Pricing Decision and Coordination Mechanism of Dual-Channel Supply Chain Dominated by a Risk-Aversion Retailer Under Demand Disruption

Bo Yan∗, Zhuo Chen, Yan-Ping Liu and Xiao-Xu Chen

Abstract. This paper studies a dual-channel supply chain composed of a retailer and a supplier, and discusses the optimal decisions of supply chain participants under decentralized decision-making without and with demand disruption, respectively. By comparing the optimal decisions in the two scenarios, we find that the optimal decision after demand disruption is a linear function of the demand disruption plus optimal decision before demand disruption. Additionally, when the demand disruption is in interval \((-\psi_u, \psi_u)\), the optimal total production of the supply chain is equal before and after demand disruption. Moreover, the profits of the supply chain members and the value of their recognizing demand disruption are largely affected by the scale of demand disruption. Finally, the results show that the improved revenue-sharing contract can effectively improve the supply chain performance.

Mathematics Subject Classification. 65D18, 90B50.

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1. Introduction

With the development of globalization, the supply chain is more vulnerable to the impact of the external environment. Especially in recent years, the impact of frequent public health security incidents on the supply chain cannot be underestimated. For example, the COVID-19 outbreak in 2019 has had a profound impact on many global supply chains, such as aviation industry, tourism industry and pharmaceutical industry. The risk of large losses from unexpected events has led many researchers to study how to build more stable supply chains (e.g., see [19, 27–29]). In recent years, an increasing number of scholars pay attention to the strategic research under the emergency. Some scholars have explained the problem in terms of supply disruption (e.g., see [1, 13, 17, 31]). However, when an emergency occurs, the supply-demand relationship of the whole supply chain will change. Severe cases can even lead to supply disruptions. For example, the outbreak of avian influenza can dramatically increase demand for medicines and dramatically reduce demand for poultry. Demand is hard to be predicted in current fast-changing market. Therefore, this work will study the optimal decision of the supply chain members under demand disruption.

Keywords. Demand disruption, risk averse, dual-channel supply chain, value of recognizing demand disruption, coordination mechanism.

School of Economics and Finance, South China University of Technology, Guangzhou 510006, P.R. China.

∗Corresponding author: yanbo@scut.edu.cn

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With the rapid development of e-commerce and logistics, suppliers have established their own direct retail channels, such as Dell, HP, Lenovo and Apple. Thus, the dual-channel supply chain has become the main type of supply chain system. Based on this fact, this work takes the dual-channel supply chain as the basic model to explore how the direct retail channel and the traditional retail channel respond to the impact of emergencies. The facility location problem and its related financial issues also have a significant impact on the configuration of supply chain structure [23]. Some scholars have paid attention to the impact of demand disruption on supply chain. Yang and Wu [43] showed that a partially vertically integrated supply chain (SC) structure has the best robustness, whereas the centralized SC structure, which is considered to be the most profitable, has the worst robustness. Soleimani et al. [30] believed that the optimal price will be affected by sharing demand for the direct channel in both centralized and decentralized problems. Yan et al. [41] demonstrated that the optimal sales decision of the disturbed supply chain is related to market share and demand disruption under decentralized decision-making. However, large retailers such as Suning and Officeworks occupy the dominant position throughout the supply chain because of their huge customer resources. The model discussed in this paper takes sales of computer products of the dominant retailer (such as Suning and Officeworks) as the prototype to study the optimal decision of dual-channel supply chain under demand disruption. Moreover, the competition may lead to conflict between the two channels in terms of cross-channel price and operation. Supply chain decision-makers face certain risks in practical operation, specifically when the external environment of the supply chain is unstable. Based on the above background, this paper will discuss the dual-channel supply chain dominated by the retailer.

This paper mainly studies the impact of demand disruption on dual-channel supply chain, focusing on the following three key issues. First, how does the supply chain make optimal decisions under demand disruption, including optimal pricing, optimal channel demand, optimal total production and optimal profit? Second, what is the difference between the direct retail channel and the traditional retail channel affected by demand disruption? Third, how to use the revenue-sharing contract to coordinate the dual-channel supply chain under demand disruption?

To answer these questions, the analysis of this paper is divided into three parts. First, a retailer-dominant Stackelberg model is established. Second, the optimal decision-making of supply chain with and without demand disruption is discussed and compared. Finally, an improved revenue-sharing contract is established to coordinate the decentralized supply chain, so as to achieve the optimal dual-channel decision under centralized decision-making.

The rest of the paper is organized as follows. Section 2 reviews relevant literature. Section 3 describes the model formulations. Sections 4 and 5 present the model analysis. Section 6 presents numerical case study results. Finally, Section 7 concludes the paper.

2. Literature review

The study of dual-channel supply chain originated from the study of price competition of dual-channel supply chain by Balasubramanian [3]. In recent years, the research mainly focuses on three aspects: the impact of channel structure on the supply chain, supply chain decision under demand disruption, and supply chains coordination in case of emergency. An overview is provided in Table 1.

This paper contributes to the literature from the following aspects. Firstly, previous studies mainly focused on the impact of direct retail channel on dual-channel supply chain. This paper further investigates the impact of demand disruption on dual-channel supply chain. Moreover, the shortage cost factor and the handling cost factor are innovatively introduced into the supplier’s expected profit function to explore the impact of demand disruption cost on the optimal decision of supply chain.

Additionally, previous studies have shown that demand disruption has a profound impact on supply chain performance. However, few studies focus on the key factors that affect the optimal decision-making of supply chain participants under demand disruption. Therefore, by comparing the scenario of the decentralized
### Table 1. Overview of literature reviews.

<table>
<thead>
<tr>
<th>Year</th>
<th>Main focus</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(1) The impact of channel structure on the supply chain</strong></td>
<td></td>
</tr>
<tr>
<td>Webb [33] 2002</td>
<td>Brand differentiation strategies between two channels can alleviate contradictions.</td>
</tr>
<tr>
<td>Hendershott and Zhang [15] 2006</td>
<td>Only when online channels have high return and transaction costs, and retailers have a large percentage of sales discounts, do suppliers benefit from maintaining a single traditional distribution channel. On the contrary, the dual-channel marketing model is more beneficial to the interests of suppliers.</td>
</tr>
<tr>
<td>Arya et al. [2] 2007</td>
<td>Dual-channel sales model could enable both suppliers and retailers to achieve Pareto optimum if both suppliers and retailers have the advantages of production costs and sales costs, respectively.</td>
</tr>
<tr>
<td>Yan [40] 2011</td>
<td>Suppliers and retailers can reduce the channel conflict of supply chains by formulating reasonable profit distribution schemes.</td>
</tr>
<tr>
<td>Pu et al. [24] 2017</td>
<td>Under deterministic demand, the sales effort level of offline stores under decentralized setting and the profit of dual channel supply chain are lower than those under centralized setting, and both decrease with the increase of free-riding consumers.</td>
</tr>
<tr>
<td>Chen et al. [10] 2017</td>
<td>Considering the price and quality decisions, it is found that the increase of new channels can improve the supply chain performance.</td>
</tr>
<tr>
<td>Guo [14] 2019</td>
<td>Channel conflict is represented in a dynamic competition between online retailer and traditional retailer, and online customer reviews affect the purchase decisions of potential customers by providing information about retail prices in both channels.</td>
</tr>
<tr>
<td>Pu et al. [25] 2019</td>
<td>Supply chain firms would prefer the distribution strategy of selling high-end and low-end products through offline (online) channels and online (offline) channels when consumers are significantly (minimally) sensitive to the product quality differentiation.</td>
</tr>
<tr>
<td>Chen et al. [11] 2020</td>
<td>Compared with centralized model, for any given selling price, the ratio of profit margins of selling one unit in the direct and retail channels determines the retailer’s service strategy; and the supplier will raise the level of direct channel service, but put less effort on quality improvement in the decentralized model.</td>
</tr>
<tr>
<td><strong>(2) Supply chain decision under demand disruption</strong></td>
<td></td>
</tr>
<tr>
<td>Huang et al. [18] 2012</td>
<td>The results indicate that the optimal output is robust under demand disruption in both centralized and decentralized dual-channel supply chains.</td>
</tr>
<tr>
<td>Lei et al. [20] 2012</td>
<td>When demand and cost disruption are private information, the linear contract menu can effectively analyze the asymmetric information supply chain under demand and cost disruption.</td>
</tr>
<tr>
<td>Cao et al. [6] 2013</td>
<td>A revenue-sharing method is proposed to coordinate a supply chain consisting of one supplier and ( n ) Cournot competing retailers when the production cost and demands are simultaneously disturbed.</td>
</tr>
<tr>
<td>Cao [5] 2014</td>
<td>An improved revenue-sharing contract is proposed to coordinate the dual-channel supply chain when the two end competition market demands are simultaneously disturbed.</td>
</tr>
<tr>
<td>Chen et al. [9] 2017</td>
<td>The retailer is reluctant to share his private information on the disrupted demand with his partner because of the fear of information leakage. Meanwhile, the performance of the whole chain may become worse off if the information of disrupted demand is shared in this chain.</td>
</tr>
<tr>
<td>Behzadi et al. [4] 2018</td>
<td>Three proposed risk management strategies, namely diversified demand market, backup demand market and flexible rerouting, improve both expected profit and risk measures in agribusiness supply chains.</td>
</tr>
<tr>
<td>Tang et al. [32] 2018</td>
<td>Suitable changes and improvements in revenue-sharing contracts can help coordinate the dual-channel supply chain with demand disruptions.</td>
</tr>
<tr>
<td>Rahmani and Yavari [26] 2019</td>
<td>The centralized scenario under demand disruptions leads to achieve the products with a higher green degree compared with the decentralized scenario.</td>
</tr>
</tbody>
</table>
### Table 1. Continued.

