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COORDINATION FOR PULL AND PUSH CONTRACTS IN DECENTRALIZED SYSTEM WITH UNCERTAIN SUPPLY

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Abstract. This paper investigates how a random supply can influence management decisions under pull and push contracts in a decentralized supply chain with one supplier and one retailer. We suppose that the supplier faces yield uncertainty, and we adopt game models to analyse the supply chain members' decisions (*i.e.*, wholesale price and order quantity) under the two commonly used contracts. Specifically, we analyse the revenue sharing mechanism and buyback mechanism with pull and push contracts, respectively, and find that the buyback contract can efficiently coordinate the supply chain with push contract, while the revenue sharing contract cannot improve the performance with a pull contract. Then we design a modified revenue sharing contract that introduces a subsidy for excess inventory and shows that for the pull case, the proposed mechanism can coordinate the supply chain effectively. Finally, the analysis results are displayed intuitively by numerical cases.

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1. Introduction

The mismatch between supply and demand is a common phenomenon in production practices and has been handled as a main problem in supply chain management. In traditional manufacturing, uncertain demand or supply is regarded as the main cause of this mismatch. In this paper, we focus on the uncertain supply problem in a supplier-retailer supply chain with different market dominance, which is depicted by bargaining power through the well-known pull and push contracts. Supply uncertainty is a concerned problem in both the industrial and academic worlds [18, 19, 30, 36, 40] and can be caused by a number of reasons, such as supply disruptions due to bad weather, natural disasters, suppliers going out of business, yield uncertainty due to product defects or batch processes in which only a certain percentage of the yield is usable or lead time uncertainty which results from stock-outs at the supplier or manufacturer or due to transit delays [35]. In this paper, we mainly focus on random production yields to investigate how supply uncertainty influences the supply chain members' decisions.

Supply or yield uncertainty poses great challenges to supply chain management. One of the typical outcomes is an increase in the inventory risk, which may consequently cause substantial revenue losses. Since most supply

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chains are unable to match supply and demand perfectly, all the supply chain members bear at least some inventory risk. However, in business negotiations, the supplier and the retailer always have different bargaining power, which is dependent on whether they have abundant alternate sources. For example, yield uncertainty is a common characteristic in electronics components' production, comparing with the computer manufacturer and retailer such as Dell Inc., in which the CPU is one of the key components, the CPU supplier has stronger bargaining power, and Dell should thus bear the risk of uncertain supply. However, the suppliers that provide less crucial components, such as the keyboard or mouse, have considerably weaker bargaining power and have to take the yield uncertainty risk. In other words, the supply chain member may be able to avoid the inventory risk completely by using his dominant position. Cachon [3] first classifies inventory risk allocation into the following two categories: a push contract, where the supplier forces the retailer to take the entire inventory risk, and a pull contact, where the downstream retailer delegates the inventory risk to the upstream supplier. Additionally, since each participant decides to maximize self-interest in a decentralized system, allocating inventory risk to different supply chain members may cause worse channel performance in practice. Therefore, designing coordination schemes to improve supply chain performance by properly allocating the inventory risk between the members is a worthwhile solution.

This paper explores how the supply uncertainty affects the supplier's and retailer's decisions in a single-period supply chain under pull and push contracts and investigates the efficiencies of the corresponding coordination mechanisms. Specifically, a retailer procures a random output product from a supplier to satisfy a deterministic market demand with zero lead time, while the supplier faces uncertain yield after deciding on a certain production level. The inventory risk is borne by different supply chain members under different power structures: In a push contract, the supplier has more bargaining power and charges a single wholesale price; the retailer decides the ordering quantity under this wholesale price and the supplier provides the retailer's booked quantity. All the produced products are stored by the retailer; therefore, the retailer bears the excess inventory risk caused by a mismatch between supply and demand. In the meanwhile, the supplier should bear the random yield risk since the actual production may be lower than the planned production level. In contrast, the retailer has more bargaining power in a pull contract and decides the wholesale price before the supplier decides the quantity of output. This scenario is similar to a Vendor Managed Inventory (VMI) system; thus, both the inventory risk and random yield risk are taken by the supplier. The equilibrium decisions under pull or push contracts will bring out the double marginalization effect. In general, the well-known double marginalization effect refers to the channel barrier that causes a lower transfer or delivery amount of products through the channel, which usually reveals that as a higher selling price is set by the retailer or a lower order quantity is submitted from the retailer (see [2, 6]). Our result shows that under the push contract, the double marginalization effect is in line with the traditional one; however, under a pull contract with uncertain yield, the effect can be translated into a lower production level in a decentralized system and leads to insufficient supply to meet the demand. While related works have sufficiently examined both uncertainty supply problems and bargaining power issues, there is a gap in the literature regarding the consideration of inventory risk with an uncertain supply under push or pull power structures. This work focuses on this research gap and investigates coordination mechanisms to eliminate double marginalization.

This study stands on the interface between supply uncertainty and supply chain coordination under different supply chain power structures. We first prove theoretically that both pull and push contracts give rise to the double marginalization effect in the supply chain and undermine the performance of the entire supply chain by lowering the supplier's production level. Then, coordination mechanisms are proposed and analysed for improving the efficiency of the supply chain. The study aims to demonstrate how the allocation of inventory risk influences a supply chain's performance and its division of profit under supply uncertainty. In particular, we attempt to answer the following questions: (1) How do the supply uncertainty and power structure affect the optimal equilibrium decisions of supply chain members? (2) Can the classic coordination mechanisms (revenue sharing and buyback) coordinate a supply chain with uncertain supply under different power structures? (3) Are there any other contracts that can coordinate the supply chain if the classic mechanisms do not work? To answer these questions, a Stackelberg model is adopted to investigate the decisions under pull and push contracts. Compared to the related papers, we delve into the effects of random supply and inventory risk under different power structures, where the leader in the game can shift the inventory risk to the follower.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. The problem description and some essential assumptions are presented in Section 3, which also takes the centralized supply chain as the base model. In Section 4, we analyse the optimal order quantity and wholesale price decisions of the retailer and supplier under pull and push contracts, respectively. A pull contract with revenue sharing and risk-pooling and a push contract with buyback are considered in Section 5. Numerical examples are provided in Section 6 to illustrate the results of the proposed models and Section 7 extends the problem to random demand. Finally, Section 8 summarizes the conclusions.

2. Literature review

Our work is closely related to three main streams of operations management literature: supply uncertainty, inventory risk allocation, and coordination mechanism design. In this section, we review the relevant research in each stream.

Most of the papers considering supply uncertainty focus on the production planning problems of uncertain production capacity, random manufacturing yield, or unreliable suppliers. Lowe and Preckel [29] point out that supply uncertainty can significantly thin the margins in perishable goods, such as the fresh food supply chain since the quality and available quantity of products usually decrease with time. Motivated by the fact that the fresh food quantity is highly dependent on the short term planning in the harvest season, Ahumada and Villalobos [1] present an operational model to handle the effect of labour costs and transportation modes on uncertain crops yield. Shen *et al.* [33] study the inventory replenishment model for perishable agricultural products and demonstrate that the supply chain cost decreases with collaborative forecasting by the supplier and retailer. Taking into account the supply uncertainty caused by the deterioration of perishable products, such as medicine and foods, Ferreira *et al.* [9] develop a continuous aid inventory management model by using the Markov Decision Process. In addition to perishable products, random yields are also a concern in the manufacturing industry. Most studies investigate the impact of an uncertain supply on decisions. Gurnani and Gerchak [19] consider an assembler that procures components from two suppliers, and investigate the assembler's order decision and two suppliers' production decisions in the presence of random component yields. Pan and So [30] and Li *et al.* [26] analyse the interactions among the assembler and suppliers in their procurement decisions in a random yield production system. He *et al.* [20] study the impacts of decision sequences on a random yield supply chain with a service level constraint. Different methods are used in the literature to describe supply randomness; two of the most widely used forms are the additive and multiplicative forms. Li *et al.* [24] explore the double marginalization effects caused by considering an uncertain supply that is formulated by both additive and multiplicative yield models. Peng *et al.* [31] assume that the final production quantity can be expressed as random variable multiplies by the ordering quantity. Without loss of generality, we use the multiplicative yield model to depict a proportional random supply in this paper. Demand is always uncertain in practice and there are plenty of studies jointly considering random supply and random demand in supply chain management. Kazaz [22] develops a single-period two-stage decision-making problem under random yield and uncertain demand to determine the optimal production quantity, as well as the optimal resource order. Fu *et al.* [10] consider a decentralized system with unreliable suppliers and an assembler who faces random demand, and they use a Bernoulli model to describe the stochastic yield of suppliers. Fuller *et al.* [12] study a more general distribution of supply uncertainty by introducing a multi-echelon supply system for the oil industry value chain, which covers the process from crude oil input through to product output. The actives, such as transportation and storage, may lead to an uncertain supply. Incorporating the random demand, an agent-based modelling and simulation is proposed to obtain profit from a complex value chain. Giri *et al.* [13] consider a three-layer supply chain with stochastic market demand and random yield from a raw-material supplier and manufacturer and design contracts to coordinate the supply chain. However, the allocation of inventory risk is not discussed in the above literatures. In this paper, we will extend our proposed model by introducing stochastic demand and investigating the allocation of the inventory risk mainly caused by the mismatch between supply and demand.

Our study is also related to the literature on inventory risk allocation. Since the seminal work of Cachon [3], pull and push contracts are widely used for allocating inventory risk in supply chain management. With a push contract, all inventory risk is pushed onto the retailer. With a pull contract, the retailer pulls inventory from the supplier, thereby leaving the supplier with all the inventory risk. VMI (the supplier decides how much inventory to stock at the retailer and owns that inventory) or drop shipping (the supplier holds the inventory and ships directly to consumers, bypassing the retailer) are two situations that can be represented by a pull contract. Cachon [3] studies how the allocation of inventory risk impacts supply chain efficiency under uncertain demand. Subsequently, Granot and Yin [15] allow suppliers to form coalitions and incorporate push and pull contracts into a random demand assembly system in which equilibrium pricing and production/procurement strategies are investigated. Prasad *et al.* [32] examine the advance selling price and inventory decisions in a two-period setting to control the inventory risk caused by uncertain demand. The above studies mainly concern the pricing and inventory problems with random demand under pull and push contracts. Fang *et al.* [7] study a push assembly system with multiple suppliers, simultaneous supplier contracts, and a wholesale price contract. Pan and So [30] make pricing and production decisions in an assembly system with uncertain supply under a pull contract. Other factors are also considered in the literatures related to pull and push contracts, such as the presence of an outside market [14], the impact of a hybrid push-pull contract [16], a supply chain risk-averse attitude [41], and different lead times for products [42].

