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A SINGLE-CONSIGNOR MULTI-CONSIGNEE MULTI-ITEM MODEL WITH PERMISSIBLE PAYMENT DELAY, DELAYED SHIPMENT AND VARIABLE LEAD TIME UNDER CONSIGNMENT STOCK POLICY

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Abstract. This article proposes a two-level fuzzy supply chain inventory model, in which a single consignor delivers multiple items to the multiple consignees with the consignment stock agreement. The lead time is incorporated into the model and is considered a variable for obtaining optimal replenishment decisions. In addition, crashing cost is employed to reduce the lead time duration. This article investigates four different cases under controllable lead time to analyze the best strategy, focusing on two delays such as delay-in-payments and delay-in-shipment. In all four cases, all associated inventory costs are treated as a trapezoidal fuzzy number, and a signed distance method is employed to defuzzify the fuzzy inventory cost. An efficient optimization technique is adopted to find the optimal solution for the supply chain. Four numerical experiments are conducted to illustrate the four cases. Any one of these experimental results will provide the best solution for the ideal performance of the business under controllable lead time in the consignment stock policy. Finally, the managerial insights, conclusion and future direction of this model are provided.

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1. Introduction

In the supply chain, the consignment stock plays a vital role to attain a higher profit. Consignment stock (CS) is a business agreement where the consignor agrees to deliver the goods/products to the consignee without getting paid for the products in advance – the consignor still owns the products. The consignee pays for those products only when it sold. Inventory management is a crucial part of the consignment partnership process. This partnership has attracted many researchers' attention, and, as a result, numerous inventory models have been studied under the CS contract. Moreover, under CS policy, the retailer/buyer is referred to as the consignee and the supplier/manufacturer as the consignor. In the industrial market, the number of new products increases day by day depending on the customers' needs. On its basis, to attract new customers, many researchers have incorporated permissible payment delay to their models, saying that such incorporation will also enable greater profitability under the integrated supply chain.

Keywords. Consignment stock, controllable lead time, delay in shipment, delay in payment, fuzzy cost.

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Many inventory issues consider infinite storage in the buyer's warehouse, but almost all businesses face warehouse space limitations. Many researchers have developed a multi-inventory model that considers the number of shipments equal or unequal. If the buyer's warehouse has low capacity, the shipment is considered unequal. Therefore, such a space limitation plays an important role. Lead time in inventory management is the time period between placing an order to replenish inventory and receiving the order. The manufacturing methods and the management of stocks can also influence the lead time. Concerning production, it may take longer to build all the elements of a finished product on-site than to complete some off-site items. Therefore, it is crucial to consider the lead time as a variable.

The fuzzy logic techniques effectively solve complex, ill-defined problems characterized by environmental uncertainty and ambiguity of information. It allows for handling uncertain and imprecise knowledge and provides a robust framework for reasoning. Therefore, it has been confirmed that fuzzy logic is compelling in overcoming such uncertainty, and it describes a phenomenon in which a mathematical model or input data is unknown. Due to the uncertainty of the information and the complexity of the decision-making process, it is difficult for decision-makers to express their preferences using the exact numbers. In such a case, it is easy for them to use linguistic labels, *i.e.*, fuzzy or vague terms, to express their preferences. Thus, the solution to these sorts of challenges can be found by considering an uncertain parameter with fuzzy numbers (see, for instance, [19,20,22]). Generally, fuzzy numbers are used to treat uncertain parameters. It is essential to understand uncertainty, to manage inventory strategies in supply chain management. In inventory models, there are uncertainties not only in demand for goods but also in the calculation of inventory-related costs, and some random (stochastic) techniques have been used to deal with these issues under such supply chain management's inventory policies. In such problems, uncertainties are shaped by probability distributions based on the past analysis; however, past data are not always accurate or reliable. Furthermore, it is difficult to determine and implement uncertainty in integrated inventory models. Therefore, fuzzy set-based techniques may be the best way to treat these uncertainties for the practical application of inventory concepts in the supply chain (see, for instance, [29]). Fuzziness describes event ambiguity, and it measures the degree to which an event occurs, not whether it occurs. Randomness describes the uncertainty of event occurrence, that is, an event occurs or not. Therefore, whether an event occurs is "random" and to what degree it occurs is "fuzzy". Moreover, randomness is an objective form of indeterminacy whose distribution function of random variables is deduced by applying statistical methods. Fuzziness is a subjective form of indeterminacy that is distinguished by the degree of belongingness to a set.

By combining all of the features as mentioned above, in this study, we have considered four different cases under controllable lead-time (CLT) in an uncertain environment, namely (i) CS policy with no delay in payment (NDIP) – no delay in the shipment (NDIS) under CLT, (ii) CS policy with delay in payment (DIP) – no delay in the shipment (NDIS) under CLT, (iii) CS policy with no delay in payment (NDIP) – delay in the shipment (DIS) under CLT, (iv) CS policy with delay in payment (DIP) – delay in the shipment (DIS) under CLT.

2. Literature review

The literature review section covers the following topics: consignment stock policy, delay in payment, delay in shipment, controllable lead time, and CS policy in a fuzzy sense. We are particularly interested in research work dealing with two concepts: the allowable payment delay with interest and the delayed delivery in an ambiguous environment, which will bridge the existing literature and the current work.

2.1. Consignment Stock policy

Braglia and Zavanella [5] were the first researchers who proposed the inventory model under the consignment stock policy between the single vendor and a single buyer. Huang and Chen [16] showed the industrial strategy model in the supply chain following the CS policy. Zavanella and Zanoni [40] addressed the production inventory model under CS policy between the single-vendor multi-buyer. Srinivas and Rao [35] investigated and optimized the CS contract-based supply chain model for single-vendor multi-buyer with the genetic algorithm. Giri *et al.* [14] developed a three-tier supply chain model based on CS policy. Sarkar *et al.* [28] analyzed CS policy with a royalty reduction under a distribution-free approach. Gharaei *et al.* [12] designed the Vendor Managed Inventory (VMI) with the CS policy model and sharing multiple items between the single-vendor multi-buyer under greenhouse gas emissions and penalty. Giri and Masanta [13] examined the CS policy model under the consideration of learning and forgetting strategy with an uncertain return. Bylka [6] formulated the CS contract model with the limited warehouse capacity on the buyer's side. Sardar and Sarkar [25] investigated the supply chain model with advanced technology to solve unreliability. Sardar *et al.* [31] considered a CS agreement based model with radio frequency identification and machine learning. Chakraborty *et al.* [7] proposed a closed-loop supply chain model with CS policy. Çömez-Dolgan *et al.* [8] developed an inventory model in two different scenarios with untimely delivery costs.

2.2. Delay in payment

Aggarwal and Jaggi [1] illustrated the inventory model for deteriorating items by implementing permissible delay in payments. Sarkar [24] examined an imperfect production model with delayed payments and stock-based demand. Zahran *et al.* [39] studied the CS case with delay-in-payments for single-vendor and single-buyer. Shabani *et al.* [32] developed an inventory model with a two-warehouse inventory, fuzzy demand rate under permissible delay in payment. Ebrahimi *et al.* [9] proposed a two-echelon supply chain model with a delay in payment contract under stochastic promotional effort dependent demand.

2.3. Delay in shipment

Hill [15] suggested an integrated production-inventory model with the optimal production and shipment policy for the single-vendor single-buyer problem. Valentini and Zavanella [36] developed a consignment stock model with kth delayed shipment under the industrial case. Yi and Sarker [37] analyzed the replenishment policy model with delayed deliveries under controllable lead time. Yu and Hsu [38] considered an integrated inventory model for defective items with unequal-sized shipments. Ganesh Kumar and Uthayakumar [10] developed an inventory model by considering the delayed shipments under VMI policy.

2.4. Controllable lead time

Jha and Shanker [18] developed the production inventory model by considering the crashing cost for multi-buyer. Jamshidi *et al.* [17] considered a flexible inventory model with controllable lead time. Sarkar *et al.* [26] examined the effects of quality improvement and price discounts in the context of controllable lead time. Shin *et al.* [34] developed an inventory model following a continuous review methodology with variable lead time. Sarkar *et al.* [27] developed a model between single-vendor multi-buyer with varying production rate and controllable lead time. Ganguly *et al.* [11] designed the supply chain model with the influence of controllable lead time. Ahmad and Benkherouf [2] investigated an inventory model with replenishment decisions under partial backorder. Sharma *et al.* [33] analyzed the supply chain model with deteriorating products under varying lead time. Sarkar *et al.* [30] illustrated the deteriorating products-inventory model with varying demand and lead time.

2.5. CS policy under fuzzy environment

Ouyang and Yao [22] examined the distribution free inventory model with fuzzy demand. Björk [4] proposed an Economic Order Quantity (EOQ) model by considering the lead time, inventory level, and demand as a triangular fuzzy number, and defuzzification is done by using the signed distance method. Kazemi *et al.* [21] developed an inventory model by considering the inventory cost as the trapezoidal fuzzy number. Ali and Nakade [3] have developed a framework for examining the disruption of the supply chain in uncertain situations. Rani *et al.* [23] illustrated a model with carbon emission depended demand under a fuzzy environment. Sarkar *et al.* [29] suggested the supply chain model by assuming the inventory associated cost as a triangular fuzzy number under the signed distance method. Karthick and Uthayakumar [19] investigated the imperfect production inventory model with triangular fuzzy demand under the signed distance method. Karthick and

TABLE 1. A comparison of the present model with related existing models.

Reference	CS policy	Multi item	Multi consignee	Delay in payment	Delay in shipment	Controllable lead time	Fuzzy environment
Braglia and Zavanella [5]	√						
Bylka [6]	√						
Gharaei *et al.* [12]	√	√	√				
Huang and Chen [16]	√						
Karthick and Uthayakumar [20]	√	√					√
Sarkar *et al.* [30]					√		
Sarkar *et al.* [29]							√
Valentini and Zavanella [36]	√						
Yi and Sarker [37]	√				√	√	
Zavanella and Zanoni [40]	√		√				
Zahran *et al.* [39]	√			√			
Present model	√	√	√	√	√	√	√

Uthayakumar [20] developed a VMI-consignment stock policy model with multiple items and trapezoidal fuzzy number under the graded mean integration method.

2.6. The literature gap in previous research

From the above discussion, we observed that the CS policy model plays an essential role in business management. Braglia and Zavanella [5], Valentini and Zavanella [36], and Huang and Chen [16] developed a CS policy model for industrial purpose. Zavanella and Zanoni [40] extend the work of [5, 16, 36] by considering the single buyer to multiple buyers. Yi and Sarker [37] examined the inventory model with CS agreement with the incorporation of variable lead time. Also, Zahran *et al.* [39] have analyzed the CS policy model with permissible payment delay. However, Zahran *et al.* [39], Yi and Sarker [37], Braglia and Zavanella [5], Valentini and Zavanella [36] and Huang and Chen [16] do not consider their models with multiple buyers with multiple products. Nevertheless, Zahran *et al.* [39] does not discuss how their model operates with controllable lead time. In trading, lead time plays a significant role in avoiding shortages, so lead time reduction is considered necessary. Furthermore, there is no inventory model in the literature for dealing with CS policy between a single consignor and multiple consignees with multiple products in a fuzzy environment. Based on that, in addition, this study presents four special cases associated with two delays: delay in shipping and delay in payment.

Contributions of various study articles from the existing literature are given in Table 1. The rest of the paper has been comprised as follows: In Section 3, notations and assumptions are given to develop the model. In Section 4, four different cases are developed under controllable lead time in the fuzzy environment. The defuzzification process for the fuzzified total profit function is developed in Section 5. In Section 6, the solution procedure has been derived to find optimal solutions. Moreover, in this paper, all basic inventory cost is treated as a trapezoidal fuzzy number, and the defuzzification process is done using the signed distance method. Four numerical examples are considered for each case to validate this model in Section 7. Numerical discussions and managerial insights are given in Sections 8 and 9, respectively. Finally, the conclusion is given in Section 10.

3. Problem definition, notations and assumptions

3.1. Problem definition

The consignor produces a certain quantity of goods and transfers them equally to each consignee. Once the goods are withdrawn from the consigned inventory, the consignee pays the consignor an equal payment in an equal interval scheme (see, for instance, [39]). Also, if the consignee's warehouse reaches the maximum stock level, the shipments will be delayed. This aside, lead time plays a crucial role in the supply chain, so lead time crashing cost is incorporated to reduce lead-time length. This study analyzes the consequences of delayed deliveries and delayed payments in four different cases with uncertain supply chain costs. In the first and second cases, the shipment is considered without delay, whereas it is considered delayed in the third and fourth cases. Similarly, in the second and fourth cases, the payment (with interest charges) to the consignor is considered with delay and in the first and third cases without delay.

In this paper, we develop a mathematical model using the notations and assumptions listed below.

3.2. Notations

The following notations will be used to develop the model.

Indices

i	The index of items and $1 \leq i \leq z$, where z is the total number of items
j	The index of consignee's and $1 \leq j \leq y$, where y is the total number of consignee
c	The index of cases and $c = 1, 2, 3, 4$

Parameters

S_{vi}	Setup cost for ith item ($\$/setup$)
p_i	Production rate of ith item ($units/year$)
c_{pri}	Production cost of ith item ($\$/unit$)
c_{pi}	Purchasing cost of raw materials for ith item ($\$/unit$)
γ_i	Number of units needed to produce ith item
t_{ij}	Time of invoice of ith item for jth consignee
d_{ij}	Demand rate of ith item from jth consignee ($units/year$)
O_{rij}	Ordering cost of ith item for jth consignee ($\$/order$)
h^f_{mij}	Consignor's financial holding cost of ith item for jth consignee ($\$/unit/year$)
h^p_{mij}	Consignor's physical holding cost of ith item for jth consignee ($\$/unit/year$)
h_{mij}	Consignor's holding cost ($\$/unit/year$), *i.e.*, $h_{mij} = h^f_{mij} + h^p_{mij}$
h^p_{rij}	Physical holding cost of ith item for jth consignee ($\$/unit/year$)
h^p_{dij}	Physical holding cost of ith item for jth consignee in transit ($\$/unit/year$)
α_{ij}	Fraction of invoice's time given to the jth consignee to settle down its payment for ith item (interest-free)
β_{ij}	Fraction of invoice's time given to the jth consignee to settle down its payment for ith item (interest-charges)
I_{vij}	Consignor's investment interest rate of ith item for jth consignee ($\%/year$)
I_{bij}	Investment interest rate of jth consignee for ith item ($\%/year$)
c_{bij}	Consignor's selling price of ith finished item for jth consignee
c_{cij}	Selling price of jth consignee for ith finished item ($\$/unit$)
c_{tij}	Transaction cost of ith item for jth consignee ($\$/transaction$)

T_{ij}	Cycle length of ith item for jth consignee ($year$)
$B(l_j)$	Lead time crashing cost for jth consignee ($\$/shipment$)

Decision variables

n_{ij}	Number of shipments of ith item for jth consignee
m_{ij}	Number of payments of ith item for jth consignee
q_{ij}	Shipment size of ith item for jth consignee
k_{ij}	Number of delayed deliveries of ith item due to the stock capacity of jth consignee
l_j	Lead time length of jth consignee ($year$)

Diagram notations

B_1	Accumulative sales of jth consignee
B_2	Profit of jth consignee
I_1	Interest-free period
I_2	Interest-charge period

3.3. Assumptions

The following assumptions are considered while developing the model.

