SOME DEGREE CONDITIONS FOR $\mathcal{P}_{\geq k}$-FACTOR COVERED GRAPHS

Guowei Dai$^1$, Zan-Bo Zhang$^2$, Yicheng Hang$^1$ and Xiaoyan Zhang$^{1,*}$

Abstract. A spanning subgraph of a graph $G$ is called a path-factor of $G$ if its each component is a path. A path-factor is called a $\mathcal{P}_{\geq k}$-factor of $G$ if its each component admits at least $k$ vertices, where $k \geq 2$. (Zhang and Zhou, Discrete Math. 309 (2009) 2067–2076) defined the concept of $\mathcal{P}_{\geq k}$-factor covered graphs, i.e., $G$ is called a $\mathcal{P}_{\geq k}$-factor covered graph if it has a $\mathcal{P}_{\geq k}$-factor covering $e$ for any $e \in E(G)$. In this paper, we firstly obtain a minimum degree condition for a planar graph being a $\mathcal{P}_{\geq 2}$-factor and $\mathcal{P}_{\geq 3}$-factor covered graph, respectively. Secondly, we investigate the relationship between the maximum degree of any pairs of non-adjacent vertices and $\mathcal{P}_{\geq k}$-factor covered graphs, and obtain a sufficient condition for the existence of $\mathcal{P}_{\geq 2}$-factor and $\mathcal{P}_{\geq 3}$-factor covered graphs, respectively.

Mathematics Subject Classification. 05C70, 05C38.

Received December 15, 2020. Accepted September 15, 2021.

1. Introduction

The graphs considered here are finite and simple, unless explicitly stated. Let $G = (V(G), E(G))$ be a graph. We denote by $V(G)$ and $E(G)$ the vertex set and the edge set of $G$, respectively. A spanning subgraph of $G$ is a subgraph $H$ of $G$ such that $V(H) = V(G)$ and $E(H) \subseteq E(G)$. A subgraph $H$ of $G$ is called an induced subgraph of $G$ if every pair of vertices in $H$ which are adjacent in $G$ are also adjacent in $H$. For $v \in V(G)$, we use $d_G(v)$ and $N_G(v)$ to denote the degree of $v$ and the set of vertices adjacent to $v$ in $G$, respectively. For $S \subseteq V(G)$, we write $N_G(S) = \bigcup_{v \in S} N_G(v)$. We use $\delta(G)$ to denote the minimum degree of a graph $G$. We refer to [5] for the notation and terminologies not defined here.

For a family of connected graphs $\mathcal{F}$, a spanning subgraph of a graph $G$ is called an $\mathcal{F}$-factor of $G$ if its each component is isomorphic to some graph in $\mathcal{F}$. In particular, an $\mathcal{F}$-factor is called a $\mathcal{P}_{\geq k}$-factor of $G$ if every component in $\mathcal{F}$ is a path of order at least $k$, where $k \geq 2$. A graph $G$ is called a $\mathcal{P}_{\geq k}$-factor covered graph if it has a $\mathcal{P}_{\geq k}$-factor covering $e$ for any $e \in E(G)$.

Since Tutte proposed the well known Tutte 1-factor theorem [15], there are many results on graph factors [1,3,8,10,16] and $\mathcal{P}_{\geq k}$-factors in claw-free graphs and cubic graphs [4,12,13]. More results on graph factors can be found in the survey papers and books in [1,14,19]. We use $\omega(G)$, $i(G)$ to denote the number of components and isolated vertices of a graph $G$, respectively. For a subset $X \subseteq V(G)$, $G - X$ denotes the graph obtained from $G$ by removing all vertices in $X$ and its incident edges.

Keywords. Graph, $\mathcal{P}_{\geq 2}$-factor covered graph, $\mathcal{P}_{\geq 3}$-factor covered graph, planar graph, Minimum degree.

1 School of Mathematical Science & Institute of Mathematics, Nanjing Normal University, Nanjing, Jiangsu 210023, P.R. China.
2 School of Statistics & Mathematics, and Institute of Artificial Intelligence & Deep Learning, Guangdong University of Finance & Economics, Guangzhou, Guangdong 510630, P.R. China.
* Corresponding author: zhangxiaoyan@njnu.edu.cn; zhang_njnu@aliyun.com

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from $G$ by deleting all the vertices of $X$. Akiyama, Avis and Era [2] proved the following theorem, which is a criterion for a graph to have a $\mathcal{P}_{\geq 2}$-factor.

