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A MULTI-RETAILER SUSTAINABLE SUPPLY CHAIN MODEL WITH INFORMATION SHARING AND QUALITY DETERIORATION

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Abstract. With the effect of increasing production rate, the probability of shifting the manufacturing process from "in-control" state to "out-of-control" increases with the passage of time. This happens due to the degradation of the mechanism which results in production of defective items. This study helps in examining the effect of changed production rate on the quality of goods produced. This research further examines the influence of manufacturing rate on "mean time to failure (MTTF)". This increased production rate is not always environmental friendly due to the emission of contaminated gases after production process. The idea of making a specific investment initiation is incorporated in this paper to attain a sustainable environment development. Also, the information exchange is assumed in the supply chain system to achieve a better profitability. The mathematical model thus created and is validated with enough data, numerical experimentation, and graphical representation. The study concluded that higher degree of quality function reduces the MTTF of machine, also setup and environmental investment has highest impact on the total cost.

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1. Introduction and motivation

A smooth conduct of supply chain requires appropriate contribution from each of its contributing sector. In other words, information flow between several parties must be continued properly. Thus, an integrated supply chain model retains a valuable contribution in forming a successful supply chain. The pioneer approach on integrated supply chain management was introduced by Goyal [16] which was later extended with a joint economic lot-size model [17]. A supply chain with various intermediate parties faces difficulties to maintain the sharing of information due to demand uncertainty. Therefore, considering random demand is a matter of concern for supply chain modelling. A very well-known approach to deal with uncertain demand is to handle with a normal distribution. A significant number of articles used this distribution to solve and obtain the managerial decisions in an integrated supply chain management [24,27,28,31,33]. An important aspect was left out of the discussion of the literatures, which was existence of multiple retailers. Inclusion of multi-retailer in an integrated channel was introduced by Banerjee and Banerjee [3]. Later on many researchers studied and extended the basic

Keywords. Supply chain management, variable production cost, quality management, controllable lead time, single-setup multi-delivery policy, sustainability.

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idea of the existence of multi-retailer in their studies [4,21,26,28]. Moreover, a smooth conduction of delivery of items is another vital parameter to cope up with customer satisfaction. One of the most useful ways to achieve this is to reduce the lead time. A lead time is composed of many components such as supplier's lead time, order preparation, order transit, delivery time, and the setup time [24,39]. Reduction of each component of lead time leads to an achievement of successful supply chain. Therefore, researchers has been creating and implementing efficient methods to reduce lead time from decades. The investment (lead time crashing cost) to shorten lead time was studied by many literatures [24,27,32,34].

The role of manufacturer in the supply chain has significant importance in maintaining the system reliability. After a certain period of time the system may shift from "in-control" to "out-of-control" state and begin producing defective items. The probability of shifting one state to another state can be reduced by an investment [35,36]. Increased production rate is one of the most crucial reasons behind this situation. As an example, in a robotic assembly manufacturing system, increasing rate of production may result the deterioration of the repeatability of robotic arm [23]. As the arm speed is increased to raise the production rate, robot repeatability deteriorates. "Repeatability is defined as the ability of the robot to return to the same point, and is critical for product quality. The deterioration of repeatability results in a decrease in the percentage of conforming units produced by the robot [29]". Offodile and Ugwu [30] also supported the idea of deterioration of robotic arm with repeatability. They induced that process variables, especially speeds and weight highly affects robot performance. Conrad and McClamrock [11] studied a drilling operation which concluded that 10% change in processing rate of the drilling machine results 50% change in tool cost. Therefore, production rate plays an extremely vital role in controlling system reliability as well as production or machine tool cost. Wang *et al.* [47] enlightened on the issue of quality deterioration during production process especially when the process reaches to "out-of-control" state. They considered the adaptation of predictive maintenance policy to prevent defective production. Cheng and Lee [9] emphasised that deterioration of machine during production process influenced quality of product. Hence, a rapid quality check and machine maintenance are required to meet the product conformance.

Excess production rate also has an impact of environmental sustainability due to which industries release additional carbon in the environment. Environmental degradation, global warming, and strict governmental rules force industries to adopt green initiatives and incorporate sustainability practices into their supply chain. An additional charge termed as environmental sustainability cost has to be incurred by the companies, which is added for accounting social welfare. This environmental cost is one of the components of total cost of entire supply chain. Though environmental impact is one of the most important concerns in sustainable development, many researchers considered economic and social impact also along with environmental sustainability [18,19]. As the sustainability in supply chain was limited to optimization of environmental factors only, researchers gradually considered joint decision-making with manufacturing, disposal, and customer service also along with sustainable development [25]. Again, sustainable order quantity (SOQ) model and economic order quantity (EOQ) model with sustainability were developed and became a matter of concern [5,7]. Later on a significant number of definitions on several aspects regarding green and sustainable supply chain were stated [1,22].

On the above context of the study, we set the objectives of this article. The objective of this research is to develop a two echelon supply chain system with single-vendor multi-buyer integrated supply chain system under demand uncertainty. Lead time plays a crucial role for customer satisfaction and uncertainty in lead time demand makes the system vulnerable towards reduced profitability. Therefore, reducing lead time is one of the most important tasks for the managers to enhance the customer's demand satisfaction. In addition to that another parameter also plays important role for customer satisfaction such as quality of product. The product quality depends on system reliability as a reliable manufacturing system produces an insignificant number of defectives. Therefore, a study on the effects of reliability under increasing rate of production along with lead time reduction strategy is another important goal of this study. Due to strict government regulations and increasing emission of greenhouse gases, a sustainable supply chain system has become a matter of concern. One of the vital objectives of this study is to analyze the effect of environmental for the sustainable development on the entire system cost of the chain. Therefore, the aim of this study is to minimize the supply chain system cost under the factors discussed above on the centralized and decentralized supply chain and to establish a comparative study between the two systems.

The whole article is divided into many sections. Section 2 describes the review of existing literatures. An author's contribution table (Tab. 1) depicting the research gaps is also added in this section. Section 3 includes problem definition, assumptions and notation to develop the mathematical model. Section 4 elaborates the mathematical model with an unconstrained nonlinear programming problem and also an efficient solution algorithm is depicted in this section to solve the model. Section 5 discusses the numerical experimentation and sensitivity analysis. Sections 6 and 7 include the managerial insights and conclusions, respectively.

2. Literature review

Quality deterioration during production process

Many researchers studied the case of the shifting of the manufacturing system from "in-control" to "out-of-control". The stage when the process enters into "out-of-control" state, the chances of producing defective items increases significantly. Thus, the quality of product deteriorates with on-going production process. Porteus [35] stated the shifting of perfect quality production to imperfect quality production due to changed production rate. Rosenblatt and Lee [36] contemplated that the shifting of state occurs after a period η. The time of shifting is a negative exponentially distributed random variable with a specified mean. "The exponential assumption is motivated by the observation that, beyond some initial age, the hazard function of a machine is relatively flat so that the failure rate is approximately constant". Khouja and Mehrez [23] stretched this work and established a relation between the mean of the random variable and production rate. They reviewed quality function in linear and quadratic polynomial.

In integrated supply chain model where quality of production does not remain same throughout the process, the increased production rate draws our attention on the safety of environment as well. Hence, Sarkar *et al.* [42] used a realistic approach for single supplier and multiple buyer by viewing production rate as decision variable instead of a parameter in machine manufacturing based system. With increased production, the machine components start dying and results in production of sub-standard goods. This condition more likely appeared in robot-based production where the robot is used repeatedly to raise production rate [29]. The increased production influences the release of vulnerable gases. With rapid climate change, sustainability is becoming a corporate social responsibility. They need to incorporate green initiates and seek effective strategies to attain sustainable development in SCM [46]. To celebrate century of the EOQ model, an honour to Ford Whitman Harris was presented by Cárdenas-Barrón *et al.* [8].

Supply chain management with single and multiple retailers

"Supply chain management (SCM) is a collaboration among suppliers, manufacturers, retailers, and customers. The supply chain model is used to minimize the total cost or to maximize the total profit throughout the network under the condition that demands of each facility have to be met" [39]. The thought of simple integrated inventory system was pioneered by Goyal [16]. Banerjee [4] reviewed this system for either both parties receive benefit or none incur losses. Goyal [17] further modified this model with a combined economic-batch-size model for retailer and vendor. Banerjee and Burton [2] discriminated "coordinated" and "independent" restocking policies for a manufacturer and for more than one customer. Lead time variable inventory model was proposed by Ben-Daya and Raouf [6]. Chung [8] scrutinized an inventory model for deteriorated items and consider pricing policy, the out-of-order production, the warranty-period, the inspection planning, and the demand depend on stock-level. In addition to this, Chung [8] established an integrated inventory model for manufacturer and retailer to obtain ideal order quantity, cycle length and total inventory cost. A supply chain model with variable back ordered was presented by Sarkar [38] and two different integrated inventory model under uncertain condition and advertising dependent demand was presented by Sarkar *et al.* [41]. Sarkar and Majumder [39] and developed integrated supply chain models in which methods were discussed to reduce setup cost which has remarkable impact on minimizing the overall expected supply chain cost. An inventory model was discussed by Sarkar and Giri [44] in uncertain demand environment.

The integrated models with imperfect quality have always been the interest area of many. Like, Sarkar *et al.* [44] worked on imperfect production manufacturing system and provided its safety stock, optimal batch size and reliability. Sana [37] offered an inventory model for manufacturer and retailer. He suggested an integrated model to cut the production of substandard items. Gao [15] developed a probabilistic models for the production and operational incitements by improving quality and coordination in supply chains with cots allocations. Majumder *et al.* [28] and Dey *et al.* [13] proposed the improvement in the production quality with the reduction in setup cost and setup time for vendor-buyer supply chain model.

