RAIRO-Oper. Res. 56 (2022) 1491–1501
https://doi.org/10.1051/ro/2022013

RAIRO Operations Research
www.rairo-ro.org

AN EXTENSION OF THE FLOWSORT METHOD BASED ON INTUITIONISTIC FUZZY SET THEORY TO SOLVE MULTICRITERIA DECISION MAKING PROBLEMS

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Abstract. Multiple attribute decision analysis (MADA) is an important part of the modern decision science. In fact, it has been increasingly used in the literature. Therefore, the study of sorting problems is an active research topic in the multiple criteria decision aid (MCDA) area. Although, it is difficult to quantitatively and precisely express the evaluation criteria to solve real-life sorting problems. To solve this fuzziness and vagueness, the intuitionistic fuzzy set (IFS) theory achieved great success in various recent researches. Therefore, this paper presents a novel extension of the FlowSort method, which is a PROMETHEE-based sorting method, based on the intuitionistic fuzzy set theory. To clarify this new extension, an illustrative example and an empirical comparison with other MCDM methods are presented.

Mathematics Subject Classification. 90B50.

Received September 30, 2019. Accepted February 5, 2022.

1. Introduction

Multiple criteria decision aid (MCDA), which is involved in supporting the structuring and modeling of the decision making problems related to multiple conflicting criteria, is the process of choosing, ranking, classifying or describing alternatives.

The classification problematic arises in terms of assigning each action (or a single isolated action) to one and only one between several categories (at least two) according to standards relating to the intrinsic value of each action [9]. Indeed, this problematic orients the decision problem to an assignment of alternatives to one of the predefined categories or classes. In fact, a class is a collection of alternatives with similar properties or even values for the same properties, when compared to alternatives of other classes. There are two types of classification problems: *nominal*, when the classes are not ordered, and *ordinal*, when they are ordered according to the decision-maker preferences.

In this paper, we studied the ordinal classification problem, also called the sorting problem, which consists in orienting the decision problem to an assignment of alternatives to one of the predefined, ordered and homogenous categories or classes. Moreover, many methods have been proposed during the previous decades. Among which

Keywords. Multicriteria Decision Making, sorting problems, Intuitionistic Fuzzy Set, FlowSort method.

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we can mention the well-known sorting methods, such the ELECTRE-TRI [29], the THESEUS [11], and the multiple attribute utility theory-based the UTADIS [27], AHPSort [18],ANPSort [17], MACBETHSort [16], etc.

Based on the PROMETHEE [5] methodology, several authors proposed the PROMETHEE-based methods, such as FlowSort [24], PROMSORT [2] and the PROMETHE-TRI [8]. In fact, PROMETHEE is one of the most famous MCDM methods since it is easy to use, simple to process and uses less parameter than the other MCDM methods, like ELECTRE [12]. Indeed, Figueira *et al.* were the pioneers of the PROMETHEE-TRI [12] method by extending it to the sorting context but it rather uses incompletely ordered categories. On the other hand, in 2007, Araz and Ozkarahan proposed the PROMSORT [2] method, which uses completely ordered categories while the assignment of the alternatives is not independent.

Developed by Nemery and Lamboray in 2007, FlowSort [24] was proposed for the purpose of assigning actions to completely ordered categories defined by limiting profiles or central profiles. In fact, this method solves the drawbacks of PROMETHEE-TRI [8] and PROMSORT [2] and treats the problematic sorting issue for independent assignments and completely ordered categories. The alternatives and preference parameters evaluation of the FlowSort method are defined as crisp values. However, in a real-world situation, decisional problems are multi-dimensional and ambiguous in nature. So, it is difficult to express the evaluation criteria precisely.

Multiple extensions of the FlowSort method have been developed to solve these problems. Indeed, Janssen and Nemery [19] proposed an extension of the FlowSort sorting method to the case of input data imprecision. Moreover, Campos *et al.* [7] extended the FlowSort method to introduce a fuzzy sorting method called the Fuzzy FlowSort (F-FlowSort). For a simplified FlowSort version, Assche and De Smet [3] set out the parameters of a sorting model using classification examples in the context of traditional and interval sorting. Moreover, Pelissari *et al.* [26] suggested a new multi-criteria method called the SMAA-Fuzzy-FlowSort to sort the problems under uncertainty by applying the Stochastic Acceptability Analysis to the Fuzzy-FlowSort method. Furthermore, Lolli *et al.* [21] introduced a group multi-criteria decision support system named the FlowSort-GDSS for sorting the failure modes into priority classes.

As clearly stated above, the fuzzy set (FS) theory [30] has been successfully applied in a good number of studies. However, this theory is not flawless as it uses only the membership degree of an element to a fuzzy set that belongs to zero and one. Actually, it is necessary to define the non-membership degree of an element to a fuzzy set because it is not necessarily equal to 1, minus the degree of membership. Therefore, to overcome this limitation, the intuitionistic fuzzy set theory concept seems to be more suitable to deal with uncertainty than other generalized fuzzy set forms [30]. Furthermore, compared to the traditional fuzzy set, the IFS can describe the fuzzy nature of the real world more efficiently [24]. Besides, it provides more flexibility to treat real life problems under an uncertain environment because when the environment changes, the intuitionistic fuzzy sets are easy to modify [31].