<table>
<thead>
<tr>
<th>Year</th>
<th>Main focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>2019</td>
<td>Hormozzadeh-Ghalati et al. [16] Inventory control is used to manage the risk of demand uncertainty. The corporate profit will be maximized if the optimal integration of suppliers and the optimal order quantities from each supplier are determined.</td>
</tr>
<tr>
<td>2020</td>
<td>Yan et al. [42] The improved revenue-sharing contract can better coordinate the RFID-based fresh agricultural product supply chain after the demand disruption.</td>
</tr>
<tr>
<td>2020</td>
<td>Zhao et al. [45] Demand disruptions can promote supply chain coordination in a fashion supply chain.</td>
</tr>
<tr>
<td></td>
<td>(3) Supply chains coordination in case of emergency</td>
</tr>
<tr>
<td>2007</td>
<td>Xiao et al. [37] In case of demand disruption, a linear or whole unit volume discount program coordination mechanism can be used to coordinate a supply chain consisting of one supplier and two competing retailers.</td>
</tr>
<tr>
<td>2009</td>
<td>Xiao et al. [36] In the supply chain composed of a supplier and a retailer, the coordination mechanism of market demand which is sensitive to retail price and service and the impact of demand interruption on the coordination mechanism are proposed.</td>
</tr>
<tr>
<td>2009</td>
<td>Chen and Xiao [7] In a supply chain consisting of one supplier, one dominant retailer and multiple fringe retailers, linear quantity discount schedule and Groves wholesale price schedule are proposed to coordinate demand disruptions.</td>
</tr>
<tr>
<td>2012</td>
<td>Zhang et al. [44] In a one-supplier–two-retailers supply chain, revenue-sharing contracts are used to coordinate demand disruptions.</td>
</tr>
<tr>
<td>2012</td>
<td>Chen et al. [8] A nonlinear Grove wholesale price scheme can fully coordinate such a supply chain even if both market demand and production cost are disturbed.</td>
</tr>
<tr>
<td>2018</td>
<td>Nobil et al. [22] This research shows that the proposed inventory model with shortage is a convex programming problem, so an accurate algorithm for solving these inventory problems is proposed.</td>
</tr>
<tr>
<td>2019</td>
<td>Xu et al. [39] In the supply of two competing suppliers and one supplier, the bilateral participation contract can achieve perfect coordination if the competition is weak or if a transfer payment policy exists.</td>
</tr>
</tbody>
</table>

...decision-making with and without demand disruption, this paper further identifies the key parameters that affect the optimal pricing and demand of each participant.

Moreover, previous studies have shown that supply chain coordination is an effective way to solve channel conflict. This paper further studies how to coordinate the dual-channel supply chain under demand disruption. This work will enrich the literature of solving supply chain channel conflict.

### 3. Model Descriptions

This paper investigates a dual-channel supply chain consisting of one supplier and one retailer. The supplier opens the direct retail channel, and the retailer opens the traditional retail channel. The operation of the supply chain is illustrated in Figure 1.

![Figure 1. The structure of the dual-channel supply chain.](image-url)
Table 2. Symbol description.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>The wholesale price, decided by the supplier.</td>
</tr>
<tr>
<td>c</td>
<td>Unit production cost.</td>
</tr>
<tr>
<td>m</td>
<td>Wholesale and retail price differential, which is called “markon”.</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Retail price of Channel $i$.</td>
</tr>
<tr>
<td>$k_r$</td>
<td>The risk-averse coefficient of the retailer.</td>
</tr>
<tr>
<td>$k$</td>
<td>The risk-averse coefficient of the supply chain.</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Demand of Channel $i$.</td>
</tr>
<tr>
<td>$D$</td>
<td>The total demand of the whole supply chain.</td>
</tr>
<tr>
<td>Q</td>
<td>Total production of the supply chain.</td>
</tr>
<tr>
<td>$\tilde{a}$</td>
<td>The potential scale of market demand. And it follows the normal distribution $N(a, \sigma^2)$.</td>
</tr>
<tr>
<td>$\Delta a$</td>
<td>The demand disruption caused by the disaster.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>The market share of the direct retail channel.</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>The individual price effects of the traditional retail channel.</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>The individual price effects of the direct retail channel.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The cross-price effects of the two channels.</td>
</tr>
<tr>
<td>$u_1$</td>
<td>The shortage cost factor.</td>
</tr>
<tr>
<td>$u_2$</td>
<td>The handling cost factor.</td>
</tr>
<tr>
<td>U</td>
<td>The utility functions.</td>
</tr>
<tr>
<td>$\pi_j$</td>
<td>Profit of member $j$ in the supply chain.</td>
</tr>
<tr>
<td>$\pi$</td>
<td>The total profit of the whole supply chain.</td>
</tr>
<tr>
<td>$S_k$</td>
<td>The market share of the direct retail channel and the demand disruption are divided into 3 intervals in decentralized decision-making.</td>
</tr>
<tr>
<td>$\psi$</td>
<td>For expositional simplicity, we denoted $\psi = \frac{\alpha_1^2 + 2\alpha_1\alpha_2 - 2\alpha_1\beta - \beta^2}{\alpha_1^2 + 2\alpha_1\alpha_2 - 2\alpha_1\beta - \beta^2}$.</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>For expositional simplicity, we denoted $\gamma_1 = \beta - \alpha_2 + \alpha_1\rho + \alpha_2\rho - 2\beta\rho$.</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>For expositional simplicity, we denoted $\gamma_2 = \beta + \alpha_1 + \alpha_1\rho - \beta\rho$.</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>For expositional simplicity, we denoted $\gamma_3 = \alpha_1^2 - 2\alpha_1\beta + 2\alpha_1\alpha_2 - \beta^2$.</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>For expositional simplicity, we denoted $\gamma_4 = 3\alpha_1^2 - 6\alpha_1\beta + 4\alpha_1\alpha_2 - \beta^2$.</td>
</tr>
<tr>
<td>w</td>
<td>The wholesale price determined after coordination.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The proportion of the retailer’s sales revenue shared with the supplier.</td>
</tr>
</tbody>
</table>

Notes. Subscripts: $i = r$ and $i = d$ represent the traditional retail channel and the direct retail channel, respectively; $j = s$ and $j = r$ represents the supplier and the retailer, respectively. $k = 1, 2, 3$.

Table 2 lists the definitions of the symbols used in the paper.

We assume that the retailer acts as the leader, who is risk averse with a coefficient of risk aversion $k_r$. The supplier acts as the follower, who is risk neutral. The reasons for applying the above assumptions are as follow. The retailer is faced with pricing decisions and channel competition from the supplier. Thus, this work focuses more on the fact that the retailer is more susceptible to demand disruption than the supplier. This two-echelon supply chain constitutes a Stackelberg game dominated by the retailer. It is also assumed that the loss of supply chain caused by demand disruption is borne by the supplier. This is because that the supplier is primarily responsible for the production of the product. When there is excess demand or demand shortage, the supplier needs to bear the loss of the whole supply chain.

$c$ denotes the unit cost of production. $w$ denotes the wholesale price. The retail price of products sold in the traditional retail channel and the direct channel is denoted as $p_r$ and $p_d$, respectively. Let $p_r = w + m$, and $m$ represents wholesale and retail price differential which is called “markon”.

The retailer plays a dominant role in the dual-channel supply chain. Therefore, the retailer can obtain a lower wholesale price by using its bargaining power. In order to attract consumers to purchase goods in the traditional retail channel the retailer sets the retail price of traditional retail channels lower than that of the direct retail
channel, that is, \( p_d > p_r \). For instance the retail price of an Apple computer sold by Officeworks during sales promotion will be lower than that of the same computer sold on Apple’s official website. The retail price should be higher than the wholesale price, i.e., \( p_r > w \) and \( p_d > w \).

Let \( D_r \) denote the demand of the traditional retail channel and \( D_d \) denote the demand of the direct retail channel. \( D = D_r + D_d \) represents the total demand of the whole supply chain [5]. We assume that the market demand of two channels is a linear function of price [17] and the form is as follow:

\[
D_r = (1 - \rho)a - \alpha_1 p_r + \beta p_d, \quad (3.1)
\]

\[
D_d = \rho a - \alpha_2 p_d + \beta p_r, \quad (3.2)
\]

where \( a \) denotes the market demand in the traditional retail channel, \( \rho \) is the market share of the direct retail channel. \( \alpha_1 \) and \( \alpha_2 \) denote the individual price effects of the traditional retail channel and the direct retail channel, respectively. The cross-price effects \( \beta \) of two retail channels are assumed as the same. Additionally, the individual price effect is greater than the cross-price effect [12], i.e., \( \alpha_1 > \beta \) and \( \alpha_2 > \beta \).

The total demand of the whole supply chain is:

\[
D = a - (\alpha_1 - \beta)p_r - (\alpha_2 - \beta)p_d. \quad (3.3)
\]

4. Comparison of decision before and after the demand disruption

In this section, the optimal decisions before and after the demand disruption of two-echelon supply chain under decentralized decision-making will be discussed. In the following discussion, the parameter with superscript “*” represents the optimal decision of supply chain participants under decentralized decision-making without demand disruption. The parameter with superscript “–#” represents the optimal decision of supply chain participants under decentralized decision-making with demand disruption.

4.1. Decentralized decisions without demand disruption

The dominant retailer first sets the retail price of the traditional retail channel. Then the supplier determines the wholesale price and retail price of the direct retail channel on the basis of the retailer’s reaction. In this paper, the backward induction method is used to solve the problem.