**Theorem 1.1.** (Akiyama et al. [1]) A graph $G$ has a $\mathcal{P}_{\geq 2}$-factor if and only if $i(G-X) \leq 2|X|$ for all $X \subseteq V(G)$.

By introducing the concept of a sun, Kaneko [9] gave a criterion for a graph with a $\mathcal{P}_{\geq 3}$-factor. Recently, a simpler proof for Kaneko’s theorem [9] was presented by Kano et al. [11].

A graph $H$ is called factor-critical if $H - \{v\}$ has a 1-factor for each $v \in V(H)$. Let $H$ be a factor-critical graph and $V(H) = \{v_1, v_2, \ldots, v_n\}$. By adding new vertices $\{u_1, u_2, \ldots, u_n\}$ together with new edges $\{v_i u_i : 1 \leq i \leq n\}$ to $H$, the resulting graph is called a sun. Note that, according to Kaneko [9], we regard

**Theorem 1.2.** (Kaneko [9]) A graph $G$ has a $\mathcal{P}_{\geq 3}$-factor if and only if $\text{sun}(G-X) \leq 2|X|$ for all $X \subseteq V(G)$.

Zhang and Zhou [20] proposed the concept of path-factor covered graph, which is a generalization of matching cover. They also obtained a characterization for $\mathcal{P}_{\geq 2}$-factor and $\mathcal{P}_{\geq 3}$-factor covered graphs, respectively.

**Theorem 1.3.** (Zhang and Zhou [20]) Let $G$ be a connected graph. Then $G$ is a $\mathcal{P}_{\geq 2}$-factor covered graph if and only if $i(G-S) \leq 2|S| - \varepsilon_1(S)$ for all $S \subseteq V(G)$, where $\varepsilon_1(S)$ is defined by

$$\varepsilon_1(S) = \begin{cases} 
2 & \text{if } S \neq \emptyset \text{ and } S \text{ is not an independent set;} \\
1 & \text{if } S \text{ is a nonempty independent set and there exists a nontrivial component of } G-S; \\
0 & \text{otherwise.}
\end{cases}$$

**Theorem 1.4.** (Zhang and Zhou [20]) Let $G$ be a connected graph. Then $G$ is a $\mathcal{P}_{\geq 3}$-factor covered graph if and only if $\text{sun}(G-S) \leq 2|S| - \varepsilon_2(S)$ for all $S \subseteq V(G)$, where $\varepsilon_2(S)$ is defined by

$$\varepsilon_2(S) = \begin{cases} 
2 & \text{if } S \neq \emptyset \text{ and } S \text{ is not an independent set;} \\
1 & \text{if } S \text{ is a nonempty independent set and there exists a non-sun component of } G-S; \\
0 & \text{otherwise.}
\end{cases}$$

For a connected graph $G$, its isolated toughness, denoted by $I_t(G)$, was first introduced by Yang et al. [18] as follows. If $G$ is complete, then $I_t(G) = +\infty$; otherwise,

$$I_t(G) = \min \left\{ \frac{|S|}{i(G-S)} : S \subseteq V(G), i(G-S) \geq 2 \right\}.$$

The binding number is introduced by Woodall [17] and defined as

$$\text{bind}(G) = \min \left\{ \frac{|N_G(S)|}{|S|} : \emptyset \neq S \subseteq V(G), N_G(S) \neq V(G) \right\}.$$

Recently, Zhou [21] and Dai [7] obtained some classes of $\mathcal{P}_{\geq 2}$-factor covered graphs, respectively.

**Theorem 1.5.** (Zhou [21]) Let $G$ be a connected graph. Then $G$ is a $\mathcal{P}_{\geq 2}$-factor covered graph if $\text{bind}(G) > 2/3$.

**Theorem 1.6.** (Dai [7]) Let $G$ be a connected graph of order at least two. Then $G$ is a $\mathcal{P}_{\geq 2}$-factor covered graph if one of the following holds: (i) $G$ is claw-free and $\delta(G) \geq 2$; (ii) $I_t(G) > 2/3$. 