Coordination is an important goal in supply chain management, and an extensive body of literature has focused on the coordination of supply chains by adopting different types of contracts. Cachon and Lariviere [4] study revenue-sharing contracts in a general supply chain model with the revenues determined by each retailer's purchase quantity and price. They demonstrate that revenue sharing coordinates a supply chain with a single retailer and arbitrarily allocates the supply chain's profit. Gurnani and Gerchak [19] design an additional penalty contract to coordinate the members of the decentralized assembly system. Wang and Chen [39] combine option contracts with supply chain risk management to investigate the management decisions in the fresh produce supply chain. Kalkancı and Erhun [21] investigate price-only contracts and complex payment contracts in an assembly supply chain under symmetric and asymmetric information. Guan *et al.* [17] study a time-based payment contract in a decentralized assembly system in which the lead-times of both suppliers are stochastic and independent. Li *et al.* [25] combine a penalty term in writing contracts with the provision of financial assistance, which is the carrot and stick approach used by a manufacturer to deal with supply disruption. We also refer the reader to Cachon [2] and Tang [37] for an excellent survey of this field. There are also some studies that focus on coordination schemes to relieve the impact of an uncertain supply. Yan *et al.* [40] introduce a positive salvage value to extend the work of Gurnani and Gerchak [19] and derive a new coordination scheme. Güler [18] proposes four contracts to study the coordination of assembly systems with random yield and random demand. Li *et al.* [27] mitigate supply uncertainty using supply diversification and responsive pricing strategies. Lin and Xiao [28] study the credit guarantee scheme used in a supply chain finance system under push and pull contracts. In a supply chain with rush orders, Chintapalli *et al.* [5] examine and demonstrate that an advance-order discounts contract cannot coordinate the supply chain, while a delegation contract coordinates the supply chain and is Pareto-improving.

In view of the existing literature, we find that few of the previous studies consider the impacts of a random supply and inventory risk in a decentralized setting under pull and push contracts, and most of the above studies neglect the inventory holding cost caused by an uncertain supply. The issues related to inventory risk allocation and the coordination mechanism problem under pull and push structures with uncertain supply have not been fully elucidated. Thus, this study investigates the decentralized system with uncertain supply and compares how the allocation of inventory risk impacts the optimal decisions under pull and push contracts. Finally, we attempt to design coordination schemes to improve the performance of the whole supply chain.

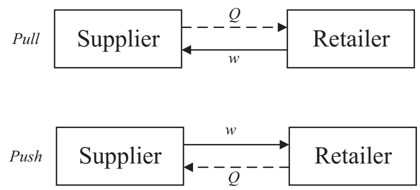


FIGURE 1. Illustration of pull and push contracts.

TABLE 1. Notations.

Q	Production quantity
c	Unit production cost
p	Sale price of retailer
w	Wholesale price
α	Random supply reliability factor with support $[a, b]$
$f(\cdot)$	Probability density function of α
$F(\cdot)$	Cumulative distribution function of α
μ	Mean of α
θ	Standard deviation of α
d	Constant demand
h	Holding cost of unsold product
π_r	Expected payoff of retailer
π_s	Expected payoff of supplier
π_c	Total expected payoff of the supply chain

3. Model description

We consider a supply chain consisting of one supplier and one retailer. The retailer orders a specific product from the supplier and sells it to the market with deterministic demand. The production yield of the supplier is random. In line with the prior literature [19,34,38], we model the supply uncertainty as a proportional random yield, *i.e.*, the final available quantity is assumed to be αQ when the planned production quantity is Q, and α is a random variable representing the supply reliability factor. Both the supplier and the retailer make decisions prior to the selling season, which means there is no opportunity for the retailer to replenish inventory once the products are sold out.

We also assume forced compliance on the side of the supplier. The suppliers can either accept the production quantity to implement the contract or reject the contract without any production [18]. The two contracts and the sequence of events are illustrated in Figure 1.

Notations used in this paper are presented in Table 1.

We focus on the allocation of inventory risk and assume the unsold products will produce a holding cost instead of a salvage value. Actually, the two items can be seen as equivalent in a sense. Our first objective is to determine the impact of an uncertain supply on the ordering or production decisions and expected profits of supply chain members. To this end, we will adopt a Stackelberg model to investigate the equilibrium decisions of the supplier and retailer under two different power contract structures, which are named pull and push contracts. The readers can refer to Fudenberg and Tirole [11] and Lau and Lau [23] for the details of Stackelberg game and its analysis procedure. To be brief, in a Stackelberg game, the players act as either the leader or follower; the leader moves first and then the follower moves sequentially. In our problem, under a pull contract, the retailer acts as the Stackelberg leader, who first determines the unit wholesale price w. Then, the supplier acts as the Stackelberg follower and chooses the production quantity Q before the market demand is realized. Under the pull contract, the supplier has to bear the excessive production risks caused by random yields [3, 8]. By contrast, under a push contract, the supplier has more bargaining power and acts as the Stackelberg leader. In this instance, the supplier first charges the retailer the wholesale price w for the product. The retailer then determines the ordering quantity Q based on the agreed wholesale prices. Because of the uncertainty of supply, at the end of the selling season, there are $\min(\alpha Q, d)$ units of product that are sold. Thus, the retailer will face the risk of overstock. In line with most supply chain models, we assume that the distribution of has the strictly increasing generalized failure rate (IGFR) property, *i.e.*, $xf(x)/\bar{F}(x)$ is increasing with x. Many distributions have the IGFR property, such as the normal, the exponential, the gamma, and the Weibull.

Another concerned issue is whether the additional supply chain contract can coordinate the whole supply chain. To investigate this question, we attempt to introduce three coordination mechanisms, revenue sharing and a buyback commitment to the pull and push cases. In the pull contract with revenue sharing, the retailer agrees to share a portion of the sales revenue with the supplier, and then, the supplier may have incentive to produce more goods. We also propose a new contract based on the classic revenue sharing mechanism for the pull contract, named revenue sharing with a subsidy. In this new contract, in addition to sharing part of the profit with the supplier, the retailer also "shares" part of the supplier's inventory risk to stimulate the supplier to produce more products. Furthermore, the buyback commitment is embedded into the push contract when the supplier has the leader status and promises the retailer that the unsold products can be returned to the supplier at a given price to motivate the retailer to order more products.

4. Centralized and decentralized supply chains

4.1. Centralized system

First, we investigate the quantity decision problem of the centralized supply chain, which performs as a benchmark for the decentralized cases. We use subscript C to denote the centralized situation.

The centralized supply chain aims to maximize the sum of the retailer's profit and the supplier's profit by choosing a moderate production quantity. The quantity of sold product is the minimum of the actual output of products and the demand of the terminal market, $\min(d, \alpha Q)$, therefore, the expected payoff of the supply chain is

$$\pi_c(Q) = pE[\min\{d, \alpha Q\}] - cQ - hE(\alpha Q - d)^+. \tag{4.1}$$

Define $S(Q) = E[\min\{d,\ \alpha Q\}]$, then

$$S(Q) = d\bar{F}(d/Q) + Q\int_a^{d/Q} xf(x)\,\mathrm{d}x, \tag{4.2}$$

$$E(\alpha Q - d)^+ = E(\alpha Q - \min\{d, \alpha Q\}) = \mu Q - S(Q), \tag{4.3}$$

and

$$\begin{aligned}\pi_c(Q) &= pS(Q) - cQ - h(\mu Q - S(Q)) \\ &= (p+h)S(Q) - (c+\mu h)Q,\end{aligned} \tag{4.4}$$

the supply chain faces a quantity decision similar to the newsvendor problem, and we can obtain the following Lemma 4.1.

Lemma 4.1. *There is a unique Q_c^* that maximizes the expected profit of the centralized system, and Q_c^* is increasing with p and decreasing with h.*

See the Appendix A for all the proof.

Under the optimal quantity, we can calculate the maximum expected profit of the total supply chain, as follows:

$$\begin{aligned}\pi_c^*(Q_c^*) &= (p+h)S(Q_c^*) - (c+\mu h)Q_c^* \\ &= (p+h)d\bar{F}(d/Q_c^*).\end{aligned} \tag{4.5}$$

It should be noted that the optimal Q_c^* satisfies $\int_a^{d/Q} xf(x)\,\mathrm{d}x = (c+\mu h)/(p+h)$, and $\int_a^{d/Q} xf(x)\,\mathrm{d}x$ is the actual expected demand of the supply chain. This equation implies that the optimal production quantity Q_c^* allows the actual expected demand to be equal to the critical fractile, which has a similar formula structure to the classic newsvendor problem.

4.2. Pull contract

In the pull contract, the retailer has more bargaining power and acts as the game leader. Within this setting, the retailer first offers a unit wholesale price w, then the supplier chooses a production quantity Q and bears the inventory risk of uncertain production. The retailer only needs to pay the supplier for the amount used to meet the actual demand. This is a classic Stackelberg game, and the supplier's expected profit with a given wholesale price w is as follows:

$$\pi_s(Q|w) = wE[\min\{d, \alpha Q\}] - cQ - hE(\alpha Q - d)^+. \tag{4.6}$$

Similar to the centralized system, we can solve the optimal Q_s^* by Lemma 4.2.

Lemma 4.2. *Under a pull contract with a given wholesale price w, the expected profit $\pi_s(Q|w)$ is concave with respect to the production quantity Q. The optimal production quantity of supplier Q_s^* can be implicitly defined by the following:*

$$Q_s^* = \arg_Q\{(w+h)S'(Q) - (c+\mu h) = 0\}. \tag{4.7}$$

The supplier's expected profit under this production quantity can be written as follows:

$$\begin{aligned}\pi_s^*(Q_s^*, w) &= (w+h)S(Q_s^*) - (c+\mu h)Q_s^* \\ &= (w+h)d\bar{F}(d/(Q_s^*).\end{aligned} \tag{4.8}$$

With the given decision of the supplier in the first stage, the retailer chooses the optimal wholesale price to maximize the expected payoff. Instead of solving the optimal wholesale price directly, we first analyse whether there is a w that can coordinate the supply chain, which means that $Q_s^* = Q_r^* = Q_c^*$, where Q_r^* can be considered as the optimal quantity that the retailer wants the supplier to choose.

