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CONSIGNMENT STOCK POLICY IN A CLOSED-LOOP SUPPLY CHAIN

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Abstract. The main feature of sustainable supply Chain Management is reuse of end of life (EOL) products to reduce the environmental pollution. This paper considers the consignment stock (CS) policy for a closed-loop supply chain (CLSC). To achieve the economic goal, this study presents a multi-echelon supply chain with a single manufacturer, a single retailer and a third-party service provider. The objective is to optimize this three-level CLSC under a Stackelberg game scenario. The other objective is to identify the environmental impact of remanufacturing EOL products. In this CLSC, the manufacturer acts as the Stackelberg leader and the retailer, the third party are the followers. Under manufacturer's leadership, the retailer and the third party acquire three different decision strategies – I, II, and III. In decision strategy I, the retailer acts as the leader and the third party acts as a follower, whereas in decision strategy II, they obtain a Nash game strategy. In the decision strategy III, they act as one unit and decide their decisions centrally. Numerical example and sensitivity analysis are used to illustrate optimal results of different decision strategies and also check the behaviour of key model-parameters. The best outcomes are found in decision strategy I. The main findings of our study show that when upper level player has more power then the supply chain gives best outcomes.

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1. Introduction

Consignment stock is the inventory that is controlled by the retailer but is still legally owned by the supplier/manufacturer. In CS policy, the manufacturer has placed some products in the retailer's warehouse and gave consent to sell or use the products directly from their stock. The manufacturer claims purchasing cost only after products sold. In the right situation, consignment inventory can have considerable benefits for both the retailer and the manufacturer. One obvious advantage to retailers does not have to pay the supplier until products are sold, the business is able to use this capital on other projects and benefit from the increase in cash flow. The major advantage to the manufacturer is that they are able exposed their products in front of end-users. Other advantage of CS policy for both players is that it reduces the lead-time on products. In recent competitive business era, many industries adopt CS agreement policy. For example, in health care industries, clothing and furniture retailers, some gas stations, auto parts industry bookstores, sports equipment and musical

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instruments, stores, have started to adopt CS agreements [27, 41]. In some cases, the retailer is responsible for any damage or deterioration [24]; the manufacturer always likes to stock as much as possible in the retailer's warehouse. So, the main feature of the consignment agreement is that both players are benefited under some conditions of their storage capacities and costs.

The manufacturer deals with several challenges in recent decades to develop coordination in a supply chain model. The shortage of resources and dreadful conditions of environment make difficulties to coordinate production efficiency and pollution reduction in a CLSC. They are interested to develop integrated optimization for new product in forward channel and used product in reverse channel to make eco-friendly supply chain. The CLSC integrates all flows of material, financial and information within the forward and reverse chains. It helps the enterpriser to overcome the challenges related to its operations and decision strategies and be acquainted with prospective benefits. A third party logistics is the main contributor in the reverse supply chain or CLSC. They are well-known for materials recycling, resources saving, and environment protecting activities.

According to the recent research Norwich University, U.S. companies produce 7.6 billion tons of non-hazardous waste each year, with 55% of the waste ending up in landfills. Meanwhile, companies like Dell are taking the lead in recycling products with commitments to reuse 10% of its discarded parts in new models. The rapid development of science and technology is driving the fast expansion of the electronics industry. New electronic products are now being introduced at an increasingly fast rate, and the abandoned waste products are burdening society and the environment. As claimed by the Environmental Protection Agency (EPA), there are 20–50 million tons of wasted electronic products generated worldwide every year, which is a considerable environmental concern. Given the danger of natural resource depletion, governments are paying more attention to environmental protection and encouraging enterprises to recycle and reuse waste electronic products to reduce environmental pollution and minimize resource waste [10].

Recent days, many companies in many countries have been familiar with the integrated eco-friendly business policies to develop modern product recovery and reuse their EOL products. The returned or recovered products are converted to a "like new" one which is equivalent to the original manufactured product by several operations (disassembly, cleaning, testing, part replacement/repair and reassembly) in remanufacturing process. Electronic industries are the main industries that are directly use the closed loop supply chain to reduce green house gasses. On the other hand, nowadays, all e-retail channels (Flipkart, Amazon) encourage the customer to return the EOL products. High value industrial products such as aircraft or automobile engines, aviation equipment, railroad locomotives equipment, medical equipment, machine tools, copiers, electrical and electronic equipment, toner cartridges, cellular telephones, single-use cameras, etc. are some examples of such remanufacturing process [15].

From the above discussion, it is clear that, in CLSC, collection of reused products is very familiar to the researchers but in a consignment context, CLSC remains conceal till now. In this circumstance, a three level closed-loop supply chain system that consists of a manufacturer, a retailer and a third-party provider was developed and a consignment stock agreement was adopted between the players to find the optimum remanufactured quantity from EOL products. The rest of the paper is organized as follows: Literature review is discussed in the next section. Notations and assumptions of the proposed model are presented in Section 3. In Section 4, two models are formulated from the perspectives of the manufacturer, the retailer and the integrated system. The solution methodology is discussed in Section 5. The model is demonstrated with a numerical example and the optimal results are compared for different game theoretic approaches in Section 6. A sensitivity analysis is carried out in Section 7. Finally, in Section 8, the paper is concluded with some remarks and future scope of research.

2. Literature review

2.1. Consignment stock policy

Consignment stock policy is now an emerging policy which is used by different industries (*e.g.*, medical instrument producer). Braglia and Zavanella [5] were the first who proposed a consignment inventory model for solving the joint economic lot size problem (JELP). Valentini and Zavanella [41] described the technique, underlining the consignment stock policy's potential benefits and pitfalls. Wang *et al.* [42] studied the consignment policy with revenue sharing contract. They derived the dependency of the demand elasticity and the retailer's share of channel cost in a supply chain. Gümüs *et al.* [19] proposed the impact of consignment inventory and vendor-managed inventory for a two-level supply chain. Zavanella and Zanoni [51] showed that consignment stock policy works better than the non-coordinated supply chain. They considered a one-vendor multiple buyers integrated production inventory model under consignment policy and obtained the optimal replenishment decisions for both the vendor and the buyer in such a situation. Zanoni *et al.* [49] considered the vendor managed inventory (VMI) under consignment with learning and forgetting effects. Wang *et al.* [43] presented a single-manufacturer single-buyer supply chain problem with a single deteriorating product under consignment policy. Bylka [8] investigated inventory pattern and cost structure of production-distribution cycle (PDC) under generalized consignment stock policy. Zanoni and Jaber [48] investigated and compared different policies for consignment stock with stock dependent demand. Supply chain composed of a single vendor and multiple retailers operating under a VMI contract was investigated by Hariga *et al.* [20]. Braglia *et al.* [6] proposed a relationship between the vendor and the buyer in a win-win situation with fixed batch manufacturing process. Comparative perspective and critical review for consignment stock policy were discussed by Sarker [27]. Safety stock management in a single-vendor single-buyer problem under VMI with consignment agrement was considered by Braglia *et al.* [7]. Bazan *et al.* [4] considered a VMI model with consignment stock agreement for a two-level supply chain with an imperfect production process with/without restoration interruptions. Zanoni *et al.* [50] presented VMI with consignment stock agreement in a joint economic lot size model for coordinated inventory replenishment decisions under under the emission-trading scheme. Jaber *et al.* [23] developed a consignment stock for a two-level supply chain with remanufacturing and waste disposal problem. Zahran *et al.* [47] investigating the effects of different payment schemes on the total profit for a two-level consignment stock supply chain system. Zahran *et al.* [46] compared different coordination scenarios in a three-level supply chain system under the light of CS policy.

