RAIRO-Oper. Res. 55 (2021) 1137–1152
https://doi.org/10.1051/ro/2021044

RAIRO Operations Research
www.rairo-ro.org

ELECTRIC POWER SUPPLY CHAIN NETWORKS DESIGN FEATURING DIFFERENTIAL PRICING AND PREVENTIVE MAINTENANCE

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Abstract. The electric power supply chain network plays an important role in the world economy. It powers our homes, offices, and industries and runs various forms of transportation. This paper considers an electric power supply chain network design problem featuring differential pricing and preventive maintenance. We demonstrate that this general model can be formulated as the centralized and decentralized supply chain models. A continuous approximation approach is used to model the problems. The objective of these models is to determine the optimal power plants' service area, electricity price, and preventive maintenance budget while maximizing the total network profit or the own organization's benefits. Our model is applied to the case of a power company in northern Vietnam. We show that the proposed approach can be used to address real-world cases effectively. The results demonstrate that the use of differential pricing policy and preventive maintenance could much enhance power company profit.

Mathematics Subject Classification. 90B05.

Received November 24, 2020. Accepted March 18, 2021.

1. Introduction

In modern society, very few goods or services do not depend directly on the use of electricity [23]. Indeed, electricity is the lifeblood of the world economy: it powers our homes, offices, and industries; provides communications, entertainment, and medical services; powers computers, technology, and the Internet; and runs various forms of transportation. As such, the electric power industry plays a critical role in our society. Electricity is transmitted from power plants to end customers through transmission lines, suppliers, and distribution lines in what is a kind of supply chain network; thus, a supply chain network perspective is useful in studying the

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Keywords. Electric power supply chain network, continuous approximation approach, differential pricing, preventive maintenance.

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Article published by EDP Sciences

electric power industry. In addition, with an increase in the world economy and the power demand also the development of renewable energy, the electric power supply chain network (EPSCN) becomes more complicated. If power markets have been only operated by the governments or government companies, nowadays, there are many outside players want to join in the EPSCN. Application supply chain management research theory with respect to the EPSCN is really fairly nascent, but it is attracting scholarly attention worldwide [20].

A continuous approximation (CA) approach is successfully used in production-distribution system design or facility-inventory allocation problem [4, 17], but there are no previous studies that use the CA approach for the power plants' location problems (see Sect. 2.1). This study introduces a CA approach for designing an EPSCN considering differential pricing and preventive maintenance. The proposed method uses continuous functions that rely on the service area to model the problems. This approach does not require as much data as mathematical programming models, and its development and implementation are therefore relatively easier and more straightforward [4].

In practice, the EPSCN is usually under the control of government, even in the developed or developing countries; it could be seen as a centralized supply chain model. However, the increase of power demand and the development of renewable energy could make the EPSCN change in the current world economic when more outside companies want to join in the power industry. Thus, we propose two models of supply chain: centralized and decentralized model. In the former model, a single decision maker owns, sets up, and operates the EPSCN himself/herself. His/Her objective is to maximize the total supply chain network profit by determining where to set up the power plants? How much should be paid for the preventive maintenance activities? How to charge the electricity price to end users? In the different ways, there are two decision-makers in the latter model (see Fig. 1 in page 9 for a visualization): the power generators who are assumed to generate and transmit electricity to the suppliers and determine where to open the power plant; the electricity suppliers who are responsible for selling, distributing the electricity to the end users, and determine how to charge the electricity price to the end users.

In addition, our model also considers differential price and preventive maintenance policies. The use of differential price could improve the economic efficiency of wholesale electricity markets, by discouraging low-value energy consumption, reducing demand volatility, and lowering peak demand [2]. In this paper, electricity demand is derived by a function of price and time, and the electricity price varies over time. We look to determine whether or not the differential pricing policy should be applied to the EPSCN. Moreover, given problems regarding network vulnerabilities, one of the most important problems that power distribution companies face is determining a suitable maintenance budget for distribution lines [5]. In our model, we not only try to find the optimal power plants' service area and electricity price but also determine the maintenance budget to obtain the optimal objective function of all models. A nonlinear optimization method is used to resolve these problems. The solutions are found directly when using a continuous approximation approach. In addition, the mathematical model is applied to the case of a power company in northern Vietnam.

In this context, the main contributions of this paper are the following:

- An EPSCN design based on continuous models.
- Two supply chain models (centralized and decentralized model) are considered for designing an electric power supply chain network (EPSCN).
- A differential pricing strategy and preventive maintenance are incorporated in the EPSCN models.
- Finally, a case study of Electricity Vietnam Company is presented to illustrate our models and our proposed method.

This paper is organized as follows. The next section reviews the previous related literature. Section 3 considers the mathematical modelling of PDN. The proposed solution method is proposed in Section 4. Examples and numerical analysis are given in Section 5. Section 6 concludes the paper.

2. Literature review

2.1. Electric power supply chain network

Wu *et al.* [23] and Nagurney *et al.* [11] develop an electric power supply chain network model that include both power plants and pollution taxes; they describe the behavior of electric power generators, suppliers, and consumers in the demand market. However, these studies focus on the determination of the carbon cost within an electric power supply chain network equilibrium model, and they do not consider location problems. Sharma Ashwani [16] uses a mixed integer nonlinear programming approach to analyze optimal power plant location; he considers cost, security, and system load ability. Nagurney *et al.* [12] propose a dynamic electric power supply-chain network model in which demand varies over time; that model uses an evolutionary variation inequality formulation. Both Sharma Ashwani [16] and Nagurney *et al.* [12] use discrete models to design power networks, in different ways; we, on the other hand, apply the continuous approximation to model our network.

Wang and Cong [20] compare the differences between electric power supply-chain management and traditional supply-chain management; they also show that electric power supply-chain management could expand into the following areas: contract management, information sharing, stock management, and risk-sharing. Zdraveski *et al.* [26] propose a novel and very straightforward algorithm for undertaking dynamic intelligent load-balancing – which reduces power losses in a power distribution network (PDN) – to identify the load-balancing problem among the phases of three-phase systems; they assume the use of an existing smart network of power meters. Khosrojerdi *et al.* [9] design power grid networks while considering the transmission system, or failures therein. They consider the location and preventive maintenance problems to satisfy a deterministic demand for electricity. This model considers power demand, which comprises time and price dimensions. Our decision variables vary over time.