In a realistic scenario, focusing on multiple retailers has more importance than a single one. Jha and Shanker [21] developed a single-vendor multi-buyer integrated production inventory model. They studied the effect of service level constraint with controllable lead time. In the same direction production rate dependent lead time was calculated by Sarkar *et al.* [45]. Banerjee and Banerjee [3] introduced a coordinated inventory model for single-vendor and multi-buyers under electronic data interchange (EDI) policy. Consequently, several studies on single-vendor multi-buyers were proposed in the literature [2, 26, 48]. Sarmah *et al.* [45] developed a centralized system of single-vendor with multiple heterogeneous buyers and focused on the negotiation to obtain the due of extra saving resulting from coordination. Recently, Dey *et al.* [12] worked on coordinated supply chain model where setup cost decreased by discrete investment and process quality was improved by a logarithmic investment function and expected total profit was optimized.

Lead time reduction

While considering uncertain demand, the role of lead time becomes a topic of concern. A little lead time improves the customer's satisfaction level. Thus, lead time reduction plays an important role to achieve a successfull supply chain though an amount may be incurred by the owner. Liao and Shyu [24] proposed a probabilistic model with the assumption that demand must follow normal distribution and the lead time divided into n-constituents with different cost for the lessening of lead time. They proposed that there is no suitable inventory model that deals with lead time as a "decision variable". In their paper, they calculated the length of lead time to reduce the expected total cost. Pan and Yang [34] assumed the homogeneous inventory model in which lead time and set-up cost was degraded to achieve profitable business. Ouyang *et al.* [32] and allowed shortage and presumed lead time as random and controllable. Ouyang and Chen [31] developed an imperfect production model by considering improvement in quality and lead-time reductions in batch size reorder point. Huang *et al.* [20] developed a sustainable homogeneous inventory model for maximizing profit by considering demand as selling-price dependent to increase the sales, and lead time demand using Poisson distribution.

Sustainability

As the concern regarding the continuous degradation of environment rises day by day, companies have begun investing on the new technologies to reduce the emission of greenhouse gases. Therefore, a sustainable development in manufacturing and supply chain became an important issue for every industrialist. Based on the supply chain sustainable development, Hacking and Guthrie [18] and Herva and Roca [19] discussed on the influence of environmental, economic and social development. Linton *et al.* [25] suggested modifying the methodology to optimize sustainability in a supply chain. The study proposed that the optimization of environmental factors should be shifted to the optimization of entire supply chain operations. Bouchery *et al.* [7] developed a multi-objective inventory model with environmental, economic, and social tradeoffs which are imposed by different regulatory bodies. Battini *et al.* [5] assumed an EOQ model by considering jointly environmental and social sustainability factors in delivery operations. A numerous definitions and discussions regarding green supply chain and sustainable SCM were elaborated by Ahi and Searcy [1]. Khan *et al.* [22] incorporated the environmental and social investment for sustainable development in a centralized supply chain model.

Table 1 depicts the author's contribution table with some of the important literatures which compares the recent study with the previous literature.

TABLE 1. Author's contribution table.

Author's name	Supply chain	Multi-retailer	Controllable lead time	Variable production rate	Defective manufacturing	Mean time to failure	Environmental sustainability
Goyal [16]	✓						
Banerjee [4]	✓						
Porteus [35]					✓		
Rosenblatt & Lee [36]					✓		
Khouja & Mehrez [23]				✓	✓	✓	
Banerjee & Burton [2]	✓	✓					
Ouyang *et al.* [32]	✓		✓				
Jha and Shankar [21]	✓	✓	✓				
Majumder *et al.* [28]	✓		✓				
Khan *et al.* [22]	✓	✓					✓
This study	✓	✓	✓	✓	✓	✓	✓

3. Problem definition, notation, and assumptions

To define the problem proposed in this article has been divided into some attributes. Below mentioned points describes all attributes considered in this paper.

- **Supply chain with single-vendor multi-buyer and SSMD policy**
  The article considers a supply chain problem with single-vendor multi-buyer (generalized any number of buyers). Single-setup multiple-delivery policy is adopted to deliver items between the parties.
- **Uncertain demand**
  The annual demand of retailers is assumed as random and follows a normal distribution with a known mean and standard deviation.
- **Variable production rate and production cost**
  Other than many existing literatures, this article relaxes the assumption of constant production rate and assumes variable rate which is potentially acceptable in real manufacturing systems. Simultaneously, production cost also has a dependency in production rate.
- **Stochastic time in which the system shifts to another state**
  Shifting time from "in-control" situation to "out-of-control" is a random variable with exponential distribution. The mean of this distribution has an impact in determining the reliability of the system by creating a relation between the mean of the exponential distribution and mean time to failure (MTTF) of the system. Moreover, dependency on the production rate with system reliability is studied.
- **Environmental sustainability**
  An environmental sustainability cost is incurred by the vendor and all buyers to sustain the environment friendly manufacturing system as well supply chain management.

Notation

q_i order quantity delivered to ith buyer by vendor in a single lot (units).
k_i safety factor for ith buyer.
r_i reorder point for ith buyer (units).
s_i safety factor for ith buyer (units).
L_i length of lead time for ith buyer (units).
d_i demand per unit time for buyer i (units per unit time).
h_{bi} holding cost for buyer i per unit per unit time ($ per unit per unit time).
n number of buyers (integer).
O_{bi} ordering cost per order ($ per order).
π_i unit backorder cost ($ per unit backordered).
σ_i standard deviation for the demand.
P rate of production per unit time (unit per unit time).
$C(P)$ cost for production ($ per unit).
Q lot size delivered by vendor.
m number of lots delivered (positive integer).
S setup cost for vendor ($ per setup).
h_v holding cost for vendor ($ per unit per unit time).
R cost for rework per unit ($ per unit).
E_v environmental cost parameter for vendors.
E_{bi} environmental cost for buyer i.
t production run time (time unit).
X_i random lead time demand of ith buyer (normal distribution) with mean d_iL_i and standard deviation $\sigma_i\sqrt{L_i}$.

Assumptions

(1) As the article develops a single-vendor multi-buyer model, to fulfil each buyer's demand, the vendor supplies a total of $Q = \sum_{i=1}^{n} q_i$ items.
(2) Vendor uses SSMD policy for transportation, *i.e.*, produces mQ (m is any positive integer) items in a single setup, just after receiving the orders from all buyers to reduce setup cost. It is considered $q_i = Q\frac{d_i}{D}$ *i.e.* the equality $\frac{q_i}{Q} = \frac{d_i}{D}$ is satisfied [27].
(3) The rate of production is a variable quantity where the range of variation is $P_{\min}(P_{\min} > D = \sum_{i=1}^{n} d_i)$ and $P_{\max}$. The unit cost for production is dependent on the rate of production P. When the rate of production is increase, the quality of the product gradually deteriorates.
(4) The lead time L_i (for buyer i) has n_i mutually n independent components. For the jth component, $a_{i,j}$ = minimum duration, $b_{i,j}$ = normal duration, and $c_{i,j}$ = crashing cost per unit time. For the sake of convenience, it is assumed $c_{i,1} \leq c_{i,2} \leq \ldots \leq c_{i,n}$.
(5) For the ith buyer, it is assumed $L_{i,0} = \sum_{i=1}^{n} b_{i,j} \cdot L_{i,r}$ is the length of lead time with components $1, 2, \ldots,$ crashed to their minimum duration. Thus, $L_{i,r}$ can be expressed as $L_{i,r} = L_{i,0} - \sum_{j=1}^{r}(b_{i,j} - a_{i,j})$, $r = 1, 2, 3, \ldots, n$, and the lead time crashing cost per cycle $R_i(L_i)$ is expressed as $R_i(L_i) = c_{i,r}(L_{i,r-1} - L_i) + \sum_{j=1}^{r-1} c_{i,j}(b_{i,j} - a_{i,j}), L\epsilon[L_{i,r}, L_{i,r-1}]$.
(6) The elapsed time after the production system goes "out-of-control" is an exponentially distributed random variable and the mean of the exponential distribution is a decreasing function of the production rate.
(7) The cost for lead time crash is totally buyer cost component and shortages are allowed with fully backlogged.
(8) An environmental sustainability cost is incurred by all buyers and the vendor.

TABLE 2. Significances of costs parameters for buyer's side.

Cost	Expression	Description
Ordering cost	$\frac{O_{bi}d_i}{q_i}$	The expected cycle lengths of buyer i is $\frac{d_i}{q_i}$. Thus, the ordering costs of buyers are denoted by $\frac{O_{bi}d_i}{q_i}$.
Holding cost	$h_{bi}\left\{\frac{q_i}{2}+k_i\sigma_i\sqrt{L_i}\right\}$	When the ith buyer's inventory level reaches to the reorder point r_i, an order of quantity q_i is placed by the buyer. The expected inventory level before receipt an order is $r_i - d_iL_i$ and the expected inventory level just immediately after the delivery q_i is $q_i + r_i - d_iL_i$, for ith buyer. Thus, the average inventory over a cycle can be written as $\frac{q_i}{2} + r_i - d_iL_i$ which implies that the buyer's expected holding cost per unit time becomes $h_{bi}\left\{\frac{q_i}{2}+r_i-d_iL_i\right\}$. Now, r_i can be expressed as $r_i = d_iL_i + k_i\sigma_i\sqrt{L_i}$. Thus, holding cost for ith buyer becomes $h_{bi}\left\{\frac{q_i}{2}+k_i\sigma_i\sqrt{L_i}\right\}$.
Shortage cost	$\frac{\pi_i d_i}{q_i}E\left(X_i-r_i\right)^+$	X_i is the stochastic lead time demand and r_i is the reorder point for ith buyer, therefore the expected shortage at the end of the cycle is expressed as $E\left(X_i-r_i\right)^+$ for buyer i resulting $\frac{\pi_i d_i}{q_i}E\left(X_i-r_i\right)^+$ as shortage cost.
Lead time crashing cost	$R\left(L_i\right)\frac{d_i}{q_i}$	$R\left(L_i\right)$ is the lead time crashing cost per cycle and $\frac{d_i}{q_i}$ is cycle length, so lead time crashing cost is expressed as $R\left(L_i\right)\frac{d_i}{q_i}$.
Environmental cost	q_iE_{bi}	E_{bi} is the environmental cost per unit item, thus the total environmental cost per cycle is q_iE_{bi}.

