A COMMON WEIGHTS MODEL FOR INVESTIGATING EFFICIENCY-BASED LEADERSHIP IN THE RUSSIAN BANKING INDUSTRY

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Abstract. In this race for productivity, the most successful leaders in the banking industry are those with high-efficiency and a competitive edge. Data envelopment analysis is one of the most widely used methods for measuring efficiency in organizations. In this study, we use the ideal point concept and propose a common weights model with fuzzy data and non-discretionary inputs. The proposed model considers environmental criteria with uncertain data to produce a full ranking of homogenous decision-making units. We use the proposed model to investigate the efficiency-based leaders in the Russian banking industry. The results show that the unidimensional and unilateral assessment of leading organizations solely according to corporate size is insufficient to characterize industry leaders effectively. In response, we recommend a multilevel, multicomponent, and multidisciplinary evaluation framework for a more reliable and realistic investigation of leadership at the network level of analysis.

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1. Introduction

Globalization has converted the banking industry and, subsequently, the financial system into a vital sector of the economy [46]. Consequently, the efficiency of the banking industry has been a key driver of financial and economic development and growth [41,90,96]. Inefficient banks threaten the stability of the financial system, and banks are under constant pressure to increase their efficiency by adopting efficient banking practices, lowering their costs, improving productivity, and avoiding risky investments [8].

In this race for productivity, the most successful leaders in the banking industry are those with high profitability [29,92], visibility [21], and competitive advantage [9]. Consequently, according to the competitive dynamics [43,72], and the neo-institutional theoretical perspectives [25,56], organizations in similar situations tend to
imitate the leader’s structure, processes, and strategies [48, 49]. Hence, studying the efficiency-based leadership among a set of competitive organizations such as banks is a critical task for managers who want to follow the leaders. In this regard, both parametric and non-parametric methods are widely used for efficiency analysis in the banking industry [22, 71, 74]. While parametric methods restrict the production function to a special parametric form before estimation them, non-parametric methods avoid a parametric production function and provide a clear understanding of the production possibility set [11].

One of the most applicable and popular non-parametric methods for performance measurement is the data envelopment analysis (DEA) method, which has been introduced by Charnes et al. [12]. The conventional DEA models consider three specific assumptions. First, they require precise input and output data. In contrast, the fuzzy and stochastic DEA models are designed to consider uncertain data in performance evaluation [57, 65]. Second, primary models with homogeneous decision-making units (DMUs) have been developed using “exogenously fixed” or non-discretionary factors for different operating environments [7, 37, 64]. Third, initial models with the ability to divide DMUs into two groups of efficient and inefficient units without providing any additional information and ranking of the efficient DMUs have been proposed for performance evaluation [4, 16, 36, 50] in the form of common weights (CW) models [18]. We consider these three assumptions and propose a CW model for investigating efficiency-based leadership in the Russian banking industry.

The remainder of this paper is organized as follows. In Section 2, we present a review of the relevant literature review, followed by a description of the fuzzy CW model with non-discretionary inputs in Section 3. In Section 4, we present a case study to demonstrate the applicability of the proposed model. Conclusions and future research are provided in Section 6.

2. Literature review

2.1. Data envelopment analysis (DEA)

DEA is a non-parametric fractional mathematical programming method for measuring and comparing the relative efficiency as a ratio of a weighted sum of the outputs to a weighted sum of the inputs among a set of homogeneous DMUs with numerous applications in airports, hospitals, universities, banks, technologies, etc. [13, 20, 78, 91]. In the following, the CCR (Charnes, Cooper, and Rhodes) input-oriented model of DEA proposed by Charnes et al. [12] is presented as the central model for development in the literature.

2.1.1. The CCR model

Using the traditional denotations in DEA and according to the research of Charnes et al. [12], we assume that there are a set of $n$ DMUs and each DMU $j$, $(j = 1, \ldots, n)$ produces $s$ different outputs using $m$ different inputs which are denoted by $x_{ij}$, $(i = 1, \ldots, m)$ and $y_{rj}$, $(r = 1, \ldots, s)$, respectively. It is assumed that $x_{ij}$ and $y_{rj}$ are all positive. For any evaluated DMU $j$, the efficiency score $E$ can be calculated by the following CCR input-oriented multiplier model:

$$E_o = \max \sum_{r=1}^{s} u_r y_{ro}$$

s.t.

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall j,$$

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$u_r \geq 0, v_i \geq 0, \quad \forall r, \forall i$$

(2.1)

where the decision variables $v_i$ and $u_r$ are the assigned weights for the $i$th input and the $r$th output, respectively. The efficiency score for DMU $o$ ($E_o$) is calculated as the weighted sum of its outputs, while the weighted sum of its inputs equals 1.

While the conventional DEA models such as CCR require accurate measurement of both inputs and outputs, crisp input and output data may not always be relevant in real-world situations. The observed values of the
input and output data in real-world problems sometimes include missing data, judgment data, or predictive data, which are generally imprecise or vague. One way to deal with the uncertain data is the consideration of fuzzy numbers in developing DEA models [6,60]. Four general approaches have been recognized in the literature for developing fuzzy DEA models [27,90]. Here we will focus on the α-level approach, which has been proposed by Saati et al. [65]. Although this approach is not computationally efficient, it is possibly the most popular method due to its linear computation for each α value [73]. In the following subsection, a particular case of fuzzy DEA with triangular fuzzy numbers for inputs and outputs has been proposed by Saati et al. [65] based on the CCR model.

2.1.2. The fuzzy CCR model

Saati et al. [65] define all inputs and outputs as triangular fuzzy numbers $\tilde{x}_{ij} = (\hat{x}_{ij}, x_{ij}^m, \hat{x}_{ij})$ and $\tilde{y}_{rj} = (\hat{y}_{rj}, y_{rj}^m, \hat{y}_{rj})$, respectively. The fuzzy CCR model using α-level approach is formulated as model (2.2) and (2.3) by defining two interval variables, including $\hat{x}_{ij} \in [\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l, \alpha x_{ij}^m + (1 - \alpha) x_{ij}^u]$ and $\hat{y}_{rj} \in [\alpha y_{rj}^m + (1 - \alpha) y_{rj}^l, \alpha y_{rj}^m + (1 - \alpha) y_{rj}^u]$ where $\alpha \in (0,1]$.

$$E_o = \max \sum_{r=1}^{s} u_r \hat{y}_{r o}$$

s.t. $$\sum_{r=1}^{s} u_r \hat{y}_{r j} - \sum_{i=1}^{m} v_i \hat{x}_{ij} \leq 0, \quad \forall j$$

$$\sum_{i=1}^{m} v_i \hat{x}_{io} = 1,$$

$$\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l \leq \hat{x}_{ij} \leq \alpha x_{ij}^m + (1 - \alpha) x_{ij}^u, \quad \forall i, \forall j,$$

$$\alpha y_{rj}^m + (1 - \alpha) y_{rj}^l \leq \hat{y}_{rj} \leq \alpha y_{rj}^m + (1 - \alpha) y_{rj}^u, \quad \forall r, \forall j,$$

$$u_r, v_i, \hat{x}_{ij}, \hat{y}_{rj} \geq 0, \quad \forall r, \forall i, \forall j$$