Define $G(w, Q_s^*) = (w+h)S'(Q) - (c+h)$, then

$$\begin{aligned}\frac{\partial Q_s^*}{\partial w} &= -\frac{\partial G(w,Q_s^*)}{\partial w}\Big/\frac{\partial G(w,Q_s^*)}{\partial Q_s^*} = \frac{Q_s^{*3}S'(Q)}{(w+h)d^2 f(d/Q_s^*)} > 0, \\ \frac{\partial Q_s^*}{\partial h} &= -\frac{\partial G(w,Q_s^*)}{\partial h}\Big/\frac{\partial G(w,Q_s^*)}{\partial Q_s^*} = \frac{Q_s^{*3}(S'(Q)-\mu)}{(w+h)d^2 f(d/Q_s^*)} < 0.\end{aligned}$$

Therefore, we further have the following conclusion:

Corollary 4.3. *Q_s^* is increasing with w and decreasing with h.*

The result is intuitive that with a higher wholesale, the supplier will choose to provide a larger quantity of product to obtain more profit. Additionally, with the increase of the holding cost h, it is evident that the supplier will reduce the production to avoid the inventory risk. Based on the monotonicity of Q_s^* with respect to w, we can also conclude that for any $w < p$, there is $Q_s^* < Q_c^*$.

Thus, it is obvious that there is a one-to-one mapping between the production quantity Q_s^* and the wholesale price w. We can use equation (4.7) to solve w in terms of Q_s^*, as follows:

$$w(Q_s^*) = \frac{c + \mu h}{S'(Q_s^*)} - h. \tag{4.9}$$

Plugging equation (4.9) into equation (4.8) and write the supplier's profit in terms of Q_s^* as follows:

$$\pi_s(Q_s^*) = \frac{c + \mu h}{S'(Q_s^*)} S(Q_s^*) - (c + \mu h)Q_s^*, \tag{4.10}$$

$$\frac{\partial \pi_s(Q_s^*)}{\partial Q_s^*} = -(c + \mu h)\frac{S(Q_s^*)S''(Q_s^*)}{(S'(Q_s^*))^2} > 0. \tag{4.11}$$

The supplier's profit is increasing in Q_s^*.

Given a wholesale price, the expected profit of the retailer is as follows:

$$\pi_r(Q) = (p - w)E[\min\{d, \alpha Q\}] = (p - w)S(Q). \tag{4.12}$$

It is obvious that given a wholesale price w, $\pi_r(Q)$ is a concave increasing function with Q. The retailer will not accept any wholesale price where $w \geq p$, hence $Q_s^* < Q_c^*$ is held.

However, which w would the retailer choose to maximize his profit? To choose w, the retailer must anticipate the quantity Q that the supplier will choose for each value of w. In other words, the retailer can entice the supplier to choose whatever Q he wishes by selecting the unique wholesale price, call it $w(Q)$, that makes Q optimal for the supplier. In particular, following equation (4.4), the retailer's profit is now a function of Q and the corresponding $w(Q)$, as follows:

$$\begin{aligned}\pi_r(Q) &= (p - w)E[\min\{d, \alpha Q\}] \\ &= (p - (c + \mu h)/S'(Q) + h)S(Q),\end{aligned} \tag{4.13}$$

and the first-order of $\pi_r(Q)$ with Q is

$$\begin{aligned}\frac{\partial \pi_r(Q)}{\partial Q} &= \left(p - \frac{c + \mu h}{S'(Q)} + h\right) S'(Q) + (c + \mu h)S(Q)\frac{S''(Q)}{(S'(Q))^2} \\ &= (p + h)S'(Q) - (c + \mu h)\left(1 - \frac{S(Q)S''(Q)}{(S'(Q))^2}\right).\end{aligned} \tag{4.14}$$

If $\pi_r(Q)$ is concave with respect to Q, then there is a unique Q to maximize $\pi_r(Q)$. Define $s(x) = E[\min\{\alpha, x\}]$, thus $S(Q) = Qs(d/Q)$, and

$$\frac{S(Q)S''(Q)}{(S'(Q))^2} = -\frac{d^2 s(d/Q)f(d/Q)}{Q^2(s(d/Q) - d/Q\bar{F}(d/Q))^2}.$$

To derive the unique and optimal condition, the following Lemma 4.4 is needed for many of the subsequent results.

Lemma 4.4. *Let* $A(x) = s(x)/(s(x)/x - \bar{F}(x))$, $B(x) = f(x)/(s(x)/x - \bar{F}(x))$, $C(x) = xB(x)$, *and* $H(x) = A(x)B(x)$. *If* $C(x)$ *is decreasing with* x, *then* $H(x)$ *is decreasing in* x.

Using the conclusion of Lemma 4.4, we have:

Proposition 4.5. *The retailer's profit with a pull contract,* $\pi_r(Q)$, *is concave with* Q *and there is a unique optimal order quantity* Q *if* $C(x)$ *is decreasing with* x. *Moreover, the optimal* Q_r^* *satisfies*

$$(p + h)S'(Q) = (c + \mu h)\left(1 - \frac{S(Q)S''(Q)}{(S'(Q))^2}\right). \tag{4.15}$$

As mentioned previously, in most supply chain management studies, the distribution of the random variable is usually assumed to have the strictly increasing generalized failure rate(IGFR) property [3, 35]. In this study, in addition to the IGFR property, we also assume that $C(x)$ is strictly decreasing with respect to x. This is a tighter condition than IGFR as the above proof's explanation. Fortunately, through the theory proof and numerical computing, we find that some common distributions have the strictly decreasing $C(x)$, such as the normal, the gamma and the lognormal distributions.

Denote Q_r^* as the retailer's most preferred quantity under a pull contract. If the retailer was able to choose any wholesale price in the pull mode, then the retailer would choose $w(Q_r^*)$. Since $S''(Q) < 0$, then we can conclude that $S'(Q_r^*) > (c + \mu h)/(p + h)$, and under this pull contract, we cannot find a wholesale price w so that $Q_r^* = Q_s^* = Q_c^*$.

Let Q_{pull}^* be the equilibrium solution under the pull contract and π_{pull}^* be the corresponding profit of the whole supply chain, Theorem 4.6 compares the optimal equilibrium solutions under the pull contract and the centralized system.

Theorem 4.6. *Under the pull contract, there is (1) $Q_{\text{pull}}^* = Q_r^* < Q_c^*$, and (2) $\pi_{\text{pull}}^* < \pi_c^*$.*

This result reflects the well-known double marginalization effect that occurs in a decentralized system. In a general case, the effect usually occurs because of a higher selling price set by the retailer or a smaller order quantity submitted from the retailer. However, in a supply chain with an uncertain supply and a more powerful retailer, compared with the optimal decision in the centralized case, the double marginalization in our model is the lower production level as shown by Theorem 4.6 and the resulting insufficient supply (see [24]).

4.3. Push contract

Under a push contract, the supplier has more bargaining power than the retailer. As a Stackelberg leader, the supplier first determines the wholesale price w. Then the retailer, who plays as the follower, determines the order quantity Q. In this setting, the retailer bears the risk of surplus inventory. Similar to the analysis in the pull contract, the retailer decides order quantity Q to maximize his expected payoff with the given wholesale price w, as follows:

$$\pi_r(Q|w) = pE[\min\{d, \alpha Q\}] - wE(\alpha Q) - hE(\alpha Q - d)^+. \tag{4.16}$$

Lemma 4.7. *Under a push contract, $\pi_r(Q|w)$ is concave in Q with a given w, and the optimal Q_r^* is decreasing with respect to w.*

We can derive the retailer's optimal expected profit as follows:

$$\begin{aligned} \pi_r(Q_r^*, w) &= (p + h)\left(Q_r^* S'(Q_r^*) + d\bar{F}(d/Q_r^*)\right) - \mu(w + h)Q_r^* \\ &= (p + h)d\bar{F}(d/Q_r^*). \end{aligned} \tag{4.17}$$

Since $\partial\pi_r(Q_r^*, w)/\partial Q_r^* = df(d/Q_r^*)/Q_r^{*2} > 0$, it is obvious that the retailer's optimal expected profit is increasing with respect to Q_r^*. According to the decreasing property of $S(Q)$, we obtain the following conclusion.

Corollary 4.8. *If $\mu w > c$, then $Q_c^* > Q_r^*$.*

Then, we turn to the decision of supplier. It is clear that if $w = c/\mu$, then $Q_c^* = Q_r^*$, *i.e.*, under this wholesale price, the retailer will choose the same ordering quantity with a centralized supply chain. However, notice that the expected profit of the supplier equals to $(\mu w - c)Q_r^*$, which has a value of zero if we set the wholesale price to c/μ. Thus, the supplier will not accept this contract. Corollary 4.8 also points out that if the supplier earns a positive profit, which means $\mu w > c$, the retailer will choose a lower ordering level to reduce the risk of a stock overage. According to Lemma 4.7, Q_r^* is decreasing with w, so there is also a one-to-one relationship between the production quantity Q_r^* and the wholesale price w. We can rewrite w in terms of Q_r^* as follows:

$$w = \frac{(p + h)S'(Q_r^*)}{\mu} - h. \tag{4.18}$$

In the first stage, the supplier anticipates the response of the retailer, then determines the optimal wholesale price w by maximizing his expected payoff as follows:

$$\max \pi_s(w, Q) = \mu w Q - cQ. \tag{4.19}$$

Furthermore, plugging equation (4.18) into equation (4.19), $\pi_s(w, Q)$ can also be formulated by Q as follows:

$$\pi_s(Q) = (p + h)S'(Q)Q - (\mu h + c)Q, \tag{4.20}$$

Proposition 4.9. *For a push contract, $\pi_s(Q)$ is concave with Q and there is a unique optimal order quantity Q and Q_s^* satisfies the following:*

$$(p + h)(S'(Q) + S''(Q)Q) - (\mu h + c) = 0. \tag{4.21}$$

Let Q^*_{push} be the equilibrium solution under a push contract, and π^*_{push} be the corresponding profit of the whole supply chain; similar to the pull case, we have the following theorem.

Theorem 4.10. *Under a push contract, (1) $Q^*_{\text{push}} = Q_s^* < Q_c^*$, (2) $\pi^*_{\text{push}} = \pi_c(Q^*_{\text{push}}) < \pi_c^*$.*

In Theorem 4.10, Q^*_{push} is the ordering quantity that the supplier who faces an uncertain yield most prefers under a push contract. If the supplier were able to choose any wholesale price in the push mode, then the supplier would choose $w(Q^*_{\text{push}})$. Additionally, in this instance, here the double marginalization occurs as a result of the smaller order quantity submitted from the retailer, which is in line with the traditional one.

