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A MATHEMATICAL MODEL ON ECO-FRIENDLY MANUFACTURING SYSTEM UNDER PROBABILISTIC DEMAND

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Abstract. The article deals with a mathematical model of production inventory system of green products in a green manufacturing industry. The main objective of this proposed model is to formulate a profit function for service level and random variable dependent demand implementing green technology in the manufacturing industry for reduction of green house gas emission. The production lotsize is considered here as an increasing function of green technology and capital invested for setup the manufacturing system which meets the market demand. As a result, green technology, capital invested for setup and service level are decision variable which are optimized to achieve maximum profit. Finally, numerical example for normal distribution and distribution free cases are illustrated to justify the proposed model.

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1. Introduction

Nowadays, sustainable and green technology are a research topic in a good and efficient management of the manufacturing industries since these topics are competitive and beneficial in both the short and long term project of the organizations. Although "Going Green" and Sustainability are not same in general but these are related to each other. These are holistic approaches of the organizations to do good things for the society by implementing environmentally cognizant improvements. Generally speaking, green technology for going green practices can strengthen the reputation of a manufacturing industry and make more marketable.

It is observed that energy resources of manufacturing system and its allied systems are driven by fossil fuels. These type of resources are non renewable and generate green house gases (GHG) which are harmful for the people of a country. Moreover, these types of resources are not unlimited so the use of alternative resources is urgent for its reservation. Although the green technology for alternative resources like solar, water and wind energy along with efficient equipment and machinery is costly for setup the manufacturing system. This makes the industry more efficient, competitive and gainful. Therefore, planning for green products out of recycled resources is main part of the green technology. Hence green technology successfully transformed

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into green manufacturing by shifting towards green energy will be more beneficial for the nation and business organizations. Green technology is important for ecological sustainablity. This includes many concerns but not limited to air, water and land pollution, energy usage and waste generation and recycling. In a nutshell, its objective is to minimize the affect of human activities on the environment. The manufacturer can attract the consumers towards green products by communicating the message of their benefits to all stakeholder through advertising, consumer awareness programme and sales teams' efforts.

The production industry is one of the leading sources of GHG emission and it is increasing day by day for unconscious industrialists as well as the customers of the products. Traditional manufacturing industries lead by fossil fuels do not emphasize on sustainable of the ecological system and human race. Many researchers have developed the production-inventory models ignoring the cost of green technology for green products. In this line of works, the works of Cardenas-Barron [4], Sarkar *et al.* [27], Taleizadeh *et al.* [31], Nobil *et al.* [19], Bhunia *et al.* [3] should be mentioned in inventory literature. Chung and Wee [6] investigated green product design and remanufacturing activities in an integrated production inventory model. Chung and Wee [7] extended a production inventory model considering remanufacturing in a green supply chain. Wee *et al.* [34] studied renewable energies based on four components: renewable energy supply chain, renewable energy performance, and limitations which are overcome by suggested strategies. This study is referred to the reader for understanding green technologies and its prospects. Chen *et al.* [5] suggested a bi-objective model to reduce carbon emission from idling truck engines at marine container terminals by minimizing both the truck waiting times and truck arrival pattern change .In this context, a literature survey on multicriteria decision making techniques proposed by Govindan *et al.* [11] for green supplier evaluation and selection should be mentioned among others. Sarkar *et al.* [25] discussed vendor-buyer integrated inventory model in which setup cost and penalty of carbon emission during transporting items is reduced by new technology. Summerbel *et al.* [29] analyzed a case study of cement industry for potential reduction of carbon emissions by performance improvement of various factors. Saxena *et al.* [28] studied a green supply chain model with mixed strategy of production and remanufacturing under the condition of permissible delay in payment. Fattahi and Govindan [10] designed the integrated forward and reverse logistics network for the stochastic demand of new as well as used products and formulated stochastic and mixed-integer linear programming model. Also, they proposed a novel simulation technique to solve the problem. Modak *et al.* [18] analyzed decentralized and centralized models of manufacturer–retailer supply chain considering GHG emissions trading schemes. Recently, Modak [18] reviewed the research works on reduction of the trend of greenhouse gas emission in supply chain management.

In practice, demand of the products is uncertain in nature. This type of demand pattern includes newsvendor and markov modulated demand. Besides uncertainty, a good management offers better services to attract the customers to compete with rivalry businessmen. Besides revenue sharing [20], buyback [9], disposal cost sharing [21], profit sharing [24], the impact of service level offered by the upstream channel member to the downstream members in a supply chain increases the demand [12–15, 23, 30, 33, 36], In newsvendor inventory literature, the research works done by He *et al.* [13], Alfares and Elmora [1], Sarkar and Chaudhuri [26], Wang [32] and Xiao *et al.* [35] are noteworthy. Lu *et al.* [16] applied game theory to achieve the equilibrium solutions for each channel member (two competitive manufacturers and their common retailer) while end customers are sensitive with retail price and service offered by the manufacturers. Recently, Roy *et al.* [22] extended a single-period newsvendor type inventory model to obtain the optimal order quantity in light of the competing retailers' strategies in which unsold items of the retailers are buyback to the manufacturer at a price.

The aim of this present article is to develop a mathematical model incorporating green technology in a manufacturing industry. As the green technology attracts the conscious customers to buy more, the manufacturer has to enhance the production rate that results in higher cost for setup, labour and advanced technology. Consequently, the production rate is a nonlinear function of green technology and capital invested for setup and other factors. The green technology generates lower emission of GHG and it indirectly decreases the penalty cost charged by the government organizations for GHG emission. So, our model considers the penalty cost for GHG is a monotonic decreasing function of technology. Recently, BCG (Boston Consulting Group; `bcg.com`) survey of consumers in both the developed and developing countries has pointed out that there is still a huge gap in consumer awareness that Green companies must strive to bridge. In this regard, we introduce the newsvendor type demand function as sales effort (like adverting, awareness programmes, promotional efforts). Here, the cost for service level by sales team efforts is a nonlinear function of service level that increases the demand of the customers by promoting their benefits towards stakeholders. Finally, we formulate an expected profit function by trading off cost for green technology, penalty of GHG emission, service level, raw materials, salvage value of used products for recycling, inventory and shortage cost of the finished products. Then, our objective is to maximize the expected profit function at optimal technology, capital cost and service level those are decision variables of the model. The schematic diagram is presented in Figure 1 as follows.

2. Assumptions and notation

The following notations are used to depict the proposed model.