Under deterministic demand conditions, the operational benefits of CS systems were presented in Gumus *et al.* [19] and Giri *et al.* [13] for two- and three-stage supply chain systems, respectively. Recently, Zahran and Jaber [45] tackled the more complex three-stage supply chain structure involving multi-suppliers, single-vendor and multi-buyers, and Giri *et al.* [13] addressed a three-stage supply chain comprised of a single-supplier, single-vendor and multi-buyers. Later, Giri *et al.* [14] developed a CS with unequal shipments and process unreliability for a two-level supply chain. Sarkar *et al.* [29] developed a distribution free model with consignment stock policy in a two-echelon supply chain model. Hariga *et al.* [21] addressed a centralized closed loop supply chain comprised of a single vendor and a single buyer operating under a consignment stock strategy. Taleizadeh and Moshtagh [33] discussed a CLSC in an imperfect manufacturing/remanufacturing system in which the collection rate of used items depends on the acceptance quality level of returns. Taleizadeh *et al.* [38] coordinated a VMI inventory model on consignment stock policy with penalty. Recently, As'ad *et al.* [1] developed two stage closed loop supply chain models under consignment stock agreement and different procurement strategies. All research works addressing CS models have dealt with the problem in the context of a centralized two-stage supply chain system. The environmental approach to the CS agreement in CLSC only considered in the last two articles in different context.

2.2. Remanufacturing models

Some of the researches in this context include inventory control and lot-sizing of remanufacturing systems [39, 40], reverse channel/network design [32]. Some CLSCs and remanufacturing models focused on considering time value of product return [18], quality consideration [11], limited durability and finite life cycles [12]. Most recently, environmental and economical assessment of closed-loop supply chain was discussed by Sarkar *et al.* [28]. Sarkar *et al.* [30] optimized production delivery policies in a constrained CLSC through meta-heuristic approach for returnable transport packaging.

TABLE 1. Comparison of the proposed model with existing literature.

Existing literature	Three-level SC	Closed loop SC	Consignment stock	Game theoretic approach
Valentini and Zavanella [41]	No	No	YES	No
Giri *et al.* [13]	Yes	No	Yes	No
Zahran and Jaber [45]	Yes	No	Yes	No
Taleizadeh and Moshtagh [33]	Yes	Yes	Yes	No
Sarkar *et al.* [28]	Yes	Yes	No	No
Sarkar *et al.* [29]	No	No	Yes	Yes
Sarkar *et al.* [30]	No	Yes	No	Yes
Sarkar *et al.* [31]	Yes	Yes	No	Yes
Maiti and Giri [25]	No	Yes	No	Yes
This article	Yes	Yes	Yes	Yes

For improving economic and environmental performances, development of forward and reverse channels together is very much essential. Savaskan *et al.* [32] addressed to choose the appropriate reverse channel structure for the collection of used products from customers. Atasu *et al.* [2] showed that remanufacturing is also a profitable business. Hong and Yeh [22] showed a retailer collection model whereby the retailer collects EOL products and the manufacturer cooperates with a third-party firm to handle used products. Choi *et al.* [9] examined the performance of different CLSCs under different channel leaderships with a retailer, a collector, and a manufacturer. They showed that the supply chain performance is the worst when the most upstream member acts as the supply chain leader. For more works on reverse logistics and CLSC, the readers can be referred to the comprehensive reviews done by Govindan *et al.* [17]. Maiti and Giri [25] considered a CLSC model with retail price and product quality dependent demand and optimized the model in three different Stackelberg games led by the manufacturer, and compared the optimal results to find out the best decentralized scenario. Giri *et al.* [16] optimized product quality and pricing strategy by Stackelberg game policy in a two period CLSC under variable markup scheme. Sarkar *et al.* [31] studied joint pricing and inventory policies for three-layer dual-channel hybrid CLSC model with remanufacturing. In order to reduce solid waste generation in supply chain, RTI is used to transport finished products from the manufacturer to the retailers.

Taleizadeh *et al.* [34] explored a dual-channel CLSC system consisting of a manufacturer and a retailer and coordinated through two-part tariff contract. Game theoretic approach for a decentralized CLSC has been discussed by Taleizadeh *et al.* [35] to optimize the price, product quality and the collection rate. discussed the issues of quality improvement, return policy and the environmental-friendly production in a CLSC. Marketing effort dependent demand for a three-echelon supply chain has been established by Sane-Zarang *et al.* [26]. Taleizadeh *et al.* [36,37] discussed a CLSC model with pricing decisions and the discounts on returned product. Recently many researchers classification of the relevant literature with respect to the manuscript presented in Table 1.

The aim of this paper is to develop a CLSC model with consignment contract. In this paper, we consider a closed-loop supply chain model where the third party collects the EOL items from the end customer. The remanufacturing of EOL items is done by the manufacturer and remanufactured items are sold in the market in direct channel. Manufacturer and the retailer follows CS agreement between them. For all models, the manufacturer is the Stackelberg leader for different strategies to find the optimal decisions.

3. Notations and assumptions

The following notations and assumptions are used to develop the proposed CLSC model:

D : demand rate of the product in the market.
P : rate of production from raw materials.
R : rate of production from returned products.
n : number of shipments.
A_v : set up cost of the manufacturer.
A_b : ordering cost of the retailer.
A_3 : set up cost of the third party
A_t : average recycling cost per unit.
h_{vs} : non-financial holding cost of the manufacturer.
h_{vf} : financial holding cost of the manufacturer.
h_{bs} : non-financial holding cost of the retailer.
h_u : holding cost of the third party.
c_f : unit transportation cost.
c_m : unit manufacturing cost from raw material.
c_r : unit manufacturing cost from returned product.
c_3 : unit cost paid by the manufacturer to the third party.
c_q : unit quality improvement cost.
α_r : minimum acceptable quality of the return product.
g : goodwill cost.

Decision variables

T : cycle length (weeks).
α : quality of the finished product.
τ : collection rate (units/unit time).

The following assumptions are made to develop the proposed model.

(i) Supply chain consists of one manufacturer, one retailer and one third party. It is also assumed that all members are interested in cooperating with each other in the integrated system and that all the information is shared among the SC players.

(ii) The manufacturer adopts CS policy to deliver the product to the retailer. The third party collects the used products from the consumers and supplies to the manufacturer for remanufacturing. The production process at the manufacturers facility transforms raw materials (RMs) into finished products and is also flexible enough to process returned products, received on a continuous basis from the third party, into finished products as well.

(iii) The manufacturer remanufactures the used products collected by the third party. The remanufactured items are as good as new. The remanufactured items are sold by the manufacturer to the end customers in a direct channel. A fraction of the total demand is collected as reusable products.

(iv) Third party pays an amount to the customer (return value) based on the quality of the return product. He can paid different amounts for different products. It is also assumed that the average cost of recycling is less than the remanufacturing cost, which is less than the manufacturing cost.

(v) Market demand is deterministic, known and constant. Manufacturing and remanufacturing rates are also constants but greater than the demand rate.

4. Model formulation

4.1. Problem description

Consider a three stage supply chain comprised a manufacturer, a retailer and a third party where the consignment stock (CS) partnership is adopted between the manufacturer and the retailer (Fig. 1). The third party collects the EOL products from the end customer and delivers to the manufacturer for the remanufacturing.

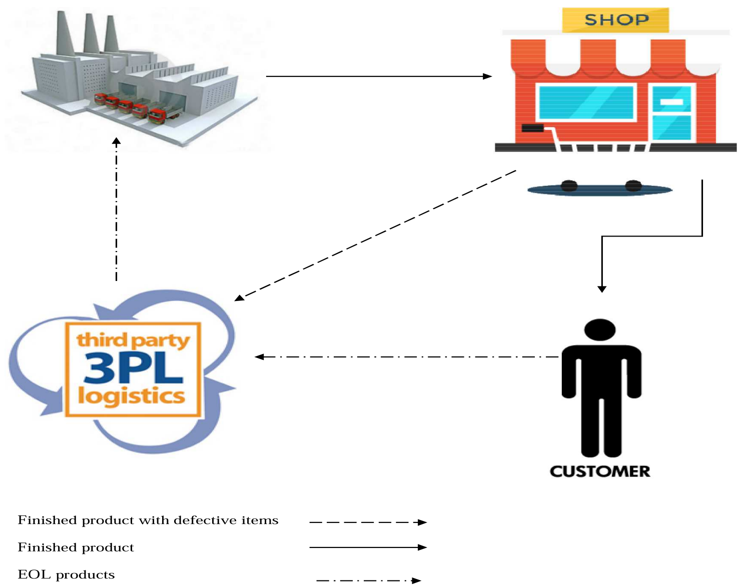


FIGURE 1. Pictorial representation of product's flow in the closed loop supply chain.

