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GROUP RANKING OF TWO-STAGE PRODUCTION UNITS IN NETWORK DATA ENVELOPMENT ANALYSIS

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Abstract. Data envelopment analysis (DEA) is a useful mathematical tool for evaluating the performance of production units and ranking their relative efficiency. In many real-world applications, production units belong to several separate groups and also consist of several sub-units. In this paper, we introduce a new method of evaluating group efficiency of two-stage production systems. To this end, some new DEA models are introduced for evaluating and ranking groups of production systems based on the average and weakest group performance criteria. Some numerical examples, including an empirical application in the banking industry, are also provided for illustration.

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1. Introduction

The efficiency evaluation of decision-making units (DMUs) is a key task in management science, to better understand the performance of those units and plan to improve them in the future. Data envelopment analysis (DEA), introduced by Charnes *et al.* [8], is a data-oriented instrument for efficiency analysis, performance measurement, and management of homogenous production units. DMUs produce outputs with inputs consumption and are divided into efficient and inefficient ones based on their efficiency score. In the input-output space, the efficient units are placed on the efficient frontier. The efficient frontier consists of at least one efficient unit. Some important questions arise in this regard: (i) "among several efficient units, which one is the best?" and (ii) "How to assign a unique ranking position to each unit (including the efficient and the inefficient)?". Conventional DEA methods such as CCR and BCC cannot answer these questions, because of lacking discrimination in their ranking criteria.

Several methods such as ranking methods, network DEA (NDEA) models, and so on have been introduced to overcome this issue and to improve the discrimination power of basic DEA models. In the last three decades, several ranking methods have been proposed in the DEA context. Adler *et al.* [1] and Chen [10] thoroughly reviewed the ranking methods. Among all the ranking methods, the cross-efficiency evaluation is the most popular one, which was first proposed by Sexton *et al.* [32]. This method uses the idea of peer-evaluation

Keywords. Data envelopment analysis (DEA), group ranking, network group efficiency (NGE) Network DEA (NDEA).

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instead of self-evaluation information in the basic DEA models such as CCR and BCC. Despite many benefits and widespread applications of the cross-efficiency model, its usefulness is likely to decrease because of the non-uniqueness of optimal inputs/outputs weights [25]. To solve this problem, Sexton *et al.* [32] and Doyle and Green [15] proposed popular aggressive and benevolent formulations as secondary goals to select a solution between multiple optimal weights.

Anderson and Peterson [3] proposed another method for ranking efficient units called super-efficiency which first eliminates the DMU under assessment from the DMUs set, then computes its distance from the new efficient border. However, this elimination might cause the infeasibility of the associated DEA models [3], [26]. To overcome this issue, Memariani *et al.* [27] modified the non-radial model presented in Mehrabian *et al.* [26] and converted it to an input-output orientation model that made the linear programming model always feasible [5]. There are also some research studies in this regard, for instance, Banker [7] and Jahanshahloo [18] among others. Torgerson [36] considered the benchmarks ranking method for comparing efficient DMUs. In this method, the ranking of efficient DMU is performed based on their importance as a criterion for other DMUs. Khoshandam *et al.* [21], [22] also used a DEA-based method to estimate marginal optimal weights in evaluating the units. Another useful method for ranking units has recently been presented as a ratio-based efficiency analysis by Salo and Punkka [31]. They determined ranking intervals, dominance relations, and efficiency bounds. Ranking intervals show the best and worst rank positions for each production unit. Dominance relations shows what other DMUs a given DMU dominates in pairwise efficiency comparisons [29], [30]. The efficiency bounds show how efficient a given DMU can be relative to other DMUs or a subset of other DMUs. Aldamak *et al.* [2] conducted a comparison review of the above-mentioned methods.

In all of the above-mentioned methods, the manufacturing process is considered as a black-box, containing only inputs/outputs data, and the internal processes are totally ignored and the system is generally investigated. Whereas, in reality, there are production systems that are formed from some manufacturing units with several internal functions and components, so that the current models do not provide accurate efficiency scores for them. The idea of dividing manufacturing systems into several internal sub-processes was first introduced by Charnes *et al.* [9]. They noticed this phenomenon in studying army enlistment, which had two processes: the first one publicizes information through advertisements, and the second closes the contracts. The term "network DEA", was first used by Färe and Grosskopf [28]. In the network DEA, instead of input-output data, a network structure is used where intermediate products and exchanges are also considered in the evaluation. In other words, rigorous evaluation is made between subunits and the overall efficiency, depending on the efficiency of each process.

There are extensive and growing studies that have been explored NDEA models and applications in recent DEA literature. Zhu [40] proposed a model to calculate the overall efficiency of units by using the arithmetic mean of divisional efficiencies. Lewis *et al.* [24] analyze the performance of organizations with complex structures based on network data envelopment analysis, and use the model for the Major Baseball League, explaining its benefits, including recognizing the inefficiencies lost in standard models. Cook *et al.* [13] showed that all approaches to two-stage network data envelopment analysis can be categorized using collaborative game concepts or Stackelberg (leader-follower). Chen *et al.* [11] presented the overall efficiency as a weighted average of the efficiency of the production stages. Kao [19] introduced series and parallel structures to measure the efficiency of production systems and those of the processes at the same time. It is shown that in a series structure, the overall efficiency score is presented as the product of the efficiencies of the sub-processes and in a parallel structure system, inefficiency slacks are the sum of the inefficiency slacks of all sub-processes. Tone *et al.* [35] proposed a slack-based model (SBM) in network systems that can be used to evaluate sectoral efficiencies along with the overall efficiency of DMUs. Despotis *et al.* [14] suggested network DEA approach for series multi-stage processes. Guo *et al.* proposed [17] additive efficiency decomposition where the overall efficiency is defined as a weighted average of stage efficiencies and the weights are used to reflect relative importance of individual stages.