2.1. Notation

Q: production lotsize per unit time.
θ: type of technology implemented in production system.
$\overline{\theta}$: upper boundary of θ.
$\underline{\theta}$: lower boundary of θ.
k: cost invested for setup the production system except the cost of materials. It includes labour, energy, light, fan, water, etc.
$D(x,s)$: demand of the product per unit time.
s: volume of service provider like advertising and repairing of defective items after sale and others.
x: random variable with probability density function $f(x)$ having mean μ and standard deviation σ.
$E[x]$: expectation of variable x.
$H(s)$: cost at service provider.
$G(\theta)$: cost or penalty charges for GHG emission due to manufacturing and transportation of the products.
$T(\theta)$: cost of technology for θ type technology.
c_r: cost of raw materials per unit item.
c_h: cost of holding per unit per unit time.
c_s: cost of penalty per unit item per unit time during stock out period.
c_0: lower bound of cost of green technology which occurs at $\theta = \underline{\theta}$.
c_1: scale parameter of cost per unit item charged for GHG emission.
c_2: salvage value per unit item earned by the customers from the manufacturer after submitting the used products.
δ: percent of the used products which are received from the customers.
p: selling price per unit item.
$\dot{u}$: the first order derivative of u with respect to the decision variable.
$\ddot{u}$: the 2nd order derivative of u with respect to the decision variable.
$\pi(\theta,k,s)$: expected profit function of the manufacturer.

2.2. Assumptions

The following assumptions are considered to investigate the proposed model.

(i) Demand of the product is assumed to be a function of random variable and service level.
(ii) Production lotsize is a function of green technology (θ) and cost (k) invested for setup, labour energy, light, fan, water and others. It is an increasing function of θ and k.
(iii) Lead time of the production is neglected for shake of simplicity, *i.e.*, the lotsize (lot-for-lot policy) is instantly available at the manufacturer.

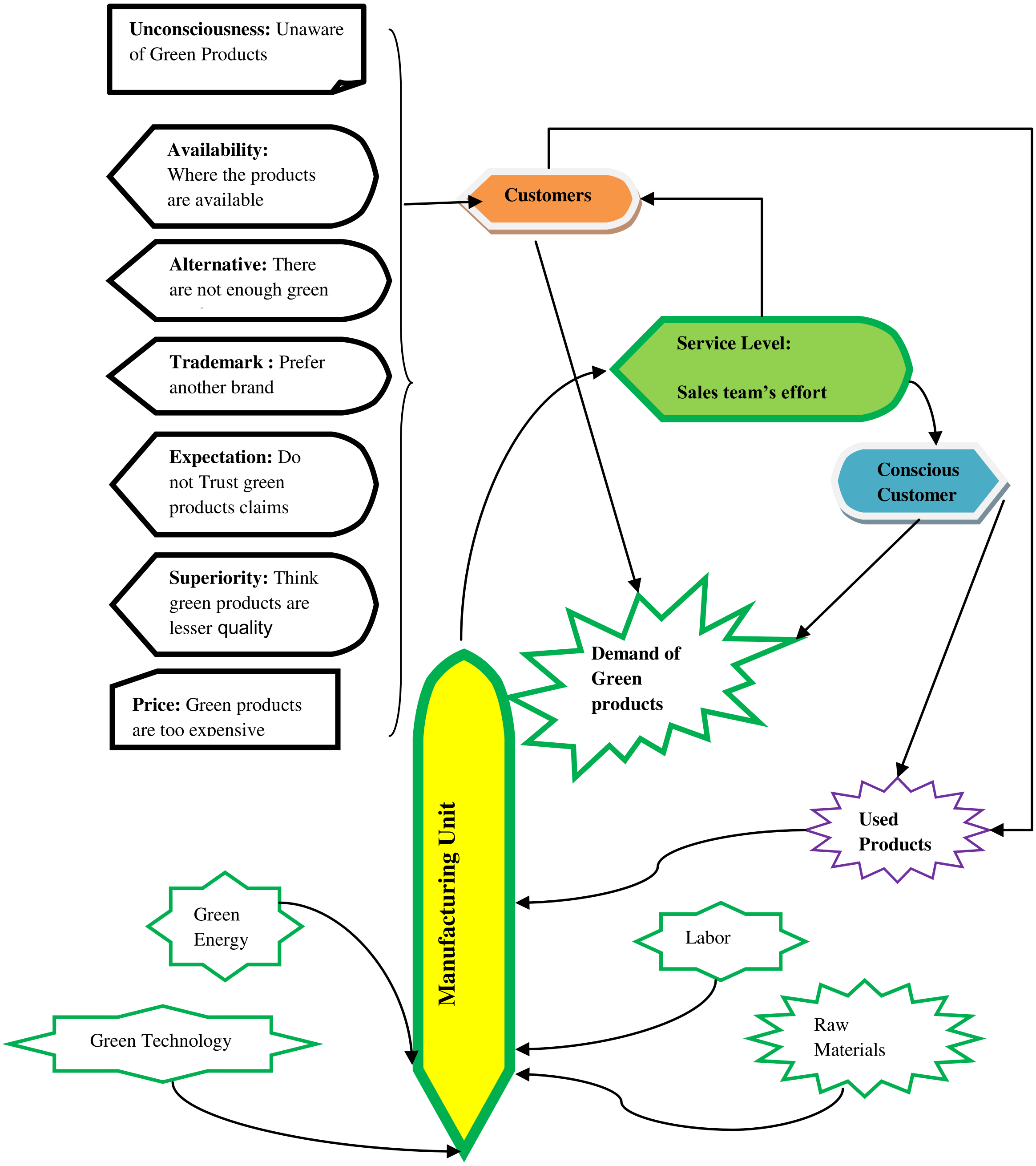

FIGURE 1. Schematic diagram of the model.

(iv) In traditional manufacturing system, fuel fossil are used as energy to run the production that generates GHG. Our model considers renewable resources like solar energy, bio-fuel, energy created from hydrology as advanced technology which generates GHG at minimum level. This type of technology generally enhance the production rate as well as decreases the cost of penalty for GHG emission. Although the cost of technologies to generate such type of renewable energy resources are costly, it is urgent to motivate the customers offering better services to buy more green products to protect our environment from a large scale of pollution. In this system, the users of green products would get salvage value from used products if they return those at the time of next purchase. The manufacturer collects the used products from the buyer for recycling the process that saves from more use of raw materials. Consequently, the cost of penalty for GHG emission is assumed to be an increasing function of production lotsize and type of technology.