On the other hand, the IFS theory achieved great success in various MCDA research studies, mainly with Park *et al.* [25], who extended the group decision making VIKOR method to an interval-valued intuitionistic fuzzy environment. In this method, the attribute weight information is partially known. As for Chen [8], he developed an extended TOPSIS method with an inclusion comparison approach to address multiple criteria group of decision-making medical problems in the interval-valued intuitionistic fuzzy set framework, etc.

Thus, the aim of our research, which is also at the heart of its originality, was to develop an extension of the FlowSort method to deal with the imprecision issue using the IFS-theory. The IFS ill-known quantity can be presented as Intuitionistic Fuzzy numbers (IFNs) through several functions, such as the Triangular Intuitionistic Fuzzy numbers (TIFNs), the Interval-Valued Intuitionistic Fuzzy numbers (IVIFNs), the Trapezoidal Intuitionistic Fuzzy numbers (TrIFNs), etc. In fact, the TIFNs seem to be the simplest one [28] as it can be easily specified and implemented by the decision maker [21]. Besides, its application in the MCDM is based on its ability to express decision information in several dimensions [20]. Therefore, in our proposed method, we will present the input data in the decision matrix as triangular intuitionistic fuzzy numbers.

Therefore, this paper is organized as follows. In the second section, we will introduce the IFS theory notations. Then, in the third section, we will present the FlowSort method using crisp evaluations. The fourth section will be devoted to the development of an extension of the FlowSort method based on the IFS theory to solve the MCDM problem. Section five will include a numerical example and a comparison of the results obtained with other MCDA methods. The final section will conclude the paper and outline further research.

2. FlowSort method

The FlowSort method is an ordinal classification method based on the ranking methodology of PROMETHEE method. In fact, it is proposed for assigning a set of alternatives $A = \{a_1, a_2, \dots, a_n\}$ to K ordered categories $C_1, C_2, \dots, C_k$ evaluated according to m criteria $G = \{g_1, g_2, \dots, g_m\}$. Each category is defined by a set of limiting profiles $R = \{r_1, r_2, \dots, r_{k+1}\}$ or by the set of K central profiles (centroids) for K ordered categories $\tilde{R} = \{\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_k\}$ defined by the DM. Therefore, to avoid conflicts, we defined each category by a set of reference profiles $R^* = \{r_1^*, r_2^*\}$ founded in [24]. For each alternative a_i for all $i \in \{1, 2, \dots, n\}$ we defined the set $R_i^* = R^* \cup \{a_i\}$ where a_i is the action to be assigned.

The assignment of alternatives is deduced based on their relative position with respect to the reference profiles, in terms of positive, negative and net flows. It depends on the comparison of the alternative with all the reference profiles simultaneously [24]. The positive, negative and net flows are computed as follows:

$$\phi^+_{R_i^*}(x) = \frac{1}{|R_i^*| - 1} \sum_{y \in R_i^*} \pi(x, y), \tag{2.1}$$

$$\phi^-_{R_i^*}(x) = \frac{1}{|R_i^*| - 1} \sum_{y \in R_i^*} \pi(y, x), \tag{2.2}$$

$$\phi_{R_i^*}(x) = \phi^+_{R_i^*}(x) - \phi^-_{R_i^*}(x), \tag{2.3}$$

where $\pi(x, y) = \sum w_j P_j(x, y)$ as in PROMETHEE [5], and $|R_i^*|$ is the number of elements included in the set R_i^*.

Indeed, there are three different assignment rules based on the positive, negative and net flows, which are defined as follows:

$$C_{\phi^+}(a_i) = C_K \qquad \text{if} \;\; \phi^+_{R_i^*}(r_K) > \phi^+_{R_i^*}(a_i) \geq \phi^+_{R_i^*}(r_{K+1}) \tag{2.4}$$

$$C_{\phi^-}(a_i) = C_k \qquad \text{if} \;\; \phi^-_{R_i^*}(r_K) \leq \phi^-_{R_i^*}(a_i) < \phi^-_{R_i^*}(r_{K+1}) \tag{2.5}$$

$$C_{\phi}(a_i) = C_K \qquad \text{if} \;\; \phi_{R_i^*}(r_K) > \phi_{R_i^*}(a_i) \geq \phi_{R_i^*}(r_{K+1}). \tag{2.6}$$

3. Intuitionistic Fuzzy Set theory

3.1. Definition

The Intuitionistic Fuzzy Set theory [4] is a generalization of the fuzzy set theory which consists in assigning a membership and a non-membership degree to each element.