4. Mathematical model

The integrated single-vendor multi-buyer model is developed in this article. The following sections describe the cost expressions for buyers, vendor, and centralized system.

4.1. Mathematical model for buyers

Since, E_{bi} is the environmental cost for each buyer i, thus, the environmental cost component of each inventory cycle for each buyer i should be q_iE_{bi}. The total cost for buyer i for every inventory cycle is given by (1).

$$\mathrm{ETC}_{bi}\left(q_i,k_i,L_i\right)=\begin{bmatrix}\frac{O_{bi}d_i}{q_i}+h_{bi}\left\{\frac{q_i}{2}+k_i\sigma_i\sqrt{L_i}\right\}\\ +\frac{\pi_i d_i}{q_i}E\left(X_i-r_i\right)^+ + R\left(L_i\right)\frac{d_i}{q_i}+q_iE_{bi}\end{bmatrix} \tag{4.1}$$

where, the cost components are illustrated in Table 2.

The expression of expected shortage at the end of the cycle can be written as $E\left(X_i-r_i\right)^+ = \sigma_i\sqrt{L_i}\Psi\left(k_i\right)$. Where, $\Psi\left(k_i\right)=\phi\left(k_i\right)-k_i\left(1-\Phi\left(k_i\right)\right)$, $\phi\left(k_i\right)$ and $\Phi\left(k_i\right)$ are standard normal probability density function and distribution function of normal variate, respectively.

Thus, (4.1) can be written as follows

$$\mathrm{ETC}_{bi}\left(Q,k_i,L_i\right)=\begin{bmatrix}\frac{O_{bi}d_i}{Q}+h_{bi}\left\{\frac{Q}{2D}d_i+k_i\sigma_i\sqrt{L_i}\right\}\\ +\pi_i\sigma_i\sqrt{L_i}\psi\left(k_i\right)\frac{D}{Q}+R\left(L_i\right)\frac{D}{Q}+Q\frac{d_i}{D}E_{bi}\end{bmatrix}. \tag{4.2}$$

4.2. Mathematical model for vendor

The expression of the expected total cost for the vendor used which is similar as Sana [37] but environmental cost of vendor. As, in a single production cycle, vendor produces, mQ number of lots, thus, the environmental

TABLE 3. Significances of costs parameters for vendor's side.

Cost	Expression	Description
Setup cost	$\frac{S_v D}{mQ}$	In SSMD policy, vendor produces integer multiple of buyer's order quantity. As, Q is the total order quantity of all buyers ($Q=\sum_{i=1}^{n} q_i$), vendor produces mQ quantity where, m is a positive integer. The expected cycle length of the vendor thus becomes $\frac{D}{mQ}$ and the setup cost of vendor becomes $\frac{S_v D}{mQ}$.
Holding cost	$\frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$	The average inventory of the vendor can be calculated as $\frac{\left[\left\{mQ\left(\frac{Q}{P}+\frac{(m-1)Q}{D}\right)-\frac{m^2Q^2}{2P}\right\}-\left\{\frac{Q^2}{D}(1+2+\ldots+(m-1))\right\}\right]D}{mQ} = \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$. Thus, the expected holding cost per unit time per unit item is $\frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$.
Rework cost	$\mathrm{RD}\alpha f(P)\frac{Q}{2P}$	The expected number of defective units in a lot size Q is given by $E(N)=\alpha P\left[\frac{Q}{P}+\frac{1}{f(P)}\exp\left(-\frac{Qf(P)}{P}\right)-\frac{1}{f(P)}\right]$ [22]). From Maclaurin's series, $\exp\left(-\frac{Qf(P)}{P}\right)=1-\frac{Qf(P)}{P}+\frac{(Qf(P))^2}{2P^2}$ (higher powers are nullified due to small $f(P)$). Which yields $E(N)=\alpha f(P)\frac{Q}{2P}$, resulting the expected rework cost as $\mathrm{RD}\alpha f(P)\frac{Q}{2P}$.
Manufacturing cost	$\mathrm{DC}(P)$	The production cost per unit is $C(P)$, implies the total expected production cost is $\mathrm{DC}(P)$.
Environmental cost	mQE_v	E_v is the environmental cost per unit item per cycle, the expected total environmental cost is mQE_v.

cost of the vendor should become mQE_v. Therefore, total cost of the vendor possesses the expression elaborated by (4.3).

$$\mathrm{ETC}_v(m,Q,P)=\left\{\begin{array}{c}\frac{S_vD}{mQ}+\frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]\\ +\mathrm{RD}\alpha f(p)\frac{Q}{2P}+\mathrm{DC}(P)+mQE_v\end{array}\right\}. \tag{4.3}$$

Moreover, every cost components of (4.3) are illustrated in Table 3.

Significance of quality function $f(P)$ and production cost $C(P)$

The time in which the manufacturing process shifts "in-control" to "out-of-control" follows an exponential distribution with mean μ [36]. To establish the relation between the production rate and product quality, the mean μ is considered as an increasing function of production rate $f(P)$, which is denoted as the "quality function" [23]. This function relates the manufacturing sustainability to the rate of production. Conventionally, $f(P)$ is increasing in P such that $1/f(P)$ becomes a decreasing function in P. This $1/f(P)$ implies MTTF of the production system. Therefore, higher production rate leads to low MTTF which results degradation of system reliability as reduced MTTF is vulnerable to production of low quality products. Moreover, the manufacturing cost $C(P)$ is considered as a convex function of P as used in many literatures [12,14,42].

4.3. Centralized supply chain

In case of information sharing a centralized model should be implemented. Thus, the total expected cost jointly for all buyers and the vendor is established. In this case, we should consider the sum of costs of all buyers. Therefore, the expected joint total cost can be expressed by (4.4).

$$\mathrm{EJTC}(Q,k_i,L_i,P,m)=\sum_{i=1}^{n}\left[\begin{array}{c}\frac{O_{bi}d_i}{Q}+h_{bi}\left\{\frac{Q}{2D}+k_i\sigma_i\sqrt{L_i}\right\}+\pi_i\sigma_i\sqrt{\mathrm{L}_i}\Psi(k_i)\frac{D}{Q}+R(L_i)\frac{D}{Q}\\ +\frac{S_vD}{mQ}+\frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+\mathrm{RD}\alpha f(P)\frac{Q}{2P}\\ +\mathrm{DC}(P)+Q\left[mE_v+\frac{d_i}{D}E_{bi}\right]\end{array}\right]. \tag{4.4}$$

Now, the objective is to minimize EJTC with five decision variables Q, k_i, L_i, P, and m. To obtain the global optimum value with respect to the decision variables, first we take first order partial derivatives of EJTC with respect to the decision variables $Q, k_i, L_{i,}, P$, and m and put equal to zero. The sufficient condition of the global minimum of the objective cost function EJTC, the Hessian matrix of EJTC should be positive definite. But, due to some conditions, obtaining the global minimum through Hessian matrix is restricted for the decision variables Q, k_i, and P only. The reasons are stated below.

(1) The number of shipment m must be a positive integer.
(2) The second order partial derivative of EJTC with respect to L_i is negative $(i = 1, 2, \ldots, n)$.

$$\frac{\partial^2\mathrm{EJTC}\left(Q,k_i,L_i,P,m\right)}{\partial L_i^2}=-\frac{D}{4Q}\pi_i\sigma_i\Psi\left(k_i\right)L_i^{-\frac{3}{2}}-\frac{1}{4}h_{bi}k_i\sigma_iL_i^{-3/2}<0.$$

Thus, $\mathrm{EJTC}\left(Q,k_i,L_i,P,m\right)$ is concave for L_i for the fixed values of Q, k_i, m, and P. Therefore, in the interval $[L_{i,j}, L_{i,j-1}]$ $\mathrm{EJTC}\left(Q,k_i,L_i,P,m\right)$ is attained minimum value for the fixed value of Q, k_i, P and m.

As, m is a positive integer, discrete optimization technique is used to obtain the optimal value of m. The method follows the following inequalities to find the value of m. For fixed values of Q, k_i, P and L_i, the below mentioned inequality holds true.

$$\mathrm{EJTC}\left(Q,k_i,L_{i,},P,m-1\right)\geq\mathrm{EJTC}\left(Q,k_i,L_{i,},P,m\right)\leq\mathrm{EJTC}\left(Q,k_i,L_{i,},P,m+1\right)$$

For optimal m the process requires to find such a value of m so that the above inequality holds.