The following Theorem 4.11 compares the performance of the supply chain with pull and push contracts and indicates that the pull contract is more attractive to the supply chain relative to the push contract.

Theorem 4.11. *Under a decentralize system, the retailer's maximum expected profit with a pull contract is greater than the supplier's expected profit with a push contract, $\pi_r^{\text{pull}}(Q^*_{\text{pull}}) > \pi_s^{\text{push}}(Q^*_{\text{push}})$, the inventory is greater, $Q^*_{\text{pull}} > Q^*_{\text{push}}$, and the total supply chain's profit is higher, $\pi_c(Q^*_{\text{pull}}) > \pi_c(Q^*_{\text{push}})$.*

5. Coordinate mechanism

As the competition is intensified in the market, many companies realize that the performance of their businesses highly depend on the collaboration and coordination across the supply chain. Unfortunately, the chain members are primarily concerned about their individual interests and that self-serving focus often results in poor supply chain performance. In this section, based on pull and push contracts, we propose some contract mechanisms to coordinate the supply chain.

5.1. Pull contract with revenue sharing

To motivate the supplier to provide more product, we assume that the retailer shares a portion of the sales revenue with the supplier, and we introduce β to represent the sharing ratio.

Following the above analysis steps, in the second stage, the supplier decides his production quantity with the given wholesale price w of the retailer, and the expected payoff can be displayed as follows:

$$\hat{\pi}_s(Q|w) = wE[\min\{d, \alpha Q\}] + \beta(p - w)E[\min\{d, \alpha Q\}] - cQ - hE(\alpha Q - d)^+, \tag{5.1}$$

where the first term is the revenue of the satisfied demand, the second term is the sharing revenue from the retailer, the third term is the production cost, and the last term is the surplus value of over-production. According to the previous notation, we can rewrite $\pi_s(Q|w)$ as follows:

$$\hat{\pi}_s(Q|w) = [\beta p + (1 - \beta)w + h]S(Q) - (c + \mu h)Q. \tag{5.2}$$

Lemma 5.1. *Under a pull contract and given wholesale price w, the expected profit of supply $\pi_s(Q|w)$ is concave with respect to production quantity Q. The optimal quantity Q_s^* can be obtained by the following equation:*

$$\hat{Q}_s^* = \arg_Q\{[\beta p + (1-\beta)w + h]S'(Q) - (c + \mu h) = 0\}. \tag{5.3}$$

Then the supplier's optimal production quantity satisfies the following:

$$S'(\hat{Q}_s^*) = \frac{c + \mu h}{\beta p + (1-\beta)w + h}. \tag{5.4}$$

Compared with supplier's choice under the pull contract without revenue sharing, $S'(Q_s^*) = (c+\mu h)/(w+h)$, we have $\hat{Q}_s^* > Q_s^*$ if $\beta > 0$ and $p > w$. This finding indicates that with the revenue sharing contract, the supplier will choose to produce more products since he can earn part of the profit of the retailer, and this part of the revenue can neutralize a portion of the inventory risk. From the perspective of the supplier, if $\hat{Q}_s^* = Q_c^*$, then the wholesale price should satisfy $\beta p + (1-\beta)w = p$, which implies $p = w$. However, the retailer's expected profit is zero under this wholesale price. Therefore, the revenue sharing mechanism cannot coordinate the supply chain under a pull contract.

At this step, we investigate whether the revenue sharing contract can improve the supply chain's profit or not. Similar to the previous analysis, we can show that $\partial\hat{Q}_s^*/\partial w > 0$, and there is a one-to-one map relationship between the wholesale price and the supplier's optimal production level, that is, as follows:

$$w(Q) = (c + \mu h)/((1-\beta)S'(Q)) - (\beta p + h)/(1-\beta). \tag{5.5}$$

Plugging $w(Q)$ into the retailer's profit function, we have the following:

$$\begin{aligned}\hat{\pi}_r(Q) &= (1-\beta)(p-w)E[\min\{d, \alpha Q\}] \\ &= \left(p + h - \frac{c+\mu h}{S'(Q)}\right) S(Q).\end{aligned} \tag{5.6}$$

Under the pull contract, the retailer's expected profit with revenue sharing has the same form when there is no revenue sharing. Therefore, the Q that the retailer prefers is also equal to the result in Section 4.2, and the total expected profit of the whole supply chain does is not improved by introducing the revenue sharing contract.

5.2. Pull contract with revenue sharing and inventory risk subsidy

To coordinate the supply chain based on a pull contract, we propose a new contract based on the classic revenue sharing mechanism for the pull contract. In this new contract, apart from sharing part of the profit with the supplier, the retailer also "shares" part of the supplier's inventory risk to stimulate the supplier to produce more products. In this section, we introduce a subsidy provided by retailer for the excess inventory of supplier and use superscript "Sub" to denote the case with inventory risk subsidy. Under this contract, the expected profit of the supplier and retailer can be written as follows:

$$\hat{\pi}_s^{\text{Sub}}(Q|w) = (w + \beta(p-w) + h - r)S(Q) - (c + \mu(h-r))Q. \tag{5.7}$$

$$\hat{\pi}_r^{\text{Sub}}(Q, w) = ((1-\beta)(p-w) + r)S(Q) - \mu r Q. \tag{5.8}$$

In line with revenue sharing contract, β is the sharing portion of the retailer, and r denotes the compensation from the retailer to the supplier for the excess inventory; to guarantee the validation, assume that $r < h$.

Lemma 5.2. *Given the wholesale price w, both $\hat{\pi}_s^{\text{Sub}}(Q, w)$and $\hat{\pi}_r^{\text{Sub}}(Q, w)$ are concave in Q.*

Define

$$w(r,\beta) = p - \frac{\mu p - c}{(1-\beta)(c+\mu h)} r, \tag{5.9}$$

the following Theorem 5.3 states that the current mechanism can coordinate the whole supply chain.

Theorem 5.3. *For any $0 \le r \le h + c/\mu$, if wsatisfies equation (5.9), then $\hat{Q}_c^* = \hat{Q}_s^{\text{Sub}*} = \hat{Q}_r^{\text{Sub}*}$.*

Let $\lambda = (c + \mu(h - r))/(c + \mu h)$, then we have:

$$\lambda = 1 - \mu r/(c + \mu h) = (\beta p + (1-\beta) w(r,\beta) + h - r)/(p + h), \tag{5.10}$$

the supplier's profit is

$$\begin{aligned} \hat{\pi}_s^{\text{Sub}}(Q, w, r) &= (w + \beta(p - w) + h - r) S(Q) - (c + \mu(h - r))Q \\ &= \lambda(p + h) S(Q) - \lambda(c + \mu h) Q \\ &= \lambda \pi_c(Q), \end{aligned} \tag{5.11}$$

where $\pi_c(Q)$ is the total supply chain profit. Since $\lambda \ge 0$, it is obvious that $\hat{\pi}_s(Q, w, r)$ and $\pi_c(Q)$ can research the maximum values at the same Q. The same argument also applies to the retailer since

$$\begin{aligned} \hat{\pi}_r^{\text{Sub}}(Q, w, r) &= ((1-\beta)(p - w) + r) S(Q) - \mu r Q \\ &= (1-\lambda)(p + h) S(Q) - (1-\lambda)(c + \mu h) Q \\ &= (1-\lambda)\pi_c(Q). \end{aligned} \tag{5.12}$$

As λ increases, the retailer's profit decreases and the supplier's profit increases. Therefore, in a sense, λ represents the division of profit between the players.

Theorem 5.4. *If $w(r)$ is set according to equation (5.9), then the retailer's (supplier's) profit is increasing (decreasing) in $r \in [0, h + c/\mu]$. Moreover,*

(1) *If $0 < r < h + c/\mu$, then the profits are shared by the players.*
(2) *If $r = 0$, which means there is no risk pooling, then the supplier earns the entire supply chain profit.*
(3) *If $r = h + c/\mu$, then the retailer earns the entire supply chain profit.*

5.3. Push contract with buyback

To motivate the retailer to order more products, the supplier can promise retailer that the products can be returned to the supplier at the price h_b. In general, h_b may be smaller than h. In this instance, $h_b > h$ indicates that the supplier gives more of a subsidy to motivate the retailer to order more product. Thus, the expected payoff of the retailer is as follows:

$$\begin{aligned} \tilde{\pi}_r(Q|w) &= pE[\min\{d, \alpha Q\}] - wE[\alpha Q] - (h - h_b)E(\alpha Q - d)^+ \\ &= pS(Q) - \mu w Q - (h - h_b)(\mu Q - S(Q)). \end{aligned} \tag{5.13}$$

Lemma 5.5. *Under a push contract with supplier buyback, $\tilde{\pi}_r(Q|w)$ is concave in Q.*

Recall that under a centralized system, the optimal production quantity Q_c^* is given by equation (4.9). Let

$$w(h_b) = \frac{c}{\mu} + \frac{h_b(\mu p - c)}{\mu(p + h)}, \tag{5.14}$$

if $\mu p > c$, it is clear that $w(h_b)$ is increasing with h_b. We can further obtain the following theorem:

Theorem 5.6. *For any $0 \leq h_b \leq p + h$, if w satisfies equation (5.14), then $Q_c^* = \tilde{Q}_s^* = \tilde{Q}_r^*$.*

Theorem 5.6 establishes that, if the parameters are set appropriately, then the optimal order quantities coincide. Moreover, according to Theorem 5.7 below, there exists such a value of w that both parties earn positive profits. Therefore, the buyback contract coordinates the supply chain.

To analyse this more intuitively, we set

$$\lambda = (p + h - h_b)/(p + h), \tag{5.15}$$

then

$$\lambda = 1 - h_b/(p + h) = \mu(w(h_b) + h - h_b)/(c + \mu h). \tag{5.16}$$

The retailer's profit under a buyback contract is as follows:

$$\begin{aligned} \tilde{\pi}_r(Q, w, h_b) &= pS(Q) - wQ - (h - h_b)(Q - S(Q)) \\ &= (p + h - h_b)S(Q) - \mu(w + h - h_b)Q \\ &= \lambda(p + h)S(Q) - \lambda(c + \mu h)Q \\ &= \lambda \pi_c(Q), \end{aligned} \tag{5.17}$$

where $\pi_c(Q)$ is the total supply chain profit. Since $\lambda \geq 0$, it is obvious that $\tilde{\pi}_r(Q, w, h_b)$ and $\pi_c(Q)$ can reach the maximum values at the same Q. The same argument applies to the supplier since

$$\begin{aligned} \tilde{\pi}_s(Q, w, h_b) &= (\mu w - c - \mu h_b)Q + h_b S(Q) \\ &= -(1 - \lambda)(c + h)Q + (1 - \lambda)(p + h)S(Q) \\ &= (1 - \lambda)\pi_c(Q). \end{aligned} \tag{5.18}$$

As λ increases, the retailer's profit increases and the supplier's profit decreases, so in a sense, λ represents the division of profit between the players.