Recently, Jabbarzadeh *et al.* [7] took into account for different unique smart grid components such as demand side management programs, microgrid structure, two-way distribution lines, and supplier–consumer nodes, while incorporating different interrelated decisions including facility location, capacity expansion, load allocation, and pricing. Sarkar *et al.* [15] considered an electric power transmission and distribution as representative of the product distribution network. The model was developed using a combination of the supply chain management technique and power transmission terminologies as a linear model. Hosseini-Motlagh *et al.* [6] designed and optimized an electricity supply chain network which took into account distributed generators. They also integrated resiliency and corporate social responsibility into the model. A novel fuzzy-robust approach was developed to deal with uncertainty. Most of the above works have used the discrete model to consider EPSCN problem. In the different way, our work uses a continuous approximation approach which defines all functions as continuous function and solves the model directly to find the optimal solutions.

2.2. The electricity price policies

There is general agreement that charging real-time electricity prices to customers could improve the economic efficiency of wholesale electricity markets, by discouraging low-value energy consumption, reducing demand volatility, and lowering peak demand [2]. In the literature, a number of studies analyze electricity price time series and market demand. Buzoianu *et al.* [1] examine a dynamic supply–demand model to simultaneously capture electricity price and usage time series. Wang *et al.* [22] consider a load-serving entity day-ahead planning problem that involves coordinating electricity users within the community *via* dynamic pricing. Tsitsiklis and Xu [18] propose a dynamic pricing mechanism that explicitly encourages customers to change their consumption patterns so as to offset the variability of demand on conventional units. Malakar *et al.* [10] discuss a novel day-ahead price-based optimal reactive power dispatch problem. The basic objective of their model is to reduce the cost of reactive power generation from generators and other sources when generators are engaged in feeding a projected megawatt demand over a certain time period. Zhou *et al.* [27] proposed a time-of-use pricing model based on power supply chain for user-side microgrid. A bullwhip effect of the EPSCN is also investigated. They show that the costs of the whole EPSCN are reduced by their model.

In general, many questions relate to real-time electricity price. In the current study, we look to incorporate real-time pricing theory into the EPSCN model as differential pricing policy and answer the question: Should we use a real-time pricing strategy to determine electricity price? The optimal price in each time period is determined so as to maximize the company's total profit.

2.3. Preventive maintenance for power supply chain networks

Power networks are vulnerable and failures are inevitable [9]. An effectively administered preventive maintenance program helps reduce the number of accidents, save lives, and preclude costly breakdowns and unplanned outages. Having in place a preventive maintenance program can reduce the risk of unplanned downtime by as much as 60% [13]. In practice, distribution companies need to pay each year several maintenance costs, such as those relating to inspection, cleaning and lubrication, adjustments, overcurrent protective device testing, insulation testing, charge/close/trip circuit testing, dielectric testing, and time and speed testing. Normally, preventive maintenance represents 15–18% of a total maintenance budget; however, limitations on maintenance budgets among distribution companies, as well as the difficulties inherent in accessing preventive maintenance budget funds, have made it necessary to conduct research into preventive maintenance budget planning [5].

Several studies have considered preventive maintenance [3, 5, 8, 14, 21, 24, 25], however, those studies focus solely on determining the optimal preventive maintenance strategy. The current study is different, in that it considers the preventive maintenance budget part of the power distribution company's total cost. An increase in the preventive maintenance budget will reduce both the failure rate and overall maintenance costs. In short, we look to determine the optimal preventive maintenance budget while maximizing company profit.

3. MATHEMATICAL MODELLING

3.1. Assumptions and notations

In this section, we develop an electric power supply chain that consists of n power generators, several power plants, transmission lines, electric power suppliers, distribution lines, and end users (Fig. 1). The power generators are located at the level-one and there is a decision-maker who owns and operates the power plants and transmission lines. Each power generator serves a given large service area, C_i. He/She is responsible for generating and transmitting the electric power to the power suppliers. In our model, the power generators are assumed to determine how to locate and allocate the power plants in level two (Fig. 1) and also obtain the optimal maintenance budget to minimize their cost.

For power plants' location problems, each power generator's service area, C_i $(i = 1, 2, \ldots, N)$, includes several power plants. Let A_i^t be the service area of each power plant in the same power generator's service area i at time $t (t = 1, 2, \ldots, T)$. The average service area of each power plant in the same power generator's service area i is $A_i = \frac{\sum_{t=1}^{T} A_i^t}{T}$. Then, the number of power plants should be opened in a power generator's service area (C_i) is C_i/A_i. The objective of this paper is to determine the service area of each power plant i at time t (Fig. 2), A_i^t. Assume that the power plants' service area is roughly circular in shape (see Fig. 2); irregularly shaped service areas could be roughly circular, hexagonal, or square-shaped, but they have little effect on the optimal solution [4]. Instead of finding the exact location of each power plant, we assume that in any case, each power plant is located at the center of its service area. Let f_r be the constant that depend on the distance metric and shape of substation service area, the outbound distance from a power plant to suppliers is $f_r\sqrt{A_i^t}$ [4].

The power suppliers are structured in the level-three and treated as decision-makers in the EPSCN (Fig. 1). The optimal order amount of electricity from each power plant in each C_i at time t is Q_i^t. The location of power suppliers are given. They act as electrical distributors which respond to distribute the electrical power to the end users.

