RAIRO-Oper. Res. 55 (2021) 1043–1061
https://doi.org/10.1051/ro/2021015

**RAIRO Operations Research**
www.rairo-ro.org

# DECISION SUPPORT SYSTEM FOR CUSTOMERS DURING AVAILABILITY OF TRADE CREDIT FINANCING WITH DIFFERENT PRICING SITUATIONS

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**Abstract.** This study introduces an inventory system with a non-instantaneous deteriorating product with credit facility and variable demand depending on the selling price. Two different selling prices are considered in the deterioration and non-deterioration periods. Shortages are partially backlogged and dependent on the length of the customers' waiting time upto the arrival of fresh lot. Alternative trade-credit policy is applied herein, and several cases, sub-cases and situations are investigated. The corresponding optimization problems of different cases, sub-cases and situations are solved using an interval-oriented multi-section technique with the help of interval mathematics and interval order relations. A numerical example with three different credit periods is studied and solved to validate the said problem. Also, two different case studies are investigated. Then to investigate the effect of changes of several system parameters on the optimal policy, post optimality analyses are performed.

**Mathematics Subject Classification.** 90B05.

Received July 11, 2020. Accepted January 25, 2021.

## 1. Introduction

Every business organization is forced to maintain a large inventory of products to fulfill the demand of retailer and customer. However, maintaining the appropriate inventory is quite difficult without proper planning and strategies because the current business situation is highly competitive. In this connection, every organization is required proper planning to efficiently run their business. Every business organization generally opts for several kinds of strategies, including controlling product deterioration, maintaining the inventory to fulfill the retailers' demand and introducing different kinds of offer to attract retailer. In the next section, inventory related literature is discussed in details.

*Keywords.* Inventory, non-instantaneous deterioration, credit policy, partial backlogging.

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### 1.1. Literature review

The deterioration effect of an item generally does not start instantly after the replenishment of product. It starts after a certain period of time. This was not considered when first inventory model was first developed (Harris–Wilson inventory model). However, in reality, most of the items continuously deteriorate with the passage of time due to several factors. Therefore, the deterioration effect cannot be discarded when studying inventory control problems. Ghare and Schrader [11] first considered this effect in formulating their inventory model. Emmons [10] then introduced Weibull (two parameter) distributions to demonstrate the deterioration rate of an item in the inventory analysis. Sarma [42] introduced an inventory model for a perishable item under a two-warehouse system. Meanwhile, Mandal and Phaujdar [30] proposed a deteriorating inventory model, where demand of the product was dependent on the stock. Pakkala and Achary [34] discussed an inventory problem with a deterioration effect under a two-storage facility. Wee [53] proposed a deteriorating inventory model under shortage. In addition, Benkherouf [3] discussed a two-warehouse system under the deterioration effect. Chang *et al.* [7] investigated an inventory problem with the deterioration effect under trade-credit policy. Papachristos and Skouri [36] introduced a quantity discount inventory model with deterioration. Balkhi and Benkherouf [2] proposed a stock-dependent demand related inventory model under deterioration. Hou [14] discussed a stock-dependent inventory problem for deteriorating items with inflation. Moreover, Dye *et al.* [9] introduced an inventory model for deteriorating item with a capacity constraint. Roy [38] investigated a time-varying holding cost-related deteriorating inventory model. Lee and Dye [20] proposed a stock-dependent demand related inventory model under deterioration. Mishra *et al.* [32] investigated a time-varying holding cost-related inventory problem along with shortages. In this connection, we may refer to the works of Jaggi *et al.* [15] and Shaikh *et al.* [45] among others.

Suppliers/retailers may offer several kinds of facilities to their customers due to globalization marketing policy. Trade-credit financing is one of the popular facility in the business world. Retailers are motivated to purchase more products for their business due to the availability of credit facilities for the procured quantity of the products. In this agreement, during the allowable time period for availability of trade-credit facility, a supplier does not charge any interest to his retailers. However, according to the terms and conditions, a supplier charges interest from his retailers if the retailers don't able to make payment within the stipulated time period. This type of offer is generally known as allowable trade credit financing or allowable delay-in-payments. This concept of delay in payments was first proposed by Haley and Higgins [13] in the area of inventory control. Goyal [12] then developed a credit policy approach in an inventory model. Aggarwal and Jaggi [1] modified Goyal's approach under the assumption of deterioration. Jamal *et al.* [19] modified Aggarwal and Jaggi's approach by introducing backlogged shortage [1], after which a number of works was performed by several other researchers. Teng *et al.* [50] proposed an inventory model with a credit policy approach. Soni *et al.* [48] introduced a comprehensive study on credit policy on inventory. Liang and Zhou [21] introduced a credit policy inventory model in a two-warehouse system for deteriorating items. Maiti [29] proposed a credit-linked promotional demand on an inventory system. Kumar *et al.* [26] introduced an inventory model with a variable demand dependent on price considering the trade-credit facility. Sarkar *et al.* [39] studied a credit policy inventory problem with a finite replenishment rate. Similarly, Liao *et al.* [22] introduced another important factor of permissible delay in payment with the capacity constraint of the warehouse in an inventory system. Pal *et al.* [35] proposed a three-stage credit concept in their inventory model. Wu *et al.* [54] introduced a maximum life time-related inventory model with a two-level credit policy. Meanwhile, Shah and Cardenas-Barron [43], Jaggi *et al.* [16], and Tayal *et al.* [49] considered both trade-credits and preservation technology in their inventory models. Singh *et al.* [47] formulated a stock-dependent demand related inventory model under the credit policy approach. Bhunia *et al.* [6] introduced a partially integrated production problem with a reliability factor and a price-dependent demand in an interval environment. They solved the corresponding optimization problem using the soft computing technique. Mashud *et al.* [31] studied a sustainable inventory model in two warehouse system under advance and trade credit facilities. Tiwari *et al.* [51] proposed a multi-item green production system under trade credit facility. Sarkar *et al.* [41] introduced multi trade credit facility in sustainable inventory management. Khanna *et al.* [24] studied a supply chain system where the credit facility is considered as customer based two level credit facilities. Kumar

TABLE 1. Trade credit policy-related literature in the area of inventory control system.

| Author(s) | Demand rate | Shortage policy | Level of trade-credit policy | Solution technique |
|---|---|---|---|---|
| Bhunia *et al.* [5] | Fixed | Partial Backlogging | Single | Lingo software |
| Khanra *et al.* [25] | Time dependent | Complete backlogging | Single | Not mentioned |
| Chung *et al.* [8] | Constant | No | Two | Not mentioned |
| Jaggi *et al.* [17] | Constant | No | No | Maple software |
| Jaggi *et al.* [18] | Selling price dependent | No | Single | Not mentioned |
| Sarkar *et al.* [40] | Constant | No | Two | Mathematica software |
| Mohsen *et al.* [33] | Constant | Complete backlogging | Two | Not mentioned |
| Pervin *et al.* [37] | Time dependent | No | Single | Newton-Raphson Method |
| Kundu *et al.* [28] | Selling price dependent | No | No | PSO |
| Shah *et al.* [44] | Quadratic demand | No | Single | Not mentioned |
| Udayakumar and Geetha [52] | Constant | No | Single | Not mentioned |
| This paper | Selling price dependent with different rates | Partial backlogging (waiting time dependent) | Single (alternative approach) | Multi-section technique |

*et al.* [27] proposed a pricing, free replacement and warranty policies in inventory system for new launching product. Shin *et al.* [46] studied an effect of human errors in trade credit financing policy.

