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OPTIMAL ORDERING POLICY IN A TWO-ECHELON SUPPLY CHAIN MODEL WITH VARIABLE BACKORDER AND DEMAND UNCERTAINTY

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Abstract. The paper investigates a two-echelon production-delivery supply chain model for products with stochastic demand and backorder-lost sales mixture under trade-credit financing. The manufacturer delivers the retailer's order quantity in a number of equal-sized shipments. The replenishment lead-time is such that it can be crashed to a minimum duration at an additional cost that can be treated as an investment. Shortages in the retailer's inventory are allowed to occur and are partially backlogged with a backlogging rate dependent on customer's waiting time. Moreover, the manufacturer offers the retailer a credit period which is less than the reorder interval. The model is formulated to find the optimal solutions for order quantity, safety factor, lead time, and the number of shipments from the manufacturer to the retailer in light of both distribution-free and known distribution functions. Two solution algorithms are provided to obtain the optimal decisions for the integrated system. The effects of controllable lead time, backorder rate and trade-credit financing on optimal decisions are illustrated through numerical examples.

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1. Introduction

Supply chain (SC) management is concerned with the coordination of material, information, and money along with a network of companies whose purpose is to achieve better performance. Supply chain can be classified into two categories-integrated (or centralized) and non-integrated (decentralized) supply chains. In a non-integrated supply chain each member decides based on its own policy, which can lead to inefficient decisions (Katok and Wu [21]). According to Giannoccaro and Pontrandolfo [10], co-ordination strategy incentivises each supply chain member in such a way that the decisions taken jointly by the members are optimal from a centralized supply chain perspective to increase the chain profit (Weng [46]). Coordination strategies involve mechanization of a company's replenishment processes as well as the connection of buyer and supplier communities with real-time forecast, inventory on-hand, optimal lot sizing, quality improvements, inspections, and shipment information to reduce inventory and eliminate unnecessary expenses. The so-called integrated supply chain models simulate today's business practices (*e.g.*, automotive, apparel, grocery) where there exists a long relationship between buyers and suppliers.

Keywords. Integrated model, lead time reduction, controllable backorder, trade-credit financing, distribution-free approach.

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In many cases it is very important to control the lead time to manage the supply chain efficiently. Typically, lead time consists of order preparation, order transit, delivery time, and set-up time. Consideration of zero or constant lead time may result in deviation of the optimal solution from the actual one. Therefore, to make the supply chain more efficient, it is important to consider variable lead time. Generally, lead time is comprised of several components, and these components can be reduced by additional crashing costs. Reducing lead time may appear to be convenient in competitive situations because it can lower the safety stock level, reduce the stock-out losses, and improve the service level to the customer so as to increase the competitive edge in business.

Stochastic inventory models should not neglect backorder-lost sale mixture. In fact, it is reasonable to assume that only a fraction of the demand is backordered during the stock-out period, while the remaining fraction is lost. For example, customers whose needs are not serious can wait (these demands are backordered); while others cannot wait and meet their demands from other sources (these demands are lost). This phenomenon indicates that, longer the lead time is, higher the shortages are, the smaller the proportion of customers can wait, and hence smaller the backorder rate would be. Hence, the assumption of lead time (or waiting time) dependent backorder rate may reflect real life situations. Numerous inventory models have been developed to include this feature. However, in integrated SC model, this feature has been rarely considered.

In a business organization, the retailer usually pay the price of purchased quantity immediately. But if the retailer is given some delay time for payment, he/she may tend to buy more items. The retailer can earn interest from the revenue generated during the delay period. The use of the credit period can be seen in many deterministic inventory models. However, the use of credit period in stochastic inventory model is rarely found in the literature.

In some practical situations, information about the demand distribution may be rather limited. That is, decision makers only know an estimate of the mean and variance, but not the specific distribution types. In this scenario, the demand is generally considered to be a normally distributed random variable over a given period of time. However, the normal distribution does not provide the best protection against the occurrence of other distributions with the same mean and variance. Therefore, it is a challenge to the inventory/supply chain managers to make decisions without having idea of lead time demand distribution. That is, the replenishment policy can be optimized considering the worst non-negative distribution with the given mean and variance. This is called "minimax distribution-free approach". Due to its practicality (easy to use) and optimality (under certain conditions), it has received great attention in the inventory management literature. The reader is referred to some of the recent works dealing with distribution free approach, *e.g.* [5, 22, 23].

Many industries have dedicated efforts to improve customer service, control order frequency and reduce costs with their business partners. In this regard, the following questions may be raised from manager's view points:

I. What would be the optimal ordering policy for an industry if demand during replenishment lead time is stochastic?
II. What impact does lead time have on the backorder rate?
III. In which condition an industry manager should invest money to reduce lead time?
IV. What steps a manager will adopt when lead time demand distribution is unknown?

In an attempt to find out the answers to the above questions, this paper presents a continuous review integrated single-manufacturer single-retailer supply chain model with stochastic demand under controllable lead time. The manufacturer delivers the retailer's order quantity in a number of equal-sized shipments. Shortages are allowed in the retailer's inventory and a fraction of shortages are partially backlogged with lead time dependent backlogging rate. Initially the model is developed for the case when lead time demand distribution is known (normal distribution) and thereafter, the model is developed for unknown distribution case. Distribution-free approach is employed to find the optimal solution when lead time demand distribution is unknown. In this study, we assume that the long-term strategic partnership between the retailer and the manufacturer is well established and, therefore, the retailer and the manufacturer cooperate and share information with each other. Our goal is to find the optimal order quantity, safety factor, backorder rate, and the number of shipments by minimizing the annual total cost of the integrated system. Rest of the paper is organized as follows: Section 2 reviews the relevant literature. Notation and assumptions are given in Section 3. In Section 4, the proposed model is formulated mathematically. Section 5 describes the solution procedure of the model. Numerical examples are given in Section 6. Sensitivity analysis is presented in Section 7 and finally the paper is concluded in Section 8.

2. Literature review

In the retailing industry, WalMart, and Proctor and Gamble received substantial collaboration benefits by implementing collaborative planning and replenishment, a business model that intends to help supply chain members to collaborate in both tactical and strategic levels. Therefore, during the last few years, research on integrated manufacturer-retailer blue supply chain problem has been primarily focused on shipment schedule in terms of the size and frequency of shipments transferred between both parties. The cooperation between the upstream and downstream players gives a far greater benefit than a non-collaborative relationship. Goyal [13] was the first researcher who introduced a single-supplier single-buyer integrated inventory model. Banerjee [3] generalized Goyal's model and presented a joint economic lot size model where the vendor produces order on lot-for-lot basis to fulfill the buyer's order quantity under deterministic condition. Further, Goyal [15] relaxed the lot-for-lot policy of the vendor to generalize Banerjee's model. Later, Ha and Kim [16] generalized Goyal's [15] model and developed an integrated lot-splitting model facilitating multiple shipments in small lots.

In the literature, most of the supply chain models have been developed assuming lead time as a fixed or deterministic parameter. Although this assumption follows JIT (just-in-time) philosophy, it is not fitted in most of the modern complex setups where overseas, containerized, and air-freight shipping are involved. Liao and Shyu [29] were the first researchers to study lead time reduction in stochastic environment. In their model, they assumed that the lead time can be decomposed into several components having different crashing costs for reducing to a specified minimum duration. Thereafter, a number of researchers have contributed significantly in controllable lead time literature under various assumptions [4, 35, 37, 47].

Pan and Yang [40] were the first authors who studied lead time reduction in a setting with more than one economic actor. They considered a system where the product is delivered to a single buyer from a single vendor and assumed that the vendor may reduce lead time according to the scheme proposed by Liao and Shyu [29]. Ouyang *et al.* [38] extended Pan and Yang's [40] model by considering shortages and taking reorder point as a decision variable. Further, they solved the model using minimax distribution-free approach where only the first and second moments of the probability of lead time demand were known and finite. They obtained total cost lower than Pan and Yang's [40] model. Further extensions can be found in Giri and Roy [11], Jha and Shanker [20], Mandal and Giri [31] who studied the effects of lead time reduction in single-vendor multi-buyer supply chain system; Yang and Pan [49], Wu *et al.* [48], Ouyang *et al.* [39], Sarkar and Giri [41], who included quality considerations in the model formulation; Jha and Shanker [19], Y. Li *et al.* [27], and G. Li *et al.* [28] who considered service level constraint with lead time reduction.

Heydari *et al.* [17] used lead time reduction as an incentive mechanism in order to convince the buyer to participate in the coordination plan. The showed that a smaller lead time is beneficial to the buyer because of a lower inventory cost. Yang *et al.* [50] developed a newsvendor model to investigate inventory competition in a dual-channel supply chain and explored the delivery lead time decision in the direct channel. Zikopoulos [51] studied a remanufacturing system and examined the advisability taking into account the stochastic remanufacturing lead-time under different scenarios of returns' quality and demand for remanufactured products. Recently, Sarkar and Giri [42] considered the case where the replenishment lead time is a function of lot size and transportation time, and determined the optimal lot size which minimizes the total cost of the supply chain.

Due to variable lead time, sometimes the vendor may fail to deliver a lot within the desired lead time. As a result, the buyer may faces stock-out situation, in which case, customers' demand is not fulfilled resulting a financial loss. Moreover, the unsatisfied customers may not turn up next time to meet their demand from the same source. This indicates that, in reality the backorder rate should not be constant. Ouyang *et al.* [37] generalized Ben-Daya and Raouf's [4] model by considering mixture of backorder and lost sales. Ouyang and Chuang [36] considered backorder rate as a control variable to generalize Ouyang *et al.*'s [37] model.

Table 1. A comparison of the present model with some related works in the literature.

