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OPTIMAL DYNAMIC PRICING, PRESERVATION TECHNOLOGY INVESTMENT AND PERIODIC ORDERING POLICIES FOR AGRICULTURAL PRODUCTS

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Abstract. This paper focuses on the inventory management of agricultural products, a specific type of perishable items carrying the deterioration property. In practice, the deterioration rate of agricultural products is varying with time and can be slowed down *via* investing in the preservation technology. This objective of this paper is to maximize the firm's total profit per unit time by simultaneously determining dynamic pricing, replenishment cycle length, replenishment quantity and preservation technology investment. We first derive pricing policy by solving a dynamic optimization problem and then propose a solution procedure to obtain the optimal strategies that maximize profit. Furthermore, numerical examples and sensitivity analysis are conducted to gain more managerial insights. We find that the firm should take a penetration pricing policy. In addition, if the shelf life of products is very long, the firm should not take preservation technology investment. When the unit holding cost is relatively small or the unit purchasing cost is relatively large, the firm should increase preservation technology investment.

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1. Introduction

Agriculture product accounts for a large proportion of the market demand, and the consumption of such kind of products in the world has the potential to increase because of a highly growing population. Different from other products, there is a continuous and significant reduction in quality and quantity over time for agricultural products. The deterioration property leads that agricultural products possess limited shelf lives. In fact, deterioration occurs due to a variety of phenomena of complicated chemical, physical and biochemical origins after products are harvested. Owing to the fact that products cannot maintain the original value or utility until they are unfit for human consumption and finally have to be discarded, a loss or waste of agricultural products naturally emerges. In recent decades, huge loss of agricultural products has been a problem of increasing severity, with rising awareness of managers. For example, it is roughly estimated by the Food and Agricultural Organization of the United Nations that one third of the food produced globally is wasted along the entire

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supply chain annually (Gustavsson *et al.* [16]). Redlingshöfer *et al.* [33] presented the data on the extent of food loss and waste in France, reporting that in the year 2013, the loss of fruits and vegetables from production to retailing sectors reaches up to 12%. In China, the average deteriorating rate of fruit and vegetable is up to 20–30% and causes more than 100 billion yuan of economics loss per year (Li and Zhu [23]). The serious circumstance that large quantities of foods which should have been eaten end up as waste makes food waste a topic of concern worldwide (Mirabella *et al.* [28]). Such challenging situation turns the proper management of agricultural product supply chain an urgent need which is crucial to profitability and drives the enterprises to adopt effective measures to cut losses. As such, various available preservation technologies or approaches are commonly applied to extend the shelf life of agricultural products since improvements in storage conditions can protect products from deteriorative effect or retard the rate of deterioration. The technologies include physical, chemical and biopreservation technologies, such as modified atmosphere packaging, pressurized inert gases, cold plasma and so forth (Ma *et al.* [27]). For instance, sliced carrots under high oxygen and high carbon dioxide can be stored for more 2–3 days in contrast with the condition in the air (Amanatidou *et al.* [3]). Sharma *et al.* [38] found that all coating formulations have the potential to improve the shelf life and preserve the quality of fresh pineapple for 15 days. Papachristodoulou *et al.* [30] reflected that application of ozonated water before packaging yields a shelf-life extension of 3 days for spinach with decreasing yellowing and maintained compositional characteristics of the leaves. Eriksson *et al.* [10] found that reducing temperature for stored meat products from 4 °C to 2 °C would potentially lead to a prolongation of shelf life in the range 28–36% and a 19% reduction in mass of wasted meat.

Agricultural products are a specific type of perishable items which capture the deterioration property. As defined in Wee [43], deterioration refers to phenomena of spoilage, pilferage, decay, evaporation, obsolescence, or damage that commonly exists in practice and be viewed as a focal factor influencing perishable inventory. Recently, researchers gradually highlight the effect of deterioration and give considerable attention to relevant management problems of perishable items since Ghare and Schrader [13] first considered a deteriorating inventory model with a constant rate of deterioration. However, in real life situations, for agricultural products, deterioration rate usually depends on time. Covert and Philip [7] further extended the model by considering a two-parameter Weibull distribution deterioration. Soon afterwards, Philip [31] developed the model associating with a three-parameter Weibull deterioration rate without shortage. Recently, Skouri *et al.* [39] formulated a deteriorating inventory model with general a ramp type demand rate and Weibull distribution deterioration. In addition, several related articles concerning of either a constant or exponential deterioration rate have been proposed by Cohen [6], Feng *et al.* [11], Hwang and Shinn [19], Lo *et al.* [26] and many others. Sana [35] determined the optimal selling price and lot size with partial backlogging. In the model, deterioration is characterized as a function of time and related with shelf life of perishable products. Sarkar and Sarkar [36] proposed an EOQ model where the suppliers entitled a trade-credit offer to the retailers for the purpose of promoting more sales with different discount rates on the purchasing costs. The time-varying deterioration in the work reflects that the quantity or utility of the product drops at an accelerating rate as it approaches closely to the maximum shelf life, which is also considered in this study.

Suffering from spoiled products in large quantities, an increasing number of firms have the incentive to reduce the loss by means of effective capital investment in warehouse equipments, *i.e.*, preservation technology investment. Accordingly, the firm will see a commensurate reduction of waste as well as an increase in profit. This is consistent with Tsao [40] who found that the retailer earns less profit when facing with a higher rate of deterioration. Geetha and Uthayakumar [12] and Ouyang *et al.* [29] discovered that a reduction of the rate of deterioration produced by the improvement of storage facility will bring out a lowered inventory cost. Hsu *et al.* [18] formed an inventory model of deteriorating products where the retailer attempts to reduce the deterioration rate of the item by taking the preservation technology investment. Then, a solution procedure was proposed to make decisions including the optimal replenishment cycle, shortage period, periodic order quantity and preservation technology cost to maximize profit. Dye [8] considered an inventory system for a non-instantaneous item to discuss the effect of preservation technology on the inventory decisions. Liu *et al.* [24] developed an inventory model for perishable foods where the demand depends on both the price and the quality of foods with the later decaying continuously overtime and then derived the joint dynamic pricing and preservation technology investment strategies to maximize profit. Saha *et al.* [34] addressed a replenishment problem of deteriorating items through determining the joint dynamic pricing, investment in green operations, preservation technology, and optimal replenishment times in presence of reference price. The demand rate in the model relays on the sales, reference prices and green concern level simultaneously. Other papers related to preservation technology investment can be found in Refs. [9, 15, 45] and so forth.