Theorem 5.7. *If $w(h_b)$ is set according to equation (5.14), then the retailer's (supplier's) profit is decreasing (increasing) in $h_b \in [0, p + h]$. Moreover,*

(1) *If $0 < h_b < p + h$, then the profits are shared by the players.*
(2) *If $h_b = 0$, which means there is no buyback contract, then the retailer earns the entire supply chain profit.*
(3) *If $h_b = p + h$, then the supplier earns the entire supply chain profit.*

As h_b is a payment made to the retailer, it seems unconventional that the supplier's profit is increasing in h_b. However, since $w(h_b)$ is increasing in h_b, the offset by the increased revenue from the wholesale price exceeds the side payment to the retailer by the buyback commitment. One consequence of Theorem 5.7 is that, for a non-coordinating push contract, there exists h_b and $w(h_b)$ such that neither player has a lower profit under the buyback contract, and at least one player has a strictly higher profit. Therefore, the supplier can always choose h_b and $w(h_b)$ values such that both players prefer the buyback contract to the status quo if the supply chain is not currently coordinated.

6. Numerical study

In this section, we perform a series of numerical experiments to generate additional insights and examine the results of the previous analysis. Assume that the supply reliability follows a gamma distribution with mean 1 and standard deviation 0.5. For our base case, we set the production cost of the product to $c = 5$ and the sale price of the end product is $p = 20$. Without loss of generality, the holding cost of surplus product is $h = 3$, and the deterministic demand $d = 100$. Figure 2 displays the expected profit under different contracts with respect to the order quantity Q.

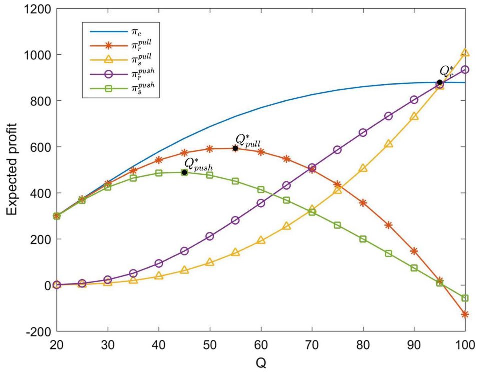
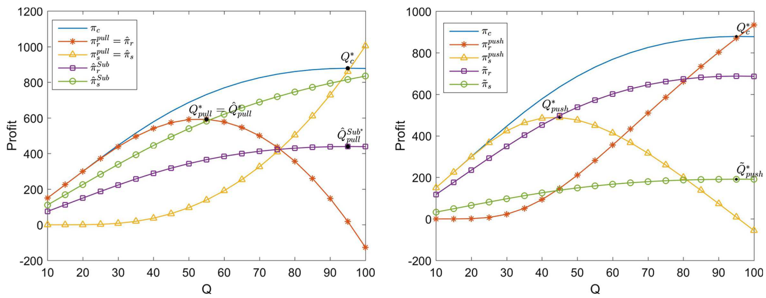


FIGURE 2. Profit under pull and push contract.



(a) (b)

FIGURE 3. Profit with different coordination mechanisms. (a) Pull contract. (b) Push contract.

As displayed in Figure 2, under a push contract, the supplier prefers a lower quantity than the retailer's most preferred quantity with a pull contract. If the quantity of the pull (push) contract equals Q_c^*, *i.e.*, the optimal quantity under the centralized case, the supplier (retailer) earns all the profit of the whole supply chain. This result is intuitive, since when $Q_{\text{pull}} = Q_c^*$, remembering equations (4.9) and (4.15), the wholesale price w should be equal to p. Additionally, if the production quantity exceeds Q_c^*, which implies $w > p$, and the retailer will not obtain a positive profit from selling one product, the corresponding profit is undoubtedly negative, which is why the profit of the centralized system is less than the profit of the supplier under the pull case when $Q > Q_c^*$. The conclusions for the push contract are similar. Additionally, notice that $\pi_c(Q_{\text{pull}}^*) > \pi_c(Q_{\text{push}}^*)$; Figure 2 also indicates that the pull mode is attractive to the supply chain relative to the push mode, which confirms our conclusion presented by Theorem 4.11.

Figure 3 compares the profit of each supply chain member with different coordination mechanisms in pull and push contracts. The figure shows that under a pull contract, the revenue sharing mechanism does not improve the efficiency of the system while the modified revenue sharing mechanism, which introduces a subsidy for the excess inventory can coordinated the supply chain. Moreover, for the push case under a buyback contract, the ordering quantity reaches the level under the centralized system. The managerial insight is that, under the classic revenue sharing coordination contract in the pull case, the entire inventory risk is borne by the supplier, who acts as a follower in the Stackelberg game and has less power than the retailer, who is the leader in the supply chain. However, for the revenue sharing mechanism with a subsidy in the pull case and the buyback contract in the push case, the overstock risk is shared between the supplier and the retailer, which means that the leader in the Stackelberg game also bears part of inventory risk. Thus, we can find that the key issue that influences the efficiency of the supply chain is the allocation of inventory risk.

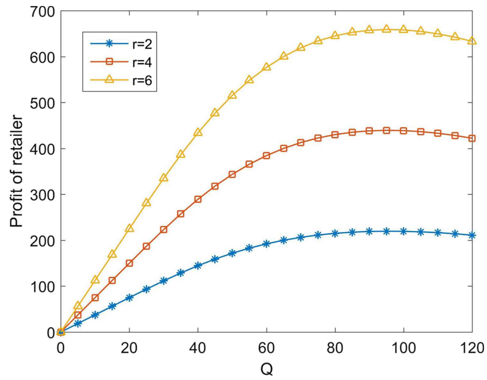


FIGURE 4. Impact of the subsidy price on the retailer's profit under a pull contract.

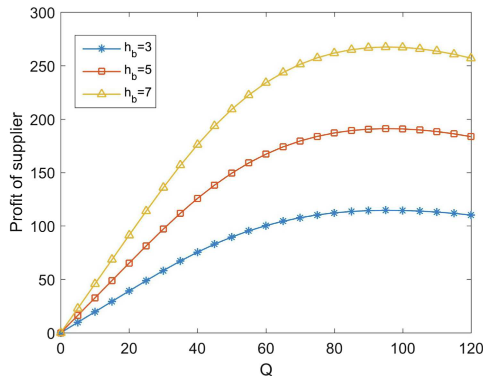


FIGURE 5. Impact of the buyback price on the supplier's profit under a push contract.

Figure 4 shows that with the increase of the subsidy for excess inventory, the retailer's profit will also increase. When $r = c/\mu + h$, the retailer shares all the profit of the entire supply chain. This numerical result is consistent with equation (5.12). Observing Figure 5, we can find that under the push case, the supplier's expected profit is also increasing with the buyback price h_b and the supplier obtains all the profit of the supply chain if $h_b = p + h$, which verifies the conclusion of Theorem 5.7.

Figures 6 and 7 present how the uncertainty of supply affects the profit of the whole supply chain and the final ordering quantity. If the random supply has a higher uncertainty, the leader in the Stackelberg game will prefer a lower inventory level to reduce the cost resulting from an overstock. Additionally, it is intuitive that the expected profit of the whole supply chain will decrease with the increase of supply uncertainty, which is why most decision makers do not want a high degree of uncertainty. Therefore, it is necessary for supply chain members to adopt some measures to reduce the uncertainty both in supply and demand, or take the advantage of historical data to forecast the demand or supply as accurately as possible, to shrink the mismatch between supply and demand.

7. Extension to random demand

We now turn our attention to the case in which the retailer faces random demand D, which is independent of supply randomness. Assume that demand D is random with CDF $G(\cdot)$ and PDF $g(\cdot)$, and use the superscript "rand" to denote the randomness of demand; then, for the centralized system, the realized profit for the supply

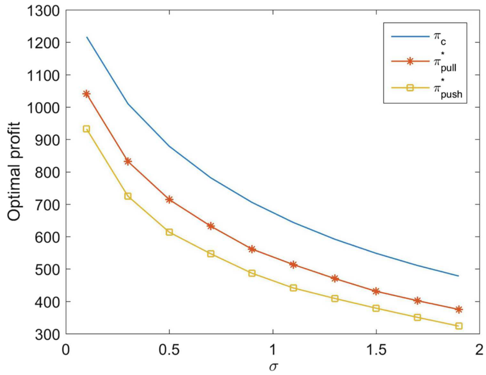


FIGURE 6. Profit of the supply chain with respect to σ.

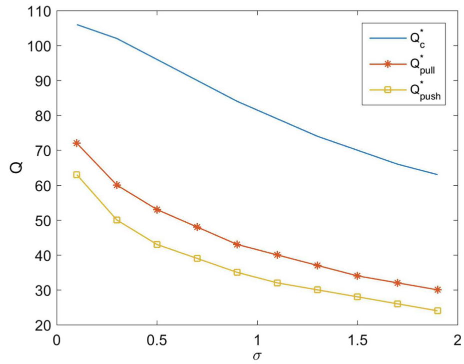


FIGURE 7. Optimal quantity with respect to σ.

chain can be written as follows:

$$\tilde{\pi}_c^{\mathrm{rand}} = \begin{cases} \alpha pQ - cQ, & \text{if } D > \alpha Q \\ pD - cQ - h(\alpha Q - D), & \text{if } D \leq \alpha Q. \end{cases}$$

The expected payoff of the supply chain is as follows:

$$\begin{aligned} \pi_c^{\mathrm{rand}}(Q) &= \int_a^b E[\tilde{\pi}_c|\alpha = x]f(x)\,\mathrm{d}x \\ &= \int_a^b f(x)\int_0^{xQ}[py - cQ - h(xQ - y)]g(y)\,\mathrm{d}y\,\mathrm{d}x + \int_a^b f(x)\int_{xQ}^{\infty}(xpQ - cQ)g(y)\,\mathrm{d}y\,\mathrm{d}x \\ &= -cQ + \int_a^b f(x)\int_0^{xQ}[py - h(xQ - y)]g(y)\,\mathrm{d}y\,\mathrm{d}x + \int_a^b f(x)\int_{xQ}^{\infty} xpQg(y)\,\mathrm{d}y\,\mathrm{d}x. \end{aligned} \tag{7.1}$$