Now, to obtain the decision variables Q, k_i, and P, equate the first order partial derivatives with respect to the variables to zero. The results obtained are stated by (4.5)–(4.7).

$$Q=\sqrt{\frac{2D\left\{\frac{S_v}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\Psi\left(k_i\right)+R\left(L_i\right))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+\frac{\mathrm{RD}\alpha f(P)}{P}+\left(mE_v+\frac{d_i}{D}E_{bi}\right)}} \tag{4.5}$$

$$\Phi\left(k_i\right)=1-\frac{h_{bi}Q}{D\pi_i} \tag{4.6}$$

$$\frac{1}{P^2}=\frac{2h_v\mathrm{DC}\left(P\right)}{2QD\left(2-m\right)+h_vR\alpha DQ\left(f(P)-Pf'(P)\right)}. \tag{4.7}$$

Two separate cases are considered with two individual functions to explain the MTTF.

Case I: $\frac{1}{f(P)}=\frac{1}{b_1P}$ (The quality function $f(P)$ is linear in P).
Case II: $\frac{1}{f(P)}=\frac{1}{b_2P+c_2P^2}$ (The quality function $f(P)$ is quadratic in P).
Where, b_1, b_2, c_2 are non-negative scaling parameters.

From Figure 1 the effect of production rate on MTTF is clearly observed. As production increases, the MTTF of the system reduces simultaneously. This is also shown that quadratic quality fucntion affects the system reliability more than the linear case.

We use a special U shaped cost function for production cost $C(P)$ as

$$C(P)=\left(\frac{a_1}{P}+a_2P\right) \tag{4.8}$$

where, a_1 and a_2 are constants which give the best fit of the function.

Now, the optimal decisions and expected joint total cost based on two cases are as follows:

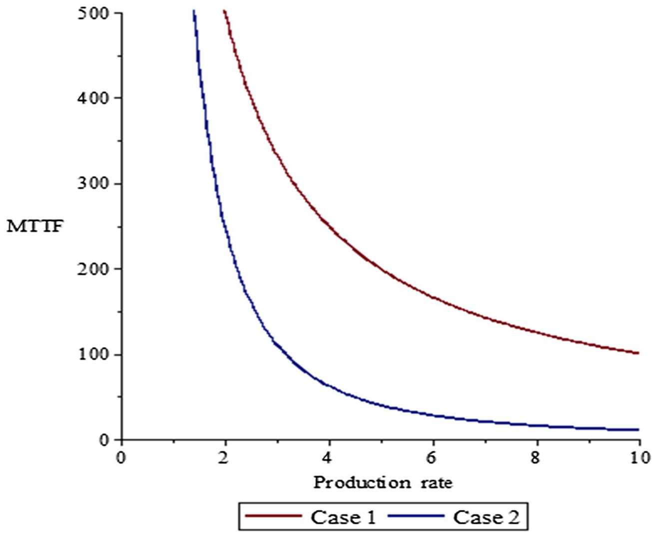

FIGURE 1. Graphical representations of MTTF.

Case I: $f(P)$ is linear in P

$$Q_1=\sqrt{\frac{2D\left\{\frac{S_v}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\Psi\left(k_i\right)+R\left(L_i\right))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P_1}\right)-1+\frac{2D}{P_1}\right]+\frac{\mathrm{RD}\alpha b_1P_1}{P_1}+\left(mE_v+\frac{d_i}{D}E_{bi}\right)}} \tag{4.9}$$

$$\Phi\left(k_i^1\right)=1-\frac{h_{bi}Q_1}{D\pi_i} \tag{4.10}$$

$$P_1=\sqrt{\frac{2a_1D-Q_1h_vD(m-2)}{2Da_2}}. \tag{4.11}$$

Then the total cost becomes

$$\mathrm{EJTC}(Q_1,k_i^1,L_i,P_1,m)=\sum_{i=1}^{n}\left[\begin{array}{l}\frac{O_{bi}d_i}{Q_2}+h_{bi}\left\{\frac{Q_2}{2D}d_i+k_i^1\sigma_i\sqrt{L_i}\right\}+\pi_i\sigma_i\sqrt{L_i}\Psi(k_i^1)\frac{D}{Q_1}\\ +R(L_i)\frac{D}{Q_1}+\frac{S_vD}{mQ_1}+\frac{Q_1}{2}h_v\left[m\left(1-\frac{D}{P_1}\right)-1+\frac{2D}{P_1}\right]\\ +\mathrm{RD}\alpha f(p)\frac{Q_1}{2p_1}+D\left(\frac{a_1}{P_1}+a_2P_1\right)+Q_1\left\{mE_v+\frac{d_i}{D}E_{bi}\right\}\end{array}\right]. \tag{4.12}$$

Case II: $f(P)$ is quadratic in P

$$Q_2=\sqrt{\frac{2D\left\{\frac{S_v}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\Psi\left(k_i\right)+R\left(L_i\right))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P_2}\right)-1+\frac{2D}{P_2}\right]+\frac{\mathrm{RD}\alpha(b_2P_2+c_2P_2^2)}{P_2}+\left(mE_v+\frac{d_i}{D}E_{bi}\right)}} \tag{4.13}$$

$$\Phi\left(k_i^2\right)=1-\frac{h_{bi}Q_2}{D\pi_i} \tag{4.14}$$

$$P_2=\sqrt{\frac{2a_1D-Q_2h_vD\left(m-2\right)}{2Da_2+R\alpha DQ_2b}}. \tag{4.15}$$

Then the total cost becomes

$$\mathrm{EJTC}(Q_1,k_i^2,L_i,P_2,m)=\sum_{i=1}^{n}\left[\begin{array}{c}\frac{O_{bi}d_i}{Q_2}+h_{bi}\left\{\frac{Q_2}{2D}d_i+k_i^2\sigma_i\sqrt{L_i}\right\}+\pi_i\sigma_i\sqrt{L_i}\Psi(k_i^2)\frac{D}{Q_2}+R(L_i)\frac{D}{Q_2}\\ +\frac{S_vD}{mQ_2}+\frac{Q_2}{2}h_v\left[m\left(1-\frac{D}{P_2}\right)-1+\frac{2D}{P_2}\right]+\mathrm{RD}\alpha(b_2P_2+c_2P_2^2)\frac{Q_2}{2P_2}\\ +D\left(\frac{a_1}{P_2}+a_1P_2\right)+Q_2\left\{mE_v+\frac{d_i}{D}E_{bi}\right\}\end{array}\right]\cdot \tag{4.16}$$

Proposition 4.1. *The joint expected total cost EJTC in Case 1 is positive definite in* Q_1, k_i^1*, and* P_1 *if the following condition*

$$\begin{aligned}&\frac{1}{Q_1}((2a_1-Q_1h_v(m-2))\sum\left(2\left(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\Phi(k_i^1)+R(L_i)\right.\right.\\&\quad\left.+\frac{S_v}{m}\right)\cdot\left(\sum\left(Q_1\pi_i\sigma_i\sqrt{L_i}k_i^1\varphi(k_i^1)\right.\right.\\&\quad\left.\left.\left.-\frac{D}{Q_1}\left(\sum\pi_i\sigma_1\sqrt{L_i}\left(1-\Phi(k_i^1)\right)\right)^2\right)\right)\right)\\&\quad>\frac{\left(\frac{m}{2}-1\right)^2h_v^2}{P_1}\cdot\sum\left(\pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)\right)\end{aligned}$$

is satisfied.

Proof. See Appendix A. □

Proposition 4.2. *The joint expected total cost EJTC in Case 2 is positive definite in* Q_2, k_i^2*, and* P_2 *if the following condition*

$$\begin{aligned}&\left(2\frac{D}{P_2^3}a_1-\frac{Q_2h_vD}{P_2^3}(m-2)\right)\cdot 2\sum\pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)\left(O_{bi}+R(L_i)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\psi(k_i^1)k_i^1\right)\\&-\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^1)\right)\right)^2>\left(m\frac{h_vD}{2P_2^2}\frac{h_vD}{P_2^2}+\frac{\mathrm{RD}\alpha c_2}{2}\right)^2\cdot\sum\left(\frac{D}{Q_2^2}\varphi_i\sigma_i\sqrt{L_i}k_i^2\varphi(k_i^2)\right)\end{aligned}$$

is satisfied.

Proof. See Appendix B. □

4.4. Solution algorithm

The iterative procedure is also applicable here as the closed form solution is unavailable. The following steps are given to develop the solution algorithm.

Step 1. Input values of all cost parameters and set $m = 1$. For each value L_i perform the following steps.
Step 1a. Obtain the values of Q from (4.9) and (4.13).
Step 1b. Obtain $\Phi(k_i)$ from (4.10) and (4.14) and find the values of k_i by inverse normal distribution.
Step 1c. Obtain P from (4.11) and (4.15).
Step 1d. Perform 1a–1c by updating the values until no changes occurs (upto a specified accuracy level) in Q, k_i, and P.
Step 2. Obtain the total cost from (4.12) and (4.16).
Step 3. Set $m = 2, 3, \ldots, p$ and perform the steps again.
Step 4. Obtain the minimum total cost for $m = j; 1 < j < p$.

5. Numerical experiments

The following two examples are given to check the applicability of the model.

Table 4. Lead time data.

Buyer i	Lead time component	Normal duration ($b_{i,r}$) (week)	Maximum duration ($a_{i,r}$) (week)	Unit crashing cost ($c_{i,r}$) ($ per unit)
1	1	20	6	0.1
	2	20	6	1.2
	3	16	6	5.0
2	1	20	6	0.5
	2	16	9	1.3
	3	13	6	5.1
3	1	25	11	0.4
	2	20	6	2.5
	3	18	11	5.0

Table 5. Optimal result table.