The model includes $(n+1)(m+s)$ decision variables. Although model (2.2) is a non-linear programming (NLP) model due to the existence of non-linear terms $v_i \hat{x}_{ij}$ and $u_r \hat{y}_{rj}$, it can be transformed into the following linear programming (LP) model (2.3) using two changes in variables $\hat{x}_{ij} = v_i \hat{x}_{ij}$ and $\hat{y}_{rj} = u_r \hat{y}_{rj}$, and substituting them in model (2.2):

$$E_o = \max \sum_{r=1}^{s} \hat{y}_{r o}$$

s.t. $$\sum_{r=1}^{s} \hat{y}_{r j} - \sum_{i=1}^{m} \hat{x}_{ij} \leq 0, \quad \forall j,$$

$$\sum_{i=1}^{m} \hat{x}_{io} = 1,$$

$$v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l\right) \leq \hat{x}_{ij} \leq v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^u\right), \quad \forall i, \forall j,$$

$$u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^l\right) \leq \hat{y}_{rj} \leq u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^u\right), \quad \forall r, \forall j,$$

$$u_r, v_i, \hat{x}_{ij}, \hat{y}_{rj} \geq 0, \quad \forall r, \forall i, \forall j$$

where $\hat{x}_{ij}$ and $\hat{y}_{rj}$ are decision variables used to convert the primary non-linear fuzzy model into a crisp parametric LP model while $\alpha \in (0,1]$ [62]. Accordingly, the model will provide an optimal solution for each $\alpha$. In this model, all evaluated DMUs must be homogeneous according to the original DEA’s fundamental assumptions. However, in many real-world problems, environmental diversity may violate the presumption of homogenous units [37,63]. Ruggiero [63] has demonstrated that the consequence of not controlling the environmental variables results in biased estimation of technical efficiency. In response, researchers have focused on the “exogenously fixed” or “non-discretionary” factors in their models to meet this assumption (e.g., [7,35]). A CCR model with non-discretionary inputs proposed by Banker and Morey [7] is presented next to demonstrate the mathematical application of these inputs in the model.
2.1.3. The CCR model with non-discretionary inputs

Homogeneity is a fundamental assumption for all basic DEA models [7]. According to the homogeneity assumption, all DMUs must agree to the following three conditions: (i) the DMUs should execute the same processes; (ii) their efficiency should be evaluated by the same input and output variables; and (iii) all DMUs operate within the same environment under the same conditions [99]. When environmental factors cause non-homogeneity, they are considered in a single model as non-discretionary inputs. Therefore, different reference sets are defined to discriminate DMUs in different environments [7,33,63]. There is no generally accepted approach for using non-discretionary factors in DEA models. Therefore, this study considers the research of Banker and Morey [7], who proposed the CCR model by applying non-discretionary inputs \( k(z_{kj}), (k = 1, \ldots, t) \) for its simplicity and popularity as the following model (2.4):

\[
E_o = \max \sum_{r=1}^{s} u_r y_{ro} - \sum_{k=1}^{t} w_k z_{ko},
\]

s.t.
\[
\sum_{r=1}^{s} u_r y_{rj} - \sum_{k=1}^{t} w_k z_{kj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall j
\]
\[
\sum_{i=1}^{m} v_i x_{io} = 1,
\]
\[
u_r, v_i, w_k \geq \varepsilon, \quad \forall r, \forall i, \forall k,
\]

where \( \varepsilon \) is a small, non-negative number used to avoid ignoring factors in calculating efficiency for DMU \( o \) [3].

This model can be extended to situations where some non-discretionary outputs are beyond the manager’s discretionary controls. In this case, increasing output is not a meaningful target for managers while there are non-controllable outputs. In other words, managers are interested in estimating the maximum possible increase in the discretionary outputs with keeping the inputs and non-discretionary outputs at their current levels. Therefore, the output-oriented objective function of the CCR model describes this situation more realistically [7].

The above DEA models evaluate the relative efficiency with favorable weights for each DMU. These efficiency scores usually lie in \((0, 1] \). While a ranking for inefficient DMUs is given using these models, they do not provide sufficient information about the efficient DMUs with an efficiency score of 1. Researchers have solved this problem by using various methods (e.g., [4, 16, 50, 94]). Among them, the CW models are more favorable and applicable according to the literature (e.g., [15, 28, 50, 76, 79, 86]). In this research, we will use the CW model based on the ideal point method proposed by Sun et al. [75]. The prominent feature of this method compared to competing methods, is its feasibility feature [66].

2.1.4. The CW model with ideal point approach

The CW models reduce the flexibility and the dispersion in the optimal weights assigned to the inputs and outputs by each DMU and make it possible to compare and rank the efficiency of all DMUs on the same basis [42,86]. In this study, we use the CW model based on the ideal point method proposed by Sun et al. [75], which provides a basic model for our final model development.

**Definition 2.1.** The (virtual) ideal DMU is a DMU that its inputs are at the minimum level, and its outputs are at the maximum level among all DMUs.

The ideal DMU is shown by \( \text{IDMU} = (\underline{x}, \underline{y}) \) where \( \underline{x} \) and \( \underline{y} \) respectively denote the inputs and outputs of the ideal unit, and \( \underline{x}_i = \min \{ x_{ij} | \forall j \} \) (\( \forall i \)) and \( \underline{y}_r = \max \{ y_{rj} | \forall j \} \) (\( \forall r \)). The CW model with ideal point method is developed next as model (2.5) based on the CCR model [44,75]:
\[ \theta = \min \sum_{j=1}^{n} \left[ \sum_{i=1}^{m} v_i (x_{ij} - \bar{x}_i) + \sum_{j=1}^{n} u_r (\bar{y}_r - y_{rj}) \right] \]

s.t.
\[ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall j \]
\[ \sum_{i=1}^{m} v_i \bar{x}_i = 1, \]
\[ \sum_{r=1}^{s} u_r \bar{y}_r = 1, \]
\[ v_i, u_r \geq 0, \quad \forall i, \forall r, \]

where \((v, u) \in \mathbb{R}^{m+s}\) is the common set of weights and the constraints \(\sum_{i=1}^{m} v_i x_{ij} = 1\) and \(\sum_{r=1}^{s} u_r y_{rj} = 1\) ensure that the IDMU is efficient. The efficiency score of DMU\(_j\) is measured by \(\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}\) for \(j = 1, \ldots, n\), which is less than or equal to one due to the first set of constraints.

In contrast to the conventional DEA models, which must be solved \(n\) times, model (2.5) is solved one time for all units.