For a connected graph $G$, its \textit{toughness}, denoted by $t(G)$, was first introduced by Chvátal [6] as follows. If $G$ is complete, then $t(G) = +\infty$; otherwise,

$$t(G) = \min \left\{ \frac{|S|}{\omega(G-S)} : S \subseteq V(G), \omega(G-S) \geq 2 \right\}.$$ 

Using Theorem 1.4, Zhou et al. [22] and Dai [7] obtained some classes of $\mathcal{P}_{\geq k}$-factor covered graphs, respectively.

\textbf{Theorem 1.7.} (Zhou et al. [22], Zhou [21]) Let $G$ be a connected graph of order at least three. Then $G$ is a $\mathcal{P}_{\geq 3}$-factor covered graph if one the following holds: (i) $\text{bind}(G) \geq 3/2$; (ii) $t(G) > 2/3$; (iii) $I_1(G) > 5/3$; (iv) $G$ is $r$-regular where $r \geq 2$.

\textbf{Theorem 1.8.} (Dai [7]) Let $G$ be a connected graph of order at least three. Then $G$ is a $\mathcal{P}_{\geq 3}$-factor covered graph if one the following holds: (i) $G$ is claw-free and $\delta(G) \geq 3$; (ii) $G$ is a 3-connected planar graph.

In this paper, we proceed to investigate $\mathcal{P}_{\geq k}$-factor covered graphs. We respectively obtain two special classes of $\mathcal{P}_{\geq 2}$-factor covered graphs and $\mathcal{P}_{\geq 3}$-factor covered graphs. Our main results will be shown in Sections 2 and 3, respectively.

\section{2. Minimum degree for $\mathcal{P}_{\geq k}$-factor covered planar graphs}

In this section, we study the relationship between planar graphs and $\mathcal{P}_{\geq k}$-factor covered graphs, and obtain a minimum degree condition for a planar graph being a $\mathcal{P}_{\geq 2}$-factor and $\mathcal{P}_{\geq 3}$-factor covered graph, respectively.

To prove our results, we will use an important lemma as following.

\textbf{Lemma 2.1.} [3] Let $G$ be a connected planar graph with at least three vertices. If $G$ does not contain triangles, then $|E(G)| \leq 2|G| - 4$.

\textbf{Theorem 2.2.} Let $G$ be a connected planar graph of order at least two. If $\delta(G) \geq 3$, then $G$ is a $\mathcal{P}_{\geq 2}$-factor covered graph.

\textit{Proof.} Suppose $G$ is not a $\mathcal{P}_{\geq 2}$-factor covered graph. By Theorem 1.3, there exists a subset $S \subseteq V(G)$ such that $i(G-S) > 2|S| - \varepsilon_1(S)$. According to the integrality of $i(G-S)$, we obtain that $i(G-S) \geq 2|S| - \varepsilon_1(S) + 1$.

\textbf{Claim 2.3.} $S \neq \emptyset$.

\textit{Proof.} Suppose $S = \emptyset$, by the definition of $\varepsilon_1(S)$, we have $\varepsilon_1(S) = 0$. Then $i(G) = i(G-S) \geq 2|S| - \varepsilon_1(S) + 1 = 1$. On the other hand, $i(G) \leq \omega(G) = 1$ since $G$ is a connected graph. So, we obtain that $G$ is an isolated vertex, a contradiction. This completes the proof of Claim 2.3.

By Claim 2.3, $S \neq \emptyset$. Then by the definition of $\varepsilon_1(S)$, we obtain $\varepsilon_1(S) \leq 2$. It follows immediately that

$$i(G-S) \geq 2|S| - \varepsilon_1(S) + 1 \geq 2|S| - 1.$$ 

Set $|S| = s$. We denote by $I(G-S)$ the set of isolated vertices in $G-S$. Then we construct a simple bipartite graph $H = H[X,Y]$ as follows. Let $X = S$ and $Y \subseteq I(G-S)$ such that $|Y| = 2s - 1$. For any $s \in X$ and $y \in Y$, $sy \in E(H)$ if and only if $sy \in E(G)$. Since $\delta(G) \geq 3$, it is clear that for each $y \in Y$, we have $|N_H(y)| \geq 3$. Hence, $|H| \geq s + (2s - 1) = 3s - 1 \geq 5$ and

$$|E(H)| \geq 3 \times (2s - 1) = 6s - 3.$$ 

(2.1)