Apart from these, some recent works along with our proposed work are shown in a tabular form in Table 1.

An inventory model is formulated herein for not instantly starting deteriorating products by taking an alternative approach of the trade-credit policy and variable demand. Shortages are partially allowed with the variable rates dependent on customer's waiting time. Several situations are studied depending on the trade-credit policy, time for zero ending inventory, cycle length, and mode of payment. The interval-oriented multi-section optimization technique (IOMOT) [23] is applied to solve the optimization problems of these situations by using interval mathematics and interval order relations [4].

## 2. Assumptions and notation

The proposed model is developed under the following assumptions and notation.

### 2.1. Assumptions

(i) The demand rate $D(p)$ dependent on the selling price $(p)$ is given by

$$D(p) = \begin{cases} D_1(p), & 0 < x \le x_d \\ D_2(p), & x > x_d \end{cases}$$

where $D_1(p) = a - bp_1 (a > bp_1), D_2(p) = a - bp_2 (a > bp_2), p_2 < p_1$ and $a$ & $b$ are positive constants.

(ii) Items start to deteriorate after a period of time $x_d$ and shortages are allowed with the backlogging rate dependent on the customer's waiting time upto the arrival of fresh lot. The mathematical form of the backlogging rate is given by $[1 + \mu(X - x)]^{-1}, \mu > 0$.

(iii) If $X \geq T$, then the credit period is settled at $T$. After this time period, the retailer must pay interest with the rate $I_p$. Before this time period, the retailer can also earn interest with the rate $I_e$ where $I_p \geq I_e$.
(iv) If $X < T$, then the credit period is fixed at $T$ and the retailer does not pay any interest during this time interval. Alternatively, the retailer may earn revenue from product sales and interest earning.
(v) The lead time is constant. The planning horizon is infinite. The replenishment rate is instantaneous.

### 2.2. 2.2 Notation

The following notations are used throughout the manuscript.

| **Notation** | **Units** | **Meanings** |
|---|---|---|
| $c_o$ | \$/unit | Setup cost/order |
| $c_p$ | \$/unit | Purchase cost/unit |
| $\lambda$ | \$/unit | Discount rate |
| $p_1$ | \$/unit | Price per unit of item in the non-deteriorating period (before $x_d$) |
| $p_2 = (1-\lambda)p_1$ | \$/unit | Price per unit of item in the deteriorating period (after $x_d$) |
| $A$ | units | Highest level of initial inventory |
| $S$ | units | Maximum shortage level |
| $c_h$ | \$/unit | Holding cost |
| $c_b$ | \$/unit | Shortage cost |
| $\theta$ | constant | Deterioration rate, where $0 < \theta \ll 1$ |
| $c_{ls}$ | \$/unit | Lost sale cost |
| $x_d$ | year | Non-deterioration time period |
| $x_1$ | year | Time period when the inventory level reaches to zero |
| $X$ | year | Cycle length |
| $U(x)$ | units | Inventory level at time $x$ |
| $T$ | year | Duration of credit period |
| $I_e$ | \$/unit/year | Interest rate |
| $I_p$ | \$/unit/year | Interest paid |
| TP | \$/year | Total profit |

## 3. Problem description

At the beginning of business time, $(A + S)$ units are received by a retailer to start his business. During the time interval $0 < x \leq x_d$, the inventory level decreases due to satisfy the customers' demand only. During the time period $x_d < x \leq x_1$, the stock decreases to meet up the joint effect of customers' demand and deterioration. Finally, at $x = x_1$, the inventory level reaches to zero. After that, backlogs occur in the time period $x_1 < x \leq X$ with a rate of $[1 + \mu(X - x)]^{-1}$ $(\mu > 0)$. The pictorial representation of the inventory level of the system is shown in Figure 1.

Two scenarios may arise according to the values of $x_d$ and $x_1$ as

**Scenario 1.** $x_d \leq x_1$.
**Scenario 2.** $x_d > x_1$.

Here we have investigated scenario 1 only.

**Scenario 1.** $x_d \leq x_1$.

In this scenario, the inventory level satisfies the following differential equations over the period $(0,\ X)$:

$$\frac{\mathrm{d}U(x)}{\mathrm{d}x} = -D_1, \qquad 0 \leq x \leq x_d \tag{3.1}$$

$$\frac{\mathrm{d}U(x)}{\mathrm{d}x} + \theta U(x) = -D_2, \qquad x_d < x \leq x_1 \tag{3.2}$$

$$\frac{\mathrm{d}U(x)}{\mathrm{d}x} = \frac{-D_2}{1 + \mu(X - x)}, \qquad x_1 < x \leq X \tag{3.3}$$

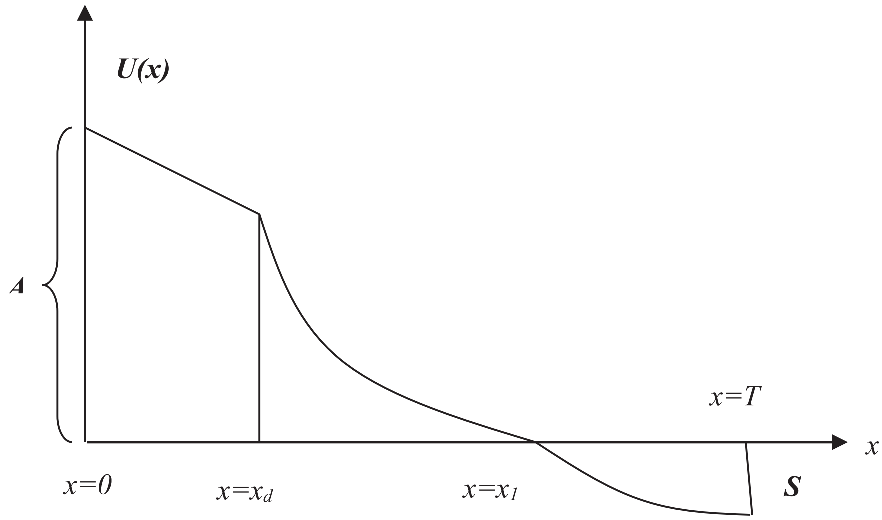


FIGURE 1. Pictorial representation of inventory situations.

using the conditions

$$\left.\begin{array}{rl} U(x) = A & \text{at } x = 0 \\ U(x) = 0 & \text{at } x = x_1 \\ \text{and } U(x) = -S & \text{at } x = X \end{array}\right\}. \tag{3.4}$$

Also, $U(x)$ is also continuous at $x = x_d$ and $x = x_1$.
The solutions of differential equations (3.1)–(3.3) with the conditions (3.4) yield

$$U(x) = A - D_1 x, \qquad 0 \le x \le x_d \tag{3.5}$$

$$U(x) = \frac{D_2}{\theta}\left(e^{\theta(x_1 - x)} - 1\right), \qquad x_d < x \le x_1 \tag{3.6}$$

$$U(x) = \frac{D_2 \log|1 + \mu(X - x)|}{\mu}, \qquad x_1 < x \le X. \tag{3.7}$$

Again, using continuity conditions of $U(x)$ at $x = x_d$ and $x = x_1$, we obtain

$$A - D_1 x_d = \frac{D_2}{\theta}\left(e^{\theta(x_1 - x)} - 1\right) \tag{3.8}$$

$$S = \frac{D_2 \log|1 + \mu(X - x_1)|}{\mu}. \tag{3.9}$$

**The cost function.** The total cost per cycle of the inventory system consists of the following components (i) Ordering cost, (ii) Purchase cost, (iii) Inventory holding cost, (iv) Shortage cost and (v) Lost sale cost.