Let a set X be fixed then, an intuitionistic fuzzy set (IFS) A in X can be defined as follows:

$$A = \big\{\langle x, \mu_A(x), \nu_A(x)\rangle \big| x \in X\big\}, \mu_A(x), \nu_A(x) \in [0, 1], \tag{3.1}$$

where $\mu_A(x)$ and $\nu_A(x)$ are defined respectively as the membership and the non-membership degrees of the element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

The fuzzy set [23] is defined by $A = \big\{\langle x, \mu_A(x)\rangle \big| x \in X\big\}$ and can be defined as IFS by $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle \big| x \in X\}$. For each IFS A in X, the degree of hesitancy of x to A is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. If $\pi_A(x) = 0$ then A is reduced to a fuzzy set.

TABLE 1. Linguistic variable for the rating [14].

Very Poor (VP)	$\langle 0, 0, 1; 0, 0, 2\rangle$
Poor (P)	$\langle 0, 1, 3; 0, 1, 4\rangle$
Medium Poor (MP)	$\langle 1, 3, 5; 0.5, 3, 5.5\rangle$
Fair (F)	$\langle 3, 5, 7; 2, 5, 8\rangle$
Medium Good (MG)	$\langle 5, 7, 9; 4.5, 7, 9.5\rangle$
Good (G)	$\langle 7, 9, 10; 6, 9, 10\rangle$
Very Good (VG)	$\langle 9, 10, 10; 8, 10, 10\rangle$

TABLE 2. Linguistic variables for the importance weight of each criterion [14].

Very Low (VL)	$\langle 0, 0, 0.1; 0, 0, 0.2\rangle$
Low (L)	$\langle 0, 0.1, 0.3; 0, 0.1, 0.4\rangle$
Medium Low (ML)	$\langle 0.1, 0.3, 0.5; 0.05, 0.3, 0.5, 0.5\rangle$
Medium (M)	$\langle 0.3, 0.5, 0.7; 0.2, 0.5, 0.8\rangle$
Medium High (MH)	$\langle 0.5, 0.7, 0.9; 0.45, 0.7, 0.95\rangle$
High (H)	$\langle 0.7, 0.9, 1; 0.6, 0.9, 1\rangle$
Very High (VH)	$\langle 0.9, 1, 1; 0.8, 1, 1\rangle$

3.2. The Triangular Intuitionistic Fuzzy number (TIFN)

The IFS describes information when uncertainty is involved. Therefore, an ill-known quantity of information may be expressed with an intuitionistic fuzzy number (IFN). Moreover, there are multiple functions that help explain the intuitionistic fuzzy numbers, such as the trapezoidal [6], the triangular [19], the interval number [27], etc. but the simplest one presents the membership and the non-membership function by the triangular fuzzy numbers.

The TIFN [19] will be represented by the following two sets of triplets $A_{(\text{TIFN})} = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ where a_2 is the mean value of the intuitionistic fuzzy numbers $\mu_A(x)$ and $\nu_A(x)$, a_1 and a_3 which are respectively the left and right boundaries of $\mu_A(x)$ while, a_1' and a_3' are respectively the left and the right boundaries of $\nu_A(x)$, and $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$. The membership and the non-membership functions of the TIFN are given as follows:

$$\mu_{A_{(\text{TIFN})}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

$$\nu_{A_{(\text{TIFN})}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & \text{for } a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2}, & \text{for } a_2 \leq x \leq a_3' \\ 1 & \text{otherwise.} \end{cases} \tag{3.3}$$

3.3. Converting linguistic terms into Triangular Intuitionistic Fuzzy numbers

In many situations of the real life, information cannot be evaluated exactly in numerical values but rather in linguistic variables, which are words, sentences or natural language. On the other hand, the linguistic intuitionistic fuzzy number (LIFN), which is a special intuitionistic fuzzy number, can more easily describe the vagueness existing in the real decision-making process [13]. For their part, Gautam *et al.* [13] expressed the linguistic variables in positive triangular intuitionistic fuzzy numbers, as shown in Tables 1 and 2.

3.4. Operations of Triangular Intuitionistic Fuzzy numbers

Mahapatra and Roy [22] introduced some arithmetic operations of TIFNs;

Let $A_{(\text{TIFN})} = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B_{(\text{TIFN})} = \{(b_1, b_2, b_3); (b'_1, b_2, b'_3)\}$. The operations on triangular intuitionistic fuzzy numbers are the following:

$$A_{(\text{TIFN})} + B_{(\text{TIFN})} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3)\}; \tag{3.4}$$

$$A_{(\text{TIFN})} - B_{(\text{TIFN})} = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1); (a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1)\}; \tag{3.5}$$

$$A_{(\text{TIFN})} * B_{(\text{TIFN})} = \{(a_1 * b_1, a_2 * b_2, a_3 * b_3); (a'_1 * b'_1, a_2 * b_2, a'_3 * b'_3)\}. \tag{3.6}$$

Let k be a scalar number:

$$\text{If } k > 0 \text{ then } k * A_{(\text{TIFN})} = \{(k * a_1, k * a_2, k * a_3); (k * a'_1, k * a_2, k * a'_3)\} \tag{3.7}$$

$$\text{If } k < 0 \text{ then } k * A_{(\text{TIFN})} = \{(k * a_3, k * a_2, k * a_1); (k * a'_3, k * a_2, k * a'_1)\}. \tag{3.8}$$