The supplier’s expected profit is determined by:

\[
E(\pi_s) = (w - c)D_r + (p_d - c)D_d. \quad (4.1)
\]

The retailer’s profit is determined by:

\[
E(\pi_r) = mD_r. \quad (4.2)
\]

First, we analyze the supplier’s decision. Because the supplier is risk-neutral, its utility function is the same as the expected revenue function. Thus, the utility function of the supplier is:

\[
U_s(\pi_s) = (w - c)D_r + (p_d - c)D_d. \quad (4.3)
\]

By substituting equations (3.1) and (3.2) for equation (4.3), we can obtain the Hessian matrix as follows:

\[
H = \begin{bmatrix}
\frac{\partial^2 U_s(\pi_s)}{\partial p_d^2} & \frac{\partial^2 U_s(\pi_s)}{\partial p_d \partial p_r} \\
\frac{\partial^2 U_s(\pi_s)}{\partial p_r \partial p_d} & \frac{\partial^2 U_s(\pi_s)}{\partial p_r^2}
\end{bmatrix} = \begin{bmatrix}
-2\alpha_2 & 2\beta \\
2\beta & -2\alpha_1
\end{bmatrix}.
\]

\( H_1 = -2\alpha_2 < 0, \quad H_2 = 4(\alpha_1\alpha_2 - \beta^2) > 0. \) Under the condition \( \alpha_1 > \beta \) and \( \alpha_2 > \beta \), \( H_2 > 0 \) is satisfied. \( U_s(\pi_s) \) is a joint concave function of \( p_d \) and \( w \). Thus, the optimal wholesale price and the optimal direct retail price are given by:

\[
p_d = \frac{(\beta + \alpha_1 \rho - \beta \rho) a}{2(\alpha_1 \alpha_2 - \beta^2)} + \frac{c}{2}. \quad (4.4)
\]
The retailer is risk-averse, the mean variance theory is applied to obtain the utility function of the retailer (e.g., see [21,35]) as follows:

\[ U_r(\pi_r) = E(\pi_r) - k_r \sqrt{\text{var}(\pi_r)}, \]

where the second term is the risk cost of the retailer, and \(k_r\) reflects the retailer’s attitude towards uncertainty. The larger the risk aversion coefficient \(k_r\), the more conservative the retailer’s behaviour. Equation (4.6) reflects that the retailer must weigh the expected profits against the uncertain risk.

After combining equations (3.1), (3.2), (4.4), (4.5), and \(p_r = w + m\), \(U_r(\pi_r)\) can be easily evaluated as a concave function of \(m\). Then the optimal added price \(m^*\) is given by:

\[ m^* = \frac{(1 - \rho)(a - k_r \sigma) - (\alpha_1 - \beta)c}{2\alpha_1} \]  

(4.7)

By substituting equation (4.7) into equations (4.4) and (4.5), the optimal wholesale price and the optimal direct retail price can be calculated. Additionally, the condition \(p_d > w\) must be satisfied. Thus, the inequality condition  
\[ \frac{\alpha_1(\alpha_2 - \beta^2) + (1 - \beta)(\alpha_1 + \beta)c + (\alpha_2 - \beta^2)k_r \sigma}{\alpha_1(\alpha_2 - \beta^2) + (\alpha_2 - \beta^2)k_r \sigma} < \rho < 1 \]

needs to be satisfied, which indicates that the market share that satisfies the condition is closely related to the risk aversion coefficient of the retailer.

**Proposition 4.1.** Optimal pricing of decentralized decisions without demand disruption is described below:

\[ w^* = \frac{a[(1 - \rho)(\alpha_1 \alpha_2 + \beta^2) + 2\alpha_1 \beta \rho]}{4\alpha_1(\alpha_1 \alpha_2 - \beta^2)} + \frac{(3\alpha_1 - \beta)c + (1 - \rho)k_r \sigma}{4\alpha_1} \]  

(4.8)

\[ p_d^* = \frac{(\beta + \alpha_1 \rho - \beta \rho) a + c}{2(\alpha_1 \alpha_2 - \beta^2)} \]  

(4.9)

\[ p_r^* = \frac{a[(1 - \rho)(3\alpha_1 \alpha_2 - \beta^2) + 2\alpha_1 \beta \rho]}{4\alpha_1(\alpha_1 \alpha_2 - \beta^2)} + \frac{(\alpha_1 + \beta)c + (1 - \rho)k_r \sigma}{4\alpha_1} \]  

(4.10)

By substituting equations (4.8)-(4.10) into equations (3.1)-(3.3), we can obtain the optimal demand in the supply chain.

**Proposition 4.2.** Optimal demand and optimal total production of decentralized decisions without demand disruption are described below:

\[ D_r^* = \frac{(1 - \rho)(a + k_r \sigma) - (\alpha_1 - \beta)c}{4} \]  

(4.11)

\[ D_d^* = \frac{\beta(1 - \rho)(a - k_r \sigma) + (\beta^2 - 2\alpha_1 \alpha_2 + \alpha_1 \beta)c + 2\alpha_1 \rho a}{4\alpha_1} \]  

(4.12)

\[ Q^* = D^* = \frac{a\gamma_2 - \gamma_3 c + (1 - \rho)(\alpha_1 - \beta)k_r \sigma}{4\alpha_1} \]  

(4.13)

Combining the equations (3.1)-(3.3) and (4.8)-(4.10) with equations (4.1) and (4.2), we can obtain the optimal expected profits of the supplier and the retailer:

\[ \pi^*_s = \frac{[4\alpha_1 \rho(\alpha_1 \rho - 2\beta \rho + 2\beta) + (1 - \rho)^2(\alpha_1 \alpha_2 + 3\beta^2)] a^2}{16\alpha_1(\alpha_1 \alpha_2 - \beta^2)} + \frac{[a\beta \rho(3\gamma_2 - 2\alpha_1)] c}{8\alpha_1(\alpha_1 \alpha_2 - \beta^2)} + \frac{(\alpha_1^2 - 2\alpha_1 \beta + 4\alpha_1 \alpha_2 - 3\beta^2) c^2}{16\alpha_1} + \frac{[(1 - \rho)(2a + k_r \sigma) - 2(\alpha_1 - \beta)c(1 - \rho)] k_r \sigma}{16\alpha_1} \]  

(4.14)

\[ \pi^*_r = \frac{[(1 - \rho)(a + k_r \sigma) - (\alpha_1 - \beta)c][(1 - \rho)(a - k_r \sigma) - (\alpha_1 - \beta)c]}{8\alpha_1} \]  

(4.15)
4.2. Decentralized decisions with demand disruption

This section discusses the decentralized supply chain with demand disruption. In the case of demand disruption, the potential demand changes from \( a \) to \( a + \Delta a \). \( \Delta a \) is the demand disruption caused by the unexpected events. The market demands for the two channels under demand disruption are:

\[
\overline{D}_r = (1 - \rho)(a + \Delta a) - \alpha_1 \overline{p}_r + \beta \overline{p}_d \\
\overline{D}_d = \rho(a + \Delta a) - \alpha_2 \overline{p}_d + \beta \overline{p}_r.
\]  

\( (4.16) \)

\( (4.17) \)

The total demand of the supply chain under demand disruption is:

\[
\overline{D} = a + \Delta a - (\alpha_1 - \beta) \overline{p}_r - (\alpha_2 - \beta) \overline{p}_d.
\]  

\( (4.18) \)

If the market demand is larger than the total production, there will be shortage costs. In turn, there will be handling costs. \( u_1 \) and \( u_2 \) denote the shortage cost factor and the handling cost factor, respectively. In addition, we assume that the supplier undertakes the losses caused by the demand fluctuations of the system. Therefore, the expected profit functions of the retailer and the supplier after the disruption are:

\[
E(\pi_r) = \overline{m} \overline{D}_r, \\
E(\pi_s) = (\varpi - c) \overline{D}_r + (\overline{p}_d - c) \overline{D}_d - u_1 (\overline{D} - Q^*)^+ - u_2 (Q^* - \overline{D})^+.
\]  

\( (4.19) \)

\( (4.20) \)

If \( \Delta a > 0 \), then \( \overline{D} \geq Q^* \). If \( \Delta a < 0 \), then \( \overline{D} \leq Q^* \).

**Case 1.** When the condition \( \Delta a > 0 \) is satisfied the market demand is larger than the total production. The expected profit function of the retailer is expressed by equation (4.19), and the expected profit function of the supplier is:

\[
E(\pi_s) = (\varpi - c) \overline{D}_r + (\overline{p}_d - c) \overline{D}_d - u_1 (\overline{D} - Q^*)^+.
\]  

\( (4.21) \)

The backward induction method is applied to solve the problem. First, the supplier’s decision is calculated. Because the supplier is risk-neutral, and the utility function of the supplier is:

\[
U_s(\pi_s) = (\varpi - c) \overline{D}_r + (\overline{p}_d - c) \overline{D}_d - u_1 (\overline{D} - Q^*) \\
\text{s.t.} \\
\overline{D} \geq Q^*.
\]  

\( (4.22) \)

Substituting equations (4.16) and (4.17) into equation (4.22), it is easy to prove that \( U_s(\pi_s) \) is a joint concave function of \( \overline{p}_d \) and \( \varpi \). Because the constraint \( \overline{p}_d > \varpi \) is linear, thus there exists a unique optimal solution. To solve the aforementioned optimization problem with constraint, the Lagrange multiplier \( \lambda \geq 0 \) is used. Then the KKT condition of the supplier is calculated as follow:

\[
\begin{cases}
\frac{\partial U_s(\pi_s)}{\partial \overline{p}_d} = 0 \\
\frac{\partial U_s(\pi_s)}{\partial \varpi} = 0 \\
\lambda (\overline{D} - Q^*) = 0 \\
\overline{D} \geq Q^*.
\end{cases}
\]  

\( (4.23) \)

(1) When condition \( \lambda = 0 \) is satisfied, the optimal decision is presented as follow:

\[
\begin{cases}
\overline{p}_d = \frac{(\beta + \alpha_1 \rho - \beta \rho)(a + \Delta a)}{2(\alpha_1 \alpha_2 - \beta^2)} + \frac{c + u_1}{2} \\
\varpi = \frac{(\alpha_2 - \alpha_1 \rho + \beta \rho)(a + \Delta a)}{2(\alpha_1 \alpha_2 - \beta^2)} + \frac{c + u_1 - \overline{m}}{2}.
\end{cases}
\]  

\( (4.24) \)

By combining equations (4.19) and (4.24), we apply the mean variance theory to solve the utility function of the expected profit of the retailer.

\[
U_r(\pi_r) = \overline{m} \overline{D}_r - k_r \sqrt{\text{var}(\overline{p}_r)}.
\]  

\( (4.25) \)
When the condition $\Delta a = \Delta a > 0$, the direct retail channel and the demand disruption are divided into 3 intervals as follow: the channel structure and demand disruption scale are applicable to different decisions. The market share of strategy is the same.

By comparing the above two cases, we find that the optimal markon is equal, which means that the optimal markon is shown here.