Nowadays, compensating for the negative impact of deterioration may be achieved by implementing efficient marketing policies. As an important marketing tool, pricing strategy is broadly adopted to maximize profit due to its ability of effectively matching supply and demand. Wang *et al.* [42] verified that dynamic pricing enjoys better performance compared to a fixed price. The rapid development of information technology and the prevalence of the Internet and e-commerce make dynamic pricing an easier and more popular measure in reality. Particularly for perishable items, dynamic pricing enables the firm to flexibly control and respond to changeable inventory circumstances. Therefore, a large number of related researches on dynamic pricing for perishable goods can be found, such as in Refs. [2, 5, 17, 25, 32, 41] and so forth. In this work, we focus on dynamic pricing for agricultural products with a time-varying deterioration rate. It is worth to note that a varying deterioration with time makes perishable inventory system more complicated which is a crucial concern for most firms. For the products with the varying deterioration, taking continuous time dynamic pricing policy is flexible and profitable. Related problems of ordering decisions are also rich research topics in the field of perishable inventory management problems. Thus, the integration problem of pricing and ordering decisions draw extensive attentions from relevant managers and scholars. Abad [1] set a generalized model of dynamic pricing and lot-sizing in which demand is partially backordered. Jia and Hu [21] studied dynamic pricing and ordering policies for a perishable product supply chain that consists of one supplier and one retailer in a finite horizon with uncertain demand. Until now, a variety of investigations continue to gush and enrich the literature of perishable items. For more related investigations, we refer to Bakker *et al.* [4], Goyal and Giri [14] and Janssen *et al.* [20] for a detailed review on the deteriorating inventory literature.

Inspired by the aforementioned studies, we present a continuous time inventory model for agricultural products where pricing and inventory decisions are to be determined while capital is to be invested in preservation technology. Regarding agricultural products, to prolong the product's shelf life has been a common practice existing in real world and an increasing number of firms are attempting to reduce a large waste caused along the agricultural product chain. In this study, with the objective of maximizing the total profit per unit time, the enterprise simultaneously determines the optimal pricing policy, replenishment quantity, replenishment cycle length and preservation technology investment with the assumption that shortage is not allowed. The demand is linearly decreasing with the selling price, and the deterioration rate is increasing with respect to time, which implies the deterioration rate sharply increases as the product approaches the expiration date. The combination of time-varying deterioration and dynamic pricing raises the complexity of this model and makes it challenging to obtain the optimal solution in an analytical approach. By virtue of dynamic optimization method, we first derive the optimal pricing strategy *via* solving an optimal control problem. Then, we propose a solution procedure to acquire the optimal strategies numerically. Furthermore, we carry out numerical examples to illustrate the effectiveness of the proposed method. Additionally, we address sensitivity analysis with respect to major parameters to probe potential managerial insights. We observe that facing a time-varying deterioration rate, the firm should take a penetration pricing policy which enables him to capture a large market demand at the beginning of the selling horizon, and then gradually raise the marginal profit with an increasing selling price. We find that the optimal price is positively impacted by market potential, unit purchasing cost and unit holding cost but negatively influenced by price sensitivity, fixed order cost, investment efficiency and shelf life of the product. We show that the investment strategy of the firm depends on the key parameters including the shelf life of the product. If the shelf life is very long, the firm has no incentive to invest. For example, storage of walnuts in light and at room temperature is common practice, and walnuts can exist for several months after being harvested. As reported in Koyuncu and Askin [22], shelled walnuts can be stored for 10–12 months at temperature of 5 °C with relative humidity of 55–65%. Regarding this stability of walnuts, many firms keep them simply instead of investing in extending the shelf life. In absence of a long shelf life, the investment decision relies on the cost coefficients. Specially, a relatively small unit holding cost or a relatively large unit purchasing cost stimulates preservation technology investment.

To summarize, the primary contributions of this paper which distinguish our study from existing literature are threefold.

1. We first investigate the problem of simultaneously determining the optimal dynamic pricing policy, preservation technology investment and periodic ordering strategies for agricultural products with limited shelf life and time-varying deterioration rate.
2. We propose a solution procedure to effectively derive the joint decision of dynamic pricing, preservation technology investment, and inventory ordering.
3. We find that wether or not the firm should extend products' shelf life *via* investing in preservation technology depends on the shelf life and the cost coefficients which provides managerial insights for the firm.

The remainder of the paper is organized as follows: In Section 2, notations and assumptions are provided. In Section 3, the model and the solution method are presented. Then, some numerical examples and sensitivity analysis are conducted to illustrate the theoretical results in Section 4. Finally, Section 5 gives the conclusion and further research directions.

2. Notations and assumptions

The following notations have been used in this study, see Table 1.

The mathematical model of the inventory control and pricing problem is based on the following assumptions:

1. Product starts to deteriorate once it gets stocked during a replenishment cycle $[0, T]$.
2. The demand is linearly decreasing with respect to price as follows

$$D(t) = \alpha - \beta p(t), \tag{2.1}$$

where $\alpha > 0$ and $\beta > 0$. It should be satisfied that $p \in [0, \frac{\alpha}{\beta}]$ to ensure a non-negative demand.

Table 1. Notations.

Parameters and system variables	
$D(t)$	The demand rate at time t
$I(t)$	The inventory level at time t
α	The basic market potential
β	The price sensitivity
$\theta(t,u)$	The time-varying deterioration rate at time t with given u
h	The unit holding cost
c	The unit purchasing cost
K	The fixed order cost
$f(u)$	The effect of investment on extending the shelf life, $f(u) \geq 1$
ω	The shelf life of products
Π	The total profit per unit time
Decision variables	
$p(t)$	The unit sales price at time t
Q	The replenishment quantity at the leadtime
T	The replenishment cycle length
u	The preservation technology investment per unit time to prolong the shelf life

3. The investment effect, $f(u)$ is a continuous, increasing and concave function of preservation technology investment u, *i.e.*, $f'(u) > 0, f''(u) < 0$ with $f(0)=1$, reflecting the effect of investment on extending the shelf life ω. Correspondingly, $f(u)\omega$ refers to the prolonged shelf life.
4. Deterioration is dependent on both time and preservation technology investment

$$\theta(t,u)=\frac{1}{1+f(u)\omega-t},\ 0\leq t\leq T, \tag{2.2}$$

by following Sana [35] with the assumption of $\theta=\frac{1}{1+\omega-t},\ 0<\theta<1$. Consistent with the reality, we consider that the length of replenishment cycle T is less than or equal to $f(u)\omega$. When $\omega\to\infty$, $\theta\to 0$ representing nonperishable items.
5. Replenishment rate is instantaneous and lead time is neglected.
6. Shortage and backlogging are not allowed to occur.