As for, Gani and Abbas [13], they defined the defuzzification of the triangular intuitionistic number to an ordinal number as follows:

$$A = \frac{(a_1 + 2a_2 + a_3) + (a'_1 + 2a_2 + a'_3)}{8}. \tag{3.9}$$

4. The Intuitionistic Fuzzy Set FlowSort method for multicriteria decision making

This research aims to develop an IFS FlowSort method where an ill-known quantity is expressed in an intuitionistic fuzzy number (IFN). In fact, this method consists in computing the TIFNs preference degree, which is the deviation between alternatives and limiting profiles according to each criterion using the arithmetic operations given above. After that, the TIFNs-preference degrees should be defuzzified to crisp values and then integrated into the two last steps of the original FlowSort method.

As presented in Figure 1 the procedure of our proposed extension is the following:

Step 1. Creating an evaluation matrix. In this step, to solve the sorting MCDM problem, it is necessary to describe:

$A = \{a_1, a_2, \ldots, a_n\}$ a set of n alternatives.

$G = \{g_1, g_2, \ldots, g_m\}$ a set of m criteria evaluated by $W = \{w_1, w_2, \ldots, w_m\}$ criteria weights.

$C = \{c_1, c_2, \ldots, c_k\}$ a set of k classes.

$X_{(l)} = \left(x_{ij(l)}\right)_{n*m}$ is the TIFN performance rating for alternative $a_i (i = 1, 2, \ldots, n)$ on criterion $c_j (j = 1, 2, \ldots, m)$.

$$x_{ij} = \left\{\left(x^1_{ij}, x^2_{ij}, x^3_{ij}\right); \left(x'^1_{ij}, x^2_{ij}, x'^3_{ij}\right)\right\}, \quad i = 1, 2, \ldots, n, \; j = 1, 2, \ldots, m$$

where x^2_{ij} is the mean value of the intuitionistic fuzzy numbers $\mu(x_{ij})$ and while $\nu(x_{ij})$, x^1_{ij} and x^3_{ij} are respectively the left and the right boundaries of $\mu(x_{ij})$ then, x'^1_{ij} and x'^3_{ij} are respectively the left and the right boundaries of $\nu(x_{ij})$, and $x'^1_{ij} \le x^1_{ij} \le x^2_{ij} \le x'^3_{ij} \le x^3_{ij}$.

On the other hand, the parameter values, such as the criteria weights, the DM weights and the preference and the indifference degrees are assumed to be crisp numbers.

Step 2. Determine the set of ordered categories $C_1 \triangleright C_2 \triangleright \ldots \triangleright C_k$, where $C_h \triangleright C_l$ with $h < l$ denoting that category C_h is preferred to category C_l. Moreover, each category is defined by one central profile or two limiting profiles.

Let $R = \{r_1, r_2, \ldots, r_{k+1}\}$ be the set of limiting profiles, where r_h and r_{h+1} is the upper and the lower of C_h. There are K central profiles (centroids) for K ordered categories $\tilde{R} = \{\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_k\}$ defined by the DM.

However, when there is no distinction between a set of limiting profiles and the set of centroid, there are reference profiles $R^* = \{r^*_1, r^*_2, \ldots\}$. Let us define the set $R^*_i = R^* \cup \{a_i\}$ where a_i is the action to be assigned.

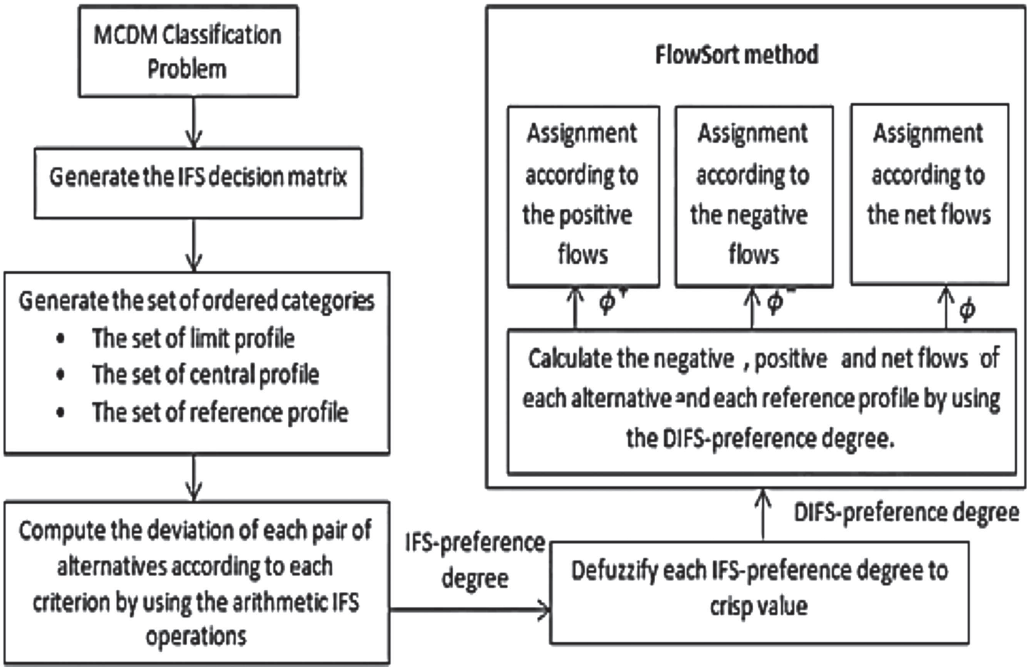

Figure 1. The IF-FlowSort procedure.