The solution process is similar to case 1. For expositional simplicity, only the result of the optimal markon is expressed as equation (4.19). The expected profit function of the supplier is:

$$E(\pi_s) = (\bar{w} - c)\overline{D}_r + (\overline{p}_d - c)\overline{D}_d - u_2(Q^* - \overline{D}).$$

Further, the supplier’s decision is analyzed first. The utility function of the supplier is:

$$U_s(\pi_s) = (\bar{w} - c)\overline{D}_r + (\overline{p}_d - c)\overline{D}_d - u_2(Q^* - \overline{D})$$

s.t. $\overline{D} \leq Q^*$.

The solution process is similar to case 1. For expositional simplicity, only the result of the optimal markon is shown here.

(3) When the condition $\lambda = 0$ is satisfied, the optimal markon is calculated as follow:

$$\overline{m}^# = \frac{(1 - \rho)(a + \Delta a - k_r \sigma) - (\alpha_1 - \beta)(c + u_1)}{2\alpha_1}. \quad (4.26)$$

By substituting equation (4.26) into equations (4.24), (4.16), (4.17) and (4.18), combined with the constraints of $\overline{p}_d > \overline{w}$ and $\overline{D} \geq Q^*$, the optimal pricing and optimal demand of the two channels and the optimal production of the whole supply chain can be calculated.

(2) When the condition $\lambda > 0$ is satisfied, similar to the case of $\lambda = 0$, the optimal added price is calculated as follow:

$$\overline{m}^# = \frac{(1 - \rho)(a + \Delta a - k_r \sigma) - (\alpha_1 - \beta)(c + u_1)}{2\alpha_1}. \quad (4.27)$$

**Case 2.** When the condition $\Delta a < 0$ is satisfied, the expected profit function of the retailer is expressed as equation (4.19). The expected profit function of the supplier is:

$$E(\pi_s) = (\bar{w} - c)\overline{D}_r + (\overline{p}_d - c)\overline{D}_d - u_2(Q^* - \overline{D}). \quad (4.28)$$

Furthermore, the supplier’s decision is analyzed first. The utility function of the supplier is:

$$U_s(\pi_s) = (\bar{w} - c)\overline{D}_r + (\overline{p}_d - c)\overline{D}_d - u_2(Q^* - \overline{D})$$

s.t. $\overline{D} \leq Q^*$.

The solution process is similar to case 1. For expositional simplicity, only the result of the optimal markon is shown here.

(3) When the condition $\lambda = 0$ is satisfied, the optimal markon is calculated as follow:

$$\overline{m}^# = \frac{(1 - \rho)(a + \Delta a - k_r \sigma) - (\alpha_1 - \beta)(c + u_2)}{2\alpha_1}. \quad (4.30)$$

(4) When the condition $\lambda > 0$ is satisfied, the optimal markon is calculated as follow:

$$\overline{m}^# = \frac{(1 - \rho)(a + \Delta a - k_r \sigma) - (\alpha_1 - \beta)(c - u_2)}{2\alpha_1}. \quad (4.31)$$

By comparing the above two cases, we find that the optimal markon is equal, which means that the optimal strategy is the same.

Applying the similar solution under decentralized decision-making without demand disruption, we find that the channel structure and demand disruption scale are applicable to different decisions. The market share of the direct retail channel and the demand disruption are divided into 3 intervals as follow:

$$S_1 = \left\{ (\rho, \Delta a) \mid \frac{(1 - \rho)(a + \Delta a) + (\alpha_1 \sigma_2 - \beta^2)(\alpha_1 - \beta^2)k_r \sigma + (\alpha_1 - \beta)(c + u_1)}{2\alpha_1} < \rho < 1, \Delta a \geq \psi_{u1} \right\};$$

$$S_2 = \left\{ (\rho, \Delta a) \mid \frac{2(\alpha_1 \sigma_2 - \beta^2)(\alpha_1 - \beta^2)k_r \sigma + (\alpha_1 - \beta)(c - u_2)}{2\alpha_1} < \rho < 1, \Delta a < \psi_{u1} \right\};$$

$$S_3 = \left\{ (\rho, \Delta a) \mid \frac{(1 - \rho)(a + \Delta a) + (\alpha_1 \sigma_2 - \beta^2)(\alpha_1 - \beta^2)k_r \sigma + (\alpha_1 - \beta)(c - u_2)}{2\alpha_1} < \rho < 1, \Delta a \leq -\psi_{u2} \right\}. $$
For expositional simplicity, we denote \( \psi = \frac{\alpha_1^2 + 2\alpha_2 \alpha_3 - 2\alpha_2 \beta - \beta^2}{\alpha_1 \beta_1 + \beta_2 + \alpha_1 \beta - \beta} \). When solving the optimal pricing decision, three solutions are found. The optimal pricing decision is related to the range of \( \rho \) and \( \Delta a \), so we divide \( \rho \) and \( \Delta a \) into three intervals: \( S_1 \), \( S_2 \), and \( S_3 \).

The scope of \( \rho \) is given by the condition \( \frac{\rho}{\Delta a} > w \), and \( \Delta a \) presents the scale of demand disruption. The scale of demand disruption is relatively large in intervals \( S_1 \) and \( S_3 \), but relatively small in region \( S_2 \).

**Proposition 4.3.** Optimal markon of decentralized decisions with demand disruption is described below:

\[
\begin{align*}
\bar{m}^\# &= \begin{cases} 
\frac{(1-\rho)(a+\Delta a-k_c \sigma)-(a_1-\beta)(c+u_1)}{2a_1} & \text{if } (\rho, \Delta a) \in S_1, \\
\frac{(a_1-\beta)(a+u_1)-\frac{\gamma_1(a+\Delta a)}{2(a_1+2\beta)}}{2a_1} & \text{if } (\rho, \Delta a) \in S_2, \\
\frac{(1-\rho)(a+\Delta a-k_c \sigma)-(a_1-\beta)(c-u_2)}{2a_1} & \text{if } (\rho, \Delta a) \in S_3.
\end{cases}
\end{align*}
\]

**Proposition 4.4.** Optimal pricing decision, optimal demand and optimal total production under decentralized decision-making with demand disruption are described below:

\[
\begin{align*}
\bar{w}^\# &= \begin{cases} 
\frac{[1-\rho]((a_1+2\beta)+2a_1 \beta)(a+\Delta a)}{4a_1(a_1+2\beta)^2} + \frac{(3a_1-\beta)(c+u_1)+(1-\rho)k_c \sigma}{4a_1} & \text{if } (\rho, \Delta a) \in S_1, \\
\frac{[2a_1-5a_1^2-3a_1 \beta+2\beta-3\beta^2-a_1^2 \beta]}{8a_1(a_1+2\beta)^2} + \frac{(1-\rho)k_c \sigma}{4a_1} & \text{if } (\rho, \Delta a) \in S_2, \\
\frac{[1-\rho]((a_1+2\beta)^2+2a_1 \beta)(a+\Delta a)}{4a_1(a_1+2\beta)^2} + \frac{(3a_1-\beta)(c-u_2)+(1-\rho)k_c \sigma}{4a_1} & \text{if } (\rho, \Delta a) \in S_3.
\end{cases}
\end{align*}
\]

\[
\bar{p}_d^\# = \begin{cases} 
\frac{2(a+\Delta a)[\gamma_4-3(1-\rho)(a_1-\beta)(a_1+2\beta)]}{8a_1(a_1+2\beta)^2} - \frac{\gamma_4(a_1+2\beta)}{8a_1(a_1+2\beta)(a_1+2\beta)} & \text{if } (\rho, \Delta a) \in S_1, \\
\frac{[3(a_1+2\beta)](1-\rho)+2a_1 \beta}{2(a_1+2\beta)^2} + \frac{c-u_2}{2} & \text{if } (\rho, \Delta a) \in S_2, \\
[3(a_1+2\beta)](1-\rho)+2a_1 \beta + \frac{(a_1+2\beta)}{2(a_1+2\beta)^2} & \text{if } (\rho, \Delta a) \in S_3.
\end{cases}
\]

\[
\bar{p}_r^\# = \begin{cases} 
\frac{\gamma_3-3(1-\rho)(a_1-\beta)(a_1+2\beta)}{8a_1(a_1+2\beta)^2} - \frac{\gamma_3(a_1+2\beta)}{8a_1(a_1+2\beta)(a_1+2\beta)} & \text{if } (\rho, \Delta a) \in S_1, \\
\frac{(a_1+2\beta)[(a_1+2\beta)](1-\rho)+2a_1 \beta}{2(a_1+2\beta)^2} & \text{if } (\rho, \Delta a) \in S_2, \\
\frac{[3(a_1+2\beta)](1-\rho)+2a_1 \beta}{4a_1(a_1+2\beta)^2} + \frac{(a_1+2\beta)(c-u_2)-(1-\rho)k_c \sigma}{4a_1} & \text{if } (\rho, \Delta a) \in S_3.
\end{cases}
\]

\[
\bar{D}_r^\# = \begin{cases} 
\frac{(1-\rho)(a_1+2\beta)+(a_1-\beta)(c+u_1)}{4a_1} & \text{if } (\rho, \Delta a) \in S_1, \\
\frac{(a_1+2\beta)[(a_1+2\beta)](1-\rho)k_c \sigma}{4a_1} - \frac{\gamma_4(a+\Delta a)}{4(a_1+2\beta)} & \text{if } (\rho, \Delta a) \in S_2, \\
\frac{(1-\rho)(a_1+2\beta)+(a_1-\beta)(c-u_2)}{4a_1} & \text{if } (\rho, \Delta a) \in S_3.
\end{cases}
\]

\[
\bar{D}_d^\# = \begin{cases} 
\frac{(\beta+2a_1 \beta)(a+\Delta a)+(\beta^2-2a_1 \beta+4a_1 \beta)(c+u_1)-\beta(1-\rho)k_c \sigma}{4a_1} & \text{if } (\rho, \Delta a) \in S_1, \\
\frac{[(\beta+2a_1 \beta)(a+\Delta a)+(\beta^2-2a_1 \beta+4a_1 \beta)(c+u_1)-\beta(1-\rho)k_c \sigma]}{4a_1} & \text{if } (\rho, \Delta a) \in S_2, \\
\frac{(\beta+2a_1 \beta)(a+\Delta a)+(\beta^2-2a_1 \beta+4a_1 \beta)(c-u_2)-\beta(1-\rho)k_c \sigma}{4a_1} & \text{if } (\rho, \Delta a) \in S_3.
\end{cases}
\]
Proposition 4.7. For expositional simplicity, we denote: \( \gamma_1 = \beta - \alpha_2 + \alpha_1 \rho + \alpha_2 \rho - 2 \beta \rho, \gamma_2 = \beta + \alpha_1 - \beta \rho, \gamma_3 = \alpha_1^2 - 2 \alpha_1 \beta + 2 \alpha_1 \alpha_2 - \beta^2, \) and \( \gamma_4 = 3 \alpha_1^2 - 6 \alpha_1 \beta + 4 \alpha_1 \alpha_2 - \beta^2. \)