Decision variables	Case 1	Case 2
m	5	5
Q	297.309370	293.020404
k_1	1.710298	1.717145
k_2	1.710298	1.717145
k_3	1.678377	1.685314
r_1	69.246906	69.370151
r_2	77.096437	72.804442
r_3	88.812848	89.020964
P	552.633327	552.487068
$C(P)$	118.596466	118.598606
MTTF	18.095181	11.658694
EJTC	206 077.212853	206 129.039444

Example

The values of parameter are taken as follows for the illustration of the model numerically $A_{b1} = \$100$/setup, $A_{b2} = \$150$/setup, $A_{b3} = \$100$/setup, $d_1 = 200$ units/year, $d_2 = 100$ units/year, $d_3 = 100$ units/year, $S_v = \$4000$/setup, $h_v = \$10$/unit/week, $h_{b1} = \$11$/unit/week, $h_{b2} = \$11$/unit/week, $h_{b3} = \$12$/unit/week, $\sigma_1 = 9, \sigma_2 = 10, \sigma_3 = 15$, $\pi_1 = \$50$/unit, $\pi_2 = \$50$/unit, $\pi_3 = \$51$/unit, $a_1 = 35\times10^3$, $a_2 = 0.1$, $E_v = \$12.5$/unit, $b_1 = 10^{-4}$, $b_2 = 10^{-4}$, $c_2 = 10^{-6}$, $R = \$60$/unit.

Therefore, according to the parameter values, the MTTF functions of Cases 1 and 2 transforms as follows.

Case I: $\frac{1}{f(P)} = \frac{1}{10P}$.
Case II: $\frac{1}{f(P)} = \frac{1}{10P+10P}$.

The lead time data is given by Table 4.

Using the parametric values and lead time data (Tab. 4) optimal decision values of the decision variable along with optimized total joint minimum cost are obtained which are illustrated in Table 5.

TABLE 6. Sensitivity analysis for Case 1.

Case 1 (Linear quality function)					
Cost parameter	% change	Sensitivity	% change	% change	Sensitivity
S_v	−10	−1.175314	h_{b3}	−10	−0.082873
	−5	−0.581071		−5	−0.041200
	+5	0.568742		+5	0.040745
	+10	1.125912		+10	0.081052
A_{b1}	−10	−0.039403	h_v	−10	−0.097448
	−5	−0.019695		−5	−0.048733
	+5	0.019681		+5	0.048748
	+10	0.039347		+10	0.097508
A_{b2}	−10	−0.059126	π_1	−10	−0.015044
	−5	−0.029547		−5	−0.007290
	+5	0.029515		+5	0.006875
	+10	0.058999		+10	0.013379
A_{b3}	−10	−0.039403	π_2	−10	−0.022565
	−5	−0.019695		−5	−0.010934
	+5	0.019681		+5	0.010313
	+10	0.039347		+10	0.020068
h_{b1}	−10	−0.077129	π_3	−10	−0.016614
	−5	−0.038351		−5	−0.008050
	+5	0.037940		+5	0.007591
	+10	0.075485		+10	0.014770
h_{b2}	−10	−0.129083	R	−10	−0.007018
	−5	−0.129083		−5	−0.003509
	+5	0.063616		+5	0.003509
	+10	0.126645		+10	0.007019
E_v	−10	−0.602509	E_b	−10	−0.525259
	v−5	−0.299646		−5	−0.261380
	v+5	0.296526		+5	0.258950
	+10	0.590024		+10	0.515533

5.1. Sensitivity analysis

The sensitivity analysis of all cost parameters for Cases 1 and 2 are performed in Tables 6 and 7, respectively. The cost parameters are varied from −10% to +10% and the changes in expected total cost is observed (Figs. 2 and 3).

5.2. Decentralization of supply chain

In this case vendor and the buyer's make their decisions independently. When buyers make decisions on their own, the following cost equation for the buyers is considered.

$$\mathrm{ETC}_{bi}\left(Q, k_i, L_i\right) = \begin{bmatrix} \frac{O_{bi} d_i}{Q} + h_{bi}\left\{\frac{Q}{2D} d_i + k_i \sigma_i \sqrt{L_i}\right\} \\ +\pi_i \sigma_i \sqrt{L_i} \psi\left(k_i\right) \frac{D}{Q} + R\left(L_i\right) \frac{D}{Q} + Q \frac{d_i}{D} E_{bi} \end{bmatrix}.$$

TABLE 7. Sensitivity analysis for Case 2.

Case 2 (Quadratic quality function)					
Cost parameter	% change	Sensitivity	Cost parameter	% change	Sensitivity
S_v	−10	−1.189376	h_v	−10	−0.102522
	−5	−0.588021		−5	−0.051257
	+5	0.575540		+5	0.051247
	+10	1.139365		+10	0.102481
A_{b1}	−10	−0.038804	π_1	−10	−0.015227
	−5	−0.019395		−5	−0.007378
	+5	0.019381		+5	0.006958
	+10	0.038749		+10	0.013538
A_{b2}	−10	−0.058227	π_2	−10	−0.022841
	−5	−0.029098		−5	−0.011067
	+5	0.029067		+5	0.010437
	+10	0.058102		+10	0.020308
A_{b3}	−10	−0.038804	π_3	−10	−0.016820
	−5	−0.019395		−5	−0.008149
	+5	0.019381		+5	0.007683
	+10	0.038749		+10	0.014949
h_{b1}	−10	−0.078281	R	−10	−0.050851
	−5	−0.038923		−5	−0.025421
	+5	0.038506		+5	0.025413
	+10	0.076609		+10	0.050818
h_{b2}	−10	−0.131507	E_v	−10	−0.616383
	−5	−0.065430		−5	−0.306505
	+5	0.064808		+5	0.303236
	+10	0.129017		+10	0.603303
h_{b3}	−10	−0.084111	E_b	−10	−0.512709
	−5	−0.041815		−5	−0.255190
	+5	0.041352		+5	0.252920
	+10	0.082259		+10	0.503627

By using the same solution methodology as used in Section 4, the decisions for buyer i are calculated as

$$Q = D\sqrt{\frac{\left\{\sum_{i=1}^{n}(O_{bi} + \pi_i\sigma_i\sqrt{L_i}\Psi\left(k_i\right) + R\left(L_i\right))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i + \left(\frac{d_i}{D}E_{bi}\right)}}.$$

$$\Phi(k_i) = 1 - \frac{h_{bi}Q}{D\pi_i}\cdot$$

The values of the decision variables of buyer, obtained by using the solution algorithm described above are illustrated in Table 8.

The decisions of buyers are used to obtain the optimal production rate, production cost, number of shipment, and expected total cost of the vendor. The vendor's optimal values are illustrated in Table 9.

5.3. Numerical discussion

The results of numerical experimentation are illustrated in Tables 4, 7, and 8. The sensitivity analysis of all cost parameters are shown in Tables 5 and 6. According to the results, the optimal lot sizes and total costs of centralized system for Cases 1 and 2 are 297.30 units; \$206 077.21and 293.02 units; \$206 129.03, respectively. As the order quantity of each buyer is defined by $q_i = Q\frac{d_i}{D}$, therefore, the order quantity for buyer 1, 2 and 3 are

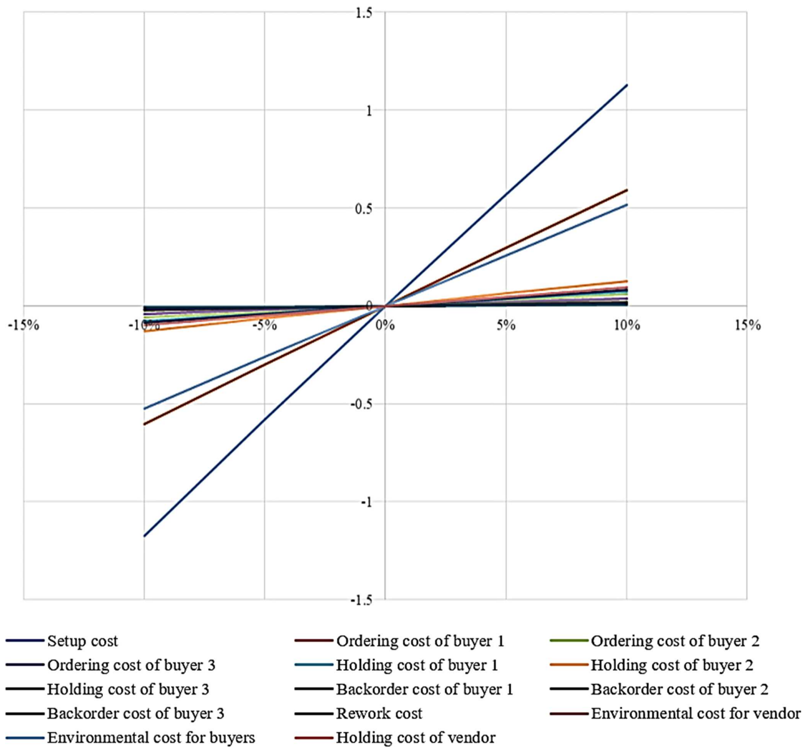

FIGURE 2. Graphical representation of sensitivity analysis for linear quality function.

calculated as 100, 98, and 99, respectively. Accordingly, for Case 2, these are 98.66, 96.68, and 97.67, respectively. MTTF for Cases 1 and 2 are 18.09 weeks and 11.65 weeks, respectively. Clearly, it is observed that quadratic quality function results lower MTTF than linear case which proves that the system following linear nature of the quality function more reliable than the system having quadratic nature.

Comparison of centralization and decentralization

Table 4 represents the decisions for centralized supply chain model whereas, Tables 7 and 8 describes the decentralized model. It is observed that expected joint total costs for centralized chain are 206 077.21 and 206 129.04 for Cases 1 and 2, respectively. On the other hand, for decentralized chain independent decisions are taken by the buyers which are then followed by the vendor. A comparison table (Tab. 10) is created to show the difference of total costs.