Author(s)	Integrated model	Controllable lead time	Variable backorder rate	Trade credit financing	Variable safety factor	Distribution free approach
Kumar and Goswami [22]						
Liao and Shyu [29]		√				
Ouyang *et al.* [37]		√				
Pan and Yang [40]	√	√				
Ouyang *et al.* [38]	√	√			√	√
Yang and Pan [49]	√	√				
Ouyang *et al.* [39]	√	√				
Ouyang and Chuang [36]		√	√			√
Sarkar *et al.* [44]		√			√	√
Sarkar and Giri [42]	√	√			√	
Ouyang and Chuang [36]		√	√			√
Lee [25]		√	√			√
Lee *et al.* [26]		√	√			√
Chung and Huang [8]				√		
Chen and Kang [7]	√			√		
Huang [18]	√	√		√		
Wu *et al.* [48]	√	√			√	√
This paper	√	√	√	√	√	√

Lee [25] and Lee *et al.* [26] analyzed two inventory models with mixture of normally distributed lead time demand and controllable negative exponential backorder rate. Sarkar *et al.* [44] studied an inventory model with quality improvement and backorder price discount under controllable lead time. Mishra and Tripathy [33] developed an inventory model with time dependent Weibull deterioration and salvage value. Further, Mishra [32] proposed an inventory model to develop an optimal pricing policy for deteriorating items with stock and price dependent demand under partially backlogged shortages. Braglia *et al.* [5] studied a periodic-review joint-replenishment problem (JRP) with stochastic demands, backorder-lost sale mixture, and controllable major ordering cost and lead times. Braglia *et al.* [6] proposed a continuous review (Q, r) inventory model for deteriorating item with random demand and partial backlogging.

In today's competitive business world, organizations are using various types of promotional tool in order to increase their sales volumes. One such tool is trade-credit or permissible delay-in-payments where the retailer need not to pay for the goods purchased until a prescribed period offered by the vendor. During the delay period, the retailer can earn interest from bank/share market by using the revenue on sales. To address the issue of trade-credit, researchers have made a great deal of efforts [1,14,24,30]. Chung and Huang [8] proposed a two-warehouse deteriorating inventory model with limited storage capacity under permissible delay in payments. Chen and Kang [7] developed an integrated vendor-buyer model with variant permissible delay in payments and imperfect quality item. Huang [18] developed an integrated inventory model to determine the optimal policy under conditions of order processing cost reduction and permissible delay in payments. Mishra *et al.* [34] developed an optimal ordering and pricing policy for perishable items under conditions of permissible delays. Tiwari *et al.* [45] investigated an inventory model for deteriorating items with unreliable supply and trade credit policy. Wu *et al.* [48] dealt with a probabilistic continuous review (Q, r) inventory policy under permissible delay in payments where the supplier offers a credit period that is less than average duration of the inventory cycle. Arkan and Hejazi [2] designed a supply chain model for the coordination between a single buyer and a single supplier considering credit period and controllable lead time.

A comparison of the present model with closely related models available in the existing literature is given in Table 1. It is seen from the literature review that the integrated supply chain models dealing with controllable lead time and stochastic demand are formulated based on the assumption of full backorder or fixed partial backorder. However, when an item is on backorder, a customer may look elsewhere for a substitute product, especially if the expected wait time until the product becomes available is long. That is, a longer lead time increases the shortage amount as well as decreases the backorder rate. The present work is close to the work of Ouyang *et al.* [38]. However, Ouyang *et al.*'s [38] model has a few distinct differences with the study at hand. First of all, their model assumes that shortages are fully backlogged, while in this study, we investigate the case where shortages are partially backlogged with backorder rate as a function of lead time through shortages quantity. In addition, we found that, trade credit inventory problem has been studied for many times in the literatures under deterministic as well as periodic review inventory system with less attention being given to probabilistic and continuous review inventory system. This paper therefore contributes to the literature by proposing a continuous review integrated supply chain model with stochastic demand under variable backorder rate and trade credit financing.

3. Notation and assumptions

We use the following notation to develop the proposed model.

	Description
Decision variables	
Q	Retailer's ordered quantity (units)
L	Retailer's lead time (week)
k	Safety factor
β	Fraction of demand which is backordered during stock-out period, $\beta \in [0, 1]$
m	Number of deliveries from the manufacturer to the retailer
Parameters	
D	Annual demand at the retailer (units/year)
S	Manufacturer's setup cost per setup ($/setup)
A	Retailer's ordering cost per order ($/order)
r_b	Retailer's holding cost rate per unit per unit time
r_v	Manufacturer's holding cost rate per unit time
π_0	Retailer's marginal profit ($/unit)
π	Unit shortage cost at the retailer ($/unit)
$C(L)$	Lead time crashing cost function
r	Reorder point at the retailer
t_c	Retailer's trade-credit period (year)
c_b	Purchasing price ($/item)
c_s	Selling price ($/item)
c_v	Unit production cost at the manufacturer ($/item)
I_c	Fixed interest rate at which the retailer has to pay to the bank for the remaining amount of stock during the period t_c to Q/D ($/year)
I_d	Fixed interest rate for the revenue earned by the retailer ($/$/year)
I_v	Fixed interest rate for calculating the manufacturer's interest (opportunity) loss due to trade-credit offer ($/year)
σ	Standard deviation of the lead time demand
u_i	ith component of lead time with u_i as minimum duration (days), $i = 1, 2, ..., n$

	Description
v_i	ith component of lead time with v_i as normal duration (days), $i = 1, 2, ..., n$
m_i	ith component of lead time with m_i as crashing cost per day, $i = 1, 2, 3, ...n$
X	Lead time demand having distribution function F, finite mean DL and standard deviation $\sigma\sqrt{L}$
$E(X)$	Mathematical expectation of X
x^+	$\max\{x, 0\}$
Ω	Class of p.d.f.s with finite mean DL and standard deviation $\sigma\sqrt{L}$
$E(X-r)^+$	Expected shortage quantity at the end of the cycle
ETC_b	Expected average cost for the retailer
ETC_v	Average cost for the manufacturer
ETC^N	Expected average cost of the integrated system in normal distribution case
ETC^W	Expected average cost of the integrated system in distribution-free case

We make the following assumptions to develop the model:

1. A supply chain consisting a single-manufacturer and a single-retailer deals with a single type of item.
2. The retailer places an order of size mQ which the manufacturer produces with a finite production rate $P(> D)$ in a single setup but ships the entire quantity to the retailer over m deliveries of equal size.
3. The retailer's inventory is continuously monitored. Replenishment is planed whenever the inventory level drops to the reorder point r. The reorder point r is defined by r = the expected demand during lead time (DL) + safety stock (k) $\times$ standard deviation of lead time demand $(\sigma\sqrt{L})$, *i.e.*, $r = DL + k\sigma\sqrt{L}$, where k is the safety factor and satisfies $Pr(X > r) = q$ where q represents the allowable stock-out probability during lead time (see Tiwari *et al.* [45], Sarkar *et al.* [43]).
4. The lead time L consists of n mutually independent components. The ith component has a minimum duration u_i days, normal duration v_i days, and a crashing cost m_i per day. Further, we rearrange m_i as $m_1 \leq m_2 \leq m_3, \dots, \leq m_n$. Then, it is clear that the reduction of lead time should first occur in component 1 (because it has the minimum unit crashing cost), and then component 2, and so on.
5. Let $L_0 = \sum_{j=1}^{n} v_j$ denote the maximum duration of lead time and L_i as the length of lead time with components $1, 2, \dots, i$ crashed to their minimum durations. Then L_i can be expressed as (see Liao and Shyu [29])
$$L_i = \sum_{j=1}^{n} v_j - \sum_{j=1}^{i} (v_j - u_j)$$
where $i = 1, 2, 3, \dots, n$, and the lead time crashing cost function $C(L)$ as
$$C(L) = m_i(L_{i-1} - L) + \sum_{j=1}^{i-1} m_j(v_j - u_j).$$
6. If a shortened lead time is requested then the extra costs incurred by the manufacturer will be fully transferred to the retailer. Therefore, lead time crash cost is the retailer's cost component.
7. The manufacturer offers a certain trade credit period t_c to attract the retailer to cooperate in the integrated strategy. Therefore, the retailer need not to pay immediately after receiving the deliveries. The offered credit period t_c is less than the reorder interval, which means that the credit period cannot be longer than the time at which another order is placed. This is in agreement with the usual practice.
8. The retailer deposits the sale income in a bank with annual interest rate I_d before the payment is due. At the payment time, the distributor pays off the purchased products' cost for all products to the manufacturer.

The retailer has a loan from a bank for the unsold products' cost. During the period of delayed payment, the manufacturer has an interest (opportunity) loss for all products with annual rate I_v where $I_v = I_d$.
9. The backorder rate is variable and it is a function of lead time.

4. Mathematical model

As mentioned in assumption (3), whenever the inventory level drops to the reorder point r, the retailer requests the manufacturer for a delivery. The manufacturer produces mQ units (where m is an integer) at one setup. Therefore, the average cycle time for the manufacturer is $\frac{mQ}{D}$ and the average length of a replenishment cycle is $\frac{Q}{D}$.

According to our assumption, the lead time demand X has a probability density function $f(x)$ with mean DL and standard deviation $\sigma\sqrt{L}$ and the reorder point $r = D + k\sigma\sqrt{L}$. Shortages occur when $X > r$. The retailer's expected shortage quantity at the end of a replenishment cycle is $E(X-r)^+$ and hence, the expected backorder quantity is $\beta E(X-r)^+$. Therefore, the expected loss in sales per shipment cycle is $(1-\beta)E(X-r)^+$ and the expected stock-out cost per replenishment cycle is $[\pi + \pi_0(1-\beta)]\, E(X-r)^+$.

Further, at the beginning of each replenishment cycle, the retailer's expected net inventory is the safety stock $(r - DL)$ plus the previous replenishment cycle's lost sales $(1-\beta)E(X-r)^+$. The expected net inventory level immediately after a replenishment is $Q + r - DL + (1-\beta)E(X-r)^+$. Therefore, the average inventory over a replenishment cycle is $\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+$. Hence, the retailer's holding cost per unit time is $h_b[\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+]$. Further, the safety stock plus the previous replenishment cycle's lost sales is $r - DL + (1-\beta)E(X-r)^+$ which is carried throughout the replenishment cycle. Therefore, the total interest charged at a rate I_c by the manufacturer to the retailer for this amount of stock is $c_b I_c[r - DL + (1-\beta)E(X-r)^+]$.

We assume that the permissible delay period is t_c which is less than the reorder interval. This assumption is realistic, as the payment for the earlier order should be cleared before another order is placed. Here, the retailer earns interest on the sales revenue at the rate I_d during the time period $(0, t_c)$. Therefore, the retailer's interest earn per unit time is $\frac{c_s I_d D}{Q}\int_0^{t_c} Dt\mathrm{d}t = \frac{D^2 t_c^2 c_s I_d}{2Q}$. Additionally, the previous replenishment cycle's backlogged items are cleared at the beginning of the cycle. Therefore, the interest earned per unit time from the backlogged items is $\frac{c_s I_d t_c D}{Q}\beta E(X-r)^+$. The retailer still has some inventory $(Q - Dt_c)$ after the credit period t_c. If he takes a short term loan from the bank at an interest rate I_c for the duration $(t_c, \frac{Q}{D})$ to finance the unsold stock then his opportunity cost (due to payment of interest) per unit time is $\frac{c_b I_c D}{Q}\int_{t_c}^{Q/D}(Q - Dt)\mathrm{d}t = \frac{(Q-Dt_c)^2 c_b I_c}{2Q}$.