3. Mathematical model

3.1. Continuous known distribution

We consider a green technology which is applied in a manufacturing firm/industry to produce the goods which have the consumers in a market. The manufacturing systems produces the production lotsize to satisfy the probabilistic and service level dependent demand of the product. Here, the production lotsize per unit time is

$$Q(\theta, k) = e^{(\theta - \underline{\theta})}\xi(k), \tag{1}$$

where θ is the type of technology having the upper limit $(\overline{\theta})$ and lower limit $(\underline{\theta})$,*i.e.*, $\theta\epsilon[\underline{\theta}, \overline{\theta}]$ and $\xi(k)$ is an increasing function of capital (k) invested for setup of the production systems, labour, water, etc. Consequently, Q is an increasing function of both the decision variables θ and K. The demand of the product per unit time at the market is

$$D(x, s) = x + \frac{\gamma s}{1+s}\cdot \tag{2}$$

where x is a random variable which follows a probability density function $f(x)$ and the 2nd term of $D(x, s)$ is a bounded increasing function of service level (s). This is an additional demand belongs to $[0, \gamma]$ where γ is the maximum demand due to the affect of the sufficiently large volume of the service level (*i.e.*, $s \to \infty$) provided by the manufacturer and additional demand is 0 when $s \to 0$. This type of services contain advertising about the quality and environmental effect of the products, promotional efforts like free gift, salvage value of the used products and free services for repairing of the defective items after sale, if needed. The cost of green technology for θ type is

$$T(\theta) = c_0\left(\frac{\overline{\theta} - \underline{\theta}}{\overline{\theta} - \theta} - 1\right). \tag{3}$$

Here, $\dot{T}(\theta) = \frac{c_0}{(\overline{\theta}-\theta)^2} \geq 0 \; \forall \; \theta \; \epsilon \; [\underline{\theta}, \overline{\theta}]$, *i.e.*, $T(\theta)$ is an increasing function of $\theta \; \epsilon \; [\underline{\theta}, \overline{\theta}]$. The cost/penlty charges for GHG emission during production and transportation of lotsize Q is

$$G(\theta, Q) = c_1(\overline{\theta} - \theta)^2 Q. \tag{4}$$

This cost is concave function of θ because $\dot{G}(\theta) \geq 0 \forall \theta \; \epsilon \; [0, \overline{\theta} - 2]$ and $\dot{G}(\theta) \leq 0 \forall \theta \; \epsilon \; [\overline{\theta} - 2, \overline{\theta}]$, and G attains maximum value at $\theta = \overline{\theta} - 2$ as $\ddot{G}(\overline{\theta} - 2) = -2(\overline{\theta})^2 e^{(\overline{\theta} - \underline{\theta} - 2)}\xi(k) \leq 0$. The cost of service provider for service level s is

$$H(s) = \alpha s + \beta s^2, \tag{5}$$

where $\alpha(\geq 0)$ and $\beta(\geq 0)$ are scale parameters. This is an increasing function of s. The cost of raw materials for the lotsize Q is

$$R_r(Q) = c_r Q. \tag{6}$$

The expected salvage value earned by the customers from the manufacturer after submitting the used products is

$$R_c(s) = \delta c_2 \left(\mu + \frac{\gamma s}{1+s} \right). \tag{7}$$

where δ is a certain percent of the used products are received from the customers. Generally speaking, δ is not 100% because of lack of consciousness and education of the customers about eco-friendly environment and reservation of natural resources. The expected holding cost is

$$\text{Inv}(Q) = c_h E[(Q-D)^+]. \tag{8}$$

The expected shortage cost is

$$\text{Shor}(Q) = c_s E[(D-Q)^+]. \tag{9}$$

The expected selling price is

$$\text{Sel}_p(s) = p \left(\mu + \frac{\gamma s}{1+s} \right). \tag{10}$$

Therefore, the expected profit of the manufacturer by trading off all cost parameters related to the firm is

$$\begin{aligned}
\pi(\theta,k,s) &= \text{Sel}_p(s) - R_c(s) - [\text{Inv}(Q) + \text{Shor}(Q) + R_r(Q) + T(\theta) + H(s) + G(\theta,Q) + k] \\
&= \text{Sel}_p(s) - R_c(s) - [c_h(Q - \frac{\gamma s}{1+s} - \mu) + (c_h + c_s)E[(D-Q)^+] \\
&\quad + R_r(Q) + T(\theta) + H(s) + G(\theta,Q) + k] \\
&= (p - \delta c_2)(\mu + \frac{\gamma s}{1+s}) - [c_h(Q - \frac{\gamma s}{1+s} - \mu) + (c_h + c_s) \int_{(Q-\frac{\gamma s}{1+s})}^{\infty} (x + \frac{\gamma s}{1+s} - Q) f(x)\mathrm{d}x \\
&\quad + c_r Q + c_0(\frac{\overline{\theta} - \theta}{\overline{\theta} - \theta} - 1) + c_1(\overline{\theta} - \theta)^2 Q + \alpha s + \beta s^2 + k].
\end{aligned} \tag{11}$$

Now, our objective is to maximize $\pi(\theta,k,s)$ for $\theta \,\epsilon\, [\underline{\theta}, \overline{\theta}]$, $k > 0$ and $s > 0$. The partial derivatives of $\pi(\theta,k,s)$ are as follows:

$$\frac{\partial \pi}{\partial \theta} = -\xi e^{(\theta - \underline{\theta})} \left[c_h + c_r + c_1(\overline{\theta} - \theta)(\overline{\theta} - \theta - 2) - (c_h + c_s) \int_{(Q-\frac{\gamma s}{1+s})}^{\infty} f(x)\mathrm{d}x \right] - \frac{c_0(\overline{\theta} - \underline{\theta})}{(\overline{\theta} - \theta)^2} \tag{12}$$

$$\frac{\partial \pi}{\partial k} = -e^{(\theta - \underline{\theta})} \dot{\xi} \left[c_h + c_r + c_1(\theta - \underline{\theta})^2 + \frac{e^{-(\theta - \underline{\theta})}}{\dot{\xi}} \right] + (c_h + c_s) e^{(\theta - \underline{\theta})} \dot{\xi} \int_{(Q-\frac{\gamma s}{1+s})}^{\infty} f(x)\mathrm{d}x \tag{13}$$

$$\frac{\partial \pi}{\partial s} = \frac{\gamma}{(1+s)^2} \left[p - \delta c_2 + c_h - \frac{1}{\gamma}(\alpha + 2\beta s)(1+s)^2 - (c_h + c_s) \int_{(Q-\frac{\gamma s}{1+s})}^{\infty} f(x)\mathrm{d}x \right]. \tag{14}$$

The necessary conditions ($\frac{\partial \pi}{\partial \theta} = 0$, $\frac{\partial \pi}{\partial k} = 0$, $\frac{\partial \pi}{\partial s} = 0$) for optimality of $\pi(\theta,k,s)$ provide the following relations:

$$(c_h + c_s) \int_{(Q-\frac{\gamma s}{1+s})}^{\infty} f(x)\mathrm{d}x = c_h + c_r + c_1(\overline{\theta} - \theta)(\overline{\theta} - \theta - 2) + \frac{c_0(\overline{\theta} - \underline{\theta}) e^{-(\theta - \underline{\theta})}}{(\overline{\theta} - \theta)^2 \xi} \tag{15}$$

$$(c_h + c_s) \int_{(Q-\frac{\gamma s}{1+s})}^{\infty} f(x)\mathrm{d}x = c_h + c_r + c_1(\overline{\theta} - \theta)^2 + \frac{e^{-(\theta - \underline{\theta})}}{\dot{\xi}} \tag{16}$$

$$(c_h + c_s) \int_{(Q-\frac{\gamma s}{1+s})}^{\infty} f(x)\mathrm{d}x = (p - \delta c_2) + c_h - \frac{1}{\gamma}(\alpha + 2\beta s)(1+s)^2. \tag{17}$$