(a) Ordering cost: Since there is only one item and one stocking point, the ordering cost per cycle is $c_o$.
(b) Inventory holding cost: The inventory holding cost at any time is the product of inventory level and holding cost per unit time. Thus the total holding $C_{\text{hol}}$ is given by

$$\begin{aligned} C_{\text{hol}} &= c_h\left(\int_0^{x_d} U(x)\mathrm{d}x + \int_{x_d}^{x_1} U(x)\mathrm{d}x\right) \\ &= c_h\left[A x_d - \frac{D_1 x_d^2}{2} + D_2\left\{e^{\theta(x_1 - x_d)} - 1\right\}/\theta^2 - D_2(x_1 - x_d)/\theta\right]. \end{aligned}$$

TABLE 2. Different cases for the proposed inventory model.

| Cases | Time period | Sub-cases | Situations |
|---|---|---|---|
| Case 1 | $0 < T \leq x_d$ | $J_1 \geq c_p(A+S)$ | Full payment |
| | | $J_1 < c_p(A+S)$ | Partial payment |
| Case 2 | $x_d < T \leq x_1$ | $J_2 \geq c_p(A+S)$ | Full payment |
| | | $J_2 < c_p(A+S)$ | Partial payment |
| Case 3 | $x_1 < T$ | | – |

(c) Purchasing cost: Since the ordering quantity is $A+S$, the total purchase cost of the entire cycle is $c_p(A+S)$.

(d) Shortage cost: The shortage cost at any time is the product of shortage level at that time and shortage cost per unit item. Hence the total shortage cost $C_{\text{Sho}}$ of the inventory system is given by

$$\begin{aligned} C_{\text{Sho}} &= c_b \int_{x_1}^{X} [-U(x)]\, \mathrm{d}x \\ &= c_b \left[ \left( S + \frac{D_2}{\mu} \right)(X - x_1) - D_2 \left\{ 1 + \mu(X - x_1) \right\} \log \left\{ 1 + \mu(X - x_1) \right\} / \mu^2 \right]. \end{aligned}$$

(e) Lost sale cost: The lost sale cost at any time is the product of unsatisfied demand and lost sale cost per unit item. Hence the total lost sale cost OCLS of the inventory system is given by

$$\begin{aligned} \text{OCLS} &= c_{ls} \int_{x_1}^{X} \left[ 1 - \frac{1}{1 + \mu(X - x)} \right] D_2 \mathrm{d}x \\ &= c_{ls} D_2 \left[ (X - x_1) - \frac{\log |1 + \mu(X - x_1)|}{\mu} \right]. \end{aligned}$$

According to the trade credit policy, different cases may arise shown in Table 2.
Now we have discussed each case separately.

**Case 1.** $0 < T \leq x_d$.

In this situation, retailer must pay $c_p(A+S)$ within the period for $x = T$. At the same time, they earn the sales revenue and interest during the entire cycle:

$$J_1 = D_1 p_1 T \left( 1 + \frac{1}{2} I_e T \right) + p_1 S \left( 1 + I_e T \right). \tag{3.10}$$

According to the values of $J_1$ and $c_p(A+S)$, two possible sub-cases may arise:

**Sub-case 1.1.** $J_1 \geq c_p(A+S)$.

**Sub-case 1.2.** $J_1 < c_p(A+S)$.

Now we have discussed each sub-case separately.

**Sub-case 1.1.** $J_1 \geq c_p(A+S)$.

In this situation, retailer needs to pay an amount $c_p(A+S)$ to the suppliers until the time period $x = T$. Thus, retailer earns interest on the rest amount $J_1 - c_p(A+S)$ during the period $[T, X]$. After the time period $x = T$, retailer earns money from sales as well as from interest earned on sales amount.
Revenue earnings throughout the period $[T, x_d]$ is given by $p_1 \int_T^{x_d} D_1 \mathrm{d}x$, while revenue earnings during the time period $[x_d, x_1]$ is given by $p_2 \int_{x_d}^{x_1} D_1 \mathrm{d}x$.
The interests earned are given by $p_1 I_e \int_T^{x_d} \int_T^{x} D_1 \mathrm{d}u \mathrm{d}x$ and $p_2 I_e \int_{x_d}^{x_1} \int_{x_d}^{x} D_1 \mathrm{d}u \mathrm{d}x$.
Hence, the profit per unit is presented as follows:

$$Z^{(1.1))}(x_1 X) = \frac{\text{TP}_1}{X} \tag{3.11}$$

where $\mathrm{TP}_1$ = ⟨rest amount after payment⟩ + ⟨interest accumulated on the rest amount after payment during the time period $[T, X]$ + ⟨sales revenue during the time period $[T, x_d]$⟩ + ⟨sales revenue during the time period $[x_d, x_1]$⟩ + ⟨interest earned during the time period $[x_1, X]$⟩ + ⟨interest earned during the time period $[x_d, x_1]$⟩ + ⟨interest earned during the time period $[x_1, X]$⟩ – ⟨cost of ordering⟩ – ⟨carrying cost⟩ – ⟨cost of Shortage⟩ - ⟨cost of lost sale⟩,

$$i.e., \mathrm{TP}_1 = \{J_1 - c_p(A+S)\}\{1 + I_e(X-T)\} + D_1p_1(x_d - T)\left\{1 + \frac{1}{2}I_e(x_d - T)\right\} + D_2p_2(x_1 - x_d)\left\{1 + \frac{1}{2}I_e(x_1 - x_d)\right\}\{1 + I_e(X - x_1)\} - c_o - C_{\text{hol}} - C_{\text{Sho}} - \text{OCLS}. \tag{3.12}$$

Hence, in this case, we can write the corresponding optimization problem as follows:

**Problem 1.** Maximize $Z^{1.1}(x_1, X) = \frac{\mathrm{TP}_1}{X}$

$$\text{subject to } 0 < T \le x_d < x_1. \tag{3.13}$$

**Sub-case 1.2.** $J_1 < c_p(A+S)$.

Here, two situations may arise:

**Situation 1.2.1.** Part payment accepted at time $x = T$.

**Situation 1.2.2.** Part payment not accepted at time $x = T$.

Now, we have studied each situation separately.

**Situation 1.2.1.** Part payment accepted at time $x = T$.

In this situation, retailer does not have sufficient money $J_1$ in $x = T$. It is assumed that the remaining amount $c_p(A+S) - J_1$ is to be paid on $x = B$ (where $B > T$). Due to this situation, retailer must pay an interest $c_p(A+S) - J_1$ throughout the time period $[T, B]$.

