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AN OPTIMAL ORDERING POLICY FOR A VISITOR-BASED PURCHASING SYSTEM WITH STOCHASTIC DELIVERY TIME AND PARTIAL PREPAYMENT FOR PROFIT MAXIMIZATION

Ata Allah Taleizadeh[1], Hamidreza Zarei[1] and Bhaba R. Sarker[2,*]

Abstract. The classical inventory control policies assume that orders are paid for at the time of their receipts, but in practice, suppliers may require retailers to pay a fraction of the purchasing cost in advance, and sometimes allow them to pay this cost in several prepayments during a predetermined period. Planning inventory replenishments and prepayments become challenging when decisions must be made under uncertainty, especially when delivery time is stochastic, and shortages may occur. This paper develops an inventory control model in a purchasing system in which a visitor sells the product of a manufacturer, and a buyer receives call from the visitor to make an order and items arrives at stochastic time. Both partial prepayments and partial backordering are assumed in the model. The main aim of the paper is to determine the optimal level of inventory of the buyer such that his total profit is maximized. A mathematical model with a general probability distribution for lead time is developed and globally optimal solutions are derived for the model. The applicability of the model is discussed through two special cases for uniform and exponential probability distributions. The results are supportive of the proposed ideas and they reflect an efficient approach.

Mathematics Subject Classification. 35-XX, 44-XX, 45-XX, 90-XX, 91-XX.

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1. Introduction

In competitive markets, usually wholesaler requires some prepayments when orders from buyers are placed. This request may be used to avoid cancelations from the buyers. So, there are situations in which wholesaler suggests prepayment, and buyers must pay the fraction of the purchasing cost. Under prepaying a fraction of purchasing cost to the retailer, the buyer sacrifices the interest on the fraction of purchasing cost prepaid to the wholesaler. Thus this payment scheme is a real life phenomenon and the decision regarding the prepayment and inventory policies have a crucial impact on the total cost and decision variables of inventory systems used by the buyers.

Classic inventory control models assume that the order is delivered to the customer in a deterministic (and often negligible) lead time, and that purchasing costs must be paid at the time of the delivery. In reality,

Keywords. Inventory control, stochastic period length, partial prepayment, advance payment, partial backordering.

[1] School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.

[2] Department of Mechanical & Industrial Engineering, Louisiana State University, Baton Rouge, LA 70803, USA.

*Corresponding author: basrker@lsu.edu

suppliers often require that the retailers pay a fraction of the purchasing cost in advance. The supplier may give the customer the opportunity to make the prepayment in a single or multiple installment. In addition, period length and delivery lead times are often uncertain especially in visitor – based selling system and have to be accounted in inventory control policies. This paper therefore develops an inventory control model that considers both prepayments and stochastic period length because of uncertain arrival time of visitor or delivery time to the buyer's store.

Advanced payments (or prepayments) have frequently been discussed in the literature. If the supplier requests an advanced payment, then the buyer is required to pay a fraction of the purchasing cost before the receipt of the order. One of the first works in this is the one of Goyal [8], who developed an economic order quantity (EOQ) model with permissible delay in payment. Taleizadeh [28] developed an EOQ model for evaporating products with shortages and partial backlogging and assumed that the supplier requests a partial advanced payment from the buyer. Taleizadeh *et al.* [32] studied a fuzzy EOQ model for deteriorating products. The authors considered a situation where the supplier grants quantity discounts, but in turn charges an advanced payment from the buyer. Taleizadeh *et al.* [33] presented an EOQ model with multiple prepayments and considered three possible states for shortages. Guria *et al.* [11] presented an inventory control model for a product under inflation and price-dependent demand. They assumed that the planning horizon can either be deterministic or stochastic and that shortages occur in case of uncertainty. Thangam [38] developed an EOQ model for a three-echelon supply chain with perishable products. The authors considered prepayments and aimed on minimizing the retailer's inventory carrying cost. Zhang *et al.* [41] studied the effect of determination of price and inventory replenishment on the system when demands are from different market segments. Maihami and Nakhai [18] developed an inventory control model for deteriorating items. The authors assumed that shortages may occur, and that partial backordering is possible. In addition, they assumed that demand is price- and time-dependent and determined the optimal selling price in addition to the inventory control policy. Sarker *et al.* [26] determined an optimal payment time under permissible delay in payment for products with deterioration. Liang and Zhou [17] considered a two-warehouse inventory model with delay in payments for deteriorating products to discover the optimal replenishment policy to minimize the aggregate inventory costs. Zhang *et al.* [42] analyzed the buyer's inventory policy when advance and delay in payment simultaneously occur and as partial. Gupta *et al.* [9] solved the mixed integer inventory problem with constant lead time, uniform demand rate and discount for prepaying a fraction of purchasing cost. They used real coded genetic algorithm to solve this problem. Ouyang *et al.* [21] developed an inventory model for deteriorating products with delay in payments. Taleizadeh [29] developed an EOQ model for a deteriorating product with multiple advance payment in the system. Taleizadeh *et al.* [30] developed an EOQ model to purchase expensive raw materials with advance payment and without shortages. Further, Wu *et al.* [40] studied an inventory model for materials with expiration date when the seller asks for advanced payments.

