backup operation is conducted after the process of the warned job is completed. The design variable under such a strategy is the warning time T. We also consider that at each backup time, only the files updated and/or created since the previous backup operation are added to the backup disks.

We assume the following:

(1) Hard disk failures occur only when the system is used, and thus it is assumed that the system processes the jobs continuously in time.

(2) The processing time for each job is independently and identically distributed (*i.i.d.*).

(3) A hard disk failure time follows an exponential distribution with a failure rate λ based on the idea that a hard disk failure seldom occurs.

(4) The hard disk failure is assumed to be detected instantly.

(5) No hard disk failure occurs during both a backup operation and a recovery operation.

We give below the notation in this study:

T : Warning time.
T_e : Excess age of the warned job.
Y : Hard disk failure time.
C_1 : Cost for a backup operation.
C_2 : Cost for losing data created/updated per unit time.
$G(t)$: Distribution function for the excess age.
$F(t)$: Distribution function for the hard disk failure time.
$\bar{F}(t)$: $1 - F(t)$.
$H(t)$: Distribution function for the processing time of each job.
$\bar{H}(t)$: $1 - H(t)$.
$h(t)$: Density function associated with $H(t)[= dH(t)/dt]$.

In the above, the excess age signifies the residual processing time of a warned job at T.

2.2. Expected cost

The processing times of jobs generate a renewal process, and therefore the distribution function $G(t)$ for excess age, T_e is given [e.g. 6] by

$$G(t_e) = H(T + t_e) - \int_0^T \bar{H}(T - x + t_e)\, m(x)\, dx, \tag{1}$$

where

$$m(t) = \sum_{n=1}^{\infty} h_n(t), \tag{2}$$

$$h_n(t) = \frac{dH_n(t)}{dt}, \tag{3}$$

$$H_n(t) = \int_0^t H_{n-1}(t-x)\, h(x)\, dx, \quad n = 2, 3, \ldots, \tag{4}$$

$$H_1(t) = H(t). \tag{5}$$

From the assumption (3), the process of the system behaviour generates a renewal reward process [6], where the renewal point is assigned to the time when one of the following two events occurs:

(i) The process of the warned job had been finished and the backup operation has been carried out, that is, $Y > T + T_e$.

(ii) A hard disk failure occurred before a backup operation, and a recovery from the hard disk failure using backup disks has been completed, that is, $Y \leqslant T + T_e$.

The cost in the case of the event (i) is C_1, while the cost in the case of the event (ii) is expressed as $C_2 Y$.

Let $A(T)$ and $B(T)$ respectively denote the expected time and the expected cost over the time between two successive renewal points, then the expected cost, $C(T)$ per unit time over an infinite time span is expressed as

$$C(T) = \frac{B(T)}{A(T)}, \tag{6}$$

where

$$A(T) = \int_0^\infty (T + t_e)\bar{F}(T + t_e)\, dG(t_e) + \int_0^\infty \int_0^{T+t_e} y\, dF(y)\, dG(t_e), \tag{7}$$

$$B(T) = C_1 \int_0^\infty \bar{F}(T + t_e)\, dG(t_e) + C_2 \int_0^\infty \int_0^{T+t_e} y\, dF(y)\, dG(t_e). \tag{8}$$

Furthermore, the assumption (3) yields

$$\begin{aligned} A(T) = & \int_T^\infty te^{-\lambda t}\, h(t)\, dt + \int_0^T \int_T^\infty te^{-\lambda t}\, h(t-x)\, m(x)\, dx\, dt \\ & + \int_T^\infty y\,\lambda\, e^{-\lambda y}\, \bar{H}(y)\, dy \\ & + \int_0^T \int_T^\infty y\,\lambda\, e^{-\lambda y}\, \bar{H}(y-x)\, m(x)\, dx\, dy \\ & + \int_0^T y\,\lambda\, e^{-\lambda y}\, dy, \end{aligned} \tag{9}$$