The amount payable at time $x = B$ is:

$$\{c_p(A+S) - J_1\}\{1 + I_p(B-T)\}.$$

The money earned from the interest on the sales amount upto time $x = B$ is $\frac{1}{2}D_1p_1I_e(B-T)^2$.

Now, the total amount on the retailer's hand = ⟨sales amount for the entire time period $[T, B]$⟩ + ⟨interest earned⟩ = $D_1p_1(B-T) + \frac{1}{2}D_1p_1I_e(B-T)^2 = D_1p_1(B-T)\left\{1 + \frac{1}{2}I_e(B-T)\right\}$.

Therefore, at $x = B$, the payable amount = available amount is on the retailer's hand

$$\{c_p(A+S) - J_1\}\{1 + I_p(B-T)\} = D_1p_1(B-T)\left\{1 + \frac{1}{2}I_e(B-T)\right\}. \tag{3.14}$$

The simplified form of (3.14) is a quadratic equation in $B$. The admissible solution of (3.14) is given by

$$B = T + \frac{-[D_1p_1 - I_p\{J_1 - c_p(B-T)\}] \pm \sqrt{[D_1p_1 - I_p\{J_1 - c_p(A+S)\}]^2 - 2D_1p_1I_e\{J_1 - c_p(A+S)\}}}{D_1p_1I_e}. \tag{3.15}$$

Hence the profit per unit is presented as follows:

$$Z^{(1.2.1)}(x_1, X) = \frac{\mathrm{TP}_2}{X} \tag{3.16}$$

where $\mathrm{TP}_2$ = ⟨sales revenue during the time period $[B, x_d]$⟩ + ⟨interest earned on sales revenue during the time period $[B, x_d]$⟩ + ⟨sales revenue during the time period $[x_d, x_1]$⟩ + ⟨interest earned on sales revenue during the time period $[x_d, x_1]\rangle$ + $\langle$interest earned on cash in hand during the time period $[x_1, X]\rangle$ – $\langle$cost of ordering$\rangle$ – $\langle$carrying cost$\rangle$ – $\langle$cost of backlog$\rangle$ – $\langle$cost of lost sale$\rangle$

$$i.e., \mathrm{TP}_2 = D_1p_1(x_d - B)\left\{1 + \frac{1}{2}I_e(x_d - B)\right\} + D_2p_2(x_1 - x_d)\left\{1 + \frac{1}{2}I_e(x_1 - x_d)\right\}\{1 + I_e(X - x_1)\} - c_o - C_{\mathrm{hol}} - C_{\mathrm{Sho}} - \mathrm{OCLS}. \tag{3.17}$$

Hence the optimization problem can be written as

**Problem 2.** Maximize $Z^{(1.2.1)}(x_1, X) = \frac{\mathrm{TP}_2}{X}$

$$\text{subject to } 0 < T \leq x_d < x_1 < X. \tag{3.18}$$

**Situation 1.2.2.** Part payment not accepted on $x = T$.

For this case, the supplier does not accept any fraction payment on $x = T$. However, retailer does not have the available amount to pay the supplier. Let the retailer be able to pay the amount at time $x = B$. In this situation, retailer must pay the interest to the suppliers for the entire time period $[T, B]$, that is, $c_p(A + S)\{1 + I_p(B - T)\}$.

Retailer earns interest by up to time $x = B$ is $J_1\{1 + I_e(B - T)\} + p_1D_1(B - T)\left\{1 + \frac{1}{2}I_e(B - T)\right\}$.

Therefore, the amount that needs to be paid is equal to the total amount available on $x = B$

$$i.e., c_p(A + S)\{1 + I_p(B - T)\} = J_1\{1 + I_e(B - T)\} + p_1D_1(B - T)\{1 + I_e(B - T)\}. \tag{3.19}$$

From equation (3.19), we have

$$B = T + \frac{-\{D_1p_1 + J_1I_e - c_p(A + S)\} \pm \sqrt{\{D_1p_1 + J_1I_e - c_p(A + S)\}^2 - 2D_1p_1I_e\{J_1 - c_p(A + S)\}}}{D_1p_1I_e}. \tag{3.20}$$

Hence the profit per unit is

$$Z^{(1.2.2)}(x_1, X) = \frac{\mathrm{TP}_3}{T} \tag{3.21}$$

where $\mathrm{TP}_3$ = $\langle$sales revenue during the time period $[B, x_d]\rangle$ + $\langle$interest earned on sales revenue during the time period $[B, x_d]\rangle$ + $\langle$selling price $[x_d, x_1]\rangle$ + $\langle$interest earned on the sales revenue during the time period $[x_d, x_1]\rangle$ + $\langle$interest earned on cash in hand during the time period $[x_1, X]\rangle$ – $\langle$cost of ordering$\rangle$ – $\langle$carrying cost$\rangle$ – $\langle$cost of shortage>$\rangle$ – $\langle$cost of lost sale$\rangle$

$$i.e., \mathrm{TP}_2 = D_1p_1(x_d - B)\left\{1 + \frac{1}{2}I_e(x_d - B)\right\} + D_1p_2(x_1 - x_d)\left\{1 + \frac{1}{2}I_e(x_1 - x_d)\right\}\{1 + I_e(X - x_1)\} - c_o - C_{\mathrm{hol}} - C_{\mathrm{Sho}} - \mathrm{OCLS}. \tag{3.22}$$

The optimization problem in this situation is given by

**Problem 3.** Maximize $Z^{(1.2.2)}(x_1, X) = \frac{\mathrm{TP}_3}{X}$

$$\text{subject to } 0 < T \leq x_d < x_1 < X. \tag{3.23}$$

**Case 2.** $x_d < T \leq x_1$.

The revenue earned on $x = T$ is given by

$$J_2 = p_1D_1x_d\left(1 + \frac{1}{2}I_ex_d\right) + p_2D_2(T - x_d)\left\{1 + \frac{1}{2}I_e(T - x_d)\right\} + p_1S(1 + I_ex_1) + p_2S\left\{1 + \frac{1}{2}I_e(T - x_d)\right\}.$$

Two possible sub-cases may arise according to the values of $J_2$ and $c_p(A + S)$:

**Sub-case 2.1.** $J_2 \geq c_p(A+S)$.
**Sub-case 2.2.** $J_2 < c_p(A+S)$.

Now we have discussed different sub-cases.

**Sub-case 2.1.** $J_2 \geq c_p(A+S)$.

In this situation, retailer must pay an amount $c_p(A+S)$ to the suppliers until the time period of $x = T$. Therefore, retailer earns interest on amount $J_2 - c_p(A+S)$ during the entire time interval of $[T, X]$. After time $x = T$, retailer also earns money from sales and interest on the sales amount.