### 3. Proposed Model

In this study, we propose a novel DEA model to overcome the shortfalls highlighted earlier and find a common set of weights in a fuzzy environment where the inputs \(\tilde{x}_{ij} = (x^l_{ij}, x^m_{ij}, x^u_{ij})\), non-discretionary inputs \(\tilde{z}_{kj} = (z^l_{kj}, z^m_{kj}, z^u_{kj})\), and outputs \(\tilde{y}_{rj} = (y^l_{rj}, y^m_{rj}, y^u_{rj})\) are triangular fuzzy numbers. To this aim, we define the fuzzy ideal DMU as FIDMU \(= (\tilde{x}, \tilde{z}, \tilde{y})\) in which:

(a) \(\tilde{x}_i = (x^l_i, x^m_i, x^u_i)\), \(\tilde{z}_k = (z^l_k, z^m_k, z^u_k)\), \(\tilde{y}_j = (y^l_j, y^m_j, y^u_j)\)
(b) \(\tilde{z}_k = (z^l_k, z^m_k, z^u_k)\), \(\tilde{z}_k = (z^l_k, z^m_k, z^u_k)\), \(\tilde{y}_j = (y^l_j, y^m_j, y^u_j)\)
(c) \(\tilde{z}_k = (z^l_k, z^m_k, z^u_k)\), \(\tilde{z}_k = (z^l_k, z^m_k, z^u_k)\)

Accordingly, the CW model with the ideal point method proposed by Sun et al. [75] is developed in a fuzzy environment by applying non-discretionary inputs in Models (2.3)–(2.5) and defining six interval variables, including \(\tilde{x}_{ij} \in [\alpha x^m_{ij} + (1 - \alpha) x^l_{ij}, x^m_{ij}, x^m_{ij} + (1 - \alpha) x^u_{ij}]\), \(\tilde{z}_k \in [\alpha z^m_k + (1 - \alpha) z^l_k, z^m_k, z^m_k + (1 - \alpha) z^u_k]\), \(\tilde{y}_j \in [\alpha y^m_j + (1 - \alpha) y^l_j, y^m_j, y^m_j + (1 - \alpha) y^u_j]\), and \(\tilde{y}_j \in [\alpha y^m_j + (1 - \alpha) y^l_j, y^m_j, y^m_j + (1 - \alpha) y^u_j]\) to propose the following NLP model:

\[ E^\ast_{\text{IDMU}}(\alpha) = \min \sum_{j=1}^{n} \left[ \sum_{i=1}^{m} v_i (\tilde{x}_{ij} - \tilde{\hat{x}}_i) + \sum_{k=1}^{t} w_k (\tilde{z}_{kj} - \tilde{\hat{z}}_k) + \sum_{r=1}^{s} u_r (\tilde{y}_r - \tilde{\hat{y}}_{rj}) \right] \]

s.t.
\[ \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} - \sum_{k=1}^{t} w_k \tilde{z}_{kj} \leq 0, \quad \forall j, \]
\[ \sum_{i=1}^{m} v_i \tilde{x}_i = 1, \]
\[ \sum_{r=1}^{s} u_r \tilde{y}_r = 1, \]
\[ \alpha x^m_{ij} + (1 - \alpha) x^l_{ij} \leq \tilde{x}_{ij} \leq \alpha x^m_{ij} + (1 - \alpha) x^u_{ij}, \quad \forall i, \forall j, \]
\[ \alpha z^m_k + (1 - \alpha) z^l_k \leq \tilde{z}_k \leq \alpha z^m_k + (1 - \alpha) z^u_k, \quad \forall k, \forall i, \]
\[ \alpha y^m_j + (1 - \alpha) y^l_j \leq \tilde{y}_j \leq \alpha y^m_j + (1 - \alpha) y^u_j, \quad \forall r, \forall j, \]
\(\alpha z_{kj}^m + (1-\alpha) \hat{x}_{kj}^l \leq \hat{z}_{kj} \leq \alpha z_{kj}^m + (1-\alpha) \hat{x}_{kj}^u,\) \(\forall k, \forall j,\)
\(\alpha z_{k}^m + (1-\alpha) \hat{x}_{k}^l \leq \hat{z}_{k} \leq \alpha z_{k}^m + (1-\alpha) \hat{x}_{k}^u,\) \(\forall k,\)
\(\alpha y_{rj}^m + (1-\alpha) y_{rj}^l \leq \tilde{y}_{rj} \leq \alpha y_{rj}^m + (1-\alpha) y_{rj}^u,\) \(\forall r, \forall v_j,\)
\(\alpha \overline{y}_{r}^m + (1-\alpha) \underline{y}_{r}^l \leq \overline{y}_{r} \leq \alpha \overline{y}_{r}^m + (1-\alpha) \underline{y}_{r}^u,\) \(\forall r,\)
\(v_i, u_r, w_k \geq \varepsilon,\) \(\forall i, \forall r, \forall k, \forall j.\)

The NP model (3.2) has \((n+2)(m+s+t)\) decision variables. We use six variable changes, including \(\hat{x}_{ij} = v_i \hat{x}_{ij}, \hat{z}_{kj} = w_k \hat{z}_{kj}, \hat{y}_{rj} = u_r \hat{y}_{rj},\) and \(\bar{y}_k = u_r \bar{y}_r\) to formulate the following linearized model:

\[E_{\text{IDMU}}^* (\alpha) = \min \sum_{j=1}^{n} \left[ \sum_{j=1}^{m} (\hat{x}_{ij} - \hat{x}_{k}) + \sum_{k=1}^{t} (\hat{z}_{kj} - \hat{z}_{k}) + \sum_{r=1}^{s} (\overline{y}_{r} - \underline{y}_{r}) \right] \]

s.t. \(\sum_{r=1}^{s} \overline{y}_{r} - \sum_{i=1}^{n} \hat{x}_{ij} - \sum_{k=1}^{t} \hat{z}_{kj} \leq 0, \forall j,\)
\(\sum_{i=1}^{n} \hat{x}_{ij} = 1,\)
\(\sum_{r=1}^{s} \overline{y}_{r} - \sum_{k=1}^{t} \hat{z}_{k} = 1,\)
\(v_i (\alpha x_{ij}^m + (1-\alpha) x_{ij}^l) \leq \hat{x}_{ij} \leq v_i (\alpha x_{ij}^m + (1-\alpha) x_{ij}^u), \forall i, \forall j,\)
\(w_k (\alpha z_{kj}^m + (1-\alpha) z_{kj}^l) \leq \hat{z}_{kj} \leq w_k (\alpha z_{kj}^m + (1-\alpha) z_{kj}^u), \forall k, \forall j,\)
\(w_k (\alpha z_{k}^m + (1-\alpha) z_{k}^l) \leq \hat{z}_{k} \leq w_k (\alpha z_{k}^m + (1-\alpha) z_{k}^u), \forall k,\)
\(u_r (\alpha \overline{y}_{r}^m + (1-\alpha) \underline{y}_{r}^l) \leq \overline{y}_{r} \leq u_r (\alpha \overline{y}_{r}^m + (1-\alpha) \underline{y}_{r}^u), \forall r,\)
\(v_i, u_r, w_k \geq \varepsilon, \forall i, \forall r, \forall k, \forall j.\)

It is evident that in this model, all DMUs consider the IDMU as the reference object. In other words, the IDMU must take the efficiency value of one, and other DMUs are compared to the IDMU for efficiency calculation in a fuzzy environment.