(1) The demand rate of ith item for jth ($j = 1, 2, 3, \ldots, y$) consignee is assumed to be constant.
(2) The production rate of ith item per year is finite, and it should be greater than the demand rate of the ith item for jth consignee ($i.e.$, $p_i > d_{ij}$) to avoid shortages.
(3) The system inventory is continuously reviewed, and the shortage is not allowed.
(4) The cycle time is common for both the consignor and consignee.
(5) The holding cost of the consignor is divided into two parts, namely financial and physical. Therefore, consignor's holding cost of the ith item for the jth consignee, $h_{mij} = h^f_{mij} + h^p_{mij}$, unit holding cost of the ith item for the jth consignee in transit, $h_{dij} = h^p_{dij} + h^f_{mij}$, and consignee's unit holding cost of the ith item for the jth consignee, $h_{rij} = h^p_{rij} + h^f_{rij}$ (refer, [37]).
(6) The consignee incurs only the physical holding cost for ith item.
(7) For the jth consignee, the lead time l_j consists of n_j components which are mutually independent. The kth component has a minimum duration $m_{j,k}$, normal duration $n_{j,k}$ and a crashing cost per unit time $e_{j,k}$ and assume that $e_{j,1} \leq e_{j,2} \leq \ldots \leq e_{j,n_{ij}}$. The lead time components are to be crashed one at a time beginning from the least component of e_i and so on.
(8) Let $l_{j,0} = \sum_{k=1}^{n_{ij}} n_{j,k}$ and $l_{j,f}$ is the length of the lead time components $1, 2, 3, \ldots, f$ crashed to their minimum duration, then expression of $l_{j,f}$ is given by $l_{j,f} = l_{j,0} - \sum_{j=1}^{f}(n_{j,k} - m_{j,k})$, where $f = 1, 2, \ldots, n_{ij}$ and crashing cost for the lead time per cycle is given by (see, for instance, [27])

$$B(l_j) = e_{j,f}\left(l_{j,f-1} - l_j\right) + \sum_{k=1}^{f-1} e_{j,k}\left(n_{j,k} - m_{j,k}\right), \; l_j \in \left[l_{j,f}, l_{j,f-1}\right].$$

4. Mathematical model

In this section, a trapezoidal fuzzy number and signed distance method are provided for a preliminary purpose, then a mathematical formulation is developed, including four cases. Basic costs related to inventory and production are unpredictable due to various factors, $i.e.$, inflation, the global energy crisis, fuel prices, and oil prices. Failing to consider these unforeseen circumstances results in an unstable supply chain model. For this reason, all the specific costs associated with the consignor and the consignee are considered to be fuzzy costs (Trapezoidal fuzzy number) in the proposed model. The signed distance method is used to solve fuzzy parameters.

4.1. Trapezoidal fuzzy number

The fuzzy number $\tilde{t}$ is said to be a non-negative trapezoidal fuzzy number (t_1, t_2, t_3, t_4) of t_i such that $t_1 < t_2 < t_3 < t_4$. The membership function of trapezoidal fuzzy number is given by

$$\mu_d(x) = \begin{cases} 0, & x \leq t_1 \\ \mathcal{B}(x) = \frac{x-t_1}{t_2-t_1}, & t_1 \leq x \leq t_2 \\ 1, & t_2 \leq x \leq t_3 \\ \mathcal{K}(x) = \frac{x-t_4}{t_3-t_4}, & t_3 \leq x \leq t_4 \\ 0, & x \geq t_4 \end{cases} \tag{4.1}$$

where, t_1 = lower limit, t_2 = lower mode, t_3 = upper mode and t_4 = upper limit of the fuzzy number $\tilde{t}$. We represent the trapezoidal fuzzy number as $\tilde{t} = (t - \varphi_1, t - \varphi_2, t + \varphi_3, t + \varphi_4)$, where φ_i, $i = 1, 2, 3, 4$ are arbitrary positive numbers with the restriction $t > \varphi_1 > \varphi_2$, $\varphi_3 < \varphi_4$. For the trapezoidal fuzzy number $\tilde{t} = (t_1, t_2, t_3, t_4)$, the left and right λ cuts of $\tilde{t}$ are respectively given by $\tilde{t}_L(\lambda) = t_1 + (t_2 - t_1)\lambda$ and $\tilde{t}_U(\lambda) = t_4 - (t_4 - t_3)\lambda$.

4.2. Signed distance method

For any $t \in R$, $d(t, 0) = t$ is named as the signed distance from t to 0. If $t > 0$, then the distance from t to 0 is $t = d(t, 0)$; if $t < 0$, the distance from t to 0 is $-t = -d(t, 0)$. Therefore, $d(t, 0) = t$ is known as the signed distance from t to 0.

For the fuzzy set $\tilde{t} \in R^+, 0 \leq \lambda \leq 1$, the following expression can be obtained as $\tilde{F} = \cup_{0\leq\lambda\leq1}\tilde{F}_\lambda = \cup_{0\leq\lambda\leq1}[L_\lambda, R_\lambda]$. The signed distance of the interval $[L_\lambda, R_\lambda]$ measured from the origin 0 is given by $d([L_\lambda, R_\lambda], \tilde{0}) = \frac{(\tilde{F}_L(\lambda)+\tilde{F}_U(\lambda))}{2}$. For the fuzzy number $\tilde{F} \in R^-$, the proposed defuzzification methods $d(\tilde{F}, 0)$ (the distance from $\tilde{F}$ to 0) is written as

$$\begin{aligned} d_0\left(\tilde{F}, \tilde{0}\right) &= \int_0^1 d\left(\tilde{F}_\lambda, \tilde{0}\right) \mathrm{d}\lambda = \int_0^1 d\left(\left[L_\lambda, R_\lambda\right], \tilde{0}\right) \mathrm{d}\lambda = \frac{1}{2}\int_0^1 \left\{\tilde{F}_L(\lambda) + \tilde{F}_U(\lambda)\right\} \mathrm{d}\lambda \\ &= \frac{1}{2}\int_0^1 \left[t_1 + \left(t_2 - t_1\right)\lambda + t_4 - \left(t_4 - t_3\right)\lambda\right] \mathrm{d}\lambda = \frac{1}{4}\left[t_1 + t_2 + t_3 + t_4\right]. \end{aligned} \tag{4.2}$$

4.3. Mathematical formulation

All four cases in this study consider the model between a single consignor and multiple consignees with multi-item based on the CS policy.

The cost associated with CS policy model are considered as the trapezoidal fuzzy number 4.1, which are given in the following:

Set-up cost: $\tilde{S}_{vi} = (S_{vi} - \varphi_{S_{vi1}}, S_{vi} - \varphi_{S_{vi2}}, S_{vi} + \varphi_{S_{vi3}}, S_{vi} + \varphi_{S_{vi4}})$,
Selling price: $\tilde{c}_{cij} = \left(c_{cij} - \varphi_{c_{cij1}}, c_{cij} - \varphi_{c_{cij2}}, c_{cij} + \varphi_{c_{cij3}}, c_{cij} + \varphi_{c_{cij4}}\right)$,
Transaction cost: $\tilde{c}_{tij} = \left(c_{tij} - \varphi_{c_{tij1}}, c_{tij} - \varphi_{c_{tij2}}, c_{tij} + \varphi_{c_{tij3}}, c_{tij} + \varphi_{c_{tij4}}\right)$,
Order cost: $\tilde{O}_{bij} = \left(O_{bij} - \varphi_{O_{bij1}}, O_{bij} - \varphi_{O_{bij2}}, O_{bij} + \varphi_{O_{bij3}}, O_{bij} + \varphi_{O_{bij4}}\right)$,
Production cost: $\tilde{c}_{pri} = \left(c_{pri} - \varphi_{c_{pri1}}, c_{pri} - \varphi_{c_{pri2}}, c_{pri} + \varphi_{c_{pri3}}, c_{pri} + \varphi_{c_{pri4}}\right)$,
Consignor's selling price: $\tilde{c}_{bij} = \left(c_{bij} - \varphi_{c_{bij1}}, c_{bij} - \varphi_{c_{bij2}}, c_{bij} + \varphi_{c_{bij3}}, c_{bij} + \varphi_{c_{bij4}}\right)$,
Transit physical holding cost: $\tilde{h}^p_{dij} = \left(h^p_{dij} - \varphi_{h^p_{dij1}}, h^p_{dij} - \varphi_{h^p_{dij2}}, h^p_{dij} + \varphi_{h^p_{dij3}}, h^p_{dij} + \varphi_{h^p_{dij4}}\right)$,
Consignor's raw material purchasing cost: $\tilde{c}_{pi} = \left(c_{pi} - \varphi_{c_{pi1}}, c_{pi} - \varphi_{c_{pi2}}, c_{pi} + \varphi_{c_{pi3}}, c_{pi} + \varphi_{c_{pi4}}\right)$,
Consignee's physical holding cost: $\tilde{h}^p_{rij} = \left(h^p_{rij} - \varphi_{h^p_{rij1}}, h^p_{rij} - \varphi_{h^p_{rij2}}, h^p_{rij} + \varphi_{h^p_{rij3}}, h^p_{rij} + \varphi_{h^p_{rij4}}\right)$,
Consigor's financial holding cost: $\tilde{h}^f_{mij} = \left(h^f_{mij} - \varphi_{h^f_{mij1}}, h^f_{mij} - \varphi_{h^f_{mij2}}, h^f_{mij} + \varphi_{h^f_{mij3}}, h^f_{mij} + \varphi_{h^f_{mij4}}\right)$, and
Consigor's physical holding cost: $\tilde{h}^p_{mij} = \left(h^p_{mij} - \varphi_{h^p_{mij1}}, h^p_{mij} - \varphi_{h^p_{mij2}}, h^p_{mij} + \varphi_{h^p_{mij3}}, h^p_{mij} + \varphi_{h^p_{mij4}}\right)$,

where $\varphi_{S_{vic}}, \varphi_{O_{bijc}}, \varphi_{h^f_{mijc}}, \varphi_{h^p_{mijc}}, \varphi_{h^p_{dijc}}, \varphi_{h^p_{rijc}}, \varphi_{c_{tijc}}, \varphi_{c_{pric}}, \varphi_{c_{bic}}, \varphi_{c_{cic}}$ and $\varphi_{c_{pic}}$, $i = 1,2,3$; $j = 1,2,3$; $c = 1,2,3,4$, are arbitrary positive numbers under the following conditions:

$$S_{vi} > \varphi_{S_{vi1}} > \varphi_{S_{vi2}}, \varphi_{S_{vi3}} < \varphi_{S_{vi4}}; O_{bij} > \varphi_{O_{bij1}} > \varphi_{O_{bij2}}, \varphi_{O_{bij3}} < \varphi_{O_{bij4}}; h^f_{mij} > \varphi_{h^f_{mij1}} > \varphi_{h^f_{mij2}}, \varphi_{h^f_{mij3}} < \varphi_{h^f_{mij4}}; h^p_{mij} > \varphi_{h^p_{mij1}} > \varphi_{h^p_{mij2}}, \varphi_{h^p_{mij3}} < \varphi_{h^p_{mij4}}; h^p_{dij} > \varphi_{h^p_{dij1}} > \varphi_{h^p_{dij2}}, \varphi_{h^p_{dij3}} < \varphi_{h^p_{dij4}}; h^p_{rij} > \varphi_{h^p_{rij1}} > \varphi_{h^p_{rij2}}, \varphi_{h^p_{rij3}} < \varphi_{h^p_{rij4}}; c_{tij} > \varphi_{c_{tij1}} > \varphi_{c_{tij2}}, \varphi_{c_{tij3}} < \varphi_{c_{tij4}}; c_{pri} > \varphi_{c_{pri1}} > \varphi_{c_{pri2}}, \varphi_{c_{pri3}} < \varphi_{c_{pri4}}; c_{bij} > \varphi_{c_{bij1}} > \varphi_{c_{bij2}}, \varphi_{c_{bij3}} < \varphi_{c_{bij4}}; c_{cij} > \varphi_{c_{cij1}} > \varphi_{c_{cij2}}, \varphi_{c_{cij3}} < \varphi_{c_{cij4}}; \quad \text{and} \quad c_{pi} > \varphi_{c_{pi1}} > \varphi_{c_{pi2}}, \varphi_{c_{pi3}} < \varphi_{c_{pi4}}.$$

The cost formulation of the consignor and consignees are described as follows.

4.3.1. Consignor's cost formulation

The costs associated with the consignor for y consignees and z items are derived as following:

Setup cost. Setup cost is the cost of purchasing and maintaining the equipment needed for the production stage before manufacturing the products,

$$\mathrm{SC} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{\tilde{S}_{vi} d_{ij}}{n_{ij} q_{ij}}. \tag{4.3}$$

Raw material cost. Spare parts are required to make a finished product, so the cost of purchasing those spare parts (raw materials) is known to be a raw material cost,

$$\mathrm{RMC} = \sum_{i=1}^{z}\sum_{j=1}^{y} \gamma_i \tilde{c}_{pi} d_{ij}. \tag{4.4}$$

Production cost. Production costs refer to the cost of producing or manufacturing an item. Also, this includes direct labour costs, direct material and overhead costs for production,

$$\mathrm{PC} = \sum_{i=1}^{z}\sum_{j=1}^{y} \tilde{c}_{pri} d_{ij}. \tag{4.5}$$

Lead time crashing cost. Lead time is the interval between when an order is placed to fill the goods and when the order is received. However, to reduce the length of lead time, the crashing cost is used as

$$\mathrm{LTCC} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{B(l_j) d_{ij}}{q_{ij}}. \tag{4.6}$$

4.3.2. Consignee's cost formulation

The costs associated with y consignees for z items are derived as follows:

Purchasing cost. Purchase cost refers to the cost of purchasing products from the consignor,

$$\mathrm{PRC} = \sum_{i=1}^{z}\sum_{j=1}^{y} \tilde{c}_{b_{ij}} d_{ij}. \tag{4.7}$$

Ordering cost. The cost required by y consignees to process the order from the consignor is said to be an ordering cost,

$$\mathrm{OC} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{\tilde{O}_{bij} d_{ij}}{q_{ij}}. \tag{4.8}$$

Transaction cost. The commission paid by y consignees for transaction per cycle is calculated as

$$\mathrm{TRC} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{m_{ij}\tilde{c}_{tij} d_{ij}}{n_{ij} q_{ij}}. \tag{4.9}$$

The total cost of the supply chain (without inventory holding cost of the consignor and y consignees) is derived by adding equations (4.3)–(4.9).

$$C_{\text{total}} = \mathrm{SC} + \mathrm{RMC} + \mathrm{PC} + \mathrm{LTCC} + \mathrm{PRC} + \mathrm{OC} + \mathrm{TRC}$$

$$C_{\text{total}} = \sum_{i=1}^{z}\sum_{j=1}^{y} \left(\gamma_i \tilde{c}_{pi} + \tilde{c}_{pri} + \tilde{c}_{bij}\right) d_{ij} + \left(\tilde{S}_{vi} + n_{ij}\tilde{O}_{bij} + m_{ij}\tilde{c}_{tij} + n_{ij} B(l_j)\right) \frac{d_{ij}}{n_{ij} q_{ij}}. \tag{4.10}$$

Case 1. CS policy with NDIP – NDIS under CLT $\left(h^p_{mij} > h^p_{rij}\right)$. The consignor produces q_{ij} of items in each n_{ij} batches within cycle with fixed setup cost $\tilde{S}_{vi}$ at finite production rate p_i. In order to avoid the shock out, the production rate is assumed to be greater than the demand rate. The consignor utilizes the consignee's warehouse space to store the manufactured items; this seems to be more advantageous for the consignor to keep fewer items in the warehouse. Moreover, there is an advantage for the consignee by holding the maximum stock level to avoid stockouts. The inventory pattern of the consignor, transit, consignees and financial behaviour of y consignees can be seen in Figure 1, and the average inventory of the system is calculated as

$$I_{\text{cs}} = \sum_{i=1}^{z}\sum_{j=1}^{y} q_{ij}\left(\frac{n_{ij}}{2} - \frac{n_{ij} d_{ij}}{2p_i} + \frac{d_{ij}}{p_i}\right) + d_{ij} l_j. \tag{4.11}$$

The average inventory of y consignees are derived by dividing the area $\sum_{i=1}^{z}\sum_{j=1}^{y} q_{ij}^2\left(\frac{n_{ij}}{2p_i} - \frac{n_{ij}^2}{2p_i} + \frac{n_{ij}^2}{2d_{ij}}\right)$ by the cycle time $\sum_{i=1}^{z}\sum_{j=1}^{y} \frac{n_{ij} q_{ij}}{d_{ij}} = \sum_{i=1}^{z}\sum_{j=1}^{y} T_{ij}$, that is,

$$I_{\text{consignee}} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{q_{ij}^2\left(\frac{n_{ij}}{2p_i} - \frac{n_{ij}^2}{2p_i} + \frac{n_{ij}^2}{2d_{ij}}\right)}{T_{ij}} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{d_{ij}}{n_{ij} q_{ij}} \times q_{ij}^2 \left(\frac{n_{ij}}{2p_i} - \frac{n_{ij}^2}{2p_i} + \frac{n_{ij}^2}{2d_{ij}}\right). \tag{4.12}$$