In the forward supply chain, the manufacturer sells the product to potential customer through the retailer. The third party (3PL) collects the EOL items from the end customers. The returned products are sent to the manufacturer for remanufacturing in the reverse supply chain. The manufacturer meets up all market demand of the product which are produced from raw materials al well as from the returned products. The manufacturer sells the remanufactured items in a direct channel to the potential customer through the retailer. The products manufactured from the return items are as good as new. It is also assumed that only a fraction τ of the market demand is meet up from the remanufactured quantity. As both (manufactured from the raw material and remanufactured from the EOL products) products are produced in the same facility, an important part is made by the third party by collecting the product for optimal cost decision.

4.2. Manufacturer's cost

Suppose that the manufacturer produces total Q quantity of finished product to fulfil the retailer's demand. Since a faction (τ) of the total demand is met from the returned products, therefore, the remaining fraction $(1-\tau)DT$ is fulfilled by products made by the manufacturer from fresh raw materials. As there are n equal shipments from the manufacturer to the retailer, the size (q_1) of each shipment is given by

$$q_1 = \frac{(1-\tau)Q}{n}.$$

Proposition 4.1. *At the retailer, the maximum inventory for n shipments is given by*

$$I_{\max} = nq_1 - (n-1)\frac{q_1 D}{P}.$$

Proof. The maximum inventory is given by n shipments each of size q_1 minus the inventory for the $(n-1)$ shipments with time interval $\frac{q_1 D}{P}$, which gives

$$I_{\max} = nq_1 - (n-1)\frac{q_1 D}{P}.$$

Thus the desired result is achieved. □

TABLE 2. Holding cost sharing in the consignment stock case.

		Position of raw-material/product	
		Manufacturer	Retailer
Relevant costs	Manufacturer	$h_{v,s} + h_{v,f}$	$h_{v,f}$
	Retailer	0	$h_{b,s}$

Consignment stock policy is an approach where a shipment is delivered to the retailer as soon as the production of that batch is completed regardless of the inventory level at the retailer's premises. In return, the manufacturer maintains ownership of this inventory (and accordingly bares the capital "financial" cost associated with tying up capital in inventory) until the product is sold to the end customer at which point in time, the manufacturer gets his share of the revenue. The retailer's holding cost per unit of stock per unit time (h_b) is shared by both parties, the capital (opportunity) cost is charged to the manufacturer as he owns the product until sold and the physical storage cost is incurred by the retailer. The concept of cost sharing under CS partnership (for both the ordering and the holding cost) was first introduced by Valentini and Zavanella [41]. The holding cost sharing in CS policy is shown in Table 2.

The manufacturer's total cost is given by

$$\begin{aligned}\text{Total cost} &= \text{set-up cost} + \text{manufacturing cost} + \text{holding cost}\\ &\quad + \text{quality up-gradation cost} + \text{goodwill cost}.\end{aligned}$$

The manufacturer produces $(1-\tau)Q$ quantity of finished products from the raw materials. His set-up cost is A_v and production cost is $c_m(1-\tau)Q$, where c_m is the unit manufacturing cost from the raw materials. A quadratic function is used to represent the quality up-gradation cost [25]. For maintaining the quality (α) of the finished product, the quality up-gradation cost of the manufacturer is given by $c_q\alpha^2(1-\tau)Q$. According to our assumption the quality of the finished product is not 100% pure, and therefore, there is an impurity $(1-\alpha)$ in each product. To maintain the goodwill band, the manufacturer incurs the goodwill cost $(1-\alpha)g$. Hence, the average cost of the manufacturer is given by

$$\mathrm{TC}_V = \frac{1}{T}\left(A_v + c_m(1-\tau)Q + (h_{vf}+h_{vs})\mathrm{HC}_v + h_{vf}\mathrm{HC}_b + c_q\alpha^2(1-\tau)Q + (1-\alpha)g\right) \tag{4.1}$$

where the manufacturer's holding cost $\mathrm{HC}_v = \frac{nq_1^2}{2P}$ and the retailer's holding cost is given by the following equations (please see Appendix A for details).

$$\begin{aligned}
&I_{\max} = nq_1 - (n-1)\frac{q_1 D}{P},\\
&\mathrm{HC}_b^1 = \frac{q_1^2}{2P}(n-1)\left(2-\frac{D}{P}\right), \quad \mathrm{HC}_b^2 = \frac{q_1^2(n-2)(n-1)}{2P}(n-1)\left(1-\frac{D}{P}\right), \quad \mathrm{HC}_b^3 = \frac{I_{\max}^2}{2D}\\
&\mathrm{HC}_b = \mathrm{HC}_b^1 + \mathrm{HC}_b^2 + \mathrm{HC}_b^3.
\end{aligned}$$

4.3. Remanufacturing cost

The manufacturer pays a collection cost c_3 to the 3PL for collecting EOL products from the end customers. On the other hand, the manufacturer invests remanufacturing cost c_r for restructuring the EOL items. So, for τQ products, the remanufacturing cost is $\tau Q(c_r + c_3)$.

The average quality of the collected product is α_r. After remanufacturing, the product quality is upgraded to α. A quadratic function is used to represent the decreasing value of investment in terms of quality return

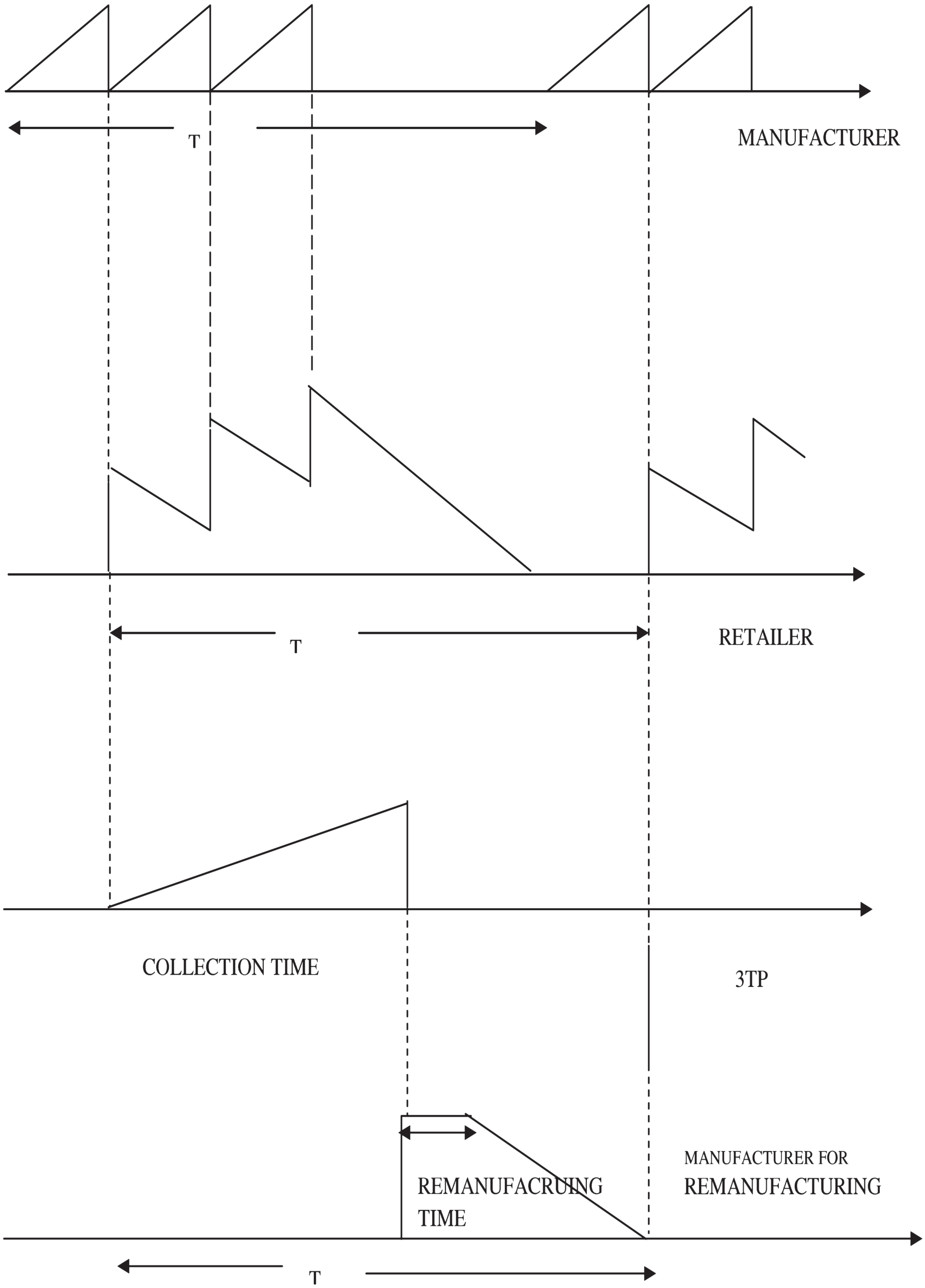


Figure 2. Graphical representation of the manufacturer, retailer and third party's inventory levels.