Therefore the average profit is given by:

$$Z^{(2.1)}(x_1, X) = \frac{\mathrm{TP}_4}{X} \tag{3.24}$$

where $\mathrm{TP}_4 = \langle$rest amount after payment$\rangle$ + $\langle$interest earned on the rest amount during the time period $[T, X]\rangle$ + $\langle$sales revenue during the time period $[T, x_1]\rangle$ + $\langle$interest earned during the time period $[T, x_1]\rangle$ + $\langle$interest earned during the time period $[x_1, X]\rangle$ − $\langle$cost of ordering $\rangle$ − $\langle$carrying cost$\rangle$ − $\langle$cost of shortage$\rangle$ − $\langle$cost of lost sale$\rangle$

$$\begin{aligned} i.e., \mathrm{TP}_4 = &\left\{J_2 - c_p(A+S)\right\}\left\{1 + I_e(X-T)\right\} \\ &+ D_2p_2(x_1 - T)\left\{1 + \frac{1}{2}I_e(x_1 - T)\right\}\left\{1 + I_e(X - x_1)\right\} - c_o - C_{\mathrm{hol}} - C_{\mathrm{Sho}} - \mathrm{OCLS}. \end{aligned} \tag{3.25}$$

Hence the optimization problem can be written as follows:

**Problem 4.** Maximize $Z^{(2.1)}(x_1, X) = \frac{\mathrm{TP}_4}{X}$

$$\text{subject to } x_d < T \leq x_1 < X. \tag{3.26}$$

**Sub-case 2.2.** $J_2 < c_p(A+S)$.

Again, two situations may arise:

**Situation 2.2.1.** Part payment accepted at $x = T$.
**Situation 2.2.2.** Part payment not accepted at $x = T$.

Now we have discussed different situations.

**Situation 2.2.1.** Part payment accepted at $x = T$.

In this situation, retailer cannot have sufficient money $c_p(A+S) - J_2$ paid at time $x = B(B > T)$. Due to this situation, retailer requires to pay an interest $c_p(A+S) - J_2$ for the entire time period $[T, B]$.

Now, on $x = B$, the total amount is $\left\{c_p(A+S) - J_2\right\} I_p(B-T)$.

Hence the total amount payable is

$$\left\{c_p(A+S) - J_2\right\} + \left\{c_p(A+S) - J_2\right\} I_p(B-T) = \left\{c_p(A+S) - J_2\right\}\left\{1 + I_p(B-T)\right\}.$$

The interest earned at $x = T$ is $\frac{1}{2}D_2p_2I_e(B-T)^2$ given as $D_2p_2(B-T)\left\{1 + \frac{1}{2}I_e(B-T)\right\}$.

Therefore at $x = B$, the total payable amount = available amount:

$$i.e., \left\{c_p(A+S) - J_2\right\}\left\{1 + I_p(B-T)\right\} = D_2(B-T)p_2 + \frac{1}{2}D_2p_2I_e(B-T)^2. \tag{3.27}$$

From equation (3.27), we have

$$B = T + \frac{-\left[D_2p_2 + I_p\left\{J_2 - c_p(A+S)\right\}\right] \pm \sqrt{\left[D_2p_2 + I_p\left\{J_2 - c_p(A+S)\right\}\right]^2 - 2D_2p_2I_e\left\{J_2 - c_p(A+S)\right\}}}{D_2p_2I_e}. \tag{3.28}$$

Hence the profit per unit can be written as

$$Z^{(2.2.1)}(x_1, X) = \frac{\mathrm{TP}_5}{X} \tag{3.29}$$

where $\mathrm{TP}_5$ = ⟨sales revenue during the time period $[B, x_1]$⟩ + ⟨interest earned on sales revenue during the time period $[B, x_1]$⟩ + ⟨interest earned on cash in hand during the time period $[x_1, X]$⟩ – ⟨cost of ordering⟩ – ⟨carrying cost⟩ – ⟨cost of shortage⟩ – ⟨cost of lost sale⟩

$$i.e., \mathrm{TP}_5 = D_2 p_2 (x_1 - B)\left\{1 + \frac{1}{2} I_e (x_1 - B)\right\}\{1 + I_e(X - x_1)\} - c_o - C_{\mathrm{hol}} - C_{\mathrm{Sho}} - \mathrm{OCLS}. \tag{3.30}$$

In this situation, the optimization problem is as follows:

**Problem 5.** Maximize $Z^{(2.2.1)}(x_1, X) = \frac{\mathrm{TP}_5}{X}$

$$\text{subject to } x_d < T \leq x_1 < X. \tag{3.31}$$

**Situation 2.2.2.** Part payment cannot be accepted at $x = T$.

In this situation, the supplier cannot agree to any fraction payment $x = T$. However, retailer does not have the available amount to pay to the supplier. Let the retailer be able to pay the amount at time $x = B$. In this situation, retailer needs to pay interest to the suppliers during the period $[T, B]$.

Here the net amount is $c_p(A + S)\left\{1 + \frac{1}{2} I_p (B - T)\right\}$; the total amount available at time $x = B$ is $J_2 \{1 + I_e(B - T)\} + p_2 D_2 (B - T)\left\{1 + \frac{1}{2} I_e (B - T)\right\}$.

Hence at $x = B$, the total amount payable = net amount available.

$$i.e., c_p(A + S)\{1 + I_p(B - T)\} = J_2\{1 + I_e(B - T)\} + p_2 D_2 (B - T)\left\{1 + \frac{1}{2} I_e (B - T)\right\}. \tag{3.32}$$

From equation (3.32), we have

$$B = T + \frac{-\{D_2 p_2 + J_2 I_e - I_p c_p (A + S)\} \pm \sqrt{\{D_2 p_2 + J_2 I_e - I_p c_p (A + S)\}^2 - 2 D_2 p_2 I_e \{J_2 - c_p (A + S)\}}}{D_2 p_2 I_e}. \tag{3.33}$$

Hence the profit per unit is

$$Z^{(2.2.2)}(x_1, X) = \frac{\mathrm{TP}_6}{X} \tag{3.34}$$

where $\mathrm{TP}_6$ = ⟨sales revenue during the time period $[B, x_1]$⟩ + ⟨interest earned on sales revenue during the time period $[B, x_1]$⟩ + ⟨interest earned on cash in hand during the time period $[x_1, X]$⟩ – ⟨cost of ordering⟩ – ⟨carrying cost⟩ – ⟨cost of shortage⟩ – ⟨cost of lost sale⟩

$$i.e., \ \mathrm{TP}_6 = D_2 p_2 (x_1 - B)\left\{1 + \frac{1}{2} I_e (x_1 - B)\right\}\{1 + I_e(X - x_1)\} - c_o - C_{\mathrm{hol}} - C_{\mathrm{Sho}} - \mathrm{OCLS}. \tag{3.35}$$

In this situation, the optimization problem can be presented as follows:

**Problem 6.** Maximize $Z^{(2.2.2)}(x_1, X) = \frac{\mathrm{TP}_6}{X}$

$$\text{subject to } x_d < T \leq x_1 < X. \tag{3.36}$$

**Case 3.** $x_1 < T$.

In this scenario, the total revenue is given by

$$J_3 = D_1 p_1 x_d \left\{1 + \frac{1}{2} I_e x_d\right\} + p_2 D_2 (x_1 - x_d) \left\{1 + \frac{1}{2} I_e (x_1 - x_d)\right\} \{1 + I_e (T - x_1)\} + p_1 S \{1 + I_e x_d\}$$
$$+ p_2 S \{1 + I_e T\}$$

where $J_3$ is always higher than $c_p(A+S)$. Thus on $x = T$, the total rest amount after payment is $J_3 - c_p(A+S)$. The profit per unit is $Z^{(3.3)}(x_1, X) = \frac{\text{TP}_7}{X}$, where $\text{TP}_7 = \langle$available amount $[T, X]\rangle - \langle$cost of purchase$\rangle - \langle$cost of ordering$\rangle - \langle$carrying cost$\rangle - \langle$cost of shortage$\rangle - \langle$cost of lost sale$\rangle$

$$i.e., \text{TP}_7 = J_3 - c_p(A + S) - c_o - C_{\text{hol}} - C_{\text{Sho}} - \text{OCLS}. \tag{3.37}$$