Step 3. Calculating the preference degree $\pi(A,B)$ of each alternative A over an alternative B using the arithmetic operation of the triangular intuitionistic fuzzy numbers for all alternatives A, B of R_i^*.

$$\pi(A,B)=\sum w_j * P_j(A,B) \tag{4.1}$$

$$\pi(A,B)=\sum w_j * P_j(f_j(A)-f_j(B)) \tag{4.2}$$

where $f_j(A)=(a_1,a_2,a_3;a_1',a_2,a_3')1$ and $f_j(B)=(b_1,b_2,b_3;b_1',b_2,b_3')1$are two types of triangular intuitionistic fuzzy numbers and w_j is a scale number.

$$\pi(A,B)=\sum w_j * P_j(\alpha_1,\alpha_2,\alpha_3;\alpha_1',\alpha_2,\alpha_3') \tag{4.3}$$

where:

$$\alpha_1=a_1-b_3,\ \alpha_2=a_2-b_2,\ \alpha_3=a_3-b_1,\ \alpha_1'=a_1'-b_3',\ \alpha_3'=a_3'-b_1'$$

$$\pi(A,B)=\sum w_j * \left(\alpha_1^{P_j},\alpha_2^{P_j},\alpha_3^{P_j};\alpha_1'^{P_j},\alpha_2^{P_j},\alpha_3'^{P_j}\right) \tag{4.4}$$

$$=\sum\left(w_j\alpha_1^{P_j},w_j\alpha_2^{P_j},w_j\alpha_3^{P_j};w_j\alpha_1'^{P_j},w_j\alpha_2^{P_j},w_j\alpha_3'^{P_j}\right) \tag{4.5}$$

$$=\left(\sum w_j\alpha_1^{P_j},\ \sum w_j\alpha_2^{P_j},\ \sum w_j\alpha_3^{P_j};\ \sum w_j\alpha_1'^{P_j},\ \sum w_j\alpha_2^{P_j},\ \sum w_j\alpha_3'^{P_j}\right). \tag{4.6}$$

Step 4. Each preference degree $\pi(A,B)$ should be defuzzified to transform the intuitionistic fuzzy number into a real number. Therefore, we suggest using Abbas and Gani's [13] operator given in (3.6) since it can be easily employed by users, as it makes them use the IFS-FlowSort method more simple.

$$\pi_d(A,B)=\frac{\left(\sum w_j\alpha_1^{P_j}+2*\sum w_j\alpha_2^{P_j}+\sum w_j\alpha_3^{P_j}\right)+\left(\sum w_j\alpha_1'^{P_j}+2*\sum w_j\alpha_2^{P_j}+\sum w_j\alpha_3'^{P_j}\right)}{8}. \tag{4.7}$$

TABLE 3. IFS-FlowSort decision matrix.

	c_1	c_2	c_3	c_4	c_5
A_1	$\langle 5.7, 7.7, 9.3; 5, 7.7, 9.7\rangle$	$\langle 5, 7, 8.7; 4.2, 7, 9.2\rangle$	$\langle 5.7, 7.7, 9; 4.7, 7.7, 9.3\rangle$	$\langle 8.3, 9.7, 10; 7.3, 9.7, 10\rangle$	$\langle 3, 5, 7; 2, 5, 8\rangle$
A_2	$\langle 3.7, 6, 7.7; 3, 6, 7.8\rangle$	$\langle 4, 5, 5; 3.7, 5, 5.3\rangle$	$\langle 4, 5, 5; 3.7, 5, 5.3\rangle$	$\langle 2.7, 4, 5; 2.3, 4, 5.3\rangle$	$\langle 2.7, 3, 4; 2.3, 3, 4.3\rangle$
A_3	$\langle 2.7, 4, 5; 2.3, 4, 5.3\rangle$	$\langle 2.7, 3, 4; 2.3, 3, 4.7\rangle$	$\langle 3.7, 4, 5.3; 3.3, 4, 5.7\rangle$	$\langle 2.7, 3, 4; 2.3, 3, 4.3\rangle$	$\langle 4.5, 5, 5.5; 4, 5, 5.5\rangle$
A_4	$\langle 6.3, 8.3, 9.7; 5.5, 8.3, 9.8\rangle$	$\langle 9, 10, 10; 8, 10, 10\rangle$	$\langle 8.3, 9.7, 10; 7.3, 9.7, 10\rangle$	$\langle 9, 10, 10; 8, 10, 10\rangle$	$\langle 7, 8.7, 9.7; 6.2, 8.7, 9.8\rangle$
A_5	$\langle 6.3, 8, 9; 5.3, 8, 9.3\rangle$	$\langle 7, 8.7, 9.7; 6.2, 8.7, 9.8\rangle$	$\langle 7, 8.7, 9.7; 6.2, 8.7, 9.8\rangle$	$\langle 7, 8.7, 9.7; 6.2, 8.7, 9.8\rangle$	$\langle 6.3, 8.3, 9.7; 5.5, 8.3, 9.8\rangle$