4.3 Comparison of decision without and with demand disruption

Proposition 4.5. The optimal pricing decisions without and with demand disruption are compared as follows:

\[
\bar{Q}^# = \begin{cases} 
\frac{\gamma_2(a+\Delta a) + (1 - \rho)(\alpha_1 - \beta)k_r - \gamma_3(s + u_1)}{4a_1} & \text{if } (\rho, \Delta a) \in S_1, \\
\frac{\gamma_2(a+\Delta a) + (1 - \rho)(\alpha_1 - \beta)k_r - \gamma_3(c + u_2)}{4a_1} & \text{if } (\rho, \Delta a) \in S_2, \\
\frac{\gamma_2(a+\Delta a) + (1 - \rho)(\alpha_1 - \beta)k_r - \gamma_3(c - u_2)}{4a_1} & \text{if } (\rho, \Delta a) \in S_3.
\end{cases}
\]

For expositional simplicity, we denote: \( \gamma_1 = \beta - \alpha_2 + \alpha_1 \rho + \alpha_2 \rho - 2 \beta \rho, \gamma_2 = \beta + \alpha_1 - \beta \rho, \gamma_3 = \alpha_1^2 - 2 \alpha_1 \beta + 2 \alpha_1 \alpha_2 - \beta^2, \) and \( \gamma_4 = 3 \alpha_1^2 - 6 \alpha_1 \beta + 4 \alpha_1 \alpha_2 - \beta^2. \)

Proposition 4.6. Optimal demand and optimal total production without and with demand disruption are compared as follows:

\[
\bar{w}^# = \begin{cases} 
\bar{w}^* + \frac{(2 \gamma_1 + \beta^2 - \alpha_1 \alpha_2)}{4(\alpha_1 \alpha_2 - \beta^2)}(1 - \rho)(\alpha_1 - \beta)u_1 & \text{if } (\rho, \Delta a) \in S_1, \\
\bar{w}^* + \frac{(3 \gamma_1 - 1 + \alpha_1 - \beta \alpha_2)}{4(\alpha_1 \alpha_2 - \beta^2)}(1 - \rho)(\alpha_1 - \beta)u_1 & \text{if } (\rho, \Delta a) \in S_2, \\
\bar{w}^* + \frac{(3 \gamma_1 - 1 + \alpha_1 - \beta \alpha_2)}{4(\alpha_1 \alpha_2 - \beta^2)}(1 - \rho)(\alpha_1 - \beta)u_1 & \text{if } (\rho, \Delta a) \in S_3,
\end{cases}
\]

Proposition 4.7. When the scale of demand disruption is in the interval \((-\psi u_2, \psi u_1)), the direct retail channel and the traditional retail channel possess the same sales stability. When the scale of the demand disruption exceeds the interval \((-\psi u_2, \psi u_1)), the sales stability of the direct retail channel is weaker than that of the traditional retail channel (see Appendix A).
Conclusion 1

(1) When the demand disruption is within the interval \((-\psi u_2, \psi u_1)\), the value of the suppliers’ recognizing demand disruption is linearly related to the cost factors \(u_1\) and \(u_2\).

(2) When the demand disruption is within the interval \((-\psi u_2, \psi u_1)\), the value of the retailers’ recognizing demand disruption is independent of the cost factors \(u_1\) and \(u_2\) due to the robustness of production.

(3) When the demand disruption exceeds interval \((-\psi u_2, \psi u_1)\), the value of the two participants’ recognizing demand disruption and the cost factors show a quadratic relationship.
Regardless of the scale of demand disruption, the value of the two participants’ recognizing demand disruption and the scale of demand disruption $\Delta a$ show a quadratic relationship. While the value of the two participants’ recognizing demand disruption show a linear relationship with the risk aversion coefficient of the retailer $k_r$.

4.4. Managerial insights of dual-channel supply chain with demand disruption

Based on the calculation results of the above model, the managerial insights of this work are summarized as follows:

1. Whether there is demand disruption or not, the retailer’s risk aversion attitude has no effect on the direct retail channel pricing, while the production cost directly affects the optimal pricing of the direct retail channel pricing. The result shows that the optimal pricing of the direct retail channel pricing is cost driven.

2. The retailer’s risk aversion coefficient $k_r$, demand disruption $\Delta a$, market share of the direct retail channel $\rho$, shortage cost factor $u_1$ and handling cost $u_2$ are the main factors influencing the optimal decision of supply chain participants under demand disruption. Therefore, to improve the supply chain performance, we should focus on the influence of the above five factors.

3. When the market share of the direct retail channel and the demand disruption are in interval $S_1$, the optimal demand and total optimal production of the traditional retail channel and the direct retail channel are inversely proportional to the shortage cost factor $u_1$. Thus, we should improve the production capacity of the whole supply chain to reduce the negative impact of shortage caused by demand disruption on the whole supply chain.

4. When the market share of the direct retail channel and the demand disruption are in interval $S_2$, the optimal production of the whole supply chain with demand disruption is same as that without demand disruption. The result shows that in interval $S_2$, demand disruption does not affect the optimal demand of the whole supply chain. The impact of demand disruption on the optimal demand of the two channels is only a zero-sum game between the traditional retail channel and the direct retail channel.

5. When the market share of the direct retail channel and the demand disruption are in interval $S_3$, the optimal demand and total optimal production of the traditional retail channel and the direct retail channel are directly proportional to the handling cost factor $u_2$. When the demand disruption is in interval $S_3$, the production capacity should be reduced to avoid the impact of overproduction on supply chain performance.

6. If the supplier’s shortage cost factor or the handling cost factor is large, then untimely recognition of the demand disruption will have a significant impact on its profit. When the scale of demand disruption is in a robust interval $(\psi u_2, \psi u_1)$, the supplier only needs to increase the cost on the basis of normal production. However, when the scale of demand disruption exceeds the robust interval $(\psi u_2, \psi u_1)$, the supplier needs to pay extra cost for the emergency overtime and the shortage of materials to complete the production task. Therefore, the value of supply chain participants’ recognizing demand disruption in time is directly proportional to the scale of demand disruption. Improving the accuracy of demand information is helpful for supply chain participants to take timely and effective response measures in case of emergency.

5. Coordination of the risk-averse supply chain without and with demand disruption

This paper uses an improved revenue-sharing contract to coordinate the risk-averse retailer-dominant supply chain under decentralized decision-making. Firstly, the supplier provides the products to the retailer at a wholesale price. Subsequently, the retailer returns a certain proportion of the sales revenue (as agreed by both parties) to the supplier to maximize the overall revenue of the supply chain. And the revenue-sharing contract ensures that the optimal pricing and order quantity of the supply chain under decentralized decision-making are as same as those under centralized decision-making. Therefore, we need to calculate the optimal variables under centralized decision-making prior to coordination. Because the retailer is risk-averse and plays a dominant role
in the supply chain. Under centralized decision-making, the retailer’s risk-aversion attitude will affect the performance of the whole supply chain. Therefore, this paper assumes that under centralized decision-making, both the supplier and retailer are risk-averse and work collectively in the supply chain. The risk aversion coefficient is $k$.

In the following discussion, the parameter with “^∧” in the superscript on the right denotes the optimal decision of supply chain participants under centralized decision-making without demand disruption. The parameter with “^′” in the superscript on the right denotes the optimal decision of supply chain participants under centralized decision-making with demand disruption.

5.1. Coordination of the risk-averse supply chain without demand disruption

This section starts with exploring centralized decisions without demand disruption. Then, the coordination mechanism is discussed based on the centralized optimal decision without demand disruption.

5.1.1. Centralized decision without demand disruption

The expected profit of the supply chain under centralized decision-making after the demand disruption is:

$$E(\pi) = (p_r - c) D_r + (p_d - c) D_d. \quad (5.1)$$

The expected utility function of the supply chain is:

$$U(\pi) = E(\pi) - k \sqrt{\text{var}(\pi)}. \quad (5.2)$$

Substituting equations (3.1), (3.2) and (5.1) into equation (5.2), it is easy to judge that $U(\pi)$ is a joint concave function of $p_d$ and $p_r$. The optimal $p_d$ and $p_r$ can be calculated by maximizing equation (5.2):

$$p_d^\wedge = \frac{(\beta + \alpha_1 \rho - \beta \rho)(a - k \sigma)}{2(\alpha_1 \alpha_2 - \beta^2)} + \frac{c}{2}, \quad (5.3)$$

$$p_r^\wedge = \frac{(\alpha_2 - \alpha_2 \rho + \beta \rho)(a - k \sigma)}{2(\alpha_1 \alpha_2 - \beta^2)} + \frac{c}{2}. \quad (5.4)$$

By substituting equations (5.3) and (5.4) into equations (3.1) and (3.2), we can calculate the optimal decision as follow:

$$D_r^\wedge = \frac{\rho(a + k \sigma) - (\alpha_2 - \beta) c}{2} \quad (5.5)$$

$$D_d^\wedge = \frac{(1 - \rho)(a + k \sigma) - (\alpha_1 - \beta) c}{2} \quad (5.6)$$

$$Q^\wedge = D^\wedge = \frac{(a + k \sigma) - (\alpha_1 + \alpha_2 - 2 \beta) c}{2} \quad (5.7)$$

$$\pi^\wedge = \frac{[\alpha_2(1 - \rho)^2 + (2 \beta + \alpha_1 \rho - 2 \beta \rho) \rho] (a^2 - k^2 \sigma^2)}{4(\alpha_1 \alpha_2 - \beta^2)} + \frac{(\alpha_1 + \alpha_2 - 2 \beta) c^2 - 2ac}{4}. \quad (5.8)$$