3. Basic model

In this section, we consider a monopolist that dynamically prices agricultural products in stock to fulfill the market demand during the finite horizon $[0,T]$. We assume that the deterioration rate of inventory is proportional to the inventory level, *i.e.*, $\theta(t,u)I$. The inventory level is depleted by the combined effect of demand and deterioration, and hence governed by the following differential equation:

$$\dot{I}(t)=-\theta(t,u)I(t)-D(t),\ 0\leq t\leq T, \tag{3.1}$$

with the initial inventory level $I(0)=Q$ and the inventory level reaching zero by the terminal time T, *i.e.*, $I(T)=0$.

It can be inferred from (3.1) that the inventory level keep nonnegative at all times, *i.e.*, $I(t)\geq 0$ for all $t\in[0,T]$ and there exists no backorder during the selling period. According to the above analysis, the dynamics of the inventory level per cycle is depicted in Figure 1.

The firm's total profit per replenishment cycle is comprised of the earnings of sales less the holding cost, the investment cost and the ordering cost that includes a variable part cQ and a fixed one K. The former refers to the total purchasing costs and the latter corresponds to the replenishment cost per order. Besides, it is assumed that the holding cost is linear in inventory. Accordingly, the total profit can be described as

$$\int_0^T (p(t)D(p(t))-hI(t))\,\mathrm{d}t-uT-(cQ+K). \tag{3.2}$$

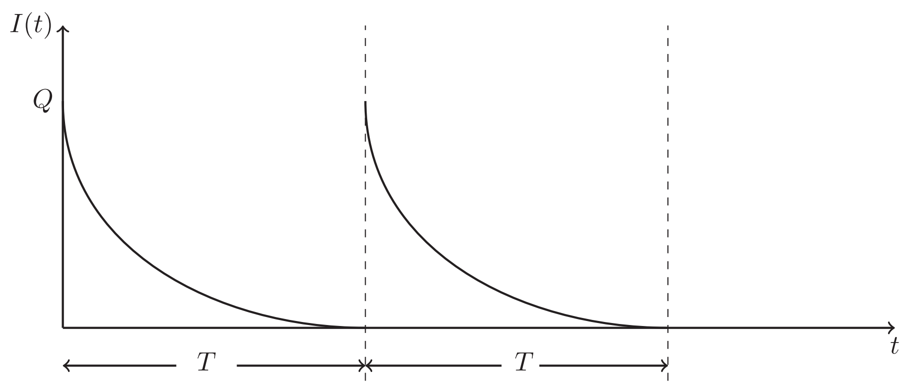

Figure 1. Graphical representation of the inventory system.

Our objective is to jointly determine dynamic pricing policy, replenishment quantity, the replenishment cycle length and the preservation technology investment to maximize the total profit per unit time, which formulates the optimization problem as

$$\begin{aligned} \max_{(p(\cdot),Q,u,T)} \Pi &= \frac{1}{T}\int_0^T (p(t)D(p(t)) - hI(t))\mathrm{d}t - u - \frac{cQ+K}{T}, \\ \text{s.t.} \quad \dot{I}(t) &= -\tfrac{I(t)}{1+f(u)\omega - t} - \alpha + \beta p(t), t \in [0,T], \\ & I(0)=Q, I(T)=0, \\ & 0 \le p(t) \le \tfrac{\alpha}{\beta}. \end{aligned} \tag{3.3}$$

The optimization problem above includes parameter optimization and optimal control problems, but the complexity of the system makes it challenging to derive the solution simultaneously in an analytical approach. Therefore, we solve problem (3.3) sequentially. At first, setting Q, u and T fixed, we derive the optimal pricing strategy p^* *via* solving the following optimal control problem

$$\begin{aligned} \max_{p(\cdot)} \tilde{\Pi} &= \int_0^T (p(t)D(p(t)) - hI(t))\mathrm{d}t, \\ \text{s.t.} \quad \dot{I}(t) &= -\tfrac{I(t)}{1+f(u)\omega - t} - \alpha + \beta p(t), t \in [0,T], \\ & I(0)=Q, I(T)=0, \\ & 0 \le p(t) \le \tfrac{\alpha}{\beta}. \end{aligned} \tag{3.4}$$

Applying the optimal control theory to obtain the optimal pricing policy p^* and associating an adjoint variable λ with the state variable I, we formulate the Hamiltonian function as

$$H(I,p,\lambda,t) = p(t)(\alpha - \beta p(t)) - hI(t) + \lambda(t)\left(-\tfrac{I(t)}{1+f(u)\omega - t} - \alpha + \beta p(t)\right). \tag{3.5}$$

According to Pontryagin's maximum principle (Sethi and Tompson [37]), the optimal control pricing strategy p^* should satisfy

$$H(I,p^*,\lambda,t) \ge H(I,p,\lambda,t). \tag{3.6}$$

The adjoint variable requires to satisfy the following adjoint equation

$$\dot{\lambda} = -\frac{\partial H}{\partial I} = \frac{\lambda}{1+f(u)\omega - t} + h. \tag{3.7}$$

Note that the initial condition $I(0)=Q$ and the terminal condition $I(T)=0$ of the system state variable are fixed. There is no constraints for the initial and terminal conditions of λ. Denoting $\lambda(0)=\lambda_0$ and integrating (3.7), the adjoint variable can be derived as

$$\lambda(t) = \frac{-ht^2 + 2ht(1+f(u)\omega) + 2\lambda_0(1+f(u)\omega)}{2(1+f(u)\omega - t)}, \tag{3.8}$$

where λ_0 is to be determined.

Noticing that $p \in [0, \frac{\alpha}{\beta}]$, we have the following proposition.

Proposition 3.1. *For any given (Q,u,T), the optimal pricing strategy is given as*

$$p^*(t) = \begin{cases} 0, & \text{if } \lambda_0 \le \underline{\lambda}, \\ \frac{\alpha}{2\beta} + \frac{-ht^2+2ht(1+f(u)\omega)+2\lambda_0(1+f(u)\omega)}{4(1+f(u)\omega-t)}, & \text{if } \underline{\lambda} < \lambda_0 < \overline{\lambda}, \\ \frac{\alpha}{\beta}, & \text{if } \lambda \ge \overline{\lambda}, \end{cases} \tag{3.9}$$

where $\underline{\lambda} = \frac{\beta ht^2 + 2(\alpha - \beta h(1+f(u)\omega))t - 2\alpha(1+f(u)\omega)}{2\beta(1+f(u)\omega)}$ *and* $\overline{\lambda} = \frac{\beta ht^2 - 2(\alpha + \beta h(1+f(u)\omega))t + 2\alpha(1+f(u)\omega)}{2\beta(1+f(u)\omega)}$.