As $G$ is a connected planar graph, it is easy to see that $H$ is also a connected planar graph. According to the fact that a bipartite graph does not contain any odd cycles, Lemma 2.1 implies that $|E(H)| \leq 2|H| - 4 = 2 \times (3s - 1) - 4 = 6s - 6$, which is a contradiction to (2.1). This completes the proof of Theorem 2.2. \qed
It is not hard to find that the conditions in Theorem 2.2 is not sufficient for a graph to be a $\mathcal{P}_{\geq 3}$-factor covered graph. However, if we strengthen the conditions on connectivity and minimum degree, then we could obtain a minimum degree condition for the existence of $\mathcal{P}_{\geq 3}$-factor covered planar graphs.

**Theorem 2.4.** Let $G$ be a connected planar graph. If $\delta(G) \geq 4$, then $G$ is a $\mathcal{P}_{\geq 3}$-factor covered graph.

**Proof.** Suppose $G$ is not a $\mathcal{P}_{\geq 3}$-factor covered graph. By Theorem 1.4, there exists a subset $S \subseteq V(G)$ such that $sun(G-S) > 2|S| - \varepsilon_2(S)$. According to the integrality of $sun(G-S)$, we obtain that $sun(G-S) \geq 2|S| - \varepsilon_2(S) + 1$. We distinguish three cases below to show that $G$ is a $\mathcal{P}_{\geq 3}$-factor covered graph.

**Case 1.** $S = \emptyset$.

In this case, by the definition of $\varepsilon_2(S)$, we have $\varepsilon_2(S) = 0$. Since $G$ is a connected graph, $sun(G) \leq \omega(G) = 1$. On the other hand, we obtain that

$$sun(G) = sun(G-S) \geq 2|S| - \varepsilon_2(S) + 1 = 1.$$ 

It follows easily that $sun(G) = 1$, i.e., $G$ is a big sun. By the definition of sun, it contradicts the fact that $\delta(G) \geq 4$. This completes the proof of Case 1.

**Case 2.** $|S| = 1$.

In this case, we obtain $\varepsilon_2(S) \leq 1$ by the definition of $\varepsilon_2(S)$. It follows immediately that

$$sun(G-S) \geq 2|S| - \varepsilon_2(S) + 1 \geq 2.$$ 

Let $C$ be sun component of $G-S$ and $x$ a vertex of $V(C)$ such that $d_C(x) \leq 1$. Since $\delta(G) \geq 4$, we have

$$|S| \geq d_G(x) - d_C(x) \geq \delta(G) - 1 \geq 3.$$ 

This contradiction completes the proof of Case 2.

**Case 3.** $|S| \geq 2$.

In this case, we obtain $\varepsilon_2(S) \leq 2$ by the definition of $\varepsilon_2(S)$. It follows immediately that

$$sun(G-S) \geq 2|S| - \varepsilon_2(S) + 1 \geq 2|S| - 1.$$ 

Set $|S| = s$. We denote by $Sun(G-S)$ the set of sun components in $G-S$. Since $sun(G-S) \geq 2|S| - 1$, let $C_1, C_2, \ldots, C_{2s-1}$ be $2s-1$ distinct sun components where $C_i \in Sun(G-S)$ for any $1 \leq i \leq 2s-1$.

Then we construct a simple bipartite graph $H = H[X,Y]$ as follows. For each $i \in [1, 2s-1]$, choose vertex $c_i \in V(C_i)$ such that $d_{C_i}(c_i) \leq 1$. Let $X = S$ and $Y = \{c_1, c_2, \ldots, c_{2s-1}\}$. For any $s \in X$ and $c_i \in Y$, $sc_i \in E(H)$ if and only if $sc_i \in E(G)$. Since $\delta(G) \geq 4$, it is clear that for each $1 \leq i \leq 2s-1$, we have $|N_H(c_i)| \geq 3$. Hence, $|H| = s + (2s-1) = 3s-1 \geq 5$ and

$$|E(H)| \geq 3 \times (2s-1) = 6s-3. \quad (2.2)$$

As $G$ is a connected planar graph, it is easy to see that $H$ is also a connected planar graph. According to the fact that a bipartite graph does not contain any odd cycles, Lemma 2.1 implies that

$$|E(H)| \leq 2|H| - 4 = 2 \times (3s-1) - 4 = 6s-6,$$

which is a contradiction to (2.2). This completes the proof of Case 3.