$$\begin{aligned} B(T) = & C_1 \int_T^\infty e^{-\lambda t}\, h(t)\, dt + C_1 \int_0^T \int_T^\infty e^{-\lambda t}\, h(t-x)\, m(x)\, dx\, dt \\ & + C_2 \int_T^\infty y\,\lambda\, e^{-\lambda y}\, \bar{H}(y)\, dy \\ & + C_2 \int_0^T \int_T^\infty y\,\lambda\, e^{-\lambda y}\, \bar{H}(y-x)\, m(x)\, dx\, dy \\ & + C_2 \int_0^T y\,\lambda\, e^{-\lambda y}\, dy. \end{aligned} \tag{10}$$

By calculating Eqs. (9) and (10), we have

$$A(T) = p\left[1 + \int_0^T e^{-\lambda t}\, m(t)\, dt\right], \tag{11}$$

$$\begin{aligned} B(T) = & C_1(1-\lambda p) + C_2\,\lambda\, Q \\ & + \lambda \int_0^T [C_2\, p\,\lambda + C_2\, Q - C_1\, p]\, e^{-\lambda t}\, m(t)\, dt, \end{aligned} \tag{12}$$

where

$$p = \int_0^\infty e^{-\lambda x}\, \bar{H}(x)\, dx\, (> 0), \tag{13}$$

$$Q = \int_0^\infty xe^{-\lambda x}\, \bar{H}(x)\, dx\, (> 0). \tag{14}$$

From the above results, we obtain

$$C(T) = \frac{C_1(1-\lambda p) + C_2 \lambda Q + \lambda \int_0^T [C_2 p \lambda + C_2 Q - C_1 p] e^{-\lambda t} m(t)\, dt}{p\left[1 + \int_0^T e^{-\lambda t} m(t)\, dt\right]}. \tag{15}$$

We have formulated the expected cost per unit time. If a warning time T minimizes $C(T)$, it is the optimum.

3. ECONOMICAL WARNING STRATEGY

This section examines the existence of an economical warning time, which minimizes $C(T)$. By differentiating $C(T)$ in Eq. (6) with respect to T, we have

$$\begin{aligned} C'(T) &= \frac{A(T)B'(T) - A'(T)B(T)}{A^2(T)} \\ &= \frac{A'(T)}{A^2(T)}\left[\frac{B'(T)}{A'(T)} A(T) - B(T)\right]. \end{aligned} \tag{16}$$

Since $A^2(T) > 0$, and we have

$$A'(T) = pe^{-\lambda T} m(T) > 0, \tag{17}$$

the sign of $C'(T)$ coincides with that of $D(T)$, which is defined by

$$D(T) = \frac{B'(T)}{A'(T)} A(T) - B(T). \tag{18}$$

In the following, therefore, we examine the sign of $D(T)$ instead.

Differentiating $D(T)$ with respect to T, we have

$$\begin{aligned} D'(T) &= \left[\frac{B'(T)}{A'(T)}\right]' A(T) \\ &= C_2 \lambda A(T) > 0, \end{aligned} \tag{19}$$

which reveals that $D(T)$ is increasing in T.

On the other hand, we have

$$\lim_{T \to +0} D(T) = -C_1 < 0, \tag{20}$$

$$\lim_{T \to +\infty} D(T) = +\infty. \tag{21}$$

Equations (19), (20) and (21) reveal that the sign of $D(T)$ varies from negative to positive. It follows that there exists a unique optimal warning time T^*, which minimizes $C(T)$.