Equating equations (15) and (17), we have

$$k = \xi^{-1}\left[\frac{c_0(\overline{\theta}-\underline{\theta})e^{-(\theta-\underline{\theta})}/(\overline{\theta}-\theta)^2}{p-\delta c_2-c_r-c_1(\overline{\theta}-\theta)(\overline{\theta}-\theta-2)-(\alpha+2\beta s)(1+s)^2/\gamma}\right]. \tag{18}$$

For feasible value of k, $p-\delta c_2 > c_r + c_1(\overline{\theta}-\theta)(\overline{\theta}-\theta-2)+(\alpha+2\beta s)(1+s)^2/\gamma$ must hold. Equating equations (15) and (16), we have an equation

$$\psi_1(\theta,s) = \frac{c_0(\overline{\theta}-\underline{\theta})}{(\overline{\theta}-\theta)^2\xi} - \frac{1}{\dot{\xi}} - 2c_1(\overline{\theta}-\theta)e^{(\theta-\underline{\theta})} = 0. \tag{19}$$

Similarly, using equations (16) and(17), we have

$$\psi_2(\theta,s) = p-\delta c_2-c_r-c_1(\overline{\theta}-\theta)^2-(\alpha+2\beta s)(1+s)^2/\gamma - \frac{e^{-(\theta-\underline{\theta})}}{\dot{\xi}} = 0. \tag{20}$$

The simultaneous equations (19) and (20) can be solve by Newton-Raphson method which is defined as follows:

$$\theta_{n+1} = \theta_n - [\psi_1(\theta_n,s_n)\frac{\partial}{\partial s}\{\psi_2(\theta_n,s_n)\} - \psi_2(\theta_n,s_n)\frac{\partial}{\partial s}\{\psi_1(\theta_n,s_n)\}]/J_n$$

$$s_{n+1} = s_n + [\psi_1(\theta_n,s_n)\frac{\partial}{\partial \theta}\{\psi_2(\theta_n,s_n)\} - \psi_2(\theta_n,s_n)\frac{\partial}{\partial \theta}\{\psi_1(\theta_n,s_n)\}]/J_n,$$

where it is being supposed that the Jacobian $J_n = \frac{\partial\psi_1}{\partial\theta}\times\frac{\partial\psi_2}{\partial s} - \frac{\partial\psi_1}{\partial s}\times\frac{\partial\psi_2}{\partial\theta} \neq 0, (n = 0,1,2,\ldots)$. The above iteration will continue until $\mid \theta_{n+1}-\theta_n \mid < \epsilon, \mid s_{n+1}-s_n \mid < \epsilon$ with ϵ as the given accuracy. Thus we obtain the optimal value of K^* from equation (18) using the feasible solution (θ^*, s^*) from equations (19) and (20). Now, using the necessary conditions of optimality of π in 2nd order partial derivatives of π, we have as follows:

$$\frac{\partial^2\pi}{\partial\theta^2} = \frac{c_0(\overline{\theta}-\underline{\theta})}{(\overline{\theta}-\theta)^3}(2+\overline{\theta}-\theta) - (c_h+c_s)(e^{(\theta-\underline{\theta})}\xi)^2 f\left(Q-\frac{\gamma s}{1+s}\right) \tag{21}$$

$$\frac{\partial^2\pi}{\partial\theta\partial k} = (c_h+c_r+c_1(\overline{\theta}-\theta)^2)\dot{\xi}e^{(\theta-\underline{\theta})} + 1 - (c_h+c_s)e^{2(\theta-\underline{\theta})}\xi\dot{\xi}f\left(Q-\frac{\gamma s}{1+s}\right) = \frac{\partial^2\pi}{\partial k\partial\theta} \tag{22}$$

$$\frac{\partial^2\pi}{\partial\theta\partial s} = \left[\frac{(c_h+c_s)\xi\gamma e^{(\theta-\underline{\theta})}}{(1+s)^2}\right] f\left(Q-\frac{\gamma s}{1+s}\right) = \frac{\partial^2\pi}{\partial s\partial\theta} \tag{23}$$

$$\frac{\partial^2\pi}{\partial k^2} = -(c_h+c_s)(\dot{\xi}e^{(\theta-\underline{\theta})})^2 f\left(Q-\frac{\gamma s}{1+s}\right) + \frac{e^{-(\theta-\underline{\theta})}}{\dot{\xi}} \tag{24}$$

$$\frac{\partial^2\pi}{\partial k\partial s} = \left[\frac{(c_h+c_s)\dot{\xi}\gamma e^{(\theta-\underline{\theta})}}{(1+s)^2}\right] f\left(Q-\frac{\gamma s}{1+s}\right) = \frac{\partial^2\pi}{\partial s\partial k} \tag{25}$$

$$\frac{\partial^2\pi}{\partial s^2} = -2\beta - \frac{2(\alpha+2\beta s)}{(1+s)} - \left[\frac{(c_h+c_s)\gamma^2}{(1+s)^2}\right] f\left(Q-\frac{\gamma s}{1+s}\right). \tag{26}$$

Now, the hessian matrix of π at (θ^*, k^*, s^*) is

$$H = \begin{pmatrix} \frac{\partial^2\pi}{\partial\theta^2} & \frac{\partial^2\pi}{\partial\theta\partial k} & \frac{\partial^2\pi}{\partial\theta\partial s} \\ \frac{\partial^2\pi}{\partial k\partial\theta} & \frac{\partial^2\pi}{\partial K^2} & \frac{\partial^2\pi}{\partial k\partial s} \\ \frac{\partial^2\pi}{\partial s\partial\theta} & \frac{\partial^2\pi}{\partial s\partial k} & \frac{\partial^2\pi}{\partial s^2} \end{pmatrix}.$$

Proposition 3.1. *The expected profit function $\pi(\theta,k,s)$ attains maximum value at (θ^*,k^*,s^*) if the hessian matrix H (θ^*,k^*,s^*) is negative definite, i.e., the three eigen values of the H are all negative.*