Hence the corresponding optimization problem can be written as follows:

**Problem 7.** Maximize $Z^{(3.3)}(x_1, X) = \frac{\text{TP}_7}{X}$

$$\text{subject to } 0 < x_d < x_1 < T < X. \tag{3.38}$$

## 4. Solution procedure

The optimization problems in (3.13), (3.18), (3.23), (3.26), (3.29), (3.36) and (3.38) are highly nonlinear in nature. Therefore, obtaining the analytic solutions of these optimization problems is a quite formidable task. So to solve the said optimization problem, we have applied the soft computing method IOMOT developed by Karmakar *et al.* [18]. The details about this technique are discussed in the next section.

### 4.1. Solution procedure of the interval-oriented multi-section optimization technique (IOMOT)

Let us assume that $f(y)$ is a function to be optimized and defined on a bounded region $a_L \le y \le a_U$, where $y = (y_1, y_2, \cdots, y_n)$, $a_L = (a_{1L}, a_{2L}, \cdots, a_{nL})$, and $a_U = (a_{1U}, a_{2U}, \cdots, a_{nU})$. $n$ is the number of variables and $a_{iL} \le y_i \le a_{iU}, \forall i = 1, 2, \cdots, n$. The given region is divided into sub-regions, say $G_1, G_2, \cdots, G_d$, to find the maximum/minimum of function $f(y)$ in the given region using the multi-section technique [23]. With the help of interval mathematics, the value of $f(y)$ is then calculated in each region in an interval form like $f(G_i) = [f_{iL}, f_{iU}], \forall i = 1, 2, \cdots, d$, where $f_{iL}$ and $f_{iU}$ are the lower and upper bounds of $f(y)$ values in the sub-regions $G_i, \forall i = 1, 2, \cdots, d$. The best sub-region is then selected by comparing the intervals $f(G_i), \forall i = 1, 2, \cdots, d$ of the function values with the help of interval order relations. Repeating this process on the selected sub-region, we obtain the best value of the given function in an interval form with a negligible width.

The pseudo code of the solution procedure of IOMOT is as follows:

*procedure IOMOT*
*begin*
*initialize the number of decision variables n;*
*initialize the number of divisions d;*
*initialize the bounds of the variables;*
***while*** *termination criterion not satisfied* ***do***
    *divide the selected region (initially it is the given region)* $a_L \le y \le a_U$ *into sub regions* $G_1, G_2, \cdots, G_d$ *with exhaustive form;*
    *calculate* $f_{iL}$ *and* $f_{iU}$ *for each region* $G_i, i = 1, 2, \cdots, d$ *by interval mathematics;*
    *compare these intervals* $[f_{iL}, f_{iU}], (i = 1, 2, \cdots, d)$ *by interval order relations and select best region;*
*end while*
*print the values of function value and all the decision variables in interval form;*
*end*

Table 3. Computational results for $T = 60/365$ year.

| Case/ sub-case | Scenario | $x_1$ (year) | $X$ (year) | $A$ (unit) | $S$ (unit) | $p_2$ ($) | Average profit ($) |
|---|---|---|---|---|---|---|---|
| 1.1 | – | $[1.5237, 1.5237]$ | $[1.7250, 1.7250]$ | 269.3817 | 30.0132 | 45.00 | $[4435.2656, 4435.2656]$ |
| 1.2 | 1.2.1 | $[1.5214, 1.5214]$ | $[1.6187, 1.6187]$ | 268.9716 | 15.5497 | 45.00 | $[5111.5937, 5111.5942]$ |
| | 1.2.2 | $[1.6111, 1.6111]$ | $[1.7125, 1.7125]$ | 285.4793 | 16.1601 | 45.00 | $[4975.2509, 5059.5927]$ |

**Notes.** – indicates no scenario. As per condition $T \le x_d$, other cases 2.1, 2.2.1, 2.2.2 and 3 are infeasible.

Table 4. Computational results for $T = 120/365$ year.

| Case/ sub-case | Scenario | $x_1$ (year) | $X$ (year) | $A$ (unit) | $S$ (unit) | $p_2$ ($) | Average profit ($) |
|---|---|---|---|---|---|---|---|
| 2.1 | – | $[1.1328, 1.1328]$ | $[1.80, 1.80]$ | 198.2465 | 78.0804 | 45.00 | $[5100.7041, 5100.7045]$ |
| 2.2 | 2.2.1 | $[0.2750, 0.2750]$ | $[0.5218, 0.5218]$ | 46.9242 | 35.7862 | 45.00 | $[6405.9604, 6405.9604]$ |
| | 2.2.2 | | | Not feasible | | | |

**Notes.** – indicates no scenario. As per condition $x_d < T \le x_1$, other cases 1.1, 1.2.1, 1.2.2 and 3 are infeasible.

Table 5. Computational results for $T = 210/365$ year.

| Case/ sub-case | Scenario | $x_1$ (year) | $X$ (year) | $A$ (unit) | $S$ (unit) | $p_2$ ($) | Average profit ($) |
|---|---|---|---|---|---|---|---|
| 3 | – | $[0.2750, 0.2750]$ | $[0.5437, 0.5437]$ | 46.9242 | 38.4500 | 45.00 | $[6580.7061, 6580.7061]$ |

**Notes.** – indicates no scenario. As per condition $x_1 < T$, other cases 1.1, 1.2.1, 1.2.2, 2.1, 2.2.1 and 2.2.2 are infeasible.

In the above procedure, the termination criterion is as follows:

If for all variables $w_i < \varepsilon$, $\varepsilon$ is a pre-assigned positive number (however small), where $w_i = a_{iU} - a_{iL}, i = 1, 2, \cdots, n$.

## 5. Computational illustration

We have considered a numerical example of a hypothetical inventory system and solved the optimization problems of different cases, sub-cases and scenarios using IOMOT to validate and illustrate the suggested model.

**Example.** The values of the different parameters are given by

$$c_h = \$5/\text{unit},\ c_o = \$300/\text{unit},\ a = 190,\ b = 0.4,\ \theta = 0.05,\ c_p = \$20/\text{unit},\ \mu = 1.6,$$
$$I_e = \$0.09/\text{year},\ I_p = \$0.12/\text{year},\ p_1 = \$50/\text{unit},\ c_b = \$35/\text{unit},\ c_{ls} = \$35/\text{unit},$$
$$\lambda = 0.1,\ x_d = 0.2\,\text{year}.$$

For different values of credit period $T$, we have solved the example using the interval-oriented multi-section optimization technique (IOMOT). The corresponding computational results are displayed in Tables 3–5.