as the quality of the finished product increases [25]. Thus the degradation cost is given by $\tau Q c_q(\alpha^2 - \alpha_r^2)$. The holding cost of the manufacturer for the remanufacturing is given by the area of the trapezium (Fig. 2) having base $\left(\frac{\tau Q}{R} + \frac{\tau Q}{D}\right)$ and height $\frac{\tau Q}{2}$. Therefore, the average cost of the manufacturer for remanufacturing is given by

$$\mathrm{TC}_R = \frac{1}{T}\left(\tau Q\left(c_r + c_3\right) + \frac{\tau Q}{2}\left(\frac{2\tau Q}{R} + \frac{\tau Q}{D}\right) h_{vf} + \tau Q c_q\left(\alpha^2 - \alpha_r^2\right)\right). \tag{4.2}$$

4.4. Retailer's cost

The retailer's total cost includes ordering cost, transportation cost and inventory holding cost. Therefore, the average cost of the retailer is given by

$$\mathrm{TC}_B = \frac{1}{T}\left(A_b + n c_f + h_{bs}\mathrm{HC}_b\right). \tag{4.3}$$

4.5. Third party's cost

The third party collects only those used products whose quality is higher than the acceptable quality α_r. Based on the quality of the returned product, the third party pays an amount to the customer. The total cost for collecting and recycling is $\tau Q A_t$. The collection time of the 3PL is the difference between the cycle time and the time for the restructuring (*i.e.*, for remanufacturing and selling the remanufactured products). The holding cost for the 3PL is $\frac{1}{2}\left(T - \frac{\tau Q}{R} - \frac{\tau Q}{D}\right)\tau Q h_u$. Therefore, the total cost of the 3PL is given by

$$\mathrm{TC}_P = \frac{1}{T}\left(A_3 + \tau Q A_t + \frac{1}{2}\left(T - \frac{\tau Q}{R} - \frac{\tau Q}{D}\right)\tau Q h_u\right). \tag{4.4}$$

Our objective in thus study is to determine the minimal cost of each individual player and also the optimal decisions on cycle length (T), collection rate (τ) and quality (α) of the product. In the next section, the solution methodology of our problem will be discussed.

5. Model analysis and solution methodology

In this section, we consider a Stackelberg game where the manufacturer is the Stackelberg leader and other players, *i.e.*, retailer and third party as the follower. In some industries, product return activities are administrated by manufacturers in parallel to the distribution of the new products. To meet the quality standard, some industries like Xerox, Canon and Hewlett Packard (HP) [34] have the privilege in reusing the high value, end of lease copiers in manufacturing new copiers. In a similar vein, IBM and Compaq encourage consumers to use their asset recovery services which provide easy disposal and replacement of end of life PCs [13]. In our model analysis, three decision strategies are considered by the retailer and the third party in order to follow the manufacturer's decision:

(i) Stackelberg game where retailer is the leader.
(ii) Nash game where both the retailer and the third party have the same decision power.
(iii) Centralized policy where both the retailer and the third party act like a single unit and optimize centrally for their decisions.

5.1. Decision strategy I (DS I)

In this strategy, under the manufacturer leadership, the retailer and the third party play another Stackelberg game between them. In this game, the retailer act as the leader and the third party as the follower. So, all the players make their decisions sequentially. In this asymmetrical relationship, the retailer is dominated by the manufacturer and the third party is dominated by the retailer. This corresponds to the common assumption in the marketing literature, which has been valid in the retail industry for many years and is still valid in the other sectors such as the automotive industry [3]. In the Stackelberg equilibrium, the manufacturer is aware of the retailer's responses on his decision. Under this commitment, the manufacturer optimizes his total cost (manufacturing cost and remanufacturing cost jointly) and declares the quality of the finished product. Then the retailer sets his optimal time period (T). Finally, the third party determines the collection rate after getting the feedback from the manufacturer and the retailer.

These decision problems is solved by setting the first order derivatives $\frac{\partial \mathrm{TC}_P}{\partial \tau}$, $\frac{\partial \mathrm{TC}_B}{\partial T}$ and $\frac{\partial(\mathrm{TC}_V+\mathrm{TC}_R)}{\partial \alpha}$ equal to zero sequentially. The second order sufficient conditions are obvious from the following:

$$\frac{\partial^2 \mathrm{TC}_P}{\partial \tau^2} = \frac{nq_1}{(1-\tau)^3} > 0, \quad \frac{\partial^2 \mathrm{TC}_B}{\partial T^2} = \frac{2(A_b + nc_f)}{T^3} > 0,$$
$$\frac{\partial^2(\mathrm{TC}_V + \mathrm{TC}_R)}{\partial \alpha^2} = 2c_q D(1-\tau) + 2c_q D\tau > 0, \quad \text{since} \quad 0 < \tau < 1.$$

Theorem 5.1. *For the decision strategy I, the optimal decisions are as follows:*

$$\tau^* = \frac{R(2A_t + h_u T)}{2h_u T(D+R)}, \quad T^* = \frac{2\sqrt{B1}}{\sqrt{B2}},$$
$$\alpha^* = \frac{g(2D+R)\sqrt{Dh_{bs}h_u^2\big(D(n-1) - nP\big)}}{4c_q D\sqrt{A_t^2 D h_{bs}\big((n-1)D - nP\big)R^2 - 2(D+R)^2 h_u^2 P\big(A_b n - c_f n^2\big)}},$$

where

$$\begin{aligned} B1 &= 2A_b D^2 h_u^2 nP + 2c_f D^2 h_u^2 n^2 P - 4A_b D h_u^2 nPR + 4c_f D h_u^2 n^2 PR \\ &\quad + A_t^2 D^2 h_{bs} R^2 - A_t^2 D^2 h_{bs} nR^2 + A_t^2 D h_{bs} nPR^2 + 2A_t^2 D h_{bs} nPR^2 + 2c_f h_u^2 n^2 PR^2, \\ B2 &= D^2 h_{bs} h_u^2\big(4 + nPR + R^2 - nR^2\big) + 4D^3 h_{bs} h_u^2\big(nP + R - nR\big) \\ &\quad - 4D^4 h_{bs} h_u^2 n + D h_{bs} h_u^2 nPR^2. \end{aligned}$$

Proof. The first order optimality conditions give the results in Theorem 5.1. □

Proposition 5.2. *For DS I, the average recycling cost per unit incurred by the third-party is given by* $A_t < h_u T\left(\frac{D}{R} + \frac{1}{2}\right)$.

Proof. The condition $\tau^* < 1$ implies $\frac{R(2A_t + h_u T)}{2h_u T(D+R)} < 1$ which gives the desired result.

This implies that this strategy gives better results if the recycling cost is under control. If this cost being high then all the optimal decisions become imbalanced. □

5.2. Decision strategy II (DS II)

In this strategy, the retailer and the third party both decide their individual decisions simultaneously. The manufacturer sets the quality of the product after optimizing his total cost. Next, the retailer and the third party optimize their individual total cost simultaneously. They follow Nash-equilibrium condition to find their optimal decisions on the cycle time and the collection rate.