Though in most of the existing literature the backorder rate is considered as constant, in this paper, we take the backorder rate β as a variable and define it as

$$\beta = \frac{1}{1 + \alpha E(X-r)^+}, \tag{4.1}$$

α $(0 < \alpha < \infty)$ being a constant. From (4.1), we see that the backorder rate is a decreasing function of shortage quantity. Further, as $\alpha \to \infty$, we have $\beta \to 0$ (complete lostsale case) and as $\alpha \to 0$, we have $\beta \to 1$ (complete backordered case).

The retailer's expected total cost per unit time is

$$\begin{aligned}\mathrm{ETC}_b(Q,r,L) &= \text{Ordering cost} + \text{Holding cost} + \text{Safety stock plus previous cycle's lostsale}\\ &\quad\ \text{cost} + \text{Stockout cost} + \text{Opportunity (interest) cost} - \text{Interest earned}\\ &= \frac{AD}{Q} + \frac{r_b c_b Q}{2} + c_b(r_b + I_c)\left[r - DL + (1-\beta)E(X-r)^+\right]\\ &\quad + \frac{D}{Q}\left[\pi + \pi_0(1-\beta)\right]E(X-r)^+ + \frac{(Q-Dt_c)^2 c_b I_c}{2Q}\\ &\quad - \frac{D^2 t_c^2 c_s I_d}{2Q} - \frac{c_s t_c I_d D\beta}{Q}E(X-r)^+. \end{aligned} \tag{4.2}$$

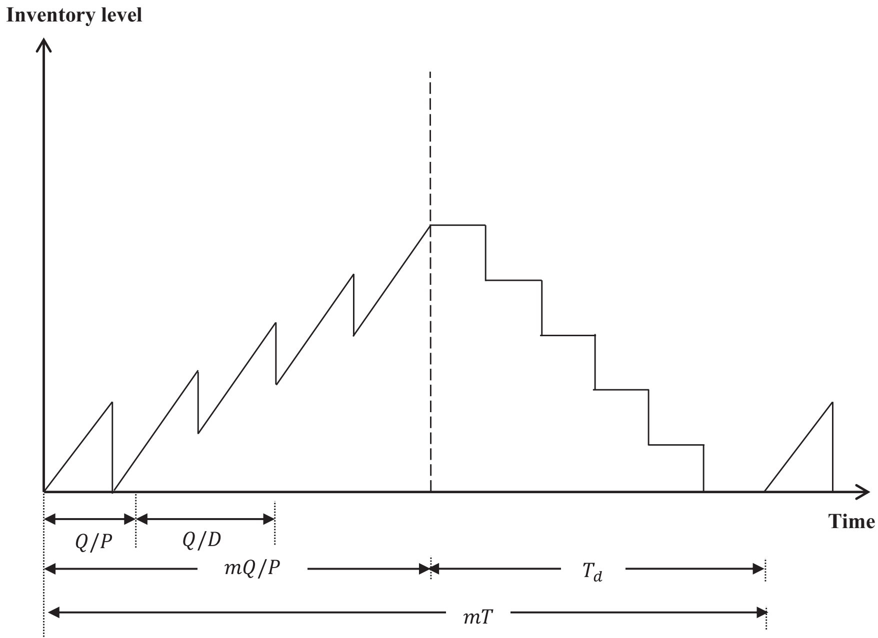

FIGURE 1. Manufacturer's inventory level in a cycle with time.

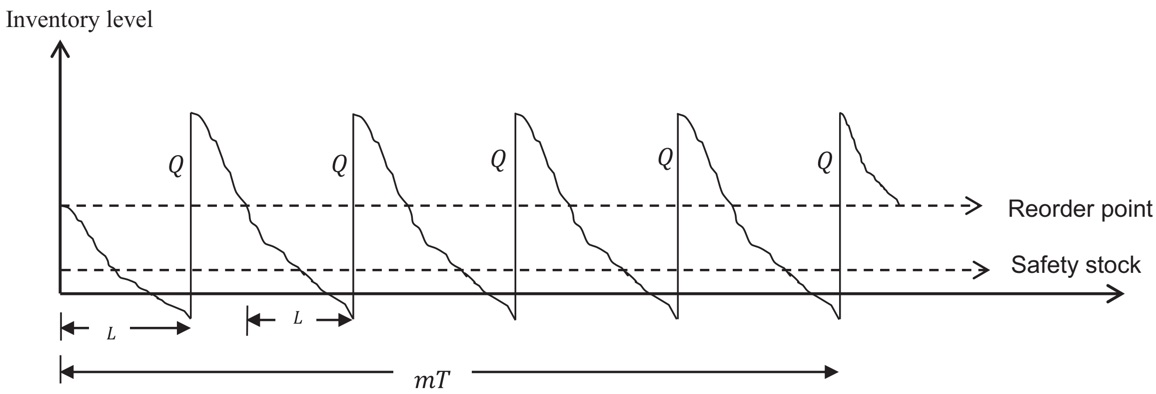

FIGURE 2. Retailer's inventory in manufacturer's one cycle.

On the other hand, the manufacturer's total cost is

$$TC_v(m) = \text{Setup cost} + \text{Holding cost} + \text{Opportunity (interest) cost}.$$

Figure 1 displays the inventory pattern of the manufacturer. The manufacturer delivers the first lot as soon as it has Q items and then starts building up the inventory as its production rate is higher than that of the demand. This build-up of inventory in the production time (T_p) is supplied in equal shipments during the non-production time (T_d). Figure 2 depicts the behavior of the retailer's inventory. Figure 3 shows the same behavior of inventory in another form. The triangle "ABG" and the rectangle "BCGE" are the total inventory

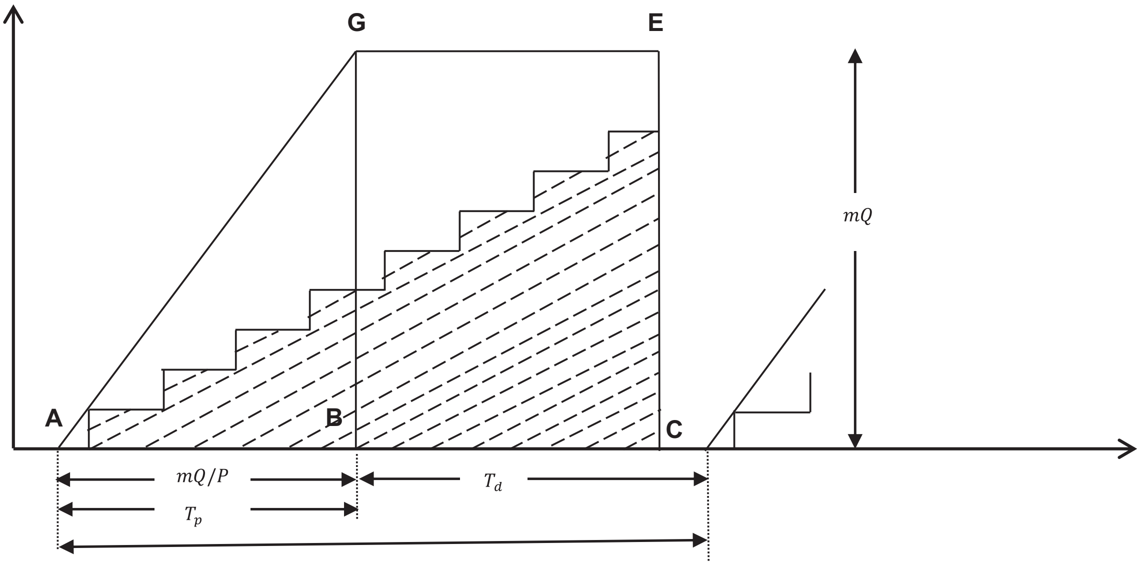

FIGURE 3. Time-weighted inventory for manufacturer and retailer.

at the manufacturer's end while the shaded rectangles are the total inventory supplied to the retailer. Hence, the total inventory or stock level at the manufacturer can be determined by using Figure 3.

Following the method in Goyal [13], the total inventory at the manufacturer in a cycle is the sum of areas of the triangle AGB and the rectangle BGCE in Figure 3, *i.e.*,

$$\text{Area}_{\text{ABG}} = \frac{1}{2}(mQ/P)mQ = \frac{m^2Q^2}{2P},$$
$$\text{Area}_{\text{BCGE}} = mQ\left[(m-1)\left(\frac{Q}{D} - \frac{Q}{P}\right)\right] = \frac{mQ^2(m-1)(P-D)}{PD}.$$

The total inventory moved to the retailer in a cycle by the manufacturer is $m(m-1)Q^2/2D$. So, the manufacturer's total inventory in a cycle is

$$= \frac{m^2Q^2}{2P} + \frac{mQ^2(m-1)(P-D)}{PD} - \frac{m(m-1)Q^2}{2D},$$
$$= \frac{mQ^2}{2D}\left\{(m-1) - (m-2)\frac{D}{P}\right\}.$$

Therefore, the manufacturer's holding cost per unit time is

$$= \frac{r_v c_v Q}{2}\left[(m-1) - (m-2)\frac{D}{P}\right].$$

The manufacturer's setup cost per unit time is $\frac{SD}{mQ}$.