On the other hand, the first work on stochastic length on this subject was performed by Ertogral and Rahim [6]. In this work they analyzed an inventory problem with periodic replenishment when the supplier visit intervals are taken to be independent identically distributed random variables. Chiang [3] considered it is possible that the review periods has a variable length. He assumed that replenishment intervals are independently and identically distributed. In another study, Chiang [4] extended his model with considering backordering and lost sales for shortages when there is a fixed cost of ordering. Further, Tang and Musa [37] worked on distinguishing risk issues and research advancements in supply chain risk management which has demonstrated attention on this subject is increasing in the world. Sarkar *et al.* [25] presented a model in which they assumed lead time is not fixed and goods can be defected with permissible delay in payments. They considered order quantity and lead time as decision variables although lead time is stochastic. Ben-Daya and Abdul [1] developed inventory model with stochastic demand and considered lead time as one of the decision variables in model. Wu [39] presented an inventory model with variable lead time and uncertain quantity of goods which was received. Taleizadeh *et al.* [31] developed an inventory problem in which they assumed the demand is stochastic and follows a uniform distribution with partial backordering shortages and considering the lead time is sensitive to the lot size. Karimi Nasab and Konstantaras [14] developed an inventory model when the time between two replenishments is stochastic with the supplier offers special sale. Taleizadeh *et al.* [34] studied optimal replenishment policy when replenishment intervals are probabilistic, and the seller increases the price of materials in a close future. Later, Taleizadeh *et al.* [35] developed a model when the buyer can pay for purchased materials with delay and the replenishment intervals are probabilistic. Hayya *et al.* [12] presented a solution to obtain the value of reorder point and order quantity when the demand and the lead time are *i.i.d.* Rahim [24] considered the demand in lead time as a probabilistic variable but lead time can be either deterministic or probabilistic, and he obtained the optimal order quantity. Maiti *et al.* [19] developed inventory model with considering probabilistic lead time and shortages and price discount when the demand depended to the price in a finite time horizon with assuming prepayment in model. In stochastic environment, for example, Das *et al.* [5] developed an inventory model with assuming delay in payment can occur and the demand vary with advertisement and selling price and Gupta *et al.* [10] presented an inventory model for deterioration products when delay in payments can occur and the demand is dependent to the stock. Also Sridevi [27] developed an inventory model in which the production rate is probabilistic and follows a Weibull distribution and the demand is sensitive to selling price. Jana *et al.* [13] developed a model for deteriorating products in which partial backordering is considered. Also they assumed that the demand rate may be random or fuzzy random variables. Ben-Daya and Hariga [2] developed a model for production system with stochastic demand and changing lead time with the amount of lot size. Panda *et al.* [23] developed a model for a single period production system with multi-product while producing imperfect products and demand are stochastic with limited budget and shortages. Eynan and Kropp [7] developed a periodic review inventory when the demand is stochastic and permissible shortage with different shortage cost. Taleizadeh *et al.* [7] considered an inventory model when there is uncertainty about the occurrence of special sales and non-zero initial inventory level.

Pal [22] optimized a production system with quality sensitive market demand, partial backlogging and permissible delay in payment. Later, Khan *et al.* [15] proposed a model for a two-warehouse inventory system for deteriorating items with partial backlogging, and advance scheme. Recently, Khara *et al.* [16] developed an imperfect production model with advance payment and credit period in a two-echelon supply chain management and Mashud *et al.* [20] studied joint pricing deteriorating inventory model that considered product life cycle and advance payment with a discount facility.