TABLE 4. The category boundaries.

	g_1	g_2	g_3	g_4	g_5
IR_1	10	10	10	10	10
IR_2	6	6	6	6	6
IR_3	4	4	4	4	4
IR_4	0	0	0	0	0

Step 5. The positive, negative and the net flows of each alternative A of R_i^* are computed according to the defuzzified outranking degree $\pi_d(A, B)$:

$$\phi^+_{R_i^*}(A) = \frac{1}{|R_i^*| - 1} \sum_{B \in R_i^*} \pi_d(A, B) \tag{4.8}$$

$$\phi^-_{R_i^*}(A) = \frac{1}{|R_i^*| - 1} \sum_{B \in R_i^*} \pi_d(B, A) \tag{4.9}$$

$$\phi_{R_i^*}(A) = \phi^+_{R_i^*}(A) - \phi^-_{R_i^*}(A). \tag{4.10}$$

Step 6. As with the FlowSort method, three different assignment rules based on the positive, negative and net flows are defined as follows:

$$C_{\phi^+}(a_i) = C_K \qquad \text{if} \;\; \phi^+_{R_i^*}(r_K) > \phi^+_{R_i^*}(a_i) \geq \phi^+_{R_i^*}(r_{K+1}) \tag{4.11}$$

$$C_{\phi^-}(a_i) = C_k \qquad \text{if} \;\; \phi^-_{R_i^*}(r_K) \leq \phi^-_{R_i^*}(a_i) < \phi^-_{R_i^*}(r_{K+1}) \tag{4.12}$$

$$C_{\phi}(a_i) = C_K \qquad \text{if} \;\; \phi_{R_i^*}(r_K) > \phi_{R_i^*}(a_i) \geq \phi_{R_i^*}(r_{K+1}). \tag{4.13}$$

5. NUMERICAL EXAMPLE

The criteria weights used in the IFS-TOPSIS are given as triangular intuitionistic fuzzy values. In our proposed method, because of the need to use the weight as a crisp number, we defuzzify and normalize the TIFN given in [13] in order to obtain the set $\{w_1 = 0.17, w_2 = 0.23, w_3 = 0.19, w_4 = 0.23, w_5 = 0.15\}$. We suppose that the indifference threshold $q_j = 0$ and the preference threshold $p_j = 6 \; \forall j = 1, \ldots, 5$ where each criterion has four category boundaries IR_1, IR_2, IR_3 and IR_4 (see Tab. 2).

We then compute the deviation of each pair of alternatives according to each criterion using the arithmetic IFS operations to obtain the intuitionistic fuzzy preference degrees given in Table 3 (see Tab. 5).

In the third step of the procedure, we apply the IFS-FlowSort, method, which consists in defuzzifying the IF-preference degrees to crisp numbers (see Tab. 6). In fact, the obtained results are shown in Tables 4.

We calculated the positive, negative and net flows values of each alternative A of R_i^* as given in the fourth step (see Tabs. 7 and 8). In fact, such in the original FlowSort method; $\phi^+_{R_i}(A_i) = \sum_{B \in R_i^*} \pi_d(A_i, \mathrm{IR}_j)$, $\phi^-_{R_i}(A_i) = \sum_{B \in R_i^*} \pi_d(\mathrm{IR}_j, A_i), \phi_{R_i}(A) = \phi^+_{R_i}(A_i) - \phi^-_{R_i}(A_i)$, $i = 1, 2, n = 5, j = 1, k = 4$. Then, by comparing the results given in column A_i to the ones given in IR_j columns, we concluded that alternatives A_1, A_4 and A_5 are

TABLE 5. The intuitionistic fuzzy preference degrees.