Hence, the manufacturer's total cost per unit time is

$$TC_v(m) = \frac{SD}{mQ} + \frac{r_v c_v Q}{2}\left\{(m-1) - (m-2)\frac{D}{P}\right\} + I_v c_b t_c D. \tag{4.3}$$

Therefore, the expected average total cost of the supply chain is the sum of the retailer's expected average total cost given by (4.2) and manufacturer's average total cost given by (4.3), *i.e.*,

$$\begin{aligned}\mathrm{ETC}(Q,r,L,m) = {} & \frac{D}{Q}\left[A + C(L) + \frac{S}{m}\right] + \frac{r_b c_b Q}{2} + c_b(r_b + I_c)\left[r - DL + (1-\beta)E(X-r)^+\right] \\ & + \frac{D}{Q}\left[\pi + \pi_0(1-\beta)\right]E(X-r)^+ + \frac{(Q-Dt_c)^2 c_b I_c}{2Q} - \frac{D^2 t_c^2 c_s I_d}{2Q} \\ & - \frac{D c_s t_c I_d}{Q}\beta E(X-r)^+ + \frac{r_v c_v Q}{2}\left\{(m-1) - (m-2)\frac{D}{P}\right\} + I_v c_b t_c D. \end{aligned} \tag{4.4}$$

5. Solution procedure

5.1. Lead time demand follows normal distribution

In this sub-section, we assume that the lead time demand X is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$. We note that $r = DL + k\sigma\sqrt{L}$ and the expected shortage quantity at the end of a cycle is

$$\begin{aligned} E(X-r)^+ &= \int_r^\infty (x-r)\, f(x)\mathrm{d}x \\ &= \int_{DL+k\sigma\sqrt{L}}^\infty \left\{x - \left(DL + k\sigma\sqrt{L}\right)\right\} \frac{1}{\sigma\sqrt{L}\sqrt{2\pi}} e^{-\frac{(x-DL)^2}{2\sigma^2 L}}\mathrm{d}x. \end{aligned}$$

After some calculations, the above expression reduces to (see Pan and Yang [40]; Ouyang *et al.* [38]; Sarkar *et al.* [44])

$$E(X-r)^+ = \sigma\sqrt{L}\Psi(k), \tag{5.1}$$

where $\Psi(k) = \phi(k) - k[1-\Phi(k)]$, and ϕ and Φ denote the standard normal probability density function and distribution function, respectively.
Substituting the value of $E(X-r)^+$ in (1), we get

$$\beta = \frac{1}{1 + \alpha\sigma\sqrt{L}\Psi(k)}\cdot \tag{5.2}$$

Therefore, when lead time demand follows normal distribution, the expected average total cost of the supply chain can be obtained by using (5.1) and (5.2) in (4.4) as

$$\begin{aligned}\mathrm{ETC}^N(Q,k,L,m) = {} & \frac{D}{Q}[G(m) + C(L)] + c_b(r_b + I_c)k\sigma\sqrt{L} + \frac{Q}{2}H(m) \\ & + \left\{\frac{D}{Q}\left(\pi - \frac{c_s t_c I_d}{1 + \alpha\sigma\sqrt{L}\Psi(k)}\right) + \frac{\alpha\sigma\sqrt{L}\Psi(k)M(Q)}{1 + \alpha\sigma\sqrt{L}\Psi(k)}\right\}\sigma\sqrt{L}\Psi(k) \\ & + \frac{(Q-Dt_c)^2 c_b I_c}{2Q} - \frac{D^2 t_c^2 c_s I_d}{2Q} + I_v c_b t_c D \end{aligned} \tag{5.3}$$

$$\begin{aligned} \text{where } G(m) &= A + \frac{S}{m} \\ M(Q) &= \frac{D\pi_0}{Q} + c_b(r_b + I_c) \\ H(m) &= r_b c_b + r_v c_v\left[(m-1) - (m-2)\frac{D}{P}\right] \end{aligned}$$

Note 1. It is clear that if $I_d = 0, I_c = 0, I_v = 0$ and $\alpha = 0$, *i.e.*, the case when trade-credit is not allowed and shortages are fully backlogged, (5.3) reducers to

$$\mathrm{ETC}^N(Q,k,L,m) = \frac{D}{Q}[G(m) + \pi\sigma\sqrt{L}\Psi(k) + C(L)] + \frac{Q}{2}H(m) + r_b c_b k\sigma\sqrt{L} \tag{5.4}$$

which is exactly same as Equation (7) of Ouyang *et al.*'s [38]. Therefore, Ouyang *et al.*'s [38] model is a special case of our model.

Note 2. If we take $I_d = 0$ and $I_c = 0$ then the retailer's cost function (4.2) becomes

$$\begin{aligned}\mathrm{ETC}_b(Q,L) = {} & \frac{D}{Q}[A + R(L)] + r_b c_b\left(\frac{Q}{2} + k\sigma\sqrt{L}\right)\\ & + \left\{\frac{r_b c_b \alpha\sigma\sqrt{L}\Psi(k)}{1+\alpha\sigma\sqrt{L}\Psi(k)} + \frac{D}{Q}\left[\pi + \frac{\pi_0\alpha\sigma\sqrt{L}\Psi(k)}{1+\alpha\sigma\sqrt{L}\Psi(k)}\right]\right\}\sigma\sqrt{L}\Psi(k)\end{aligned} \tag{5.5}$$

which is same as the expected average cost derived by Ouyang and Chuang [36] (taking $r_b c_b = h$, $\Psi(k) = G(k)$, and $C(L) = R(L)$). This indicates that Ouyang and Chuang's [36] model is also a special case of our model.

In order to show that the expected cost function (5.3) is strictly convex *i.e.*, it has a unique minimum, we derive the following propositions:

Proposition 5.1. *For given values of $Q, k,$ and m, $ETC^N(Q,k,L,m)$ is concave in $L \in [L_i, L'_{i-1}]$, where $L'_{i-1} = min\ \left\{L_{i-1}, \left(\frac{\sqrt{\pi c_s t_c I_d} - \pi}{\pi\alpha\sigma\Psi(k)}\right)^2\right\}$.*

Proof. See Appendix A. Hence, the minimum value of the expected total cost $\mathrm{ETC}^N(Q,k,L,m)$ will occur at the end points of the interval $[L_i, L'_{i-1}]$ (see Liao and Shyu [29] and Ouyang *et al.* [38]). □

Proposition 5.2. *If $I_c > \frac{c_s I_d}{c_b}$, then for fixed values of m and $L \in [L_i, L_{i-1}]$, $ETC^N(Q,k,L,m)$ is convex in Q and k for all $Q > 0$ and $k > 0$ such that $\Psi(k) > \frac{c_s t_c I_d}{\pi_0\alpha\sigma\sqrt{L}}$.*

Proof. See Appendix B. □

Proposition 5.3. *For given values of m and $L \in [L_i, L_{i-1}]$, $ETC^N(Q,k,L,m)$ has a unique minimum provided that $I_c > \frac{c_s I_d}{c_b}$ and $k > 0$ such that $\Psi(k) > \frac{c_s t_c I_d}{\pi_0\alpha\sigma\sqrt{L}}$, and the corresponding values of Q and k are given by*

$$Q = \sqrt{\frac{2D[G(m) + \sigma\sqrt{L}\Psi(k)\Delta(k,L) + C(L)] + F_1}{F_2 + H(m)}}, \tag{5.6}$$

$$k = \Phi^{-1}\left(1 - \frac{F_2[V(k,L)]^2 Q}{v(k,L)QM(Q)[1+V(k,L)] + D\pi[V(k,L)]^2 - Dst_c I_d}\right). \tag{5.7}$$

Proof. See Appendix C.

The optimal value of m *i.e.* m^* can be obtained from

$$\mathrm{ETC}^N(m^*-1) \geq \mathrm{ETC}^N(m^*) \leq \mathrm{ETC}^N(m^*+1). \tag{5.8}$$

The following algorithm is suggested to obtain numerically the optimal values of Q and k for specific values of m and L. □

Algorithm 1.

Step 1 Set $m = 1$.

Step 2 For each L_i, $i = 0, 1, 2, \ldots, n$, perform 2a to 2c.

2a Set $k_{i1} = 0$ (implies $\Psi(k_{i1}) = 0.39894$).

2b Substituting $\Psi(k_{i1})$ into (10), evaluate Q_{i1}.

2c Utilize Q_{i1} to obtain the value of k_{i2} from (11) by checking the normal table, and evaluate $\Psi(k_{i2})$.

2d Repeat 2b to 2c until no change occurs in the values of Q_i and k_i.

Step 3 For each set of values (Q_i, k_i, L_i, m), find $\mathrm{ETC}^N(Q_i, k_i, L_i, m), i = 1, 2, \ldots, n$.

Step 4 Find $\min_{i=0,1,2,\ldots,n} \mathrm{ETC}^N(Q_i, k_i, L_i, m)$.

If $\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m) = \min_{i=0,1,2,\ldots,n} \mathrm{ETC}^N(Q_i, k_i, L_i, m)$, then (Q_m^*, k_m^*, L_m^*, m) is the optimal solution for fixed m.

Step 5 Set $m = m + 1$ and repeat Steps 2, 3, and 4 to get $\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m)$.

Step 6 If $\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m) \leq \mathrm{ETC}^N(Q_{m-1}^*, k_{m-1}^*, L_{m-1}^*, m)$, then go to Step 5; otherwise, go to Step 7.

Step 7 Set $\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m) = \mathrm{ETC}^N(Q_{m-1}^*, k_{m-1}^*, L_{m-1}^*, m)$. Then (Q^*, k^*, L^*, m) is the optimal solution.

After substituting the values of k^* and L^*, the optimal backorder rate and the reorder point can be obtained as

$$\beta^* = \frac{1}{1 + \alpha\sigma\sqrt{L^*}\Psi(k^*)} \text{ and } r^* = DL^* + k^*\sigma\sqrt{L^*}.$$

5.2. Lead time demand is distribution-free

In many practical situations, the information about the probability distribution of the lead time demand is limited or unavailable. In this section, we relax the assumption of normally distributed lead time demand. We assume that the density function of the lead time demand belongs to Ω with finite mean DL and standard deviation $\sigma\sqrt{L}$. If the distributional form of lead time demand X is unknown, the exact value of $E(X - r)^+$ cannot be determined. Therefore, the min-max distribution-free approach is used to solve this problem (Gallego and Moon [9], Ouyang *et al.* [38], and Lee [25]):

$$\text{min-max}_{F\in\Omega} \quad \mathrm{ETC}^W(Q, k, L, m). \tag{5.9}$$

The following proposition which was proposed by Gallego and Moon [9] is used to approximate the value of $E(X - r)^+$.

