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PERFORMANCE EVALUATION OF TWO-STAGE PRODUCTION SYSTEMS WITH TIME-LAG EFFECTS: AN APPLICATION IN THE HORTICULTURE INDUSTRY

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Abstract. In standard data envelopment analysis (DEA), it is assumed that inputs of a specific production period are used to generate outputs of the same period. However, in some practical examples, time-lag effects exist between inputs and outputs. The inputs of one period are used to generate outputs for several periods, or inputs of several periods are used to create outputs for one period. In this paper, we present some new DEA models for performance assessment of network production systems with time-lag effects. An empirical application in the horticulture sector in Iran shows the usefulness and capabilities of our proposed approach.

Mathematics Subject Classification. 90B30.

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1. Introduction

Data envelopment analysis (DEA), initially introduced by Farrell [14], is a non-parametric mathematical-programming method for evaluating the efficiency of a set of decision-making units (DMUs) with multiple inputs and outputs. Charnes *et al.* [4] presented the CCR model by extending Farrell's work to evaluate DMUs considering multiple inputs and multiple outputs. Later on, Banker *et al.* [2] extended the CCR model to the BCC model by assuming variable returns to scale into the evaluation. In addition to these basic DEA models, other evaluation approaches, such as Additive and Slack Base Measure (SBM) models, have also been proposed in the DEA literature [40]. In the last four decades, an impressive number of methods and applications have been reported in the DEA framework. For further information, the interested readers could refer to Emrouznejad *et al.* [10] and Kaffash *et al.* [19] for comprehensive surveys and analysis of related studies in DEA theory and applications.

In classic DEA models, DMUs are considered black-boxes; the intrinsic activity and internal structures of sub-processes do not account for the unit's assessment (see [16, 17, 20, 26, 30] for more details). In a production unit, the inputs may pass through multiple processes to produce outputs. As a result of applying traditional DEA models, a black-box DMU may be seen efficient while its subunits are performing inefficiently [12, 24].

Keywords. Network Data Envelopment Analysis (NDEA), time lag, efficiency, horticultural industry.

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In recent DEA literature, considerable efforts have been devoted to developing new models for analyzing multi-stage production units. Seiford and Zhu [39] attempted to abandon the traditional perspective in the performance evaluation of bank branches and consider the internal structure by separating the bank's operations into two successive stages of profiting and marketing. However, they used two DEA models to assess the two stages and ignored the transfer of information between the two stages.

According to Cook *et al.* [7], in two-stage models where the first stage outputs are used as the second stage inputs, if the second stage becomes inefficient, we have to reduce the input to make it efficient. However, reducing the second stage input results in decreasing the output of the first stage, which yields to the inefficiency of the first stage. Although they looked at the internal structures in these types of research, they did an incomplete evaluation due to overlooking subunit communications and how to transfer information from one stage to the next. To troubleshoot the issues raised in independent and classic DEA models, Färe and Groskopf [13] introduced network data envelopment analysis (NDEA) models to evaluate the processes operations in assessing the efficiency of the DMUs with multiple-stage structure. Unlike the classic models, the NDEA models of Kao [20] depend on the structure of the DMU, relations of its subdivisions, and the type of inputs and outputs. Despotis *et al.* [9] used a weak-link approach to provide a simple two-stage model for evaluating the efficiency of DMUs. Khoveyni *et al.* [28] examined the concept of variations effect in a two-stage NDEA to see how output products would change if the intermediate products rise due to the increasing inputs in the first stage. Research in this area is still of interest to many researchers [5, 6, 21, 23, 31, 32].

In most DEA models, the efficiency of the companies and organizations is usually evaluated for a specific time, which is not an accurate assessment of the performance of the whole system. It is more realistic to assess and compare the efficiency of these kinds of operations over several periods. In this regard, there are many extensions of the original DEA models that take more than a one-time period to evaluate the efficiency. Some examples of these models are window analysis and Malmquist productivity evaluation. Regardless of minor differences between these models, they all have the common purpose of evaluating DMUs over multiple periods. Nemoto and Goto [34] proposed a dynamic DEA model for measuring the performance of multiple-time production systems. Golany *et al.* [15] presented an efficiency measurement framework for systems composed of two subsystems arranged in series that simultaneously computes the efficiency of the aggregate system and each subsystem. Park and Park [36] introduced a two-stage approach to measure aggregative efficiency over several periods. Amirteimoori and Kordrostami [1] presented a model to measure cumulative efficiency across all periods and showed that cumulative efficiency is a convex combination of periods' efficiency scores. Kao and Liu [25] proposed a method to measure the cumulative efficiency values of several periods. In other words, they used a network approach to assess the efficiency of periods and the overall efficiency of DMUs. Jablonsky [18] analyzed the performance of multiple-time systems and presented the efficiency and hyper-efficiency concepts in multiple-time DEA models. Razavi *et al.* [37] introduced a two-stage approach based on Chebyshev inequality bounds related to multiple-time production systems. Kordrostami and Jahani [29] presented a method for evaluating the efficiency of multi-time production systems with negative data. Recently, Esmaeilzadeh and Kazemi Matin [11] expanded the concept of multiple-time production in the NDEA. Their models were based on series and parallel approaches in network data envelopment analysis.

An essential issue in traditional DEA is that the inputs for one period are used for generating outputs of the same period, but in practice, the input of one period may be used for the output generation of several periods, or the input of several periods is used to generate the output of one period. This condition is called production processes with time-lag effects. For example, in the horticultural industry, the costs incurred for a garden as input in one year are presented as output in subsequent years. For the first time, Özpeynici and Köksalan [35] introduced DEA models with time-lag effects. They presented two multiple models with input and output lag times. On the other hand, there are evaluation models of stocks portfolio performance in which variables such as returns of funds are considered time-lag components. In this field, the black-box models have also been proposed to evaluate the performance of stocks portfolio (see, *e.g.*, [3, 27, 33, 38]).

In some real-world applications, we may encounter a network process that one of its stages has a time delay in transforming inputs to outputs. For example, in the horticultural industry, we have a delay of a few years to produce fruits, and if we want to compote the product, we will have a network whose first stage has a time delay. Given that no previous research has evaluated the performance of DMUs in a network structure with a time lag, the question is how to deal with a time lag in a network process.

This paper aims to answer the research question and provide models for NDEA with a time lag. For this purpose, we consider a simple two-stage network whose first-stage inputs have a time lag. We first evaluate the efficiency of the stages independently, and then we provide a multiplier DEA model for the overall performance assessment of the two-stage production units. We will also present an envelopment DEA model for evaluating the efficiency of production processes in both black-box and network cases, by considering time-lag effects. Besides, the applicability of the proposed models is demonstrated by applying them in a real-world case in the horticulture industry in Iran.

The rest of the paper is organized as follows. Section 1 introduces basic multiple period models of DEA with time lag effects in the black-box case. Section 2 presents the new approach in modeling the time lag effect in a two-stage network DEA framework. The new suggested models are presented in both multiple and envelopment forms. An empirical application of performance assessments of horticulture sectors in Iran is presented in Section 3, as well as data analysis and discussions. Section 4 contains conclusions and suggestions.

1. Multiple period DEA models for time lag effects

Suppose that all DMUs use m different inputs to generate s different outputs in each period. Also, suppose $x_{ijt}(i = 1, \dots, m)$ and $y_{rjt}(r = 1, \dots, s)$ represent the ith input and the rth output in the period t $(t = 1, \dots, T)$, respectively. The input and output vectors of DMU_j in period t are also denoted by $\mathbf{x}_{jt} = (x_{1jt}, x_{2jt}, \dots, x_{mjt})$ and $\mathbf{y}_{jt} = (y_{1jt}, y_{2jt}, \dots, y_{sjt})$, respectively.