Combining Case 1–3, Theorem 2.4 is proved. \qed
3. DEGREE CONDITIONS FOR $\mathcal{P}_{\geq k}$-FACTOR COVERED GRAPHS

In this section, we mainly investigate the relationship between the maximum degree of any pairs of non-adjacent vertices and $\mathcal{P}_{\geq k}$-factor covered graph, and obtain a degree condition for the existence of $\mathcal{P}_{\geq 2}$-factor and $\mathcal{P}_{\geq 3}$-factor covered graphs, respectively.

**Theorem 3.1.** Let $G$ be a connected graph of order at least two. If
\[
\max\{d_G(u), d_G(v)\} > \left\lceil \frac{n + 1}{3} \right\rceil
\]
for all pairs of non-adjacent vertices $u$ and $v$ of $G$, then $G$ is a $\mathcal{P}_{\geq 2}$-factor covered graph.

**Proof.** Suppose $G$ is not a $\mathcal{P}_{\geq 2}$-factor covered graph. By Theorem 1.3, there exists a subset $S \subseteq V(G)$ such that $i(G-S) > 2|S| - \varepsilon_1(S)$. Let $I(G-S)$ be the set of isolated vertices of $G-S$. According to the integrality of $i(G-S)$, we obtain that
\[
i(G-S) \geq 2|S| - \varepsilon_1(S) + 1.
\]
(3.1)

**Claim 3.2.** $|S| \geq 2$.

**Proof.** If $S = \emptyset$, then $\varepsilon_1(S) = 0$. By (3.1), $i(G) = i(G-S) \geq 1$. On the other hand, $i(G) \leq \omega(G) = 1$. So, we obtain that $G$ is an isolated vertex, a contradiction.

Thus, we may assume $|S| = 1$, then $\varepsilon_1(S) \leq 1$. By (3.1), we have that $i(G-S) \geq 2|S| - \varepsilon_1(S) + 1 \geq 2|S| \geq 2$. As $I(G-S)$ is independent in $G$, there is a vertex $x \in I(G-S)$ such that $d_G(x) > \lceil \frac{n+1}{3} \rceil > \frac{n+1}{3}$. Then we have that $|S| \geq d_G(x) > \frac{n+1}{3}$ since $N_G(x) \subseteq S$. It follows that $i(G-S) \geq 2|S| > 2\frac{n+2}{3}$ and thus
\[
n \geq |S| + i(G-S) > \frac{n+1}{3} + \frac{2n+2}{3} = n+1,
\]
a contradiction. This completes the proof of Claim 3.2. □

By Claim 3.2 and (3.1), we have $\varepsilon_1(S) \leq 2$ and
\[
i(G-S) \geq 2|S| - \varepsilon_1(S) + 1 \geq 2|S| - 1 \geq 3.
\]
(3.2)

Since $I(G-S)$ is an independent set of $G$, there exists $x \in I(G-S)$ such that $d_G(x) > \lceil \frac{n+1}{3} \rceil > \frac{n+1}{3}$. Then we have $|S| \geq d_G(x) > \frac{n+1}{3}$ since $N_G(x) \subseteq S$. It follows from (3.2) that $i(G-S) \geq 2|S| - 1 > 2\frac{n-1}{3}$ and thus
\[
n \geq |S| + i(G-S) > \frac{n+1}{3} + \frac{2n-1}{3} = n.
\]

This contradiction completes the proof of Theorem 3.1. □

**Theorem 3.3.** Let $G$ be a connected graph of order $n \geq 7$. Then $G$ is a $\mathcal{P}_{\geq 3}$-factor covered graph if
\[
\max\{d_G(u), d_G(v)\} > \left\lceil \frac{n+2}{3} \right\rceil
\]
for all pairs of non-adjacent vertices $u$ and $v$ of $G$.