Sotiros *et al.* [33] introduced dominance at the divisional efficiency level in two-stage network DEA. They proposed the dominance property as a minimum requirement that the two-stage NDEA methods should satisfy, regardless of the optimality criterion used to evaluate the overall and the divisional efficiencies of a system.

Guo *et al.* [17] investigated the impact of weight changes on the sub-process efficiency and overall efficiency. Azizi and Kazemi Matin [6] proposed a new approach to rank two-stage production systems based on the performance of each stage. Esmaeilzadeh *et al.* [16] employed novel multi-period network DEA models developed for performance evaluation of overall and specific period efficiencies with parallel and series internal structures in the sub-processes for each period. Tavana *et al.* [34] introduced a two-stage network model for measuring performance in three-level supply chains.

All of the mentioned methods have a common feature which could be called individual evaluation of DMUs. In many real cases, individual DMUs are members of several distinct groups. For example, consider the sales units of a particular brand. These units operate independently but under the supervision of single management to achieve the group's common targets. In this condition, the management team more likely emphasizes the overall performance of the whole group instead of the performance of a specific member within the group.

To compare one group of DMUs with another, group-efficiency evaluation is needed. Ang *et al.* [4] looked at this evaluation from two perspectives: group efficiency evaluation based on average performance and weakest performance. When the average performance of members of a group is the criterion for evaluation, effective members have a positive influence and ineffective members damage the overall performance of the group. In some cases, negative effects are offset by positive effects. According to this idea, the overall performance of the group can be based on the average performance of the members. But when the criterion of evaluation is the weakest performance of the members of a group, according to the "cask principle" in management, we measure the overall performance of a group by evaluating the performance of the worst-performing member of the group. In other words, if a member of the group renders a poor service or causes a major scandal, it will damage the brand's reputation as well as the reputation of the other members of the group. In this case, customers are reluctant to use the services of that brand in any branch.

In this paper, we present a novel two-stage network group efficiency evaluation method. Because of its broader insight, network DEA always yields more accurate results compared with the conventional black-box case. Our goal is to determine the true ranking of two-stage network production groups. Some new network DEA models are presented for efficiency evaluation of the group of production units in both average and weakest performance criteria. For illustration purposes, we considered 20 branches of the Mellat bank in Iran [23], divided into four groups. Using the provided models, we calculated the network group efficiency score (NGE) of each group. The data analysis shows that the results obtained from the new method are more reliable than the results by the conventional DEA setting. However, determining grouping criteria is not the only objective of this study, and the proposed model evaluates pre-grouped units and ranking as well.

The rest of this paper is organized as follows. Section 2 provides a brief introduction of the group efficiency evaluation method and its various approaches. The newly proposed method is explained in Section 3. In Section 4, some illustrative examples are presented, and the efficiency scores obtained from the various methods of group efficiency evaluation are reported and analyzed. The last section includes conclusions and possible future research.

2. Group efficiency evaluation

In most DEA models, the focus is on ranking individual DMUs. In other words, in these methods, the performance of the individual DMU is important, not the group to which that unit belongs to. Now, if somebody decides to compare one group of production units with another one, while each group's unit is trying to improve the group status, a new method should be applied to evaluate the groups and to calculate their efficiency scores, separately.

Cook *et al.* [12] were the first, who mentioned the concept of group efficiency evaluation in the DEA framework. Later on, Ang *et al.* [4] explored this issue comprehensively and presented models for calculating group efficiency. They considered two approaches for evaluating group efficiency: one based on average and another on the weakest performance criteria. In this section, we examine both approaches.

Suppose, n DMUs are organized into K groups with D_k members for each group k, $(k = 1, \ldots, K)$, and each DMU_{d_k},$(d_k = 1, \ldots, D_k)$, has m inputs $\mathbf{x}_{d_k} = (x_{id_k})$ and s outputs $\mathbf{y}_{d_k} = (y_{rd_k})$ [4]. For each group, t, $(t = 1, \ldots, K)$ group efficiency score based on average performance is obtained by solving the optimization model (2.1):

$$
\begin{aligned}
E_t^A = \max & \frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \\
\text{s.t. } & \frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \leq 1, \qquad k = 1, \ldots, K\,, d_k = 1, \ldots, D_k, \\
& \frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \leq 1, \\
& v_{it}, u_{rt} \geq 0, \qquad i = 1, \ldots, m,\ r = 1, \ldots, s.
\end{aligned}
\tag{2.1}
$$