Proposition 5.4. *For any $F \in \Omega$,*

$$E(X - r)^+ \leq \frac{1}{2}\left\{\sqrt{\sigma^2 L + (r - DL)^2} - (r - DL)\right\}. \tag{5.10}$$

Substituting $r = DL + k\sigma\sqrt{L}$ in (5.10), the following inequality is obtained:

$$E(X - r)^+ \leq \frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1 + k^2} - k\right). \tag{5.11}$$

Using the above inequality, the backorder rate β can be expressed as

$$\beta \geq \frac{1}{1 + \frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1 + k^2} - k\right)}. \tag{5.12}$$

Using (4.4) and (5.12), Equation (5.9) becomes

$$\begin{aligned} ETC^W(Q,L,k,m) = {} & \frac{D}{Q}[G(m)+C(L)] + c_b(r_b+I_c)k\sigma\sqrt{L} + \frac{r_b c_b Q}{2} + \frac{Q}{2}H(m) \\ & + \left[\left(\frac{\frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)}{1+\frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)} \right) M(Q) \right. \\ & \left. + \frac{D}{Q}\left(\pi - \frac{c_s t_c I_d}{1+\frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)} \right) \right] \frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right) \\ & + \frac{(Q-Dt_c)^2 c_b I_c}{2Q} - \frac{D^2 t_c^2 c_s I_d}{2Q} + I_v c_b t_c D. \end{aligned} \tag{5.13}$$

Similar to the case of normally distributed demand, it can be easily verified that, for fixed (Q,k,m), $\mathrm{ETC}^W(Q,L,k,m)$ is convex in $L \in [L_i, L_{i-1}]$. Therefore, the minimum expected average cost will occur at the end point of the interval $[L_i, L_{i-1}]$. Further, keeping m and $L \in [L_i, L_{i-1}]$ fixed, it can also be verified that $\mathrm{ETC}^W(Q,L,k,m)$ is convex in Q and k. Therefore, for fixed values of m and $L \in [L_i, L_{i-1}]$, the expected average cost will be minimum at the point (Q,k) which satisfies $\partial\mathrm{ETC}^W(Q,L,k,m)/\partial Q = 0$ and $\partial\mathrm{ETC}^W(Q,L,k,m)/\partial k = 0$, simultaneously. This gives

$$Q = \sqrt{\frac{2D[G(m)+\sigma\sqrt{L}\Psi(k)\Upsilon(k,L)+C(L)]+F_1}{F_2+H(m)}}, \tag{5.14}$$

$$[1+\omega(k,L)]^2 = \frac{\omega[QM(Q)+DsI_d t_c]}{D\omega(\pi+\pi_0) - QF_2\left(\omega+\alpha k\sigma\sqrt{L}\right)}, \tag{5.15}$$

where $\omega(k,L) = \frac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)$, $\hat{\pi} = \pi + \frac{\pi_0\omega(k,L)}{1+\omega(k,L)}$ and $\Upsilon(k,L) = \hat{\pi} - \frac{c_s t_c I_d}{1+\omega(k,L)}$. The following algorithm is developed to obtain the optimal values of Q and k for a specific values of m and $L \in [L_i, L_{i-1}]$.

Algorithm 2.
Step 1 Set $m = 1$.
Step 2 For each L_i, $i = 0,1,2,\ldots,n$, perform Steps 2a–2c.
 2a Set $k_{i1} = 0$.
 2b Evaluate Q_{i1} from (5.14).
 2c Utilize Q_{i1} to obtain the value of k_{i2} from (5.15).
 2d Repeat Steps 2b and 2c until no change occurs in the values of Q_i and k_i.
Step 3 For each set of values (Q_i,k_i,L_i,m), compute $\mathrm{ETC}^W(Q_i,k_i,L_i,m), i = 1,2,\ldots,n$.
Step 4 Find $\min_{i=0,1,2,\ldots,n}\mathrm{ETC}^W(Q_i,k_i,,L_i,m)$.
If $\mathrm{ETC}^W(Q_m^{**},k_m^{**},L_m^{**},m) = \min_{i=0,1,2,\ldots,n}\mathrm{ETC}^W(Q_i,k_i,L_i,m)$, then $(Q_m^{**},k_m^{**},L_m^{**},m)$ is the optimal solution for fixed m.
Step 5 Set $m = m+1$, repeat Steps 2–4 to get $\mathrm{ETC}^W(Q_m^{**},k_m^{**},L_m^{**},m)$.
Step 6 If $\mathrm{ETC}^W(Q_m^{**},k_m^{**},L_m^{**},m) \le \mathrm{ETC}^W(Q_{m-1}^{**},k_{m-1}^{**},L_{m-1}^{**},m)$, then go to Step 5; otherwise, go to Step 7.
Step 7 Set $\mathrm{ETC}^W(Q_m^{**},k_m^{**},L_m^{**},m) = \mathrm{ETC}^W(Q_{m-1}^{**},k_{m-1}^{**},L_{m-1}^{**},m)$. Then (Q^{**},k^{**},L^{**},m) is the optimal solution.

After substituting the values of k^{**} and L^{**}, the optimal backorder rate and the reorder point can be obtained as

$$\beta^{**} = \frac{1}{1+\frac{1}{2}\alpha\sigma\sqrt{L^{**}}\left(\sqrt{1+k^{**2}}-k^{**}\right)} \text{ and } r^{**} = DL^{**} + k^{**}\sigma\sqrt{L^{**}}.$$

Table 2. Parameter values.

Parameters	Values	Parameters	Values	Parameters	Values
D	600 units/year	P	2000 units	S	\$1500/set up
A	\$200/order	r_b	\$0.2/unit/year	r_v	\$0.2/unit/year
c_b	\$100/unit/year	c_s	\$110/unit/year	c_v	\$70/unit/year
π_0	\$150/unit	π	\$50/unit	σ	7 units/week

Table 3. Lead time data.

Lead time component i	Normal duration v_i (days)	Minimum duration u_i (days)	Unit crashing cost m_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 4. Optimal results in Example 6.1.

m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$\text{ETC}^N(Q_m^*, k_m^*, L_m^*, m)$
1	3	1.00	47	264	0.91	\$8349
2	4	1.20	63	174	0.93	\$7311
3	**4**	**1.31**	**64**	**136**	**0.94**	**\$7094**
4	4	1.39	66	114	0.95	\$7105

Notes. The bold values indicate the optimal solution of the decision variables.

6. Numerical examples

Example 6.1. In order to illustrate the solution procedure of the model, we consider in Table 2 the data which are used by Ouyang *et al.* [38]. For controllable backorder rate and trade-credit financing, we take some additional parameter-values as: $\pi_0 = \$150$/unit, $\alpha = 0.1$, $t_c = 0.2$ years, $I_d = \$0.04/\$/$ year, $I_c = \$0.08/\$/$ year, $I_v = \$0.04/\$/$ year. The lead time has three components with data given in Table 3. Using the lead time data and Algorithm 1, we obtain the results for the case when lead time demand follows normal distribution. The summary of optimal results is given in Table 4. Variation of the expected average cost with respect to number of shipments m is depicted in Figure 4. From Table 4, we obtain the optimal order quantity $Q^* = 136$ units, safety stock $k^* = 1.31$, reorder point $r^* = 64$ units, lead time $L = 4$ weeks, number of lots (delivered from the manufacturer to the retailer) $m^* = 3$, backorder rate $\beta^* = 0.94$, and the minimum expected average cost $\text{ETC}^N = \$7094$.

We now examine the case when both parties take decisions independently to determine their own optimal policies. When the retailer takes decision independently, his optimal policy is as follows: order quantity $Q_b^* = 112$ units, safety stock $k_b^* = 1.39$, reorder point $r_b^* = 66$ units, lead time $L_b^* = 4$ weeks, backorder rate $\beta^* = 0.95$, and the minimum expected average cost is \$2735.67. Also, the manufacturer's optimal production quantity is $m_v^* Q_b^* = 448$ units and the minimum average cost \$4370.53. Therefore, when the manufacturer and the retailer do not cooperate with each other, the expected average cost of the supply chain is \$7106, see Table 5. However, when both parties cooperate with each other, the expected average cost is \$7094, which is less than the expected average cost of the supply chain in the decentralized system. From Table 5, we can observe that the retailer's expected average cost in the decentralize model is lower than that of the integrated model, which implies that

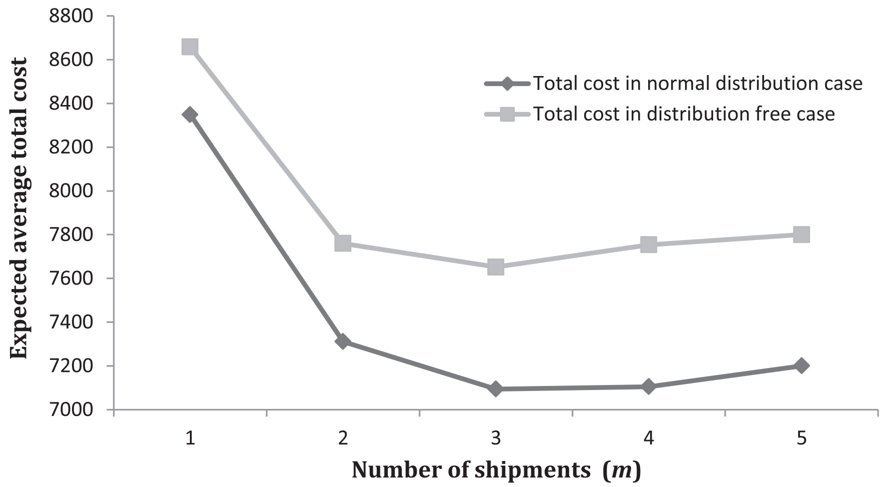

FIGURE 4. ETC *vs*. number of shipments (m).

TABLE 5. Allocation of expected average cost.

Model type	Retailer		Manufacturer	
Independent	Order quantity	112	Number of shipments	4
	Lead time (weeks)	4	Production quantity	448
	Safety factor	1.39		
	Reorder point	66		
	Backorder rate	0.95		
	Expected average cost	\$2735.67	Average cost	\$4370.53
Integrated	Order quantity	136	Number of shipments	3
	Lead time (weeks)	4	Production quantity	408
	Safety factor	1.31		
	Reorder point	64		
	Backorder rate	0.94		
	Expected average cost	\$2789.92	Average cost	\$4304.28
	Allocated average cost	\$2731.00	Allocated average cost	\$4363.20

the retailer may not prefer integrated decision making model unless there is some cost sharing mechanism. Goyal [13] suggested the following method to allocate the expected cost among the retailer and the manufacturer:

Retailer's cost $= \rho \times \text{ETC}^N(Q^*, r^*, L^*)$ and manufacturer's cost $= (1-\rho) \times \text{ETC}^N(Q^*, r^*, L^*)$, where

$$\rho = \frac{\text{ETC}_b(Q_b^*, k_b^*, L_b^*)}{\text{ETC}_b(Q_b^*, k_b^*, L_b^*) + \text{ETC}_v(Q_b^* m_v^*)}, \tag{6.1}$$

The allocated costs for the retailer and the manufacturer are shown in Table 5.