$d\pi$	IR_1	IR_2	IR_3	IR_4
$(A_1 - Ir_i)$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0.09, 0.3, 0.47; 0.05, 0.3, 0.56 \rangle$	$\langle 0.3, 0.6, 0.8; 0.2, 0.6, 0.88 \rangle$	$\langle 0.84, 0.9, 1; 0.75, 0.9, 1 \rangle$
$(Ir_i - A_1)$	$\langle 0.18, 0.4, 0.66; 0.09, 0.4, 0.76 \rangle$	$\langle 0, 0.02, 0.13; 0, 0.02, 0.22 \rangle$	$\langle 0, 0, 0.02; 0, 0, 0.05 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$
$(A_2 - Ir_i)$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0, 0, 0.05; 0, 0, 0.05 \rangle$	$\langle 0, 0.13, 0.2; 0, 0.13, 0.26 \rangle$	$\langle 0.6, 0.7, 0.8; 0.5, 0.7, 0.85 \rangle$
$(Ir_i - A_2)$	$\langle 0.76, 0.8, 0.97; 0.71, 0.8, 0.97 \rangle$	$\langle 0.16, 0.22, 0.4; 0.12, 0.22, 0.48 \rangle$	$\langle 0, 0.02, 0.09; 0, 0.02, 0.16 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$
$(A_3 - Ir_i)$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0.01, 0.02, 0.1; 0, 0.02, 0.17 \rangle$	$\langle 0.5, 0.6, 0.75; 0.4, 0.6, 0.8 \rangle$
$(Ir_i - A_3)$	$\langle 0.86, 0.9, 0.95; 0.8, 0.9, 0.97 \rangle$	$\langle 0.22, 0.37, 0.46; 0.16, 0.37, 0.5 \rangle$	$\langle 0, 0.08, 0.15; 0, 0.08, 0.2 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$
$(A_4 - Ir_i)$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0.34, 0.55, 0.63; 0.2, 0.55, 0.63 \rangle$	$\langle 0.66, 0.9, 0.95; 0.5, 0.9, 0.96 \rangle$	$\langle 1, 1, 1; 0.95, 1, 1 \rangle$
$(Ir_i - A_4)$	$\langle 0.02, 0.09, 0.3; 0.01, 0.09, 0.46 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$
$(A_5 - Ir_i)$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0.12, 0.4, 0.58; 0.02, 0.4, 0.6 \rangle$	$\langle 0.4, 0.7, 0.9; 0.3, 0.7, 0.92 \rangle$	$\langle 1, 1, 1; 0.93, 1, 1 \rangle$
$(Ir_i - A_5)$	$\langle 0.06, 0.2, 0.42; 0.04, 0.2, 0.54 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$	$\langle 0, 0, 0; 0, 0, 0 \rangle$

TABLE 6. The defuzzified preference degrees.

$\mathrm{d}\pi$	IR_1	IR_2	IR_3	IR_4
$(A_1 - Ir_i)$	0	0.29	0.56	0.91
$(Ir_i - A_1)$	0.4	0,05	0.009	0
$(A_2 - Ir_i)$	0	0.01	0.12	0.71
$(Ir_i - A_2)$	0.84	0.26	0.04	0
$(A_3 - Ir_i)$	0	0	0.05	0.6
$(Ir_i - A_3)$	0.92	0.36	0.08	0
$(A_4 - Ir_i)$	0	0.5	0.8	0.97
$(Ir_i - A_4)$	0.14	0.002	0	0
$(A_5 - Ir_i)$	0	0.37	0.69	0.965
$(Ir_i - A_5)$	0.23	0.004	0	0

assigned to the first category so that they will be the candidates to be selected. For instance, candidate A_2 which is assigned to the second category will be discussed and while candidate A_3, which is assigned to the third category will be rejected.

In order to compare our results, we used the same input data and we applied it in the FlowSort, the F-FlowSort, the TOPSIS [15] and the IFS-TOPSIS [14] methods. As can be seen in Table 9, assignments are closely similar except for the second alternative. In fact, when using the F-FlowSort by considering the assignment based on the positive flows, the 2nd candidate will be assigned to the third category. However, when using the F-FlowSort method by considering the assignment based on the negative and net flows the 2nd candidate will be assigned to the second category. Therefore, alternative 2 can be unambiguously assigned to category 2. Consequently, the IFS-FlowSort can successfully correct this ambiguous assignment by using the perfect information given by the IFS values and assign it to the second category when considering the assignment based on the three flows. In addition, we found identical results when applying the FlowSort method.

Moreover, a relationship can be observed when the results are obtained using the ranking methods. In fact, the results given by using the PROMETHEE [5], the TOPSIS [15] and the IFS-TOPSIS [14] methods and by our IFS-FlowSort method are almost the same. As it can be seen in Figure 2, if we can group alternatives in three ordered categories to the best from the worst; the 1st, 4th and 5th alternatives remain the most preferred therefore, they can logically be assigned to the first category. On the other hand, the 2nd alternative can be moderately preferred while the 3th alternative is always the worst one. Therefore, this observation cannot be generalized since many studies are wanted in this area.

6. CONCLUSION

The FlowSort method treats the MCDM ordinal classification problem by classifying the inventory items into ordered categories using the limiting and the centroid profiles based on exact values. In fact, this simple is

TABLE 7. The positive, negative and the net flow values.