The average inventory in transit is calculated as.

$$I_{\text{transit}} = \sum_{i=1}^{z}\sum_{j=1}^{y} n_{ij} q_{ij} l_j \times \frac{1}{T_{ij}} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{d_{ij}}{n_{ij} q_{ij}} \times n_{ij} q_{ij} l_j = \sum_{i=1}^{z}\sum_{j=1}^{y} d_{ij} l_j. \tag{4.13}$$

The average inventory of the consignor $I_{\text{consignor}}$ is derived by subtracting $I_{\text{consignee}}$ and I_{transit} from the system average inventory I_{cs}.

$$I_{\text{consignor}} = I_{\text{cs}} - I_{\text{consignee}} - I_{\text{transit}} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{q_{ij} d_{ij}}{2p_i} \tag{4.14}$$

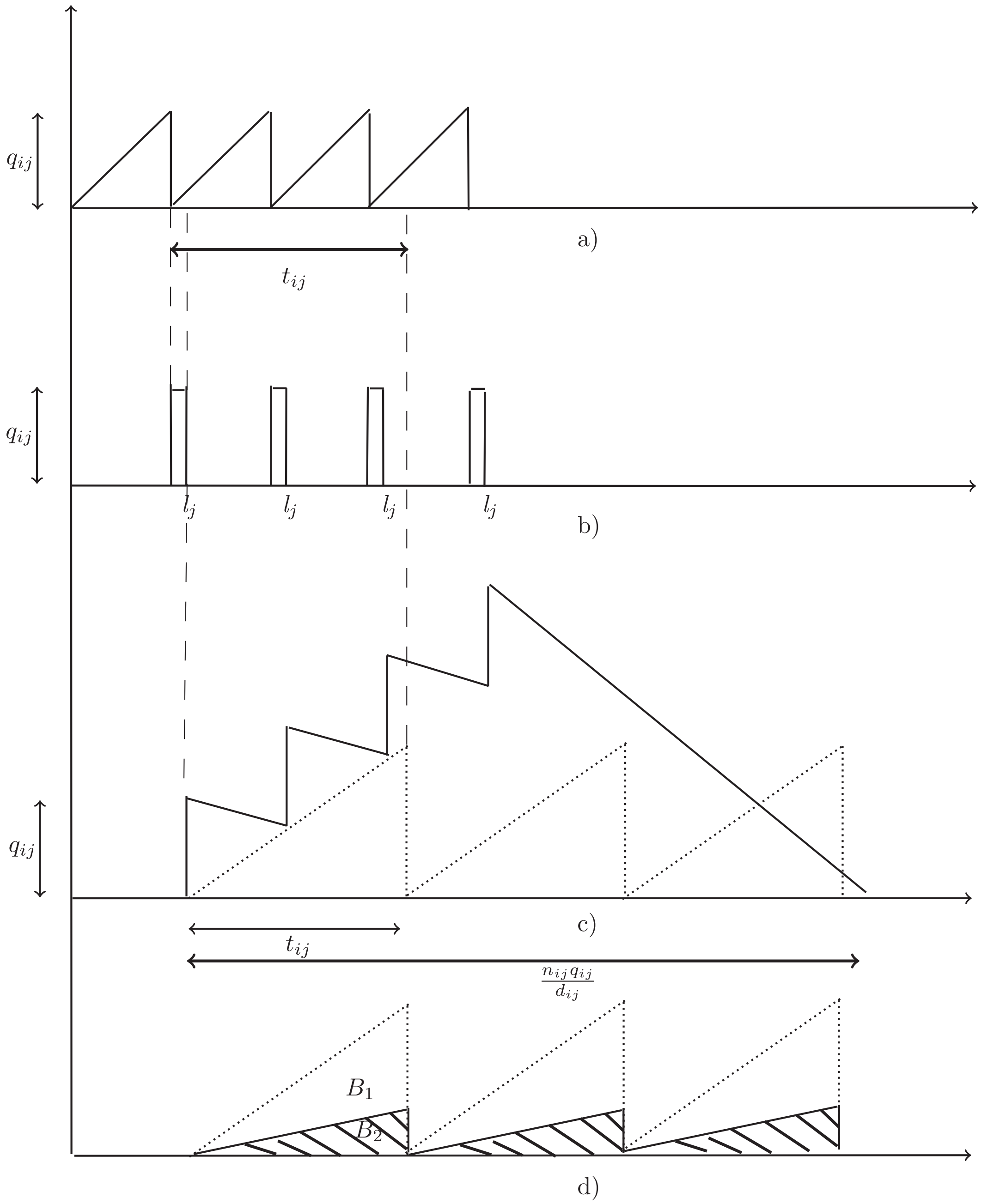

Figure 1. Inventory pattern of consignment stock policy with no delay in payment and no delay in shipment under controllable lead time (Case 1). (a) Consignor inventory. (b) Transit inventory. (c) Inventory of jth consignee. (d) Financial behaviour of jth consignee.

therefore, the physical holding cost of the consignor for y consignees are obtained as

$$\sum_{i=1}^{z}\sum_{j=1}^{y}\tilde{h}^{p}_{mij}\frac{q_{ij}d_{ij}}{2p_i}. \tag{4.15}$$

The financial holding cost of the consignor is formulated as (see, for instance, [39])

$$\sum_{i=1}^{z}\sum_{j=1}^{y}\tilde{h}^{f}_{mij}\left(\frac{(m_{ij}+1)n_{ij}q_{ij}}{2m_{ij}}-(n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) \tag{4.16}$$

where, $\tilde{h}^{f}_{mij}=\tilde{c}_{bij}I_{vij}$. The physical holding cost of y consignees are derived as

$$\sum_{i=1}^{z}\sum_{j=1}^{y}\tilde{h}^{p}_{rij}\left(\frac{n_{ij}q_{ij}}{2}-(n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) \tag{4.17}$$

and the transit holding cost is given as

$$\sum_{i=1}^{z}\sum_{j=1}^{y}\left(\tilde{h}^{p}_{dij}+\tilde{h}^{f}_{mij}\right)d_{ij}l_j. \tag{4.18}$$

The total cost of the supply chain with y consignees for z items are calculated by adding the equations (4.10), (4.15)–(4.18).

$$\begin{aligned}\tilde{C}_1\left(m_{ij},n_{ij},q_{ij},l_j\right)=&\sum_{i=1}^{z}\sum_{j=1}^{y}\left(\gamma_i\tilde{c}_{pi}+\tilde{c}_{pri}+\tilde{c}_{bij}\right)d_{ij}+\left(\tilde{S}_{vi}+n_{ij}\tilde{O}_{bij}+m_{ij}\tilde{c}_{tij}+n_{ij}B(l_j)\right)\frac{d_{ij}}{n_{ij}q_{ij}}\\&+\tilde{h}^{f}_{mij}\left(\frac{\left(m_{ij}+1\right)n_{ij}q_{ij}}{2m_{ij}}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(\tilde{h}^{p}_{mij}+\tilde{h}^{f}_{mij}\right)\frac{q_{ij}d_{ij}}{2p_i}\\&+\tilde{h}^{p}_{rij}\left(\frac{n_{ij}q_{ij}}{2}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(\tilde{h}^{p}_{dij}+\tilde{h}^{f}_{mij}\right)d_{ij}l_j.\end{aligned} \tag{4.19}$$

The revenue of the consignor is obtained as

$$\tilde{R}^{1}_{\text{consignor}}=\sum_{i=1}^{z}\sum_{j=1}^{y}\tilde{c}_{bij}d_{ij} \tag{4.20}$$

and the revenue of y consignees for z items are obtained by adding the selling price and investment interest rate of y consignees for z items.

$$\tilde{R}^{1}_{\text{consignee}}=\sum_{i=1}^{z}\sum_{j=1}^{y}\tilde{c}_{cij}\left(d_{ij}+I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right). \tag{4.21}$$

Then the total revenue of the supply chain is calculated by adding the equations (4.20) and (4.21), that is,

$$\tilde{R}^{1}_{\text{total}}(m_{ij},n_{ij},q_{ij})=\sum_{i=1}^{z}\sum_{j=1}^{y}\tilde{c}_{bij}d_{ij}+\tilde{c}_{cij}\left(d_{ij}+I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right). \tag{4.22}$$

Hence, the annual profit function $\tilde{P}_1(m_{ij},n_{ij},q_{ij},l_j)$ for case 1 can be written as

$$\text{Max }\tilde{P}_1\left(m_{ij},n_{ij},q_{ij},l_j\right)=\tilde{R}^{1}_{\text{total}}\left(m_{ij},n_{ij},q_{ij}\right)-\tilde{C}_1\left(m_{ij},n_{ij},q_{ij},l_j\right) \tag{4.23}$$

$$\begin{aligned}\text{Max } \tilde{P}_1(m_{ij}, n_{ij}, q_{ij}, l_j) = {} & \sum_{i=1}^{z}\sum_{j=1}^{y} \tilde{c}_{bij} d_{ij} + \tilde{c}_{cij}\left(d_{ij} + I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right) - \Bigg(\left(\gamma_i \tilde{c}_{pi} + \tilde{c}_{pri} + \tilde{c}_{bij}\right) d_{ij} \\ & + \left(\tilde{S}_{vi} + n_{ij}\tilde{O}_{bij} + m_{ij}\tilde{c}_{tij} + n_{ij}B(l_j)\right)\frac{d_{ij}}{n_{ij}q_{ij}} \\ & + \tilde{h}^{f}_{mij}\left(\frac{(m_{ij}+1)\, n_{ij}q_{ij}}{2m_{ij}} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) + \left(\tilde{h}^{p}_{mij} + \tilde{h}^{f}_{mij}\right) \\ & \times \frac{q_{ij}d_{ij}}{2p_i} + \tilde{h}^{p}_{rij}\left(\frac{n_{ij}q_{ij}}{2} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) + \left(\tilde{h}^{p}_{dij} + \tilde{h}^{f}_{mij}\right) d_{ij} l_j\Bigg) \qquad (4.24)\end{aligned}$$

subject to

$$\left(n_{ij}q_{ij} - (n_{ij}-1)q_{ij}\frac{d_{ij}}{p_i}\right) \le I_{\max},$$
$$q_{ij} > 0,$$
$$m_{ij},\ n_{ij} \text{ and } l_j \text{ are positive integers.}$$

Case 2. CS policy with DIP – NDIS under CLT $\left(h^{p}_{mij} > h^{p}_{rij}\right)$. In this case, the consignor offers the allowable payment delay to the consignee under the CS policy, *i.e.*, the consignee pays the invoice amount to the consignor by the end of the permissible period $\tau_{ij} = t_{ij} + \alpha_{ij}t_{ij}$, where $\alpha_{ij}t_{ij}$ is delay period offered by the consignor without interest. Sometimes, in reality, the consignee may not pay the invoice amount within the delay period $\alpha_{ij}t_{ij}$. Therefore, the consignee may pay the invoice amount with interest charges by the end of the period $\delta_{ij} = t_{ij} + \alpha_{ij}t_{ij} + \beta_{ij}\tau_{ij}$, where $\beta_{ij} > 0$. The inventory pattern for this case is given by Figure 2, and the system inventory is the same as in the case 1. The opportunity loss of the consignor is written as (see, for instance, [39])

$$\sum_{i=1}^{z}\sum_{j=1}^{y} \tilde{h}^{f}_{mij}\left(\frac{(m_{ij} + 1 + 2\alpha_{ij} + 2\beta_{ij}(1+\alpha_{ij}))n_{ij}q_{ij}}{2m_{ij}} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right). \qquad (4.25)$$

The consignee pays the invoice amount by the end of δ_{ij} period, therefore the interest charges for the extra delay by $\beta_{ij}\tau_{ij}$, which incurs the cost of $\sum_{i=1}^{z}\sum_{j=1}^{y}(\delta_{ij} - \tau_{ij})c_{bij}I_{vij}d_{ij}T_{ij}$, where $\sum_{i=1}^{z}\sum_{j=1}^{y} T_{ij} = \sum_{i=1}^{z}\sum_{j=1}^{y}\frac{\beta_{ij}(1+\alpha_{ij})n_{ij}q_{ij}}{m_{ij}d_{ij}}$. The total cost for case 2 is obtained by adding (4.10), (4.15), (4.17), (4.18) and (4.25).

$$\begin{aligned}&\tilde{C}_2(m_{ij}, n_{ij}, q_{ij}, l_j) \\ &= \sum_{i=1}^{z}\sum_{j=1}^{y}\Bigg(\left(\gamma_i\tilde{c}_{pi} + \tilde{c}_{pri} + \tilde{c}_{bij}\right) d_{ij} + \left(\tilde{S}_{vi} + n_{ij}\tilde{O}_{bij} + m_{ij}\tilde{c}_{tij} + n_{ij}B(l_j)\right)\frac{d_{ij}}{n_{ij}q_{ij}} \\ &\quad + \left(\tilde{h}^{p}_{mij} + \tilde{h}^{f}_{mij}\right)\frac{q_{ij}d_{ij}}{2p_i} + \tilde{h}^{f}_{mij}\left(\frac{(m_{ij} + 1 + 2\alpha_{ij} + 2\beta_{ij}(1+\alpha_{ij}))\, n_{ij}q_{ij}}{2m_{ij}} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) \\ &\quad + \tilde{h}^{p}_{rij}\left(\frac{n_{ij}q_{ij}}{2} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) + \tilde{c}_{bi}I_{vij}\left(\frac{\beta_{ij}(1+\alpha_{ij})\, n_{ij}q_{ij}}{m_{ij}}\right) + \left(\tilde{h}^{p}_{dij} + \tilde{h}^{f}_{mij}\right) d_{ij} l_j\Bigg). \qquad (4.26)\end{aligned}$$

The consignor's revenue is adding obtained by selling cost of the z items to the y consignees and the interest charged for unsettled balances, which is written as

$$\tilde{R}^{2}_{\text{consignor}} = \sum_{i=1}^{z}\sum_{j=1}^{y} c_{bij}\left(d_{ij} + I_{vij}\frac{\beta_{ij}(1+\alpha_{ij})\, n_{ij}q_{ij}}{m_{ij}}\right). \qquad (4.27)$$

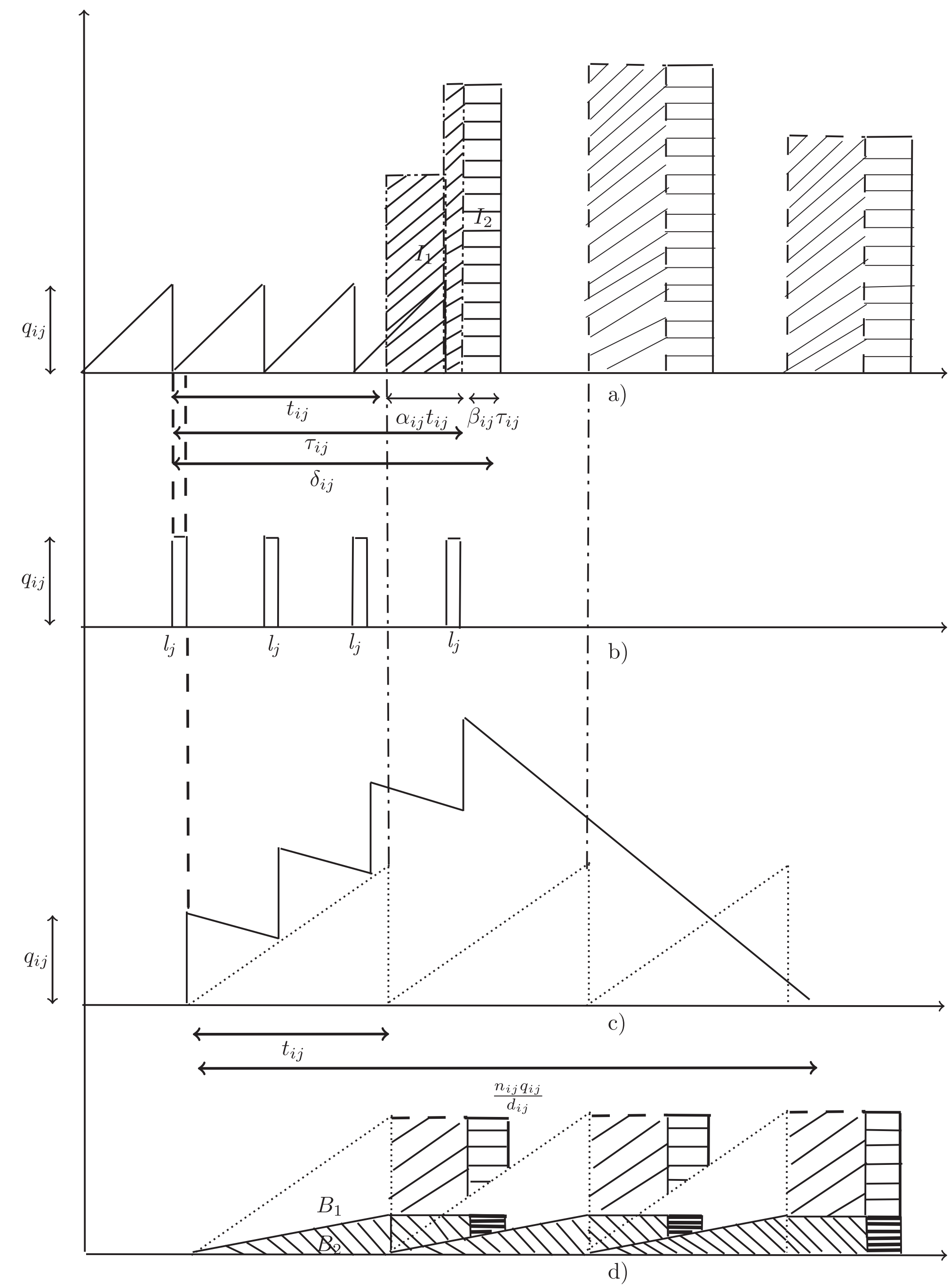

FIGURE 2. Inventory pattern of CS policy with delay in payment and no delay in shipment under controllable lead time (Case 2). (a) Consignor inventory. (b) Transit inventory. (c) Inventory of jth consignee. (d) Financial behaviour of jth consignee.