The CCR multiplier model to evaluate the efficiency of DMU_k in period t is stated as follows:

$$
\begin{aligned}
\max\ \theta_k = & \sum_{r=1}^{s} u_r y_{rkt} \\
\text{s.t.}\ & \sum_{i=1}^{m} v_i x_{ikt} = 1 && t = 1, \dots, T \\
& \sum_{r=1}^{s} u_r y_{rjt} - \sum_{i=1}^{m} v_i x_{ijt} \le 0 && t = 1, \dots, T, \quad j = 1, \dots, n \\
& u_r \ge 0 && r = 1, \dots, s \\
& v_i \ge 0 && i = 1, \dots, m
\end{aligned} \tag{1.1}
$$

where $u_r(r = 1, \dots, s)$ and $v_i(i = 1, \dots, m)$ are the output and input weights for evaluating DMU_k, respectively. Also, at optimality, $0 < \theta_k^* \le 1$ is the efficiency score of DMU_k. Note that in most traditional DEA models, it is assumed that the inputs in a specific production period are used to generate outputs of the same period.

The time lag may occur at inputs or outputs. For example, Figure 1a displays the time lag in the inputs. In this figure, a time lag of three years $(D = 3)$ has been considered for 6 years $(T = 6)$. As seen in Figure 1a, the outputs start from the third year, and we can use the inputs of the first to the third year for the third year output. For the fourth year's output, we can use the inputs of the second to the fourth year. Similarly, for the fifth year's output, the inputs of the third to the fifth year and, for the sixth year's outputs, the inputs of the fourth to the sixth year can be used. However, we do not have outputs for the first two years.

A similar analysis can be performed for output time lags, as shown in Figure 1b. Here the first-year input is only used for the outputs of the first three years. Similarly, the fourth year's inputs will generate outputs of the fourth to the sixth year. Also, we will not have input in the fifth and sixth years.

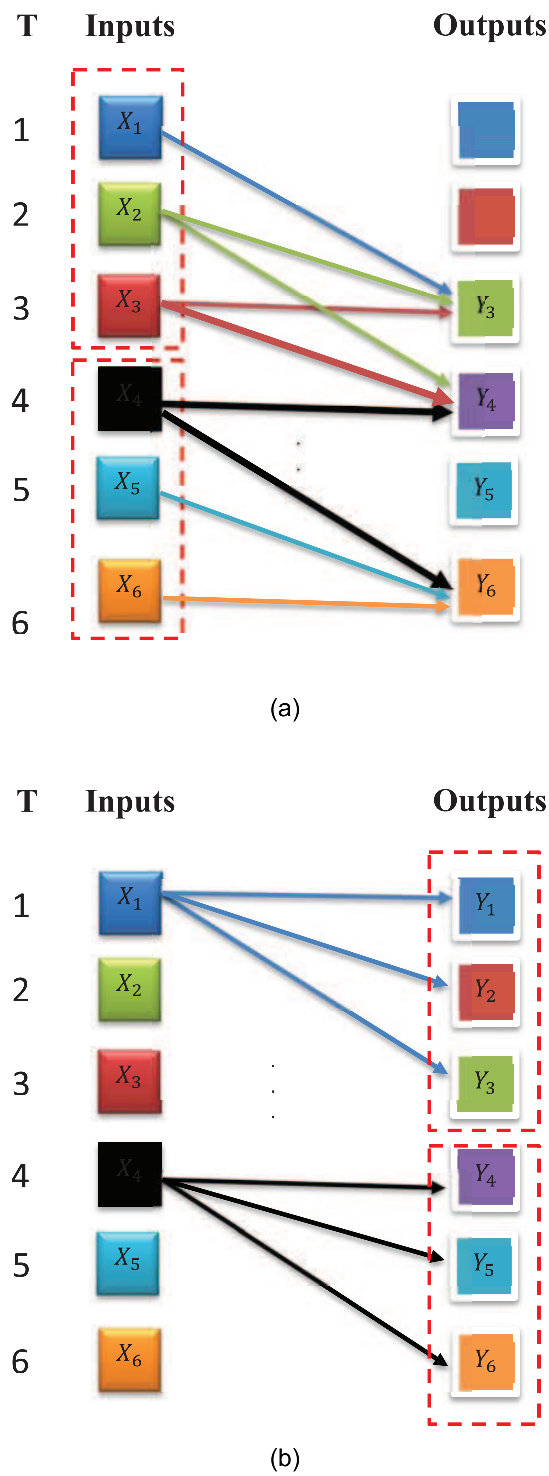

FIGURE 1. (a) The time lag effects for inputs. (b) The time lag effects for outputs.

The following multiplier DEA model (MPI) is proposed by Özpeynici and Köksalan [35] for evaluating the efficiency of the DMU_k with the time lag in inputs (shown typically in Fig. 1a).

$$\begin{aligned}
E_k^{\text{MPI}} &= \max \sum_{t=D}^{T}\sum_{r=1}^{S} y_{rkt}u_{rt} \\
\text{s.t.} &\sum_{p=0}^{D-1}\sum_{i=1}^{m} x_{ik(t-p)}v_{it}^{p} = 1 && t = D, D+1, \ldots T \\
&\sum_{r=1}^{s} y_{rjt}u_{rt} - \sum_{p=0}^{D-1}\sum_{i=1}^{m} x_{ij(t-p)}v_{it}^{p} \leq 0 && j = 1,\ldots,n, t = D, D+1,\ldots,T \\
&v_{it}^{p} \geq 0 && i = 1,\ldots,m, p = 0,1,\ldots,D-1, t = D, D+1,\ldots T \\
&u_{rt} \geq 0 && r = 1,\ldots,s, t = D, D+1,\ldots,T. \qquad (1.2)
\end{aligned}$$

Here, $x_{ik(t-p)}$ shows the ith input of DMU_k in the period $(t-p)$. Also, D is the time lag duration. For the case of the output delay, shown in Figure 1b, the multiplier model for efficiency evaluation in the presence of output time lag (MPO) is represented as follows:

$$\begin{aligned}
E_k^{\text{MPO}} &= \max \sum_{t=1}^{T-D+1}\sum_{r=1}^{s}\sum_{p=0}^{D-1} y_{rk(t+p)}u_{rt}^{p} \\
\text{s.t.} &\sum_{i=1}^{m} x_{ikt}v_{it} = 1 && t = 1,\ldots,T-D+1 \\
&\sum_{r=1}^{s}\sum_{p=0}^{D-1} y_{rj(t+p)}u_{rt}^{p} - \sum_{i=1}^{m} x_{ijt}v_{it} \leq 0 && j = 1,\ldots,n, t = 1,\ldots,T-D+1 \\
&v_{it} \geq 0 && i = 1,\ldots,m, t = 1,\ldots,T-D+1, \\
&u_{rt}^{p} \geq 0 && r = 1,\ldots,s, p = 0,1,\ldots,D-1, t = 1,\ldots,T-D+1. \qquad (1.3)
\end{aligned}$$

As explained in the previous section, the traditional DEA models consider a production unit a black box, and the internal processes of the units are ignored. Inputs may go through several stages to produce outputs. The overall efficiency depends on the efficiency of these stages. For accurate performance evaluation, the efficiency values of these stages and the system as a whole must be calculated, and their relationships should be determined. As a result, the efficiency value obtained is more reliable, and the sources of inefficiency are better realized [22]. To resolve this issue, we will present a new two-stage DEA model with time lag effects in the next section.

2. New two-stage multi-time DEA modeling with time lag effects

In this section, we present some novel two-stage DEA models with time lag effects in both multiplier and envelopment forms.

2.1. A two-stage multiplier network DEA model with a time lag

For the simplicity of presentation, the network is considered a two-stage process in a series case as follows.

The research question is, when there is a time lag in the first stage of a two-stage network production system, how the evaluation process is done. The main purpose of this section is to answer this question in detail. In other words, we aim to present new network DEA models that evaluate the efficiency with a time lag in the first stage; the extension to the case of time lag in both production stages would be straightforward.

Suppose that all DMUs use m different inputs to generate q intermediate products in the first stage, and the second stage consumes the intermediate products as inputs to generate s final outputs in the second stage.