5.1.2. Coordination without demand disruption

Referring to the relevant literature by Zhang et al. [41], the improved revenue-sharing contract $(w^\circ, \phi)$ is adopted in this paper in order to coordinate the risk-averse supply chain. Based on the contract, the expected profits of the supplier and the retailer under the decentralized model are as follows:

$$\pi^\circ_s = (\phi p_r^\circ + w^\circ - c) D_r + (p_d^\circ - c) D_d \quad (5.9)$$

$$\pi^\circ_r = [(1 - \phi) p_r^\circ - w^\circ] D_r. \quad (5.10)$$
When the retailer set the retail price, the variance of its expected return is considered. Thus, the utility function of the retailer is:

$$U_r(\pi_r) = [(1 - \phi) p_r^o - w^o] D_r - k_r \sqrt{\text{var}(\pi_r)}.$$  \hfill (5.11)

By maximizing the utility function of the retailer, the response function of the retail price on the wholesale price and the direct retail price is:

$$p_r^* = \frac{(1 - \phi) [(1 - \rho) a + \beta p_d^o] + \alpha_1 w^o - (1 - \rho) (1 - \phi) k_r \sigma}{2\alpha_1 (1 - \phi)}.$$  \hfill (5.12)

After adopting the revenue-sharing contract, the pricing strategy of the supply chain under decentralized decision-making is as same as that under centralized decision-making. Thus, the conditions \(\Delta a > 0\) and \(\Delta \rho > 0\) should be satisfied. Meanwhile, the revenue distribution coefficient \(\phi\) is determined by \(\pi^*_s \geq \pi^*_a\) and \(\pi^*_r \geq \pi^*_a\). Then, the wholesale price under the contract can be calculated as follows:

$$w^* = (1 - \phi^*) \left\{ \frac{\beta + \alpha_1 \rho - \beta \rho(a + \Delta a)}{2\alpha_1 (\alpha_1 + \beta^2)} + \frac{(1 - \rho) k_r \sigma}{\alpha_1} - \frac{\varphi(1 - \rho)(2\alpha_1 \alpha_2 - \beta^2) + \alpha_1 \beta \rho k_r \sigma}{2\alpha_1 (\alpha_1 + \beta^2)} \right\}.$$  \hfill (5.13)

### 5.2. Coordination of the risk-averse supply chain with demand disruption

In this section, this work begins with exploring centralized decisions with demand disruption. Then, the coordination mechanism is discussed based on the centralized optimal decision with demand disruption.

#### 5.2.1. Centralized decision with demand disruption

Under demand disruption, the demand of the traditional retail channel, the direct retail channel and the total demand are presented in equations (4.16)–(4.18).

Under centralized decision-making after demand disruption, the expected profit of the supply chain is:

$$E(\pi) = (\bar{p}_r - c) \bar{D}_r + (\bar{p}_d - c) \bar{D}_d - u_1(\bar{D} - Q^\wedge) - u_2(\bar{Q} - \bar{D})^+.$$  \hfill (5.14)

Similar to Section 4.2, we need to discuss \(\Delta a\).

**Case 3.** When the condition \(\Delta a > 0\) is satisfied, the expected profit of the supply chain is:

$$E(\pi) = (\bar{p}_r - c) \bar{D}_r + (\bar{p}_d - c) \bar{D}_d - u_1(\bar{D} - Q^\wedge).$$  \hfill (5.15)

The variance of the expected profit of the supply chain is:

$$\text{var}(\pi) = \frac{(2c - 2\bar{p}_r + u_1 - 2\rho \bar{p}_d + 2\rho \bar{p}_r)^2}{4}.$$

The expected utility function of the supply chain is:

$$U(\pi) = E(\pi) - k \sqrt{\text{var}(\pi)}$$  

s.t.  \(\bar{D} \geq Q\). \hfill (5.17)

\(U(\pi)\) is the joint concave function of \(\bar{p}_d\) and \(\bar{p}_r\). The constraint condition \(\bar{D} \geq Q^\wedge\) is linear. Thus, equation (5.17) has the unique optimal solution. After applying the Lagrange multiplier \(\lambda \geq 0\) to equation (5.17), the result is calculated as follows:

$$\begin{cases}
\alpha_2 \bar{p}_d - \rho(a + \Delta a) - \beta \bar{p}_r + \lambda \alpha_2(\alpha_2 - \beta) - \alpha_2(c - \bar{p}_d) + \beta(c - \bar{p}_r) - u_1(\alpha_2 - \beta) + \rho k \sigma = 0 \\
\alpha_1 \bar{p}_r - \beta \bar{p}_d + (\lambda - u_1)(\alpha_1 - \beta) - \alpha_1(c - \bar{p}_r) + \beta(c - \bar{p}_d) - (\alpha + \Delta a - k \sigma)(1 - \rho) = 0 \\
\lambda(\bar{D} - Q^\wedge) = 0 \\
\bar{D} \geq Q^\wedge.
\end{cases}$$  \hfill (5.18)
Case 4. When the condition $\Delta a < 0$ is satisfied, the expected profit of the supply chain is:

$$E(\pi) = (\bar{p}_r - c)D_r + (\bar{p}_d - c)D_d - u_2(Q^\lambda - D).$$

(5.19)

The variance of the expected profit of the supply chain is:

$$var(\pi) = \frac{(-2c + 2\bar{p}_r - u_2 + 2\rho \bar{p}_d - 2\rho \bar{p}_r)^2}{4}.$$  

(5.20)

The expected utility of the supply chain is:

$$U(\pi) = E(\pi) - k\sqrt{var(\pi)}$$

s.t. \( D \leq Q^\lambda \).

(5.21)

Similar to case 3, equation (5.21) is solved as follows:

$$\begin{cases} 
\alpha_2 \bar{p}_d - \rho (a + \Delta a) - \beta \bar{p}_r - \lambda (\alpha_2 - \beta) - \alpha_2 (c - \bar{p}_d) + \beta (c - \bar{p}_r) + u_2 (\alpha_2 - \beta) + \rho k \sigma = 0 \\
\alpha_1 \bar{p}_r - \beta \bar{p}_d + (u_2 - \lambda) (\alpha_1 - \beta) - \alpha_1 (c - \bar{p}_r) + \beta (c - \bar{p}_d) - (a + \Delta a - k \sigma)(1 - \rho) = 0 \\
\lambda (Q^\lambda - D) = 0 \\
\bar{D} \leq Q^\lambda 
\end{cases}$$

(5.22)

The potential scale of market demand affects supply chain decisions more than risk aversion, which means $a + \Delta a - k \sigma > 0$.

By solving equations (5.18) and (5.22), the optimal decision under centralized decision-making with demand disruption is calculated as follows:

$$\bar{p}_d = \begin{cases} 
\frac{(\beta + \alpha_1 \rho - \beta \rho)(a + \Delta a + k \sigma)}{2(\alpha_1 \alpha_2 - \beta \rho)} + \frac{c + u_1}{2} & \text{if } \Delta a \geq (\alpha_1 + \alpha_2 - 2\beta) u_1, \\
\frac{(\beta + \alpha_1 \rho - \beta \rho)(a + \Delta a - k \sigma)}{2(\alpha_1 \alpha_2 - \beta \rho)} + \frac{\Delta a}{2 (\alpha_1 + \alpha_2 - 2 \beta)} + \frac{c}{2} & \text{if } - (\alpha_1 + \alpha_2 - 2 \beta) u_2 < \Delta a < (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{(\beta + \alpha_1 \rho + \alpha_1 \beta)(a + \Delta a + k \sigma)}{2(\alpha_1 \alpha_2 - \beta \rho)} + \frac{c - u_2}{2} & \text{if } \Delta a \leq - (\alpha_1 + \alpha_2 - 2 \beta) u_2, 
\end{cases}$$

$$\bar{p}_r = \begin{cases} 
\frac{(\alpha_2 - \alpha_1 \rho + \alpha_1 \beta)(a + \Delta a - k \sigma)}{2(\alpha_1 \alpha_2 - \beta \rho)} + \frac{c + u_1}{2} & \text{if } \Delta a \geq (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{(\alpha_2 - \alpha_1 \rho + \alpha_1 \beta)(a + \Delta a + k \sigma)}{2(\alpha_1 \alpha_2 - \beta \rho)} + \frac{\Delta a}{2 (\alpha_1 + \alpha_2 - 2 \beta)} + \frac{c}{2} & \text{if } - (\alpha_1 + \alpha_2 - 2 \beta) u_2 < \Delta a < (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{(\alpha_2 - \alpha_1 \rho + \alpha_1 \beta)(a + \Delta a - k \sigma)}{2(\alpha_1 \alpha_2 - \beta \rho)} + \frac{c - u_2}{2} & \text{if } \Delta a \leq - (\alpha_1 + \alpha_2 - 2 \beta) u_2, 
\end{cases}$$

$$\bar{D}_d = \begin{cases} 
\frac{\rho (a + \Delta a + k \sigma) - (a + \Delta a - k \sigma)}{2} & \text{if } \Delta a \geq (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{\rho (a + \Delta a + k \sigma) - (a + \Delta a - k \sigma)}{2} + \frac{(a + \Delta a - k \sigma)}{2} & \text{if } - (\alpha_1 + \alpha_2 - 2 \beta) u_2 < \Delta a < (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{\rho (a + \Delta a + k \sigma) - (a + \Delta a - k \sigma)}{2} + \frac{(a + \Delta a - k \sigma)}{2} & \text{if } \Delta a \leq - (\alpha_1 + \alpha_2 - 2 \beta) u_2, 
\end{cases}$$

$$\bar{D}_r = \begin{cases} 
\frac{(a + \Delta a + k \sigma) - (a + \Delta a - k \sigma)}{2} & \text{if } \Delta a \geq (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{(a + \Delta a + k \sigma) - (a + \Delta a - k \sigma)}{2} + \frac{(a + \Delta a - k \sigma) - (a + \Delta a - 2 \beta)}{2} & \text{if } - (\alpha_1 + \alpha_2 - 2 \beta) u_2 < \Delta a < (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{(a + \Delta a + k \sigma) - (a + \Delta a - 2 \beta) c}{2} & \text{if } \Delta a \leq - (\alpha_1 + \alpha_2 - 2 \beta) u_2, 
\end{cases}$$