3.2. Discrete distribution

When the decision variables are discrete in nature, the production lot size is

$$Q = [e^{(\theta_i - \underline{\theta})}\xi(k_j)], \tag{27}$$

where [.] is a box function. The random variable of the demand m follows discrete distribution with probability f_m such that $\sum_{m=0}^{\infty} f_m = 1$ and $\mu = \sum_{m=0}^{\infty} m f_m$. The expected profit function is

$$\pi(\theta_i, k_j, s_l) = (p - \delta c_2)\left(\mu + \left[\frac{\gamma s_l}{1+s_l}\right]\right) - \left\{c_h\left(Q - \left[\frac{\gamma s_l}{1+s_l}\right] - \mu\right) + (c_h + c_s) \sum_{m=(Q-[\frac{\gamma s_l}{1+s_l}])+1}^{\infty} \left(m + \left[\frac{\gamma s_l}{1+s_l}\right] - Q\right) f_m \right.$$
$$\left. + c_r Q + c_0\left(\frac{\overline{\theta} - \underline{\theta}}{\overline{\theta} - \theta_i} - 1\right) + c_1(\overline{\theta} - \theta_i)^2 Q + \alpha s_l + \beta s_l^2 + K_j \right\}. \tag{28}$$

subject to the constraints : $\underline{\theta} \leq \theta_i \leq \overline{\theta}$ for $i = 1, 2, \ldots (\overline{\theta} - \underline{\theta})$; $a \leq k_j \leq b$ for $j = 1, 2, \ldots (b - a)$; $0 \leq s_l \leq s_{\max}$ for $l = 1, 2, \ldots s_{\max}$. Now, our objective is to maximize $\pi(\theta_i, K_j, s_l)$ subject to the above constraints. This problem can be easily solved by the following algorithm.

Algorithm

Step 1: Set $i = 1$, $\theta_{\text{old}} = \underline{\theta}$, $k_{\text{old}} = a$

Step 2: Now maximize $\pi_{\text{old}}(\theta_{\text{old}}, K_{\text{old}}, s_l)$ for $s_l \epsilon [0, s_{\max}]$. So, π_{old} attains maximum at $s_l = s^* \epsilon [0, s_{\max}]$, *i.e.*, $\pi^m_{\text{old}} = \pi_{\text{old}}(\theta_{\text{old}}, k_{\text{old}}, s_{\text{old}})$ where $s_{\text{old}} = s^*$. Then go to Step 3.

Step 3: Set $k_{\text{new}} = k_{\text{old}} + 1$ and maximum value of $\pi_{\text{new}}(\theta_{\text{old}}, k_{\text{new}}, s_l)$ is $\pi^m_{\text{new}}(\theta_{\text{old}}, k_{\text{new}}, s_{\text{new}})$ at $s_{\text{new}} = s^* \epsilon [0, s_{\max}]$ and let $\pi^i_{\text{opt}} = \pi^m_{\text{old}}$, $\theta^i_{\text{opt}} = \theta_{\text{old}}$, $k^i_{\text{opt}} = k_{\text{old}}$, $s^i_{\text{opt}} = s_{\text{old}}$. Go to next step.

Step 4: If $\pi^m_{\text{new}} \geq \pi^i_{\text{opt}}$ then $\pi^i_{\text{opt}} = \pi^m_{\text{new}}$, $\theta^i_{\text{opt}} = \theta_{\text{old}}$, $k^i_{\text{opt}} = k_{\text{new}}$, $s^i_{\text{opt}} = s_{\text{new}}$. Now go to step 5

Step 5: If $k_{\text{new}} < b$ then set $k_{\text{old}} = k_{\text{new}}$ and go to step 3 otherwise go to step 6.

Step 6: Set $i = i + 1$, $\theta_{\text{new}} = \theta_{\text{old}} + 1$. If $i > (\overline{\theta} - \underline{\theta})$ then go to step 7 otherwise go to step 2 and set $k_{\text{old}} = a$.

Step 7: Now, we have the suboptimal solutions $\{(\pi^i_{\text{opt}}, \theta^i_{\text{opt}}, k^i_{\text{opt}}, s^i_{\text{opt}}), i = 1, 2, \ldots (\overline{\theta} - \overline{\theta})\}$ and find out the maximum value among those by the following rule presented in the next step.

Step 8: Set $i = 1$, $\pi_{\max} = \pi^i_{\text{opt}}, \theta^* = \theta^i_{\text{opt}}, k^* = k^i_{\text{opt}}, s^* = s^i_{\text{opt}}$.

Step 9: If $\pi^{i+1}_{\text{opt}} > \pi_{\max}$ then $\pi_{\max} = \pi^{i+1}_{\text{opt}}$, $\theta^* = \theta^{i+1}_{\text{opt}}, k^* = k^{i+1}_{\text{opt}}, s^* = s^{i+1}_{\text{opt}}$

Step 10: If $i > (\overline{\theta} - \underline{\theta})$ then go to step 11 otherwise set $i = i + 1$ and go to step 9.

Step 11: The required maximum expected profit is $(\pi_{\max}, \theta^*, k^*, s^*)$.

Step 12: Stop.

3.3. Distribution free case

When distribution of the random variable x is unknown, then we may approximate the profit function $\pi(\theta, k, S)$ in terms of mean (μ) and standard deviation (σ) which are calculated from previous knowledge of data. Now,$E_{f_i}(x_i - A_i)^+ = \frac{1}{2}\{E_{f_i} \mid x_i - A_i \mid + E_{f_i}(x_i - A_i)\}$ $\forall x_i \sim f_i(\mu_i, \sigma_i^2)$. Using Cauchy–Schwarz inequality, we have $E_{f_i} \mid x_i - A_i \mid \leq \sqrt{E_{f_i}(x_i - A_i)^2} = \sqrt{\sigma_i^2 + A_i^2}$. Therefore, we have

$$E_{f_i}(x_i - A_i)^+ \leq \frac{1}{2}\left\{\sqrt{\sigma_i^2 + A_i^2} - (A_i - \mu_i)\right\}.$$

Thus, we have $E[(x - Q + \frac{\gamma s}{1+s})^+] \leq \frac{1}{2}\left\{\sqrt{\sigma^2 + (Q - \frac{\gamma s}{1+s} - \mu)^2} - (Q - \frac{\gamma s}{1+s} - \mu)\right\}$ and using in equation (11) the approximated profit function is

$$\operatorname{Min} \pi(\theta, k, s) = (p - \delta c_2)\left(\mu + \frac{\gamma s}{1+s}\right) - \left[\frac{1}{2}(c_h - c_s)\left(Q - \frac{\gamma s}{1+s} - \mu\right) + \frac{1}{2}(c_h + c_s)\sqrt{\sigma^2 + (Q - \frac{\gamma s}{1+s} - \mu)^2}\right.$$
$$\left. + c_r Q + c_0\left(\frac{\overline{\theta} - \theta}{\overline{\theta} - \theta} - 1\right) + c_1(\overline{\theta} - \theta)^2 Q + \alpha s + \beta s^2 + k\right] = Y(\theta, k, s). \tag{29}$$