Suppose x_{ijt}, z_{qjt}, and y_{rjt} $(r = 1, \dots s, i = 1, \dots m, q = 1, 2, \dots, Q)$ represent ith input, qth intermediate, and rth output, respectively during time period t for DMU_j. The input, intermediate, and output vectors of DMU_j in period t are respectively presented by $\mathbf{x}_{jt} = (x_{1jt}, x_{2jt}, \dots, x_{mjt})$, $\mathbf{z}_{jt} = (z_{1jt}, z_{2jt}, \dots, z_{qjt})$, and $\mathbf{y}_{jt} = (y_{1jt}, y_{2jt}, \dots, y_{sjt})$.

We suggest the following multiplier DEA models for evaluating the efficiency values of individual stages of DMU_k:

(**Stage 1**)

$$\begin{aligned}
&E_1^k = \max \sum_{t=D}^{T} \sum_{q=1}^{Q} z_{qkt} w_{qt} \\
&\text{s.t.} \sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ik(t-p)} v_{it}^p = 1 && t = D, D+1, \dots, T \\
&\quad \sum_{q=1}^{Q} z_{qjt} w_{qt} - \sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ij(t-p)} v_{it}^p \leq 0 && j = 1, \dots, n, t = D, D+1, \dots, T \\
&\quad v_{it}^p \geq 0 && i = 1, \dots, m, t = D, D+1, \dots, T, p = 0, 1, \dots, D-1 \\
&\quad w_{qt} \geq 0 && q = 1, \dots, Q, t = D, D+1, \dots, T
\end{aligned} \tag{2.1}$$

(Stage 2)

$$\begin{aligned}
&E_2^k = \max \sum_{t=D}^{T} \sum_{r=1}^{s} y_{rkt} u_{rt} \\
&\text{s.t.} \sum_{q=1}^{Q} z_{qkt} w_{qt} = 1 && t = D, D+1, \dots, T \\
&\quad \sum_{r=1}^{s} y_{rjt} u_{rt} - \sum_{q=1}^{Q} z_{qjt} w_{qt} \leq 0 && t = D, D+1, \dots, T, j = 1, \dots, n \\
&\quad u_{rt} \geq 0 && r = 1, \dots, s, t = D, D+1, \dots, T \\
&\quad w_{qt} \geq 0 && q = 1, \dots, Q, t = D, D+1, \dots, T.
\end{aligned} \tag{2.2}$$

Here, $(v_{it}^p), (w_{qt})$ and (u_{rt}) are input, intermediate, and output weights, respectively. Also, in this Model, the time lag is applied to inputs of the first stage, and k denotes the index of the unit under evaluation.

The following multiplier model is also suggested for calculating system efficiency by considering constraints for both stages.

$$\begin{aligned}
&E_k = \max \sum_{t=D}^{T} \sum_{r=1}^{s} y_{rk} u_{rt} \\
&\text{s.t.} \sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ik(t-p)} v_{it}^p = 1 && t = D, D+1, \dots, T \\
&\quad \sum_{q=1}^{Q} z_{qjt} w_{qt} - \sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ij(t-p)} v_{it}^p \leq 0 && t = D, D+1, \dots, T, j = 1, \dots, n \\
&\quad \sum_{r=1}^{s} y_{rjt} u_{rt} - \sum_{q=1}^{Q} z_{qjt} w_{qt} \leq 0 && t = D, D+1, \dots, T, j = 1, \dots, n
\end{aligned}$$

$$
\begin{aligned}
&v_{it}^{p} \geq 0 && t = D, D+1, \ldots, T, i = 1, \ldots, m, p = 0, 1, \ldots, D-1 \\
&w_{qt} \geq 0 && q = 1, \ldots, Q, t = D, D+1, \ldots, T \\
&u_{rt} \geq 0 && r = 1, \ldots, s, t = D, D+1, \ldots, T.
\end{aligned} \tag{2.3}
$$

The system and individual efficacy values of the stages are achieved by solving the linear programming of the Model (2.3). If $\left(v_{it}^{p*}, w_{qt}^{*}, u_{rt}^{*}\right)$ is an optimal solution of this model, then the system (overall) efficiency and the efficiency of the stages in the presence of time lag for inputs of the first stage are calculated as follows:

$$
E_k^* = \frac{\sum_{t=D}^{T} \sum_{r=1}^{s} y_{rk} u_{rt}^*}{\sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ik(t-p)} v_{it}^{p*}}, \quad E_1^{k*} = \frac{\sum_{t=D}^{T} \sum_{q=1}^{Q} z_{qkt} w_{qt}^*}{\sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ik(t-p)} v_{it}^{p*}}, \quad E_2^{k*} = \frac{\sum_{t=D}^{T} \sum_{r=1}^{s} y_{rkt} u_{rt}^*}{\sum_{t=D}^{T} \sum_{q=1}^{Q} z_{qkt} w_t^*}.
$$

Note that the efficiency decomposition is provided as $E_k^* = E_1^{k*} \times E_2^{k*}$ for system efficiency evaluation.

Using the following theorem, we note that system efficiency evaluation in Model (2.3) provides a more accurate assessment than the traditional black-box approach.

Theorem 2.1. *The optimal value of Model (2.3) is always less than or equal to the optimal value of Model (1.2).*

Proof. Assume that $\left(v_{it}^{p*}, w_{qt}^{*}, u_{rt}^{*}\right)$ is an optimal solution for Model (2.3), so for the second and third sets of constraints, we have:

$$
\begin{aligned}
\sum_{q=1}^{Q} z_{qjt} w_{qt}^* - \sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ij(t-p)} v_{it}^{p*} \leq 0 \quad & t = D, D+1, \ldots, T, j = 1, \ldots, n \\
\sum_{r=1}^{s} y_{rjt} u_{rt}^* - \sum_{q=1}^{Q} z_{qjt} w_{qt}^* \leq 0 \quad & t = D, D+1, \ldots, T, j = 1, \ldots, n.
\end{aligned}
$$

By integrating these two constraints, we will obtain the following constraint:

$$
\sum_{r=1}^{s} y_{rjt} u_{rt}^* - \sum_{p=0}^{D-1} \sum_{i=1}^{m} x_{ij(t-p)} v_{it}^{p*} \leq 0 \quad t = D, D+1, \ldots, T, j = 1, \ldots, n.
$$

Because the other constraints of the two models are the same, then we infer that $\left(v_{it}^{p*}, u_{rt}^{*}\right)$ is a feasible solution for Model (1.2). In other words, we showed that the optimal solution of Model (2.3) is a feasible solution for Model (1.2). Since both objective functions of Model (1.2) and Model (2.3) are the same and of the maximization form, it can be concluded that the optimal value of Model (2.3) is less than or equal to the optimal value of Model (1.2). □

2.2. A DEA two-stage envelopment model with a time lag

In DEA, there are two different views to evaluate efficiency. One is the multiplier form, and the other is the envelopment form. Each one has unique features that can complement each other. Although these forms are generated with different views, it is possible to express the relationships between these models using the properties of primal-dual models in linear programming [8]. For example, the envelopment CCR form of the dual model is the multiplier CCR form. However, the envelopment form can present the vector point for an inefficient DMU while the multiplier form cannot.

Here, we present models to evaluate the efficiency with a time lag in envelopment form. The envelopment form makes it possible to assess the system and individual efficiencies, as well as benchmark targets. To do so, we will present the following DEA model, which is the extension of the dual form for Model (1.2) to evaluate the efficiency of observed units with inputs time lag:

$$\begin{aligned}
\min\ \theta = & \sum_{t=D}^{T} \theta^t \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j^t x_{ij(t-p)} \leq \theta^t x_{ik(t-p)} && i=1,\ldots,m, p=0,1,\ldots,D-1, t=D,\ldots,T \\
& \sum_{j=1}^{n} \lambda_j^t y_{rjt} \geq y_{rkt} && r=1,\ldots,s, t=D,\ldots,T \\
& \lambda_j^t \geq 0 && j=1,\ldots,n, t=D,\ldots,T \\
& 0 \leq \theta^t \leq 1 && t=D,\ldots,T.
\end{aligned} \tag{2.4}$$

In this model, λ_j^t is the intensity weights for constructing a non-negative combination of the observed DMUs, and θ^t is the ratio of the proportional decrease in inputs of DMU_k for time period t.