Proof. We derive the optimal solution p^* by virtue of (3.6) together with the constraint of the control variable. Specially, we have

$$p^*(t) = \begin{cases} 0, & \text{if } \lambda(t) \leq -\frac{\alpha}{\beta}, \\ \frac{\alpha+\beta\lambda(t)}{2\beta}, & \text{if } -\frac{\alpha}{\beta} < \lambda(t) < \frac{\alpha}{\beta}, \\ \frac{\alpha}{\beta}, & \text{if } \lambda(t) \geq \frac{\alpha}{\beta}. \end{cases} \tag{3.10}$$

It is concluded that $\lambda(t) \leq -\frac{\alpha}{\beta}$ refers to $\lambda_0 \leq \underline{\lambda}$, that $-\frac{\alpha}{\beta} < \lambda(t) < \frac{\alpha}{\beta}$ implies $\underline{\lambda} < \lambda_0 < \overline{\lambda}$, and that $\lambda(t) > \frac{\alpha}{\beta}$ can be transferred as $\lambda_0 \geq \overline{\lambda}$. Hence, substituting (3.8) into (3.10), we obtain (3.9). Since necessary conditions (3.6) and (3.7) for the optimal control problem (3.4) hold, and the Hamilton function is concave, p^* is optimal. The proof is complete. □

It is inferred from Proposition 3.1 that the optimal pricing policy is determined on the basis of different values of λ_0. In fact, for the real inventory management system, $p = 0$ generates no revenue margin, and $p = \frac{\alpha}{\beta}$ results in zero demand rate, which clearly indicates that the cases of $p = 0$ and $p = \frac{\alpha}{\beta}$ will not appear in the set of the optimal solutions. Therefore, we just focus on the case of $\underline{\lambda} < \lambda_0 < \overline{\lambda}$. Under this case, substituting p^* into (2.1) and solving the differential Equation (3.1) with the initial condition $I(0) = Q$, we have

$$I(t) = -\frac{1}{2}\alpha(1+f(u)\omega - t)\ln\frac{1+f(u)\omega}{1+f(u)\omega - t} + \frac{1}{2}\beta\lambda_0 t + \frac{1}{4}\beta h t^2 + \frac{(1+f(u)\omega - t)Q}{(1+f(u)\omega)}. \tag{3.11}$$

Then, together with the terminal condition $I(T) = 0$, we can finally calculate

$$\lambda_0 = \frac{\alpha(1+f(u)\omega - T)}{\beta T}\ln\frac{1+f(u)\omega}{1+f(u)\omega - T} - \frac{2(1+f(u)\omega - T)Q}{\beta T(1+f(u)\omega)} - \frac{1}{2}hT. \tag{3.12}$$

We denote

$$\begin{aligned} \overline{T} &= \min\left\{\omega, \frac{2\alpha}{\beta h}\right\}, \\ \underline{Q} &= \frac{1}{2}\alpha(1+f(u)\omega)\ln\frac{1+f(u)\omega}{1+f(u)\omega - T} - \frac{1}{4}T(2\alpha - \beta hT), \\ \overline{Q} &= \min\left\{\frac{1}{2}\alpha(1+f(u)\omega)\ln\frac{1+f(u)\omega}{1+f(u)\omega - T} + \frac{1}{4}T(2\alpha + \beta hT),\right. \\ &\quad \left.\frac{1}{2}\alpha(1+f(u)\omega)\ln\frac{1+f(u)\omega}{1+f(u)\omega - T} + \frac{(1+f(u)\omega)T}{4((1+f(u)\omega - T)}(2\alpha - \beta hT)\right\}. \end{aligned} \tag{3.13}$$

Considered $T < \overline{T}$ and $\underline{Q} < Q < \overline{Q}$, it follows that $\underline{\lambda} < \lambda_0 < \overline{\lambda}$ is satisfied at all points $t \in [0, T]$. We can further derive the pricing policy and the corresponding inventory level as below:

$$\begin{aligned} p(t) = \frac{\alpha}{2\beta} + \frac{1}{4\beta T(1+f(u)\omega - t)}\Bigg(&-2\alpha(1+f(u)\omega)(1+f(u)\omega - T)\ln\left(1 - \frac{T}{1+f(u)\omega}\right) \\ &- \beta h(1+f(u)\omega)T^2 + (2\beta h f(u)\omega t - \beta h t(t-2) + 4Q)T - 4Q(1+f(u)\omega\Bigg), \end{aligned} \tag{3.14}$$

$$\begin{aligned} I(t) = \frac{1}{4T}\Bigg(&2\alpha T(1+f(u)\omega - t)\ln\left(1 - \frac{t}{1+f(u)\omega}\right) \\ &- 2\alpha t(1+f(u)\omega - T)\ln\left(1 - \frac{T}{1+f(u)\omega}\right) - (T-t)(\beta hTt - 4Q)\Bigg). \end{aligned} \tag{3.15}$$

Substituting (3.14) and (3.15) into the objective function in problem (3.3) yields the total profit per unit time Π as follows

$$\begin{aligned}\Pi = \frac{1}{48\beta T^2(1+f(u)\omega)}\Bigg(\Bigg(&-12\alpha^2(1+f(u)\omega)^2(1+f(u)\omega-T)\ln^2\left(1-\frac{T}{1+f(u)\omega}\right)\\&+12\alpha(1+f(u)\omega)(1+f(u)\omega-T)\Bigg)(\beta hT(1+f(u)\omega)-4Q)\ln\left(1-\frac{T}{1+f(u)\omega}\right)\\&+12\alpha^2T^2(1+f(u)\omega)+6\alpha\beta hT^2(1+f(u)\omega)(2+2f(u)\omega-T)\\&+\beta^2h^2T^4(1+f(u)\omega)-48\beta KT(1+f(u)\omega)-48(1+f(u)\omega-T)Q^2\\&-24\beta T(1+f(u)\omega)(2c+hT)\Bigg)-u.\end{aligned}\tag{3.16}$$

Note that the total profit per unit time is a function of Q, u and T. The constraints of $\underline{Q} < Q < \overline{Q}$ and $T < \overline{T}$ correspondingly result from the constraint of $0 < p < \frac{\alpha}{\beta}$ in the optimization problem (3.3). Therefore, the original optimization problem (3.3) can be transferred as the following optimization problem

$$\begin{aligned}&\max_{(Q,u,T)} \Pi(Q,u,T)\\&\text{s.t. } \underline{Q} < Q < \overline{Q}, 0 < T < \overline{T}, u \geq 0.\end{aligned}\tag{3.17}$$

Furthermore, we obtain the following proposition.