In this decision problem, the optimal solution can be archived by setting the first order derivatives $\frac{\partial \mathrm{TC}_P}{\partial \tau}$ and $\frac{\partial \mathrm{TC}_B}{\partial T}$ each equal to zero simultaneously. The optimal values of T and τ can be put to the joint cost $\mathrm{TC}_V + \mathrm{TC}_R$. The optimal quality of the finished product then can be achieved from $\frac{\partial(\mathrm{TC}_V+\mathrm{TC}_R)}{\partial \alpha} = 0$. The second order sufficient conditions for the unique solution of τ, T and α are $\frac{\partial^2 \mathrm{TC}_P}{\partial \tau^2} > 0$, $\frac{\partial^2 \mathrm{TC}_B}{\partial T^2} > 0$ and $\frac{\partial^2(\mathrm{TC}_V+\mathrm{TC}_R)}{\partial \alpha^2} > 0$.

Theorem 5.3. *For decision strategy II, the optimal decisions are as follows:*

$$\tau^* = \frac{A1}{A2}, \quad T^* = \frac{A3}{A4},$$
$$\alpha^* = \frac{g h_{bs} h_u^2 (nP - (n-1)D)(2D+R)}{4c_q \left[A_t D h_{bs} h_u \big(nP - (n-1)D\big) R - \sqrt{2}(D+R) h_u^2 \sqrt{D h_{bs} n (A_b + n c_f) P \big(nP - (n-1)D\big)}\right]},$$

where

$$\begin{aligned} A1 &= R\Big[2A_b h_u^3 nP(D+R)^2 + 2c_f h_u^3 n^2 P(D+R)^2 + A_t(2A_t D h_{bs} h_u D(n-1) - nP)R(D+R) \\ &\quad - \sqrt{2}\sqrt{D h_{bs} h_u^4 n(A_b + n c_f)P(D - Dn + nP)(2D^2 + 3DR + R^2)^2}\Big], \\ A2 &= 2h_u(D+R)\left(A_t^2 D h_{bs}(D(n-1) - nP)R^2 + 2A_b h_u^2 nP(D+R)^2 + 2c_f h_u^2 n^2 P(D+R)^2\right), \\ A3 &= 2\Big[A_t D h_{bs} h_u (D(n-1) - nP)R(2D+R), \\ &\quad + \sqrt{2}\sqrt{D h_{bs} h_u^4 n(A_b + n c_f)P(D - Dn + nP)(2D^2 + 3DR + R^2)^2}\Big], \\ A4 &= D h_{bs} h_u^2 (D(n-1) - nP)(2D+R)^2. \end{aligned}$$

Proof. The first order optimality conditions give the results in Theorem 5.3. □

5.3. Decision strategy III (DS III)

The previous strategies discuss two non-cooperative games between the retailer and the third party. In this strategy, the third party and the retailer work together as a single unit. It is called as a hybrid supply chain model denoted by $\mathrm{TC}_{BP} = \mathrm{TC}_B + \mathrm{TC}_P$. This type of strategies has studied by Yang and Zhou [44] in their model between two retailers. The manufacturer optimizes his total cost to declare the quality of the product. After getting the optimal reaction from the manufacturer, the third party and the retailer jointly act as a single player and find the optimal decisions on the cycle length and the collection rate by optimizing the total cost together. The necessary conditions for minimum of TC_{BP} are $\frac{\partial \mathrm{TC}_{BP}}{\partial T} = 0$ and $\frac{\partial \mathrm{TC}_{BP}}{\partial \tau} = 0$. The optimal quality of the finished product can be achieved from the condition $\frac{\partial(\mathrm{TC}_V + \mathrm{TC}_R)}{\partial \alpha} = 0$, after getting the responses of T and τ. The sufficient condition for the convexity of the cost function TC_{BP} is that the associated Hessian matrix H must be positive definite where H is given by

$$H = \begin{pmatrix} \frac{\partial^2 \mathrm{TC}_{BP}}{\partial \tau^2} & \frac{\partial^2 \mathrm{TC}_{BP}}{\partial \tau \partial T} \\ \frac{\partial^2 \mathrm{TC}_{BP}}{\partial T \partial \tau} & \frac{\partial^2 \mathrm{TC}_{BP}}{\partial T^2} \end{pmatrix}.$$

Theorem 5.4. *The optimal decisions in the decision strategy III are as follows:*

$$\tau^* = \frac{A5}{A6}, \quad T^* = \frac{2\sqrt{(n(A_t^2 D - 2(h_{bs} - h_u)L4)P - 2D h_{bs}(n-1)L4)\,R - 2D h_u n L4 P}}{\sqrt{L5}},$$
$$\alpha^* = \frac{g\sqrt{D h_u\left(4D^2 h_{bs}(n-1) + 4D h_{bs} nP + h_u nPR\right)}}{4c_q D\sqrt{(n(A_t^2 D - 2(h_{bs} - h_u)L4)P - 2D h_{bs}(n-1)L4) - 2D h_u n L4 P}},$$

where

$$\begin{aligned}
A5 = R\Big[&2A_3L3\sqrt{L1}L2 + 2A_bL3\sqrt{L1}L2 + n(2c_fL3\sqrt{L1}L2 + A_tDP(A_tL3\sqrt{L1})\\
&+ h_u(4Dh_{bs}(D - Dn + nP)h_unPR)\Big[-2Dh_unL4P - (2Dh_{bs}(n-1)L4)\\
&+ n(A_t^2D - 2(h_{bs} - h_u)L4)P)R))\Big]^{\frac{1}{2}}\Big],\\
A6 = 2&\sqrt{L1}\,(Dh_unP + Dh_{bs}(n-1)R + (h_u - h_{bs})nPR(2Dh_unPL4))\\
&+ \left(2Dh_{bs}(n-1)L4 + n\left(A_t^2D - 2(h_{bs} - h_u)L3P\right)R\right).
\end{aligned}$$

$$\begin{aligned}
L1 &= Dh_u(4Dh_{bs}((n-1)D - nP) - h_unPR\\
L2 &= Dh_unP + Dh_{bs}(n-1)R + (h_u - h_{bs})nPR\\
L3 &= 2Dh_{bs}(n-1) + (h_u - h_{bs})nPR\\
L4 &= A_3 + Ab + nc_f
\end{aligned}$$

Proof. The first order condition gives the result. □

Special case. There is no financial and non financial holding costs and all members are involved in the supply chain having the same holding cost h.

Theorem 5.5. *The optimal decisions in decision strategy I for the above mentioned special case are as follows:*

$$\begin{aligned}
T^* &= 2\left[\frac{2A_bhnP(D+R)^2 + 2c_fhn^2P(D+R)^2 - A_t^2D((n-1)D - nP)R^2}{Dh^2(D(n-1) - nP)(2D+R)^2}\right]^{\frac{1}{2}},\\
\alpha^* &= \frac{g(2D+R)\sqrt{Dh^2(D(n-1) - nP)}}{4c_qD\sqrt{2A_bhnP(D+R)^2 + 2c_fhn^2P(D+R)^2 - A_t^2D((n-1)D - nP)R^2}},\\
\tau^* &= \frac{R(2A_t + hT)}{2hT(D+R)}.
\end{aligned}$$

Proof. The first order optimality conditions give the required result. □

6. Numerical analysis

To demonstrate the proposed model numerically, consider the following data set: $D = 1500$ units/unit time, $P = 8000$ units/unit time, $R = 5000$, $n = 10$ shipments, $A_v =$ Rs. 300/unit, $A_b =$ Rs. 400/unit, $A_3 =$ Rs. 100/unit, $h_{vs} =$ Rs. 2.7/unit, $h_{vf} =$ Rs. 3.4/unit, $h_{bs} =$ Rs. 2.6/unit, $h_u =$ Rs. 3/unit, $c_f =$ Rs. 30/unit, $c_m =$ Rs. 100/unit, $c_r =$ Rs. 30/unit, $c_3 =$ Rs. 20/unit, $c_q =$ Rs. 5/unit quality, $g =$ Rs. 2000, $\alpha_r = 0.2$, $A_t =$ Rs. 0.5/unit.

Table 3 shows that the optimum results for different strategies.

From the numerical results given in Table 3, we conclude that DS I is the best among the three strategies from cost perspective. Although, the return rate is higher in DS I, the third party is gainer when he follows DS III. Also, DS III is the best from product quality perspective. It is true in real market that, to produce the best quality product, the manufacturer has to invest high capital. As a result, product price will be high and product quantity will be low. Decision strategy II gives the worst quality product with higher quantity, and it incurs cost higher than the DS I. From the manufacturer's and the retailer's cost perspectives, the sequence of the decision strategies is given by DS I < DS II < DS III whereas from the third party's cost perspective, the sequence of decision strategies would be DS III < DS I < DS II. The sequence DS II < DS I < DS III from

TABLE 3. Optimal results for different strategies.