Differentiating $\pi_c^{\mathrm{rand}}(Q)$ with respect to Q yields the following:

$$\frac{\partial \pi_c^{\mathrm{rand}}(Q)}{\partial Q} = -c + p\mu - (p + h)\int_a^b xf(x)G(xQ)\,\mathrm{d}x.$$

It is clear that $\partial^2 \pi_c^{\mathrm{rand}}(Q)/\partial Q^2 < 0$, then we obtain the following conclusion:

Proposition 7.1. *(1) The expected profit $\pi_c^{\text{rand}}(Q)$ is concave in Q, and there is a unique $Q_c^{\text{rand}*}$ that maximizes the expected profit of the centralized supply chain; (2) $Q_c^{\text{rand}*}$ satisfies $\int_a^b f(x)xG(xQ_c^{\text{rand}*})\,\mathrm{d}x = (p\mu - c)/(p+h)$.*

In line with the deterministic case, we define $R(Q) = E[\min\{\alpha Q, D\}]$, and the conclusion in Proposition 7.1 is equivalent to

$$R'(Q_c^{\text{rand}*}) = \frac{\mu h + c}{p+h}, \tag{7.2}$$

where $R'(Q) = \int_a^b xf(x)\bar{G}(xQ)\,\mathrm{d}x$. Proposition 7.1 illustrates the uniqueness of the optimal production level for the centralized supply chain under uncertain supply and random demand. We can also show that $Q_c^{\text{rand}*}$ is increasing in p and decreasing in h. The results are similar to the case of deterministic demand.

For the decentralized case, we will discuss the problem under a pull contract. The retailer has more bargaining power and first offers a wholesale price w and then the supplier chooses a production level Q. The supplier's expected profit is as follows:

$$\pi_s^{\text{rand}}(Q|w) = wE[\min\{D, \alpha Q\}] - cQ - hE(\alpha Q - D)^+, \tag{7.3}$$

Given the wholesale price w, we can show that the expected profit of the supplier $\pi_s^{\text{rand}}(Q|w)$ is concave with respect to Q; thus, the optimal production quantity of the supplier can be obtained by solving the first-order-condition of $\pi_s^{\text{rand}}(Q|w)$. The optimal $Q_s^{\text{rand}*}$ satisfies

$$\int_a^b xf(x)G(xQ_s^{\text{rand}*})\,\mathrm{d}x = \mu - R'(Q_s^{\text{rand}*}) = \frac{w\mu - c}{w+h},$$

and we can solve w in terms of $Q_s^{\text{rand}*}$ as follows:

$$w(Q_s^{\text{rand}*}) = \frac{c+\mu h}{R'(Q_s^{\text{rand}*})} - h.$$

The retailer's expected profit can be expressed as a function of Q:

$$\begin{aligned}\pi_r^{\text{rand}}(Q) &= (p-w)E[\min\{d, \alpha Q\}] \\ &= \left(p - \frac{c+\mu h}{R'(Q)} + h\right) R(Q).\end{aligned} \tag{7.4}$$

Proposition 7.2. *If $\pi_r^{\text{rand}}(Q)$ is quasi-concave with respect to Q, then there is a unique $Q_{\text{pull}}^{\text{rand}*}$ to maximize $\pi_r^{\text{rand}}(Q)$, and $Q_{\text{pull}}^{\text{rand}*} < Q_c^{\text{rand}*}$.*

Note that

$$\frac{\partial \pi_r^{\text{rand}}(Q)}{\partial Q} = (p+h)R'(Q) - (c+\mu h)\left(1 - \frac{R(Q)R''(Q)}{(R'(Q))^2}\right),$$

and the concavity of $\pi_r^{\text{rand}}(Q)$ is equivalent to the monotonic decrease in $\partial\pi_r^{\text{rand}}(Q)/\partial Q$. However, the characteristics of $\pi_r^{\text{rand}}(Q)$ depend not only on the distribution of α but also on the distribution of the stochastic demand d; thus, it is difficult to theoretically prove the concavity of $\pi_r^{\text{rand}}(Q)$. Once we can demonstrate the concavity of $\pi_r^{\text{rand}}(Q)$, according to our above analysis, the properties of the optimal solution should be similar those to when demand is deterministic. For the sake of simplicity, we only present the theoretical analysis under the assumption that demand is known and a constant. The extensive numerical experiments also show that the concavity of $\pi_r^{\text{rand}}(Q)$ holds for many common distributions of random demand D, such as normal, lognormal and gamma distributions.

Additionally, we can easily find that the solutions in the random demand case have the same structure as those in the deterministic demand case. For the push case and the design of the coordination mechanism, the analysis is also analogous, so we omit it for brevity.

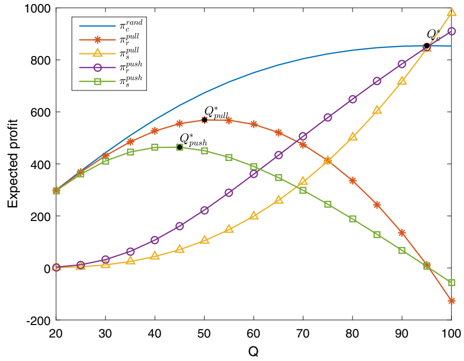


FIGURE 8. Optimal quantity with respect to σ.

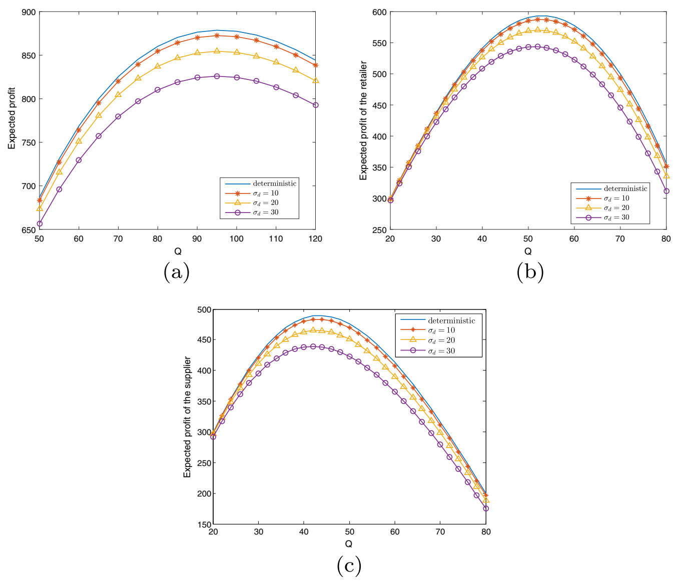


FIGURE 9. Expected profit with respect to the demand uncertainty. (a) Centralized system. (b) Pull contract. (c) Push contract.

Figure 8 provides the corresponding expected profits when demand is random and follows a gamma distribution with a mean of 100 and a standard deviation of 20. The remaining parameters are the same as in the base case in Section 6. We find that the result is very similar to Figure 2, in which the demand is deterministic. We can then conclude that once we can show the quasi-concavity of the retailer's expected profit under a pull contract $\pi_r^{\text{pull}}(Q)$ and the supplier's expected profit under a push contract $\pi_s^{\text{push}}(Q)$, the results have similar properties to the deterministic demand case.

Figure 9 shows how the uncertainty of demand affects the expected profit and the ordering quantity in different cases. It is intuitive that demand uncertainty reduces the expected profit of the whole supply chain under a centralized system, as well as the expected profit of the game leader under pull and push contracts. The result also indicates the double marginalization effect under pull and push contracts, since the optimal ordering/production quantity is less than that under the centralized system. From Figures 9b and 9c, we find that compared to the push contract, the pull contract is more attractive for the supply chain since the equilibrium quantity under the pull contract is larger than that under the push contract. This conclusion is also in line with the results of the analysis when demand is deterministic.

8. Conclusion

This paper studies a supply chain with a random supply and investigates the impact of an uncertain supply on the equilibrium order quantity and wholesale price in a decentralized system. From the perspective of inventory risk allocation, we consider two basic contracts named pull and push contracts, which have opposite decision sequence and power structures between the supplier and retailer. Under the pull contract, the retailer first determines the wholesale price and then the suppliers decide the production quantity, and all the inventory risk is undertaken by the supplier. In contrast, under the push contract, the supplier acts as the leader and decides the wholesale price, and subsequently, the retailer determines the production quantity. The inventory risk transfers to the retailer in this case. We find the existence of double marginal effects in decentralized systems, and proof that the equilibrium ordering/production quantities under the two contracts are less than the optimal quantity of a centralized system. We also show that the pull mode is attractive to the supply chain relative to the push mode, since the total profit under the pull contract is higher than that under the push contract. To eliminate double marginal effects, we propose some coordination mechanisms to improve the performance of the supply chain. We discover that the classic revenue-sharing mechanism cannot coordinate the supply chain. To solve this issue, we modified the revenue-sharing mechanism by introducing a subsidy for excess inventory, which can coordinate a supply chain with a pull contract. For the push contract, we illustrate that the buyback commitment can coordinate the supply chain efficiency, in terms of both the total profit and the ordering quantity.

There are some limitations to this study and several potential directions for future work. First, we suppose the demand is deterministic in our model, which can be extend to stochastic situation. Second, other coordination mechanisms, such as a two-part tariff contract, quantity discount contract, are worth exploring. Third, supply chain with multiple retailers and multiple suppliers, multiple selling periods, information sharing under pull and push contracts are interesting points for future work.

Appendix A.