This model is now a crisp parametric LP problem and provides an optimal solution table for different \(\alpha\) values, \(\alpha \in (0,1].\) The model possesses \((n+2)(m+s+t)\) decision variables and \((2n+3)(m+s+t)+(n+2)\) constraints. Accordingly, if \((v, u, w, \hat{x}^*, \hat{y}^*, \bar{y}^*)\) is the optimal solution for model (3.3), then, we have \(E_j^{\alpha} = \sum_{i=1}^{m} \hat{y}_{ij} - \sum_{j=1}^{n} \hat{x}_{ij}, \) \(E_j^{\alpha} \) is \(\alpha\)-efficiency score of DMU \(j\) and the value of \(\alpha\) affects efficiency scores. Also, according to model (3.3), the value of \(\epsilon\) is important for its impact on the calculated weights of inputs and outputs. Note that an unsuitable value for the \(\epsilon\) may lead to infeasibility [3,67]. In addition, its optimal value to reach maximum weights is another problem that needs further investigation.

**Definition 3.1.** DMU \(j\) is said to be efficient at given \(\alpha \in (0,1]\) if \(E_j^{\alpha} = 1.\)

**Theorem 3.2.** \(E_j^{\alpha_1} \leq E_j^{\alpha_2}\) for \(\alpha_1 \leq \alpha_2.\)

**Proof.** Let \(S(\alpha)\) be the feasible region of the model (3.3) for a given \(\alpha.\) It is easy to verify that \(S(\alpha_2) \subseteq S(\alpha_1).\) This fact that model (3.3) is a minimization problem that completes the proof. \(\square\)

The proposed model has several innovative features. Our model:
(1) provides investigators with the opportunity to address three potential concerns collectively. The first concern is uncertainty and fuzzy variables. The second concern is related to the conventional DEA limitation of not providing sufficient information for evaluating and ranking the efficient DMUs. Finally, the third concern is the homogeneity of the DMUs as a central premise in DEA modeling.

(2) provides the leadership literature with a quantitative model for measuring leadership from an efficiency perspective, which has been emphasized in previous studies [19, 30, 47, 97]. This characteristic of leadership illustrates the fact that an organization with a higher leadership index is more efficient and performs better than competing organizations [53, 61]. As a result, these high-performing organizations become role models for other competitors [29, 48].

Finally, we suggest the following model (3.4) for finding a suitable value for the epsilon in model (3.3):

\[
\varepsilon^* (\alpha) = \max \varepsilon \\
\text{s.t.} \\
\sum_{r=1}^{s} \tilde{y}_{rj} - \sum_{i=1}^{m} \tilde{x}_{ij} - \sum_{k=1}^{t} \tilde{z}_{kj} \leq 0, \quad \forall j, \\
\sum_{i=1}^{m} \tilde{x}_{i} = 1, \\
\sum_{r=1}^{s} \tilde{y}_{r} - \sum_{k=1}^{t} \tilde{z}_{k} = 1, \\
v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l\right) \leq \dot{x}_{ij} \leq v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^u\right), \quad \forall i, \forall j, \\
v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l\right) \leq \dot{z}_{ij} \leq v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^u\right), \quad \forall i, \\
w_k \left(\alpha z_{kj}^m + (1 - \alpha) z_{kj}^l\right) \leq \dot{z}_{kj} \leq w_k \left(\alpha z_{kj}^m + (1 - \alpha) z_{kj}^u\right), \quad \forall k, \forall j, \\
w_k \left(\alpha z_{kj}^m + (1 - \alpha) z_{kj}^l\right) \leq \dot{\bar{z}}_{kj} \leq w_k \left(\alpha z_{kj}^m + (1 - \alpha) z_{kj}^u\right), \quad \forall k, \\
u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^u\right) \leq \dot{y}_{rj} \leq u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^u\right), \quad \forall r, \forall j, \\
u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^u\right) \leq \dot{\bar{y}}_{rj} \leq u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^u\right), \quad \forall r, \\
\varepsilon - v_i \leq 0, \quad \forall i, \\
\varepsilon - u_r \leq 0, \quad \forall r, \\
\varepsilon - w_k \leq 0, \quad \forall k, \\
\varepsilon \geq 0, \quad \forall \dot{\bar{z}}_{kj}, \dot{y}_{rj}, \dot{\bar{y}}_{rj}, \varepsilon \geq 0, \quad \forall i, \forall r, \forall k, \forall j,
\]

where \(\varepsilon\) is a decision variable. This parametric LP model presents the maximum epsilon (\(\varepsilon^*\)) which applies to model (3.3) and all other values higher than \(\varepsilon^*\) cause infeasible results.

Choosing a suitable value for \(\varepsilon\) is a challenging problem in DEA (see [80, 81]). The value of \(\varepsilon\) selected in the epsilon-based DEA model influences the size of multipliers. In other words, different values of \(\varepsilon\) may lead to various efficiency assessments. Cook et al. [17] explained that letting \(\varepsilon = \varepsilon^*\), results in an identical assessment and, more importantly, the resulting DEA model has a sharper discriminating power.

**Theorem 3.3.** Model (3.4) is always feasible.

**Proof.** Let \(\tilde{y}_{rj}^0 = \frac{1}{s} \forall r, \forall j, \quad \dot{x}_{ij}^0 = \frac{1}{m} \forall i, \forall j, \quad \dot{x}_{ij}^0 = \frac{1}{s} \forall k, \forall j, \quad \dot{z}_{ij}^0 = \frac{1}{s} \forall i, \quad \dot{y}_{rj}^0 = \frac{1}{s} \forall r, \quad \dot{z}_{kj}^0 = \frac{1}{s} \forall k, \quad v_i^0 \in \left[\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l, \alpha x_{ij}^m + (1 - \alpha) x_{ij}^u\right], \quad \forall i, \quad w_k^0 \in \left[\alpha z_{kj}^m + (1 - \alpha) z_{kj}^l, \alpha z_{kj}^m + (1 - \alpha) z_{kj}^u\right], \quad \forall k, \quad u_r^0 \in \left[\alpha y_{rj}^m + (1 - \alpha) y_{rj}^u, \alpha y_{rj}^m + (1 - \alpha) y_{rj}^l\right], \quad \forall r, \quad \text{and} \quad \varepsilon = \min \{v_i^0, w_k^0, u_r^0, \forall i, \forall k, \forall r\}. \quad \text{Since} \quad \alpha x_{ij}^m + (1 - \alpha) \dot{x}_{ij}^l \leq \alpha x_{ij}^m + (1 - \alpha) x_{ij}^u \quad \text{and} \quad \alpha x_{ij}^m + (1 - \alpha) \dot{x}_{ij}^l \leq \alpha x_{ij}^m + (1 - \alpha) x_{ij}^l \quad \forall i, \forall j \text{it is easy to verify that:}

\[
v_i^0 \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l\right) \leq \frac{1}{m} \leq v_i^0 \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^u\right), \quad \forall i, \forall j,
\]
and
\[ \psi^*_i (\alpha \dot{z}_i^m + (1 - \alpha) \ddot{z}_i^l) \leq \frac{1}{m} \leq \psi^*_i (\alpha \dot{z}_i^m + (1 - \alpha) \ddot{z}_i^u), \quad \forall i. \]