To present the efficiency scores, we suggest taking an average of the input efficiency scores of the periods. So, E_k^* as the optimal efficiency score for the DMU_k, is calculated as $E_k^* = \frac{1}{T-D+1}\theta^*$, where θ^* is the optimal value of Model (2.4).

Regarding Model (2.4) and for considering the time lag for the inputs of the first stage, we consider the following two-stage envelopment DEA model:

$$\begin{aligned}
\min\ \theta = & \sum_{t=D}^{T} \theta^t \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j^t x_{ij(t-p)} \leq \theta^t x_{ik(t-p)} && i=1,\ldots,m, p=0,1,\ldots,D-1, t=D,\ldots,T \\
& \sum_{j=1}^{n} \lambda_j^t z_{qjt} \geq \sum_{j=1}^{n} \mu_j^t z_{qjt} && q=1,\ldots,Q, t=D,\ldots,T \\
& \sum_{j=1}^{n} \mu_j^t y_{rjt} \geq y_{rkt} && r=1,\ldots,s, t=D,\ldots,T \\
& \theta^t \text{ free} && t=D,\ldots,T \\
& \lambda_j^t \geq 0 && j=1,\ldots,n, t=D,\ldots,T \\
& \mu_j^t \geq 0 && j=1,\ldots,n, t=D,\ldots,T.
\end{aligned} \tag{2.5}$$

Similar to Model (2.4), λ_j^t and μ_j^t are respectively the first and second stage intensity weights. Also, θ^t denotes the proportional decrease ratio in the inputs of the first stage of the unit under evaluation at the period t. The second set of constraints indicates that the non-negative combination of intermediate products of the first stage must be greater than or equal to the inputs of the second stage.

This Model calculates the system efficiency of the network for the DMU under evaluation. E_k^*, the overall efficiency score for DMU_k, is suggested as $E_k^* = \frac{1}{T-D+1}\theta^*$, where θ^* is the optimal value of Model (2.5).

Need to be noted that the efficiency obtained from Model (2.5) does not suffer from existence of multiple optimal solutions. Although, it is possible that each θ^t in the objective function has multiple optimal solutions, but the optimal value of the objective function, θ^* is always unique. So, the aggregate efficiency score E_k^* for DMU_k is well-defined and unique.

Besides the system efficiency score, a key feature of the new proposed envelopment model is to provide an ideal or benchmark point for the unit under evaluation. So, an ideal or benchmark point for evaluating DMU_k

TABLE 1. Data for the four DMUs over 3 periods.

DMU	Period	x	z	y
A	1	3	–	–
	2	5	7	2
	3	8	6	5
B	1	2	–	–
	2	3	10	1
	3	4	3	7
C	1	1	–	–
	2	4	5	6
	3	7	8	10
D	1	7	–	–
	2	8	5	4
	3	9	10	5

could be presented as $(\mathbf{x}^*, \mathbf{y}^*) = \left(\sum_{j=1}^{n} \mu_j^{t*} y_{rjt}, \sum_{j=1}^{n} \lambda_j^{t*} x_{ij(t-p)}\right)$, in which $\left(\lambda_j^{t*}, \mu_j^{t*}\right)$ is the optimal solution of Model (2.5) for estimating DMU_k.

Similar to the multiplier form, the following proposition can be stated for the envelopment models.

Theorem 2.2. *The optimal value of Model* (2.5) *is always smaller than or equal to the optimal value of Model* (2.4).

Proof. The proof of this theorem is similar to that of Theorem 2.1. □

Here, a simple numerical example is presented to illustrate the discussed approach.

Example 2.3. Suppose four DMUs with one input, one intermediate product, and one output for 3 time periods. Also, suppose that $D = 2$, *i.e.*, the output is produced after two years. The information on these indices is given in Table 1 below.

The multiplier DEA models for black-box efficiency evaluation of DMU_C is as follows:

$$\begin{aligned}
\max\ & 6u_2 + 10u_3 \\
\text{s.t. } & 4v_2^0 + v_2^1 = 1 \\
& 7v_3^0 + 4v_3^1 = 1 \\
& 2u_2 - 5v_2^0 - 3v_2^1 \leq 0 \\
& u_2 - 3v_2^0 - 2v_2^1 \leq 0 \\
& 6u_2 - 4v_2^0 - v_2^1 \leq 0 \\
& 4u_2 - 8v_2^0 - 7v_2^1 \leq 0 \\
& 5u_3 - 8v_3^0 - 5v_3^1 \leq 0 \\
& 7u_3 - 4v_3^0 - 3v_3^1 \leq 0 \\
& 10u_3 - 7v_3^0 - 4v_3^1 \leq 0 \\
& 5u_3 - 9v_3^0 - 8v_3^1 \leq 0 \\
& v_2^0, v_2^1, v_3^0, v_3^1, u_2, u_3 \geq 0.
\end{aligned} \tag{2.6}$$

TABLE 2. E_k^* efficiency obtained for 4 DMUs.

DMU	Efficiency stage 1	Efficiency stage 2	Network	Black-box
A	0.56	0.30	0.17	0.33
B	0.82	0.43	0.36	0.61
C	1	0.76	0.76	1
D	0.57	0.26	0.15	0.32

By solving this linear programming (LP) optimization model, the efficiency score is obtained as $E_C^* = \frac{1}{T-D+1}\theta^* = \frac{1}{3-2+1}(2) = 1$, where θ^* is optimal value of the above model. The proposed network model for performance evaluation of DMU_C in the presence of output time lag could be stated as follows:

$$
\begin{aligned}
\max\ & 6u_2 + 10u_3 \\
\text{s.t. } & 4v_2^0 + v_2^1 = 1 \\
& 7v_3^0 + 4v_3^1 = 1 \\
& 7w_2 - 5v_2^0 - 3v_2^1 \leq 0 \\
& 10w_2 - 3v_2^0 - 2v_2^1 \leq 0 \\
& 5w_2 - 4v_2^0 - v_2^1 \leq 0 \\
& 5w_2 - 8v_2^0 - 7v_2^1 \leq 0 \\
& 6w_3 - 8v_3^0 - 5v_3^1 \leq 0 \\
& 3w_3 - 4v_3^0 - 3v_3^1 \leq 0 \\
& 8w_3 - 7v_3^0 - 4v_3^1 \leq 0 \\
& 10w_3 - 9v_3^0 - 8v_3^1 \leq 0 \\
& 2u_2 - 7w_2 \leq 0 \\
& u_2 - 10w_2 \leq 0 \\
& 6u_2 - 5w_2 \leq 0 \\
& 4u_2 - 5w_2 \leq 0 \\
& 5u_3 - 6w_3 \leq 0 \\
& 7u_3 - 3w_3 \leq 0 \\
& 10u_3 - 8w_3 \leq 0 \\
& 5u_3 - 10w_3 \leq 0 \\
& v_2^0, v_2^1, v_3^0, v_3^1, u_2, u_3, w_2, w_3 \geq 0.
\end{aligned} \tag{2.7}
$$

By solving this LP model, the network efficiency score is calculated as $E_C^* = 0.76$. Similarly, one can obtain the black-box and network delayed efficiency and the stages efficiency scores. The results are presented in Table 2.

As you can see in Table 2, the black-box efficiency for DMU_C is 1, and the other units are classified as inefficient. But it cannot be guaranteed that the network efficiency will reach 1. It means that in the network mode, the inefficiency sources are better recognized, and the efficiency values of the DMUs are more accurate than the black-box model. Also, the traditional and network efficiency of DMUs, as well as the efficiency values of the first and second stages of DMUs, have been presented. As columns 1 and 2 show, the network efficiency value is equal to the product of the efficiency values of the stages. For example, for DMU_C, the efficiency value of stage 1 equals 1, and the efficiency value of stage 2 is 0.76, and their product is 0.76.

In the next section, we present a practical application of the new proposed approach.