Similarly, the revenue of y consignees is obtained by adding sales and investment (*i.e.*,)

$$\tilde{R}^2_{\text{consignee}} = \sum_{i=1}^{z}\sum_{j=1}^{y} c_{cij}\left(d_{ij} + I_{bij}\frac{(2\alpha_{ij}+1+2\beta_{ij}(1+\alpha_{ij}))\,n_{ij}q_{ij}}{2m_{ij}}\right). \tag{4.28}$$

Therefore, the total revenue is calculated by adding (4.27) and (4.28).

$$\begin{aligned}\tilde{R}^2_{\text{total}}(m_{ij},n_{ij},q_{ij}) = &\sum_{i=1}^{z}\sum_{j=1}^{y} c_{bij}\left(d_{ij} + I_{vij}\frac{\beta_{ij}(1+\alpha_{ij})\,n_{ij}q_{ij}}{m_{ij}}\right)\\ &+ c_{cij}\left(d_{ij} + I_{bij}\frac{(2\alpha_{ij}+1+2\beta_{ij}(1+\alpha_{ij}))\,n_{ij}q_{ij}}{2m_{ij}}\right).\end{aligned} \tag{4.29}$$

Hence, the annual profit function $\tilde{P}_2(m_{ij},n_{ij},q_{ij},l_j)$ for case 2 is calculated by subtracting the total cost (4.26) from the total revenue (4.29) of the supply chain.

$$\text{Max } \tilde{P}_2(m_{ij},n_{ij},q_{ij},l_j) = \tilde{R}^2_{\text{total}}(m_{ij},n_{ij},q_{ij}) - \tilde{C}_2(m_{ij},n_{ij},q_{ij},l_j) \tag{4.30}$$

$$\begin{aligned}\text{Max } \tilde{P}_2(m_{ij},n_{ij},q_{ij},l_j) = &\sum_{i=1}^{z}\sum_{j=1}^{y}\tilde{c}_{bij}\left(d_{ij} + I_{vij}\frac{\beta_{ij}(1+\alpha_{ij})\,n_{ij}q_{ij}}{m_{ij}}\right) + \tilde{c}_{cij}\Bigg(d_{ij} + I_{bij}(2\alpha_{ij}+1\\ &+ 2\beta_{ij}(1+\alpha_{ij}))\times\frac{n_{ij}q_{ij}}{2m_{ij}}\Bigg) - \Bigg((\gamma_i\tilde{c}_{pi} + \tilde{c}_{pri} + \tilde{c}_{bij})\,d_{ij}\\ &+ \left(\tilde{S}_{vi} + n_{ij}\tilde{O}_{bij} + m_{ij}\tilde{c}_{tij} + n_{ij}B(l_j)\right)\frac{d_{ij}}{n_{ij}q_{ij}} + \left(\tilde{h}^p_{mij} + \tilde{h}^f_{mij}\right)\frac{q_{ij}d_{ij}}{2p_i}\\ &+ \tilde{h}^f_{mij}\left(\frac{(m_{ij}+1+2\alpha_{ij}+2\beta_{ij}(1+\alpha_{ij}))n_{ij}q_{ij}}{2m_{ij}} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right)\\ &+ \tilde{h}^p_{rij}\left(\frac{n_{ij}q_{ij}}{2} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) + \tilde{c}_{bi}I_{vij}\left(\frac{\beta_{ij}(1+\alpha_{ij})\,n_{ij}q_{ij}}{m_{ij}}\right)\\ &+ \left(\tilde{h}^p_{dij} + \tilde{h}^f_{mij}\right)d_{ij}l_j\Bigg)\end{aligned} \tag{4.31}$$

subject to

$$\left(n_{ij}q_{ij} - (n_{ij}-1)q_{ij}\frac{d_{ij}}{p_i}\right) \le I_{\max},$$
$$q_{ij} > 0,$$
$$m_{ij},\ n_{ij} \text{ and } l_j \text{ are positive integers.}$$

Case 3. CS policy with NDIP – DIS under CLT $\left(h^p_{mij} < h^p_{rij}\right)$. In this case, the consignor faces the problem of limited storage in the consignee's warehouse. The consignor will not offer any payment delay to the consignee. The inventory system with maximum delayed shipment is given in Figure 3, and the average inventory of the system is calculated as the same as in case 1. The average inventory of consignee is derived by dividing the area $\frac{q_{ij}^2}{2}\left(\frac{n_{ij}}{p_i} + \left(\frac{1}{p_i} - \frac{1}{d_{ij}}\right)(k_{ij}^2 - n_{ij}^2 + k_{ij})\right)$ by the cycle time $\frac{n_{ij}q_{ij}}{d_{ij}} = T_{ij}$, where, $k_{ij} = n_{ij} - 1$. The calculation is on the following:

$$I_{\text{consignee}} = \sum_{i=1}^{z}\sum_{j=1}^{y}\frac{d_{ij}}{n_{ij}q_{ij}}\times\frac{q_{ij}^2}{2}\left(\frac{n_{ij}}{p_i} + \left(\frac{1}{p_i} - \frac{1}{d_{ij}}\right)(k_{ij}^2 - n_{ij}^2 + k_{ij})\right). \tag{4.32}$$

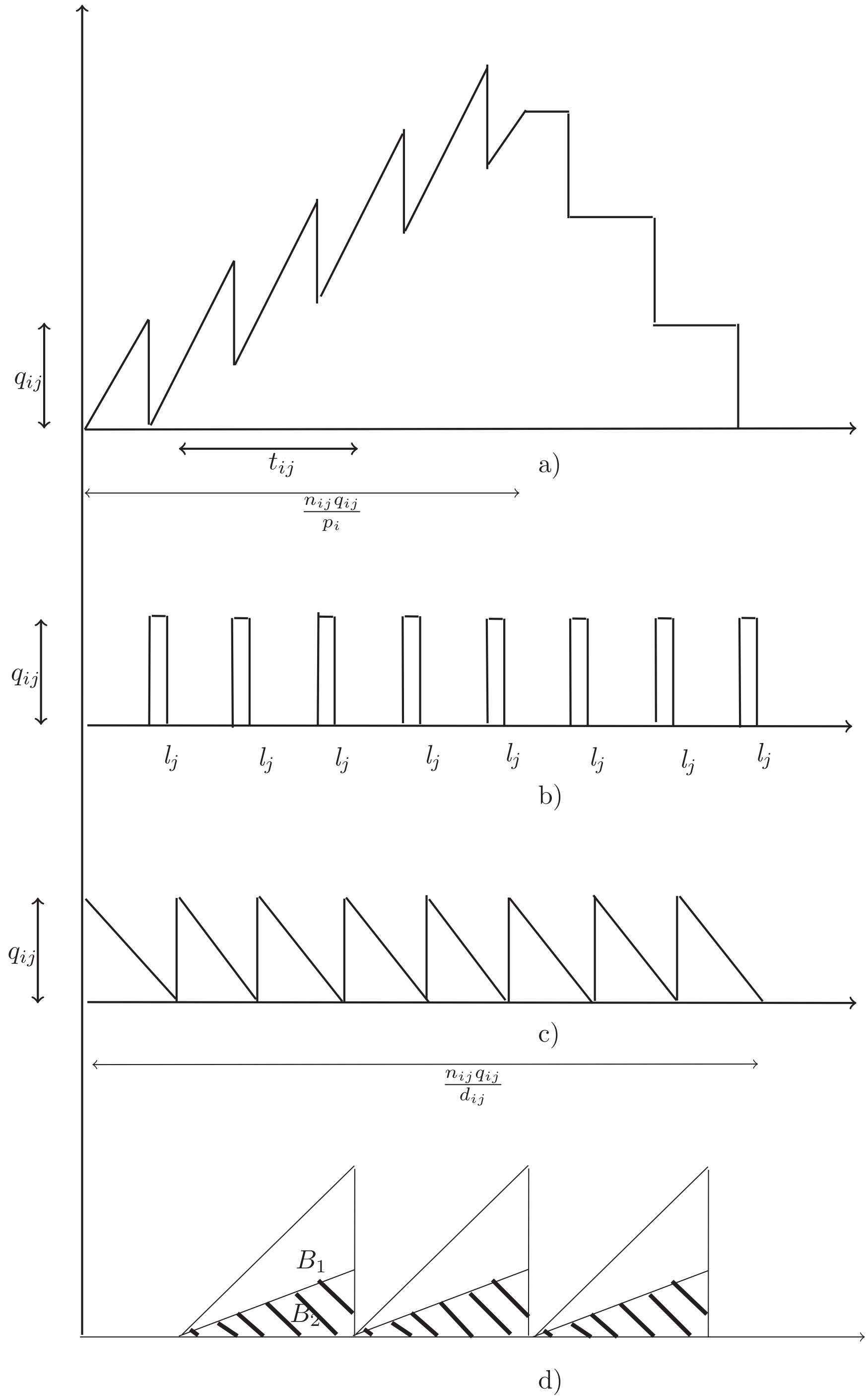

FIGURE 3. Inventory pattern of CS policy with no delay in payment and delay in shipment (Case 3). (a) Consignor inventory. (b) Transit inventory. (c) Inventory of jth consignee. (d) Financial behaviour of jth consignee.

The average inventory of the consignor $I_{\text{consignor}}$ is equal to the system average inventory I_{cs} (4.11) minus the consignee's inventory $I_{\text{consignee}}$ (4.32) minus transit inventory I_{transit} (4.18).

$$I_{\text{consignor}} = I_{\text{cs}} - I_{\text{consignee}} - I_{\text{transit}} = \sum_{i=1}^{z}\sum_{j=1}^{y} \frac{q_{ij}d_{ij}}{2p_i} + \frac{q_{ij}}{2}\left(1-\frac{d_{ij}}{p_i}\right)\frac{k_{ij}^2+k_{ij}}{n_{ij}}. \tag{4.33}$$

The total cost of the supply chain is derived as

$$\begin{aligned}\tilde{C}_3\left(m_{ij}, n_{ij}, q_{ij}, k_{ij}, l_j\right) = \sum_{i=1}^{z}\sum_{j=1}^{y}\Bigg(&\left(\gamma_i\tilde{c}_{pi}+\tilde{c}_{pri}+\tilde{c}_{bij}\right)d_{ij} + \left(\tilde{S}_{vi}+n_{ij}\tilde{O}_{bij}+m_{ij}\tilde{c}_{tij}+n_{ij}B(l_j)\right)\frac{d_{ij}}{n_{ij}q_{ij}} \\ &+ \tilde{h}^f_{mij}\left(\frac{(m_{ij}+1)n_{ij}q_{ij}}{2m_{ij}} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) + \left(\tilde{h}^p_{mij}+\tilde{h}^f_{mij}\right)\frac{q_{ij}d_{ij}}{2p_i} \\ &+ \tilde{h}^p_{rij}\left(\frac{n_{ij}q_{ij}}{2} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) + \left(\tilde{h}^p_{mij}-\tilde{h}^p_{rij}\right) \\ &\times\left(\frac{q_{ij}(p_i-d_{ij})}{2p_i}\right)(n_{ij}-1) + \left(\tilde{h}^p_{dij}+\tilde{h}^f_{mij}\right)d_{ij}l_j\Bigg)\end{aligned} \tag{4.34}$$

and the revenue of the consignor and consignee seems to be same as the case 1. Hence, the annual profit $\tilde{P}_3(m_{ij}, n_{ij}, q_{ij}, k_{ij}, l_j)$ can be written as

$$\begin{aligned}\text{Max } \tilde{P}_3\left(m_{ij}, n_{ij}, q_{ij}, k_{ij}, l_j\right) = \sum_{i=1}^{z}\sum_{j=1}^{y}&\tilde{c}_{bij}d_{ij} + \tilde{c}_{cij}\left(d_{ij}+I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right) - \Bigg(\left(\gamma_i\tilde{c}_{pi}+\tilde{c}_{pri}+\tilde{c}_{bij}\right)d_{ij} + \Big(\tilde{S}_{vi} \\ &+ n_{ij}\tilde{O}_{bij}+m_{ij}\tilde{c}_{tij}+n_{ij}B(l_j)\Big)\frac{d_{ij}}{n_{ij}q_{ij}} \\ &+ \tilde{h}^f_{mij}\left(\frac{\left(m_{ij}+1\right)n_{ij}q_{ij}}{2m_{ij}} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) \\ &+ \left(\tilde{h}^p_{mij}+\tilde{h}^f_{mij}\right)\frac{q_{ij}d_{ij}}{2p_i} + \tilde{h}^p_{rij}\left(\frac{n_{ij}q_{ij}}{2} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i}\right) + \left(\tilde{h}^p_{mij}-\tilde{h}^p_{rij}\right) \\ &\times\left(\frac{q_{ij}(p_i-d_{ij})}{2p_i}\right)(n_{ij}-1) + \left(\tilde{h}^p_{dij}+\tilde{h}^f_{mij}\right)d_{ij}l_j\Bigg)\end{aligned} \tag{4.35}$$

subject to

$$\begin{gathered}\left(\left(n_{ij}-k_{ij}\right)q_{ij} - \left(n_{ij}-k_{ij}-1\right)q_{ij}\frac{d_{ij}}{p_i}\right) \le I_{\max}, \\ q_{ij} > 0, \\ k_{ij} \le n_{ij}-1, \\ m_{ij},\ n_{ij} \text{ and } l_j \text{ are positive integers.}\end{gathered}$$