We now investigate the effects of controllable backorder and trade-credit financing on the average costs of the manufacturer and the retailer. In Table 6, we compare the results of our model with those of Ouyang *et al.* [38] where shortages were fully backlogged and trade-credit financing was not considered. From Table 6, we observe that the expected average cost of the supply chain is greater than that of the Ouyang *et al.*'s [38] model by 6.11%. This is due to consideration of variable backorder and trade-credit financing. In our model, the retailer's

TABLE 6. A comparative study.

	Integrated model		Independent model	
	Present model	Ouyang *et al.* [38] ($I_d = 0, I_c = 0,$ $I_v = 0, \alpha = 0$)	Present model	Ouyang *et al.* [38] ($I_d = 0, I_c = 0,$ $I_v = 0, \alpha = 0$)
Retailer's cost	\$2789.92	\$2862.7	\$2735.67	\$2832.0
Manufacturer's cost	\$4304.28	\$3797.7	\$4370.53	\$3893.9
Joint cost	\$7094.20	\$6660.4	\$7106.20	\$6725.9
Retailer's allocated cost	\$2731.00	\$2804.4	–	–
Manufacturer's allocated cost	\$4363.20	\$3856.0	–	–

TABLE 7. Optimal results in distribution-free case (Example 6.2).

m	L_m^{**}	k_m^{**}	R_m^{**}	Q_m^{**}	β_m^{**}	$\mathrm{ETC}^N(Q_m^{**}, k_m^{**}, L_m^{**}, m)$
1	3	1.16	49	271	0.82	\$8658
2	3	1.44	52	184	0.84	\$7760
3	**3**	**1.62**	**54**	**146**	**0.85**	\$**7652**
4	3	1.77	56	124	0.86	\$7754

Notes. The bold values indicate the optimal solution of the decision variables.

allocated cost which is 60% of the integrated cost is 3% more than that of Ouyang *et al.*'s [38] model. On the other hand, the manufacturer's allocated cost which is 40% of the integrated cost is 3% less than that of Ouyang *et al.*'s [38] model. This indicates that the manufacturer is beneficial in our model.

Example 6.2. In this example, we use the same data as given in Table 1. Applying Algorithm 2, we obtain the results of the model when lead time demand does not follow any specific distribution. The results are given in Table 7.

6.1. Evaluation of expected value of additional information (EVAI)

Now, we compare the results of the distribution-free model with those of the normal distribution model. We see from Tables 4 to 10 that, in the normal distribution model, the set of optimal values of the decision variables is $(Q^*, k^*, L^*, m^*) = (136, 1.31, 4, 3)$, and that in the distribution-free model is $(Q^{**}, k^{**}, L^{**}, m^{**}) = (146, 1.62, 3, 3)$. If we utilize the solution obtained by the distribution-free approach instead of utilizing the normal distribution model, then the added cost will be $\mathrm{ETC}^N(Q^{**}, k^{**}, L^{**}, m^{**}) - \mathrm{ETC}^N(Q^*, k^*, L^*, m^*)$ $= \mathrm{ETC}^N(146, 1.62, 3, 3) - \mathrm{ETC}^N(136, 1.31, 4, 3)$=7200 − 7094 = \$106. This amount is said to be the expected value of additional information (EVAI) for the retailer that he would be willing to pay to collect the information to know the form of lead time demand distribution.

Additionally, we consider the same problem with negative exponential backorder rate as $\beta = \theta e^{-\epsilon B(r)}$, $B(r) = E(X - r)^+$ (Lee *et al.* [26]). Results are given in Table 8.

7. SENSITIVITY ANALYSIS

In this section, we perform sensitivity analysis to investigate the effects of the key parameters on the optimal solutions.

TABLE 8. Summary of results for negative exponential backorder rate.

ϵ	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m)$
0.00	3	1.58	68	136	0.97	\$7208
2.00	3	1.78	71	136	0.98	\$7252
10.0	3	1.89	73	136	0.98	\$7292
20.0	3	1.89	73	136	0.98	\$7303
40.0	3	1.87	72	136	0.98	\$7305
80.0	3	1.87	72	136	0.98	\$7305
∞	3	1.87	72	136	0.98	\$7305

TABLE 9. Effect of trade-credit period t_c on optimal results.

t_c	m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m)$
0.1	4	4	1.40	66	111	0.95	\$7174
0.2	3	4	1.31	64	136	0.94	\$7094
0.3	3	4	1.29	64	141	0.94	\$7087
0.4	3	4	1.27	64	147	0.94	\$7160
0.5	3	4	1.25	64	155	0.93	\$7306
0.6	2	4	1.13	62	201	0.92	\$7452
0.7	2	4	1.11	62	211	0.91	\$7619
0.8	2	4	1.08	61	222	0.91	\$7827
0.9	2	4	1.06	61	234	0.91	\$8070

7.1. Effect of trade-credit period (t_c)

Table 9 presents the effect of credit period t_c ranging from 0.1 to 0.9 on optimal solutions. From Table 7, it is observed that a higher value of credit period increases the retailer's order quantity. Safety factor and reorder point both tend to decrease as credit period increases. Furthermore, the expected average cost of the supply chain tends to decrease for $t_c \in [0.1, 0.3]$ and increase for $t_c \in [0.4, 0.9]$. The expected average cost and order quantity are more sensitive for higher value of t_c, whereas the reorder point and the safety factor are less sensitive to t_c (see Figs. 5 and 6).

7.2. Effect of backorder parameter (α)

Table 10 indicates that an increase in the value of α increases the expected average cost whereas it decreases the backorder rate. This is due to the fact that, as shortage quantity becomes more sensitive to backorder parameter α, shortage quantity increases resulting an increase in the average cost. Even for small value of α, the expected average cost, safety factor, and the reorder point are highly sensitive. When α takes a very high value, the expected average cost represents the lost sale case (*i.e.*, $\beta \to 0$), and when α takes a very small value, the expected average cost represents the fully backorder case (*i.e.*, $\beta \to 1$). However, α has no effect on lead time and number of shipments (see Figs. 7 and 8).

7.3. Effect of lead time demand standard deviation (σ)

We investigate the effect of lead time standard deviation on the optimal results. From Table 11, we see that an increase in the value of σ decreases the backorder rate. This is due to the fact that a higher value of σ implies higher amount of shortages which decreases the backorder rate. Also, we see that, as σ increases, the order quantity and the safety factor also increase. This is because shortages increase for higher value of σ, which leads

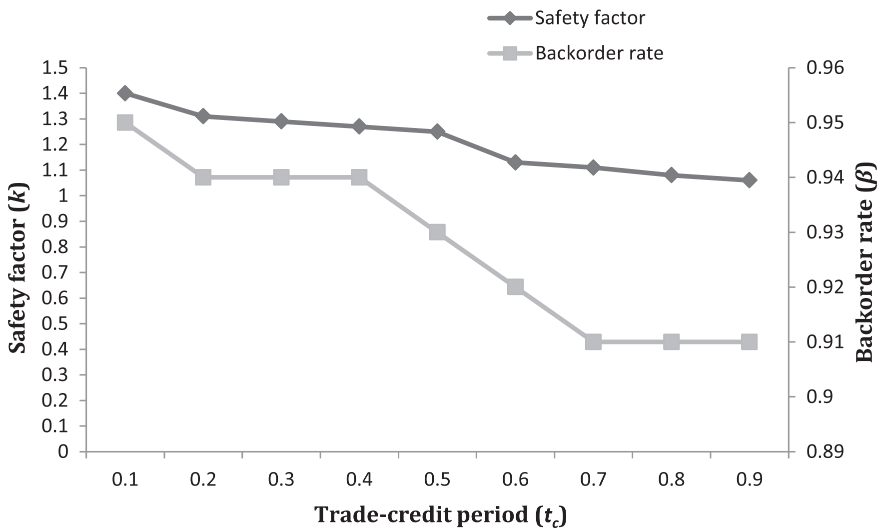

FIGURE 5. Trade-credit period (t_c) *vs*. safety factor (k) and backorder rate (β).

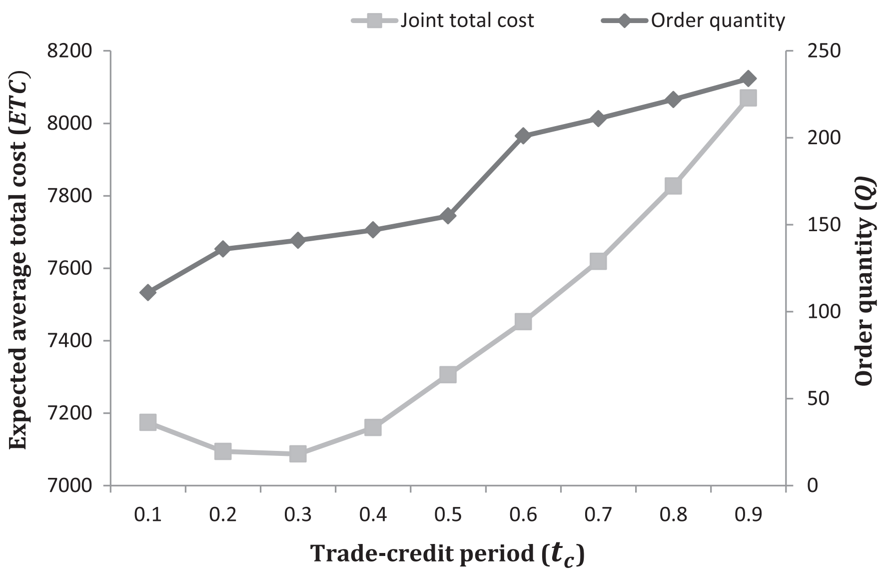

FIGURE 6. Trade-credit period (t_c) *vs*. expected average total cost (ETC) and order quality (Q).

to larger order quantity and safety stock. Figures 9 and 10 also indicate that σ impacts the expected average cost significantly.

8. Conclusions

Customer's demand, replenishment lead time, and time gap between placing and receiving of an order play vital roles in economic order quantity (EOQ) modelling. In most cases, customers do not have patience to wait during shortage period to meet their demands from the next replenishment which results in lost sales.