Users' demand for each supplier in the power generator's service area i is derived as a function of time and price, $D_i^t = M_i - \beta_i^t (P_i^t)^{\alpha_i}$, for $i = 1, 2, 3, \ldots, N$ and $t = 1, 2, 3, \ldots, T$ [22]. Here, M_i is the maximum users demand for each supplier in power generator's service area; β_i^t is the time-dependent demand shift parameter

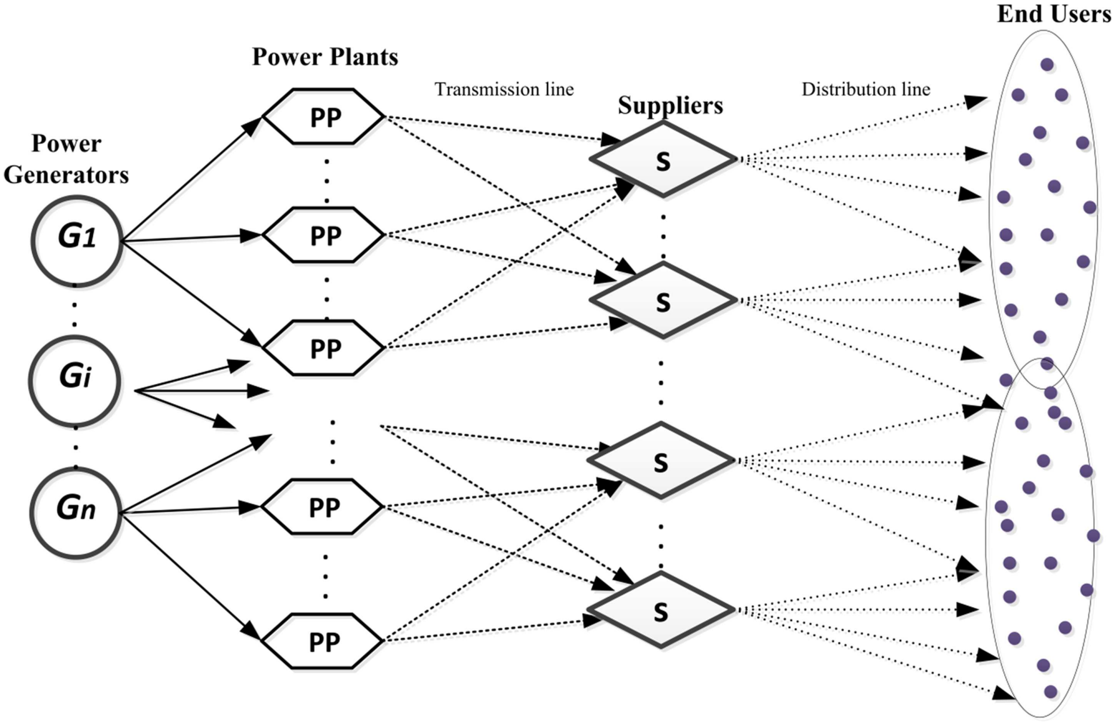

Figure 1. The electric power supply chain.

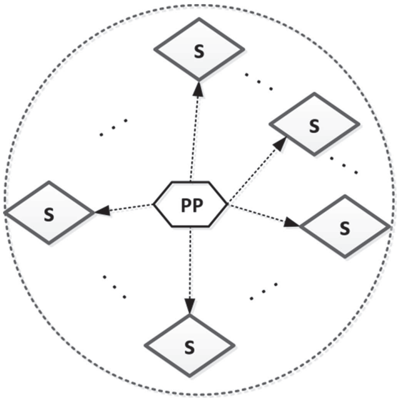

Figure 2. Service area of each power plant, A_i^t.

by the time ($\beta_i^t > 0$); α_i is the price elasticity of demand for each supplier in power generator's service area. In practice, the electricity price has to be equal to or larger than the electricity generation cost, *i.e.*, $P_i^t \geq C_g$.

We consider two cases of supply chain model: centralized decentralized model. In the centralized case, the electric power supply chain is assumed to be set up and operated by a single manager who makes all decisions: where to set up the power plants, how to manage the preventive maintenance budget, and how to charge the electricity to the end users to maximize the total network. In the second model, the decentralized case, the power generators and power suppliers serve as decision-makers in the electric power supply chain. In this case, a

Table 1. Notations for PDN design.

Notations	Definition
Indexes	
i	Set of power generator indexes, $i = 1, 2, \ldots, N$
t	Set of period time indexes, $t = 1, 2, \ldots, T$
Parameters	
F	Fixed cost for setting up and operating an power plant
C_i	Service area for power generator i
D_i^t	Electricity customer demand for each supplier in power generator i in period t
f_r	The constant that depend on the distance metric and shape of substation service area
C_{ce}	Cost of carbon emission per unit
C_f	Fixed cost of transmitting electricity distribution to the users per bound
C_g	Cost of electricity generation per unit
C_{CM}	Cost of corrective maintenance per power plant per time
C_{OM}	Cost of operating and maintenance the electrical substation in level two
C_p	Cost of purchasing the electricity from power generators (paid by suppliers)
C_t	Cost of transmitting the electricity to the suppliers per unit per distance
C_v	Variable cost of the electricity distribution to the users per unit
L	Maximum accepted percentage of electricity loss (from power plants to suppliers in level two)
l	Maximum accepted percentage of electricity loss (from suppliers to the end users)
M_i	Maximum electricity demand for generator i
G_i^t	The amount of electricity generated from each power plant in C_i
O	The ordering per order
Q_i^t	The ordered amount of electricity from suppliers to power plants in C_i
β_i^t	The shift parameter of demand curve in period t
α_i	Price elasticity of electricity demand in C_i
λ_i	The failure rate for the critical outage causes
σ	The emission factor for generating electricity per unit
a_i, b_i, γ	The coefficients of failure rate
Decision variables	
A_i^t	Service area of each electrical distributor served by substation i in period t
P_i^t	Electricity price of power plant i in period t
PM_i^t	Preventive maintenance budget for distribution line served by power plant i in period t

game-theoretical decision making is applied wherein the power generators are treated as leaders who determine the preventive maintenance budget and the service area of each power plant to minimize their total cost; the suppliers are followers who decide the optimal electric price P_i^t to maximize their own profits.

In addition, our model also considers how to obtain the preventive maintenance budget which pays for maintenance, energy not supplied, repairing, and human resources. We assume that the relationship between the failure rate λ_i^t and the preventive maintenance budget PM_i has the following nonlinear relationship, $\lambda_i^t = a_i + b_i \mathrm{e}^{-\gamma \mathrm{PM}_i}$, where a_i, b_i and γ are the coefficients of the failure rate PM_i is the preventive maintenance budget for the distribution line served by power plant i in period t. The failure rate decreases as the preventive maintenance budget increases [5]. However, an increase in the preventive maintenance budget gives rise to a higher total network cost. Therefore, companies should determine a suitable maintenance budget for distribution lines, in order to minimize costs and concurrently maximize total profit (Tab. 1).