4. NUMERICAL EXAMPLES

This section presents numerical examples in which the underlying processing time distribution is assumed to be a gamma distribution with shape parameter 2. By this assumption, we have

$$H(t) = 1 - (1 + \mu t) e^{-\mu t}. \tag{22}$$

From this, we have

$$p = \frac{\lambda + 2\mu}{(\lambda + \mu)^2}, \tag{23}$$

$$Q = \frac{\lambda + 3\mu}{(\lambda + \mu)^3}. \tag{24}$$

Furthermore,

$$m(t) = \frac{\mu}{2}(1 - e^{-2\mu t}). \tag{25}$$

From Eqs. (23), (24) and (25), we have

$$\begin{aligned} C(T) = & -C_1 \lambda \\ & + \frac{C_2 \lambda [(2+\lambda) T e^{-(2+\lambda)T} + e^{-(2+\lambda)T}] - C_2 [(2+\lambda)^2 (\lambda T e^{-\lambda T} + e^{-\lambda T}) - 4(1+\lambda)]}{(2+\lambda)[e^{-\lambda T}(\lambda e^{-2T} - \lambda - 2) + 2(\lambda+1)^2]} \\ & + \frac{2 C_1 \lambda (2+\lambda)(\lambda+\mu)^2}{(\lambda + 2\mu)[e^{-\lambda T}(\lambda e^{-2T} - \lambda - 2) + 2(\lambda+1)^2]} \\ & + \frac{C_2 \lambda (\lambda + 3\mu)}{(\lambda+\mu)(\lambda+2\mu)}. \end{aligned} \tag{26}$$

In the following, we consider the case where $(C_1, \mu) = (1.0, 0.1)$, and C_2 is set to 0.3, 0.5 and 1.0. Figure 1 reveals optimal warning times when λ varies. It can be seen in Fig. 1 that the optimal warning thime increases with decreasing C_2. Since C_2 can be regarded as the value of files, it is intuitively understandable. It is also seen in Fig. 1 that the optimal warning time decreases as the failure rate increases. This is also intuitively understandable.

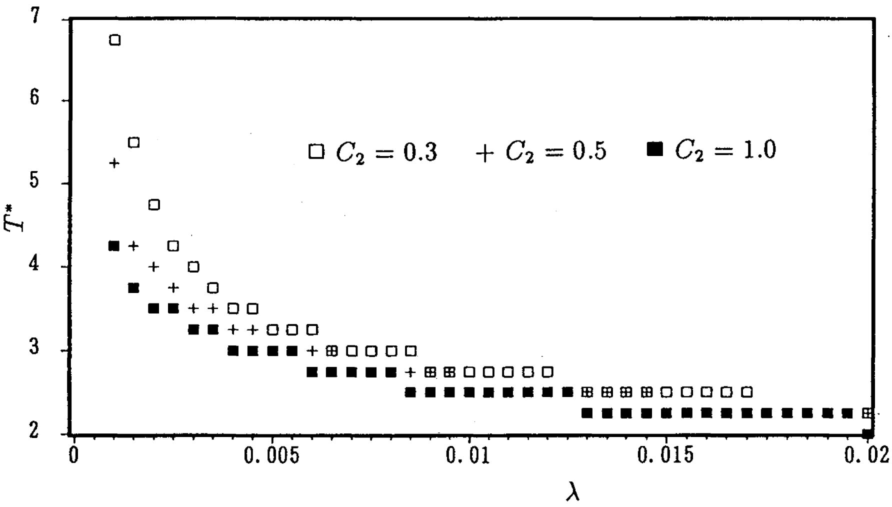

Figure 1. – Optimal warning times

5. CONCLUDING REMARKS

This study has proposed an economical backup warning strategy for a hard disk, which is used for a personal computer as well as a work station. Under the proposed strategy, a warning to backup files is given at age T. When a job accessing the hard disk is being processed at the warning time, the backup operation is carried out after the process of the job is finished. The expected cost per unit time of the proposed strategy has been formulated, which is to be minimized. It has been shown that there exists a unique optimal warning time.

Our model assumes that the cost for losing data is proportional to the hard disk failure time. An extension would be possible where the cost for losing data includes the cost proportional to the number of jobs processed up to the hard disk failure as well.

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