Here, u_{rt} and v_{it} are weights for the rth output and the ith inputs of DMUs in group t, respectively. Note that $\frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \leq 1$ is redundant and this constraint can be removed in computations. To avoid non-linearity, this the model can be transformed into the following linear form:

$$
\begin{aligned}
E_t^A = \max & \sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t} \\
\text{s.t. } & \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \leq 0, \qquad k = 1, \ldots, K\,, d_k = 1, \ldots, D_k, \\
& \sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t} = 1, \\
& v_{it}, u_{rt} \geq 0, \qquad i = 1, \ldots, m,\ r = 1, \ldots, s.
\end{aligned}
\tag{2.2}
$$

The optimal solution for model (2.2) provides the average group efficiency score for group t as follows:

$$
E_t^{A*} = \sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt}^{*} y_{rd_t}. \tag{2.3}
$$

When the group efficiency reaches the optimal level, the efficiency values of each DMU in group t could be calculated as follows:

$$
e_{d_t}^{A*} = \frac{\sum_{r=1}^{s} u_{rt}^{*} y_{rd_t}}{\sum_{i=1}^{m} v_{it}^{*} x_{id_t}},\ d_t = 1, \ldots, D_t. \tag{2.4}
$$

Ang *et al.* [4] used the cask principle to evaluate group efficiency based on the weakest performance. The Cask principle illustrates that "no matter how high the cask is, the shortest board of the cask and not the longest one, determines how much water the cask can hold". According to this idea, group efficiency score based on the weakest performance is determined by the group members with the worst performance in operations. In other words, the overall efficiency score of the group is based on the weakest performance of the worst member of the group. If the efficiency of each unit in a group p $(p = 1, \ldots, K)$ be denoted by e_{d_p} $(d_p = 1, \ldots, D_p)$, then efficiency score based on the weakest performance, E_p^W, is as follows:

$$
E_p^W = \min_{d_p} e_{d_p}. \tag{2.5}
$$

In the weakest performance scenario of group efficiency evaluation, the efficiency ratio of the weakest member of the group is maximized in a max-min approach. This approach is frequently used to maximize the minimum objective for all potentially feasible weights in DEA methods; see for example Wu *et al.* [39] and [38]. So, for each group t, $(t = 1, \ldots, K)$, group efficiency score based on the weakest performance is obtained by solving the following max-min model:

$$\begin{aligned} E_t^W &= \max \min_{d_t} \frac{\sum_{r=1}^s u_{rt} y_{rd_t}}{\sum_{i=1}^m v_{it} x_{id_t}} \\ &\text{s.t. } \frac{\sum_{r=1}^s u_{rt} y_{rd_k}}{\sum_{i=1}^m v_{it} x_{id_k}} \le 1, \qquad k = 1, \ldots, K, d_k = 1, \ldots, D_k, \\ &\quad v_{it}, u_{rt} \ge 0, \qquad i = 1, \ldots, m,\ r = 1, \ldots, s. \end{aligned} \tag{2.6}$$

The optimal solution of the model (2.6) and the best group efficiency for group t is:

$$E_t^{W*} = \min_{d_t} \frac{\sum_{r=1}^s u_{rt}^* y_{rd_t}}{\sum_{i=1}^m v_{it}^* x_{id_t}}. \tag{2.7}$$

The efficiencies of units in group t are calculated as follows:

$$e_{d_t}^{W*} = \frac{\sum_{r=1}^s u_{rt}^* y_{rd_t}}{\sum_{i=1}^m v_{it}^* x_{id_t}},\ d_t = 1, \ldots, D_t. \tag{2.8}$$

Based on the unit invariant property of basic CCR model, and to avoid dissimilarity and dispersions of weights, we can compute the exact optimal values of u_{rt}^* and v_{it}^* by adding the new constraint $\sum_{r=1}^s u_r = 1$ into the above model

$$\begin{aligned} E_t^W &= \max \min_{d_t} \frac{\sum_{r=1}^s u_{rt} y_{rd_t}}{\sum_{i=1}^m v_{it} x_{id_t}} \\ &\text{s.t. } \frac{\sum_{r=1}^s u_{rt} y_{rd_k}}{\sum_{i=1}^m v_{it} x_{id_k}} \le 1, \qquad k = 1, \ldots, K, d_k = 1, \ldots, D_k, \\ &\quad \sum_{r=1}^s u_{rt} = 1, \\ &\quad v_{it}, u_{rt} \ge 0, \qquad i = 1, \ldots, m,\ r = 1, \ldots, s. \end{aligned} \tag{2.9}$$

Model (2.9) is nonlinear and it is difficult to solve. To transform it into a linear equivalent form, the following auxiliary variable δ_t for group t is used:

$$\delta_t = \min_{d_t} \frac{\mathbf{u}_t \mathbf{y}_{d_t}}{\mathbf{v}_t \mathbf{x}_{d_t}}. \tag{2.10}$$