TABLE 10. Effect of backorder parameter α on optimal results.

α	m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m)$
0.00	3	4	1.12	62	137	1.00	$7059
0.50	3	4	1.51	67	136	0.83	$7145
1.00	3	4	1.60	68	136	0.75	$7173
10.0	3	4	1.83	72	136	0.35	$7261
20.0	3	4	1.85	72	136	0.22	$7278
40.0	3	4	1.86	72	136	0.13	$7290
80.0	3	4	1.87	72	136	0.07	$7297
100	3	4	1.87	72	136	0.06	$7299
∞	3	4	1.87	72	136	0.00	$7305

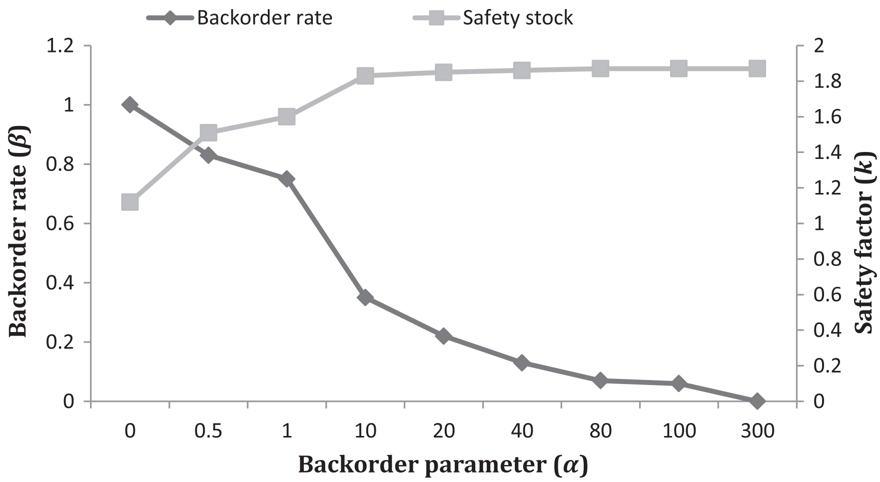

FIGURE 7. Backorder parameter (α) *vs*. safety factor (k) and backorder rate (β).

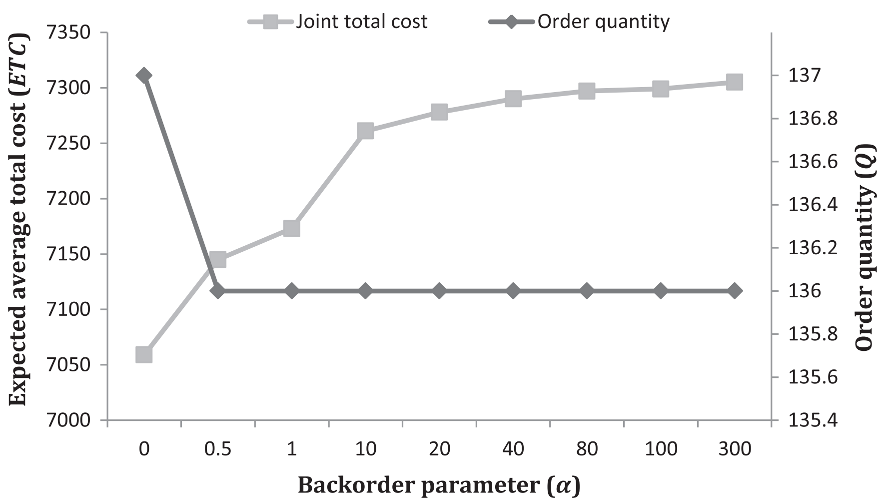

FIGURE 8. Backorder parameter (α) *vs*. expected average total cost (ETC) and order quality (Q).

Table 11. Effect of standard deviation σ on optimal results.

σ	m	L_m^*	k_m^*	R_m^*	Q_m^*	β_m^*	$\mathrm{ETC}^N(Q_m^*, k_m^*, L_m^*, m)$
1	4	6	1.29	72	111	0.99	\$6430
3	4	6	1.34	79	112	0.97	\$6676
5	3	4	1.28	59	135	0.95	\$6896
7	3	4	1.31	64	136	0.94	\$7094
9	3	4	1.34	70	137	0.93	\$7295
14	3	3	1.36	67	141	0.91	\$7762
20	3	3	1.40	83	143	0.89	\$8297

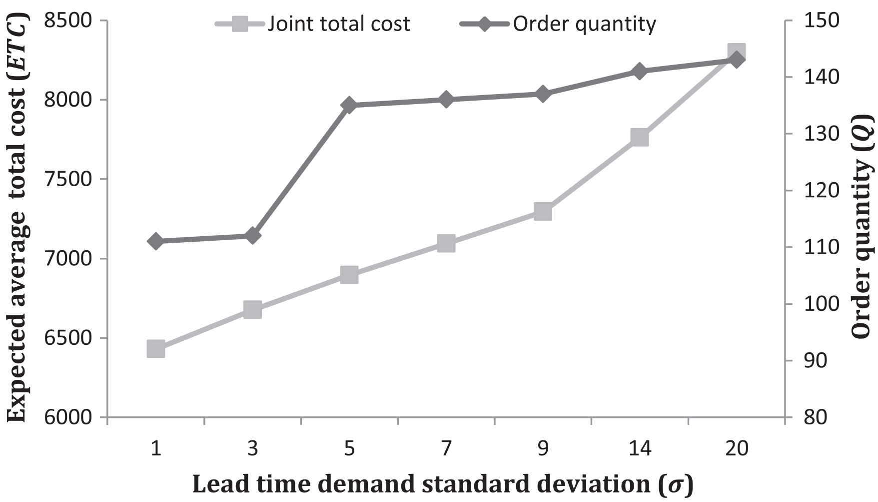

Figure 9. Lead time standard deviation (σ) *vs*. expected average total cost (ETC) and order quality (Q).

Consequently, partial backordering rate varies with replenishment lead time. Moreover, a larger lead time creates a negative impression that reduces customers' demand. The aim of this paper was to determine the inventory policy when a system faces lead time dependent backlogging rate along with stochastic lead time demand. To investigate the problem, an integrated manufacturer-retailer supply chain model is considered where the market demand is uncertain, shortages are partially backlogged, and the lead time is controllable. Also, a trade credit period offered by the manufacturer to the retailer is incorporated. To reduce the replenishment lead time, we have decomposed the lead time into several components with normal and minimum durations having different crashing costs for reducing to a specified minimum duration. First, we have formulated the model considering known lead time demand distribution and then studied extensively for unknown distribution case. A comparative study on the results of non-integrated approach and integrated approach is conducted. The proposed model can be utilized to different kinds of firms, such as electronic assembling frameworks, the garments fabricating industries, food industries and so on.

The results of the paper indicate that, in case when lead time demand deviation is high, choosing lead time reduction strategy may reduce the total expected system cost. Further, it is seen that the increment of lead time dependency parameter shifts the model to full lost sale case where expected total cost is maximum. Cases with smaller lead time demand deviation may be interpreted as a restriction on the invest amount on lead time reduction. In such a situation, a company may search for cost-optimal solutions without investing in lead

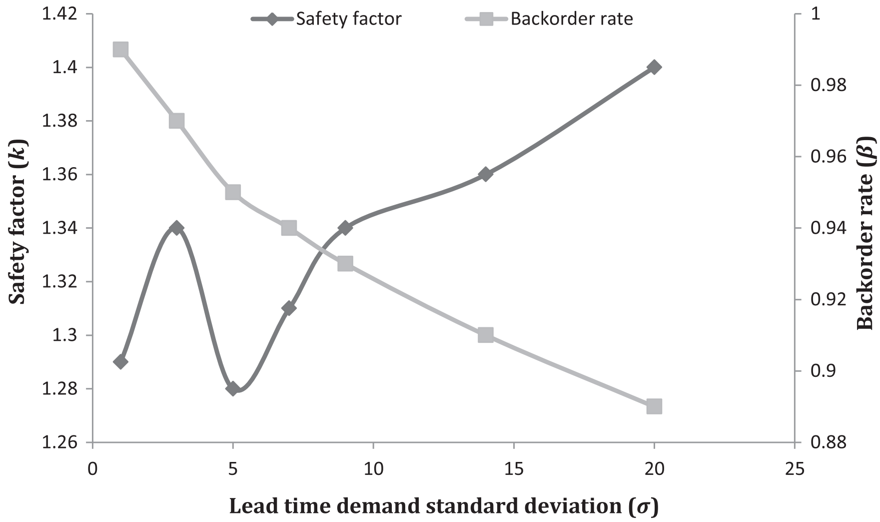

FIGURE 10. Lead time standard deviation (σ) *vs.* safety factor (k) and backorder rate (β).

time reduction. Accordingly, it can be seen that trade credit offer increases the order quantity which helps in establishing coordination between the supply chain players.

One limitation of our work is clearly the consideration of single manufacturer single retailer situation, which restricts its applicability to industries. Future research may focus on studying some other backorder rate functions (*e.g.*, exponential) in a multiple sourcing environment. One can consider imperfect production and investment in quality improvement to extend the present work. Further, the entire replenishment lead time can be taken as an addition of different lead time components (*e.g.*, setup time, production time, transportation time, etc.) and investment can be made to reduce a specific component (Glock [12]).