In this case, our aim is to maximize $Y(\theta, k, s)$. Now, 1st order partial derivatives of $Y(\theta, k, s)$ are as follows:

$$\frac{\partial Y}{\partial \theta} = -\xi e^{(\theta-\underline{\theta})}\left[\frac{1}{2}(c_h - c_s) + \frac{1}{2}(c_h + c_s)\frac{(q - \frac{\gamma s}{1+s} - \mu)}{\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2}} + c_r + c_1(\overline{\theta} - \theta)(\overline{\theta} - \theta - 2)\right] - \frac{c_0(\overline{\theta} - \underline{\theta})}{(\overline{\theta} - \theta)^2} \tag{30}$$

$$\frac{\partial Y}{\partial k} = -e^{(\theta-\underline{\theta})}\dot{\xi}\left[\frac{1}{2}(c_h - c_s) + \frac{1}{2}(c_h + c_s)\frac{(q - \frac{\gamma s}{1+s} - \mu)}{\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2}} + c_r + c_1(\theta - \underline{\theta})^2 + \frac{e^{-(\theta-\underline{\theta})}}{\dot{\xi}}\right] \tag{31}$$

$$\frac{\partial Y}{\partial s} = \frac{\gamma}{(1+s)^2}\left[p - \delta c_2 - \frac{1}{\gamma}(\alpha + 2\beta s)(1+s)^2 + \frac{1}{2}(c_h - c_s) + \frac{1}{2}(c_h + c_s)\frac{(q - \frac{\gamma s}{1+s} - \mu)}{\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2}}\right]\cdot \tag{32}$$

The necessary conditions ($\frac{\partial Y}{\partial \theta} = 0$, $\frac{\partial Y}{\partial k} = 0$, $\frac{\partial Y}{\partial s} = 0$) for optimality of $\pi(\theta, k, s)$ provide the following relations:

$$\frac{1}{2}(c_h + c_s)\frac{(q - \frac{\gamma s}{1+s} - \mu)}{\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2}} = -\frac{1}{2}(c_h - c_s) - c_r - c_1(\overline{\theta} - \theta)(\overline{\theta} - \theta - 2) - \frac{c_0(\overline{\theta} - \underline{\theta})e^{-(\theta-\underline{\theta})}}{(\overline{\theta} - \theta)^2\xi} \tag{33}$$

$$\frac{1}{2}(c_h + c_s)\frac{(q - \frac{\gamma s}{1+s} - \mu)}{\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2}} = -\frac{1}{2}(c_h - c_s) - c_r - c_1(\overline{\theta} - \theta)^2 - \frac{e^{-(\theta-\underline{\theta})}}{\dot{\xi}} \tag{34}$$

$$\frac{1}{2}(c_h + c_s)\frac{(q - \frac{\gamma s}{1+s} - \mu)}{\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2}} = -\frac{1}{2}(c_h - c_s) - c_r - (p - \delta c_2) + \frac{1}{\gamma}(\alpha + 2\beta s)(1+s)^2\cdot \tag{35}$$

Equating equations (33) and (35), we have

$$k = \xi^{-1}\left[\frac{c_0(\overline{\theta} - \underline{\theta})e^{-(\theta-\underline{\theta})}/(\overline{\theta} - \theta)^2}{p - \delta c_2 - c_r - c_1(\overline{\theta} - \theta)(\overline{\theta} - \theta - 2) - (\alpha + 2\beta s)(1+s)^2/\gamma}\right]\cdot \tag{36}$$

Equating equations (33) and (34), we have an equation

$$\psi_1(\theta, s) = \frac{c_0(\overline{\theta} - \underline{\theta})}{(\overline{\theta} - \theta)^2\xi} - \frac{1}{\dot{\xi}} - 2c_1(\overline{\theta} - \theta)e^{(\theta-\underline{\theta})} = 0. \tag{37}$$

Similarly, using equations (34) and(35), we have

$$\psi_2(\theta, s) = p - \delta c_2 - c_r - c_1(\overline{\theta} - \theta)^2 - (\alpha + 2\beta s)(1+s)^2/\gamma - \frac{e^{-(\theta-\underline{\theta})}}{\dot{\xi}} = 0. \tag{38}$$

The simultaneous equations (37) and (38) can be solve by Newton-Raphson method which is defined as before. Substituting the solution (θ, s) from equations (37) and (38) in equation (36), we have the value of k. Thus, we have the stationary point (θ^*, k^*, s^*). Now, the 2nd order derivatives using the necessary conditions of optimality of $Y(\theta, k, s)$ are as follows:

$$\frac{\partial^2 Y}{\partial \theta^2} = -\frac{c_0(\overline{\theta} - \underline{\theta})}{(\overline{\theta} - \theta)^3}(2 + \overline{\theta} - \theta) - \frac{(c_h + c_s)}{2(\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2})^3}(e^{(\theta-\underline{\theta})}\xi\sigma)^2 - 2c_1(1 - \overline{\theta} + \theta)\xi e^{\theta-\underline{\theta}} \tag{39}$$

$$\frac{\partial^2 Y}{\partial \theta \partial k} = -\frac{(c_h + c_s)}{2(\sqrt{\sigma^2 + (q - \frac{\gamma s}{1+s} - \mu)^2})^3}(e^{\theta-\underline{\theta}}\ \sigma)^2\xi\dot{\xi} + c_0\left(\frac{\dot{\xi}}{\xi}\right)\frac{(\overline{\theta} - \underline{\theta})}{(\overline{\theta} - \theta)^2} = \frac{\partial^2 Y}{\partial k \partial \theta} \tag{40}$$

$$\frac{\partial^2 Y}{\partial\theta\partial s} = \frac{(c_h+c_s)}{2(\sqrt{\sigma^2+(q-\frac{\gamma s}{1+s}-\mu)^2})^3}\left[\frac{\xi\gamma\sigma^2 e^{(\theta-\underline{\theta})}}{(1+s)^2}\right] = \frac{\partial^2 Y}{\partial s\partial\theta} \tag{41}$$

$$\frac{\partial^2 \pi}{\partial k^2} = -\frac{(c_h+c_s)}{2(\sqrt{\sigma^2+(q-\frac{\gamma s}{1+s}-\mu)^2})^3}(\dot{\xi}\sigma e^{(\theta-\underline{\theta})})^2 - \frac{\ddot{\xi}}{\dot{\xi}} \tag{42}$$

$$\frac{\partial^2 \pi}{\partial k\partial s} = \frac{(c_h+c_s)}{2(\sqrt{\sigma^2+(q-\frac{\gamma s}{1+s}-\mu)^2})^3}\left[\frac{\dot{\xi}\gamma\sigma^2 e^{(\theta-\underline{\theta})}}{(1+s)^2}\right] = \frac{\partial^2 \pi}{\partial s\partial k} \tag{43}$$

$$\frac{\partial^2 \pi}{\partial s^2} = -2\beta - \frac{2(\alpha+2\beta s)}{(1+s)} - \frac{(c_h+c_s)}{2(\sqrt{\sigma^2+(q-\frac{\gamma s}{1+s}-\mu)^2})^3}\left[\frac{\gamma^2\sigma^2}{(1+s)^2}\right] \tag{44}$$