It is observed that for same parametric values, the entire supply chain cost is lower for centralized chain than that of decentralized chain.

6. Managerial implications

The managerial opinions drawn by analysing the numerical experimentation are given as follows:

– The system containing the quadratic quality function incurs higher cost and lower supply chain profitability than the system having linear quality function.
– The MTTF is higher if the quality deterioration is a linear function of production rate than the quadratic case. Therefore, the system is more reliable in linear case.

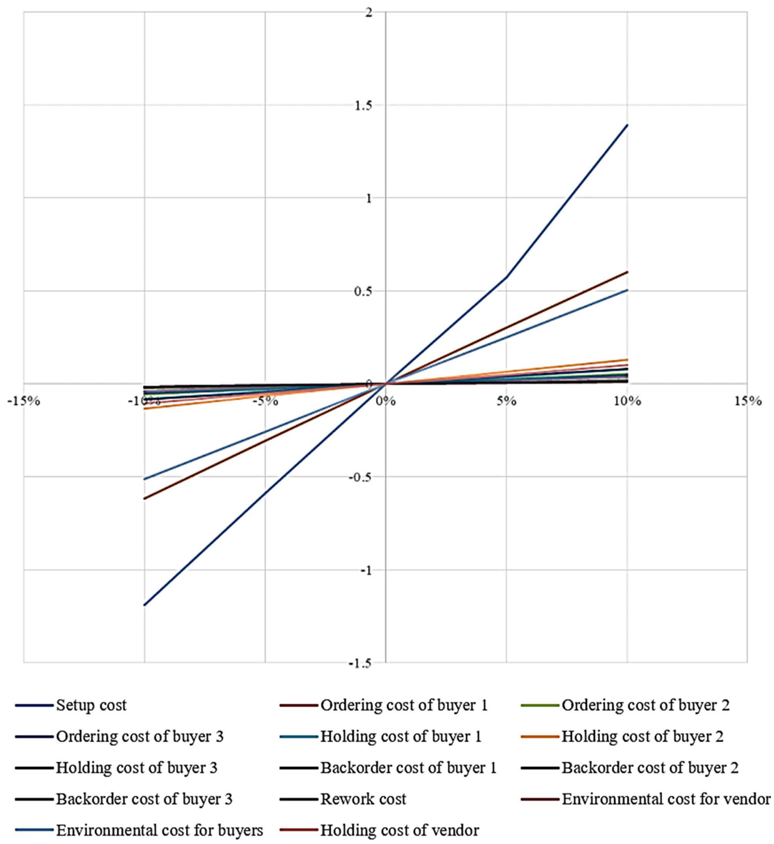

FIGURE 3. Graphical representation of sensitivity analysis for quadratic quality function.

TABLE 8. Buyer's decisions for decentralization.

Parameters	Values
Q	182.01
k_1	1.93
k_2	1.93
k_3	1.90
r_1	73.23
r_2	77.09
r_3	95.54
L_1	4
L_2	4
L_3	4
TERC	9379.68

TABLE 9. Vendor's decisions for decentralization.

Parameters	Case 1	Case 2
m	3	3
P	583.86	583.38
$C(P)$	118.33	118.33
MTTF	17.12	10.82
TECV	210 072.30	210 215.71

TABLE 10. Comparison between centralized and decentralized chain.

	Case 1		Case 2	
Centralized	206 077.21		206 129.04	
Decentralized	Buyer's cost	9379.68	Buyer's cost	9379.68
	Vendor's cost	210 072.30	Vendor's cost	210 215.71
	Total	219 451.98	Total	219 595.39

- Setup cost of vendor and environmental costs for vendor and buyers are two of the most sensitive costs which influences the expected total cost of the chain significantly.
- Rework cost, holding costs (vendor and buyers), and ordering costs are three of the less sensitive costs in the chain.
- It is better to opt for centralization between every party involved in the chain to obtain better profitability and reduced supply chain cost.

7. Conclusions

The study analyzed the reliability of a manufacturing system under two echelon supply chain management with a number of retailers. They considered a special type of quality function in their single echelon model whereas in this model similar function was analyzed under a centralized supply chain with environmental cost parameter for both vendor and buyers. In this paper, uncertainty in demand was assumed with a normal distribution.

The study concluded that for higher degree of quality function the system reliability is diminished which results early MTTF. As production rate varies and reliability is directly considered as a function of production which deteriorates with increased production rate, quadratic nature of quality function is more sensitive to quality deterioration than linear one. The investments were considered to achieve the environmental sustainability for each buyer and vendor. Due to centralization of the model, each buyer's environmental investment was added and considered to be acted like single investment which is beneficial to achieve reduced system cost. Moreover, analysis of sensitivity disclosed that the impacts of changing cost parameters occur for setup and environmental investments. Another vital conclusion was obtained, which infers that better profitability is obtain by adopting a centralized chain. The supply chain cost is less for centralization than a decentralized chain.

The model can further be extended a 3PL supply chain model. As inspection is negligible, the model can also be revised with inspection which can help reducing the rework of defective goods. A smart autonomation technology can be used for inspection. Moreover, instead of single item, a multi-item and multi-stage production model can be a great deal of attention.

Appendix A.

The second order derivatives with respect to Q_1, k_i^1, and P_1 are as follows.

$$\begin{aligned}
\frac{\partial^2(\mathrm{EJTC})}{\partial Q_1 \partial k_i^1} &= \sum\left(\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi\left(k_i^1\right)\right)\right)\\
\frac{\partial^2(\mathrm{EJTC})}{\partial Q_1 \partial P_1} &= m\frac{h_v D}{2P_1^2}-\frac{h_v D}{P_1^2}\\
\frac{\partial^2(\mathrm{EJTC})}{\partial k_i^1} &= \sum\left(\frac{D}{Q_1}\pi_i\sigma_i\sqrt{L_i}\left\{\left(\Phi\left(k_i^1\right)-1\right)\right\}+h_{bi}\sigma_i\sqrt{L_i}\right)\\
\frac{\partial^2(\mathrm{EJTC})}{\partial k_i^1 \partial Q_1} &= \sum\left(\frac{D}{Q_1^2}\pi_i\sigma_i\sqrt{L_i}\left\{\left(1-\Phi\left(k_i^1\right)\right)\right\}\right)\\
\frac{\partial^2(\mathrm{EJTC})}{\partial {k_i^1}^2} &= \sum\left(\frac{D}{Q_1^2}\pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)\right)\\
\frac{\partial^2(\mathrm{EJTC})}{\partial k_i^1 \partial P_1} &= 0\\
\frac{\partial^2(\mathrm{EJTC})}{\partial P_1 \partial k_i^1} &= 0\\
\frac{\partial^2(\mathrm{EJTC})}{\partial P_1 \partial Q_1} &= m\frac{h_v D}{2P_1^2}-\frac{h_v D}{P_1^2}\\
\frac{\partial^2(\mathrm{EJTC})}{\partial P_1^2} &= -\frac{Q_1 h_v D}{P_1^3}(m-2)+2\frac{D}{P_1^3}a_1.
\end{aligned}$$

The Hessian matrix is defined as

$$H=\begin{bmatrix}\frac{\partial^2\mathrm{EJTC}}{\partial Q_1^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_1\partial k_i^1} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_1\partial P_1}\\ \frac{\partial^2\mathrm{EJTC}}{\partial k_i^1\partial Q_1} & \frac{\partial^2\mathrm{EJTC}}{\partial\left(k_i^1\right)^2} & \frac{\partial^2\mathrm{EJTC}}{\partial k_i^1\partial P_1}\\ \frac{\partial^2\mathrm{EJTC}}{\partial P_1\partial Q_1} & \frac{\partial^2\mathrm{EJTC}}{\partial P_1\partial k_i^2} & \frac{\partial^2\mathrm{EJTC}}{\partial P_1^2}\end{bmatrix}.$$

The first principle minor is $\frac{\partial^2\mathrm{EJTC}}{\partial Q_1^2}>0$ in all cases.

The second principle minor is

$$\begin{aligned}
\begin{vmatrix}\frac{\partial^2\mathrm{EJTC}}{\partial Q_1^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_1\partial k_i^1}\\ \frac{\partial^2\mathrm{EJTC}}{\partial k_i^1\partial Q_1} & \frac{\partial^2\mathrm{EJTC}}{\partial\left(k_i^1\right)^2}\end{vmatrix} &= \begin{vmatrix}\sum\left(\frac{2O_{bi}D}{Q_1^3}+\pi_i\sigma_i\sqrt{L_i}\frac{\psi\left(k_i^1\right)D}{Q_1^3}+\frac{2\,R(L_i)D}{Q_i^3}\right)+\frac{2\,S_vD}{mQ_i^3} & \sum\left(\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi\left(k_i^1\right)\right)\right)\\ \sum\left(\frac{D}{Q_1^2}\pi_i\sigma_i\sqrt{L_i}\left\{\left(1-\Phi\left(k_i^1\right)\right)\right\}\right) & \sum\left(\frac{D}{Q_1^2}\pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)\right)\end{vmatrix}\\
&= \left(\sum\left(\frac{2O_{bi}D}{Q_1^3}+2\pi_i\sigma_i\sqrt{L_i}\frac{\psi\left(k_i^1\right)D}{Q_1^3}+\frac{2R\left(L_i\right)D}{Q_i^3}\right)+\frac{2S_vD}{mQ_i^3}\right)\\
&\quad\times\sum\left(\frac{D}{Q_1^2}\pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)\right)-\left(\sum\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi\left(k_i^1\right)\right)\right)^2>0\quad\text{holds true.}
\end{aligned}$$

Since, $2\sum\pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)(O_{bi}+R\left(L_i\right)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\psi\left(k_i^1\right)k_i^1)>\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)\right)^2$. Note that $0<1-\Phi\left(k_i^1\right)<1$, thus, $\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)<1$ implies, $\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)\right)^2\ll 1$.