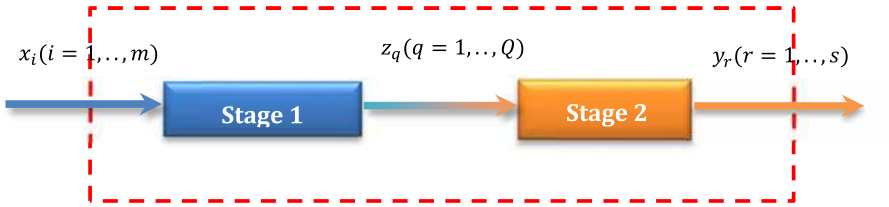

FIGURE 2. A simple two-stage process.

3. A practical example

This section will provide an empirical example of applying the models presented in this article in the horticulture industry to illustrate the importance and applicability of the proposed method.

3.1. Data

Qazvin Agriculture Jihad Organization[3] offered 150 hectares of land to boost production and reduce youth unemployment. Cherry, sour cherry, and walnut trees will be cultivated in this land. Also, the bank facilities as loans are available to 15 people chosen by lot. Because of the bank limited resources, this loan will be paid to farmers within 5 years as the work progresses. Every farmer can have a maximum of ten acres of land. We know that these trees will yield after 6 years. That is, after six years of spending money and work, the trees will bear fruit. We want to calculate and compare the efficiency of these farmers for over ten years. We consider the whole period as 6 years. Each farmer can develop the land until the fifth year and then stop the development. This action aims to create conversion industries, add value to agricultural products, and prevent waste. This example is solved in two stages. Inputs, intermediate, and output variables in this evaluation are considered as follows:

- **The inputs to the first stage: (x_{ij})**
  (1) Land area (ha)
  (2) Water consumption (L)
  (3) Fertilizer consumed (kg)
  (4) Human resources (number)
  (5) Saplings (number)
- **The intermediate products: (z_{dj})**
  (1) Cherry (ton)
  (2) Sour cherry (ton)
  (3) Walnuts (ton)
- **The outputs of the second stage: (y_{rj})**
  (1) Cherry compote (number)
  (2) Sour cherry compote (number)
  (3) Packed walnut kernels (number)

The weight of each compote or package is half a kilo, and their unit is number. The time lag diagram is also shown in Figure 2 below.

It is noteworthy that each farmer should consume the first stage inputs in 5 years and does not produce any output during these 5 years; the output is produced after 5 years. This is the reason for this time lag in the output of the first stage. In other words, this farmer uses water and fertilizer for the trees as inputs for 5 years, and after this time, he will have the fruits of walnuts, cherry, sour cherry as delayed output.

[3] https://qazvin-ajo.ir/

TABLE 3. Farmer No. 1's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1	400 000	90	200	450						
2	2	800 000	180	400	900						
3	4	1 600 000	360	800	1800						
4	6	2 400 000	540	1200	2700						
5	10	4 000 000	900	2000	4500						
6	10	4 000 000	900	200	4500	1.5	1.8	0.3	6000	7200	150
7	10	4 000 000	1000	200	4500	4	5	0.8	16 000	20 000	400
8	10	4 000 000	1000	2000	4500	12	14	1.5	48 000	56 000	750
9	10	4 000 000	1000	2000	4500	14	16	3	56 000	64 000	1500
10	10	4 000 000	1000	2000	4500	16	18	5	64 000	72 000	2500

As shown in Figure 3, there is a time lag in the first stage for 6 years, *i.e.*, the trees will yield after 6 years. But in the second stage, we have no time lag. The input of the first six-year is spent and used to produce the output of the first stage in the sixth year. The intermediate output of the sixth year is used as the input for the second stage to produce the output of this stage. The inputs of the second to the seventh year are used for the output production of the first stage of the seventh year. Also, these intermediate measures are used to produce the second stage output for the seventh year and so on until the tenth year. Besides, Table 3 presents the first farmer's input, intermediate, and output data for ten years.

Data for other farmers are given in the appendix.

3.2. Results

To evaluate these 15 farmers, we used two DEA models of (2.4) and (2.5) and two multiplier models of (2.1) and (2.2) (presented in this paper). Model (2.4) is a traditional model with a black-box structure with only input and output. But Model (2.5) has a network structure and includes intermediate products. GAMS software was used to solve all linear programming models in this study.

The results of the model implementation are reported in Table 4 below.

According to the traditional model, nine farmers are classified as efficient. But in the network model, none of the farmers were fully efficient, and farmer 1 has the highest efficiency value. This farmer has the highest efficiency in both models. Consider farmer number 4. This farmer has the lowest efficiency value in the traditional model, which means he has the worst performance. But in the network model, he is higher than seven farmers because the traditional model does not consider the intermediate measures in calculating this farmer's efficiency. Besides, the efficiency values of stages 1 and 2 are presented in the fifth and sixth columns. The product of efficiency values of the stages yields the exact value of network efficiency. As you can see, the number of efficient units is very high in the first stage. But in the second stage, none have worked well. So, the second stage has a significant impact on the inefficiency of the farmers. Therefore, in the network mode, inefficiency resources are better analyzed than the traditional model, which increases the accuracy and validity of the efficiency.

As we expected, the efficiency of the network case does not exceed the efficiency of black-box modeling. As a result, the new approach provided more discriminant power in the performance evaluation of the production units. There is no efficiency score interference in the network model, and a unique ranking is provided for the farmers. But for black-box efficiency, the efficiency score interference is not negligible, especially in score one. In the sixth column of Table 4, farmers' ranking is presented based on network efficiency scores. This ranking is based on the fact that any farmer with a high network efficiency score has a better position. Farmers 1, 15, and 6 are in the first to third ranks, and farmers 9, 8, and 12 are in the last three positions, respectively. There is also no ranking interference in network efficiency.