Analogously, we obtain:
\[ \psi^*_k (\alpha \dot{z}_k^m + (1 - \alpha) \ddot{z}_k^l) \leq \frac{1}{l} \leq \psi^*_k (\alpha \dot{z}_k^m + (1 - \alpha) \ddot{z}_k^u), \quad \forall k, \forall j, \]
\[ \psi^*_r (\alpha \dot{y}_r^m + (1 - \alpha) \ddot{y}_r^l) \leq \frac{1}{s} \leq \psi^*_r (\alpha \dot{y}_r^m + (1 - \alpha) \ddot{y}_r^u), \quad \forall r, \forall j, \]
\[ \psi^*_r (\alpha \dot{y}_r^m + (1 - \alpha) \ddot{y}_r^l) \leq \frac{1}{s} \leq \psi^*_r (\alpha \dot{y}_r^m + (1 - \alpha) \ddot{y}_r^u), \quad \forall r. \]

Hence, the vector \((\varepsilon^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0, \psi^0)\) is a feasible solution for model (3.4). This completes the proof. \(\square\)

Theorem 3.4. \(\varepsilon^* \in (0, \infty).\)

Theorem 3.5. model (3.3) is feasible for \(\varepsilon = \varepsilon^*\) (see [82]).

4. Case study

In this section, we study the efficiency-based leadership in 20 independent banks\(^1\) in the Russian Federation using the method proposed in this study. The Russian banking sector has experienced considerable disorder in a highly centralized economy with the collapse of the Soviet Union. Russian banking system operates in an adverse economic environment and is dominated by large state-owned banks, which are highly fragmented and free of financial repression. The most significant feature of the modern Russian banking system is that the rules and regulations do not apply to all banks equally [45, 54, 95]. Among emerging and transition economies, the Russian banking industry has been rarely studied for performance management and efficiency.

4.1. Measurement inputs and outputs

DEA does not provide any guidelines for selecting input and output variables. Many researchers have suggested regression analysis and principal component analysis for selecting input and output variables. Deposit is a factor widely used for DEA applications in the banking industry with a dual role [31, 77, 88]. There are three approaches for designating deposits as inputs, outputs, or both (dual role). These approaches include the production approach with the aim of deposit producing, intermediation approach with the aim of profit earning, and intermediate product approach with both aims through two processes [31, 62]. Appropriate inputs are those variables that managers would like to minimize, and appropriate outputs are those with the maximizing purpose [59]. We found employees, fixed assets, and interest expenses are regularly defined as input variables (e.g., [34, 58]) while loans and incomes are regularly defined as output variables (e.g., [24, 40]). We performed a comprehensive review of the recent DEA applications in the banking industry presented in Table 1 to select the most suitable input and output variables for our study. We chose three discretionary inputs \((x_1: \text{number of branches, } x_2: \text{interest expense, and } x_3: \text{total expenses});\) two outputs \((y_1: \text{net profit and } y_2: \text{total assets as the ultimate outputs});\) and one non-discretionary input \((z_1: \text{branch density, which is defined as the number of branches per square kilometer and is an indicator of the space dimension for each national market}).\) \(z_1\) represents the availability of banking services for clients [52].

\(^1\) The names are changed to protect the anonymity of the banks.
<table>
<thead>
<tr>
<th>No.</th>
<th>Analysis</th>
<th>Inputs</th>
<th>Outputs</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A multi-period and multi-stage DEA model using triangular type-2 fuzzy numbers for measuring the efficiencies over consecutive periods.</td>
<td>Employees’ salaries, Fixed assets, Interest payments</td>
<td>Net interest incomes, Non-performing loans</td>
<td>Zhou et al. [98]</td>
</tr>
<tr>
<td>2</td>
<td>DEA utilized for examining the effects of risk determinants on efficiency considering the Malmquist Productivity Index. Double Bootstrapped Truncated Regression for obtaining bias-corrected scores.</td>
<td>Interest expenses, Operating expenses</td>
<td>Total income</td>
<td>Fernandes et al. [24]</td>
</tr>
<tr>
<td>3</td>
<td>A copula-based econometric model for identifying parameters of the structural equations and estimating technical efficiencies of the stochastic production and cost frontiers.</td>
<td>Labor physical capital, Investmen</td>
<td>Total loans</td>
<td>Huang et al. [34]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t Non-interest income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>A new version of the modified Semi-Oriented Radial Measure model, using directional distance function and choosing a relevant direction to efficiently deal with variables with both positive and negative values.</td>
<td>Total non-interest expenses, Other operating expenses, Fixed assets, Equity</td>
<td>Gross interest and dividend income, Total non-interest operating income, Loans, Net income</td>
<td>Kaffash et al. [46]</td>
</tr>
<tr>
<td>5</td>
<td>A new DEA-based analysis framework with a regression-based feedback mechanism for providing DEA with feedback about the relevance of the inputs and the outputs.</td>
<td>Personnel expenses, Fixed assets, Equity</td>
<td>Gross loans, Total customer deposits, Gross income</td>
<td>Ouenniche and Carrales [58]</td>
</tr>
<tr>
<td>6</td>
<td>A fuzzy two-stage Game-DEA approach was proposed using a bargaining game model.</td>
<td>Personnel costs, Operating costs, Interest costs, Fixed assets, Employees, Loanable funds</td>
<td>Interest income, Fee income, Fund transfer income, Investments</td>
<td>Tavana et al. [77]</td>
</tr>
<tr>
<td>7</td>
<td>Two-stage network DEA model and bootstrapped truncated regression for measuring overall bank efficiency and its decomposition in intermediation and operating efficiencies.</td>
<td>Total interest expenses, Loan loss provisions</td>
<td>Deposits, Net interest income</td>
<td>Gulati and Kumar [26]</td>
</tr>
<tr>
<td>8</td>
<td>An input-oriented profit bootstrap DEA for investigating homogeneous and heterogeneous branches according to branch size and location.</td>
<td>Total interest expenses, Loan loss provisions</td>
<td>Deposits, Net interest income</td>
<td>Aggelopoulos and Georgopoulos [2]</td>
</tr>
<tr>
<td>9</td>
<td>DEA and stochastic frontier approach for investigating the reliability of the single frontier model</td>
<td>Total interest expenses, Total non-interest expenses</td>
<td>Deposits, Loans</td>
<td>Silva et al. [70]</td>
</tr>
<tr>
<td>10</td>
<td>Two approaches for selecting inputs and outputs in DEA</td>
<td>Employees, Number of branches, Assets, Equity, Expenses</td>
<td>Deposits, Loans, Liquid assets</td>
<td>Toloo and Tichy [83]</td>
</tr>
<tr>
<td>11</td>
<td>Fuzzy multi-objective two-stage DEA model for providing a common scale for comparing performance.</td>
<td>Total liability ratio, Total equity ratio, Unit employee cost</td>
<td>Profit ratio, ROA, ROE</td>
<td>Wang et al. [87]</td>
</tr>
<tr>
<td>No.</td>
<td>Analysis</td>
<td>Inputs</td>
<td>Outputs</td>
<td>Reference</td>
</tr>
<tr>
<td>-----</td>
<td>--------------------------------------------------------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>12</td>
<td>Additive two-stage network DEA for disaggregating, evaluating, and testing the efficiencies.</td>
<td>Fixed assets, Employees</td>
<td>Non-interest income, Interest income, Non-performing loans</td>
<td>Wang et al. [88]</td>
</tr>
<tr>
<td>13</td>
<td>The network-DEA centralized efficiency model for optimizing two stages simultaneously.</td>
<td>Number of branches, Number of employees, Personnel expenses</td>
<td>Equity, Performance assets</td>
<td>Wanke and Barros [89]</td>
</tr>
<tr>
<td>14</td>
<td>A statistical test in the network DEA framework for assessing the importance of the risk metrics in evaluating income efficiency.</td>
<td>Operational costs, Fixed assets, Deposits, Interest earnings, Personnel expenses</td>
<td>Non-interest earnings, Interest earnings, Non-performing loans</td>
<td>Matthews [55]</td>
</tr>
<tr>
<td>15</td>
<td>Three-stage data envelopment analysis with adjustment of environmental factors and statistical noise for measuring managerial efficiency and highlighting the effect of environmental criteria.</td>
<td>Number of operational staff, Number of business personnel, Branch office rent, Operating expenses</td>
<td>Net interest spread income, Net fee income</td>
<td>Shyu and Chiang [69]</td>
</tr>
<tr>
<td>16</td>
<td>An alternative DEA model that treats deposits as an intermediate product.</td>
<td>Fixed assets, Number of employees</td>
<td>Total loans, Other earning assets</td>
<td>Holod and Lewis [31]</td>
</tr>
<tr>
<td>17</td>
<td>DEA utilized for investigating the effect of the “First Financial Restructuring” on the operating efficiency.</td>
<td>Interest expense, Non-interest expense, Total deposits</td>
<td>Interest revenue, Non-interest revenue, Total loans</td>
<td>Hsiao et al. [32]</td>
</tr>
</tbody>
</table>