Strategy	T	α	τ	Q	$\mathrm{TC}_V + \mathrm{TC}_R$	TC_B	TC_P	TC
I	1.088	0.6127	0.502	1632.01	115 735.4	1079.97	895.33	117 711
II	1.2762	0.5223	0.4850	1914.31	11 711.4	1097	956.70	119 165
III	0.7083	0.9412	0.0256	1062.45	151 757.98	2078.17	200.01	154 036

TABLE 4. Holding cost variation in decision strategy I.

Variation	$h_{bs}(3) > h_{vs}(2.7)$	$h_{bs} = h_{vs}(= 2.7)$	$h_{vs} = h_{vf} = h_{bs} = h_u(= 3)$
T^*	1.0157 (−6.64%)	1.0684 (−1.80%)	1.0157 (−6.64%)
α^*	0.6563 (7.12%)	0.6240 (1.84%)	0.6563 (7.12%)
τ^*	0.5108 (1.75%)	0.5046 (0.52%)	0.5108 (1.75%)
$\mathrm{TC}_V + \mathrm{TC}_R$	115 089 (−0.56%)	115 568 (−0.14%)	114 901 (−0.72%)
TC_B	1143.73 (5.90%)	1096.53 (1.53%)	1143.73 (5.90%)
TC_p	873.744 (−2.41%)	889.353 (−0.67%)	873.744 (−2.41%)
TC	117 107 (−0.51%)	117 554 (−0.13%)	116 919 (−0.67%)

quality perspective and the sequence DS III < DS I < DS II from quantity perspective are opposite, which proves the real market case that quality and quantity are *vice versa*. The return rate is very low in DS III, which is the another reason to produce high quality products in DS III.

In CS model, retailer's non-financial holding cost is usually lower than the manufacturer's non-financial holding cost [41]. Table 4 shows the result when the retailer's non-financial holding cost is higher than the manufacturer's non-financial holding (in usual shipment policy) or both equal or all components are equal. All the results are examined on the DS I. The percentages given in the Table 4 show the change of the decisions with respect DS I which are given in Table 3. From the table, it is clear that, in all situations, the manufacturer incurs lower cost, which is predictable as his holding cost is lower. On the other hand, the retailer incurs more cost in all situations. Cost of total supply chain is lower for the higher value of holding cost in the retailer's section. The optimal quality and return rate are higher when h_{bs} increases. The cycle length becomes lower as h_{bs} increases to prevent the total cost's increase value.

7. Sensitivity analysis

In this section, the effects of changes in the parameter-values on the optimal decisions of the proposed model have been examined. The value of one parameter at a time keeping the other parameter-values unchanged.

7.1. Sensitivity on optimal cost function

In this subsection, we observe the sensitivity of some key parameters on the optimal cost function in all three cases which are given in the Table 5.

– The changes of vendor's non-financial holding cost h_{vs} always effect positive impact on the cost of supply chain. Cost always increases linearly for all decision strategies but the rate of changes is very nominal.
– The changes of buyer's non-financial holding cost effect reversely on the cost of supply chain in first two decision strategies. But in the rate of change of cost in third decision strategy is very high and has a direct impact. It signifies that, for less non financial holding cost in buyer's place reduces the cost of supply chain. Whereas, in other case the impact of the change in h_{bs} is less sensitive.

Table 5. Sensitivity of model parameters.

Parameter	Change	Change in TC (change in %)		
		DS I	DS II	DS III
h_{vs}	−50%	117 706 (−0.0042%)	119 159 (−0.0050%)	154 023 (−0.0084%)
	−25%	117 708 (−0.0025%)	119 162 (−0.0025%)	154 029 (−0.0045%)
	0%	117 711 (0%)	119 165 (0%)	154 036 (0%)
	25%	117 713 (0.0017%)	119 168 (0.0025%)	154 042 (0.0039%)
	50%	117 716 (0.0042%)	119 172 (0.0058%)	154 048 (0.0078%)
h_{bs}	−50%	120 340 (2.23%)	121 280 (1.77%)	130 701 (−15.15%)
	−25%	118 852 (0.97%)	120 054 (0.75%)	139 999 (−9.11%)
	0%	117 711 (0%)	119 165 (0%)	154 036 (0%)
	25%	116 757 (−0.81%)	118 456 (−0.59%)	177 591 (15.29%)
	50%	115 924 (−1.52%)	117 859 (−1.10%)	225 757 (46.56%)
R	−50%	123 981 (5.33%)	125 098 (4.98%)	154 664 (0.41%)
	−25%	120 049 (1.99%)	121 378 (1.86%)	154 300 (0.17%)
	0%	117 711 (0%)	119 165 (0%)	154 036 (0%)
	25%	116 162 (−1.32%)	117 699 (−1.23%)	153 834 (−0.13%)
	50%	115 062 (−2.25%)	116 657 (−2.10%)	153 676 (−0.23%)
c_m	−50%	116 422 (−1.10%)	80 545.7 (−32.41%)	80 962.2 (−47.44%)
	−25%	117 114 (−0.51)	99 855.4 (−16.20%)	117 499 (−23.72%)
	0%	117 711 (0%)	119 165 (0%)	154 036 (0%)
	25%	118 236 (0.45%)	138 475 (16.20%)	190 572 (23.72%)
	50%	118 704 (0.84%)	157 785 (32.41%)	227 109 (47.43%)
c_q	−50%	117 163 (−0.47%)	118 770 (−0.33%)	152 708 (−0.86%)
	−25%	117 530 (−0.15%)	119 036 (−0.11%)	153 593 (−0.29%)
	0%	117 711 (0%)	119 165 (0%)	154 036 (0%)
	25%	117 816 (0.09%)	119 240 (0.063%)	154 301 (0.17%)
	50%	117 883 (0.15%)	119 287 (0.10%)	154 478 (0.29%)
c_r	−50%	106 406 (−9.60%)	108 251 (−9.16%)	153 458 (−0.38%)
	−25%	112 058 (−4.80%)	113 708 (−4.58%)	153 747 (−0.19%)
	0%	117 711 (0%)	119 165 (0%)	154 036 (0%)
	25%	123 363 (4.80%)	124 622 (4.58%)	154 325 (0.19%)
	50%	129 016 (9.60%)	130 079 (9.16%)	154 614 (0.38%)
c_f	−50%	116 422 (−1.10%)	118 138 (−0.86%)	150 781 (−2.11%)
	−25%	117 114 (−0.51%)	118 684 (−0.40%)	152 510 (−0.99%)
	0%	117 711 (0%)	119 165 (0%)	154 036 (0%)
	25%	118 236 (0.45%)	119 595 (0.36%)	155 398 (0.88%)
	50%	118 704 (0.84%)	119 984 (0.69%)	156 627 (1.68%)

– The rate of production of returned product R reversely impact on the cost functions. This implies that more we produce product from the returned product we can save the cost of the system or we can gain our profit which is realistic in the real market. But more production of returned product will be directly impact on the product quality.
– The effect of unit manufacturing cost is linearly increasing the cost for every decision strategies when we increase the c_m. From the real market situation it is obvious. But it significantly impacts on cost function in DS III. Here retailer and third party acts as a unit which can impact on the manufacturer's leadership so the cost increases significantly.
– The effect of c_q, c_r and c_f is same for every decision strategies on the supply chain cost. The cost will increase for higher values of these cost parameter values.

7.2. Comparative study on optimal decisions

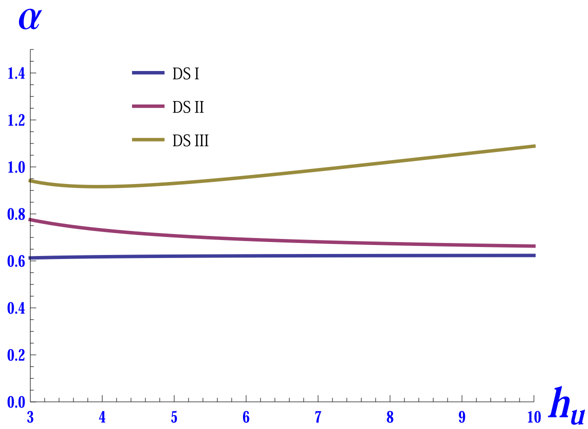
In this subsection of sensitivity analysis consists of a comparative study of the effective parameters on the the optimal decisions in three different models. This study is described with the help of Figures 3–6.