3.2. Mathematical modelling

Before developing two supply chain models, we first formulate two separate mathematical models for power generators and the suppliers.

3.2.1. The model of power generator

In our model, the components of the total power generators' cost are calculated as follows:

- *Total facility cost*: the cost of opening and operating one power plant is F/per power plant. The number of power plants should be opened in a power generator's service area (C_i) is C_i/A_i. Thus, the cost of setting up and operating all power plants is calculated as follows:

$$\mathrm{TF} = \sum_{i=1}^{N} F\frac{C_i}{A_i} = \sum_{i=1,t=1}^{N,T} F\frac{\mathrm{TC}_i}{A_i^t}\cdot \tag{3.1}$$

- *Generation cost*: let G_i^t be the amount of electricity generated from the power plant, $G_i^t = (1+L)Q_i^t$. Here, L is the maximum accepted power loss rate for transmitting electricity from power plant to the supplier in level three and Q_i^t is optimal order amount of electricity from each power plant in each C_i at time t. The total generation cost function is calculated as:

$$\mathrm{TG} = \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_g G_i^t = \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_g (1+L) Q_i^t. \tag{3.2}$$

- *Maintenance cost*: each time, the company has to pay an amount for maintenance activities. In our model, assume that the company is please to take a preventive maintenance budget PMi to prevent the failure system. When the failure occurs, the company has to pay the cost of corrective maintenance for the critical outage, repairs, energy supplied, and human resource (CPM). Then, the total cost of maintenance includes two parts: the corrective cost and the preventive maintenance budget as follows:

$$\mathrm{TPM} = \sum_{i=1}^{N} \frac{C_i}{A_i^t}\left(\lambda_i^t C_{\mathrm{CM}} + \mathrm{PM}_i\right) = \sum_{i=1}^{N} \frac{C_i}{A_i^t}\left(\left(a_i + b_i \mathrm{e}^{-\gamma \mathrm{PM}_i}\right) C_{\mathrm{CM}} + \mathrm{PM}_i\right). \tag{3.3}$$

- *Electricity transmission cost*: the cost which purchases the transmission service from the power plants to the suppliers. It depends on how far the electricity has to be transmitted. This cost is calculated as follows.

$$\mathrm{TT} = \sum_{i=1}^{N} C_t f_r \sqrt{A_i^t} D_i^t, \tag{3.4}$$

  where C_t is the cost of transmission per unit per distance; $f_r\sqrt{A_i^t}$ is the outbound distance from the power plants to the suppliers.
- *Carbon cost*: assume that the company has to pay for carbon emission cost. This cost is calculated based on the carbon emission from generating the electricity at power plants:

$$\mathrm{TE} = \sum_{i=1}^{N} C_{ce}\sigma G_i^t, \tag{3.5}$$

  where C_{ce} is the cost of carbon emission per unit; σ is the emission factor for generating electricity per unit.

3.2.2. The model of power suppliers

- *Total revenue*: the total revenue is calculated based on the users' demand as follows:

$$\mathrm{TR} = \sum_{i=1,t=1}^{N,T} P_i^t D_i^t. \tag{3.6}$$

- *Ordering cost for all suppliers:* let O is the ordering cost for each order of supplier. The total ordering for all suppliers is:

$$\mathrm{TO} = \sum_{i=1}^{N} O \frac{D_i^t}{Q_i^t}\cdot \tag{3.7}$$

- *Purchasing cost*: let C_p is the purchasing cost per unit, charged by power generators. The total purchasing cost for all suppliers is:

$$\mathrm{TPP} = \sum_{i=1}^{N} C_P Q_i^t. \tag{3.8}$$

- *Operating and maintenance cost of all suppliers*: let C_{OM} be the cost of operating and maintaining the substation in level two per electricity unit; l is the maximum accepted power loss rate for transmitting electricity from suppliers to the end users ($l > L$). The total operating and maintenance cost of all substations in level two is formulated as:

$$\mathrm{TOM} = \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_{\mathrm{OM}} Q_i^t = \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_{\mathrm{OM}} (1+l) D_i^t. \tag{3.9}$$

- *Electricity distribution cost*: the cost which pays for the distribution service from the suppliers to the end users. It depends on the amount electricity has to be distributed. This cost is calculated as follows.

$$\mathrm{TD} = \sum_{i=1}^{N} \left(C_f + C_v Q_i^t\right) \frac{D_i^t}{Q_i^t} = \sum_{i=1}^{N} \left(C_f + C_v (1+l) D_i^t\right) \frac{1}{(1+l)} \tag{3.10}$$

  where C_f is the fixed cost of electricity distribution; C_v is the variable cost of electricity distribution per bound; D_i^t/Q_i^t is the number of outbound shipments from the suppliers to end suppliers at time t.

4. Solution approach

In this section, we develop two supply chain models: centralized and decentralized model. The optimal solutions to the two models are derived by using the nonlinear optimization technique.

4.1. The centralized case

The total network profit is calculated as the sum of the total revenue, the facility cost, the generation cost, the preventive maintenance cost, electricity transmission cost, carbon cost, the operating and maintenance cost, and the cost of distributing electricity.

Maximize:

$$\begin{aligned}\mathrm{TP}\left(A_i^t, \mathrm{PM}_i^t, P_i^t\right) &= \mathrm{TR} - \mathrm{TF} - \mathrm{TG} - \mathrm{TPM} - \mathrm{TT} - \mathrm{TE} - \mathrm{TOM} - \mathrm{TD} \\ &= \sum_{i=1,t=1}^{N,T} P_i^t D_i^t - \sum_{i=1,t=1}^{N,T} f \frac{C_i}{A_i^t} - \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_g G_i^t - \sum_{i=1}^{N} \frac{C_i}{A_i^t} \left(\lambda_i^t C_{\mathrm{CM}} + \mathrm{PM}_i^t\right) \\ &\quad - \sum_{i=1}^{N} C_t f_r \sqrt{A_i^t} D_i^t - \sum_{i=1}^{N} C_{ce} \sigma G_i^t - \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_{\mathrm{OM}} Q_i^t - \sum_{i=1}^{N} \left(C_f + C_v Q_i^t\right) \frac{D_i^t}{Q_i^t}\cdot \end{aligned} \tag{4.1}$$