As the result, the model (2.6) could be stated as follows:

$$\begin{aligned} E_t^W &= \max \delta_t \\ &\text{s.t. } \sum_{r=1}^s u_{rt} y_{rd_t} - \delta_t \sum_{i=1}^m v_{it} x_{id_t} \ge 0, \qquad d_t = 1, \ldots, D_t, \\ &\quad \sum_{r=1}^s u_{rt} y_{rd_k} - \sum_{i=1}^m v_{it} x_{id_k} \le 0, \qquad k = 1, \ldots, K, d_k = 1, \ldots, D_k, \\ &\quad \sum_{r=1}^s u_{rt} = 1, \\ &\quad v_{it}, u_{rt} \ge 0, \qquad i = 1, \ldots, m,\ r = 1, \ldots, s. \end{aligned} \tag{2.11}$$

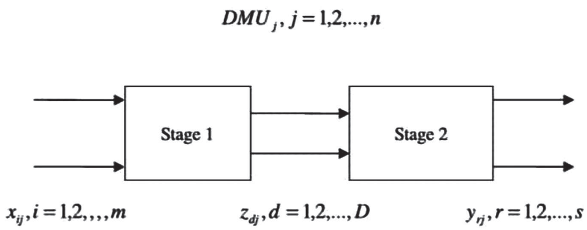

FIGURE 1. Two-stage Process (Series Structure).

Treating $0 \leq \delta_t \leq 1$ as a parameter, model (2.11) becomes linear and could be solved by provided parametric optimization methods like bisection search for calculating optimal inputs/outputs weights. see [12] and [37] for more details.

Both ideas of average and the weakest performance will be extended for the case of two-stage production systems in the Section 3.

3. Network group efficiency evaluation

Now we introduce new network DEA models for group efficiency evaluation. This method is based on a two-stage network model that assigns a reliable ranking to each group by considering the internal structure of the group members. To do so, we first briefly present a two-stage network method with series structure, then a newly proposed method from the two perspectives of average performance and weakest performance is presented.

3.1. Two-stage network DEA

The main contribution of this method is decomposing the overall efficiency score of a unit to the product of the efficiency scores of its stages. It is assumed that returns to scale are constant in the production technology. Suppose each DMU_j, $(j = 1, \ldots, n)$ has m inputs x_{ij}, $(i = 1, \ldots, m)$ and q outputs z_{pj}, $(p = 1, \ldots, q)$ in the first stage. These q outputs then will become the inputs to the second stage, hence behaving as intermediate products. The second stage outputs are y_{rj}, $(r = 1, \ldots, s)$, (See Figure 1). For DMU_k, the efficiency score of the first stage is denoted as θ_k^1 and the second stage as θ_k^2. We also denote the overall system efficiency score with θ_k^o and we use the input-oriented CCR model presented by Charnes *et al.* [8] to compute θ_k^1, θ_k^2 and θ_k^o. Model (3.1) determines the overall relative efficiency of DMU_k, if $\theta_k^o = 1$, DMU_k is efficient and otherwise $(\theta_k^o < 1)$ inefficient, [20].

$$
\begin{aligned}
\theta_k^o = \max & \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \\
\text{s.t.} \ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad && j = 1, \ldots, n, \\
& u_r, v_i \geq 0, && r = 1, \ldots, s, \ i = 1, \ldots, m.
\end{aligned} \tag{3.1}
$$

Models (3.2) and (3.3) compute the efficiency scores of the first and second stages, respectively.

$$
\begin{aligned}
\theta_k^1 = \max & \frac{\sum_{p=1}^{q} w_p z_{pk}}{\sum_{i=1}^{m} v_i x_{ik}} \\
\text{s.t.} \ & \frac{\sum_{p=1}^{q} w_p z_{pj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad && j = 1, \ldots, n, \\
& v_i, w_p \geq 0, && i = 1, \ldots, m, \ p = 1, \ldots, q,
\end{aligned} \tag{3.2}
$$

$$\begin{aligned}
\theta_k^2 = \max & \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{p=1}^{q} w_p z_{pk}} \\
\text{s.t.} & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{p=1}^{q} w_p z_{pj}} \leq 1, \qquad j = 1, \ldots, n, \\
& u_r, w_p \geq 0, \qquad r = 1, \ldots, s,\ p = 1, \ldots, q.
\end{aligned} \tag{3.3}$$

These models are similar to model (3.1) and the overall system efficiency and stages efficiencies are calculated separately. Suppose u_r^*, v_i^* and w_p^* are optimal weights for DMU_k. Therefore, the optimal values of the objective function of models (3.1), (3.2) and (3.3) are as follows:

$$\begin{aligned}
\theta_k^o &= \frac{\sum_{r=1}^{s} u_r^* y_{rk}}{\sum_{i=1}^{m} v_i^* x_{ik}}, \\
\theta_k^1 &= \frac{\sum_{p=1}^{q} w_p^* z_{pk}}{\sum_{i=1}^{m} v_i^* x_{ik}}, \\
\theta_k^2 &= \frac{\sum_{r=1}^{s} u_r^* y_{rk}}{\sum_{p=1}^{q} w_p^* z_{pk}}.
\end{aligned} \tag{3.4}$$