APPENDIX A. PROOF OF PROPOSITION 5.1

The expected average cost for normally distributed lead time demand is

$$\begin{aligned}\mathrm{ETC}^N(Q,k,L,m) = {} & \frac{D}{Q}[G(m)+C(L)] + c_b(r_b+I_c)k\sigma\sqrt{L} + \frac{Q}{2}H(m) \\ & + \left\{\frac{D}{Q}\left(\pi - \frac{c_s t_c I_d}{1+\alpha\sigma\sqrt{L}\Psi(k)}\right) + \frac{\alpha\sigma\sqrt{L}\Psi(k)M(Q)}{1+\alpha\sigma\sqrt{L}\Psi(k)}\right\}\sigma\sqrt{L}\Psi(k) \\ & + \frac{(Q-Dt_c)^2 c_b I_c}{2Q} - \frac{D^2 t_c^2 c_s I_d}{2Q} + I_v c_b t_c D. \end{aligned} \tag{A.1}$$

For fixed m, taking the first order partial derivative of $\mathrm{ETC}^N(Q,k,L,m)$ with respect to $L \in [L_i, L_{i-1}]$ we have

$$\begin{aligned}\frac{\partial \mathrm{ETC}^N}{\partial L} = {} & \frac{c_b(r_b+I_c)k\sigma L^{-1/2}}{2} + M(Q)\frac{(2+v)v^2}{2\alpha L(1+v)^2} \\ & + \frac{Dv}{2\alpha LQ}\left\{\pi - \frac{c_s t_c I_d}{(1+v)^2}\right\} - \frac{D}{Q}c_i. \end{aligned} \tag{A.2}$$

The second order derivative of $\mathrm{ETC}^N(Q,k,L,m)$ with respect to $L \in [L_i, L_{i-1}]$ is

$$\begin{aligned}\frac{\partial \mathrm{ETC}^N}{\partial L^2} &= -\frac{1}{4}c_b(r_b+I_c)k\sigma L^{-3/2} - M(Q)\frac{(3+v)v^3}{4\alpha L^2(1+v)^3}\\ &\quad - \frac{D\pi v}{4\alpha L^2 Q}\left\{\pi - \frac{c_s t_c I_d(1+3v)}{(1+v)^3}\right\} < 0 \\ &\text{if } L < \left(\frac{\sqrt{\pi c_s t_c I_d} - \pi}{\pi\alpha\sigma\Psi(k)}\right)^2.\end{aligned} \tag{A.3}$$

Hence, $\mathrm{ETC}^N(Q,k,L,m)$ is concave in $L \in [L_i, L_{i-1}^{'}]$ where $L_{i-1}^{'} = \min\left\{L_{i-1}, \left(\frac{\sqrt{\pi c_s t_c I_d}-\pi}{\pi\alpha\sigma\Psi(k)}\right)^2\right\}$. Therefore, Proposition 5.1 is proved.

Appendix B. Proof of Proposition 5.2

For fixed m and $L \in [L_i, L_{i-1}]$, taking first and second order partial derivatives of $\mathrm{ETC}^N(Q,k,L,m)$ with respect to Q and k, we have

$$\begin{aligned}\frac{\partial \mathrm{ETC}^N}{\partial Q} &= -\frac{D}{Q^2}[G(m)+C(L)] - \frac{D^2 t_c^2 c_b I_c}{2Q^2} + \frac{D^2 t_c^2 c_s I_d}{2Q^2}\\ &\quad - \frac{Dv}{\alpha Q^2}\left\{\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right\} + \frac{H(m)}{2}\end{aligned} \tag{B.1}$$

$$\begin{aligned}\frac{\partial \mathrm{ETC}^N}{\partial k} &= c_b(r_b+I_c)\sigma\sqrt{L} + \alpha\sigma^2 L\Psi(k)\lambda(k)\left\{\frac{2+v}{(1+v)^2}\right\}M(Q)\\ &\quad + \frac{D\lambda(k)\sigma\sqrt{L}}{Q}\left(\pi - \frac{c_s t_c I_d}{(1+v)^2}\right)\end{aligned} \tag{B.2}$$

$$\begin{aligned}\frac{\partial^2 \mathrm{ETC}^N}{\partial Q^2} &= \frac{2D}{Q^3}\left[G(m)+C(L)+\sigma\sqrt{L}\Psi(k)\left\{\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right\}\right]\\ &\quad + \frac{D^2 t_c^2}{Q^3}(c_b I_c - c_s I_d) > 0 \text{ if } I_c > \frac{c_s I_d}{c_b} \text{ and } \Psi(k) > \frac{c_s t_c I_d}{\pi_0\alpha\sigma\sqrt{L}}\end{aligned} \tag{B.3}$$

$$\begin{aligned}\frac{\partial^2 \mathrm{ETC}^N}{\partial k^2} &= (r_b+I_c)c_b\left(\frac{v\sigma\sqrt{L}\phi(k)(2+v)}{(1+v)^2}\right) + \frac{2\alpha\sigma^2 L[\Phi(k)-1]^2}{(1+v)^3}\\ &\quad \left\{\frac{D}{Q}(\pi_0 + c_s t_c I_d) + c_b(I_c+r_b)\right\} + \frac{D\sigma\sqrt{L}\phi(k)}{Q}\\ &\quad \left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right) > 0 \text{ if } \Psi(k) > \frac{c_s t_c I_d}{\pi_0\alpha\sigma\sqrt{L}}\end{aligned} \tag{B.4}$$

where $\overline{\pi} = \pi + \frac{\pi_0 v}{1+v}, \lambda(k) = \Phi(k) - 1, v = \alpha\sigma\sqrt{L}\Psi(k)$.

From (B.3) and (B.4), we can say that if $I_c > \frac{c_s I_d}{c_b}$, $\mathrm{ETC}^N(Q,k,L,m)$ is convex in Q and k for all $Q > 0$ and $k > 0$ such that $\Psi(k) > \frac{c_s t_c I_d}{\pi_0\alpha\sigma\sqrt{L}}$. Hence Proposition 5.2 is proved.

Appendix C. Proof of Proposition 5.3

For convexity of the cost function given in (5.3), the Hessian matrix for $\mathrm{ETC}^N(Q,k,L,m)$ must be positive definite. The Hessian matrix is given by

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial Q^2} & \frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial Q \partial k} \\ \frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial k \partial Q} & \frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial k^2} \end{bmatrix}$$

where $\frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial Q^2}$ and $\frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial k^2}$ are given in (B.3) and (B.4), and

$$\frac{\partial^2 \mathrm{ETC}^N}{\partial Q \partial k} = \frac{\partial^2 \mathrm{ETC}^N}{\partial k \partial Q} = \frac{D\sigma\sqrt{L}[1-\Phi(k)]}{Q^2}\left[\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right] \tag{C.1}$$

Now

$$\begin{aligned}
|H| &= \frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial Q^2} \times \frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial k^2} - \left[\frac{\partial^2 \mathrm{ETC}^N(\cdot)}{\partial Q \partial k}\right]^2 \\
&= \left\{\frac{2D}{Q^3}\left[G(m) + C(L) + \sigma\sqrt{L}\Psi(k)\left\{\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right\}\right] + \frac{D^2 t_c^2}{Q^3}(c_b I_c - c_s I_d)\right\} \\
&\quad \times \left\{(r_b + I_c)c_b\left(\frac{v\sigma\sqrt{L}\phi(k)(2+v)}{(1+v)^2}\right) + \frac{2\alpha\sigma^2 L[\Phi(k)-1]^2}{(1+v)^3}\left\{\frac{D}{Q}(\pi_0 + c_s t_c I_d) + c_b(I_c + r_b)\right\}\right. \\
&\quad \left. + \frac{D\sigma\sqrt{L}\phi(k)}{Q}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right)\right\} - \left\{\frac{D\sigma\sqrt{L}[1-\Phi(k)]}{Q^2}\left[\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right]\right\}^2 \\
&> \frac{2D}{Q^3}\sigma\sqrt{L}\Psi(k)\left\{\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right\}\frac{D\sigma\sqrt{L}\phi(k)}{Q}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right) \\
&\quad - \left\{\frac{D\sigma\sqrt{L}[1-\Phi(k)]}{Q^2}\left[\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right]\right\}^2 \text{ (after deleting the positive terms)} \\
&= \frac{2D^2\sigma^2 L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi} - \frac{c_s t_c I_d}{1+v}\right)\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right) \\
&\quad - \frac{D^2\sigma^2 L[1-\Phi(k)]^2}{Q^4}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right)^2 \\
&= \frac{2D^2\sigma^2 L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right)\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right) \\
&\quad - \frac{2D^2\sigma^2 L\phi(k)\Psi(k)}{Q^4}\left(\frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2} + \frac{c_s t_c I_d}{1+v}\right)\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right) \\
&\quad - \frac{D^2\sigma^2 L[1-\Phi(k)]^2}{Q^4}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right)^2 \\
&< \frac{2D^2\sigma^2 L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right)^2 \\
&\quad + \frac{2D^2\sigma^2 L\phi(k)\Psi(k)}{Q^4}\left(\frac{c_s t_c I_d}{(1+v)^2}\right)\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right) \\
&\quad - \frac{D^2\sigma^2 L[1-\Phi(k)]^2}{Q^4}\left(\overline{\pi} + \frac{\pi_0 v}{(1+v)^2} - \frac{c_s t_c I_d}{(1+v)^2}\right)^2 \text{ (after deleting the negative terms)}
\end{aligned}$$

$$
\begin{aligned}
&> \frac{2D^2\sigma^2 L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_s t_c I_d}{(1+v)^2}\right)^2 \\
&\quad -\frac{D^2\sigma^2 L[1-\Phi(k)]^2}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_s t_c I_d}{(1+v)^2}\right)^2 \text{ (after deleting the positive terms)} \\
&= \frac{2D^2\sigma^2 L\phi(k)\Psi(k)}{Q^4}\left(\overline{\pi}+\frac{\pi_0 v}{(1+v)^2}-\frac{c_s t_c I_d}{(1+v)^2}\right)^2 \{2\phi(k)\Psi(k)-[\Phi(k)-1]^2\} \\
&> 0,
\end{aligned}
$$

because $\phi(k) > 0$, $\Psi(k) > 0$ and $2\phi(k)\Psi(k) - [\Phi(k)-1]^2 > 0$, for all $k > 0$ (see Ouyang *et al.* [37]). Therefore, if the Hessian matrix for $\mathrm{ETC}^N(Q,k,L,m)$ is positive definite then there exists a unique optimal solution which can be obtained from the first order necessary conditions $\frac{\partial \mathrm{ETC}^N}{\partial Q} = 0$ and $\frac{\partial \mathrm{ETC}^N}{\partial k} = 0$ as

$$
Q = \sqrt{\frac{2D[G(m)+\sigma\sqrt{L}\Psi(k)\Delta(k,L)+C(L)]+F_1}{F_2+H(m)}}, \tag{C.2}
$$

$$
k = \Phi^{-1}\left(1-\frac{F_2[V(k,L)]^2 Q}{v(k,L)QM(Q)[1+V(k,L)]+D\pi[V(k,L)]^2-Dst_c I_d}\right), \tag{C.3}
$$

where $F_1 = D^2 t_c^2 (c_b I_c - c_s I_d), F_2 = c_b(r_b + I_c), \Delta(k,L) = \overline{\pi} - \frac{c_s t_c I_d}{1+v(k,L)}, V(k,L) = 1 + v(k,L)$.

Hence the proposition is proved.

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