**Proof.** Suppose $G$ is not a $\mathcal{P}_{\geq 3}$-factor covered graph. By Theorem 1.4, there exists a subset $S \subseteq V(G)$ such that $\text{sun}(G-S) > 2|S| - \varepsilon_2(S)$. According to the integrality of $\text{sun}(G-S)$, we obtain that
\[
\text{sun}(G-S) \geq 2|S| - \varepsilon_2(S) + 1.
\]
(3.3)
Claim 3.4. $S \neq \emptyset$.

Proof. Suppose $S = \emptyset$, then $\varepsilon_2(S) = 0$. By (3.3), $\text{sun}(G) = \text{sun}(G - S) \geq 1$. On the other hand, $\text{sun}(G) \leq \omega(G) = 1$. So, we obtain that $G$ is a big sun containing at least 7 vertices. It follows that there exist two vertices of degree one, denoted by \{u, v\}, which contradicts that $\max\{d_G(u), d_G(v)\} > \lceil \frac{n+2}{3} \rceil \geq 3$. This completes the proof of Claim 3.4. \hfill $\square$

By Claim 3.4 and (3.3), we have $|S| \geq 1$. If $|S| = 1$, then $\varepsilon_2(S) = 1$ and

$$\text{sun}(G - S) \geq 2|S| - \varepsilon_2(S) + 1 \geq 2|S| \geq 2. \tag{3.4}$$

If $|S| = 1$, then $\varepsilon_2(S) = 2$ and

$$\text{sun}(G - S) \geq 2|S| - \varepsilon_2(S) + 1 \geq 2|S| - 1 \geq 3. \tag{3.5}$$

**Case 1.** $i(G - S) \geq 2$.

Let \{x, y\} be two distinct isolated vertices of $G - S$. Since $\max\{d_G(x), d_G(y)\} > \lceil \frac{n+2}{3} \rceil \geq \frac{n+2}{3}$ and $N_G(x) \cup N_G(y) \subseteq S$, we have that

$$|S| \geq \max\{d_G(x), d_G(y)\} > \frac{n+2}{3}.$$ 

It follows from (3.4) and (3.5) that $\text{sun}(G - S) \geq 2|S| - 1 > \frac{2n+1}{3}$ and thus

$$n \geq |S| + \text{sun}(G - S) > \frac{n+2}{3} + \frac{2n+1}{3} = n + 1,$$

a contradiction.

**Case 2.** $i(G - S) \leq 1$.

In this case, by (3.4) and (3.5), there exist at least two sums of $G - S$, denoted by $C_1, C_2, \ldots, C_t$ where $t \geq 2$. We choose $c_i \in V(C_i)$ such that $d_G(c_i) \leq 1$, where $i = 1, 2$. Obviously, $c_1c_2 \notin E(G)$. Then $\max\{d_G(c_1), d_G(c_2)\} > \lceil \frac{n+2}{3} \rceil \geq \frac{n+2}{3}$. Without of generality, we assume $d_G(c_1) > \frac{n+2}{3}$. Since $d_S(c_1) = d_G(c_1) - d_C(c_1) > \frac{n+2}{3} - 1 = \frac{n-1}{3}$, we have that $|S| \geq d_S(c_1) > \frac{n-1}{3}$. It follows from (3.4) and (3.5) that

$$\text{sun}(G - S) \geq 2|S| - 1 > \frac{2n-2}{3} - 1,$$

and thus

$$n \geq |S| + 2 \times \text{sun}(G - S) - i(G - S)$$

$$> \frac{n-1}{3} + 2 \times \left(\frac{2n-2}{3} - 1\right) - 1$$

$$= \frac{5n-5}{3} - 3 \geq n.$$

This contradiction completes the proof of Theorem 3.3. \hfill $\square$

Acknowledgements. This work was supported by the National Natural Science Foundation of China (Grant Nos. 11871280, 11971349 and U1811461), the Natural Science Foundation of Guangdong Province (Grant No. 2020B1515310009) and Qinglan Project of Jiangsu Province.
REFERENCES


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