Obviously, the overall efficiency is the resultant of the efficiency of the stages [20]:

$$\theta_k^o = \theta_k^1 \times \theta_k^2. \tag{3.5}$$

Accordingly, by adding the constraints of models (3.2), (3.3) to (3.1) and also considering the series relationship between the two stages, the following model is achieved to calculate the overall efficiency:

$$\begin{aligned}
\theta_k^o = \max & \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \\
\text{s.t.} & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad j = 1, \ldots, n, \\
& \frac{\sum_{p=1}^{q} w_p z_{pj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad j = 1, \ldots, n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{p=1}^{q} w_p z_{pj}} \leq 1, \qquad j = 1, \ldots, n, \\
& u_r, v_i, w_p \geq 0, \qquad r = 1, \ldots, s,\ i = 1, \ldots, m,\ p = 1, \ldots, q.
\end{aligned} \tag{3.6}$$

It should be noted that, in model (3.6), the corresponding weights of the intermediate products z_{pj} must be the same in both stages. Model (3.6) can also be written in the following linear form:

$$\begin{aligned}
\theta_k^o = \max & \sum_{r=1}^{s} u_r y_{rk} \\
\text{s.t.} & \sum_{i=1}^{m} v_i x_{ik} = 1, \\
& \sum_{p=1}^{q} w_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad j = 1, \ldots, n, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{p=1}^{q} w_p z_{pj} \leq 0, \qquad j = 1, \ldots, n, \\
& u_r, v_i, w_p \geq 0, \qquad r = 1, \ldots, s,\ i = 1, \ldots, m,\ p = 1, \ldots, q.
\end{aligned} \tag{3.7}$$

3.2. Network group efficiency based on average performance

Suppose a brand competes with its rival brands in the market. Subordinate units do not probably function at the same level. Some units have positive or negative effects on group performance. The average performance of group members is a criterion that a manager can use to evaluate group performance. Now, if a manager, with a more meticulous look, makes the internal structure of production a criterion for evaluation, then it is necessary to define the group performance evaluation model using the network structure. These new models are suggested as follows:

$$\begin{aligned}
\theta_t^{oA} = \max & \frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \\
\text{s.t.} & \frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \leq 1, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k, \\
& \frac{\sum_{p=1}^{q} w_{pt} z_{pd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \leq 1, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k, \\
& \frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{p=1}^{q} w_{pt} z_{pd_k}} \leq 1, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k, \\
& u_{rt}, v_{it}, w_{pt} \geq 0, \qquad r=1,\ldots,s,\ i=1,\ldots,m,\ p=1,\ldots,q,
\end{aligned} \tag{3.8}$$

where, n DMUs are organized into K groups with D_k members for each group k, $(k = 1,\ldots,K)$, and each DMU_{d_k}, $(d_k = 1,\ldots,D_k)$, has m inputs $\mathbf{x}_{d_k} = (x_{id_k})$ and q intermediate products $\mathbf{z}_{d_k} = (z_{pd_k})$ and s outputs $\mathbf{y}_{d_k} = (y_{rd_k})$.

The optimal solution of the model (3.8) is interpreted as the overall group efficiency score based on average performance for group t. In this model, the first constraint is redundant and can be removed in computations. The group efficiency scores of the first and second stages are then obtained by solving the following models, respectively.

$$\begin{aligned}
\theta_t^{1A} = \max & \frac{\sum_{p=1}^{q}\sum_{d_t=1}^{D_t} w_{pt} z_{pd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \\
\text{s.t.} & \frac{\sum_{p=1}^{q} w_{pt} z_{pd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \leq 1, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k, \\
& v_{it}, w_{pt} \geq 0, \qquad i=1,\ldots,m,\ p=1,\ldots,q,
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\theta_t^{2A} = \max & \frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}}{\sum_{p=1}^{q}\sum_{d_t=1}^{D_t} w_{pt} z_{pd_t}} \\
\text{s.t.} & \frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{p=1}^{q} w_{pt} z_{pd_k}} \leq 1, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k, \\
& w_{pt}, u_{rt} \geq 0, \qquad p=1,\ldots,q,\ r=1,\ldots,s.
\end{aligned} \tag{3.10}$$

Note that the efficiencies of the whole process and the two sub-processes are calculated independently. Consider u_{rt}^*,v_{it}^* and w_{pt}^* as the optimal multipliers of the group t, chosen to compute its overall group efficiency and the efficiency of the two sub-processes. So we have:

$$\begin{aligned}\theta_t^{oA} &= \max \frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt}^* y_{rd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it}^* x_{id_t}},\\ \theta_t^{1A} &= \max \frac{\sum_{p=1}^{q}\sum_{d_t=1}^{D_t} w_{pt}^* z_{pd_t}}{\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it}^* x_{id_t}},\\ \theta_t^{2A} &= \max \frac{\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt}^* y_{rd_t}}{\sum_{p=1}^{q}\sum_{d_t=1}^{D_t} w_{pt}^* z_{pd_t}}.\end{aligned} \tag{3.11}$$

Obviously, the overall group efficiency is the product of the group efficiencies of its stages:

$$\theta_t^{oA} = \theta_t^{1A} \times \theta_t^{2A}. \tag{3.12}$$

Model (3.8) is nonlinear and can be transformed into the following linear program:

$$\begin{aligned}\theta_t^{oA} = \max &\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}\\ \text{s.t.} &\sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \le 0, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k,\\ &\sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \le 0, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k,\\ &\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t} = 1,\\ &u_{rt}, v_{it}, w_{pt} \ge 0, \qquad r=1,\ldots,s,\ i=1,\ldots,m,\ p=1,\ldots,q.\end{aligned} \tag{3.13}$$