The company/government determines the service area of each power plant (A_i^t), the electricity price (P_i^t), and the preventive maintenance budget (PM_i), in order to maximize its total profit. To solve problems, we first address the decision PM_i. Given P_i^t and A_i^t, we have

$$\frac{\partial^2 \mathrm{TP}\left(\mathrm{PM}_i \,|A_i^t, P_i^t\right)}{\partial \left(\mathrm{PM}_i\right)^2} = -\frac{\gamma^2 b_i C_i C_{\mathrm{CM}} \mathrm{e}^{-\gamma \mathrm{PM}_i}}{A_i^t} < 0. \tag{4.2}$$

Therefore, $\mathrm{TP}(\mathrm{PM}_i\,|A_i^t, P_i^t)$ is the concave function of PM_i. This means that the optimal PM_i, the solution of $\frac{\partial \mathrm{TP}(\mathrm{PM}_i\left|A_i^t, P_i^t\right)}{\partial \mathrm{PM}_i} = 0$ is the optimal solution for TP. Solving $\frac{\partial \mathrm{TP}(\mathrm{PM}_i\left|A_i^t, P_i^t\right)}{\partial \mathrm{PM}_i} = 0$, we have:

$$\mathrm{PM}_i^* = -\frac{\log\left[\frac{1}{\gamma b_i C_{\mathrm{CM}}}\right]}{\gamma}. \tag{4.3}$$

Substituting equation (4.3) into (4.1) yields

$$\begin{aligned}\mathrm{TP}'' = {} & \sum_{i=1,t=1}^{N,T} P_i^t D_i^t - \sum_{i=1,t=1}^{N,T} f\frac{C_i}{A_i^t} - \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_g G_i^t - \sum_{i=1}^{N} \frac{C_i}{A_i^t}\left(\left(a_i + \frac{1}{\gamma C_{\mathrm{CM}}}\right) - \frac{\log\left[\frac{1}{\gamma b_i C_{\mathrm{CM}}}\right]}{\gamma}\right) \\ & - \sum_{i=1}^{N} C_t f_r \sqrt{A_i^t} D_i^t - \sum_{i=1}^{N} C_{ce}\sigma G_i^t - \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_{\mathrm{OM}} Q_i^t - \sum_{i=1}^{N}\left(C_f + C_v Q_i^t\right)\frac{D_i^t}{Q_i^t}.\end{aligned} \tag{4.4}$$

The model becomes a nonlinear function with $2N * T$ variables (A_i^t and P_i^t, for $i = 1, 2, 3, \ldots, N$ and $t = 1, 2, 3, \ldots, T$). Given P_i^t, the second-order derivative of $\mathrm{TP}''(A_i^t\,|P_i^t\,)$ with respect to A_i^t is

$$\frac{\partial^2 \mathrm{TP}''\left(A_i^t\,|P_i^t\,\right)}{\partial\left(A_i^t\right)^2} = -\frac{2C_i f}{\left(A_i^t\right)^3} + \frac{C_t f_r\left(M - \beta_i^t (P_i^t)^{-\alpha_i}\right)}{4\left(A_i^t\right)^{3/2}} - \frac{2C_i\left(C_{\mathrm{CM}}\left(a_i + \frac{1}{\gamma C_{\mathrm{CM}}}\right) - \frac{\log\left[\frac{1}{\gamma b_i C_{\mathrm{CM}}}\right]}{\gamma}\right)}{\left(A_i^t\right)^3}. \tag{4.5}$$

In general case, $\frac{\partial^2 \mathrm{TP}''(A_i^t\left|P_i^t\right)}{\partial (A_i^t)^2} < 0\,\forall A_i^t$; therefore, $\mathrm{TP}''(A_i^t\,|P_i^t)$ is a concave function of A_i^t. This means that the optimal solution can be obtained by solving $\frac{\partial \mathrm{TP}''(A_i^t\left|P_i^t\right)}{\partial A_i^t} = 0$:

$$A_i^{t^*} = \frac{2^{2/3} C_i^{2/3}\left(P_i^t\right)^{2\alpha_i/3}\left(\frac{\left(1 + \gamma a_i C_{\mathrm{CM}} + f\gamma - \log\left[\frac{1}{\gamma b_i C_{\mathrm{CM}}}\right]\right)^2}{\gamma^2}\right)^{1/3}}{C_t^{2/3} f_r\left(\left(-M_i\left(P_i^t\right)^{\alpha_i} + \beta_i^t\right)^2\right)^{1/3}}. \tag{4.6}$$

Substituting equation (4.6) into (4.4), the model becomes a nonlinear function with $N * T$ variables (P_i^t, for $i = 1, 2, 3, \ldots, N$ and $t = 1, 2, 3, \ldots, T$). An enumerative search is proposed to determine $P_i^{t^*}$, and Proposition 1 can be used to reduce the search area.

Proposition 4.1. *If the valid optimal* $P_i^{t^*}$ *is found, then* $P_i^{t^*} \in \Phi$ *where* $\Phi = [C_g, (\frac{M_i}{\beta_i^t})^{-1/\alpha_i}]$.

Based on the discussion above, the following algorithm determines the optimal value for all decision variables and objective function.

Algorithm.

Step 1. Searching the optimal value of P_i^t which satisfies $P_i^t \in \Phi$, $\Phi = [C_g, (\frac{M_i}{\beta_i^t})^{-1/\alpha_i}]$.

  Step 1.1. For $j = 1$, start with $(P_i^t)^{j=1} = C_g$ and increase $(P_i^t)^j$ by 1 each time; for each $(P_i^t)^j$, using equation (4.6) to obtain the optimal $(A_i^t)^{j=1}((P_i^t)^{j=1})$, and then using equation (4.3) to calculate the corresponding $\mathrm{TP}''^{(j)}((A_i^t)^j, (P_i^t)^j)$. Repeat this step until $(P_i^t)^{j=J} = (\frac{M_i}{\beta_i^t})^{-1/\alpha_i}$.

  Step 1.2. Let $\mathrm{TP}''^*\left(A_i^{t^*}, P_i^{t^*}\right) = \max_{P_i^t \in \Phi}\{\mathrm{TP}'' j((A_i^t)^j, (P_i^t)^j)\}$ be the value found when the iteration stops.