Case 4. CS policy with DIP – DIS under CLT $\left(h^p_{mij} < h^p_{rij}\right)$. In this case, the consignor offers a payment delay to the consignee and simultaneously, there was a limited storage space in the consignee's warehouse. The inventory of consignor, consignee, transit is depicted in Figure 4. The revenue of the consignor (*i.e.*, Eq. (4.27)) and consignee (*i.e.*, Eq. (4.28)) in case 2 are taken in this case. Then, the consignor's opportunity loss is given in (*i.e.*, Eq. (4.25)). The total cost of the supply chain is calculated as

$$\tilde{C}_4\left(m_{ij}, n_{ij}, q_{ij}, l_j\right) = \sum_{i=1}^{z}\sum_{j=1}^{y}\Bigg(\left(\gamma_i\tilde{c}_{pi}+\tilde{c}_{pri}+\tilde{c}_{bij}\right)d_{ij} + \left(\tilde{S}_{vi}+n_{ij}\tilde{O}_{bij}+m_{ij}\tilde{c}_{tij}+n_{ij}B(l_j)\right)\frac{d_{ij}}{n_{ij}q_{ij}}$$

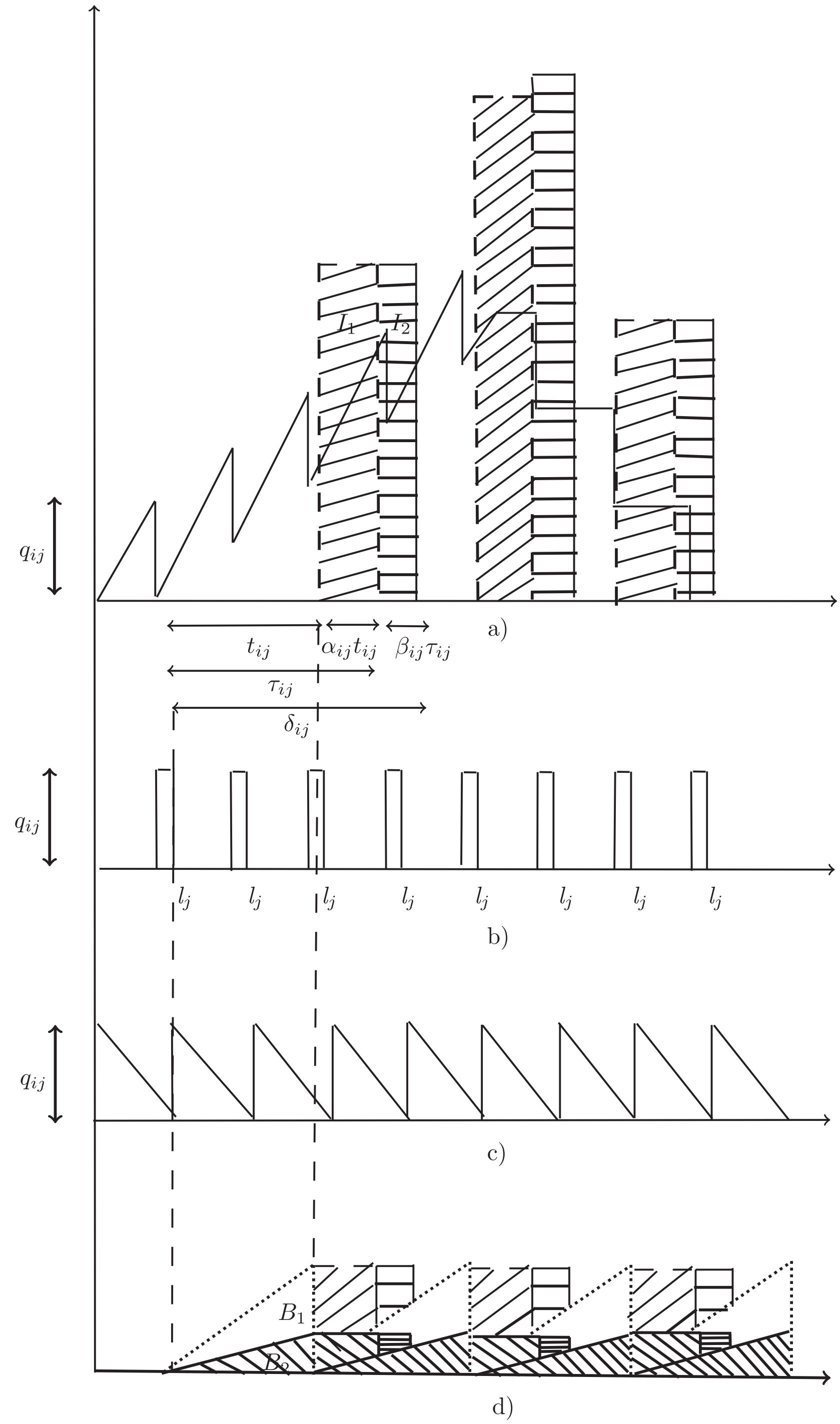

FIGURE 4. Inventory pattern of CS policy with delay in payment and delay in shipment under controllable lead time (Case 4). (a) Consignor inventory. (b) Transit inventory. (c) Inventory of jth consignee. (d) Financial behaviour of jth consignee.

$$+ \left(\tilde{h}^p_{mij} + \tilde{h}^f_{mij}\right) \frac{q_{ij}d_{ij}}{2p_i} + \tilde{h}^f_{mij} \left(\frac{(m_{ij} + 1 + 2\alpha_{ij} + 2\beta_{ij}(1+\alpha_{ij}))\, n_{ij}q_{ij}}{2m_{ij}} \right.$$
$$\left. - (n_{ij} - 1)\frac{q_{ij}d_{ij}}{2p_i} \right) + \tilde{h}^p_{rij} \left(\frac{n_{ij}q_{ij}}{2} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i} \right) + \tilde{c}_{bij} I_{vij} \left(\frac{\beta_{ij}(1+\alpha_{ij})\, n_{ij}q_{ij}}{m_{ij}} \right)$$
$$\left. + \left(\tilde{h}^p_{mij} - \tilde{h}^p_{rij}\right) \times \left(\frac{q_{ij}(p_i - d_{ij})}{2p_i} \right)(n_{ij}-1) + \left(\tilde{h}^p_{dij} + \tilde{h}^f_{mij}\right) d_{ij} l_j \right) \tag{4.36}$$

and the total revenue is same as the case 3. Hence, the annual profit $\tilde{P}_4(m_{ij}, n_{ij}, q_{ij}, k_{ij}, l_j)$ can be written as

$$\text{Max } \tilde{P}_4(m_{ij}, n_{ij}, q_{ij}, k_{ij}, l_j) = \sum_{i=1}^{z}\sum_{j=1}^{y} \tilde{c}_{bij}\left(d_{ij} + I_{vij}\frac{\beta_{ij}(1+\alpha_{ij})\, n_{ij}q_{ij}}{m_{ij}} \right)$$
$$+ \tilde{c}_{cij}\left(d_{ij} + I_{bij}(2\alpha_{ij} + 1 + 2\beta_{ij}(1+\alpha_{ij}))\frac{n_{ij}q_{ij}}{2m_{ij}} \right)$$
$$- \left((\gamma_i\tilde{c}_{pi} + \tilde{c}_{pri} + \tilde{c}_{bij})\, d_{ij} + \left(\tilde{S}_{vi} + n_{ij}\tilde{O}_{bij} + m_{ij}\tilde{c}_{tij} + n_{ij}B(l_j) \right)\frac{d_{ij}}{n_{ij}q_{ij}} \right.$$
$$+ \left(\tilde{h}^p_{mij} + \tilde{h}^f_{mij}\right)\frac{q_{ij}d_{ij}}{2p_i} + \tilde{h}^f_{mij}\left(\frac{(m_{ij} + 1 + 2\alpha_{ij} + 2\beta_{ij}(1+\alpha_{ij}))n_{ij}q_{ij}}{2m_{ij}} \right.$$
$$\left. - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i} \right) + \tilde{h}^p_{rij}\left(\frac{n_{ij}q_{ij}}{2} - (n_{ij}-1)\frac{q_{ij}d_{ij}}{2p_i} \right)$$
$$+ \tilde{c}_{bij}I_{vij}\left(\frac{\beta_{ij}(1+\alpha_{ij})\, n_{ij}q_{ij}}{m_{ij}} \right) + \left(\tilde{h}^p_{mij} - \tilde{h}^p_{rij}\right)$$
$$\left. \times \left(\frac{q_{ij}(p_i - d_{ij})}{2p_i} \right)(n_{ij}-1) + \left(\tilde{h}^p_{dij} + \tilde{h}^f_{mij}\right) d_{ij}l_j \right) \tag{4.37}$$

subject to

$$\left((n_{ij} - k_{ij})\, q_{ij} - (n_{ij} - k_{ij} - 1)\, q_{ij}\frac{d_{ij}}{p_i} \right) \le I_{\max},$$
$$q_{ij} > 0,$$
$$k_{ij} \le n_{ij} - 1,$$
m_{ij}, n_{ij} and l_j are positive integers.

5. Defuzzification methodology

Defuzzification is the method of generating a quantifiable result in crisp logic, from fuzzy sets and corresponding membership functions (*i.e.*, the process of reducing the fuzzy set to a crisp set or converting the fuzzy quantity to a crisp quantity). There may be situations in which the output of a fuzzy process must be a single scalar quantity as opposed to a fuzzy set. The left and right λ cuts of $S_{vi}, O_{bij}, h^f_{mij}, h^p_{mij}, h^p_{dij}, h^p_{rij}, c_{tij}, c_{pri}, c_{bi}, c_{ci}$, and c_{pi} are given below, $\tilde{S}_{Lvi}(\lambda) = S_{vi} - \varphi_{S_{vi1}} + (\varphi_{S_{vi1}} - \varphi_{S_{vi2}})\,\lambda$; $\tilde{S}_{Uvi}(\lambda) = S_{vi} - \varphi_{S_{vi4}} + (\varphi_{S_{vi4}} - \varphi_{S_{vi3}})\,\lambda$, $\tilde{O}_{Lbij}(\lambda) = O_{bij} - \varphi_{O_{bij1}} + (\varphi_{O_{bij1}} - \varphi_{O_{bij2}})\,\lambda$; $\tilde{O}_{Ubij}(\lambda) = O_{bij} - \varphi_{O_{bij4}} + (\varphi_{O_{bij4}} - \varphi_{O_{bij3}})\,\lambda$, $\tilde{h}^f_{Lmij}(\lambda) = h^f_{mij} - \varphi_{h^f_{mij1}} + \left(\varphi_{h^f_{mij1}} - \varphi_{h^f_{mij2}}\right)\lambda$; $\tilde{h}^f_{Umij}(\lambda) = h^f_{mij} - \varphi_{h^f_{mij4}} + \left(\varphi_{h^f_{mij4}} - \varphi_{h^f_{mij3}}\right)\lambda$, $\tilde{h}^p_{Lmij}(\lambda) = h^p_{mij} - \varphi_{h^p_{mij1}} + \left(\varphi_{h^p_{mij1}} - \varphi_{h^p_{mij2}}\right)\lambda$; $\tilde{h}^p_{Umij}(\lambda) = h^p_{mij} - \varphi_{h^p_{mij4}} + \left(\varphi_{h^p_{mij4}} - \varphi_{h^p_{mij3}}\right)\lambda$, $\tilde{h}^p_{Ldij}(\lambda) = h^p_{dij} - \varphi_{h^p_{dij1}} + \left(\varphi_{h^p_{dij1}} - \varphi_{h^p_{dij2}}\right)\lambda$; $\tilde{h}^p_{Udij}(\lambda) = h^p_{dij} - \varphi_{h^p_{dij4}} + \left(\varphi_{h^p_{dij4}} - \varphi_{h^p_{dij3}}\right)\lambda$, $\tilde{h}^p_{Lrij}(\lambda) = h^p_{rij} - \varphi_{h^p_{rij1}} + \left(\varphi_{h^p_{rij1}} - \varphi_{h^p_{rij2}}\right)\lambda$; $\tilde{h}^p_{Urij}(\lambda) = h^p_{rij} - \varphi_{h^p_{rij4}} + \left(\varphi_{h^p_{rij4}} - \varphi_{h^p_{rij3}}\right)\lambda$, $\tilde{c}_{Ltij}(\lambda) = c_{tij} - \varphi_{c_{tij1}} + (\varphi_{c_{tij1}} - \varphi_{c_{tij2}})\,\lambda$; $\tilde{c}_{Utij}(\lambda) = c_{tij} - \varphi_{c_{tij4}} +$

$\left(\varphi_{c_{tij4}}-\varphi_{c_{tij3}}\right)\lambda$, $\tilde{c}_{Lpri}(\lambda)=c_{pri}-\varphi_{c_{pri1}}+\left(\varphi_{c_{pri1}}-\varphi_{c_{pri2}}\right)\lambda$; $\tilde{c}_{Upri}(\lambda)=c_{pri}-\varphi_{c_{pri4}}+\left(\varphi_{c_{pri4}}-\varphi_{c_{pri3}}\right)\lambda$, $\tilde{c}_{Lbi}(\lambda)=c_{bi}-\varphi_{c_{bi1}}+\left(\varphi_{c_{bi1}}-\varphi_{c_{bi2}}\right)\lambda$; $\tilde{c}_{Ubi}(\lambda)=c_{bi}-\varphi_{c_{bi4}}+\left(\varphi_{c_{bi4}}-\varphi_{c_{bi3}}\right)\lambda$, $\tilde{c}_{Lci}(\lambda)=c_{ci}-\varphi_{c_{ci1}}+\left(\varphi_{c_{ci1}}-\varphi_{c_{ci2}}\right)\lambda$; $\tilde{c}_{Uci}(\lambda)=c_{ci}-\varphi_{c_{ci4}}+\left(\varphi_{c_{ci4}}-\varphi_{c_{ci3}}\right)\lambda$, and $\tilde{c}_{Lpi}(\lambda)=c_{pi}-\varphi_{c_{pi1}}+\left(\varphi_{c_{pi1}}-\varphi_{c_{pi2}}\right)\lambda$; $\tilde{c}_{Upi}(\lambda)=c_{pi}-\varphi_{c_{pi4}}+\left(\varphi_{c_{pi4}}-\varphi_{c_{pi3}}\right)\lambda$. Therefore, by using the signed distance method 4.2,

$$d_0\left(\tilde{P}_c(m_{ij},n_{ij},q_{ij},k_{ij},l_j),\tilde{0}\right)=\frac{1}{2}\int_0^1\left[\tilde{P}_c\left(m_{ij},n_{ij},q_{ij},k_{ij},l_j\right)_L(\lambda)+\tilde{P}_c\left(m_{ij},n_{ij},q_{ij},k_{ij},l_j\right)_U(\lambda)\right]\mathrm{d}\lambda$$

if $c=1,2$ then $k_{ij}=0$ and if $c=3,4$ then $k_{ij}=n_{ij}-1$,

$$d_0\left(\tilde{S}_{vi},\tilde{0}\right)=S_{vi}+\frac{1}{4}\left(\varphi_{S_{vi4}}+\varphi_{S_{vi3}}-\varphi_{S_{vi2}}-\varphi_{S_{vi1}}\right)>0 \tag{5.1}$$

$$d_0\left(\tilde{O}_{bij},\tilde{0}\right)=O_{bij}+\frac{1}{4}\left(\varphi_{O_{bij4}}+\varphi_{O_{bij3}}-\varphi_{O_{bij2}}-\varphi_{O_{bij1}}\right)>0 \tag{5.2}$$

$$d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)=h^f_{mij}+\frac{1}{4}\left(\varphi_{h^f_{mij4}}+\varphi_{h^f_{mij3}}-\varphi_{h^f_{mij2}}-\varphi_{h^f_{mij1}}\right)>0 \tag{5.3}$$

$$d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)=h^p_{mij}+\frac{1}{4}\left(\varphi_{h^p_{mij4}}+\varphi_{h^p_{mij3}}-\varphi_{h^p_{mij2}}-\varphi_{h^p_{mij1}}\right)>0 \tag{5.4}$$

$$d_0\left(\tilde{h}^p_{dij},\tilde{0}\right)=h^p_{dij}+\frac{1}{4}\left(\varphi_{h^p_{dij4}}+\varphi_{h^p_{dij3}}-\varphi_{h^p_{dij2}}-\varphi_{h^p_{dij1}}\right)>0 \tag{5.5}$$

$$d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)=h^p_{rij}+\frac{1}{4}\left(\varphi_{h^p_{rij4}}+\varphi_{h^p_{rij3}}-\varphi_{h^p_{rij2}}-\varphi_{h^p_{rij1}}\right)>0 \tag{5.6}$$

$$d_0\left(\tilde{c}_{tij},\tilde{0}\right)=c_{tij}+\frac{1}{4}\left(\varphi_{c_{tij4}}+\varphi_{c_{tij3}}-\varphi_{c_{tij2}}-\varphi_{c_{tij1}}\right)>0 \tag{5.7}$$

$$d_0\left(\tilde{c}_{pri},\tilde{0}\right)=c_{pri}+\frac{1}{4}\left(\varphi_{c_{pri4}}+\varphi_{c_{pri3}}-\varphi_{c_{pri2}}-\varphi_{c_{pri1}}\right)>0 \tag{5.8}$$

$$d_0\left(\tilde{c}_{bij},\tilde{0}\right)=c_{bij}+\frac{1}{4}\left(\varphi_{c_{bij4}}+\varphi_{c_{bij3}}-\varphi_{c_{bij2}}-\varphi_{c_{bij1}}\right)>0 \tag{5.9}$$

$$d_0\left(\tilde{c}_{cij},\tilde{0}\right)=c_{cij}+\frac{1}{4}\left(\varphi_{c_{cij4}}+\varphi_{c_{cij3}}-\varphi_{c_{cij2}}-\varphi_{c_{cij1}}\right)>0 \tag{5.10}$$

$$d_0\left(\tilde{c}_{pi},\tilde{0}\right)=c_{pi}+\frac{1}{4}\left(\varphi_{c_{pi4}}+\varphi_{c_{pi3}}-\varphi_{c_{pi2}}-\varphi_{c_{pi1}}\right)>0. \tag{5.11}$$