Also, models (3.9) and (3.10) can be converted into the following linear forms:

$$\begin{aligned}\theta_t^{1A} = \max &\sum_{p=1}^{q}\sum_{d_t=1}^{D_t} w_{pt} z_{pd_t}\\ \text{s.t.} &\sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \le 0, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k,\\ &\sum_{i=1}^{m}\sum_{d_t=1}^{D_t} v_{it} x_{id_t} = 1,\\ &v_{it}, w_{pt} \ge 0, \qquad i=1,\ldots,m,\ p=1,\ldots,q,\end{aligned} \tag{3.14}$$

$$\begin{aligned}\theta_t^{2A} = \max &\sum_{r=1}^{s}\sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}\\ \text{s.t.} &\sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \le 0, \qquad k=1,\ldots,K,\ d_k=1,\ldots,D_k,\\ &\sum_{p=1}^{q}\sum_{d_t=1}^{D_t} w_{pt} z_{pd_t} = 1,\\ &w_{pt}, u_{rt} \ge 0, \qquad p=1,\ldots,q,\ r=1,\ldots,s.\end{aligned} \tag{3.15}$$

In the new approach, based on the method proposed by Ang *et al.* [4], we have presented a network group efficiency evaluation model. In the case of two-stage production, we have achieved the group efficiency score of the first and second stages, and the overall group efficiency score as well.

3.3. Network group efficiency based on weakest performance

In the previous section, based on the average performance of the group members, we obtained the network group efficiency score to rank the groups. In this section, we consider the performance of the weakest member of the groups as a criterion to obtain the groups' efficiency scores.

For the group under evaluation t, $(t = 1, \ldots, K)$, simila r to the black-box case in model 2.11, we suggest solving the following optimization model to obtain group efficiency score of the first stage based on the weakest performance:

$$\begin{aligned}
\theta_t^{1W} = &\max \delta_t^1 \\
&\text{s.t.} \sum_{p=1}^{q} w_{pt} z_{pd_t} - \delta_t^1 \sum_{i=1}^{m} v_{it} x_{id_t} \geq 0, && d_t = 1, \ldots, D_t, \\
&\quad \sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \leq 0, && k = 1, \ldots, K,\ d_k = 1, \ldots, D_k, \\
&\quad \sum_{p=1}^{q} w_{pt} = 1, \\
&\quad v_{it}, w_{pt} \geq 0, && i = 1, \ldots, m,\ p = 1, \ldots, q,
\end{aligned} \tag{3.16}$$

while $\delta_t^1 = \min_{d_t} \frac{\mathbf{w}_t \mathbf{z}_{d_t}}{\mathbf{v}_t \mathbf{x}_{d_t}}$ and δ_t^1 are auxiliary variables for group t in the first stage.

In the same way, for the second stage, group efficiency score is calculated by solving the following model:

$$\begin{aligned}
\theta_t^{2W} = &\max \delta_t^2 \\
&\text{s.t.} \sum_{r=1}^{s} u_{rt} y_{rd_t} - \delta_t^2 \sum_{p=1}^{q} w_{pt} z_{pd_t} \geq 0, && d_t = 1, \ldots, D_t, \\
&\quad \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \leq 0, && k = 1, \ldots, K,\ d_k = 1, \ldots, D_k, \\
&\quad \sum_{r=1}^{s} u_{rt} = 1, \\
&\quad w_{pt}, u_{rt} \geq 0, && p = 1, \ldots, q,\ r = 1, \ldots, s.
\end{aligned} \tag{3.17}$$

Eventually, the overall group efficiency score is the optimal solution of the model (3.18).

$$
\begin{aligned}
\theta_t^{oW} &= \max \delta_t \\
\text{s.t.} & \sum_{r=1}^{s} u_{rt} y_{rd_t} - \delta_t \sum_{i=1}^{m} v_{it} x_{id_t} \geq 0, \qquad d_t = 1, \ldots, D_t, \\
& \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{p=1}^{q} w_{pt} z_{pd_k} \leq 0, \qquad k = 1, \ldots, K,\ d_k = 1, \ldots, D_k, \\
& \sum_{p=1}^{q} w_{pt} z_{pd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \leq 0, \qquad k = 1, \ldots, K,\ d_k = 1, \ldots, D_k, \\
& \sum_{r=1}^{s} u_{rt} = 1, \\
& u_{rt}, v_{it}, w_{pt} \geq 0, \qquad r = 1, \ldots, s,\ i = 1, \ldots, m,\ p = 1, \ldots, q.
\end{aligned} \tag{3.18}
$$

For further explanation and more thorough comparison, we will present some examples in the next section.

4. Illustrative examples

In the previous sections, we presented some new models for group ranking of two-stage network production systems. In this section, we apply our provided approach for illustration and comparison purposes. First, we present a simple example with unrealistic data to compare the results of the black-box and the new models in ranking groups. Then we apply our provided approach in the group ranking of 20 branches of the Mellat bank in Iran.