4.2. Data collection

We developed a database using the 2018 financial statements of the 20 banks selected for this study. In addition, we used the annual reports from the SPARK database, provided by the Interfax news agency and the Central Bank of the Russian (CBR) Federation. Russia is ideal for this study because of its largest market among the Commonwealth of Independent States countries with bank-based economies. We used the websites of the Russian banks and the CBR site to collect data on banks in this study. While the collected data were in crisp form, there were some uncertainties concerning the accuracy of the data. We also needed to consider the problem of income smoothing in financial statements [10]. In response, we decided to use fuzzy sets [93] to incorporate these uncertainties and ambiguities into our model [62]. We used triangular fuzzy numbers \((a_l, a_m, a_u)\) to represent the uncertainties and vagueness in our data [14]. Accordingly, the collected crisp was converted into triangular fuzzy data through the following steps [62]:

1. Considering crisp data as \(a_m\).
2. \(a_l\) is equal to \(a_m - 1\%a_m\).
3. \(a_u\) is equal to \(a_m + 0.01\%a_m\).

The fuzzy input and output data for the Russian banks considered in this study are presented in Table 2.

5. Results and discussion

We used model (3.3) to calculate the efficiencies of 20 banks (DMUs) and normalized the results. The normalization of the efficiency scores is intended to produce efficiencies between 0 and 1 with at least one efficient unit [39, 85]. We used the GAMS program with different \(\alpha\) values and an epsilon value of \(10^{-7}\). We selected
Table 2. Fuzzy input and output data for the Russian banks.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Fixed assets 1</th>
<th>Number of employees 1</th>
<th>Total interest expenses 1</th>
<th>Branch density 1</th>
<th>Total loans 1</th>
<th>Total income 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yekaterinburg Savings Bank</td>
<td>4279.59</td>
<td>2483.87</td>
<td>4284.30</td>
<td>2942</td>
<td>2495</td>
<td>2498</td>
</tr>
<tr>
<td>Bryansk Capital Bank</td>
<td>4849.40</td>
<td>4854.34</td>
<td>4854.74</td>
<td>3126</td>
<td>3129</td>
<td>3132</td>
</tr>
<tr>
<td>Novosibirsk People Bank</td>
<td>5489.06</td>
<td>5495.70</td>
<td>5499.46</td>
<td>4155</td>
<td>4159</td>
<td>4163</td>
</tr>
<tr>
<td>Northwest People Bank</td>
<td>6159.76</td>
<td>6165.9</td>
<td>6166.55</td>
<td>1597</td>
<td>1599</td>
<td>1601</td>
</tr>
<tr>
<td>Northern Kazan Trust</td>
<td>4566.72</td>
<td>4551.5</td>
<td>4551.72</td>
<td>2499</td>
<td>2499</td>
<td>2499</td>
</tr>
<tr>
<td>Volga National Bank</td>
<td>2175.86</td>
<td>2178.0</td>
<td>2178.25</td>
<td>2356</td>
<td>2356</td>
<td>2360</td>
</tr>
<tr>
<td>Saint Petersburg Savings Bank</td>
<td>3814.93</td>
<td>3818.8</td>
<td>3819.13</td>
<td>402</td>
<td>402</td>
<td>402</td>
</tr>
<tr>
<td>Krasnodar Financial Bank</td>
<td>22049.82</td>
<td>22062.9</td>
<td>22065.09</td>
<td>1098</td>
<td>1099</td>
<td>1100</td>
</tr>
<tr>
<td>Nizhny Tagil Bank</td>
<td>171073.95</td>
<td>171245.2</td>
<td>171262.32</td>
<td>27514</td>
<td>27524</td>
<td>27570</td>
</tr>
<tr>
<td>Khabarovsk Capital Bank</td>
<td>206665.65</td>
<td>20687.3</td>
<td>20690</td>
<td>27284</td>
<td>27311</td>
<td>27338</td>
</tr>
<tr>
<td>Barnaul Bankcorp</td>
<td>156387.31</td>
<td>156543.9</td>
<td>156559.14</td>
<td>17014</td>
<td>17031</td>
<td>17048</td>
</tr>
<tr>
<td>Omsk Financial Group</td>
<td>4445.53</td>
<td>4455.0</td>
<td>4455.43</td>
<td>1131</td>
<td>1132</td>
<td>1133</td>
</tr>
<tr>
<td>Far East Bank</td>
<td>4544.69</td>
<td>4549.2</td>
<td>4549.69</td>
<td>1075</td>
<td>1076</td>
<td>1077</td>
</tr>
<tr>
<td>Siberia State Bank</td>
<td>2934.42</td>
<td>2937.4</td>
<td>2937.65</td>
<td>2139</td>
<td>2141</td>
<td>2143</td>
</tr>
<tr>
<td>Union Bank of Tyumen</td>
<td>109875.52</td>
<td>10998.5</td>
<td>10999.62</td>
<td>5412</td>
<td>5417</td>
<td>5417</td>
</tr>
<tr>
<td>Ural Trust Bank</td>
<td>617.13</td>
<td>617.7</td>
<td>617.80</td>
<td>1337</td>
<td>1338</td>
<td>1339</td>
</tr>
<tr>
<td>First Citizens Samara Bank</td>
<td>13021.10</td>
<td>13034.1</td>
<td>13035.44</td>
<td>3796</td>
<td>3800</td>
<td>3804</td>
</tr>
<tr>
<td>Mahkachkala Federal</td>
<td>13330.38</td>
<td>13336.7</td>
<td>13338.03</td>
<td>3173</td>
<td>3176</td>
<td>3179</td>
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<tr>
<td>First Chiia Bank</td>
<td>12494.20</td>
<td>12508.7</td>
<td>12508.95</td>
<td>10949</td>
<td>10960</td>
<td>10971</td>
</tr>
<tr>
<td>Cherepovets Bank</td>
<td>2505.83</td>
<td>2508.3</td>
<td>2508.59</td>
<td>3528</td>
<td>3532</td>
<td>3536</td>
</tr>
</tbody>
</table>