Proposition 3.2. *For any given preservation technology investment u and replenishment cycle length $T < \overline{T}$, there uniquely exists an optimal replenishment quantity*

$$Q^* = \frac{1}{2}\alpha(1+f(u)\omega)\ln\frac{1+f(u)\omega}{1+f(u)\omega-T} - \frac{1}{4}\beta T(2c+hT)\frac{1+f(u)\omega}{1+f(u)\omega-T},\tag{3.18}$$

which satisfies $\underline{Q} < Q^* < \overline{Q}$ and the second order conditions for maximum.

Proof. Taking the first order derivative of Π in (3.16) with respect to Q, we have

$$\begin{aligned}\frac{\partial\Pi(Q,u,T)}{\partial Q} = \frac{1}{2\beta T^2(1+f(u)\omega)}\Bigg(&2\alpha(1+f(u)\omega)(1+f(u)\omega-T)\ln\frac{1+f(u)\omega}{1+f(u)\omega-T}\\&-\beta hT^2(1+f(u)\omega)-4(1+f(u)\omega-T)Q-2\beta cT(1+f(u)\omega)\Bigg).\end{aligned}$$

Let $\frac{\partial\Pi(Q,u,T)}{\partial Q} = 0$. We obtain Q^* in (3.18) which can be shown to satisfy $\underline{Q} < Q^* < \overline{Q}$.

Furthermore, taking the second order derivative of (3.16) with respect to Q yields

$$\frac{\partial^2\Pi(Q,u,T)}{\partial Q^2} = \frac{-2(1+f(u)\omega-T)}{\beta T^2(1+f(u)\omega)} < 0.$$

Thus, there exists a unique Q^* in (3.18) maximizing $\Pi(Q,u,T)$. The proof is complete. □

Although we can get the expression of $\Pi(Q,u,T)$ in (3.16), the concavity of Π with respect to (Q,u,T) is challenging to be proved through an analytical approach. Thus the optimal (Q,u,T) cannot be derived by the first-order conditions. We design an algorithm to obtain the optimal solution. We first divide the three dimensional decision space (Q,u,T) into $Q\bigcup\{u,T\}$ and then design an iterative algorithm to find the optimal (Q,u,T), which is inspired by the algorithm proposed in Zhang *et al.* [44].

Algorithm
Step 1: Set $i=0$ and initialize the value of Q_i and calculation accuracy ϵ.
Step 2: For a given replenishment quantity Q_i, search the optimal preservation technology investment and (u_i,T_i) by solving the optimization problem (3.17) *via* a two-dimensional search algorithm.
Step 3: Based on (u_i,T_i) obtained from Step 2, calculate the optimal Q_{i+1} by virtue of (3.18).
Step 4: If $|Q_{i+1}-Q_i|<\epsilon$, set $Q^*=Q_{i+1}$, then output the solution (Q^*,u^*,T^*) and stop. Otherwise, set $i=i+1$ and return to Step 2.

It should be mentioned that although the concavity of total profit per unit time Π with respect to (u,T) cannot be proved in an analytical approach, we have done a large number of numerical examples and find that the corresponding concavity holds due to the assumption that $f(u)$ is a concave function. In the above algorithm, we first provide an initial value of Q_i and calculation accuracy ϵ. In Step 2, for the current Q_i, a global maximum solution of (u^*,T^*) of $\Pi(u,T)$ is obtained. Next, the value of $\Pi(Q,u,T)$ can be further promoted by an optimal Q_{i+1} in Step 3. Note that Steps 2 and 3 address the corresponding convex optimization problems. The iteration will be repeatedly executed until it arrives at the calculation accuracy. On the basis of that the sequence of performance indices is monotonically increasing and upper bounded, hence, the convergence of the iterative algorithm can be guaranteed by a given calculation accuracy.

4. Numerical study

4.1. Numerical examples

In this subsection, we employ numerical examples to illustrate the theoretical results and obtain managerial insights.

Let $\alpha=50$, $\beta=5$, $h=0.5$, $c=1.0$, $K=15$, $\omega=5$ and $\epsilon=10^{-4}$, and consider a concave function $f(u)=\sqrt{1+\gamma u}$ that is consistent with the previously mentioned assumption. We here denote γ as the coefficient referring to the investment efficiency and set $\gamma=1.0$. We choose the parameters according to prior literature on perishable inventory systems (*e.g.*, Saha *et al.* [34] and Zhang *et al.* [45]) for a comprehensive illustration. By applying the algorithm proposed in Section 3, we calculate $Q^*=32.2546$, $u^*=0.4316$, $T^*=1.3639$, $\Pi^*=79.4216$. As shown clearly in Figure 2, the numerical results present that $\Pi(Q,u,T)$ is jointly concave with respect to (u,T) for given $Q=Q^*$ and the total profit per unit time reaches the maximum at the point of $(u,T)=(0.4316,1.3639)$. In the next moment, we run the computational result with different feasible values of Q. Figure 3 illustrates the concavity of Π with respect to Q and it can be found that the total profit per unit time reaches the maximum at the point of $Q=32.2546$ for given $(u,T)=(u^*,T^*)$.

Therefore, the optimal pricing policy and the corresponding optimal inventory level can be obtained as

$$p^*(t)=\frac{-38.4034+3.2544t+0.1250t^2}{t-6.9825},$$
$$I^*(t)=(174.5619-25.0000t)\ln(6.9825-t)+46.4656t-306.9895+0.6250t^2,$$

for $t\in[0,1.3639]$. As clearly shown in Figure 4, optimal price is increasing with time which indicates that the firm adopts a penetration pricing policy. This is because facing an accelerating deterioration rate overtime, the firm intends to capture a large enough market by setting a lower initial price at the beginning of the selling horizon. Then, a quick fall in the inventory level resulted from the large demand rate allows the firm to significantly reduce the loss of deterioration and the corresponding inventory cost which in the meanwhile enables the firm to gain more profit with increasing price in later period when the inventory level is relatively low.