Third principle minor is

$$\begin{vmatrix} \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1 \partial k_i^1} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1 \partial P_1} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial k_i^1 \partial Q_1} & \frac{\partial^2 \mathrm{EJTC}}{\partial \left(k_i^1\right)^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial k_i^1 \partial P_1} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial P_1 \partial Q_1} & \frac{\partial^2 \mathrm{EJTC}}{\partial P_1 \partial k_i^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial P_1^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1 \partial k_i^1} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1 \partial P_1} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial k_i^1 \partial Q_1} & \frac{\partial^2 \mathrm{EJTC}}{\partial \left(k_i^1\right)^2} & 0 \\ \frac{\partial^2 \mathrm{EJTC}}{\partial P_1 \partial Q_1} & 0 & \frac{\partial^2 \mathrm{EJTC}}{\partial P_1^2} \end{vmatrix} = \frac{\partial^2 \mathrm{EJTC}}{\partial P_1 \partial Q_1} \begin{vmatrix} \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1 \partial k_i^1} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1 \partial P_1} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial \left(k_i^1\right)^2} & 0 \end{vmatrix}$$

$$+ \frac{\partial^2 \mathrm{EJTC}}{\partial P_1^2} \begin{vmatrix} \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_1 \partial k_i^1} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial k_i^1 \partial Q_1} & \frac{\partial^2 \mathrm{EJTC}}{\partial \left(k_i^1\right)^2} \end{vmatrix} = -\left(m\frac{h_v D}{2P_1^2} - \frac{h_v D}{P_1^2}\right)^2 \cdot \sum \left(\frac{D}{Q_1^2} \pi_i \sigma_i \sqrt{L_i} k_i^1 \varphi(k_i^1)\right)$$

$$+ \left(2\frac{D}{P_1^3} a_1 - \frac{Q_1 h_v D}{P_1^3}(m-2)\right) \cdot \left(\sum \left(\frac{2O_{bi} D}{Q_1^3} + 2\pi_i \sigma_i \sqrt{L_i} \frac{\psi\left(k_i^1\right) D}{Q_1^3} + \frac{2R\left(L_i\right) D}{Q_i^3}\right)\right.$$

$$\left. + \frac{2S_v D}{mQ_i^3}\right) \sum \left(\frac{D}{Q_1^2} \pi_i \sigma_i \sqrt{L_i} \varphi(k_i^1\right) - \left(\sum \left[\pi_i \sigma_i \sqrt{L_i} \frac{D}{Q_1^2}\left(1 - \Phi\left(k_i^1\right)\right)\right]\right)^2 > 0$$

if the following condition holds.

$$\left(2\frac{D}{P_1^3} a_1 - \frac{Q_1 h_v D}{P_1^3}(m-2)\right) \cdot \left(\sum \left(\frac{2O_{bi} D}{Q_1^3} + 2\pi_i \sigma_i \sqrt{L_i} \frac{\psi\left(k_i^1\right) D}{Q_1^3} + \frac{2R\left(L_i\right) D}{Q_i^3} + \frac{2S_v D}{mQ_1^3}\right)\right.$$

$$\left. \cdot \sum \left(\frac{D}{Q_1^2} \pi_i \sigma_i \sqrt{L_i} k_i^1 \varphi(k_i^1)\right)\right) - \left(\sum \pi_i \sigma_i \sqrt{L_i} \frac{D}{Q_1^2}\left(1 - \Phi\left(k_i^1\right)\right)\right)^2 > \left(m\frac{h_v D}{2P_1^2} - \frac{h_v D}{P_1^2}\right)^2$$

$$\cdot \sum \left(\frac{D}{Q_1^2} \pi_i \sigma_i \sqrt{L_i} \varphi(k_i^1)\right).$$

Or,

$$\frac{1}{Q_1}\left(2a_1 - Q_1 h_v (m-2)\right)\left(\sum \left(2\left(O_{bi} + \pi_i \sigma_i \sqrt{L_i} \psi\left(k_i^1\right) + R\left(L_i\right) + \frac{S_v}{m}\right)\right) \cdot \left(\sum \left(Q_1 \pi_i \sigma_i \sqrt{L_i} k_i^1 \varphi(k_i^1)\right)\right.\right.$$

$$\left.\left. - \frac{D}{Q_1}\left(\sum \pi_i \sigma_i \sqrt{L_i}\left(1 - \Phi\left(k_i^1\right)\right)\right)^2\right)\right) > \frac{\left(\frac{m}{2} - 1\right)^2 h_v^2}{P_1} \sum \left(\pi_i \sigma_i \sqrt{L_i} \varphi(k_i^1)\right).$$

Which proves the proposition.

Appendix B.

Like Appendix A, the Hessian matrix is

$$H = \begin{bmatrix} \frac{\partial^2 \mathrm{EJTC}}{\partial Q_2^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 \mathrm{EJTC}}{\partial \left(k_i^2\right)^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial k_i^2 \partial P_2} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial P_2 \partial Q_2} & \frac{\partial^2 \mathrm{EJTC}}{\partial P_2 \partial k_i^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial P_2^2} \end{bmatrix}.$$

The first principle minor

$$\frac{\partial^2 (\mathrm{EJTC})}{\partial Q_2^2} = \sum \left(\frac{2O_{bi} D}{Q_2^3} + 2\pi_i \sigma_i \sqrt{L_i} \frac{\psi\left(k_2^1\right) D}{Q_2^3} + \frac{2R\left(L_i\right) D}{Q_2^3}\right) + \frac{2S_v D}{mQ_2^3} > 0.$$

The second principle minor

$$\begin{vmatrix} \frac{\partial^2 \mathrm{EJTC}}{\partial Q_2^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial Q_2 \partial k_i^2} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial k_i^3 \partial Q_2} & \frac{\partial^2 \mathrm{EJTC}}{\partial \left(k_i^2\right)} \end{vmatrix} > 0.$$

As,

$$2\sum \pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)(O_{bi}+R\left(L_i\right)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\psi\left(k_i^1\right)k_i^1)>\left(\sum \pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)\right)^2.$$

The third principle minor

$$\begin{vmatrix} \frac{\partial^2\mathrm{EJTC}}{\partial Q_2^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_2\partial k_i^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_2\partial P_2} \\ \frac{\partial^2\mathrm{EJTC}}{\partial k_i^2\partial Q_2} & \frac{\partial^2\mathrm{EJTC}}{\partial\left(k_i^2\right)^2} & \frac{\partial^2\mathrm{EJTC}}{\partial k_i^2\partial P_2} \\ \frac{\partial^2\mathrm{EJTC}}{\partial P_2\partial Q_2} & \frac{\partial^2\mathrm{EJTC}}{\partial P_2\partial k_i^2} & \frac{\partial^2\mathrm{EJTC}}{\partial P_2^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2\mathrm{EJTC}}{\partial Q_2^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_2\partial k_i^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_2\partial P_2} \\ \frac{\partial^2\mathrm{EJTC}}{\partial k_i^2\partial Q_2} & \frac{\partial^2\mathrm{EJTC}}{\partial\left(k_i^2\right)^2} & 0 \\ \frac{\partial^2\mathrm{EJTC}}{\partial P_2\partial Q_2} & 0 & \frac{\partial^2\mathrm{EJTC}}{\partial P_2^2} \end{vmatrix} = \frac{\partial^2\mathrm{EJTC}}{\partial P_2\partial Q_2}\begin{vmatrix} \frac{\partial^2\mathrm{EJTC}}{\partial Q_2\partial k_i^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_2\partial P_2} \\ \frac{\partial^2\mathrm{EJTC}}{\partial\left(k_i^2\right)^2} & 0 \end{vmatrix} + \frac{\partial^2\mathrm{EJTC}}{\partial P_2^2}$$

$$\times\begin{vmatrix} \frac{\partial^2\mathrm{EJTC}}{\partial Q_2^2} & \frac{\partial^2\mathrm{EJTC}}{\partial Q_2\partial k_i^2} \\ \frac{\partial^2\mathrm{EJTC}}{\partial k_i^2\partial Q_2} & \frac{\partial^2\mathrm{EJTC}}{\partial\left(k_i^2\right)^2} \end{vmatrix} = -\left(m\frac{h_vD}{2P_2^2}-\frac{h_vD}{P_2^2}+\frac{\mathrm{RD}\alpha c_2}{2}\right)^2\cdot\sum\left(\frac{D}{Q_2^2}\pi_i\sigma_i\sqrt{L_i}k_i^2\varphi(k_i^2)\right)$$
$$+\left(2\frac{D}{P_2^3}a_1-\frac{Q_2h_vD}{P_2^3}\left(m-2\right)\right)\cdot\left(2\sum \pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)\left(O_{bi}+R\left(L_i\right)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\psi\left(k_i^1\right)k_i^1\right)\right.$$
$$\left.-\left(\sum \pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)\right)^2\right)>0.$$

Only if,

$$\left(2\frac{D}{P_2^3}a_1-\frac{Q_2h_vD}{P_2^3}\left(m-2\right)\right)\cdot 2\sum \pi_i\sigma_i\sqrt{L_i}\varphi(k_i^1)\left(O_{bi}+R\left(L_i\right)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\psi\left(k_i^1\right)k_i^1\right)$$
$$-\left(\sum \pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)\right)^2>\left(m\frac{h_vD}{2P_2^2}-\frac{h_vD}{P_2^2}+\frac{\mathrm{RD}\alpha c_2}{2}\right)^2\cdot\sum\left(\frac{D}{Q_2^2}\pi_i\sigma_i\sqrt{L_i}k_i^2\varphi(k_i^2)\right)$$

holds true which proves the proposition.