FDIMU = (x, y, z) = 617.13 617.7 617.8 402 402 402 1194.31 1195.51 1195.63 0.103 0.103 0.103 2259.961 2262.223 2262.250 12 299.868 210.078 210.099 27

Notes. *In Millions of Russian Rubles. **In 10⁻³ number per km².
Table 3. Normalized efficiency scores and rankings of the Russian banks.

<table>
<thead>
<tr>
<th>j</th>
<th>DMU</th>
<th>Efficiency scores*</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \alpha = 0.25 )</td>
<td>( \alpha = 0.5 )</td>
<td>( \alpha = 0.75 )</td>
<td>( \alpha = 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Yekaterinburg Savings Bank</td>
<td>0.069623 18</td>
<td>0.06962 18</td>
<td>0.06962 18</td>
<td>0.06962 18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Bryansk Capital Bank</td>
<td>0.243749 6</td>
<td>0.243748 6</td>
<td>0.243748 6</td>
<td>0.243748 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Novosibirsk People Bank</td>
<td>0.453649 5</td>
<td>0.45365 5</td>
<td>0.453651 5</td>
<td>0.453652 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Northwest People Bank</td>
<td>0.222637 7</td>
<td>0.222618 7</td>
<td>0.222599 7</td>
<td>0.22258 7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Northern Kazan Trust</td>
<td>0.192283 9</td>
<td>0.192289 9</td>
<td>0.192959 9</td>
<td>0.192302 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Volga National Bank</td>
<td>0.187337 10</td>
<td>0.187341 10</td>
<td>0.187343 10</td>
<td>0.187347 10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Saint Petersburg Savings Bank</td>
<td>0.612904 2</td>
<td>0.613196 2</td>
<td>0.613342 2</td>
<td>0.613342 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Krassnodar Financial</td>
<td>0.222145 8</td>
<td>0.222146 8</td>
<td>0.222146 8</td>
<td>0.222146 8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Nizhny Tagil Bank</td>
<td>0.111105 15</td>
<td>0.111104 15</td>
<td>0.111105 15</td>
<td>0.11111 15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Khabarovsk Capital Bank</td>
<td>0.152525 13</td>
<td>0.152527 13</td>
<td>0.152529 13</td>
<td>0.152531 13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Barnaul Bancorp</td>
<td>0.165635 11</td>
<td>0.165631 11</td>
<td>0.165626 11</td>
<td>0.165626 11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Omsk Financial Group</td>
<td>0.07534 17</td>
<td>0.075341 17</td>
<td>0.075341 17</td>
<td>0.075343 17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Far East Bank</td>
<td>0.587567 3</td>
<td>0.58755 3</td>
<td>0.587544 3</td>
<td>0.587544 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Siberia State Bank</td>
<td>0.159976 12</td>
<td>0.15997 12</td>
<td>0.159965 12</td>
<td>0.159958 12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Union Bank of Tyumen</td>
<td>0.045352 19</td>
<td>0.045356 19</td>
<td>0.045356 19</td>
<td>0.045358 19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Ural Trust Bank</td>
<td>0.08212 16</td>
<td>0.082128 16</td>
<td>0.082135 16</td>
<td>0.082143 16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>First Citizens Samara</td>
<td>0.571191 4</td>
<td>0.571175 4</td>
<td>0.571144 4</td>
<td>0.571144 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Makhachkala Federal</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>First Chita Bank</td>
<td>0.135226 14</td>
<td>0.135223 14</td>
<td>0.13522 14</td>
<td>0.135216 14</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>20</td>
<td>Cherepovets Bank</td>
<td>0.031122 20</td>
<td>0.031122 20</td>
<td>0.031122 20</td>
<td>0.031121 20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. *Normalized efficiency scores are calculated for \( \alpha \in (0, 1] \) and \( \varepsilon = 10^{-7} \).

In this non-maximum value for epsilon arbitrarily to achieve feasible solutions due to the large values. However, epsilon’s optimal value is influential in measuring the weights and producing results with more discriminating power [17].

Table 3 illustrates the normalized efficiency scores of the 20 banks and their ranks for different values of alpha in columns 3–12. The results demonstrate that all 20 banks are inefficient, and their efficiency values have decreased substantially because of using the CW approach and the ideal point method in efficiency evaluation. This method will consider a virtual ideal unit as the reference object with the lowest inputs and highest outputs. This ideal unit will be considered our ideal efficient DMU with an efficiency score of one [75]. There is a large difference between the efficiency of the ideal unit and all other DMUs.

In the last step, model (3.4) is used to obtain the maximum value of epsilon for model (3.3). Other values greater than \( \varepsilon^* \) produce infeasible results [82]. Accordingly, the epsilon’s optimal values (maximum epsilon) were estimated for \( \alpha \in (0, 1] \) and applied to recalculate the efficiency scores in the case study, and their value is presented in Table 4. Again, we have normalized the efficiency scores to avoid small efficiencies derived from the implementation of the CW model and the ideal unit method.

A graphical representation of the recalculated efficiency scores for the optimal epsilon values is presented in Figure 1. The majority of the DMUs have the same rankings for different alpha values due to the adjustments of the weights for achieving optimal answer for \( E^*_{IDMU}(\alpha) \) in a CW model when there is no flexibility for weights.

As shown in Tables 3 and 4, the efficiency scores of the proposed model decrease with the implementation of \( \varepsilon^* \), and all DMUs obtain different rankings. Also, the efficiency scores in Table 4 follows a different trend. The efficiency scores for the majority of the DMUs decrease with increasing alpha from zero to 0.5 and then follow an increasing trend.