4.1. Example 1.

Assume 9 DMUs in three groups labeled as A, B, and C. Each group has three members; each member receives two inputs and produces two outputs. We solved the related models with hypothetical data. The results are presented and then compared.

4.1.1. Group efficiency

To estimate the group efficiency score of the groups under evaluation, we solved the models proposed by Ang *et al.* [4] with both the average performance and the weakest performance approach and then compared the results that are presented in Table 1. According to Table 1, the highest AGE score is 0.89 in the group C, and this group is ranked the best among all groups based on the average performance. The lowest AGE score is 0.55 in the group A, so this is the worst group. Therefore, if the average performance is the evaluation criterion, then the group C is the best, and the group A the worst. While based on the weakest performance, the group B has the highest WGE score and gets the highest rank. Still, the group A has the worst situation in both evaluations. As can be seen, the two approaches offer different results. Decision-makers can choose one of these methods to determine the ranking of the groups. In the seventh column of the Table, CCR efficiency scores of individual DMUs in their groups are presented. The overall CCR efficiency score of individual DMUs is presented in the eighth column.

4.1.2. Network group efficiency: average scenario

In this section, we use the network group efficiency model based on average performance to evaluate and rank the three groups. The results of solving the relevant models are reported in Table 2.

As seen in Table 2, the results of group efficiencies and network group efficiencies based on average performance are similar. According to the results, the proposed model identifies the group C as the best group with a score of $\theta_C^{oA} = 0.73$, and the group A the worst with a score of $\theta_A^{oA} = 0.43$. Scores of stage 1 and stage 2 are presented in the 10th and 11th columns, respectively. Group D is in a good position in both the first and second stages,

Table 1. Group efficiency score based on average (AGE) and weakest performance (WGE).

Group	DMU	X_1	X_2	Y_1	Y_2	$IE(ig)$	$IE(og)$	AGE	WGE
A	1	2	5	3	1	0.38	0.38	0.55	0.38
	2	2	3	4	6	0.67	0.65		
	3	1	2	4	6	1	1		
B	1	3	5	9	8	0.96	0.87	0.85	0.75
	2	2	4	6	9	1	0.75		
	3	1	2	4	3	1	1		
C	1	2	2	3	2	0.64	0.64	0.89	0.64
	2	1	2	4	3	1	1		
	3	4	2	6	7	1	1		

Table 2. Network group efficiency score based on average performance.

Group	DMU	X_1	X_2	Z_1	Z_2	Y_1	Y_2	AGE	θ_t^{1A}	θ_t^{2A}	θ_t^{oA}
A	1	2	5	3	4	3	1	0.55	0.65	0.72	0.43
	2	2	3	3	5	4	6				
	3	1	2	2	4	4	6				
B	1	3	5	5	6	9	8	0.85	0.71	0.92	0.64
	2	2	4	4	7	6	9				
	3	1	2	2	3	4	3				
C	1	2	2	2	5	3	2	0.89	1	0.73	0.73
	2	1	2	3	4	4	3				
	3	4	2	5	3	6	7				

Table 3. Network group efficiency score based on weakest performance.

Group	DMU	X_1	X_2	Z_1	Z_2	Y_1	Y_2	WGE	θ_t^{1W}	θ_t^{2W}	θ_t^{oW}
A	1	2	5	3	4	3	1	0.38	0.5	0.52	0.26
	2	2	3	3	5	4	6				
	3	1	2	2	4	4	6				
B	1	3	5	5	6	9	8	0.75	0.63	0.83	0.63
	2	2	4	4	7	6	9				
	3	1	2	2	3	4	3				
C	1	2	2	2	5	3	2	0.64	1	0.51	0.51
	2	1	2	3	4	4	3				
	3	4	2	5	3	6	7				

while Group A is not in a good position in both stages and the efficiency of the stages must be improved to increase its ranking.

4.1.3. Network group efficiency: weakest scenario

Assuming the same data and groups, by solving the network group models based on the weakest performance, the following results are obtained.

Given the weakest member of the group is the criterion for evaluation, we solved the relevant models. The provided results are presented in Table 3. Contrary to what we saw in Table 2, the weakest approach selects the

TABLE 4. Descriptive statistics data of 20 branches of the Mellat bank (million dollars).

	X_1	X_2	Z_1	Z_2	Y_1	Y_2
Max	1259	7867	0.0076	5 15 388	4 09 531	11 613
Min	12	457	0.000002	49 697	28 596	305
Average	328	2955	0.002085	2 30 282	1 85 643	3366
S.D.	311.92	2270.15	0.002243	163642.12	140669.68	3114.14

group B with a score of $\theta_B^{oW} = 0.63$ as the best group. Similar to the previous method, group A with a score of $\theta_A^{oW} = 0.26$ is identified as the worst group. As we can see, this group is in a bad situation in both stages, and it needs to be improved. Notably, the results of group efficiencies and network group efficiencies based on the weakest performance scenario are highly correlated.