Step 2. Calculating $\mathrm{TP}(\mathrm{PM}_i, P_i^t, A_i^t) = \mathrm{TP}^*(\mathrm{PM}_i^*, P_i^{t^*}, A_i^{t^*})$ by using equation (4.1) and $\mathrm{TP}^*(\mathrm{PM}_i^*, P_i^{t^*}, A_i^{t^*})$ is the optimal network profit.

4.2. The decentralized case

In this section, a game-theoretical decision-making scheme is applied to solve the problems. The power generators make their decisions based on suppliers' reaction.

4.2.1. Suppliers' reaction

The objective function of suppliers is formulated as follows:
Maximize:

$$\begin{aligned}\mathrm{TPS}\left(P_i^t\right) &= \mathrm{TR} - \mathrm{TPC} - \mathrm{TO} - \mathrm{TOM} - \mathrm{TD}\\ &= \sum_{i=1,t=1}^{N,T} P_i^t D_i^t - \sum_{i=1,t=1}^{N,T} C_p Q_i^t - \sum_{i=1}^{N} O\frac{D_i^t}{Q_i^t} - \sum_{i=1,t=1}^{N,T} \frac{1}{C_i} C_{\mathrm{OM}} Q_i^t - \sum_{i=1}^{N}\left(C_f + C_v Q_i^t\right)\frac{D_i^t}{Q_i^t}\cdot \end{aligned} \tag{4.7}$$

To obtain the optimal of P_i^t, we first check if the second-order derivative of $\mathrm{TPS}(P_i^t)$ with respect to P_i^t is negative or positive.

$$\frac{\partial^2 \mathrm{TPS}(P_i^t)}{\partial (P_i^t)^2} = \frac{1}{C_i}\alpha_i \beta_i^t (P_i^t)^{-2-\alpha}\left(C_{\mathrm{OM}}(1+l)(1+\alpha_i) + C_i\left(C_v + P_i^t + \alpha_i C_v - \alpha_i P_i^t + C_p(1+l)(1+\alpha_i)\right)\right). \tag{4.8}$$

In the general case, the electricity price elasticity $\alpha_i < 0$ and $\frac{\partial^2 \mathrm{TPS}(P_i^t)}{\partial (P_i^t)^2} < 0 \forall P_i^t$; therefore, $\mathrm{TPS}(P_i^t)$ is a concave function of P_i^t. This means that the optimal solution can be obtained by solving $\frac{\partial \mathrm{TPS}(P_i^t)}{\partial P_i^t} = 0$. Because the form of $P_i^{t^*}$ is too complicated, we don't show here. We will illustrate it in the numerical section.

4.2.2. Power generators' decision making

The objective function of power generators is given as:
Minimize:

$$\begin{aligned}\mathrm{TC}(A_i^t, \mathrm{PM}_i) &= \mathrm{TF} + \mathrm{TG} + \mathrm{TPM} + \mathrm{TT} + \mathrm{TE}\\ &= \sum_{i=1,t=1}^{N,T} f\frac{C_i}{A_i^t} + \sum_{i=1,t=1}^{N,T}\frac{1}{C_i}C_g G_i^t + \sum_{i=1}^{N}\frac{C_i}{A_i^t}(\lambda_i^t C_{\mathrm{CM}} + \mathrm{PM}_i^t)\\ &\quad + \sum_{i=1}^{N} C_t f_r \sqrt{A_i^t} D_i^t + \sum_{i=1}^{N} C_{ce}\sigma G_i^t. \end{aligned} \tag{4.9}$$

Proposition 4.2. *The total cost function* TC *is convex function with respect to* (A_i^t, PM_i).

Thus, we can find the strict global minimum point of TC *by taking the first order derivative of* TC *with respect to* A_i^t *and* PM_i, *and set the results equal to zero, then obtain the following equations:*

$$A_i^{t^*} = \frac{2^{2/3} C_i^{2/3} \mathrm{e}^{-\frac{2\gamma \mathrm{PM}_i}{3}} \left(P_i^t\right)^{2\alpha/3}\left(\left(b_i C_{\mathrm{CM}} + \mathrm{e}^{\gamma \mathrm{PM}_i}\left(a_i C_{\mathrm{CM}} + f + \mathrm{PM}_i\right)\right)^2\right)^{1/3}}{C_t^{2/3} f_r^{2/3}\left(\left(-MP^{\alpha} + \beta_i^t\right)^2\right)^{1/3}} \tag{4.10}$$

$$\mathrm{PM}_i^* = -\frac{\log\left[\frac{1}{\gamma b_i C_{\mathrm{CM}}}\right]}{\gamma}\cdot \tag{4.11}$$

FIGURE 3. Clustering service area.

TABLE 2. The input parameters.

	Value ($i = 1, 2, 3$)		Value ($i = 1, 2, 3$)
f	\$1128/power plant/quarter	a_i	$(39, 41, 59)$
C_i	(37 533.4, 64 025.2, 14 841.7) (km^2)	b_i	$(66.1, 123.9, 162.3)$
α_i	-1.5	γ	0.034
C_t	\$0.015/km/kw	f_r	0.3
C_g	\$0.05/kw	L	0.05
C_{OM}	\$0.05/kw	l	0.1
C_{PM}	\$123/power plant	t	$(1, 2, 3, 4)$

5. NUMERICAL ANALYSIS

To demonstrate the effectiveness of the proposed method, we applied it to the design of the northern Vietnamese power supply chain network. The customer service area is divided into three clusters, with each cluster i being served by each power generator i ($N = 3$): northwest Vietnam (C_1), northeast Vietnam (C_2), and the Red River delta (C_3) (Fig. 3). Although the Red River delta is the smallest area ($C_3 < C_1 < C_2$), it still has highest power demand rate because of the high population density there. Northwest Vietnam, located in the mountainous northwestern part of the country, has the lowest population density, and so it is reasonable that this area has the lowest power demand rate. We consider one-year demand which is divided into four periods ($T = 4$): spring ($t = 1$), summer ($t = 2$), fall ($t = 3$), and winter ($t = 4$). In the summertime, the electricity demand increases because of using more electricity utility ($D_i^2 > D_i^1$, D_i^3 and D_i^4). The input data parameters were selected from previous research [4,5,19,22], as shown in Table 2.