$$\bar{Q} = \begin{cases} 
\frac{(a + \Delta a + k \sigma) - (a + \Delta a - 2 \beta)(c + u_1)}{2} & \text{if } \Delta a \geq (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{(a + \Delta a + k \sigma) - (a + \Delta a - 2 \beta)(c + u_1)}{2} & \text{if } - (\alpha_1 + \alpha_2 - 2 \beta) u_2 < \Delta a < (\alpha_1 + \alpha_2 - 2 \beta) u_1, \\
\frac{(a + \Delta a + k \sigma) - (a + \Delta a - 2 \beta)(c + u_1)}{2} & \text{if } \Delta a \leq - (\alpha_1 + \alpha_2 - 2 \beta) u_2. 
\end{cases}$$
The optimal expected profit for a centralized decision-making supply chain is:

\[
\begin{align*}
\bar{\pi} &= \begin{cases} 
\frac{[(\alpha_1+\alpha_2-2\beta)r^2+(\alpha_2-2\alpha_2\rho+2\beta\rho)(\Delta a)^2-k^2\sigma^2]}{4(\alpha_1\alpha_2-\beta^2)} & \text{if } \Delta a \geq (\alpha_1+\alpha_2-2\beta)u_1, \\
-\frac{(\alpha+\Delta a)\sigma+(\Delta a-k\sigma)u_1}{2} & \text{if } -\alpha_1+\alpha_2-2\beta)u_2 < \Delta a < (\alpha_1+\alpha_2-2\beta)u_1, \\
\frac{[(\alpha_1+\alpha_2-2\beta)r^2+(\alpha_2-2\alpha_2\rho+2\beta\rho)(\Delta a)^2-k^2\sigma^2]}{4(\alpha_1\alpha_2-\beta^2)} + \frac{(\alpha_1+\alpha_2-2\beta)(c^2+u_2^2)}{4} & \text{if } \Delta a \leq -(\alpha_1+\alpha_2-2\beta)u_2.
\end{cases}
\end{align*}
\]

Managerial insights

Under centralized decision-making with demand disruption, we find that:

1. When the demand disruption satisfies the condition \( \Delta a \geq (\alpha_1+\alpha_2-2\beta)u_1 \), the supply chain should pay more attention to the risk of product shortage.
2. When the demand disruption satisfies the condition \( \Delta a \leq -(\alpha_1+\alpha_2-2\beta)u_2 \), the supply chain should pay more attention to the risk of product surplus.
3. When the demand disruption satisfies the condition \( -(\alpha_1+\alpha_2-2\beta)u_2 < \Delta a < (\alpha_1+\alpha_2-2\beta)u_1 \), the profit of supply chain is not affected by shortage cost and handling cost, which indicates the supply chain performance is not affected by shortage or overcapacity.

5.2.2. Coordination with demand disruption

Referring to the relevant literature by Zhang et al. [41], the improved revenue-sharing contract \((w^o, \phi)\) is adopted in this paper in order to coordinate the risk-averse supply chain. Among them, \( w^o \) represents the wholesale price determined after coordination. \( \phi \) represents the proportion of the retailer’s sales revenue shared with the supplier to obtain the wholesale price \( w^o \). When the demand is disrupted, the expected revenue function under the coordination of the supplier and the retailer is:

\[
\begin{align*}
\bar{\pi}_s &= (\phi p^s_w + w^o - c) \bar{D}_r + (p^d_w - c) \bar{D}_d - u_1(\bar{D} - Q^s)^+ - u_2(Q^s - \bar{D})^+ \quad (5.23) \\
\bar{\pi}_r &= [(1 - \phi) p^r_w - w^o] \bar{D}_r. \quad (5.24)
\end{align*}
\]

The expected utility function of the retailer is:

\[
U(\bar{\pi}_r) = [(1 - \phi) p^r_w - w^o] \bar{D}_r - k_r \sqrt{\text{var}(\bar{\pi}_r)}. \quad (5.25)
\]

To maximize the expected utility function of the retailer, the optimal pricing of the traditional retail channel can be calculated as follow:

\[
\bar{p}_r^o = \frac{(1 - \phi) [(1 - \rho)(a + \Delta a) + \beta \rho^s] + \alpha_1 \bar{w}_o - (1 - \phi)(1 - \rho) k_r \sigma}{2\alpha_1 (1 - \phi)}. \quad (5.26)
\]

Only when \( \bar{p}_s^o = \bar{p}_r^o \) and \( \bar{p}_d^o = \bar{p}_d^r \) are satisfied at the same time. The dual-channel supply chain under decentralized decision-making can achieve coordination. After adopting the improved revenue-sharing contract, the optimal revenue distribution coefficient \( \bar{\phi} \) is determined by the following conditions:

\[
\bar{\pi}_s^o \geq \bar{\pi}_w^o \quad \text{and} \quad \bar{\pi}_r^o \geq \bar{\pi}_w^o. \quad (5.27)
\]

Then, the relationship between the wholesale price and the profit distribution coefficient in the coordination contract is determined, as shown in Theorem 5.1.
Theorem 5.1. After demand disruption, the improved revenue-sharing contract $(\overline{w}^\circ, \overline{\phi})$ is used to coordinate the dual-channel supply chain dominated by a risk-averse retailer as follows:

$$\overline{w}^\circ = \begin{cases} (1-\overline{\phi}) \left\{ \frac{\beta(\beta+\alpha_1\beta-\alpha_1)(\alpha+\Delta a)}{2a_1(a_1+2\beta^2)} + \frac{(1-\rho)c}{a_1} - \frac{[(1-\rho)(2a_1\beta^2-\alpha_1\beta)+\alpha_1\beta]\sigma}{2a_1(a_1+2\beta^2)} \right\} & \text{if } \Delta a > \psi u_1, \\
\frac{\beta(\beta+\alpha_1\beta-\alpha_1)(\alpha+\Delta a)}{2a_1(a_1+2\beta^2)} + \frac{(1-\rho)c}{a_1} + \frac{[(1-\rho)(2a_1\beta^2-\alpha_1\beta)+\alpha_1\beta]\sigma}{2a_1(a_1+2\beta^2)} & \text{if } \psi u_2 < \Delta a < \psi u_1, \\
\frac{\beta(\beta+\alpha_1\beta-\alpha_1)(\alpha+\Delta a)}{2a_1(a_1+2\beta^2)} + \frac{(1-\rho)c}{a_1} - \frac{[(1-\rho)(2a_1\beta^2-\alpha_1\beta)+\alpha_1\beta]\sigma}{2a_1(a_1+2\beta^2)} & \text{if } \Delta a < -\psi u_2. \end{cases}$$

Theorem 5.1 shows that the optimal wholesale price under the revenue-sharing contract is positively correlated with the risk aversion coefficient $k_r$, while the optimal wholesale price is negatively correlated with the risk aversion coefficient $k$. In addition, the optimal wholesale price varies with the range of demand disruption.

Theorem 5.2. When the risk-averse dual-channel supply chain implements the improved revenue-sharing contract for coordination, the relationship between the wholesale prices without and with demand disruption is as follows:

$$\overline{w}^\circ = \begin{cases} w^\circ + \frac{(2a_1-\beta)\psi u_1}{2a_1} & \text{if } \Delta a \geq \psi u_1, \\
w^\circ - \frac{(2a_1-\beta)\psi u_1}{2a_1} & \text{if } -\psi u_2 < \Delta a < \psi u_1, \\
w^\circ - \frac{(2a_1-\beta)\psi u_1}{2a_1} & \text{if } \Delta a \leq -\psi u_2. \end{cases}$$

Theorem 5.2 shows that the wholesale price under demand disruption is independent of the risk aversion coefficient $k$ and $k_r$. In addition, when the demand disruption is within the robust interval $(\psi u_2, \psi u_1)$, the optimal wholesale price after demand disruption is based on the optimal wholesale price before demand disruption plus a linear function of demand disruption $\Delta a$. When the scale of demand disruption exceeds the robust interval $(\psi u_2, \psi u_1)$, the optimal wholesale price after demand disruption is based on the optimal wholesale price before demand disruption plus a linear function of the shortage cost factor $u_1$ or the handling cost factor $u_2$.

6. Numerical analysis

We have referred to the related literature (e.g., see [34, 38]) to implement the simulation. The relevant parameters are set as $a \sim N(100, 20^2)$, $\alpha_1 = 3$, $\alpha_2 = 5$, $\beta = 1$, $\rho = 0.4$, $c = 4$, $u_1 = u_2 = 2$ and $k_r > 0$ to conduct the numerical analysis.

In a dual-channel supply chain dominated by the risk-averse retailer the simulation data is substituted into the robust interval $(-\psi u_2, \psi u_1)$ of demand disruption to calculate the robust interval $(-40/3, 40/3)$.

(1) Impact of $\Delta a$ and $k_r$ on pricing decisions and demand decisions.

After substituting the simulation data into Proposition 4.5, the demand disruption is set as $\Delta a = -25, -10, 0, 10, 25$. Figure 2 exhibits the effects of $\Delta a$ and $k_r$ on the direct retail price $p_d$, the traditional retail price $p_r$, the demand of the direct retail channel $D_d$, and the demand of the traditional retail channel $D_r$.

Figure 2 shows that the direct retail price is independent of the risk aversion coefficient of the retailer $k_r$. However, the traditional retail price and the optimal demand of direct channel are negatively correlated with the risk aversion coefficient of the retailer $k_r$. While the optimal demand of the traditional retail channel is positively correlated with the risk aversion coefficient of the retailer $k_r$. In addition, Figure 2 also shows that the lines generated by the decision variables with demand disruption are parallel to the
Figure 2. Impact of $\Delta a$ and $k_r$ on pricing decisions and demand decisions.

Figure 3. Impact of $\Delta a$ and $k_r$ on the optimal production under decentralized decision-making.

lines generated by the decision variables without demand disruption. Which shows the optimal decision of supply chain under demand disruption is calculated by adding a linear function of demand disruption $\Delta a$ to the optimal decision variable before demand disruption.