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Conflict of interest

The authors do not have any conflict of interest with anyone.

References

[1] P. Ahi and C. Searcy, A comparative literature analysis of definitions for green and sustainable supply chain management. *J. Cleaner Prod.* **52** (2013) 329–341.

[2] A. Banerjee, A joint economic-lot-size model for purchaser and vendor. *Decis. Sci.* **17** (1986) 292–311.

[3] A. Banerjee and S. Banerjee, Coordinated order less inventory replenishment for a vendor and multiple buyers. *Int. J. Technol. Manage.* **7** (1992) 328–336.

[4] A. Banerjee and J.S. Burton, Coordinated vs. independent inventory replenishment policies for a vendor and multiple buyers. *Int. J. Prod. Econ.* **35** (1994) 215–222.

[5] D. Battini, A. Persona and F. Sgarbossa, A sustainable EOQ model: theoretical formulation and applications. *Int. J. Prod. Econ.* **149** (2014) 145–153.

[6] M. Ben-Daya and A. Raouf, Inventory models involving lead time as a decision variable. *J. Oper. Res. Soc.* **45** (1994) 579–582.

[7] Y. Bouchery, A. Ghaffari, Z. Jemai and Y. Dallery, Including sustainability criteria into inventory models. *Eur. J. Oper. Res.* **222** (2012) 229–240.

[8] L.E. Cárdenas-Barrón, K.J. Chung and G. Treviño-Garza, Celebrating a century of the economic order quantity model in honour of Ford Whitman Harris. *Int. J. Prod. Econ.* **155** (2014) 1–7.

[9] G. Cheng and L. Li, Joint optimization of production, quality control and maintenance for serial-parallel multistage production systems. *Reliab. Eng. Syst. Saf.* **204** (2020) 107–146.

[10] C.J. Chung, An integrated production-inventory deteriorating model for pricing policy considering imperfect production, inspection planning and warranty-period-and stock-level-dependent demand. *Int. J. Syst. Sci.* **39** (2008) 823–837.

[11] C.J. Conrad and N.H. McClamrock, The drilling problem: a stochastic modeling and control example in manufacturing. *IEEE Trans. Autom. Control* **32** (1987) 947–958.

[12] B.K. Dey, B. Sarkar and S. Pareek, A two-echelon supply chain management with setup time and cost reduction, quality improvement and variable production rate. *Mathematics* **7** (2019) 328.

[13] B.K. Dey, B. Sarkar, M. Sarkar and S. Pareek, An integrated inventory model involving discrete setup cost reduction, variable safety factor, selling-price dependent demand, and investment. *RAIRO:OR* **53** (2019) 39–57.

[14] B.K. Dey, S. Pareek, M. Tayyab and B. Sarkar, Autonomation policy to control work-in-process inventory in a smart production system. *Int. J. Prod. Res.* (2020). DOI: 10.1080/00207543.2020.1722325.

[15] C. Gao, T.C. Cheng, H. Shen and L. Xu, Newdevelopments in stochastic models of manufacturing and service operations incentives for quality improvement efforts coordination in supply chains with partial cost allocation contract. *Int. J. Prod. Res.* **54** (2016) 6216–6231.

[16] S.K. Goyal, An integrated inventory model for a single supplier-single customer problem. *Int. J. Prod. Res.* **15** (1976) 107–111.

[17] S.K. Goyal, A joint economic-lot-size model for purchaser and vendor: a comment. *Decis. Sci.* **19** (1988) 236–241.

[18] T. Hacking and P. Guthrie, A framework for clarifying the meaning of triple bottom-line, integrated, and sustainability assessment. *Environ. Impact Assess. Rev.* **28** (2008) 73–89.

[19] M. Herva and E. Roca, Review of combined approaches and multi-criteria analysis for corporate environmental evaluation. *J. Cleaner Prod.* **39** (2013) 355–371.

[20] C.K. Huang, T.L. Cheng, T.C. Kao and S.K. Goyal, An integrated inventory model involving manufacturing setup cost reduction in compound Poisson process. *Int. J. Prod. Res.* **49** (2011) 1219–1240.

[21] J.K. Jha and K. Shankar, Single-vendor multi-buyer integrated production-inventory model with controllable lead time and service level constraints. *Appl. Math. Model.* **37** (2013) 1753–1767.

[22] M. Khan, M. Hussain and H.M. Saber, Information sharing in a sustainable supply chain. *Int. J. Prod. Econ.* **181(A)** (2016) 201–214.

[23] M. Khouja and A. Mehrez, Economic production lot size model with variable production rate and imperfect quality. *J. Oper. Res. Soc.* **45** (1994) 1405–1417.

[24] C.J. Liao and C.H. Shyu, An analytical determination of lead time with normal demand. *Int. J. Oper. Prod. Manage.* **11** (1991) 72–78.

[25] J.D. Linton, R. Klassen and V. Jayaraman, Sustainable supply chains: an introduction. *J. Oper. Manage.* **25** (2007) 1075–1082.

[26] L. Lu, A one-vendor multi-buyer integrated inventory model. *Eur. J. Oper. Res.* **81** (1995) 312–323.

[27] A. Majumder, R. Guchhait and B. Sarkar, Manufacturing quality improvement and setup cost reduction in a vendor-buyer supply chain model. *Eur. J. Ind. Eng.* **11** (2017) 588–612.

[28] A. Majumder, C. Jaggi and B. Sarkar, A multi-retailer supply chain model with backorder and variable production cost. *RAIRO:OR* **52** (2018) 943–954.

[29] A. Mehrez, O.F. Offodile and B.H. Ahn, A decision analysis view on the effect of robot repeatability on profit. *IIE Trans.* **27** (1995) 60–71.

[30] O.F. Offodile and K. Ugwu, Evaluating the effect of speed and payload on robot repeatability. *Rob. Comput.-Integr. Manuf.* **8** (1991) 27–33.

[31] L.Y. Ouyang and C.K. Chen, Quality improvement, setup cost and lead-time reductions in lot size reorder point models with an imperfect production process. *Comput. Oper. Res.* **29** (2002) 1701–1717.

[32] L.Y. Ouyang, N.C. Yeh and K.S. Wu, Mixture inventory model with backorders and lost sales for variable lead time. *J. Oper. Res. Soc.* **47** (1996) 829–832.

[33] L.Y. Ouyang, K.S. Wu and C.H. Ho, Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time. *Int. J. Prod. Econ.* **92** (2004) 255–266.

[34] J.C.H. Pan and J.S. Yang, A study of an integrated inventory with controllable lead time. *Int. J. Prod. Res.* **40** (2002) 1263–1273.

[35] E.L. Porteus, Optimal lot sizing, process quality improvement and setup cost reduction. *Oper. Res.* **34** (1986) 137–144.

[36] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect productions processes. *IIE Trans.* **18** (1986) 48–54.

[37] S.S. Sana, A production-inventory model of imperfect quality products in a three-layer supply chain. *Decis. Support Syst.* **50** (2011) 539–547.

[38] B. Sarkar, Supply chain coordination with variable backorder, inspections, and discount policy for fixed lifetime products. *Math. Prod. Eng.* **14** (2016) 6318737: 1–20.

[39] B. Sarkar and A. Majumder, Integrated vendor-buyer supply chain model with vendor's setup cost reduction. *Appl. Math. Comput.* **224** (2013) 362–371.

[40] B. Sarkar, S.S. Sana and K.S. Chaudhuri, Optimal reliability, production lot size and safety stock in an imperfect production system. *Int. J. Math. Oper. Res.* **2** (2010) 467–490.

[41] B. Sarkar, A. Majumder, M. Sarkar, N. Kim and M. Ullah, Effects of variable production rate on quality of products in a single-vendor multi-buyer supply chain management. *Int. J. Adv. Manuf. Technol.* **99** (2018) 567–581.

[42] B. Sarkar, M. Omir and N.H. Kim, A cooperative advertising collaboration policy in supply chain management under uncertain conditions. *Appl. Soft Comput.* **88** (2020) 105984.

[43] S. Sarkar and B.C. Giri, Optimal ordering policy in a two-echelon supply chain model with variable backorder and demand uncertainty. *RAIRO:OR* (2020) DOI: 10.1051/ro/2020007.

[44] S. Sarkar, B.C. Giri and A.K. Sarkar, A vendor-buyer inventory model with lot-size and production rate dependent lead time under time value of money. *RAIRO:OR* **54** (2020) 961–979

[45] S.P. Sarmah, D. Acharya and S.K. Goyal, Coordination of a single-manufacturer/multi-buyer supply chain with credit option. *Int. J. Prod. Econ.* **111** (2008) 676–685.

[46] S. Singh, N. Holvoet and V. Pandey, Bridging sustainability and corporate social responsibility: culture of monitoring and evaluation of CSR initiatives in India. *Sustainability: Sci. Pract. Policy* **10** (2018) 2353.

[47] L. Wang, Z. Lu and Y. Ren, Joint production control and maintenance policy for a serial system with quality deterioration and stochastic demand. *Reliab. Eng. Syst. Safety* (2020) 106918.

[48] Y.Y. Woo, S.L. Hsu and S. Wu, An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction. *Int. J. Prod. Econ.* **73** (2001) 203–215.