In this case, the expected profit function attains maximum at the stationary point (θ^*, k^*, s^*) if the hessian matrix (H) is negative definite like as before. If the required optimal solution (θ^*, k^*, s^*) violates the restrictions: $\underline{\theta} \le \theta \le \overline{\theta}$, $k \ge 0$ and $s \ge 0$, the maximization problem is to be considered as constraint maximization problem as below.

$$\text{Max } \pi(\theta, k, s)$$

subject to the constraints:

$$\begin{aligned} \underline{\theta} - \theta &\le 0,\\ \theta - \overline{\theta} &\le 0,\\ -k &\le 0,\\ -s &\le 0. \end{aligned}$$

Now, this problem can be solved by SUMT(Sequential Unconstrained Maximization Technique) Algorithm which is stated as follows:

SUMT Algorithm

Step 1: We consider the new unconstrained objective function

$$f(\theta, k, s, t) = \pi(\theta, k, s) + t\left[\frac{1}{(\underline{\theta}-\theta)} + \frac{1}{(\theta-\overline{\theta})} - \frac{1}{k} - \frac{1}{s}\right],$$

where t is a nonnegative parameter.

Step 2: Select an initial nonnegative value t_0 for t and initial point (θ_0, k_0, s_0) as the first trial solution which must be an interior point of feasible region, but not lie on the boundary of the region.

Step 3: Maximize $f(\theta, k, s, t_0)$ by steepest ascent method.

Step 4: Once the optimum solution corresponding to a given value t_0 is obtained, then new value of t is to be selected such that $0 < t_1 < t_0$.

Step 5: This repeated procedure is to be terminated when two successive values of t and the corresponding optimum value of (θ, k, s) obtained by maximizing $f(\theta, k, s, t)$ are approximately the same.

4. Numerical illustration

Example 4.1. For continuous distribution, we consider normal distribution with the values of the parameters in appropriate units which are as follows: $\mu = 650$, $\sigma = 12$, $\xi(k) = k^v$, $v = 0.5$, $\underline{\theta} = 1$, $\overline{\theta} = 5$, $\gamma = 100$, $p = 50$, $c_h = 3$, $c_s = 10$, $c_r = 6$, $c_0 = 5000$, $c_1 = 3$, $c_2 = 2$, $\alpha = 10$, $\beta = 5$, $\delta = 0.7$. Then, the optimal results of decision variables

TABLE 1. Optimal values in known distribution and distribution free scenario.

Optimal	Normal Distribution $N(\mu, \sigma)$	Distribution free (μ, σ)
Technology variable (θ^*)	3.66	3.66
Capital investment (k^*)	2633.85	2633.85
Service level (s^*)	5.70	5.70
Production lotsize	733.70	735.07
Eigen values of H at (θ^*, k^*, s^*)	(-ve,-ve,-ve)	(-ve,-ve,-ve)
Income from sale	36 753.70	36 753.61
Salvage value of used product	1029.10	1029.10
Cost of inventory holding	12.39	18.00
Cost of stock out	55.05	60.00
Cost of raw materials	4402.20	4410.43
Cost of technology	9925.37	9946.20
Penalty for GHG emission	3952.30	3948.66
Cost of service level	219.45	219.38
Expected profit	14 523.99	14 487.99

and dependent cost and profit variables are displayed in Table 1. In both the cases (Normal distribution and distribution free cases) ,the required optimal solutions of decision variables $(\theta^* = 3.66, k^* = 2633.85, s^* = 5.70)$ are identical as necessary conditions for optimality are same. The maximum expected profits are different because of approximation of the function. Therefore, the distribution free case is more realistic for unknown distribution that often arises in the marketing system. The optimal values of costs and profit factors are shown in Table 1.

Example 4.2. For discrete distribution, We consider poisson distribution with the same values of parameters given in Example 4.1 including $\sigma = \sqrt{\mu}$, $a = 5000$, $b = 15\,000$, $s_{\max} = 10$. Then, the optimal solution sets of the decision variables $(\theta^* = 3.00, k^* = 9962.00, s^* = 7.00)$ are same for both poisson and distribution cases, and the expected profits are different due to approximation of the expected profit function. All costs and income from sales items in detail are given in Table 2.

5. Sensitivity analysis

We find out the optimal solutions for changes $(-50\%, -25\%, +25\%, +50\%)$ of one key parameter, keeping other parameters as fixed. From Table 3, we observe the following features which are compatible with real scenarios.

(1) When lower bound of the cost of green technology increases, the optimal value of θ^*(type of technology) decreases to mitigate the hike of cost of technology. The optimal production quantity (Q^*) decreases slightly but controlled by increasing value of set up cost (k^*) to adjust the expected demand of the customers. In this case, the volume of service provider (s^*) decreases as stock Q^* decreases. As a result, the expected profit (π^*) decreases with increasing values of c_0.
(2) When the scale parameter of cost per unit item for GHG emission(c_1) increases, Q^*, s^* and π^* decrease but θ^* increases to step down the total penalty for GHG emission.
(3) The optimal production quantity (Q^*) decreases by decreasing value of k^* while the salvage value (c_2) per unit used product increases. The optimal values of θ^* are unchanged with changes in c_2, α, β, σ, p, c_h, c_s and c_r because θ^* is dominated by apparently large value of c_0.
(4) The optimal values of Q^*, s^*, k^* and Π^* decrease with the increasing values of scale parameters (α, β), *i.e.*, higher values of cost of service provider.

Table 2. Optimal values in known discrete distribution and distribution free scenario.

Optimal	Poisson Distribution Pois$(\mu, \sqrt{\mu})$	Distribution free $(\mu, \sqrt{\mu})$
Technology variable (θ^*)	3.00	3.00
Capital investment (k^*)	9962.00	9962.00
Service level (s^*)	7.00	7.00
Production lotsize	738.00	738.00
Income from sale	36 875.00	36 875.00
Salvage value of used product	1032.50	1032.50
Cost of inventory holding	30.51	38.24
Cost of stock out	101.70	127.48
Cost of raw materials	4425.00	4425.00
Cost of technology	5000.00	5000.00
Penalty for GHG emission	8850.00	8850.00
Cost of service level	315.00	315.00
Expected profit	7158.00	7128.00

Table 3. Sensitivity analysis of key parameters for distribution free scenario.