The results in Table 3 show that by using a lower value for the epsilon in model (3.3), the Makhachkala Federal Bank with an efficiency score of 1 is the best DMU even with lower corporate size (around 3170 employees) in
Table 4. Normalized recalculated efficiency scores and rankings of the Russian banks.

<table>
<thead>
<tr>
<th>j</th>
<th>DMU</th>
<th>Efficiency scores*</th>
<th>Rank</th>
<th>Efficiency scores*</th>
<th>Rank</th>
<th>Efficiency scores*</th>
<th>Rank</th>
<th>Efficiency scores*</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\alpha = 0.25$</td>
<td></td>
<td>$\alpha = 0.5$</td>
<td></td>
<td>$\alpha = 0.75$</td>
<td></td>
<td>$\alpha = 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = 4.047 \times 10^{-1}$</td>
<td></td>
<td>$\varepsilon = 4.422 \times 10^{-1}$</td>
<td></td>
<td>$\varepsilon = 4.421 \times 10^{-1}$</td>
<td></td>
<td>$\varepsilon = 4.420 \times 10^{-1}$</td>
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<tr>
<td>1</td>
<td>Yekaterinburg Savings Bank</td>
<td>0.128967</td>
<td>14</td>
<td>0.127221</td>
<td>13</td>
<td>0.127222</td>
<td>13</td>
<td>0.127222</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>Bryansk Capital Bank</td>
<td>0.431073</td>
<td>7</td>
<td>0.426641</td>
<td>7</td>
<td>0.426641</td>
<td>7</td>
<td>0.426641</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Novosibirsk People Bank</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Northwest People Bank</td>
<td>0.129593</td>
<td>13</td>
<td>0.123205</td>
<td>14</td>
<td>0.123205</td>
<td>14</td>
<td>0.123206</td>
<td>14</td>
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<tr>
<td>5</td>
<td>Northern Kazan Trust</td>
<td>0.271678</td>
<td>8</td>
<td>0.265887</td>
<td>8</td>
<td>0.265887</td>
<td>8</td>
<td>0.265887</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>Volga National Bank</td>
<td>0.486234</td>
<td>5</td>
<td>0.468991</td>
<td>5</td>
<td>0.468993</td>
<td>5</td>
<td>0.468996</td>
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</tr>
<tr>
<td>7</td>
<td>Saint Petersburg Savings Bank</td>
<td>0.179926</td>
<td>12</td>
<td>0.178515</td>
<td>12</td>
<td>0.178515</td>
<td>12</td>
<td>0.178515</td>
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</tr>
<tr>
<td>8</td>
<td>Krasnodar Financial</td>
<td>0.026062</td>
<td>20</td>
<td>0.025027</td>
<td>20</td>
<td>0.025027</td>
<td>20</td>
<td>0.025027</td>
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<tr>
<td>9</td>
<td>Nizhny Tagil Bank</td>
<td>0.059153</td>
<td>18</td>
<td>0.049827</td>
<td>18</td>
<td>0.049827</td>
<td>18</td>
<td>0.049827</td>
<td>18</td>
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<tr>
<td>10</td>
<td>Khabarovsk Capital Bank</td>
<td>0.755002</td>
<td>3</td>
<td>0.782661</td>
<td>3</td>
<td>0.782663</td>
<td>3</td>
<td>0.782665</td>
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<td>11</td>
<td>Barnaul Bancorp</td>
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<td>19</td>
<td>0.031521</td>
<td>19</td>
<td>0.031521</td>
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<tr>
<td>12</td>
<td>Omsk Financial Group</td>
<td>0.053232</td>
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<td>0.052798</td>
<td>17</td>
<td>0.052798</td>
<td>17</td>
<td>0.052798</td>
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<tr>
<td>13</td>
<td>Far East Bank</td>
<td>0.265247</td>
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<td>0.242162</td>
<td>10</td>
<td>0.242163</td>
<td>10</td>
<td>0.242164</td>
<td>10</td>
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<tr>
<td>14</td>
<td>Siberia State Bank</td>
<td>0.106244</td>
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<td>0.067849</td>
<td>16</td>
<td>0.067849</td>
<td>16</td>
<td>0.067849</td>
<td>16</td>
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<tr>
<td>15</td>
<td>Union Bank of Tyumen</td>
<td>0.118847</td>
<td>15</td>
<td>0.118905</td>
<td>15</td>
<td>0.118905</td>
<td>15</td>
<td>0.118906</td>
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<tr>
<td>16</td>
<td>Ural Trust Bank</td>
<td>0.827162</td>
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<td>0.877927</td>
<td>2</td>
<td>0.877942</td>
<td>2</td>
<td>0.877958</td>
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<tr>
<td>17</td>
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<td>0.434996</td>
<td>6</td>
<td>0.434997</td>
<td>6</td>
<td>0.434998</td>
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<tr>
<td>18</td>
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<tr>
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<td>0.264653</td>
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<td>0.251886</td>
<td>9</td>
<td>0.251886</td>
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<tr>
<td>20</td>
<td>Cherepovets Bank</td>
<td>0.210726</td>
<td>11</td>
<td>0.224387</td>
<td>11</td>
<td>0.224389</td>
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<td>0.22439</td>
<td>11</td>
</tr>
</tbody>
</table>

Notes. *Normalized recalculated efficiency scores are obtained with $\alpha \in (0, 1]$ and the maximum $\varepsilon$. 

Figure 1. Efficiency scores with different alpha levels.
comparison with Nizhny Tagil Bank (around 27,550 employees) and Khabarovsk Capital Bank (around 27,300 employees). However, this result is different when we selected $\epsilon^*$ as the optimal value of the epsilon in model (3.3). In this situation, the Novosibirsk People Bank, with an efficiency of 1 has the highest efficiency among the 20 banks. The results demonstrate that focusing on unidimensional and unilateral attributes like the firm size [23, 68] is not sufficient for successfully characterizing leaders. Consequently, most literature reviews have concluded that trait theories have fallen out of interest between researchers in the leadership area [38]. We advocate a multilevel, multicomponent, and multidisciplinary approach to leadership [1, 5, 51, 84] for achieving robust and reliable results.

6. Conclusions and future research

In this paper, we highlighted three shortcomings in the existing DEA models and used non-discretionary inputs and fuzzy data in a CW model with an ideal point method to measure efficiency in the Russian banking industry. We also considered uncertainties inherent in real-world data and used fuzzy sets to take into account these uncertainties. In addition, we considered non-discretionary inputs to incorporate the homogeneity of the DMUs in our model. We used the proposed CW model and ranked 20 independent banks in the Russian Federation. Finally, we used our model to find the efficiency-based leaders in the Russian banking industry. The results show a unidimensional and unilateral assessment of leading organizations merely according to corporate size is not sufficient to effectively characterize industry leaders.

As for further research, we suggest developing a multidimensional DEA model considering different weights for different dimensions. This will allow us to include various characteristics of leadership based on the existent theories in the forms of different dimensions of inputs and outputs.

Declaration of interest. The authors declare no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

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REFERENCES