The below subsection shows the calculated results for two cases: centralized model (refer to the character "C") and decentralized model (refer to the character "D"). The total network profit of centralized case is $\text{TP}^* = 9.72 \times 10^6$. In the decentralized case, the optimal total profit of suppliers $\text{TPS}^* = 8.11 \times 10^6$ is and the optimal total cost of power generators is $\text{TC}^* = 6.83 \times 10^5$. The result shows that the centralized case obtains the higher profit than the decentralized case. Actually, in Vietnam, Vietnam Electricity (EVN) generates more than 23 580 MW/year-supplies about 61.2% of power for country. EVN also owns the entire national power transmission and distributions system, which covers all provinces and cites, and sells electricity to end-users countrywide. Our centralized model could be well applied in the northern Vietnamese power supply chain network (EVN, 2017). However, nowadays, the development of renewable energy and the increasing in power demand, there are many companies from private section who want to join in the Vietnamese power supply network as generators or suppliers. Our model could be applied to help the companies make their decisions: where to set up the power plants and how to charge the electricity price to optimize their own benefits.

TABLE 3. The results of optimal preventive maintenance in two cases.

Decisions	Cluster i		
	$i = 1$	$i = 2$	$i = 3$
PM_i($)-Centralized case	165.352	183.831	191.772
PM_i($)-Decentralized case	165.352	183.831	191.772

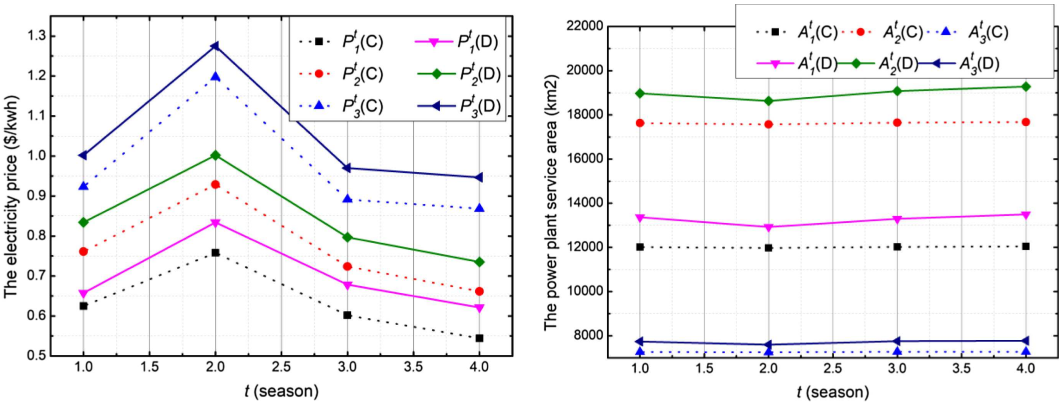

FIGURE 4. The optimal electricity price and power service area for two cases (C&D).

The optimal preventive maintenance for two cases is similar (Tab. 3). The results are reasonable because the optimal preventive maintenance is not affected by other decision variables (as shown in Eqs. (4.3)–(A.3)). In Figure 4, the dot lines illustrate the results of centralized model; otherwise, the results of the decentralized model are shown as the dash lines. The electricity price and power plants' service area vary over time in both cases in Figure 4. However, the suppliers charge a higher price in decentralized case than centralized case. In addition, companies tend to charge higher prices in the summer ($t = 2$) and lower prices in the spring ($t = 4$). Figure 4 also shows that Cluster 3 (Red River delta area) has the highest power demand to charge the highest electricity price and lower service area for each electrical distributor in both cases. We also calculated the number of power plant should be opened in each cluster (C_i/A_i) in Figure 5. The results show that the number of opening power plants is almost the same in both cases.

The demand response for differential price in both cases is shown in Figure 6. After the company applied the differential pricing strategy, the demand pattern is more stable. That means the end users may adjust their consumer behavior to reduce their electricity bills. This behavior could also help the power generators decrease the bad effects of peak-load demand and maintenance fee.

In this sub-section, we try to examine whether the differential price policy benefits the network or not. All parameters are kept as the same as the base scenario but the value of demand shift parameter does not vary over time. The optimal solutions for A_i^*, P_i^*, and PM_i^* for the case of non-differential pricing are shown in Table 4. Here, one can see that the total network profit is 4% lower in the case of applying the differential pricing policy (centralized case). This means that charging real-time prices to electricity customers could reduce the company profit, as the end users reduces their electricity bills. However, in the decentralized case, the differential pricing policy could increase the suppliers' profit and also increase the power generators' cost. An increase in the supplier's profit may cause of charging higher prices in peak hours and increasing demand in off-peak hours by lowering prices during that time. In practice, the differential pricing policy may benefit the suppliers but not the generators, the company want to join in the power supply chain network should consider that point.

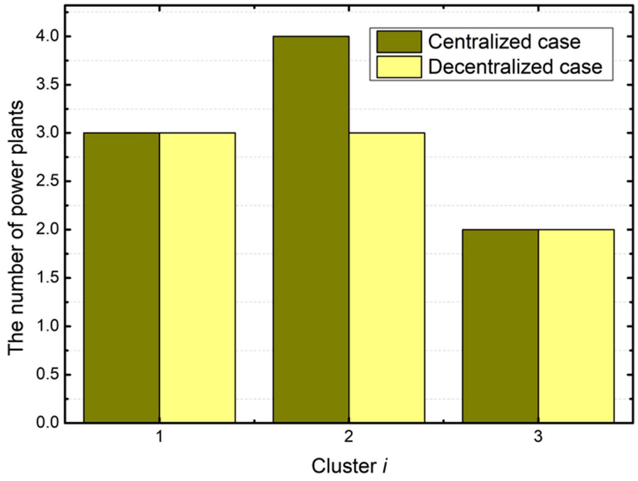

Figure 5. The number of power plants should be opened in both cases.