A.1. Proof of Lemma 4.1

Proof. According to the notations above, it is obvious that

$$\frac{\partial \pi_c}{\partial Q} = (p+h)S'(Q) - (c+\mu h). \tag{A.1}$$

For

$$S'(Q) = \int_a^{d/Q} x f(x)\,\mathrm{d}x = \frac{S(Q)}{Q} - \frac{d}{Q}\bar{F}\left(\frac{d}{Q}\right), \tag{A.2}$$

$$S''(Q) = -\frac{d^2}{Q^3} f\left(\frac{d}{Q}\right) < 0, \tag{A.3}$$

we can obtain the second derivative of π_c with respect to Q as follows:

$$\frac{\partial^2 \pi_c(Q)}{\partial Q^2} = (p+h)S''(Q) = -(p+h)\frac{d^2}{Q^3} f\left(\frac{d}{Q}\right) < 0. \tag{A.4}$$

Thus $\pi_c(Q)$ is concave in Q, and there is a unique Q_c^* that can maximize π_c, where Q_c^* satisfies

$$S'(Q_c^*) = \frac{c+\mu h}{p+h} > 0. \tag{A.5}$$

Define $L(Q,p,h) = (p+h)S'(Q) - (c+\mu h)$, according to the first-order condition of $\pi_c(Q)$, we have

$$\frac{dQ_c^*}{dp} = -\frac{dL(Q_c^*,p,h)}{dp} \Big/ \frac{dL(Q_c^*,p,h)}{dQ_c^*} = \frac{Q^3 S'(Q_c^*)}{(p+h)d^2 f(d/Q_c^*)}$$
$$\frac{dQ_c^*}{dh} = -\frac{dL(Q_c^*,p,h)}{dh} \Big/ \frac{dL(Q_c^*,p,h)}{dQ_c^*} = \frac{Q^3 (S'(Q_c^*) - \mu)}{(p+h)d^2 f(d/Q_c^*)}\cdot$$

According to equation (A.2), $0 < S'(Q_c^*) \le \int_a^b x f(x)\,\mathrm{d}x = \mu$, thus, $dQ_c^*/dp > 0$, $dQ_c^*/dh < 0$, which means that the optimal Q_c^* is increasing with p and decreasing with h. □

A.2. Proof of Lemma 4.2

Proof. Take $S(Q)$ into equation (4.11), and the differential of $\pi_s(Q|w)$ with respect to Q is

$$\frac{\partial \pi_s(Q|w)}{\partial Q} = (w+h)S'(Q) - (c+\mu h), \tag{A.6}$$

the second-order condition is

$$\frac{\partial^2 \pi_s(Q|w)}{\partial Q^2} = -(w+h)\frac{d^2}{Q^3} f\left(\frac{d}{Q}\right) < 0. \tag{A.7}$$

Hence, the supplier's optimal production quantity Q_s^* can be obtained by solving the first-order condition, *i.e.*, Q_s^* satisfies

$$S'(Q_s^*) = \frac{c+\mu h}{w+h}\cdot \tag{A.8}$$

$Q_s^* = \arg_Q\{(w+h)S'(Q) - (c+\mu h) = 0\}$, the proof is complete. □

A.3. Proof of Lemma 4.4

Proof. $C(x)$ can be rewritten as

$$C(x) = -\frac{x f(x)}{1 - F(x) - \frac{s(x)}{x}} = -\frac{\frac{x f(x)}{1-F(x)}}{1 - \frac{s(x)}{x\bar{F}(x)}} = -g(x).$$

Since $C(x)$ is decreasing with x, then $C'(x) < 0$, which means $g(x)$ is increasing.

$$g'(x) = \frac{\left(\frac{x f(x)}{1-F(x)}\right)' \left[1 - \frac{s(x)}{x\bar{F}(x)}\right] + \frac{x f(x)}{1-F(x)} \frac{x s'(x)\bar{F}(x) - s(x)\bar{F}(x) + x s(x) f(x)}{[x\bar{F}(x)]^2}}{\left[1 - \frac{s(x)}{x\bar{F}(x)}\right]^2} > 0.$$

Remember that the distribution of α has an increasing generalized failure rate, and $1 - s(x)/x\bar{F}(x) < 0$, so if $g'(x) > 0$, there must be $x s'(x)\bar{F}(x) - s(x)\bar{F}(x) + x s(x) f(x) > 0$. Then, the first-order derivate of $H(x)$ is

$$\frac{\partial H(x)}{\partial x} = A(x)C'(x) + A'(x)C(x), \tag{A.9}$$

for $C'(x) < 0$ and $A(x), C(x) > 0$, thus, equation (5.1) is negative if $A'(x) < 0$. Additionally,

$$A'(x) = \frac{\bar{F}(x)s(x) - x f(x) s(x) - x\bar{F}^2(x)}{(s(x) - x\bar{F}(x))^2},$$

where $\bar{F}(x) = s'(x)$, so $\bar{F}(x)s(x) - x f(x)s(x) - x\bar{F}^2(x) < 0$ is established. $H(x)$ is decreasing in x. □

A.4. Proof of Proposition 4.5

Proof. $\pi_r(Q)$ is concave in Q if $\partial\pi_r(Q)/\partial Q$ is decreasing in Q. Since $S'(Q)$ is a decreasing function with Q, from equation (5.1), we have that $\pi_r(Q)$ is concave if $S(Q)S''(Q)/(S'(Q))^2$ is decreasing with respect to Q.

According to the definition of $s(x)$ and $H(x)$, $S(Q)S''(Q)/(S'(Q))^2 = -H(d/Q)$. Then the problem to prove $\pi_r(Q)$ is concave with respect to Q is equivalent to show that $H(x)$ is decreasing with x. The conclusion is obvious based on the deduction of Lemma 4.4. □

A.5. Proof of Theorem 4.6

Proof. Under a pull contract, the supplier's optimal production is decided by equation (4.15), thus

$$S'(Q^*_{\text{pull}}) = \frac{c+\mu h}{p+h}\left(1 - \frac{S(Q)S''(Q)}{(S'(Q))^2}\right), \tag{A.10}$$

take the properties that $S''(Q) < 0$ and $S(Q) > 0$, it is obviously that

$$S'(Q^*_{\text{pull}}) > \frac{c+\mu h}{p+h} = S'(Q^*_c).$$

Along with $S''(Q) < 0$, $S'(Q)$ is decreasing with Q, so $Q^*_{\text{pull}} < Q^*_c$.

For the expected profit of the whole supply chain, we have proved in Section 4.1 that π_c is concave in Q, and the optimal profit is obtained at Q^*_c. On the basis of the concavity of π_c and the conclusion $Q^*_{\text{pull}} < Q^*_c$, we have $\pi^*_c = \pi_c(Q^*_c) > \pi_c(Q^*_{\text{pull}}) = \pi^*_{\text{pull}}$. □

A.6. Proof of Lemma 4.7

Proof. The expected profit of retailer $\pi_r(Q|w)$ can be reformulated as follows:

$$\pi_r(Q|w) = (p+h)S(Q) - \mu(w+h)Q, \tag{A.11}$$

the first-order condition with respect to Q is

$$\frac{\partial\pi_r(Q|w)}{\partial Q} = (p+h)S'(Q) - \mu(w+h), \tag{A.12}$$

and the second order condition is

$$\frac{\partial^2\pi_r(Q|w)}{\partial Q^2} = -(p+h)\frac{d^2}{Q^3}f\left(\frac{d}{Q}\right) < 0. \tag{A.13}$$

It is clear that $\partial^2\pi_r(Q|w)/\partial Q^2 < 0$, which indicates that given the wholesale price w, $\pi_r(Q|w)$ is concave with respect to Q, and the optimal ordering quantity of retailer Q^*_r can be obtained by solving $\partial\pi_r(Q|w)/\partial Q = 0$, Q^*_r satisfies the following:

$$S'(Q^*_r) = \frac{\mu(w+h)}{p+h}. \tag{A.14}$$

Let $F(Q,w) = (p+h)S'(Q) - \mu(w+h)$, then

$$\frac{\partial Q^*_r}{\partial w} = -\frac{\partial F(Q^*_r,w)}{\partial w}\Big/\frac{\partial F(Q^*_r,w)}{\partial Q^*_r} = -\frac{\mu Q^{*3}_r}{(p+h)d^2 f(d/Q^*_r)} < 0,$$

Q^*_r is decreasing with respect to w. □

A.7. Proof of Proposition 4.9

Proof. Differentiating $\pi_s(Q)$ with respect to Q,

$$\frac{\partial \pi_s(Q)}{\partial Q} = (p+h)(S'(Q) + S''(Q)Q) - (\mu h + c).$$

We know that $S'(Q)$ is decreasing with Q, so if $S''(Q)Q/S'(Q)$ is decreasing with Q, then $\pi_s(Q)$ holds the concavity of Q. Since

$$\frac{S''(Q)Q}{S'(Q)} = -\frac{\frac{d^2}{Q^2} f\left(\frac{d}{Q}\right)}{\frac{S(Q)}{Q} - \frac{d}{Q}\bar{F}\left(\frac{d}{Q}\right)} = -\frac{\frac{d^2}{Q^2} f\left(\frac{d}{Q}\right)}{s\left(\frac{d}{Q}\right) - \frac{d}{Q}\bar{F}\left(\frac{d}{Q}\right)}, \tag{A.15}$$

as shown in Lemma 4.4, the right hand of equation (5.15) is exactly $-C(d/Q)$, since $C(x)$ is decreasing with x and d/Q is decreasing with Q, we have $-C(d/Q)$, *i.e.* $S''(Q)Q/S'(Q)$ is decreasing with Q. Similar to the previous proof, under a push contract, the optimal quantity Q_s^* can be solved by using the first-order condition of $\pi_s(Q)$. □

A.8. Proof of Theorem 4.11

Proof. Recall that the retailer's expected profit with a pull contract and the supplier's expected profit with a push contract (Eqs. (4.13) and (4.20)), $\pi_r^{\text{pull}}(Q) > \pi_r^{\text{push}}(Q)$ can be written as follows:

$$\left((p+h) - \frac{c+\mu h}{S'(Q)}\right) d\bar{F}\left(\frac{d}{Q}\right) > 0,$$

which holds with $S'(Q) > (c+\mu h)/(p+h)$. Remembere that $S'(Q)$ is decreasing in Q, and $S'(Q_c^*) = (c+\mu h)/(p+h)$, so for any $Q < Q_c^*$, $\pi_r^{\text{pull}}(Q) > {}_s^{\text{push}}(Q)$.

According to Theorems 4.6 and 4.10, we know that neither Q_{pull}^* nor Q_{push}^* is larger than Q_c^*, so we have $\pi_r^{\text{pull}}(Q_{\text{pull}}^*) > \pi_r^{\text{pull}}(Q_{\text{push}}^*) > \pi_s^{\text{push}}(Q_{\text{push}}^*)$.