- For increasing value of the third party's holding cost h_u, the quality of the product increases in DS III, whereas it decreases in DS II and almost remains unchange in DS I. The increasing rate for the quality in DS III is much higher than the rate in other strategies. It implies that for DS III, the manufacturer increases the quality of the product to reduce the collection rate by the third party (see Fig. 3a).
- The cycle length T increases with respect to the third party's holding cost (h_u) for DS I and DS II. But the cycle length first slightly increases and then decreases in DS III (see Fig. 3b).
- The collection rate decreases with respect to increasing holding cost of the third party for DS I and DS II. But in DS III, the collection rate increase for higher value of h_u. In this strategies DSI and DS II, the third party individually optimize his cost function. As a result, direct impact of higher holding cost on third party inventory. So, he bounds to reduce his collection rate to keep his cost minimization priority. In strategy DS III, the third party and the retailer jointly optimize their total cost (see Fig. 3c). In spite of higher holding cost at third party's place, their main priority is to minimize their total cost. Also in real life situation if retailer control the third party to collect the used product they can share warehouse facility to third party. As a result, the third party keep his collection higher in spite of higher holding cost.
- For increasing number of shipment size, the quality of the product decreases for all strategies. However, for more shipments, the cycle length increases for all strategies. From real business perspectives, it is true that if the number of shipments increase to transport any product then the time cycle must be increased in some sense. For being higher cycle length, the product quality must be less than the shorter cycle length's products (see Figs. 4a and 4b). The rate of change of the collection rate with respect to shipment size is negligible for all strategies (see Fig. 4c).
- For higher non-financial holding cost (h_{bs}) of the retailer, the quality set by the manufacturer increases for all strategies. The rate of change of the quality is higher in DS III (see Fig. 5a). The behaviors of the cycle length is just opposite to the quality of the product with respect to h_{bs}. For DS III, the cycle length tends to zero *i.e.*, the cost of the individual supply chain members tends to much higher side (see Fig. 5b). The collection rate slightly increases for higher value of h_{bs} in DS I and DS II. On the other hand, the collection rate is rapidly decreasing for increasing value of h_{bs} in DS III but in DS I and II, it increases nominally (see Fig. 5c).
- The optimal decisions are less sensitive to the transportation cost (c_f). The quality and the collection rate decrease for increasing c_f, whereas the cycle length increases for higher transportation cost (see Figs. 6a–6c).

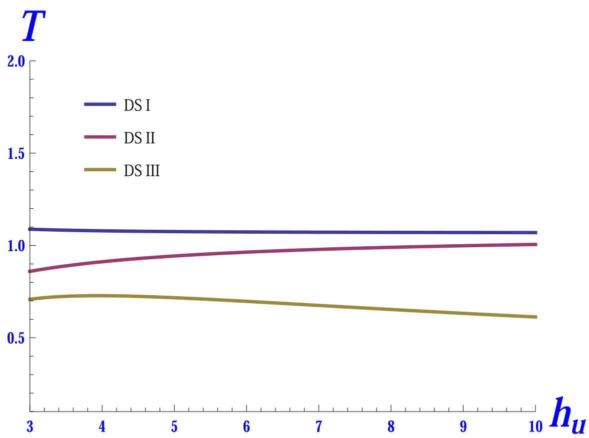
8. Managerial insights

The work stimulates to help the manufacturer for more better coordination between the members of the supply chain where the manufacturer and the remanufacturer activities utilize the same production facility. The relevance of this work describes in the following managerial implications, which mainly based on the results presents in the previous sections.

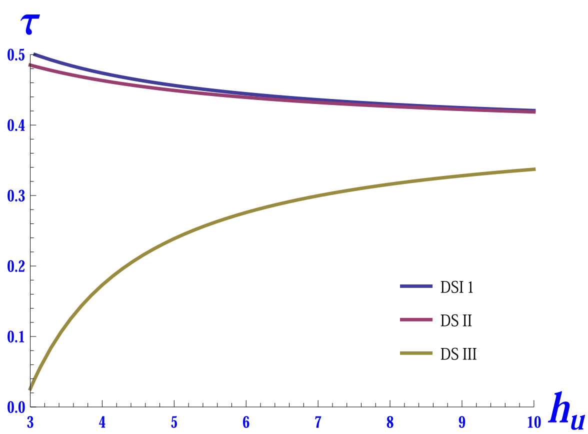
- Better coordination policies are achieved for more cost effective supply chain between the manufacturer and the third party.
- Under the manufacturer Stackelberg game, the manufacturer and the retailer prefer the DS I *i.e.*, the decision sequence must be in the order manufacturer – retailer – third party. The third part prefers DS III, where the retailer and third party play a collaborative decision strategy under manufacturer's leadership.
- The manufacturer produces higher quality product when the retailer and the third party collaboratively decide their optimum decisions. The return rate becomes very low when the retailer and the third party collaborative, which is one reason to produce high quality products.



(a) h_u vs. α

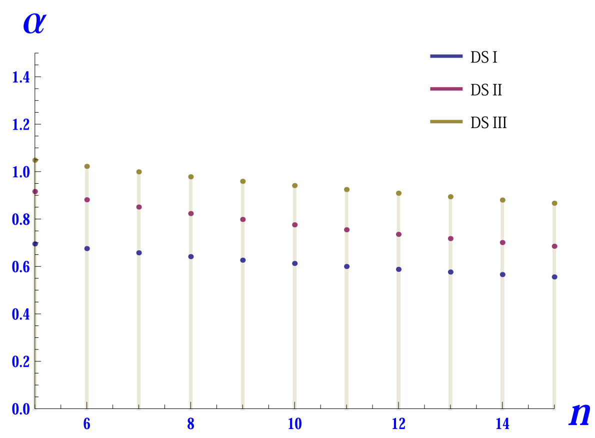


(b) h_u vs. T

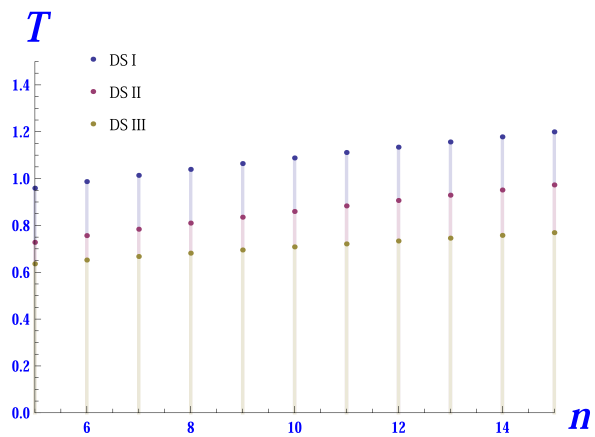


(c) h_u vs. τ

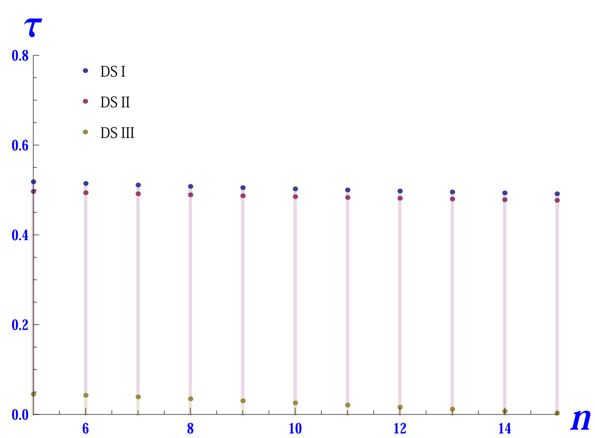
FIGURE 3. Sensitivity of decision variables w.r.t. h_u.