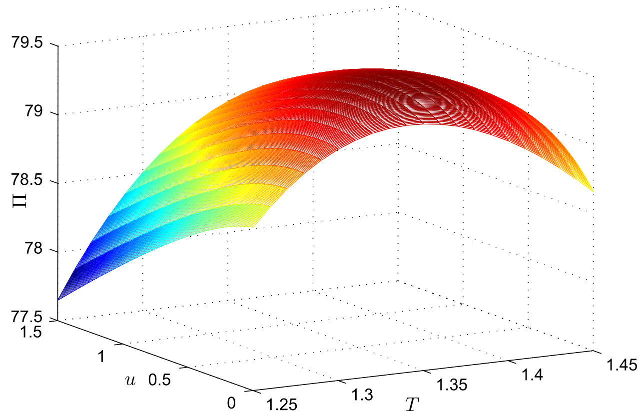

FIGURE 2. Total profit per unit time with respect to u and T for fixed Q.

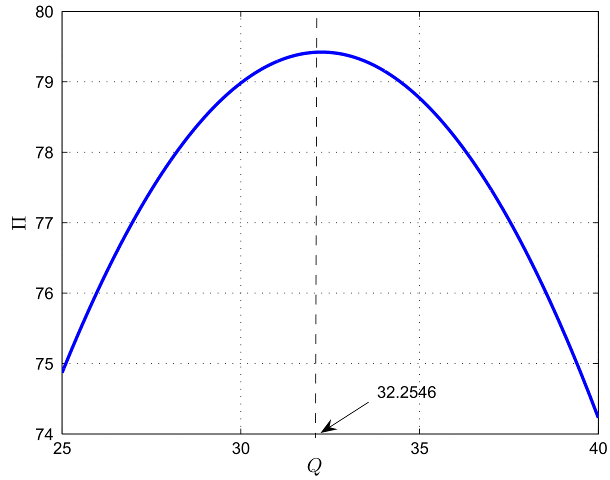

FIGURE 3. Total profit per unit time with respect to Q for fixed u and T.

4.2. Sensitivity analysis

Now, we study the impacts of changing the values of the system parameters of α, β, h, c, K and γ. The sensitivity analysis is addressed by varying one parameter by -20%, -10%, $+10\%$, and $+20\%$ at a time and keeping the remaining parameters fixed. Using the previously proposed algorithm, the computational results are reported in Tables 2 and 3. The former concludes the effects on optimal pricing policy of the parameters, and the latter illustrates the impacts of key parameters on other optimal decisions.

From Table 2, it is easy to find that market potential, unit purchasing cost and unit holding cost lift price, whereas price sensitivity, fixed order cost and investment efficiency force price down. Put differently, we find that the optimal price is positively impacted by market potential, unit purchasing cost and unit holding cost but negatively influenced by price sensitivity, fixed order cost, investment efficiency and shelf life of the product.

From Table 3, we obtain the following conclusions.

1) When the market potential α increases and other parameters keep unchanged, it can be seen in Table 2 that the optimal replenishment quantity Q^*, the optimal preservation technology investment u^* as well as the

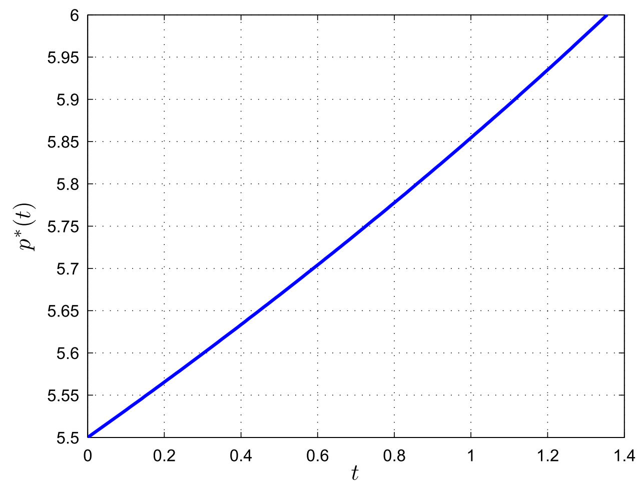

FIGURE 4. Optimal pricing policy p^*.

TABLE 2. Impacts on the optimal price p^* of key parameters.

	α	β	c	K	h	γ
$p^*(t)$	+	−	+	−	+	−

TABLE 3. Impacts of key parameters on optimal strategies and total profit.

		Q^*	u^*	T^*	Π^*			Q^*	u^*	T^*	Π^*
	40	28.1152	0.3392	1.5442	41.9960		4.0	32.7599	0.4458	1.3402	110.1074
	45	30.2647	0.3877	1.4449	59.4114		4.5	32.5080	0.4388	1.3518	93.0274
α	50	32.2546	0.4316	1.3639	79.4216	β	5.0	32.2546	0.4316	1.3639	79.4216
	55	34.1244	0.4723	1.2962	102.0085		5.5	31.9907	0.4241	1.3762	68.3432
	60	35.9110	0.5111	1.2389	127.1590		6.0	31.7280	0.4165	1.3892	59.1609
	0.8	33.3448	0.2985	1.3679	84.2234		12.0	28.7184	0.2835	1.2161	81.7475
	0.9	32.7926	0.3670	1.3659	81.8043		13.5	30.5322	0.3594	1.2919	80.5513
c	1.0	32.2546	0.4316	1.3639	79.4216	K	15.0	32.2546	0.4316	1.3639	79.4216
	1.1	31.7389	0.4931	1.3623	77.0743		16.5	33.8793	0.5001	1.4319	78.3486
	1.2	31.2433	0.5517	1.3611	74.7617		18.0	35.4493	0.5663	1.4975	77.3246
	0.40	35.1820	0.5115	1.4710	81.0197		0.8	31.9732	0.2673	1.3438	79.3426
	0.45	33.6283	0.4693	1.4143	80.2033		0.9	32.0892	0.3589	1.3532	79.3798
h	0.50	32.2546	0.4316	1.3639	79.4216	γ	1.0	32.2546	0.4316	1.3639	79.4216
	0.55	31.0336	0.3979	1.3189	78.6706		1.1	32.3618	0.4866	1.3719	79.4654
	0.60	29.9367	0.3674	1.2783	77.9470		1.2	32.4584	0.5300	1.3791	79.5097

total profit per unit time Π^* increase while the optimal length of replenishment cycle T^* declines. This is because a large market potential α has positive effect on the demand rate. Hence, such situation encourages the firm to order more agricultural products to fulfill the larger demand. Obviously, a larger demand rate expedites the reduction of the stocks. Put differently, the length of replenishment cycle is shortened by the fast decline of the inventory level. As shown in Table 1, price is promoted by the firm to produce a higher profit margin. Therefore, the total profit per unit time rises. It is intuitive to note that the increase in replenishment quantity will consequently generate more losses of deteriorated units. As such, this case requires the firm to invest more in preservation technology to slow down the rate of deterioration.