To demonstrate $Q_{\text{pull}}^* > Q_{\text{push}}^*$, from the supplier's first-order condition in push contract, Q_{push}^* satisfies

$$\frac{\partial \pi_s^{\text{push}}(Q)}{\partial Q} = (p+h)(S'(Q) + QS''(Q)) - (\mu h + c) = 0,$$

which implies

$$c + \mu h = (p+h)S'(Q_{\text{push}}^*)\left(1 + \frac{S''(Q_{\text{push}}^*)Q_{\text{push}}^*}{S'(Q_{\text{push}}^*)}\right). \tag{A.16}$$

Substitute equation (A.16) into $\left(\pi_r^{\text{pull}}(Q_{\text{push}}^*)\right)'$, then

$$\begin{aligned}\left(\pi_r^{\text{pull}}(Q_{\text{push}}^*)\right)' &= (p+h)S'(Q) - (c+\mu h)\left(1 - \frac{S(Q)S''(Q)}{(S'(Q))^2}\right)\Big|_{Q=Q_{\text{push}}^*} \\ &= \frac{(p+h)S''(Q)(S'(Q)S(Q) + QS(Q)S''(Q) - Q(S'(Q))^2)}{(S'(Q))^2}\Big|_{Q=Q_{\text{push}}^*}.\end{aligned}$$

Take $S(Q) = Qs(d/Q)$ and $S'(Q) = s(d/Q) - d/Q\bar{F}(d/Q)$ into the above equation, by using the assumption in Lemma 4.4, we have

$$S'(Q)S(Q) + QS(Q) - Q(S'(Q))^2 = d[\bar{F}(x)s(x) - (x)f(x)s(x) - x\bar{F}^2(x)]|_{x=d/Q} < 0.$$

Since $S''(Q) < 0$, thus $\left(\pi_r^{\text{pull}}(Q_{\text{push}}^*)\right)' > 0$. Using the concavity of $\pi_r^{\text{pull}}(Q)$, it is easy to confirm that $Q_{\text{push}}^* < Q_{\text{pull}}^*$; hence, $\pi_c(Q_{\text{pull}}^*) > \pi_c(Q_{\text{push}}^*)$. □

A.9. Proof of Lemma 5.1

Proof. The differential of $\pi_s(Q|w)$ with respect to Q is

$$\frac{\partial \hat{\pi}_s(Q|w)}{\partial Q} = [\beta p + (1-\beta)w + h]S'(Q) - (c + \mu h),$$

and the second-order condition is

$$\frac{\partial^2 \hat{\pi}_s(Q|w)}{\partial Q^2} = -[\beta p + (1-\beta)w + h]\frac{d^2}{Q^3} f\left(\frac{d}{Q}\right).$$

Hence $\partial^2 \hat{\pi}_s(Q|w)/\partial Q^2 < 0$. Therefore, the optimal production quantity is obtained by solving the first-order condition. □

A.10. Proof of Lemma 5.2

Proof. Given w, differentiating $\hat{\pi}_s^{\mathrm{Sub}}(Q,w)$ and $\hat{\pi}_r(Q,w)$, individually, we have

$$\begin{aligned}
\frac{\partial \hat{\pi}_s^{\mathrm{Sub}}(Q,w)}{\partial Q} &= (w + \beta(p-w) + h - r)S'(Q) - (c + \mu(h-r)),\\
\frac{\partial^2 \hat{\pi}_s^{\mathrm{Sub}}(Q,w)}{\partial Q^2} &= (w + \beta(p-w) + h - r)S''(Q),\\
\frac{\partial \hat{\pi}_r^{\mathrm{Sub}}(Q,w)}{\partial Q} &= ((1-\beta)(p-w) + r)S'(Q) - \mu r,\\
\frac{\partial^2 \hat{\pi}_r^{\mathrm{Sub}}(Q,w)}{\partial Q^2} &= ((1-)(p-w) + r)S''(Q).
\end{aligned}$$

Since $S''(Q) < 0$, and $r < h < w$, we have that both $\partial^2 \hat{\pi}_s^{\mathrm{Sub}}(Q,w)/\partial Q^2$ and $\partial^2 \hat{\pi}_r^{\mathrm{Sub}}(Q,w)/\partial Q^2$ are negative, *i.e.*, $\hat{\pi}_s^{\mathrm{Sub}}(Q,w)$ and $\hat{\pi}_r^{\mathrm{Sub}}(Q,w)$ are concave in Q. □

A.11. Proof of Theorem 5.3

Proof. In line with Lemma 5.2, from the perspective of the supplier, the optimal production quantity $\hat{Q}_s^{\mathrm{Sub}*}$ satisfies

$$S'(\hat{Q}_s^{\mathrm{Sub}*}) = \frac{c + \mu(h-r)}{w + \beta(p-w) + h - r}, \tag{A.17}$$

according to the assumption on r, $S'(Q) > 0$. Substituting the expression of $w(r)$ into (A.17), then

$$S'(\hat{Q}_s^{\mathrm{Sub}*}) = \frac{c + \mu h}{p + h},$$

which is consistent with the quantity that maximizes the total supply chain's expected profit; thus, $\hat{Q}_s^{\mathrm{Sub}*} = Q_c^*$. Therefore, from the supplier's perspective, the supply chain is coordinated. It remains to show that the retailer also prefers this same Q.

As for the retailer, given w, the optimal quantity $\hat{Q}_r^{\mathrm{Sub}*}$ satisfies

$$S'(\hat{Q}_r^{\mathrm{Sub}*}) = \frac{\mu r}{(1-\beta)(p-w) + r}, \tag{A.18}$$

similar to the proof of the supplier, we have $\hat{Q}_r^{\mathrm{Sub}*} = Q_c^*$. □

A.12. Proof of Theorem 5.4

Proof. According to equations (5.11) and (5.12), the supplier's profit is an increasing function of λ and the retailer's profit is a decreasing function of λ. According to equation (5.10), λ is a decreasing function of r. Therefore, the retailer's (supplier's) profit is an increasing (decreasing) function of r.

If $r = 0$, then $\lambda = 1$, $\hat{\pi}_s^{\text{Sub}}(Q, w, r) = \pi_c(Q)$, $\hat{\pi}_r^{\text{Sub}}(Q, w, h_b) = 0$, and if $r = h + c/\mu$, we have $\hat{\pi}_r^{\text{Sub}}(Q, w, h_b) = \pi_c(Q)$, $\hat{\pi}_s^{\text{Sub}}(Q, w, h_b) = 0$. Since the retailer's (supplier's) profit is an increasing (decreasing) function of r, when $0 < r < h + c/\mu$, it is clear that both $\hat{\pi}_s^{\text{Sub}}(Q, w, h_b)$ and $\hat{\pi}_r^{\text{Sub}}(Q, w, h_b)$ are positive. □

A.13. Proof of Lemma 5.5

Proof. Taking the derivative of $\tilde{\pi}_r(Q|w)$ with respect to Q, we have

$$\frac{\partial \tilde{\pi}_r(w, Q, h_b)}{\partial Q} = pS'(Q) - \mu w - (h - h_b)(\mu - S'(Q)),$$

where

$$\begin{aligned} S'(Q) &= \frac{d}{Q} F\left(\frac{d}{Q}\right) - \int_a^{d/Q} F(x)\,\mathrm{d}x, \\ \frac{\partial^2 \tilde{\pi}_r(w, Q, h_b)}{\partial Q^2} &= pS''(Q) + (h - h_b)S''(Q) \\ &= -(p + h - h_b)\frac{d^2}{Q^3} f\left(\frac{d}{Q}\right)\cdot \end{aligned}$$

Since $p + h - h_b > 0$, then $\partial^2 \tilde{\pi}_r(w)/\partial Q^2 < 0$, which indicates that $\tilde{\pi}_r(Q|w)$ is concave with respect to Q with the given w. □

A.14. Proof of Theorem 5.6

Proof. Since $\tilde{\pi}_r(Q|w)$ is concave with respect to Q, the optimal solution can be obtained by solving the first-order condition of $\hat{\pi}_r(w, Q, h_b)$. Since

$$\frac{\partial \hat{\pi}_r(w, Q, h_b)}{\partial Q} = (p + h - h_b)S'(Q) - \mu(w + h - h_b) = 0,$$

then

$$S'(\tilde{Q}_r^*) = \frac{\mu(w + h - h_b)}{p + h - h_b}\cdot \tag{A.19}$$

Substituting the expression of $w(h_b)$ into (A.19), we obtain

$$S'(\tilde{Q}_r^*) = \frac{c + \frac{h_b(\mu p - c)}{p+h} + \mu(h - h_b)}{p + h - h_b} = \frac{c + \mu h}{p + h}, \tag{A.20}$$

there is the same Q that maximizes the total supply chain expected profit, so $\tilde{Q}_r^* = Q_c^*$. Therefore, from the retailer's perspective, the supply chain is coordinated. It remains to show that the supplier also prefers this same Q.

The supplier's profit function is

$$\begin{aligned} \tilde{\pi}_s(w, Q, h_b) &= wQE[\alpha] - cQ - h_b E(\alpha Q - d)^+ \\ &= \mu w Q - cQ - h_b(\mu Q - S(Q)). \end{aligned} \tag{A.21}$$

We can show that $\partial^2 \tilde{\pi}_s(w, Q, h_r)/\partial Q^2 = -h_b d f(d/Q)/Q^3 < 0$, thus $\tilde{\pi}_s(w, Q, h_b)$ is concave with respect to Q. The Q that maximizes $\tilde{\pi}_s$ satisfies

$$\frac{\partial \tilde{\pi}_s(w, Q, h_b)}{\partial Q} = \mu w - c - \mu h_b + h_b S'(Q) = 0$$

$$\Leftrightarrow S'(\tilde{Q}_s^*) = \frac{c + \mu h_b - \mu w}{h_b} \cdot \tag{A.22}$$

Letting $w = w(h_b)$ as defined in (5.14), we obtain

$$S'(\tilde{Q}_s^*) = \frac{c + \mu h}{p + h},$$

therefore, $\tilde{Q}_s^* = Q_c^*$. □

A.15. Proof of Theorem 5.7

Proof. According to equations (5.17) and (5.18), the retailer's profit is an increasing function of λ and the supplier's profit is a decreasing function of λ. According to equation (5.15), λ is a decreasing function of h_b. Therefore, the retailer's (supplier's) profit is a decreasing (increasing) function of h_b.

If $h_b = 0$, then $\lambda = 1$, $\tilde{\pi}_r(Q, w, h_b) = {}_c(Q)$, $\tilde{\pi}_s(Q, w, h_b) = 0$, and if $h_b = p + h$, we have $\tilde{\pi}_s(Q, w, h_b) = \pi_c(Q)$, $\tilde{\pi}_r(Q, w, h_b) = 0$. Since he retailer's (supplier's) profit is a decreasing (increasing) function of h_b, when $0 < h_b < p + h$, it is clear that both $\tilde{\pi}_r(Q, w, h_b)$ and $\tilde{\pi}_s(Q, w, h_b)$ are positive. □

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