By inserting (5.1)–(5.11) into the equations (5.12)–(5.15) yields the defuzzified profit function,

For Case 1.

$$\begin{aligned}
&d_0\left(\tilde{P}_1(m_{ij},n_{ij},q_{ij},l_j),\tilde{0}\right)\\
&=\sum_{i=1}^{z}\sum_{j=1}^{y}d_0\left(\tilde{c}_{bij},\tilde{0}\right)d_{ij}+d_0\left(\tilde{c}_{cij},\tilde{0}\right)\left(d_{ij}+I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right)-\left(\left(\gamma_i d_0\left(\tilde{c}_{pi},\tilde{0}\right)+d_0\left(\tilde{c}_{pri},\tilde{0}\right)\right.\right.\\
&\quad\left.+\,d_0\left(\tilde{c}_{bij},\tilde{0}\right)\right)d_{ij}+\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)+n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)+n_{ij}B\left(l_j\right)\right)\frac{d_{ij}}{n_{ij}q_{ij}}\\
&\quad+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(\frac{\left(m_{ij}+1\right)n_{ij}q_{ij}}{2m_{ij}}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\right)\frac{q_{ij}d_{ij}}{2p_i}\\
&\quad\left.+\,d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(\frac{n_{ij}q_{ij}}{2}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(d_0\left(\tilde{h}^p_{dij},\tilde{0}\right)+d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)\right)d_{ij}l_j\right)
\end{aligned} \tag{5.12}$$

subject to $\left(n_{ij}q_{ij}-\left(n_{ij}-1\right)q_{ij}\frac{d_{ij}}{p_i}\right)\leq I_{\max}$, $q_{ij}>0$, m_{ij}, n_{ij} and l_j are positive integers.

For Case 2.

$$d_0\left(\tilde{P}_2\left(m_{ij},n_{ij},q_{ij},l_j\right),\tilde{0}\right)$$

$$= \sum_{i=1}^{z}\sum_{j=1}^{y} d_0\left(\tilde{c}_{bij},\tilde{0}\right)\left(d_{ij}+I_{vij}\frac{\beta_{ij}\left(1+\alpha_{ij}\right)n_{ij}q_{ij}}{m_{ij}}\right)+d_0\left(\tilde{c}_{ci},\tilde{0}\right)\left(d_{ij}+I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right.$$
$$\left.\times\left(2\alpha_{ij}+1+2\beta_{ij}\left(1+\alpha_{ij}\right)\right)\right)-\left(\left(\gamma_i d_0\left(\tilde{c}_{pi},\tilde{0}\right)+d_0\left(\tilde{c}_{pri},\tilde{0}\right)+d_0\left(\tilde{c}_{bij},\tilde{0}\right)\right)d_{ij}+\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)\right.\right.$$
$$\left.+\,n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)+n_{ij}B\left(l_j\right)\right)\frac{d_{ij}}{n_{ij}q_{ij}}+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(\left(m_{ij}+1+2\alpha_{ij}\right.\right.$$
$$\left.\left.+2\beta_{ij}\left(1+\alpha_{ij}\right)\right)\frac{n_{ij}q_{ij}}{2m_{ij}}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\right)\frac{q_{ij}d_{ij}}{2p_i}$$
$$+\,d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(\frac{n_{ij}q_{ij}}{2}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+d_0\left(\tilde{c}_{bij},\tilde{0}\right)I_{vij}\left(\frac{\beta_{ij}\left(1+\alpha_{ij}\right)n_{ij}q_{ij}}{m_{ij}}\right)$$
$$\left.+\left(d_0\left(\tilde{h}^p_{dij},\tilde{0}\right)+d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)\right)d_{ij}l_j\right) \tag{5.13}$$

subject to $\left(n_{ij}q_{ij}-\left(n_{ij}-1\right)q_{ij}\frac{d_{ij}}{p_i}\right)\leq I_{\max}$, $q_{ij}>0$, m_{ij}, n_{ij} and l_j are positive integers.

For Case 3.

$$d_0\left(\tilde{P}_3\left(m_{ij},n_{ij},q_{ij},k_{ij},l_j\right),\tilde{0}\right)$$
$$= \sum_{i=1}^{z}\sum_{j=1}^{y} d_0\left(\tilde{c}_{bij},\tilde{0}\right)d_{ij}+d_0\left(\tilde{c}_{cij},\tilde{0}\right)\left(d_{ij}+I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right)-\left(\left(\gamma_i d_0\left(\tilde{c}_{pi},\tilde{0}\right)+d_0\left(\tilde{c}_{pri},\tilde{0}\right)\right.\right.$$
$$\left.+\,d_0\left(\tilde{c}_{bij},\tilde{0}\right)\right)d_{ij}+\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)+n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)+n_{ij}B(l_j)\right)\frac{d_{ij}}{n_{ij}q_{ij}}$$
$$+\,d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(\frac{\left(m_{ij}+1\right)n_{ij}q_{ij}}{2m_{ij}}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\right)$$
$$\times\frac{q_{ij}d_{ij}}{2p_i}+\left(d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)-d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\right)\left(\frac{q_{ij}\left(p_i-d_{ij}\right)}{2p_i}\right)\left(n_{ij}-1\right)+d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)$$
$$\left.\times\left(\frac{n_{ij}q_{ij}}{2}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(d_0\left(\tilde{h}^p_{dij},\tilde{0}\right)+d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)\right)d_{ij}l_j\right) \tag{5.14}$$

subject to $\left(\left(n_{ij}-k_{ij}\right)q_{ij}-\left(n_{ij}-k_{ij}-1\right)q_{ij}\frac{d_{ij}}{p_i}\right)\leq I_{\max}, q_{ij}>0, k_{ij}\leq n_{ij}-1, m_{ij}, n_{ij}$ and l_j are positive integers.

For Case 4.

$$d_0\left(\tilde{P}_4\left(m_{ij},n_{ij},q_{ij},k_{ij},l_j\right),\tilde{0}\right)$$
$$= \sum_{i=1}^{z}\sum_{j=1}^{y} d_0\left(\tilde{c}_{bij},\tilde{0}\right)\left(d_{ij}+I_{vij}\frac{\beta_{ij}\left(1+\alpha_{ij}\right)n_{ij}q_{ij}}{m_{ij}}\right)+d_0\left(\tilde{c}_{cij},\tilde{0}\right)\left(d_{ij}+I_{bij}\frac{n_{ij}q_{ij}}{2m_{ij}}\right.$$
$$\left.\times\left(2\alpha_{ij}+1+2\beta_{ij}\left(1+\alpha_{ij}\right)\right)\right)-\left(\left(\gamma_i d_0\left(\tilde{c}_{pi},\tilde{0}\right)+d_0\left(\tilde{c}_{pri},\tilde{0}\right)+d_0\left(\tilde{c}_{bij},\tilde{0}\right)\right)d_{ij}\right.$$
$$+\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)+n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)+n_{ij}B\left(l_j\right)\right)\frac{d_{ij}}{n_{ij}q_{ij}}+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)$$
$$\times\left(\frac{\left(m_{ij}+1+2\alpha_{ij}+2\beta_{ij}\left(1+\alpha_{ij}\right)\right)n_{ij}q_{ij}}{2m_{ij}}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+\left(d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)\right.$$
$$\left.+\,d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\right)\frac{q_{ij}d_{ij}}{2p_i}+d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(\frac{n_{ij}q_{ij}}{2}-\left(n_{ij}-1\right)\frac{q_{ij}d_{ij}}{2p_i}\right)+d_0\left(\tilde{c}_{bij},\tilde{0}\right)I_{vij}$$

$$\times\left(\frac{\beta_{ij}\left(1+\alpha_{ij}\right)n_{ij}q_{ij}}{m_{ij}}\right)+\left(d_0(\tilde{h}^p_{mij},\tilde{0})-d_0(\tilde{h}^p_{rij},\tilde{0})\right)\left(\frac{q_{ij}\left(p_i-d_{ij}\right)}{2p_i}\right)\left(n_{ij}-1\right)$$
$$+\left(d_0\left(\tilde{h}^p_{dij},\tilde{0}\right)+d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)\right)d_{ij}l_j\Bigg) \tag{5.15}$$

subject to $\left(\left(n_{ij}-k_{ij}\right)q_{ij}-\left(n_{ij}-k_{ij}-1\right)q_{ij}\frac{d_{ij}}{p_i}\right)\leq I_{\max}$, $q_{ij}>0$, $k_{ij}\leq n_{ij}-1$, m_{ij}, n_{ij} and l_j are positive integers.

6. Solution procedure

In this section, we derive the optimal value of q_{ij} and demonstrate the concavity of the integrated profit function with respect to the decision variable q_{ij}. In this model, the number of shipment n_{ij}, the number of payment m_{ij} and the number of delayed deliveries k_{ij} are assumed to be positive integer variables. The given integrated profit function (5.12)–(5.15) seems to be non-linear. To solve this non-linear programming problem, we have focused on some property to obtain the optimal solutions.

Property 6.1. For given values of m_{ij}, n_{ij} and $l_j\in[l_{j,f},l_{j,f-1}]$, $d_0(\tilde{P}_c,\tilde{0})$ is concave in q_{ij}.

Proof. On taking the first and second order partial derivatives of (5.12)–(5.15) with respect to q_{ij}, we obtain

For Case 1.

$$\frac{\partial d_0\left(\tilde{P}_1\left(q_{ij}|m^*_{ij},n^*_{ij},l^*_j\right),\tilde{0}\right)}{\partial q_{ij}}$$
$$=\frac{B(l_j)d_{ij}}{q^2_{ij}}-\frac{d_{ij}}{2p_i}\left(d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)+d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)-d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(n_{ij}-1\right)\right)$$
$$-d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(\frac{n_{ij}}{2}-\frac{d_{ij}\left(n_{ij}-1\right)}{2p_i}\right)+\frac{d_{ij}}{q^2_{ij}}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+\frac{n_{ij}I_{bij}}{2m_{ij}}d_0\left(\tilde{c}_{cij},\tilde{0}\right)$$
$$+\frac{d_{ij}}{n_{ij}q^2_{ij}}d_0\left(\tilde{S}_{vi},\tilde{0}\right)-\frac{n_{ij}\left(m_{ij}+1\right)}{2m_{ij}}d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)+\frac{m_{ij}d_{ij}}{n_{ij}q^2_{ij}}d_0\left(\tilde{c}_{tij},\tilde{0}\right) \tag{6.1}$$

and

$$\frac{\partial^2 d_0\left(\tilde{P}_1\left(q_{ij}|m^*_{ij},n^*_{ij},l^*_j\right),\tilde{0}\right)}{\partial q^2_{ij}}=-\frac{2d_{ij}B\left(l_j\right)}{q^3_{ij}}-\frac{2d_{ij}}{q^3_{ij}}d_0\left(\tilde{O}_{bij},\tilde{0}\right)-\frac{2d_{ij}}{n_{ij}q^3_{ij}}d_0\left(\tilde{S}_{vi},\tilde{0}\right)-\frac{2m_{ij}d_{ij}}{n_{ij}q^3_{ij}}d_0\left(\tilde{c}_{tij},\tilde{0}\right)<0. \tag{6.2}$$

For Case 2.

$$\frac{\partial d_0\left(\tilde{P}_2\left(q_{ij}|m^*_{ij},n^*_{ij},l^*_j\right),\tilde{0}\right)}{\partial q_{ij}}=\frac{B\left(l_j\right)d_{ij}}{q^2_{ij}}-\frac{d_{ij}}{2p_i}\left(d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)+d_0\left(\tilde{h}^p_{mij},\tilde{0}\right)-d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(n_{ij}-1\right)\right)$$
$$-d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(\frac{n_{ij}}{2}-\frac{d_{ij}\left(n_{ij}-1\right)}{2p_i}\right)+\frac{d_{ij}}{q^2_{ij}}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+\frac{n_{ij}I_{bij}}{2m_{ij}}d_0\left(\tilde{c}_{cij},\tilde{0}\right)$$
$$\times\left(2\alpha_{ij}+2\beta_{ij}\left(\alpha_{ij}+1\right)+1\right)+\frac{d_{ij}}{n_{ij}q^2_{ij}}d_0\left(\tilde{S}_{vi},\tilde{0}\right)-\frac{n_{ij}}{2m_{ij}}d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)$$
$$\times\left(2\alpha_{ij}+m_{ij}+2\beta_{ij}\left(\alpha_{ij}+1\right)+1\right)+\frac{m_{ij}d_{ij}}{n_{ij}q^2_{ij}}d_0\left(\tilde{c}_{tij},\tilde{0}\right) \tag{6.3}$$

and

$$\frac{\partial^2 d_0\left(\tilde{P}_2\left(q_{ij}|m_{ij}^*, n_{ij}^*, l_j^*\right), \tilde{0}\right)}{\partial q_{ij}^2} = -\frac{2d_{ij}B\left(l_j\right)}{q_{ij}^3} - \frac{2d_{ij}}{q_{ij}^3}d_0\left(\tilde{O}_{bij}, \tilde{0}\right) - \frac{2d_{ij}}{n_{ij}q_{ij}^3}d_0\left(\tilde{S}_{vi}, \tilde{0}\right) - \frac{2m_{ij}d_{ij}}{n_{ij}q_{ij}^3}d_0\left(\tilde{c}_{tij}, \tilde{0}\right) < 0. \tag{6.4}$$

For Case 3.

$$\begin{aligned}\frac{\partial d_0\left(\tilde{P}_3\left(q_{ij}|m_{ij}^*, n_{ij}^*, k_{ij}^*, l_j^*\right), \tilde{0}\right)}{\partial q_{ij}} = {} & \frac{B\left(l_j\right)d_{ij}}{q_{ij}^2} - \frac{d_{ij}}{2p_i}\left(d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right) + d_0\left(\tilde{h}_{mij}^p, \tilde{0}\right) - d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right)\left(n_{ij}-1\right)\right) \\ & - d_0\left(\tilde{h}_{rij}^p, \tilde{0}\right)\left(\frac{n_{ij}}{2} - \frac{d_{ij}\left(n_{ij}-1\right)}{2p_i}\right) + \frac{d_{ij}}{q_{ij}^2}d_0\left(\tilde{O}_{bij}, \tilde{0}\right) + \frac{n_{ij}I_{bij}}{2m_{ij}}d_0\left(\tilde{c}_{cij}, \tilde{0}\right) \\ & + \frac{d_{ij}}{n_{ij}q_{ij}^2}d_0\left(\tilde{S}_{vi}, \tilde{0}\right) - \frac{n_{ij}\left(m_{ij}+1\right)}{2m_{ij}}d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right) + \frac{m_{ij}d_{ij}}{n_{ij}q_{ij}^2}d_0\left(\tilde{c}_{tij}, \tilde{0}\right) \\ & + \frac{\left(d_{ij}-p_i\right)\left(n_{ij}-1\right)}{2p_i}\left(d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right) - d_0\left(\tilde{h}_{rij}^p, \tilde{0}\right)\right)\end{aligned} \tag{6.5}$$

and

$$\begin{aligned}\frac{\partial^2 d_0\left(\tilde{P}_3\left(q_{ij}|m_{ij}^*, n_{ij}^*, k_{ij}^*, l_j^*\right), \tilde{0}\right)}{\partial q_{ij}^2} = & -\frac{2d_{ij}B\left(l_j\right)}{q_{ij}^3} - \frac{2d_{ij}}{q_{ij}^3}d_0\left(\tilde{O}_{bij}, \tilde{0}\right) - \frac{2d_{ij}}{n_{ij}q_{ij}^3}d_0\left(\tilde{S}_{vi}, \tilde{0}\right) \\ & - \frac{2m_{ij}d_{ij}}{n_{ij}q_{ij}^3}d_0\left(\tilde{c}_{tij}, \tilde{0}\right) < 0.\end{aligned} \tag{6.6}$$