**Case study 1**

In this example, we have considered a paddy stock-holder who collects paddy from farmers. During stock-in period, deterioration starts at rate $\theta = 6\%$ after a certain time $x_d = 1.2$ months from the initial stock time. The paddy stock-holder meets up the demand continuously to the rice factory. The demand increases with the

TABLE 6. Computational results for case study 1.

| Case/ sub-case | Scenario | $x_1$ (year) | $X$ (year) | $A$ (unit) | $S$ (unit) | $p_2$ (\$) | Average profit (\$) |
|---|---|---|---|---|---|---|---|
| 2.1 | – | [6.397651, 6.397651] | [12.350001, 12.350001] | 1379.600682 | 355.048741 | 45.000000 | [6008.680176, 6008.680176] |
| 2.2 | 2.2.1 | [7.818086, 7.818086] | [12.349993, 12.349993] | 1762.795215 | 313.878596 | 45.000000 | [8349.174805, 8349.174805] |
| | 2.2.2 | Not feasible | | | | | |

**Notes.** – indicates no scenario. As per condition $x_d < T \leq x_1$, other cases 1.1, 1.2.1, 1.2.2 and 3 are infeasible.

TABLE 7. Computational results for case study 2.

| Case/ sub-case | Scenario | $x_1$ (year) | $X$ (year) | $A$ (unit) | $S$ (unit) | $p_2$ (\$) | Average profit (\$) |
|---|---|---|---|---|---|---|---|
| 1.1 | – | [6.126179, 6.126179] | [11.800001, 11.800001] | 1189.099559 | 346.026368 | 40.500000 | [319.881012, 319.881042] |
| 1.2 | 1.2.1 | [9.244313, 9.244313] | [12.899999, 12.899999] | 1890.998439 | 281.722200 | 40.500000 | [2112.952393, 2112.952393] |
| | 1.2.2 | [9.233523, 9.233523] | [12.899998, 12.899998] | 1888.415549 | 282.136616 | 40.500000 | [1933.456909, 1972.654663] |

**Notes.** – indicates no scenario. As per condition $T \leq x_d$, other cases 2.1, 2.2.1, 2.2.2 and 3 are infeasible.

decreasing of selling price. Here, the demand rate without selling price is $a = 200$ quintal/month and selling price sensitive parameter $b = 0.24$. Again, paddy stock-holder gets an offer to a credit period $T = 1.97$ months to a finance organization or bank. He/she earns interest at the rate $I_e = 9\%$ due to default of credit period. Also the paddy stock-holder paid interest at the rate $I_p = 12\%$ against bank loan. The purchase cost of paddy per quintal is $c_p = \$20$ and the set up cost of paddy stock-holder is $c_0 = \$250$ per cycle. The paddy stock-holder pays the holding cost is $c_h = \$0.75$/quintal/month. Here the paddy stock-holder allows partially backlogged shortages during stock-out period and the value of backlogged parameter is $\mu = 1.06$. The backlogged cost and lost sale cost of the paddy stock-holder are $c_b = \$22$ and $c_{ls} = \$16$ respectively. The paddy stock-holder set up the selling price as $p_1 = \$50$ per quintal before deterioration and offers $\lambda = 10\%$ discount on selling price during deterioration period. Now, the paddy stock-holder wants to determine the optimal values of initial stock, stock-in period, business period and maximum back order quantity which is to be maximized the corresponding average profit (Tab. 6).

### Case study 2

Here, we have considered a potato stock-holder who collects potatoes from farmers' field directly. The potato stock-holder stocks his potatoes in a store room. During stock-in period, deterioration starts after a certain time $x_d = 3.5$ months from the initial stock time at rate $\theta = 4\%$. The potato stock-holder meets up the demand to a market. The demand increases with the decreasing of selling price. The demand without selling price is $a = 200$ quintal/month and selling price sensitive parameter $b = 0.24$. Again, the potato stock-holder gets advantage of credit period $T = 1.97$ months to the supplier and he/she earns interest at the rate $I_e = 9\%$ due to default of credit period. Also the potato stock-holder would pay the interest at the rate $I_p = 12\%$ against bank loan. The purchase cost per quintal potatoes is $c_p = \$20$ and set up cost of potatoes stock-holder is $c_0 = \$250$ per cycle. The potatoes stock-holder pays the holding cost as $c_h = \$5$/quintal/month. Here the potato stock-holder allows partially backlogged shortages during stock-out period and the value of backlogged parameter is $\mu = 1.08$. The backlogged cost and lost sale cost of the potato stock-holder are $c_b = \$12$ and $c_{ls} = \$14$ respectively. The potato stock-holder sets up the selling price as $p_1 = \$50$ per quintal during non-deterioration period and offers $\lambda = 10\%$ discount on the selling price during deterioration period. Now, the potato stock-holder wants to determine the optimal values of initial stock, stock-in period, business period and maximum back order quantity which is to be maximized the corresponding average profit (Tab. 7).

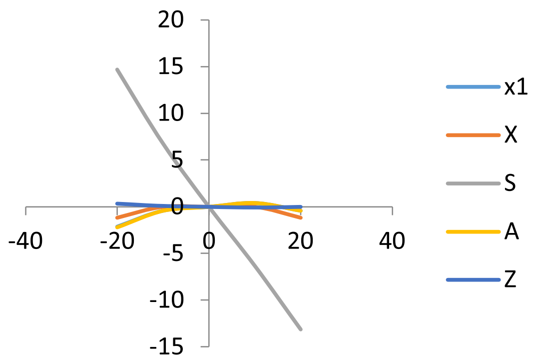


FIGURE 2. Post optimality study of $\delta$.

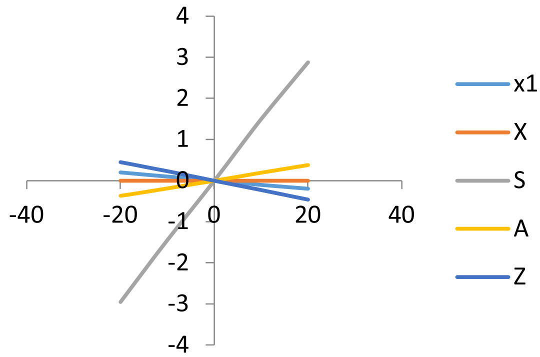


FIGURE 3. Post optimality study of $\theta$.

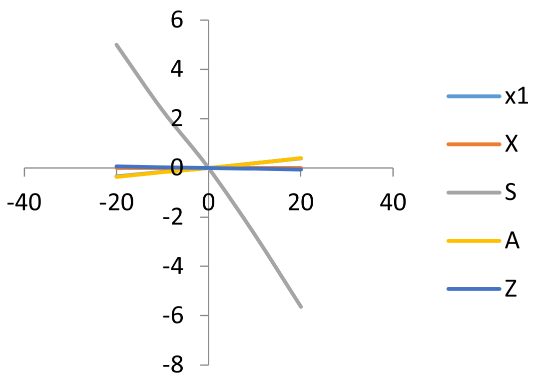


FIGURE 4. Post optimality study of $c_b$.

## 6. SENSITIVITY ANALYSIS

Considering first example for $T = 60/365$, we have measured the effects of the changes by studying the values of system parameters for the time of zero inventory ($x_1$), cycle length ($X$), highest initial inventory level ($A$), highest shortage level ($S$) and average profit ($Z$). For this purpose, a sensitivity analysis is performed graphically by altering one parameter ($-20\%$ to $20\%$) and keeping the other parameters as fixed (*cf.* Figs. 2–10).