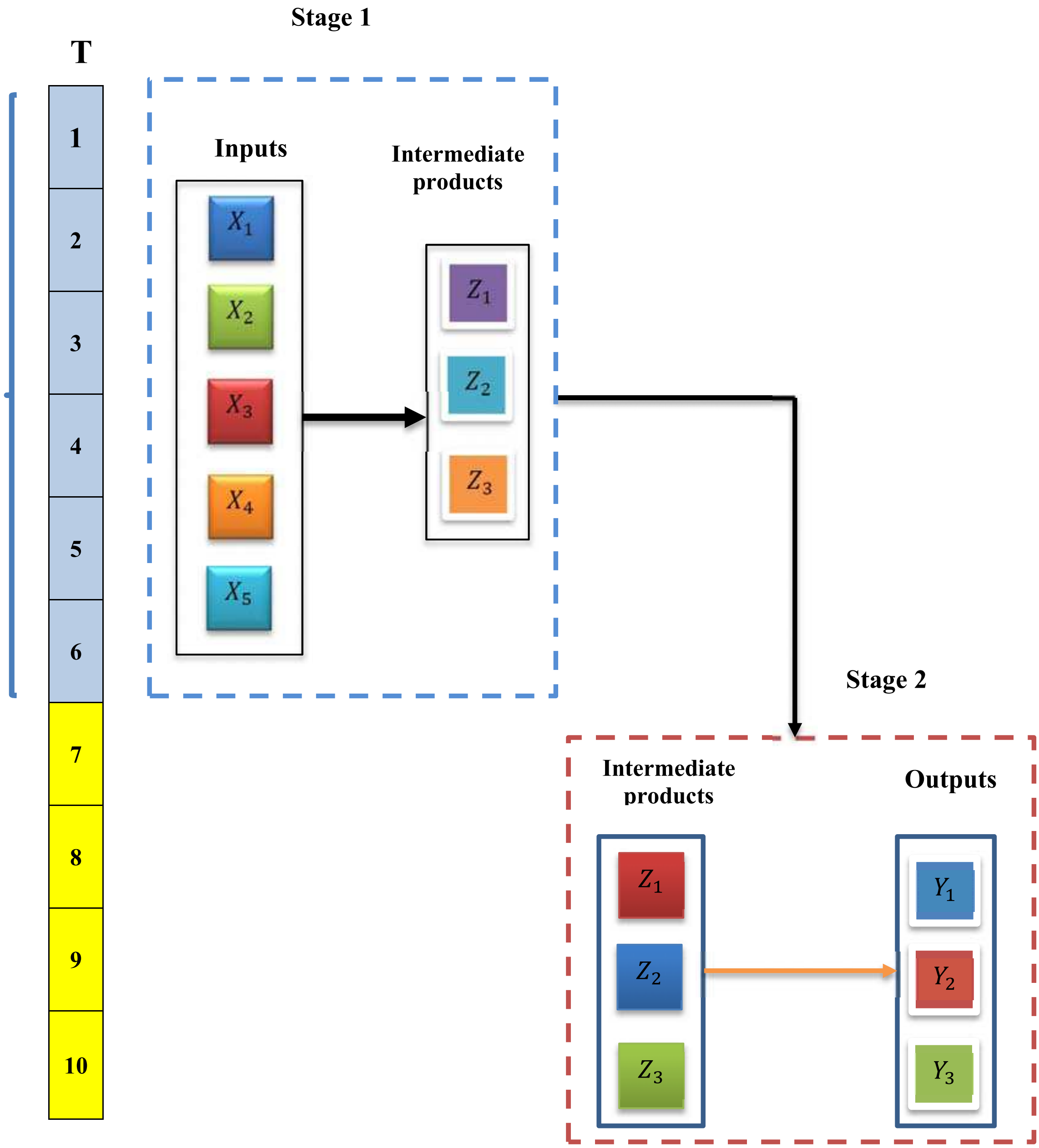

FIGURE 3. The diagram of the network production for the practical example.

Besides, 15 farmers without time lag are evaluated and compared. In this regard, the corresponding output with time lag is assumed 0, and we use a regular two-stage DEA [20] to evaluate the farmers. The results are presented in the third column of Table 4. Using this method, farmers 1, 14, and 15 are in the first, second, and third ranks, respectively. Compared with positions resulting from network efficiency with a time lag, only farmer 1 gets the same rank, and these two methods yield different results; it shows the importance of considering time lag in evaluating the farmers.

Table 4. The efficiency values of 15 farmers with a time lag.

Farmer	Black-box efficiency	Network efficiency without time lag	Network efficiency with time lag	Efficiency stage 1	Efficiency stage 2	Ranking by Network efficiency
1	1	0.485	0.998	1	0.998	1
2	0.998	0.457	0.948	0.992	0.955	7
3	0.997	0.452	0.944	0.998	0.945	9
4	0.99	0.436	0.946	0.991	0.954	8
5	0.999	0.437	0.958	0.999	0.959	3
6	0.993	0.431	0.922	0.947	0.973	11
7	1	0.445	0.938	1	0.938	10
8	1	0.432	0.908	1	0.908	14
9	1	0.405	0.918	1	0.918	13
10	1	0.413	0.92	1	0.92	12
11	1	0.453	0.953	1	0.953	6
12	0.997	0.417	0.85	1	0.85	15
13	1	0.453	0.958	1	0.958	4
14	1	0.463	0.958	1	0.957	5
15	1	0.461	0.978	0.996	0.981	2

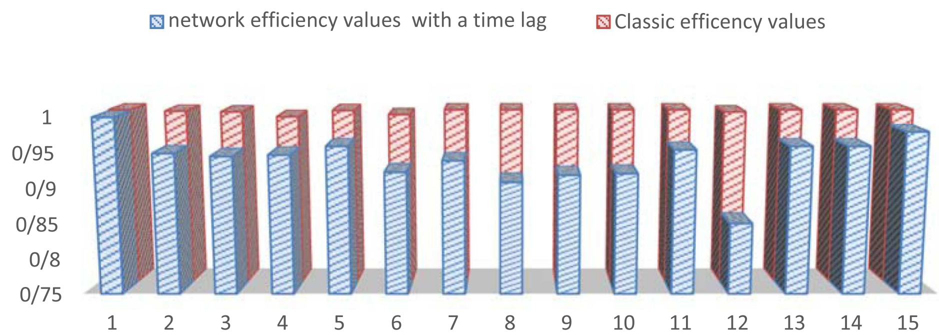

Figure 4. Network efficiency scores with time lag for farmers.

Figure 4 displays the network efficiency values of the farmers with a time lag.

As you can see, the height of the efficiency score in the network model is always less than that in the black box. The reason for this is ignoring the intermediate structure for DMUs in black-box evaluation. Therefore, the traditional model is incapable of comprehensively evaluating DMUs with time-lag. In this case, it seems essential to consider the time-lagged network models, including the model presented in this paper.

3.3. Managerial implications

As mentioned in the practical example, a time delay always happens in the production process. For instance, trees lack any output at first, and after a few years, they start to bear fruits. So if a time delay is missed in the evaluation, we may incorrectly evaluate the performance, leading to faulty management decisions by the decision-maker. The next thing seen in the numerical example is the network structure of performance appraisal. According to the practical example, in the black-box structure, most farmers were efficient; that renders an incorrect evaluation of them. However, this shortcoming was eliminated in the envelopment data analysis model of the presented network data, and the farmers were ranked using the network model. Even in the network model, the efficiency of each step is given for further analysis of management results. In general, the proposed model can be used as the main model for evaluating performance in manufacturing industries with a time delay factor, in which other models of data envelopment analysis do not adequately assess them.

4. Conclusion and suggestions

In this paper, we present a network DEA model with a time lag. Based on a real-world example and a two-stage production system with a time lag for stage 1, some new models for efficiency evaluation were presented in both multiplier and envelopment forms. Finally, to illustrate the importance of the issue, we evaluated the efficiency in the Iran horticulture industry using the models presented. It is shown that the new approach provides successful modeling of time lag effects in the performance evaluation of two-stage production systems.

Some industries may have time delay indicators, which leads to incorrect results in performance evaluation using traditional approaches. Therefore, developing performance appraisal models is essential in dealing with these conditions. But the type of model expansion should always match the needs of the real world. A real-world example discussed in this article has a two-stage network structure with a time delay at the input of the first stage. For this reason, the extended model of the input version had a two-stage with a time delay in the first stage. The results showed that a complete evaluation of decision-making units could be done using the proposed model. So, DMUs were ranked using the results of the proposed model. In contrast, the traditional model was not able to do this.

The model presented in this paper was proposed based on the type of time delay in the practical example. But in real-world applications of evaluation, other structures of time delay may be seen. The different appearances of these structures are one of the limitations of this research. However, researchers can introduce new time-delay structures in future studies based on practical examples in the real world and offer other new models for evaluations. In addition to these unfavorable output indicators, uncontrollable inputs can be considered performance evaluation indicators in dealing with time delays. Even we may need to present multi-period models. All of these issues can be viewed by researchers in future studies.

Appendix A. Data for farmers in the horticulture industry

Table A.1. Farmer No. 2's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	0.9	380 000	100	160	400						
2	2	810 000	200	350	800						
3	3.8	1 630 000	300	750	1700						
4	6.2	2 400 000	500	1170	2800						
5	10	3 970 000	850	2050	4600						
6	10	4 000 000	900	2010	4600	1.2	1.5	0.4	6500	7000	100
7	10	4 000 000	900	2030	4600	4.3	4.8	0.7	16 000	19 000	300
8	10	4 000 000	950	2040	4600	12	13	1.6	47 000	55 000	750
9	10	3 980 000	970	2050	4600	13	15	4	57 000	65 000	1600
10	10	3 980 000	1000	2070	4600	17	19	6	63 000	70 000	2400

Table A.2. Farmer No. 3's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1.2	360 000	90	150	410						
2	2.3	780 000	170	310	850						
3	4.2	1 570 000	280	720	1600						
4	6.5	2 300 000	510	1100	2600						
5	10	3 900 000	820	2000	4700						
6	10	4 500 000	910	2100	4700	1.1	1.6	0.45	6400	6900	100
7	10	4 500 000	920	2100	4700	4	5	0.8	15 000	20 000	310
8	10	4 500 000	900	2100	4700	11	13.5	1.5	48 000	53 000	780
9	10	4 500 000	900	2100	4700	12	15	4.1	56 000	64 000	1610
10	10	4 500 000	900	2100	4700	16	19.5	5.4	62 000	71 000	2430