For Case 4.

$$\begin{aligned}\frac{\partial d_0\left(\tilde{P}_4\left(q_{ij}|m_{ij}^*, n_{ij}^*, k_{ij}^*, l_j^*\right), \tilde{0}\right)}{\partial q_{ij}} = {} & \frac{B\left(l_j\right)d_{ij}}{q_{ij}^2} - \frac{d_{ij}}{2p_i}\left(d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right) + d_0\left(\tilde{h}_{mij}^p, \tilde{0}\right) - d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right)\left(n_{ij}-1\right)\right) \\ & - d_0\left(\tilde{h}_{rij}^p, \tilde{0}\right)\left(\frac{n_{ij}}{2} - \frac{d_{ij}\left(n_{ij}-1\right)}{2p_i}\right) + \frac{d_{ij}}{q_{ij}^2}d_0\left(\tilde{O}_{bij}, \tilde{0}\right) + \frac{n_{ij}I_{bij}}{2m_{ij}} \\ & \times\left(2\alpha_{ij} + 2\beta_{ij}\left(\alpha_{ij}+1\right)+1\right)d_0\left(\tilde{c}_{cij}, \tilde{0}\right) + \frac{d_{ij}}{n_{ij}q_{ij}^2}d_0\left(\tilde{S}_{vi}, \tilde{0}\right) \\ & - \frac{n_{ij}\left(2\alpha_{ij} + m_{ij} + 2\beta_{ij}\left(\alpha_{ij}+1\right)+1\right)}{2m_{ij}}d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right) + \frac{m_{ij}d_{ij}}{n_{ij}q_{ij}^2}d_0\left(\tilde{c}_{tij}, \tilde{0}\right) \\ & + \frac{\left(d_{ij}-p_i\right)\left(n_{ij}-1\right)}{2p_i}\left(d_0\left(\tilde{h}_{mij}^f, \tilde{0}\right) - d_0\left(\tilde{h}_{rij}^p, \tilde{0}\right)\right)\end{aligned} \tag{6.7}$$

and

$$\begin{aligned}\frac{\partial^2 d_0\left(\tilde{P}_4\left(q_{ij}|m_{ij}^*, n_{ij}^*, k_{ij}^*, l_j^*\right), \tilde{0}\right)}{\partial q_{ij}^2} = & -\frac{2d_{ij}B\left(l_j\right)}{q_{ij}^3} - \frac{2d_{ij}}{q_{ij}^3}d_0\left(\tilde{O}_{bij}, \tilde{0}\right) \\ & - \frac{2d_{ij}}{n_{ij}q_{ij}^3}d_0\left(\tilde{S}_{vi}, \tilde{0}\right) - \frac{2m_{ij}d_{ij}}{n_{ij}q_{ij}^3}d_0\left(\tilde{c}_{tij}, \tilde{0}\right) < 0.\end{aligned} \tag{6.8}$$

Therefore, for fixed m_{ij}, n_{ij} and $l_j \in [l_{j,f}, l_{j,f-1}]$, $d_0(\tilde{P}_c, \tilde{0})$ is concave in q_{ij}. Hence, this completes the proof of Property 6.1. □

Result. From the Property 6.1,

For Case 1. We obtain the optimal value of q_{ij} (6.9) by equating (6.1) to zero, which maximize the $d_0(\tilde{P}_1,\tilde{0})$.

$$q_{ij}=\sqrt{\frac{2m_{ij}d_{ij}p_i\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)+n_{ij}B\left(l_j\right)+n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)\right)}{n_{ij}\left(d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(m_{ij}d_{ij}-m_{ij}n_{ij}d_{ij}+m_{ij}n_{ij}p_i\right)+\kappa_1+\Gamma_1\right)}} \tag{6.9}$$

where $\kappa_1 = d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(2m_{ij}d_{ij}+n_{ij}p_i-m_{ij}n_{ij}d_{ij}+m_{ij}n_{ij}p_i\right)$ and $\Gamma_1 = m_{ij}d_{ij}d_0\left(\tilde{h}^p_{mij},\tilde{0}\right) - n_{ij}I_{bij}p_id_0\left(\tilde{c}_{cij},\tilde{0}\right)$.

For Case 2. We obtain the optimal value of q_{ij} in (6.10) by equating (6.3) to zero, which maximize the $d_0(\tilde{P}_2,\tilde{0})$.

$$q_{ij}=\sqrt{\frac{2m_{ij}d_{ij}p_i\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)+n_{ij}B\left(l_j\right)+n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)\right)}{n_{ij}\left(d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(m_{ij}d_{ij}-m_{ij}n_{ij}d_{ij}+m_{ij}n_{ij}p_i\right)+\kappa_2+\Gamma_2\right)}} \tag{6.10}$$

where $\kappa_2 = d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(2d_{ij}m_{ij}-d_{ij}m_{ij}n_{ij}+2p_i\alpha_{ij}n_{ij}+2p_i\beta_{ij}n_{ij}+p_im_{ij}n_{ij}+2p_i\alpha_{ij}\beta_{ij}n_{ij}\right)$ and $\Gamma_2 = m_{ij}d_{ij}d_0\left(\tilde{h}^p_{mij},\tilde{0}\right) - I_{bij}p_in_{ij}d_0(\tilde{c}_{cij},0)\left(1+2\alpha_{ij}+2\beta_{ij}+2\alpha_{ij}\beta_{ij}\right)$.

For Case 3. We obtain the optimal value of q_{ij} in (6.11) by equating (6.5) to zero, which maximize the $d_0\left(\tilde{P}_3,\tilde{0}\right)$.

$$q_{ij}=\sqrt{\frac{2m_{ij}d_{ij}p_i\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)+n_{ij}B\left(l_j\right)+n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)\right)}{n_{ij}\left(d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(m_{ij}p_i\right)+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(2m_{ij}d_{ij}+n_{ij}p_i-m_{ij}n_{ij}d_{ij}+m_{ij}n_{ij}p_i\right)+\Gamma_3\right)}} \tag{6.11}$$

where $\Gamma_3 = \left(2d_{ij}m_{ij}-p_im_{ij}-d_{ij}m_{ij}n_{ij}+p_im_{ij}n_{ij}\right)d_0\left(\tilde{h}^p_{mij},\tilde{0}\right) - I_{bij}p_in_{ij}d_0\left(\tilde{c}_{cij},\tilde{0}\right)$.

For Case 4. We obtain the optimal value of q_{ij} in (6.12) by equating (6.7) to zero, which maximize the $d_0\left(\tilde{P}_4,\tilde{0}\right)$.

$$q_{ij}=\sqrt{\frac{2m_{ij}d_{ij}p_i\left(d_0\left(\tilde{S}_{vi},\tilde{0}\right)+n_{ij}B\left(l_j\right)+n_{ij}d_0\left(\tilde{O}_{bij},\tilde{0}\right)+m_{ij}d_0\left(\tilde{c}_{tij},\tilde{0}\right)\right)}{n_{ij}\left(d_0\left(\tilde{h}^p_{rij},\tilde{0}\right)\left(m_{ij}p_i\right)+d_0\left(\tilde{h}^f_{mij},\tilde{0}\right)\left(2d_{ij}+p_in_{ij}-d_{ij}m_{ij}n_{ij}+\kappa_4\right)+\Gamma_4\right)}} \tag{6.12}$$

where $\kappa_4 = 2p_i\alpha_{ij}n_{ij}+2p_i\beta_{ij}n_{ij}+p_im_{ij}n_{ij}+2p_i\alpha_{ij}\beta_{ij}n_{ij}$ and $\Gamma_4 = \left(2d_{ij}m_{ij}-p_im_{ij}-d_{ij}m_{ij}n_{ij}+p_im_{ij}n_{ij}\right)d_0\left(\tilde{h}^p_{mij},\tilde{0}\right) - I_{bij}p_in_{ij}\left(1+2\alpha_{ij}n_{ij}+2\beta_{ij}+2\alpha_{ij}\beta_{ij}\right)d_0\left(\tilde{c}_{cij},\tilde{0}\right)$.

Property 6.2. For fixed values of m_{ij}, n_{ij} and q_{ij}, the integrated profit function $d_0\left(\tilde{P}_c\left(l_j\right),\tilde{0}\right)$, where $c = 1,2,3,4$ is linear on l_j.

Proof. On taking the first order derivatives of profit functions (5.12)–(5.15) will lead to

$$\frac{\partial d_0\left(\tilde{P}_c\left(l_j\right),\tilde{0}\right)}{\partial l_j} = d_{ij}\left[\frac{e_j}{q_{ij}} - \left(h^p_{dij}+\frac{1}{4}\left(\varphi_{h^p_{dij4}}+\varphi_{h^p_{dij3}}-\varphi_{h^p_{dij2}}-\varphi_{h^p_{dij1}}\right)+h^f_{mij} + \frac{1}{4}\left(\varphi_{h^f_{mij4}}+\varphi_{h^f_{mij3}}-\varphi_{h^f_{mij2}}-\varphi_{h^f_{mij1}}\right)\right)\right] \tag{6.13}$$

TABLE 2. Input parameter for trapezoidal fuzzy number.

Fuzzy parameters of S_{vi}	Item i			Fuzzy parameters of c_{pri}	Item i			Fuzzy parameters of c_{pi}	Item i		
	1	2	3		1	2	3		1	2	3
$\varphi_{S_{vi1}}$	300	300	300	$\varphi_{c_{pri1}}$	0.7	0.7	0.7	$\varphi_{c_{pi1}}$	2.6	2.3	2
$\varphi_{S_{vi2}}$	200	200	200	$\varphi_{c_{pri2}}$	0.4	0.4	0.4	$\varphi_{c_{pi2}}$	1.6	1.3	1
$\varphi_{S_{vi3}}$	150	150	150	$\varphi_{c_{pri3}}$	0.4	0.4	0.4	$\varphi_{c_{pi3}}$	2	1.8	1.5
$\varphi_{S_{vi4}}$	250	250	250	$\varphi_{c_{pri4}}$	0.9	0.9	0.9	$\varphi_{c_{pi4}}$	2.5	2.3	2.2

where $c = 1, 2, 3, 4$. Equation (6.13) result as a constant. If $\frac{e_j}{q_{ij}} > \left[d_0\left(\tilde{h}^p_{dij}, \tilde{0}\right) + d_0\left(\tilde{h}^f_{mij}, \tilde{0}\right)\right]$, then the $d_0\left(\tilde{P}_c\left(l_j\right), \tilde{0}\right)$ is linear increase on l_j. If $\frac{e_j}{q_{ij}} < \left[d_0\left(\tilde{h}^p_{dij}, \tilde{0}\right) + d_0\left(\tilde{h}^f_{mij}, \tilde{0}\right)\right]$, then the $d_0\left(\tilde{P}_c(l_j), \tilde{0}\right)$ is linear decrease on l_j. If $\frac{e_j}{q_{ij}} = \left[d_0\left(\tilde{h}^p_{dij}, \tilde{0}\right) + d_0\left(\tilde{h}^f_{mij}, \tilde{0}\right)\right]$, then the $d_0\left(\tilde{P}_c(l_j), \tilde{0}\right)$ is flat on l_j. Therefore, under each case, the profit function $d_0\left(\tilde{P}_c(l_j), \tilde{0}\right)$ is linear on l_j. Hence, this completes the proof of Property 6.2. □

For the fixed values of m_{ij}, n_{ij}, and q_{ij}, the maximum $d_0\left(\tilde{P}_c(l_j), \tilde{0}\right)$ always occurs only at the end points of $[l_{j,f}, l_{j,f-1}]$.

7. Numerical analysis

Let us consider a two-tier supply chain model for three items and three consignees, that is z=3, y=3. For simplicity the parameters for three items and three consignee are arranged in the row matrix, *i.e.*, production rate p_i=$[p_1, p_2, p_3]$ is the production rate of ith item, demand $d_{ij} = [d_{11}, d_{12}, d_{13}; d_{21}, d_{22}, d_{23};\ d_{31}, d_{32}, d_{33}]$ is the demand of the ith item for jth consignee.

The numerical data was taken from Zavanella and Zanoni [40], Sarker [37], Sarkar *et al.* [27] and Zahran *et al.* [39].

Example 7.1. *For Case 1.* $\left(h^p_{mij} > h^p_{rij}\right)$. Parameters related to Consignor: $S_{vi} = [400, 370, 345]$ (\$/*setup*), $c_{pri} = [3, 2.5, 1]$(\$/*unit*), $h^p_{mij} = [15, 14.5, 16; 14.8, 16.7, 15.9; 15, 15.7, 16]$(\$/*unit*/*year*), h^f_{mij} = [0.54, 0.54, 0.54; 0.51, 0.51, 0.51; 0.48, 0.48, 0.48] (\$/*unit*/*year*), c_{bij} = [5.4, 5.4, 5.4; 5.4, 5.4, 5.4;5.4, 5.4,5.4] (\$/*unit*), c_{pi} = [3, 2.8, 2.4] (\$/*unit*). Parameters related to Consignee: O_{bij} = [29, 26, 25; 27, 26.5, 25.5; 26, 29, 26] (\$/*order*), $h^p_{rij} = [7, 6.8, 7; 8, 7, 8; 8.5, 7, 7.8]$(\$/*unit*/*year*), I_{bij} = [0.1, 0.1, 0.1; 0.1, 0.1, 0.1; 0.1, 0.1, 0.1] (\$/*year*), $c_{cij} = [13.9, 14, 12.29; 13.9, 14, 12.29; 13.9, 14, 12.29]$(\$/*unit*), c_{tij} = [0.7, 0.6, 0.75; 0.7, 0.5, 0.6; 0.65, 0.75, 0.66] (\$/*transaction*). General parameters: d_{ij} = [900, 600, 650; 600, 650, 700; 350, 400, 430] (*units*/*year*), p_i = [3200, 2000, 2500] (*units*/*year*), h^p_{dij} = [7, 7.8, 6.2; 8.8, 7.5, 6.8; 6.4, 6.8, 7] (\$/*unit*/*year*), γ_i = [1, 1, 1]. The fuzzy parameteric values are listed out in the Tables 2 and 3. In addition, the lead time has three components with data shown in Table 4 as well as the summarized lead time components information is given in Table 5. The optimal solution, $l_j = [0.0767, 0.0767, 0.0959]$, $B(l_j) = [18.2000, 16.1000, 40.6000]$, $m_{ij} = [1, 1, 1]$, $n_{ij} = [3, 3, 3]$, the order quantity of three items for first consignee, $[q_{11}, q_{21}, q_{31}] = [128.84105, 96.193021, 68.123938]$, for second consignee, $[q_{12}, q_{22}, q_{32}] = [104.70503, 102.54538, 80.836474]$, for third consignee, $[q_{13}, q_{23}, q_{33}] = [111.80159, 106.69594, 82.509551]$, profit of first consignee on three items, $[P^{11}_1, P^{21}_1, P^{31}_1] = [3637.5498, 2382.1809, 1706.8041]$, for second consignee, $[P^{12}_1, P^{22}_1, P^{32}_1] = [2172.4448, 2847.7495, 2207.3677]$, for third consignee, $[P^{13}_1, P^{23}_1, P^{33}_1] = [1044.9304, 1678.4404, 1419.3938]$, the profit of three consignee is $[7726.5348, 7227.5621, 4142.7646]$, the overall profit of the supply chain is 19096.861.