2) When the price sensitivity β increases, the optimal Q^*, u^* and Π^* decrease while the optimal length of replenishment cycle T^* increases. When consumers become more sensitive to price, the demand accordingly shrinks which leads to a smaller amount of products ordered per cycle. For the sake of keeping enough sales volume, the firm needs to charge a lower price so as to enlarge demand. However, it cannot compensate for the lost demand caused by a larger price sensitivity. Thus, lower price and less sales jointly result in a decrease in revenue. In order to save cost, the firm prefers to cut the expenditures of investment. In addition, the replenishment cycle length will be prolonged due to the decrease in the amount of product sales.
3) When the unit purchasing cost c increases, the optimal u^* increases whereas the optimal Q^*, T^* and Π^* decline. The rise in unit purchasing cost decreases the net margin per unit product that burdens the firm. Hence, a larger cost alters the firm to replenish less products per cycle. Obviously, replenishing less products implies a shorter replenishment cycle. In order to compensate for the reduction in profit margin resulted from a larger purchasing cost, the firm is willing to increase price as shown in Table 1 which in turn incurs a shrinkage of demand. What's more, the increase in marginal cost stimulates the firm to invest more to reduce the deterioration loss. It is obvious that a larger marginal cost will reduce the profit of the firm.
4) When the fixed order cost K increases, the optimal Q^*, u^* and T^* increase while the total profit per unit time Π^* decreases. It is inferred that a relatively large fixed order cost K discourages the firm to replenish frequently. Hence, the firm is enticed to order more units per replenishment cycle. That is to say, a larger fixed order cost decreases the order frequency and prolongs the optimal length of replenishment cycle which in turn generates more deterioration loss. For this reason, a long selling horizon induces the firm to increase investment so as to mitigate the deteriorative effect, *i.e.*, postpone the deteriorating process of each unit. Aiming to stimulate demand associated with a larger replenishment quantity, the firm intentionally forces price down which in turn cuts the profit margin per unit. Thus, it can be concluded that a larger K will damage the interests of the enterprise.
5) When the unit inventory holding cost h increases, the optimal Q^*, u^* and T^* and Π^* all decrease. Intuitively, a larger h adds to the inventory cost and lowers the profit margin. Facing this situation, the firm tends to order a smaller amount of products per cycle to avoid a relatively larger holding cost which further produces a shorter replenishment cycle length. Furthermore, the firm has no incentive to enhance the investment in preservation technology since it will produce more cost in two aspects, one referring to the investment cost and the other one stemming from the added holding cost with a larger h. As seen in Table 1, the firm will charge a higher price so as to attain more profit but this cannot offset the decrease in profit margin which results from the increasing holding cost. For this reason, a larger inventory cost of agricultural products finally whittles down the interests of the firm.
6) When the efficiency of the preservation technology investment γ increases, the optimal strategies Q^*, u^*, T^* and the total profit per unit time Π^* are increasing. It is easy to see that a higher efficiency γ requires a lower expenditure of preservation technology investment. That is to say, the firm is able to retard the deteriorative effect by spending a lower cost. With this regard, it will be advantageous for the enterprise to reinforce the investment so as to reduce the deterioration loss to a greater extant. Thus, it will lead to a significant reduction of spoiled units. Meanwhile, in presence of high efficiency, the enterprise intentionally orders more agricultural products per replenishment cycle to enlarge sales. Then, a large amount of stocks may accordingly prolong the replenishment cycle length. The firm prefers to set a lower price so as to expand demand as shown in Table 1 and replenishes more agricultural products. Then, the decline in deterioration loss associated with a large amount of sales volume boosts the firm's total profit per unit time under a larger investment efficiency.

Next, we analyze the impact of shelf life on the optimal strategy. We set the same parameters as above except for the given shelf life ($\omega = 1, 2, \ldots, 10$). Hence, we calculate the optimal values Q^*, u^*, T^*, and Π^* respectively and show them in Table 4.

TABLE 4. Optimal strategies and total profit under different ω.

	ω									
	1	2	3	4	5	6	7	8	9	10
Q^*	26.7864	29.5056	30.8536	31.6820	32.2546	32.6732	32.9894	33.2587	33.4596	33.6714
u^*	1.6959	1.1937	0.8453	0.6060	0.4316	0.2981	0.1918	0.1057	0.0331	0
T^*	1.0128	1.1773	1.2659	1.3231	1.3639	1.3946	1.4185	1.4368	1.4545	1.4710
Π^*	72.6987	76.1142	77.7519	78,7442	79.4216	79.9190	80.3026	80.6092	80.8610	81.0716

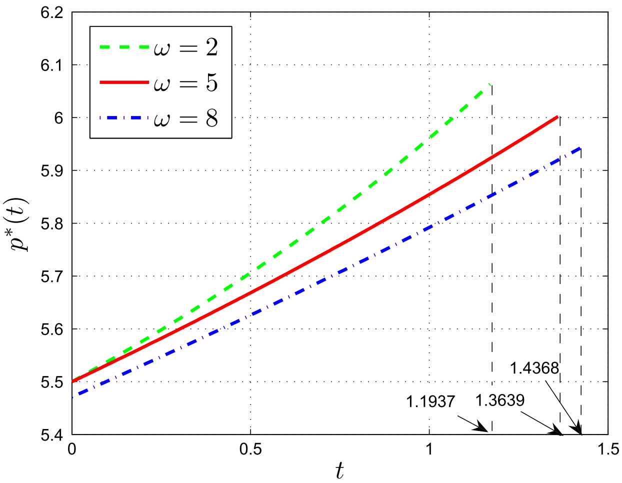

FIGURE 5. Impact of ω on optimal price.

From Table 4, we find that when the shelf life of each product ω increases, the optimal replenishment quantity Q^*, the optimal replenishment cycle length T^* and the total profit per unit time Π^* increase whereas the optimal investment u^* drops. This reflects that in presence of a relatively long shelf life of the product, the firm is allowed to cut investment in preservation technology that helps to save investment cost since the deterioration rate is relatively low. Put differently, a longer shelf life corresponds to a reduced u^*. As clearly shown in Table 4, when the product's shelf life ω approaches 10, the firm just stops investing in preservation technology since at this point the positive effect brought out by investment on the reduction in deterioration rate is not obvious. The shelf life represents the quality attribute of the product and a longer shelf life represents a higher quality and less waste of products. Facing this, the firm is likely to order a larger quantity of products per replenishment cycle to squeeze more profit, which extends the length of the replenishment cycle accordingly. Besides we assess the impact of the shelf life on optimal price in Figure 5. It is shown that the optimal price is negatively impacted by the shelf life suggesting that the firm would like to enlarge demand *via* charging a lower price when the rate of the product's deterioration is relatively low. On the contrary, the firm will charge a higher price aiming to raise the profit margin per unit product under the situation in which the agricultural products highly decay. It is obvious to note that a longer shelf life contributes to benefit the firm.