Table A.3. Farmer No. 4's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1.4	330 000	86	120	400						
2	2.5	769 000	169	300	820						
3	4.5	1 560 000	257	700	1580						
4	6	2 279 000	514	1200	2610						
5	10	3 810 000	800	2010	4620						
6	10	4 489 000	905	2114	4600	1	1.5	0.5	6300	6800	90
7	10	4 500 000	915	2200	4700	3.8	4.9	0.9	15 100	19 000	300
8	10	4 600 000	926	2200	4700	10.2	13	1.3	48 900	51 000	800
9	10	4 600 000	930	2200	4750	11.5	14	4	55 600	65 000	1600
10	10	4 600 000	910	2160	4700	15.4	20	5.5	61 000	70 000	2500

Table A.4. Farmer No. 5's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1.3	325 000	83	130	420						
2	2.6	759 000	200	310	850						
3	4.9	1 541 000	214	720	1600						
4	6.2	2 283 000	502	1240	2700						
5	10	3 900 000	720	2020	4600						
6	10	4 529 000	890	2200	4610	0.9	1.7	0.6	6000	6700	100
7	10	4 600 000	900	2250	4600	3.5	5.1	1	15 000	18 000	280
8	10	4 610 000	910	2250	4600	10	14	1.4	49 000	50 000	725
9	10	4 650 000	915	2270	4600	11	15	3.9	53 000	62 000	1510
10	10	4 500 000	915	2280	4600	15	19	5	60 000	73 000	2391

TABLE A.5. Farmer No. 6's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1	300 000	85	135	440						
2	02-Feb	720 000	196	308	810						
3	06-Apr	1 420 000	209	729	1530						
4	05-Jun	2 300 000	486	1270	2810						
5	10	3 700 000	719	2000	4521						
6	10	4 200 000	859	2150	4600	0.8	1.9	0.7	5000	6500	90
7	10	4 100 000	910	2169	4720	3.2	5	1.2	14 000	17 000	300
8	10	4 100 000	920	2170	4700	9.2	13.5	1.5	50 000	45 000	749
9	10	4 000 000	900	2180	4650	10.8	14.8	4	54 000	60 000	1520
10	10	4 000 000	900	2100	4600	3.5	19.2	5.3	59 000	70 000	2401

TABLE A.6. Farmer No. 7's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	0.8	250 000	80	140	400						
2	2	610 000	190	310	800						
3	05-Apr	1 400 000	220	700	1500						
4	03-Jun	2 500 000	456	1200	2750						
5	10	3 400 000	725	2150	4520						
6	10	4 000 000	812	2100	4520	1	2	0.1	5500	6000	100
7	10	4 200 000	920	2170	4500	3.5	6	1	13 000	18 000	310
8	10	4 100 000	930	2185	4500	10	14	1.6	49 000	42 000	750
9	10	4 100 000	935	2200	4500	11	15	5	58 000	59 000	1620
10	10	400 000	935	2125	4500	14	20	6	62 000	75 000	2570

TABLE A.7. Farmer No. 8's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1	260 000	89	139	420						
2	2	600 000	196	300	810						
3	4	1 200 000	200	709	1580						
4	6	2 300 000	471	1210	2830						
5	10	3 100 000	742	2143	4510						
6	10	3 900 000	826	2050	4500	1.2	1.5	0.7	5000	5700	80
7	10	4 000 000	907	2185	4500	3.3	5.7	1.2	12 800	17 500	250
8	10	4 000 000	910	2225	4450	9.5	13.5	1.5	47 000	40 000	681
9	10	4 000 000	915	2300	4450	10.5	14	5.5	57 000	57 000	1520
10	10	4 100 000	920	2310	4400	13	19	7	65 000	76 000	2608

Table A.8. Farmer No. 9's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	0.7	270 000	80	140	400						
2	1.6	630 000	170	289	800						
3	3.9	1 310 000	195	717	1600						
4	5.8	2 200 000	450	1360	2800						
5	10	3 000 000	780	2220	4700						
6	10	4 000 000	830	2100	4650	1.1	1.3	0.8	4920	5200	70
7	10	4 500 000	920	2185	4600	1.3	5.2	1.4	12 810	15 800	200
8	10	4 500 000	930	2325	4600	9	12.7	1.9	48 200	38 000	700
9	10	4 400 000	931	2400	4600	11	13.9	5.3	55 300	43 000	1600
10	10	4 600 000	932	2380	4600	13.8	18.5	8.2	64 100	59 000	2700

Table A.9. Farmer No. 10's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	0.9	280 000	70	170	420						
2	1.9	600 000	150	300	830						
3	3.5	1 200 000	180	700	1700						
4	5.2	2 100 000	420	1400	2900						
5	10	2 900 000	750	2300	4600						
6	10	3 700 000	900	2400	4550	1.3	1.1	1	4950	4100	60
7	10	3 900 000	910	2350	4500	2.9	4.8	1.2	13 100	11 800	150
8	10	3 900 000	950	2400	4500	9.3	11.2	1.7	49 200	35 000	710
9	10	3 910 000	940	2500	4500	11.5	12.7	5.5	54 500	40 000	1500
10	10	3 950 000	930	2400	4500	13.9	14.5	9	67 300	57 000	2600

Table A.10. Farmer No. 11's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1.2	290 000	60	190	400						
2	2.1	580 000	160	310	840						
3	3.7	1 100 000	170	720	1650						
4	5.8	2 000 000	400	1500	3000						
5	10	2 600 000	800	2400	4700						
6	10	3 400 000	1000	2300	4600	1.5	1.3	0.9	5000	4000	70
7	10	3 500 000	950	2400	4600	2.3	4.9	1.5	14 000	12 000	160
8	10	3 500 000	940	2500	4550	9	11.8	1.8	50 000	34 000	720
9	10	3 600 000	930	2550	4550	11	12.3	5.3	52 000	43 000	1510
10	10	3 700 000	920	2610	4550	14	15.1	9.8	64 000	52 000	2680

TABLE A.11. Farmer No. 12's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1.1	320 000	80	200	350						
2	2	600 000	170	300	750						
3	3.8	1 000 000	160	750	1500						
4	6	1 900 000	390	1400	2800						
5	10	2 700 000	820	2350	4300						
6	10	3 500 000	980	2400	4300	1.5	1.5	1	4000	4000	70
7	10	3 600 000	1000	2500	4250	2.4	5	1.3	13 000	13 000	150
8	10	3 500 000	1000	2600	4250	10	12	1.9	45 000	37 000	700
9	10	3 500 000	1100	2550	4250	12	13	5.2	53 000	42 000	1400
10	10	3 500 000	1100	2600	4250	15	14	9.9	67 000	50 000	2700

TABLE A.12. Farmer No. 13's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1.3	370 000	100	140	400						
2	2.4	720 000	180	300	800						
3	4.4	1 600 000	250	700	1400						
4	6.7	2 200 000	500	1000	2600						
5	10	3 800 000	800	1500	4500						
6	10	4 200 000	900	2000	4500	1.2	1.5	0.5	6000	7000	90
7	10	4 300 000	900	2100	4500	4.1	4.5	0.9	14 000	19 000	300
8	10	4 300 000	900	2200	4500	11.5	13	2.5	49 000	54 000	800
9	10	4 300 000	900	2200	4500	12	14	4.5	57 000	65 000	1500
10	10	4 300 000	900	2200	4500	15	20	6	63 000	73 000	2600

TABLE A.13. Farmer No. 14's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1.5	400 000	120	165	410						
2	2	800 000	210	340	820						
3	3.5	1 600 000	310	700	1710						
4	6	2 300 000	508	1100	2890						
5	10	4 000 000	840	2000	4500						
6	10	4 100 000	920	2020	4500	1	2	0.8	7000	7500	90
7	10	4 200 000	900	2010	4500	4	5	0.9	17 000	20 000	400
8	10	400 000	900	2050	4400	11	12	1.7	45 000	54 000	800
9	10	800 000	910	2050	4400	12	14	4.3	56 000	62 000	1500
10	10	1 600 000	910	2050	4400	16	20	7	60 000	70 000	2700