(2) Impact of $\Delta a$ and $k_r$ on the optimal production under decentralized decision-making.

By substituting simulation data into Propositions 4.2 and 4.5, the optimal total production is affected by demand disruption $\Delta a$ and risk aversion coefficient $k_r$ (in Fig. 3). The risk aversion coefficient is set as $k_r = 2, 5, 8, 10$.

Figure 3 verifies the robustness of the optimal production under decentralized decision-making with demand disruptions. When the demand disruption is within the robust interval $(-40/3, 40/3)$, the optimal production with the demand disruption is the same as the optimal production without demand disruption. When the demand disruption exceeds the robust interval $(-40/3, 40/3)$, the optimal production of the supply
chain changes in the same direction as the market demand. In addition, Figure 3 also shows that the optimal production of the supply chain is proportional to the retailer’s risk aversion coefficient $k_r$.

(3) Impact of $\Delta a$ and $k_r$ on the profit of the supplier and the retailer.

The impacts of $\Delta a$ and $k_r$ on the profit of the supplier and the retailer are shown in Figures 4 and 5. The left $(\pi_s, \pi_r)$ of the two graphs is the profit without demand disruption, and the right $(\pi_s, \pi_r)$ of the two graphs is the profit with demand disruption.

Figures 4 and 5 show that the profit of the supplier is directly proportion to the risk aversion coefficient of the retailer $k_r$. While the profit of the retailer is inversely proportional to the risk aversion coefficient of the retailer $k_r$. When the condition $\Delta a \geq 40/3$ is satisfied, the supplier’s profit with demand disruption is larger than that without demand disruption. When the condition $\Delta a < 40/3$ is satisfied, the supplier’s profit with demand disruption is smaller than that without demand disruption. The supplier’s profit with demand disruption is the smallest in the robust interval $(-40/3, 40/3)$. When the condition $\Delta a > -40/3$ is satisfied, the retailer’s profit with demand disruption is larger than that without demand disruption. When the condition $\Delta a \leq -40/3$ is satisfied, the retailer’s profit under demand disruption is smaller than that without demand disruption. The retailer’s profit under demand disruption reaches the maximum in the robust interval $(-40/3, 40/3)$.

(4) Impact of $\Delta a$ and $k_r$ on the value of two participants’ recognizing demand disruption.
Figure 6. Impact of $\Delta a$ and $k_r$ on the value of two participants’ recognizing demand disruption.

Table 5. The parameter combination of decentralized decision-making supply chain under different demand disruption scale.

<table>
<thead>
<tr>
<th>$\Delta a$</th>
<th>$w^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$(1-\phi)(4k_r - 4.4k + 9.8)$</td>
</tr>
<tr>
<td>10</td>
<td>$(1-\phi)(4k_r - 4.4k + 9.2)$</td>
</tr>
<tr>
<td>0</td>
<td>$(1-\phi)(4k_r - 4.4k + 7.6)$</td>
</tr>
<tr>
<td>-10</td>
<td>$(1-\phi)(4k_r - 4.4k + 6)$</td>
</tr>
<tr>
<td>-25</td>
<td>$(1-\phi)(4k_r - 4.4k + 5.4)$</td>
</tr>
</tbody>
</table>

Figure 6 depicts the impact of $\Delta a$ and $k_r$ on the value of two participants’ recognizing demand disruption. Figure 6 shows that the larger the magnitude of the demand disruption, the larger the value of suppliers’ recognizing demand disruption. When the condition $\Delta a \geq -40/3$ is satisfied, the larger $k_r$, the larger the value of retailers’ recognizing demand disruption. When the condition $\Delta a < -40/3$ is satisfied, the smaller $k_r$, the larger the value of retailers’ recognizing demand disruption.

(5) Coordination of the risk-averse supply chain with demand disruption.

We set $\Delta a = -25, -10, 0, 10, 25$ and calculate the parameters of the revenue-sharing contract under decentralized decision-making, as shown in Table 5.

Table 5 shows that when the improved revenue-sharing contract is adopted under decentralized decision-making, the contract parameters are inversely proportional to the risk aversion coefficient of the supply chain system $k$ and directly proportional to the risk aversion coefficient of the retailer $k_r$. Different revenue distribution coefficients $\phi$ and risk aversion coefficients will determine different wholesale prices. Finally, the revenue-sharing contract makes the supply chain under decentralized decision-making consistent with the optimal pricing and demand of the supply chain under centralized decision-making before and after the demand disruption.

7. Conclusions and discussion

This paper analyzes the optimal pricing and demand decision of a dual-channel supply chain dominated by the risk-averse retailer without and with demand disruption. In addition, this paper also discusses how to use the improved revenue-sharing contract to coordinate supply chain under decentralized decision-making. Several findings are derived and summarized as follow.
First, in the case of demand disruption, the optimal decisions of participants in the dual-channel supply chain are affected by the retailer’s risk aversion coefficient \( k_r \), demand disruption \( \Delta a \), market share of the direct retail channel \( \rho \), shortage cost factor \( u_1 \) and handling cost \( u_2 \). Second, when the demand disruption is within the robust interval \((\psi u_2, \psi u_1)\), the supply chain under decentralized decision-making does not need to adjust the optimal production plan. Thus, the supply chain only needs to maintain the same optimal production as the supply chain without demand disruption. Third, the value of two participants’ recognizing demand disruption varies with the scale of demand disruption and the degree of risk aversion. The value of supply chain management with demand disruption increases with the scale of demand disruption. Fourth, the results show that the improved revenue-sharing contract can coordinate the dual-channel supply chain under decentralized decision-making.

However, there are still some limitations in our paper. First, the linear demand function is used in this paper. This demand function is relatively mature. However, the linear demand function may not be applicable to some products, such as agricultural products which require freshness. Therefore, the future research can consider how to construct a generalized demand function. Second, this paper only considers the dual-channel supply chain consisting of one supplier and one retailer. Thus, the multi-channel and omni-channel supply chains can be considered in future research. Additionally, this paper considers that the retailer is risk-averse, and further research can consider the retailer has more risk preference. The impact of supplier’s risk aversion attitude on supply chain performance is also a potential topic worth studying.

**APPENDIX A.**

**Proof of Proposition 4.7.** The coefficients of traditional retail channel changed demands and direct retail channel changed demands affected by demand disruption in interval \((-\psi u_2, \psi u_1)\) are \( \xi_1 \) and \( \xi_2 \) respectively. Exceeds the interval \((-\psi u_2, \psi u_1)\), let \( \xi_3 \) and \( \xi_4 \) represent the coefficients of the changed sales the direct retail channel and traditional retail channel affected by demand disruption, respectively. We can calculate the relationship as follow:

\[
\xi_1 = \xi_2 = |\gamma_1| = \left| \frac{a - \alpha_0 + \alpha_1 \rho + \alpha_2 \rho - 2 \beta \rho}{4(\alpha_1 + \alpha_2 - 2 \beta)} \right|, \quad \xi_3 = \frac{1 - \rho}{4}, \quad \xi_4 = \frac{\beta + 2 \alpha_1 \rho - \beta \rho}{4 \alpha_1}.
\]

(1) When the condition \(-\psi u_2 < \Delta a < \psi u_1\) is satisfied, we can calculate \( \xi_1 - \xi_2 = 0 \). Thus, the stability of the two channels is the same at this time.

(2) When the condition \( \Delta a \geq \psi u_1 \) or \( \Delta a \leq -\psi u_2 \) is satisfied, we can calculate

\[
\xi_3 - \xi_4 = \frac{1 - \rho}{4} - \frac{\beta + 2 \alpha_1 \rho - \beta \rho}{4 \alpha_1} = \frac{(\alpha_1 - \beta) - (3 \alpha_1 - \beta) \rho}{4 \alpha_1}.
\]

If \( \rho_1 < \frac{\alpha_1 - \beta}{3 \alpha_1 - \beta} \), then \( \xi_3 > \xi_4 \). If \( \rho_1 = \frac{\alpha_1 - \beta}{3 \alpha_1 - \beta} \), then \( \xi_3 = \xi_4 \). If \( \rho_1 > \frac{\alpha_1 - \beta}{3 \alpha_1 - \beta} \), then \( \xi_3 < \xi_4 \). Moreover, because the condition

\[
\frac{(\alpha_1 \alpha_2 - 2 \alpha_1 \beta + \beta^2)(a + \Delta a) + (\alpha_1 \alpha_2 - \beta^2)k_r \sigma + (\alpha_1 - \beta)(c + u_1)}{(a + \Delta a)(2 \alpha_1^2 - 4 \alpha_1 \beta + \alpha_1 \alpha_2 + \beta^2) + (\alpha_1 \alpha_2 - \beta^2)k_r \sigma} < \rho < 1
\]

should be satisfied, we can calculate

\[
\rho_1 = \rho = \frac{\alpha_1 - \beta}{3 \alpha_1 - \beta} - \frac{(\alpha_1 \alpha_2 - 2 \alpha_1 \beta + \beta^2)(a + \Delta a) + (\alpha_1 \alpha_2 - \beta^2)k_r \sigma + (\alpha_1 - \beta)(c + u_1)}{(a + \Delta a)(2 \alpha_1^2 - 4 \alpha_1 \beta + \alpha_1 \alpha_2 + \beta^2) + (\alpha_1 \alpha_2 - \beta^2)k_r \sigma} = \frac{-((\alpha_1 - \beta)(3 \alpha_1 - \beta)(\alpha_1 \alpha_2 - \beta^2)(c + u_1) - 2 \alpha_1^2(\alpha_2 - \alpha_1)(a + \Delta a) - 2 \alpha_1(\alpha_1 \alpha_2 - \beta^2)k_r \sigma)}{(3 \alpha_1 - \beta) [(a + \Delta a)(2 \alpha_1^2 - 4 \alpha_1 \beta + \alpha_1 \alpha_2 + \beta^2) + (\alpha_1 \alpha_2 - \beta^2)k_r \sigma]} < 0.
\]

Thus \( \rho_1 < \rho \), and \( \xi_3 < \xi_4 \). When demand disruption exceeds intervals \((-\psi u_2, \psi u_1)\), the sales stability of the direct retail channel is weaker than that of the traditional retail channel.

\[\Box\]

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