In the following, we concentrate on whether or not the firm takes preservation technology investment for agricultural products with different shelf lives under two distinct cost coefficients h and c respectively. Also, we consider the impacts of investment efficiency γ on the corresponding results. The results are shown in Figures 6 and 7.

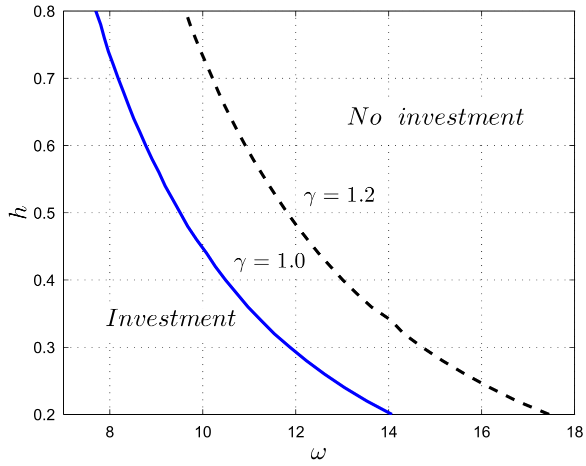

FIGURE 6. Investment decisions *via* h and ω under different efficiencies.

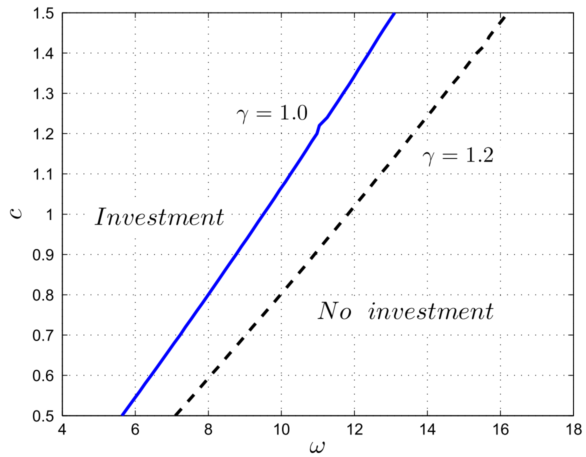

FIGURE 7. Investment decisions *via* c and ω under different efficiencies.

Notice that the solid lines that are depicted in Figures 6 and 7 refer to the threshold curves that distinguish the firm's different investment strategies when $\gamma = 1.0$, while the dashed ones represent the cases of $\gamma = 1.2$. We observe that the investment strategy of the firm depends on the key parameters including the shelf life of the product. If the shelf life is vary long, the firm has no incentive to invest. Otherwise, the investment decision relies on the cost coefficients. Specially, a relatively small unit holding cost or a relatively large unit purchasing cost stimulates preservation technology investment. To be specific, it can be clearly found in Figure 6 that when the shelf life of the product is very short, the firm tends to take investment because when facing a large deterioration rate, investing in preservation technology can greatly mitigate the deteriorative effect of products which accordingly improves profit. When ω is large, no investment appears to be a better choice for the firm when the unit holding cost is relatively large. It is shown that as ω goes up, whether to inhibit investment relies on the magnitude of h. Specially, for a moderate shelf life such as $\omega = 9$, if the unit holding cost is below about 0.56, the firm adopts an investment. Otherwise, the firm has no incentive to invest since investing creates more costs including the inventory cost as well as the investment cost. Specially, as the shelf life of the product becomes relatively long, the firm does not take preservation technology investment due to the fact that the improvement in deterioration rate brought out by the investment is slight compared with the investment cost.

Figure 7 describes the impact of the unit purchasing cost c on the firm's investment decision. We observe that when agricultural products have a relatively short shelf life, the firm intentionally invests to reduce the deterioration waste. Furthermore, as the shelf life of the product elongates, the firm's investment decision hinges on the size of the unit purchasing cost, and taking investment is no longer beneficial when the firm is confronted with a relatively small c. For example, as clearly shown in Figure 7 that if unit purchasing cost of the product of $\omega = 9$ is lower than around 0.94, the firm will not invest. This is because a relatively small unit purchasing cost implies a high profit margin per unit product, at this point, it is unnecessary for the firm to undertake the extra cost on investing *via* trading off between the improvement in the originally low deterioration rate and the expenditure of the investment. Therefore, as ω increases, the investment becomes more disadvantageous. Above all, it is concluded that a long shelf life, a high unit holding cost, or a small unit purchasing cost will make the firm not to invest in preservation technology. Otherwise, investing is beneficial for the firm. In addition, from Figures 6 and 7, we find that a higher investment efficiency stimulates the firm to invest in technology investment because it reduces the corresponding cost.

Finally, in order to verify the robustness of our results with regard to the form of demand function, we further numerically analyze the corresponding results under the case for a non-linear demand $D(t) = Ap(t)^{-2}$ where A is a constant. The additional numerical studies verify that our main results remain qualitatively. Here, we do not present the detailed computational results for saving space.

5. Conclusion and further research

In this paper, we establish an inventory model for agricultural products in which the deterioration rate of units is increasing with time and the demand is linearly decreasing with respect to price without shortage. Through applying dynamic optimization method and designing a solution procedure, we derive the optimal dynamic pricing strategy, the replenishment cycle length, replenishment quantity and the preservation technology investment simultaneously so that we can maximize the total profit per unit time. It is observed through numerical studies that it is advantageous for the firm to take the penetration pricing policy. Importantly, the preservation technology investment strategy of the firm relies on the key parameters, especially the shelf life of each product. That is, if the shelf life is long enough, the firm will not take the investment. Otherwise, whether to invest in preservation technology depends on the cost coefficients, *i.e.*, when the unit holding cost is relatively small or the unit purchasing cost is relatively large, investing in preservation technology may benefit the firm. Additionally, we observe that a higher investment efficiency encourages the firm to take preservation technology investment.

There exist some further researches to extend this inventory model. For instance, we can consider the situation that consumer demand is time dependent as well, since the demand for agricultural products may change as time goes by. Besides, we can extend the model in the context of an uncertain setting where the demand is random, at this point, allowing the shortage and determining related replenishment policies may be another potential extension. In addition, it can be further discussed that the product's shelf life or freshness may influence consumer demand or product goodwill which is in line with the reality. With this regard, it is interesting to study the corresponding impacts on the firm's strategies.

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