Parameter	k^*	θ^*	Q^*	s^*	π^*
−50%	1655.47	3.89	735.73	6.01	20 123.47
−25%	2159.10	3.76	735.40	5.85	17 115.07
$c_0 = 5000$	2633.85	3.66	735.07	5.70	14 487.99
+25%	3088.64	3.58	734.74	5.55	12 112.43
+50%	3528.08	3.51	734.41	5.41	9919.75
−50%	3232.03	3.60	735.18	5.74	16 612.45
−25%	2904.41	3.61	735.12	5.72	15 511.08
$c_1 = 3$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2409.42	3.71	735.07	5.68	13 533.80
+50%	2221.99	3.75	734.98	5.66	12 640.22
−50%	2634.24	3.66	735.19	5.75	15 002.58
−25%	2634.05	3.66	735.13	5.72	14 745.27
$c_2 = 2$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2633.65	3.66	735.01	5.67	14 230.72
+50%	2633.45	3.66	734.95	5.65	13 973.48
−50%	2635.13	3.66	735.44	5.87	14 516.91
−25%	2634.49	3.66	735.26	5.78	14 502.34
$\alpha = 10$	2633.85	3.66	735.07	5.70	14 487.99
+25%	26331.21	3.66	734.89	5.62	14 473.84
+50%	2632.57	3.66	734.70	5.54	14 459.90
−50%	2642.86	3.66	737.69	7.12	14 587.99
−25%	2637.85	3.66	736.23	6.26	14 532.50
$\beta = 5$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2630.47	3.66	734.09	5.28	14 450.39
+50%	2627.52	3.66	733.23	4.96	14 417.63
0.2	5898.79	4.22	650.00	0.0	508.45
0.4	7124.36	4.05	731.39	4.37	5846.40
$v = 0.5$	2633.85	3.66	735.07	5.70	14 487.99

TABLE 3. continue

Parameter	k^*	θ^*	Q^*	s^*	Π^*
0.6	991.59	3.46	735.56	5.93	16 808.38
0.7	410.64	3.39	735.67	5.98	17 470.23
−50%	1567.94	3.34	409.46	5.43	5174.27
−25%	2087.27	3.53	572.36	5.60	9696.80
$\mu = 650$	2633.85	3.66	735.07	5.70	14 487.99
+25%	3207.24	3.76	897.70	5.76	19 442.84
+50%	3806.69	3.84	1060.29	5.80	24 505.54
−50%	2633.85	3.66	735.07	5.70	14 526.99
−25%	2633.85	3.66	735.07	5.70	14 507.49
$\sigma = 12$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2633.85	3.66	735.07	5.70	14 468.49
+50%	2633.85	3.66	735.07	5.70	14 448.99
−50%	2481.62	3.63	690.57	4.30	13 240.22
−25%	2556.93	3.64	712.66	5.08	13 855.46
$\gamma = 100$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2712.04	3.68	757.70	6.23	15 133.08
+50%	2791.31	3.69	780.50	6.69	15 788.16
−50%	1926.84	3.48	522.97	2.20	195.79
−25%	2623.75	3.66	732.14	4.60	5315.74
$p = 50$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2639.48	3.66	736.71	6.52	23 687.40
+50%	2643.25	3.66	737.80	7.20	32 903.46
−50%	2633.85	3.66	735.07	5.70	14 496.99
−25%	2633.85	3.66	735.07	5.70	14 492.49
$c_h = 3$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2633.85	3.66	735.07	5.70	14 483.49
+50%	2633.85	3.66	735.07	5.70	14 478.99
−50%	2633.85	3.66	735.07	5.70	14 517.99
−25%	2633.85	3.66	735.07	5.70	14 502.99
$c_s = 12$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2633.85	3.66	735.07	5.70	14 472.99
+50%	2633.85	3.66	735.07	5.70	14 457.99
−50%	2635.46	3.66	735.54	5.91	16 693.92
−25%	2634.68	3.66	735.31	5.81	15 590.78
$c_r = 6$	2633.85	3.66	735.07	5.70	14 487.99
+25%	2632.97	3.66	734.81	5.58	13 385.57
+50%	2632.02	3.66	734.54	5.46	12 283.55

(5) The increasing value of index (v) increases the values of Q^* and s^*. Consequently, k^* and θ^* decrease to make more profit (π^*). In this case, π^* increases with increasing values of v.
(6) The expected profit (Π^*) decreases with increasing values of holding cost per unit (c_h), shoratge cost per unit (c_s) and material cost per unit (c_r). The optimal values of k^*, θ^*, Q^* and s^* are insensitive with changes in σ, c_h, c_s and c_r because these cost parameters are dominated by larger values of technological cost, GHG emission cost and selling price (p).
(7) the optimal values of k^*, θ^*, Q^*, s^* and π^* increase with increasing values of μ and γ. This is quite rational because μ and γ increase the expected demand of the customers that results in more production, higher set up cost, higher technology and volume of service provider.

(8) When selling price per unit item (p) increases, the production quantity increases to earn more from sales items. In this situation, k^*, θ^* and s^* increase to adjust the market demand by producing more lotsize Q^*. As a whole, the expected profit (π^*) increases remarkably by increasing values of selling price (p) per unit product.

6. Conclusion

In manufacturing management paradigm, implications of three R (Resource, reservation, reused for recycling) are more relevant to save our natural non-renewed resources as well as protect our environment from a large scale of pollution. Recently, green technology for green products is not only introduced in management science but also applied in many manufacturing industries like textile, paper mills soft drinks , electronic equipments, etc. In practice, more use of fossil fuels as traditional energy resources to run the manufacturing and transportation systems emission more GHG (Co_2, Co, So_2, CH_4). These green house gases are main factors of the global warming that begets frequent natural hazards. Moreover, pollution due to more use of fossil fuels in industries causes physical and mental hazards. As a result, green technology for renewable resources of energy such as solar, wind, stream of water bodies and bio-fuels is being used to run the production as well as transportation systems. The products made by green technology are market as green products. Then, marketing management attracts the customers to buy more providing better services, free gifts and awareness programmes related to the green products and its affects on the environment. In this proposed article, we develop a new mathematical model to sustain eco-friendly environment addressing the above vital issues of modern civilization. In this model, the decision variables are technology (θ), capital (k) invested for setup and others cost factors (labour, light fan, water, etc.) and service level (s) which are optimized to achieve maximum expected profit (π) by trading off profit and cost parameters of the manufacturing system. It is quite rational that the awareness programmes (advertising, seminars and workshops) by GO (Government Organizations), NGO (Non-Government Organizations) and Academic Institutions motivate the customers to purchase green products more. Consequently, service level (s) increases directly the demand of the products. Although the retail price of the green products is high compared to the others due to more investment in new green technologies, the conscious customers are eager to purchase the products to protect their environment where they live. In a nutshell, we may say that the marketing management has a great responsibility to motivate the customers to buy more the green products certified by the GO as this authentication is accepted by the citizen of a country.

The proposed model can be extended immediately considering variable cost and profit parameters unlike the deterministic values in a competitive marketing system. This model may be extended further assuming the demand function as fuzzy stochastic in nature. Moreover, this model might be used to develop a multi-channel supply considering different bargaining issues.

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