Table A.14. Farmer No. 15's input, intermediate, and output data for 10 years.

	x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	y_1	y_2	y_3
1	1	450 000	100	170	400						
2	2	750 000	200	350	820						
3	4	1 700 000	300	650	1700						
4	6	2 400 000	500	1000	2800						
5	10	4 000 000	800	2000	4400						
6	10	4 500 000	900	2050	4400	1.2	1.3	1	6000	7000	100
7	10	4 500 000	900	2040	4400	4.3	4.8	1.5	16 000	19 000	300
8	10	4 500 000	900	2100	4400	10.5	10.6	1.9	50 000	55 000	900
9	10	4 500 000	900	2100	4400	11.5	13	5	60 000	63 000	1600
10	10	4 500 000	900	2150	4400	15.5	18	6	68 000	72 000	2800

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References

[1] A. Amirteimoori and S. Kordrostami, Multi-period efficiency analysis in data envelopment analysis. *Int. J. Math. Oper. Res.* **2** (2010) 113–128.

[2] R.D. Banker, A. Charnes and W.W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Manage. Sci.* **32** (1984) 18–70.

[3] W. Briec and K. Kerstens, Multi-horizon Markowitz portfolio performance appraisals: a general approach. *Omega* **37** (2009) 50–62.

[4] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units. *Eur. J. Oper. Res.* **2** (1978) 429–444.

[5] Y. Chen, W.D. Cook, C. Kao and J. Zhu, Network DEA pitfalls: divisional efficiency and frontier projection. *Eur. J. Oper. Res.* **226** (2014) 507–515.

[6] L. Chen, F. Lai, Y.M. Wang, Y. Huang and F.M. Wu, A two-stage network data envelopment analysis approach for measuring and decomposing environmental efficiency. *Comput. Ind. Eng.* **119** (2018) 388–403.

[7] W.D. Cook, J. Zhu, G.B. Bi and F. Yang, Network DEA: additive efficiency decomposition. *Eur. J. Oper. Res.* **207** (2010) 1122–1129.

[8] W.W. Cooper, L.M. Seiford and K. Tone, Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software. Springer, New York (2007).

[9] D.K. Despotis, G. Koronakos and D. Sotiros, The "weak-link" approach to network DEA for two-stage processes. *Eur. J. Oper. Res.* **254** (2016) 481–492.

[10] A. Emrouznejad and G.L. Yang, A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. *Soc.-Econ. Plan. Sci.* **61** (2017) 4–8.

[11] A. Esmaeilzadeh and R. Kazmi Matin, Multi-period efficiency measurement of network production systems. *Measurement* **134** (2019) 835–844.

[12] R. Färe and S. Grosskopf, Productivity and intermediate products: a frontier approach. *Econ. Lett.* **50** (1996) 65–70.

[13] R. Färe and S. Grosskopf, Network DEA. *Soc. Econ. Plan. Sci.* **34** (2000) 35–49.

[14] M.J. Farrell, The measurement of productive efficiency. *J. R. Stat. Soc. Ser. A Gen.* **120** (1957) 253–281.

[15] B. Golany, S.T. Hackman and U. Passy, An efficiency measurement framework for multi-stage production systems. *Ann. Oper. Res.* **145** (2006) 51–68.

[16] A. Hadi-Vencheh and R. Kazemi Matin, An application of IDEA to wheat farming efficiency. *Agric. Econ.* **42** (2011) 487–493.

[17] D. Holod and H.F. Lewis, Resolving the deposit dilemma: a new DEA bank efficiency model. *J. Banking Finance* **35** (2011) 2801–2810.

[18] J. Jablonsky, Efficiency analysis in multi-period systems: an application to performance evaluation in Czech higher education. *Cent. Eur. J. Oper. Res.* **24** (2016) 283–296.

[19] S. Kaffash, R. Azizi, Y. Huang and J. Zhu, A survey of data envelopment analysis applications in the insurance industry 1993–2018. *Eur. J. Oper. Res.* **248** (2019) 801–813.

[20] C. Kao, Efficiency decomposition in network data envelopment analysis: a relational model. *Eur. J. Oper. Res.* **192** (2009) 949–962.

[21] C. Kao, Efficiency measurement for parallel production systems. *Eur. J. Oper. Res.* **196** (2009) 1107–1112.

[22] C. Kao, Efficiency decomposition for general multi-stage systems in data envelopment analysis. *Eur. J. Oper. Res.* **232** (2014) 117–124.

[23] C. Kao, Inefficiency identification for closed series production systems. *Eur. J. Oper. Res.* **275** (2019) 599–607.

[24] C. Kao and S.N. Hwang, Efficiency measurement for network systems: IT impact on firm performance. *Decis. Support Syst.* **48** (2010) 437–446.

[25] C. Kao and S.T. Liu, Multi-period efficiency measurement in data envelopment analysis: the case of Taiwanese commercial banks. *Omega* **47** (2014) 90–98.

[26] R. Kazemi Matin, G.R. Jahanshahloo and A.H. Vencheh, Inefficiency evaluation with an additive DEA model under imprecise data, an application on IAUK departments. *J. Oper. Res. Soc. Jpn.* **50** (2007) 163–177.

[27] K. Kerstens, P. Mazza, T. Ren and I. Van de Woestyne, Multi-Time and Multi-Moment Nonparametric Frontier-Based Fund Rating: Proposal and Buy-and-Hold Backtesting Strategy (No. 2021-EQM-03) (2021).

[28] M. Khoveyni, H. Fukuyama, R. Eslami and G.L. Yang, Variations effect of intermediate products on the second stage in two-stage processes. *Omega* **85** (2019) 35–48.

[29] S. Kordrostami and M. Jahani Sayyed Noveiri, Evaluating the efficiency of firms with negative data in multi-period systems: an application to bank data. *Int. J. Ind. Math.* **9** (2017) 27–35.

[30] H. Lewis and T. Sexton, Network DEA: efficiency analysis of organizations with complex internal structure. *Comput. Oper. Res.* **31** (2004) 1365–1410.

[31] S. Lim and J. Zhu, Primal-dual correspondence and frontier projections in two-stage network DEA models. *Omega* **83** (2019) 236–248.

[32] J.S. Liu and W.M. Lu, Network-based method for ranking of efficient units in two-stage DEA models. *J. Oper. Res. Soc.* **63** (2012) 1153–1164.

[33] M.R. Morey and R.C. Morey, Mutual fund performance appraisals: a multi-horizon perspective with endogenous benchmarking. *Omega* **27** (1999) 241–258.

[34] J. Nemoto and M. Goto, Measuring dynamic efficiency in production: an application of data envelopment analysis to Japanese electric utilities. *J. Prod. Anal.* **19** (2003) 191–210.

[35] O. Özpeynici and M. Köksalan, Performance evaluation using data envelopment analysis in the presence of time lags. *J. Prod. Anal.* **27** (2007) 221–229.

[36] K.S. Park and K. Park, Measurement of multi period aggregative efficiency. *Eur. J. Oper. Res.* **193** (2009) 567–80.

[37] S.H. Razavi Hajiagha, S.S. Hashemi, H. Amoozed Mahdiraji and J. Azaddel, Multi-period data envelopment analysis based on Chebyshev inequality bounds. *Expert Syst. App.* **42** (2015) 7759–7767.

[38] T. Ren, Z. Zhou and H. Xiao, Estimation of portfolio efficiency considering social responsibility: evidence from the multi-horizon diversification DEA. *RAIRO: Oper. Res.* **55** (2021) 611–637.

[39] L.M. Seiford and J. Zhu, Profitability and marketability of the top 55 US commercial banks. *Manage. Sci.* **45** (1999) 1270–1288.

[40] K. Tone, A slacks-based measure of efficiency in data envelopment analysis. *Eur. J. Oper. Res.* **130** (2001) 498–509.

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