TABLE 3. Input parameter for trapezoidal fuzzy number.

Fuzzy parameter	g	j	i = 1	2	3	g	i = 1	2	3	g	i = 1	2	3	g	i = 1	2	3
$\varphi_{O_{bijg}}$	1	1	25	24	22	2	12	18	16	3	20	17	11	4	24	21	18
		2	26	22	20		24	18	14		17	16	10		25	19	15
		3	21	17	23		17	15	18		15	18	14		19	24	22
$\varphi_{c_{cijg}}$	1	1	6	6	6	2	4	4	4	3	3	3	3	4	5	5	5
		2	6	6	6		4	4	4		3	3	3		5	5	5
		3	6	6	6		4	4	4		3	3	3		5	5	5
$\varphi_{c_{bijg}}$	1	1	5	5	5	2	3	3	3	3	1.5	1.5	1.5	4	3	3	3
		2	4.8	4.8	4.8		4	4	4		1.2	1.2	1.2		2	2	2
		3	3.8	3.8	3.8		2	2	2		0.4	0.4	0.4		1	1	1
$\varphi_{h^p_{rijg}}$	1	1	6	6	6	2	3	3	3	3	2	2	2	4	5	5	5
		2	6	6	6		3	3	3		2	2	2		5	5	5
		3	6	6	6		3	3	3		2	2	2		5	5	5
$\varphi_{h^p_{mijg}}$	1	1	14	14	14	2	10	10	10	3	9	9	9	4	13	13	13
		2	14	14	14		10	10	10		9	9	9		13	13	13
		3	14	14	14		10	10	10		9	9	9		13	13	13
$\varphi_{h^p_{dijg}}$	1	1	6.9	6.6	6	2	6	5.8	4.8	3	5	4	6	4	6	5.6	6.7
		2	7	6.5	6		4.8	3.7	5.7		6	6.7	4		8	7	5.6
		3	6	6.6	6.4		4.4	4.7	5.4		5.2	6	5.5		6	6.3	6.8
$\varphi_{h^f_{mijg}}$	1	1	0.45	0.40	0.35	2	0.38	0.35	0.28	3	0.34	0.34	0.30	4	0.48	0.44	0.40
		2	0.48	0.35	0.25		0.38	0.28	0.18		0.27	0.35	0.25		0.30	0.40	0.35
		3	0.45	0.38	0.30		0.38	0.28	0.26		0.38	0.34	0.32		0.40	0.46	0.47
$\varphi_{c_{tijg}}$	1	1	0.6	0.4	0.57	2	0.45	0.34	0.35	3	0.35	0.25	0.34	4	0.48	0.58	0.68
		2	0.65	0.43	0.58		0.28	0.36	0.42		0.34	0.25	0.45		0.68	0.43	0.57
		3	0.61	0.72	0.62		0.31	0.45	0.45		0.25	0.19	0.28		0.48	0.46	0.58

TABLE 4. Lead time components.

Consignee j	Lead time components k	Normal duration $n_{j,k}$ (year)	Minimum duration $m_{j,k}$ (year)	Unit crashing cost $e_{j,k}$ (\$/year)
1	1	$20/365 = 0.05479$	$6/365 = 0.01644$	$0.1 \times 365 = 36.5$
	2	$20/365 = 0.05479$	$6/365 = 0.01644$	$1.2 \times 365 = 438$
	3	$16/365 = 0.04383$	$9/365 = 0.02465$	$5.0 \times 365 = 1825$
2	1	$20/365 = 0.05479$	$6/365 = 0.01644$	$0.5 \times 365 = 182.5$
	2	$16/365 = 0.04383$	$9/365 = 0.02465$	$1.3 \times 365 = 474.5$
	3	$13/365 = 0.035616$	$6/365 = 0.01644$	$5.1 \times 365 = 1861.5$
3	1	$25/365 = 0.06849$	$11/365 = 0.03013$	$0.4 \times 365 = 146$
	2	$20/365 = 0.05479$	$6/365 = 0.01644$	$2.5 \times 365 = 912.5$
	3	$18/365 = 0.04931$	$11/365 = 0.03013$	$5.0 \times 365 = 1825$

Example 7.2. *For Case 2.* $\left(h^p_{mij} > h^p_{rij}\right)$. Parameters related to Consignor: I_{vij} = [0.1, 0.1, 0.1; 0.1, 0.1, 0.1; 0.1, 0.1, 0.1] (\$/*year*), General parameters: α_{ij} = [0.2, 0.2, 0.2; 0.2, 0.2, 0.2; 0.2, 0.2, 0.2], β_{ij} = [0.1, 0.1, 0.1; 0.1, 0.1, 0.1; 0.1, 0.1, 0.1]. The rest of the parameteric values are same as in the Example 7.1. The optimum solution, l_j = [0.0767, 0.0767, 0.0959], $B(l_j)$ = [18.2000, 16.1000, 40.6000], m_{ij} = [1, 1, 1], n_{ij} = [3, 3, 3], the optimal order quantity is given in the Table 6 the profit on three items for three consignee are $[P_2^{11}, P_2^{21}, P_2^{21}]$ = [3738.91, 2467.3379, 1764.9678], $[P_2^{12}, P_2^{22}, P_2^{22}]$ = [2254.8835, 2929.1406, 2273.6092], $[P_2^{13}, P_2^{23}, P_2^{23}]$ = [1112.6775, 1743.5042, 1470.9505], the profit of three consignee is [7971.2157, 7457.6333, 4327.1322] the overall profit of the supply chain is 19 755.981.

TABLE 5. Summarized lead time data.

Consignee j	Lead time (year)	$B(l_j)$ (\$/*shipment*)
1	$56/365 = 0.15342$	0
	$42/365 = 0.11506$	1.4
	$28/365 = 0.076712$	18.2
	$21/365 = 0.05753$	53.20
2	$49/365 = 0.13424$	0
	$35/365 = 0.095890$	7
	$28/365 = 0.076712$	16.1
	$21/365 = 0.057534$	51.8
3	$63/365 = 0.1726$	0
	$49/365 = 0.13424$	5.6
	$35/365 = 0.09589$	40.6
	$28/365 = 0.076712$	75.6

TABLE 6. Optimal values.

Example/ Case	Consignee j	l_j	k_{ij}	Item m_{ij}	$i = [1, 2, 3]$ n_{ij}	q_{ij}	Profit
1	1	0.0767	–	[1, 1, 1]	[3, 3, 3]	[128.84105, 96.193021, 68.123938]	19 096.861
	2	0.0767		[1, 1, 1]	[3, 3, 3]	[104.70503, 102.54538, 80.836474]	
	3	0.0959		[1, 1, 1]	[3, 3, 3]	[111.80159, 106.69594, 82.509551]	
2	1	0.0767	–	[1, 1, 1]	[3, 3, 3]	[134.41367, 100.4769, 70.809769]	**19 755.981**
	2	0.0767		[1, 1, 1]	[3, 3, 3]	[109.4138, 106.88374, 84.518569]	
	3	0.0959		[1, 1, 1]	[3, 3, 3]	[115.34919, 109.70609, 84.961654]	
3	1	0.1151	[4, 4, 4]	[1, 1, 1]	[5, 5, 5]	[76.399709, 57.672644, 39.242718]	17 179.512
	2	0.0959	[4, 4, 4]	[1, 1, 1]	[5, 5, 5]	[61.565756, 62.568235, 47.485165]	
	3	0.1342	[4, 4, 4]	[1, 1, 1]	[5, 5, 5]	[60.549037, 61.549789, 44.872583]	
4	1	0.1151	[5, 5, 5]	[1, 1, 1]	[6, 6, 6]	[70.557926, 53.168928, 35.554063]	17832.472
	2	0.0959	[5, 5, 5]	[1, 1, 1]	[6, 6, 6]	[56.473436, 58.181526, 43.846176]	
	3	0.1342	[5, 5, 5]	[1, 1, 1]	[6, 6, 6]	[54.848228, 56.1366, 40.588412]	

Notes. The results of the four examples/cases are given in Table 6. Of those four results, we have bolded the value (i.e., "19755.981") to show that Example 7.2/Case 2 results are more profitable compared to the other three examples/cases results.

Example 7.3. *For Case 3.* $\left(h^p_{mij} < h^p_{rij}\right)$. This example takes the data from the Example 7.1 excluding the physical holding cost of consignor h^p_{mij} and consignee h^p_{rij}. Instead we take h^p_{mij} = [7, 6.8, 7; 8, 7, 8; 8.5, 7, 7.8] (\$/*unit*/*year*), h^p_{rij} = [15, 14.5, 16; 14.8, 16.7, 15.9; 15, 15.7, 16] (\$/*unit*/*year*), $\varphi_{h^p_{rij4}}$ = [13, 13, 13; 13, 13, 13; 13, 13, 13], $\varphi_{h^p_{rij3}}$ = [9, 9, 9; 9, 9, 9; 9, 9, 9], $\varphi_{h^p_{rij2}}$ = [10, 10, 10; 10, 10, 10; 10, 10, 10], $\varphi_{h^p_{rij1}}$ = [14, 14, 14; 14, 14, 14; 14, 14, 14], $\varphi_{h^p_{mij4}}$ = [5, 5, 5; 5, 5, 5; 5, 5, 5], $\varphi_{h^p_{mij3}}$ = [2, 2, 2; 2, 2, 2; 2, 2, 2], $\varphi_{h^p_{mij2}}$ = [3, 3, 3; 3, 3, 3; 3, 3, 3], $\varphi_{h^p_{mij1}}$ = [6, 6, 6; 6, 6, 6; 6, 6, 6]. The optimum values, l_j = [0.1151, 0.0959, 0.1342], $B(l_j)$ = [1.4000, 7.0000, 5.6000], $k_{ij} = [4, 4, 4]$, $m_{ij} = [1, 1, 1]$, $n_{ij} = [5, 5, 5]$, the profit on three items for three consignee are $[P^{11}_3, P^{21}_3, P^{21}_3] = [3299.3444, 2145.4999, 1539.6257]$, $[P^{12}_3, P^{22}_3, P^{22}_3] = [1902.2614, 2615.3974, 1949.3164]$, $[P^{13}_3, P^{23}_3, P^{23}_3] = [834.41271, 1615.2576, 1278.3967]$, the total profit earned by the each conisgnee is [6984.4699, 6466.9752, 3728.067], the overall profit of supply chain is 17 179.512.

Example 7.4. *For Case 4.* $\left(h^p_{mij} < h^p_{rij}\right)$. The parametric values of $I_{vij}, \alpha_{ij}, \beta_{ij}$ are same as in the Example 7.2 and the values of h^p_{mij}, h^p_{rij} are taken in account from Example 7.3. The remaining data are same as from the Example 7.1. The optimum values, $l_j = [0.1151, 0.0959, 0.1342]$, $B(l_j)$ =

$[1.4000, 7.0000, 5.6000]$, $k_{ij} = [5, 5, 5]$, $m_{ij} = [1, 1, 1]$, $n_{ij} = [6, 6, 6]$, the profit on three items for three consignee are $[P_4^{11}, P_4^{21}, P_4^{21}] = [3410.6661, 2239.1973, 1581.7952]$, $[P_4^{12}, P_4^{22}, P_4^{22}] = [1980.9267, 2716.1003, 1999.9991]$, $[P_4^{13}, P_4^{23}, P_4^{23}] = [901.17809, 1689.6391, 1312.9704]$, the total profit of three consignee is [7231.6586, 6697.0261, 3903.7877], the overall profit of supply chain is 17 832.472.

The optimal solution for all four examples (four cases) are given in Table 6 and the profit obtained from the above four examples are compared in graphical representation 5.

8. Discussion of the results

This section describes the discussion of the numerical results of the proposed model.

(1) The results of the four Examples 7.1–7.4 demonstrate that the supply chain achieves more profitability if the consignor has a higher physical holding cost h^p_{mij} than the consignee's physical holding cost h^p_{rij}.
(2) If h^p_{mij} is less than h^p_{rij}, the proposed model yields a lower profit, as is clear from the results of Cases 3 and 4.
(3) The delay in shipment affects the profitability of the supply chain, which can be clearly seen in Figure 5.
(4) From the results of Example 7.2, it is evident that delaying the payment strategy can lead to higher profits.

9. Managerial insights

This article analyzes the best strategy to maximize profits through late shipments and late payments. Moreover, it gives a comparison in four cases with respect to delay in payment and delay in shipment under a controllable lead time, and the managerial insights from those comparisons are as follows:

(1) The CS agreement policy favours both the consignor and the consignee, who can save funds by sharing the cost of holding the goods physically and financially.
(2) The manager will get more profit if case 2 (CS policy with DIP – NDIS under CLT) is established than the other three cases.
(3) In order to relate this model to reality, all basic inventory costs for y items and z consignees are considered to be imprecise, which can be very helpful for managers in dealing with ambiguous situations.
(4) Under the CS policy, the consignee is not required to pay until the products are sold. Whereas, if the consignee is unable to sell all those products, they can return the products to the consignor, therefore, the consignor has to face the risks and rewards of ownership.

10. Conclusion and future directions

This article has considered a single-consignor multi-consignee for multi-item under controllable lead time in a fuzzy environment. This paper adopts the CS policy, in particular, which is more beneficial for the consignor. This paper compares four different cases to show which cases are the most profitable for the supply chain. Moreover, this paper studies the impacts of controllable lead time for multiple consignees, which is a more critical and practical factor, and this never been studied under CS policies. The numerical results showed that the supply chain players (consignor and consignee) attain the highest profit in case 4 compared to case 3, and case 1 attains higher profit than case 4. However, case 2 was shown to be the most profitable when compared to all other cases. This model can be extended in many ways; basically, this model has some limitations, so it is possible to develop this model by resolving these limitations. Primarily in this model, we assume that the production process is perfect, so it can be extended by turning this model into an imperfect production process. Another extension is possible by relaxing the equal-sized shipment and fixed demand rate in the proposed model (refer, [11,38] and Ganesh Kumar and Uthayakumar [10] for unequal-sized shipments, Sarkar *et al.* [29] for price and advertisement-dependent demand, Karthick and Uthayakumar [19, 20] for fuzzy demand). Exploring the changes that occur in this model by combining concepts such as learning and forgetting can be considered an extension (refer, [13]). The production rate is assumed to be constant in this model, so considering the production

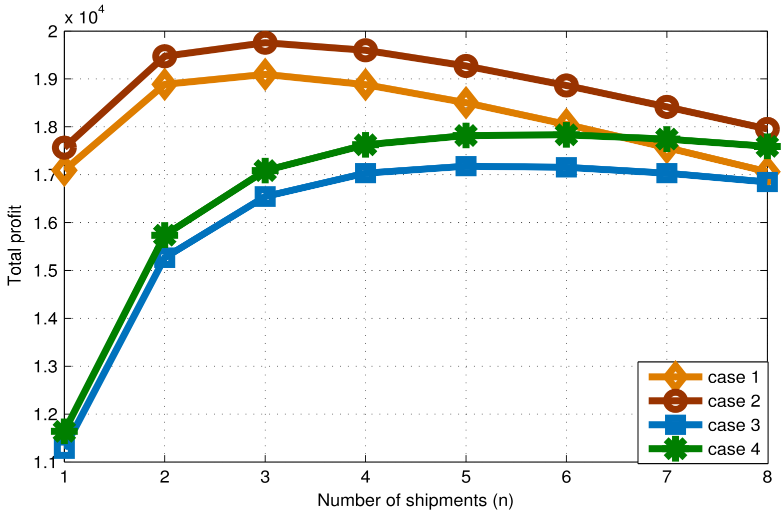

FIGURE 5. Total profit $d_0(\tilde{P}_c, \tilde{0})$ with respect to number of shipments (n_{ij}).

rate as a variable for a flexible production process is another extension (refer, [17,27]). The incorporation of the consignee's royalty reduction (refer, [28]) and radio frequency identification (refer, [31]) in the CS policy would be a reasonable extension of this model.

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