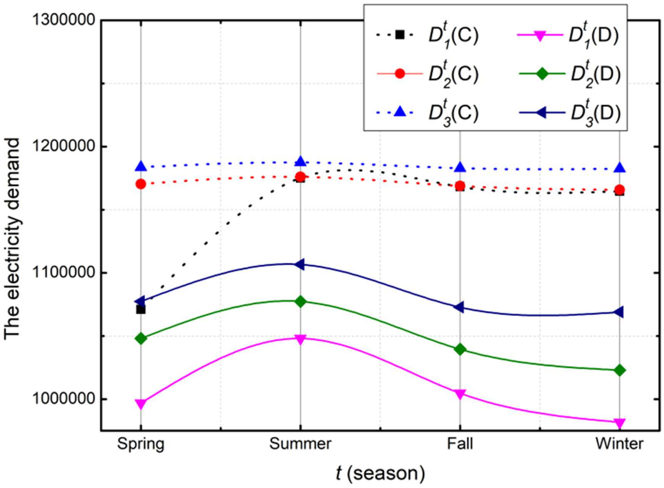

Figure 6. The demand response for differential price in both cases.

6. Conclusions

In this study, we proposed a continuous approximation approach to model the electric power supply chain network (EPSCN) design problems featuring differential pricing and preventive maintenance. Then, the nonlinear optimizations and enumerative search methods are applied to find the optimal solution quickly and easily. This method can be applied to cases involving large datasets.

We considered two supply chain models: centralized and decentralized case. Actually, the power industry is very distinctive and plays an important role in each economy. Almost countries over the world, this industry is operated and controlled by the governments. The centralized supply chain model and its advantages could be a good application in this case. However, the developments of technology and the increase of renewable energy make more outside companies who are pleased to join in the power industry. They could play as many different

Table 4. The results of non-differential price case example.

Decisions	Centralized case (cluster i)			Decentralized case (cluster i)		
	$i=1$	$i=2$	$i=3$	$i=1$	$i=2$	$i=3$
P_i($)	0.61	0.75	0.95	0.69	0.83	1.03
$A_i(\mathrm{km}^2)$	12017.6	17630.4	7264.4	12764.3	16238.9	6122.59
PM_i($)	165.351	183.831	191.772	165.352	183.831	191.772
Total profit	1.0110764×10^7			7.891012×10^6		
Total cost				7.90634×10^5		

roles like generators, suppliers, aggregators. The decentralized model is more efficient to consider and analyse the EPSCN in the future.

In addition, both differential and non-differential pricing cases were presented in our numerical analysis. We also examine if the differential price policy is applicable in two cases: centralized and decentralized model. The results show that the differential price policy could benefit the suppliers' profit but not the power generators. Additionally, EPSCNs are sensitive and vulnerable, and so while an increase in the preventive maintenance budget will increase the network cost, it will also reduce the failure rate. We determined the optimal preventive maintenance budget while maximizing the total network profit; the results also show that a company's preventive maintenance budget may increase as the power plants' service area increases.

Generally, the results show that our model and solution approach could be applicable in case of simple power distribution network where the power company is large enough to own and implement all the system himself, *i.e.* Vietnam Electricity company as an example. Nowadays, more and more companies want to join the power supply chain network in some countries, which makes the power network more complicated. Additionally, the participant of renewable energy and its generation can also complicate the network. Our model could be extended to consider more players and more sources of energy in the future works. Besides, the model is built based on the assumption that all consumers use smart meters. That means they can change the real demand through smart meters screening. Our results suggest that the smart meters could be used to instead of the old meters that benefits the suppliers' profit. However, for the power generators, the dynamic pricing (real time price) scheme design needs to be considered more factors such as investment, risk sharing, information, etc., in the future works.

Appendix A.

Proof of Proposition 4.1. As the above assumption, the lower bound of electricity price is C_g. For upper bound, from the electricity demand function, we have $\frac{\partial D_i^t(P_i^t)}{\partial P_i^t} = \alpha_i \beta_i^t P^{-1-\alpha_i} < 0 \forall P_i^t$ and $\alpha_i < 0$. This means the electricity demand function is a decreasing function in P_i^t. Since $P_i^t \geq 0$, the upper bound of the value of P_i^t is $P_i^t \leq (M_i/\beta_i^t)^{-1/\alpha_i}$. This completes the proof. □

Proof of Proposition 4.2. From total network profit function TC, the Hessian matrix of TC can be formulated as

$$H = \begin{bmatrix} \partial^2 \mathrm{TC}/\partial(\mathrm{PM}_i)^2 & \partial^2 \mathrm{TC}/\partial A_i^t \partial \mathrm{PM}_i \\ \partial^2 \mathrm{TC}/\partial \mathrm{PM}_i \partial A_i^t & \partial^2 \mathrm{TC}/\partial (A_i^t)^2 \end{bmatrix}. \tag{A.1}$$

To verify whether the matrix given in (4.10) is positive or not, we evaluate its principal minors:

$$D_1 = \frac{\gamma^2 b_i C_{\mathrm{CM}} C_i \mathrm{e}^{-\gamma \mathrm{PM}_i}}{A_i^t} > 0 \forall A_i^t, \mathrm{PM}_i \tag{A.2}$$

$$D_2 = \left(\begin{array}{l} \frac{C_i^2 \mathrm{e}^{-2\gamma \mathrm{PM}_i} \left(b_i^2 C_{\mathrm{CM}}^2 \gamma^2 + 2\gamma b_i C_{\mathrm{CM}} (1 + \gamma a_i C_{\mathrm{CM}} + \gamma f + \gamma \mathrm{PM}_i) - \mathrm{e}^{2\gamma \mathrm{PM}_i} \right)}{(A_i^t)^4} \\ + \frac{b_i C_{\mathrm{CM}} C_i C_t f_r \mathrm{e}^{-\gamma \mathrm{PM}_i} P^{-\alpha} (-MP^{\alpha} + \beta)^2 \gamma^2}{4(A_i^t)^{5/2}} \end{array} \right) \tag{A.3}$$

$$D_2 > 0 \forall A_i^t, \mathrm{PM}_i.$$

Thus, we know that the Hessian Matrix is positive definite since $D_1 < 0$ and $D_2 > 0$. The total network profit TC is concave function in A_i^t and PM_i. This completes the proof. □

Acknowledgements. This paper is supported in part by the Ministry of Science and Technology, Taiwan under grant MOST 110-2636-E-011-005 and 109-2636-E-011-006.

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