(a) n vs. α

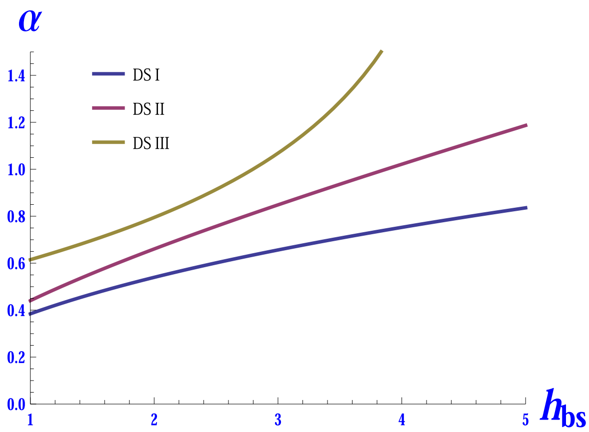


(b) n vs. T

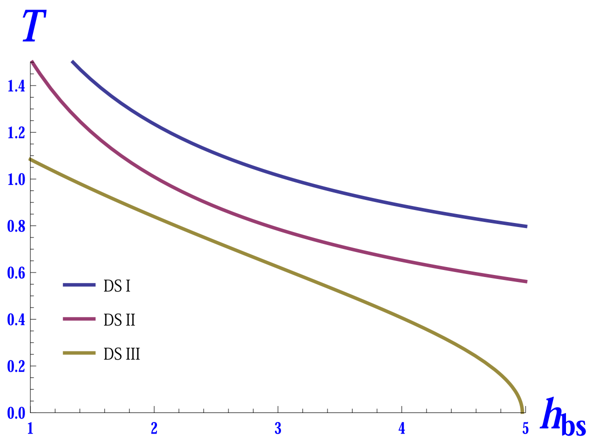


(c) n vs. τ

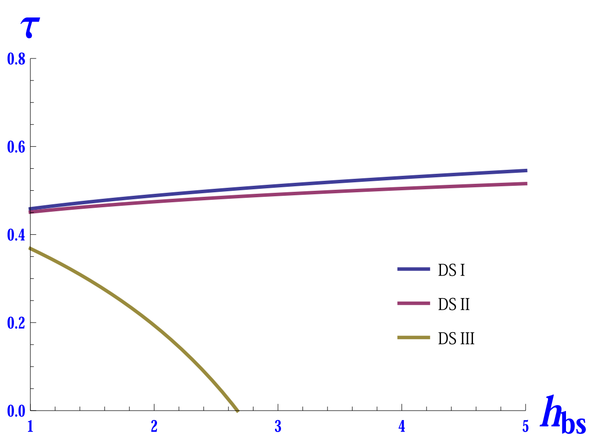
FIGURE 4. Sensitivity of decision variables w.r.t. n.



(a) h_{bs} vs. α

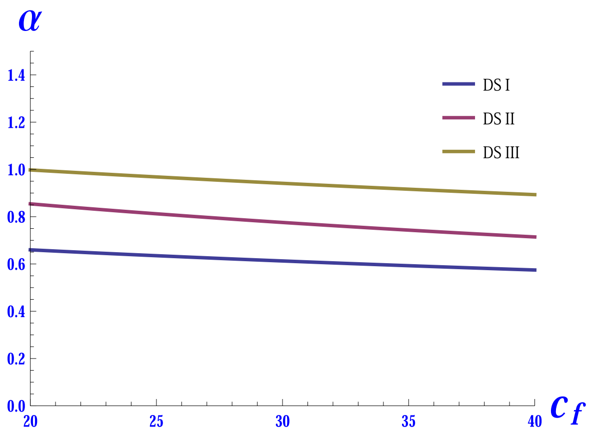


(b) h_{bs} vs. T

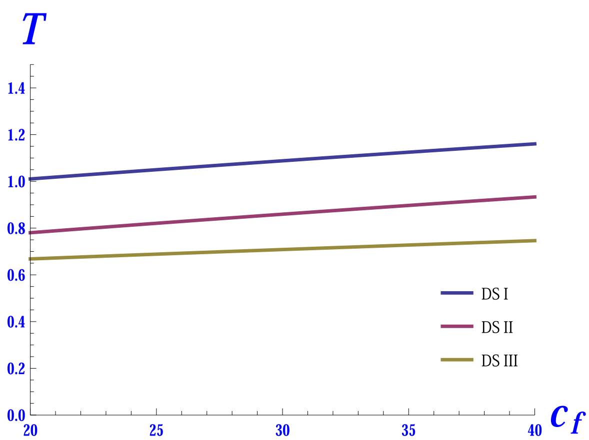


(c) h_{bs} vs. τ

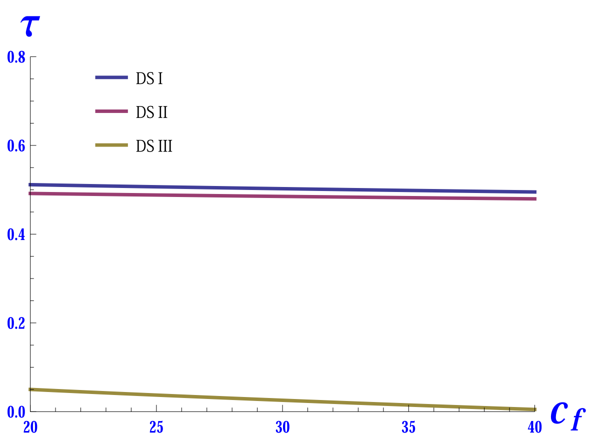
FIGURE 5. Sensitivity of decision variables w.r.t. h_{bs}.



(a) c_f vs. α



(b) c_f vs. T



(c) c_f vs. τ

FIGURE 6. Sensitivity of decision variables w.r.t. c_f.

– From our numerical study, it is also clear that if any manager wants to produce larger quantity in quick time, it will give low quality products and *vice versa*.
– The results obtained from the sensitivity analysis indicate that third strategy (DS III) has a reverse behavior rather than the other two strategies.

9. Conclusions and future research directions

This paper introduced a new aspects to literature that the consignment stock policy can be implemented in a closed loop supply chain. More over the model discusses three different types of decision strategies with their managerial aspect. The comparison of three strategies gives various aspects. The system is gainer when they follow the Stackelberg game *i.e.*, DS I. But the product quality is not best in this situation. In the DS III, the quality of product is higher with higher cost in the system. One another aspect has made in the this paper that repairing the EOL items is also profitable for the CLSC which is also environmental friendly. This study also reveals that if the retailer collaborates or insists to third party to work with him/her the product quality will be increased. This study gives the manager to choose their strategy with their different requirements. The present work is suitable for the manufactures where the manufacturing and remanufacturing activities utilize the same production facility.

Since in this study the demand has been taken as constant demand, the immediate extension of the paper would be to consider the variable deterministic or stochastic demand. The shortage may be allowed with low re-manufacturer rate in the CS policy. Since deterioration plays a vital role in the optimal decisions of any kind of inventory, it would be interesting to study the impact of deterioration in consignment stock policy for CLSC. The present model could also be extended by assuming learning in manufacturing and remanufacturing. Finally, the green house gas emission for the given model will be very interesting extension of the paper [4]. A multi-retailer case is more real problem for the extension of this paper. One can considered a dual-channel problem in forward or reverse direction of the system.

Appendix A.

To find the retailer's holding area, we divide the region in three parts. First part consists of $n-1$ similar type of trapeziums same as the first shipment area for n shipments. This area is given by

$$\mathrm{HC}_b^1 = \frac{q_1}{2P}\left(q_1 + q_1 - \frac{q_1}{P}D\right)(n-1).$$

For the second part, we use increment for successive shipments. Clearly, the increment for kth shipment in the retailer's inventory is given by

$$I_k = kq_1 - \frac{(k-1)q_1 D}{P}.$$

Therefore, the area which consists of $(n-2)$ rectangles is given by

$$\mathrm{HC}_b^2 = \frac{q_1}{P}I_2 + \frac{q_1}{P}I_2 + \ldots + \frac{q_1}{P}I_{n-1} = \frac{q_1^2(n-2)(n-1)}{2P}\left(1 - \frac{D}{P}\right).$$

The third part is the area for last shipment. It consists of one triangle with height $I_{\max}$. Therefore, the area is given by $\mathrm{HC}_b^3 = \frac{(I_{\max})^2}{2D}$.

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