		IR_1	IR_2	IR_3	IR_4	A_i
	$\phi^+_{R_1}$	2.5	1.6	1.24	0.9	1.76
R_1	$\phi^-_{R_1}$	0.4	0.65	1.30	2.62	0.47
	ϕ_{R_1}	2.09	0.96	−0.06	−1.7	1.29
	$\phi^+_{R_2}$	2.5	1.33	0.8	0.7	0.85
R_2	$\phi^-_{R_2}$	0.85	0.85	1.34	2.62	1.15
	ϕ_{R_2}	1.65	0.49	−0.5	−1.9	−0.3
	$\phi^+_{R_3}$	2.5	1.32	0.7	0.61	0.66
R_3	$\phi^-_{R_3}$	0.92	0.95	1.35	2.62	1.36
	ϕ_{R_3}	1.58	0.38	−0.65	−2	−0.7
	$\phi^+_{R_4}$	2.5	1.83	1.5	0.97	2.3
R_4	$\phi^-_{R_4}$	0.15	0.59	1.29	2.6	0.15
	ϕ_{R_4}	2.36	1.23	0.2	−1.65	2.150
	$\phi^+_{R_5}$	2.5	1.69	1.36	0.97	2.02
R_5	$\phi^-_{R_5}$	0.23	0.59	1.29	2.615	0.24
	ϕ_{R_5}	2.27	1.097	0.07	−1.65	1.79

TABLE 8. The assignments of alternatives.

Scenarios	$K_{\text{positive flows}}$	$K_{\text{negative flows}}$	$K_{\text{net flows}}$
A_1	K_1	K_1	K_1
A_2	K_2	K_2	K_2
A_3	K_3	K_3	K_3
A_4	K_1	K_1	K_1
A_5	K_1	K_1	K_1

TABLE 9. Comparison of the IFS-FlowSort with other ranking and sorting method.

Scenarios	Ranking			Sorting								
				FlowSort			F-FlowSort			IFS-FlowSort		
	F-TOPSIS	IFS-TOPSIS	PROMETHEE	K_{ϕ^+}	K_{ϕ^-}	K_{Φ}	K_{ϕ^+}	K_{ϕ^-}	K_{Φ}	K_{ϕ^+}	K_{ϕ^-}	K_{Φ}
A_1	1	2	3	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_2	4	4	4	K_2	K_2	K_2	K_3	K_2	K_2	K_2	K_2	K_2
A_3	5	5	5	K_3	K_3	K_3	K_3	K_3	K_3	K_3	K_3	K_3
A_4	3	3	1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_5	2	1	2	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1

practical and requires less rigid inputs from the decision makers. The DM who is familiar with PROMETHEE methods can easily understand the FlowSort. It solves the drawbacks of the other extensions of the PROMETHEE method (PROMSORT, PROMETHEE TRI) and treats the sorting problematic for independent assignments and completely ordered categories [19]. However, it is hard for the decision makers to precisely express their preferences. Therefore, motivated by the fact that the fuzzy scientific community has been attending with great interest the recent dispute about Atanassov Intuitionistic Fuzzy Sets [10], we propose to develop an IFS-FlowSort method based on the intuitionistic fuzzy set numbers to describe the imprecise evaluations. Furthermore, because of the simplicity and easiness of the triangular fuzzy numbers, we suggest using them

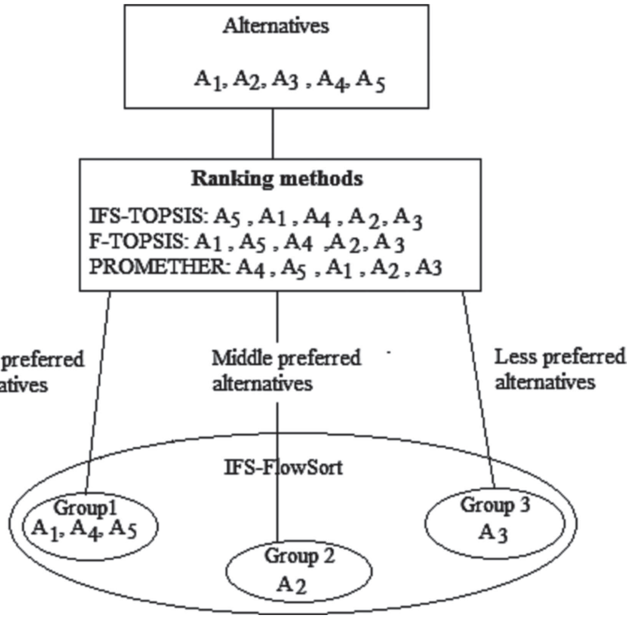

Figure 2. Comparison of the IFS-FlowSort with ranking methods.

to describe the ill-known quantity. In our proposed method, the parameter values, such as the weights or the thresholds are assumed to be crisp numbers so that they can reduce the imprecision. To illustrate this extension, a practical example is treated and validated by comparing it to the two sorting methods, the FlowSort [24] and the F-FlowSort [6] and with the ranking methods F-TOPSIS [23], IF-TOPSIS [14] and PROMETHEE [5]. As a result, all the alternatives, excepting one, are assigned to the same category. Thus, we can conclude that our extension seems to be coherent in a sorting context and in the uncertainty logic. Such as the FlowSort method it is Simple to process easy to use the use of IFS-theory makes it more abundant and flexible to express and to describe information than other extensions of fuzzy set theory when uncertain information is involved.

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