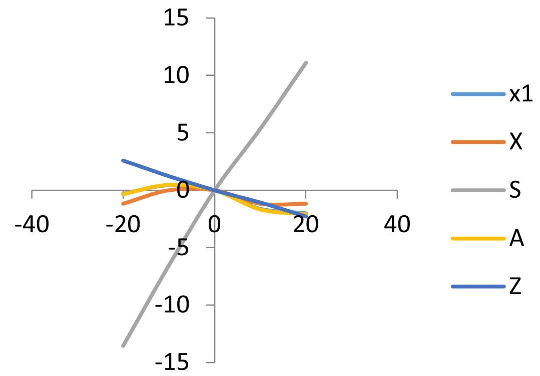


FIGURE 5. Post optimality study of $c_h$.

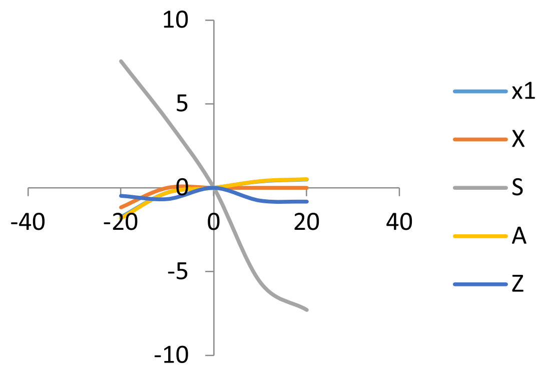


FIGURE 6. Post optimality study of $c_{ls}$.

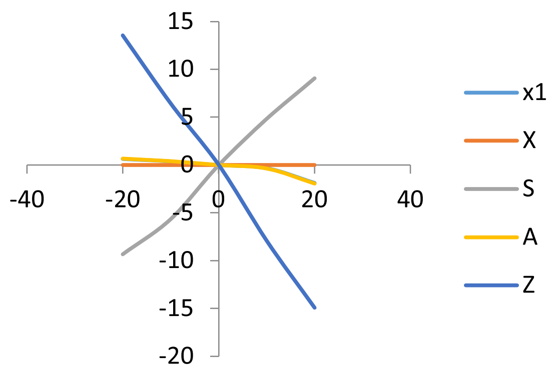


FIGURE 7. Post optimality study of $c_p$.

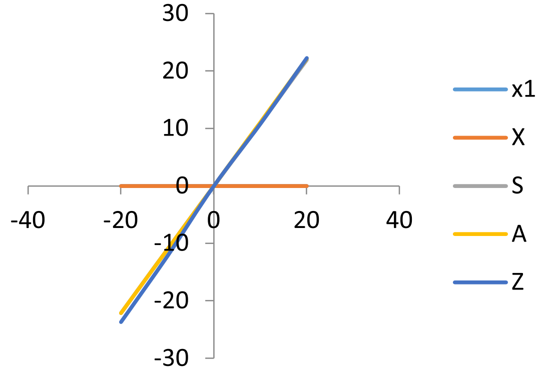


FIGURE 8. Post optimality study of $a$.

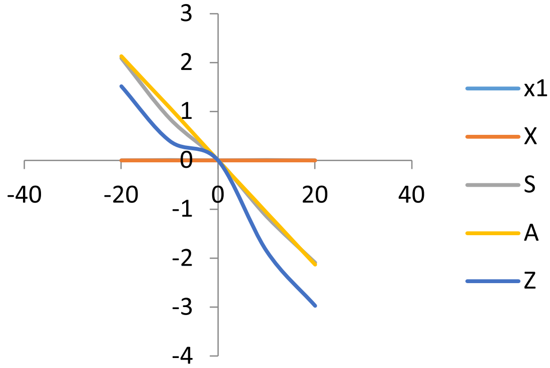


FIGURE 9. Post optimality study of $b$.

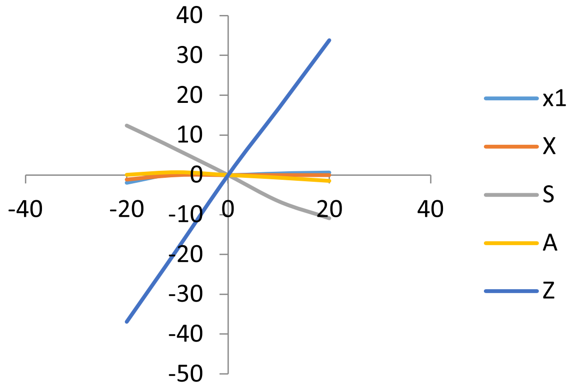


FIGURE 10. Post optimality study of $p_1$.

From Figures 2 to 10, the following observations can be made:

(i) The total profit is highly sensible with respect to parameters $c_p, a, b$ and $p_1$. However, it is less sensitive with respect to the remaining parameters.
(ii) The highest stock is extremely sensitive with regard to parameters $a$ and $b$, which are demand-related parameters. Therefore, the change in the value of these two parameters significantly affected the demand and stock is dependent on demand. However, it is not sensitive with respect to the remaining parameters.
(iii) The maximum shortage is extremely sensible with respect to parameters $c_{ls}, c_b, \theta, c_h$ and $\mu$, but is not much sensible with respect to the other parameters.
(iv) The cycle length is strictly sensible with regard to parameter $b$, but it is not much sensible with respect to the remaining parameters.

## 7. Managerial insight

(i) Demand parameters have large impact on profit of the inventory system. Decision maker can think about demand of the product in order to increase their profit.
(ii) Purchase cost has reverse effect of profit of the inventory system. Decision maker may think about the optimal order size of the product.
(iii) Decision maker may think about initial price of the product because after certain time period they will offer price discount of the product. So, decision maker needs to take the appropriate decision about the initial price in order to avoid their loss.

## 8. Concluding remarks

For the first time, an application of IOMOT [23] is studied in an inventory problem with variable demand and credit facility. Different cases, sub-cases and situations are investigated in the model formulation. Then the model is validated through a numerical example and two case studies. From the computational results of the example, it is observed that case 1.2.1 exhibits the best successful case among others.

Two different selling price dependent demands are considered for deterioration and non-deterioration periods of the item. In the non-deterioration period, the commodities are fresh in nature. As a result, the selling price would be high, and the demand would be less. The freshness of the commodities decreases when the deterioration effects start. A discount facility is then offered, and the selling price eventually becomes less and the demand would be high.

Finally, the proposed inventory problem can further be modified in several ways by adding some realistic features. Anyone can extend this model by taking the nonlinear holding cost and nonlinear demand. Other realistic extensions may also be used to consider the credit policy (single level, two level, or partial). Again one may extend this model by taking the inventory parameters as fuzzy or interval valued.

*Acknowledgements.* All authors are very much thankful to the anonymous Reviewers for their constructive comments and suggestions. The first author is very much thankful to University Grants Commission (UGC) for funding the research. The fund number is F.30-425/2018(BSR) dt. 04.01.2019. This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (NRF-2020R1F1A1064460).

*Conflict of interest.* There are no conflicts of interest to declare.

*Availability of data and material.* There is no data available.

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