4.2. Empirical application

In the present era of economic systems, the role of financial systems, money and capital markets, and consequently financial and credit institutions, headed by banks, is of great importance. Facing increasing competition, financial institutions should promote their competitiveness by implementing performance evaluation, and analyzing the operating efficiency of individual branches. Researchers have focused on evaluating and ranking the performance of banks from the past to the present. Most studies have paid less attention to the internal structure of banking operations. In this example, we selected 20 branches of Mellat Bank in Iran for our evaluation. This bank is an Iranian financial service and banking company with 1448 branches throughout Iran. These 20 branches have been selected from different areas of Tehran and we have considered each area as a group. We implemented the proposed model with both approaches (the weakest and average performance) and compared these areas (groups) with each other and assigned a rank to each area. Personnel costs and paid interests enter the model as inputs, raised funds exit as an intermediate variable (output in the first stage and input in the second stage), and loans and common income are produced as the final optimum outputs.

We divided these 20 branches into four groups and applied the newly proposed models of group efficiency evaluation on the data set. The inputs, intermediate products, and outputs are as follows:

Inputs:

- x_1 (personnel costs)
- x_2 (paid interests)

Intermediate products:

- z_1 (raised funds)
- z_2 (raised funds related to the foreign currency transactions)

Outputs:

- y_1 (loans)
- y_2 (common incomes)

Table 4 provides the descriptive statistics for the data.

By solving group efficiency evaluation models, we evaluated the groups and determined their ranks. The results are presented in Table 5. The obtained results by all methods are quite stable, but subtle differences can also be detected. By applying model (3.13), the network group efficiency scores are calculated based on the average performance (θ_t^{oA}) of the groups. Also, (θ_t^{1A}) and (θ_t^{2A}) are network group efficiency scores of stage 1 and stage 2 based on average performance, respectively, obtained by solving models (3.14) and (3.15). Group efficiency score based on average performance is obtained from solving model (2.2). Group efficiency scores based on the weakest performance are also obtained by solving models (3.16), (3.17), and (3.18).

TABLE 5. Network group efficiency scores for the empirical application.

Group	DMUs	EFF	AGE	WGE	θ_t^{1A}	θ_t^{2A}	θ_t^{oA}	θ_t^{1W}	θ_t^{2W}	θ_t^{oW}
A	1	0.96	0.51(4)	0.36(4)	0.5	0.82	0.41(2)	0.25	0.5	0.22(2)
	2	0.36								
	3	0.63								
	4	0.9								
	5	0.67								
B	6	0.59	0.75(1)	0.59(2)	0.38	0.92	0.33(4)	0.13	0.58	0.13(4)
	7	1								
	8	1								
	9	0.9								
	10	0.62								
C	11	0.89	0.68(3)	0.87(1)	0.52	0.66	0.34(3)	0.23	0.57	0.16(3)
	12	1								
	13	0.9								
	14	1								
	15	0.87								
D	16	1	0.7(2)	0.43(3)	0.65	0.85	0.55(1)	0.33	0.77	0.31(1)
	17	1								
	18	0.43								
	19	0.58								
	20	0.95								

4.2.1. Results analysis (average scenario)

In terms of group efficiency evaluation, group B with a score of AGE = 0.75 is the best group, and group A with a score of AGE = 0.51 is the worst one. While from a network perspective, group B with a score of $\theta_B^{oA} = 0.33$ is the worst group. After considering the internal structure of financial processes, the group with the highest efficiency score is ranked the worst group among the groups under evaluation. The reason for this ranking is the inefficiency in the first stage. In other words, the group's raised funds is not in a favorable position compared to personnel costs. To improve this situation, Units 6 and 9 must compensate for the weakness in the fund raising system that occurred in the first stage of the process. Group D with a score of $\theta_D^{oA} = 0.55$ is the best group among all groups. As the results show, this group has an acceptable performance in both the first and second stages.

4.2.2. Results analysis (weakest scenario)

Regarding the weakest performance point of view, group C with a score of WGE = 0.87 is the best group among the others, and group A with a score of WGE = 0.36 is the worst group. While from the network point of view, group D with a score of $\theta_D^{oW} = 0.31$ is the best group, and group B with a score of $\theta_B^{oW} = 0.13$ is the worst group. As we can see from Table 5, group B, compared to other groups in the first stage of raised funds, is not in a good position compared to personnel costs. In order to improve the situation, this shortcoming must be eliminated. As we can see, the new models offer different results from the previous models because they also consider intermediate processes, and the results of the new models are definitely more reliable. Identifying weaknesses and suggestions for improvement can also help managers make future decisions.

5. CONCLUSION

In this study, we developed a new network group efficiency evaluation method for the case of two-stage production systems. To evaluate network group efficiency, we considered two different strategies: the average and the weakest performance. The average performance considers the network group efficiency as the average of its members' performances. The weakest performance uses the worst member's efficiency to indicate the network group efficiency based on the cask principle. We presented some new network DEA models for both strategies. The results are then compared in some numerical examples, including an application in the performance evaluation of 20 branches of the Mellat bank in Iran. The provided analysis shows the usefulness and applicability of the proposed methods in providing more powerful discrimination tools in the performance evaluation of group production processes. Extending the proposed approach for the more general case of network production processes such as series-parallel cases and also for the centralized modeling in which the stages work jointly to achieve maximum efficiency of profit in terms of aggregate efficiency, can be considered as interesting challenges for future studies.

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