Based on comparison performed in Table 1 there is no work in which both stochastic length and advanced payments are considered. So in this paper an optimal control model with stochastic length and advance payment is developed to elucidate a realistic situation where the lead time is manifested by the visitor's arrival time.

The Problem

This paper considers a situation in which a manufacturer sells a product to a buyer, but receiving orders on phone and delivering the products are both performed by a visitor from the manufacturer-side. The visitor when goes to receive the orders asks the buyer to prepay α percent of the purchasing cost at a specific time before delivering the items to avoid cancelation from the buyer. Since the delivery time will be stochastic, so the buyer makes an expected order quantity and pays α percent of the purchasing cost as prepayment. Then the cost of remaining amount of the purchasing cost should be paid at the time of delivery of products. On the other hand, the manufacturer's lead time is a stochastic variable, and two possible cases may occur: (a) the time-period between replenishments is less than the amount of time required for the inventory level to reach zero, so at the time of delivery of last order the inventory level is positive, and (b) the time-period between replenishments is greater the amount of time required for the inventory level to reach zero, so at the time of delivery of last order the inventory level is negative. Assume that the prepayment is made in multiple installments at equal intervals and the maximum number of prepayments, n is a parameter suggested to the buyer by the manufacturer. In this situation, because of the buyer has already paid α percent of the capital cost and didn't get the goods, the capital cost of buyer increases. Further, it is also assumed that the shortage is permitted and a fraction of them will be backordered which gives more latitude to the decision maker. Therefore, the main aim of this paper is to determine the order quantity of the buyer such that its profit is maximized.

Table 1. A skeletal examination of the related literature.

References	Shortage			Advanced payment	Stochastic demand	Stochastic periodic length	Deterioration	Decision variables				Objective function	
	Backordering	Lost sale	Partial backordering					Order quantity	Shortage quantity	Period length	Inventory level	Convex	Concave
Ben-Daya and Hariga [2]					Y			Y				Y	
Ertogral and Rahim [6]			Y			Y					Y		Y
Rahim [35]			Y		Y	Y		Y	Y		Y		Y
Ouyang *et al.* [21]							Y	Y		Y		Y	
Eynan and Kropp [7]	Y				Y					Y		Y	
Panda *et al.* [23]	Y				Y					Y			Y
Taleizadeh *et al.* [30]				Y									
Hayya *et al.* [12]					Y	Y		Y				Y	
Taleizadeh *et al.* [39]			Y		Y	Y					Y	Y	
Liang and Zhou [17]							Y			Y		Y	
Thangam [38]				Y			Y			Y		Y	
Maihami and Nakhai [18]			Y				Y	Y		Y			Y
Sarker *et al.* [26]			Y	Y			Y	Y		Y	Y		
Taleizadeh *et al.* [31]			Y		Y			Y			Y	Y	
Karimi-Nasab and Konstantaras [14]			Y			Y		Y	Y		Y	Y	
Taleizadeh *et al.* [32]			Y			Y			Y		Y		Y
Chiang [4]	Y	Y			Y	Y					Y	Y	
Guria *et al.* [11]	Y			Y						Y		Y	
Jana *et al.* [13]			Y			Y	Y	Y	Y	Y			Y
Taleizadeh *et al.* [33]	Y		Y	Y			Y	Y	Y	Y		Y	
Taleizadeh [28]			Y	Y			Y	Y	Y	Y		Y	
Taleizadeh [29]	Y			Y			Y	Y	Y	Y		Y	
Sarkar *et al.* [25]			Y		Y	Y		Y		Y		Y	
Zhang *et al.* [42]				Y				Y		Y		Y	
Pal [23]	Y	Y		Y				Y				Y	
Khan *et al.* [15]	Y		Y	Y			Y	Y				Y	
Khara *et al.* [16]				Y		Y	Y				Y		
Mashud *et al.* [20]				Y			Y			Y			
Current paper			Y	Y		Y		Y	Y		Y		Y

Notes. Y: Yes.

2. The problem formulation

According to the problem description, all components of the profit function need to be modeled are revenue, purchasing cost, holding cost, shortage cost both lost sale and backordering, and finally the capital cost occurs because of prepayments. The following notations are used to model the problem.

(a) Parameters

D Demand rate (units/period)
γ The fraction of shortages that will backordered (Percent)
α The fraction of purchasing cost that paid as multiple prepayment (Percent)

P	Unit selling price (\$/unit)
C	Unit purchasing price (\$/unit)
n	The number of equally spaced prepayments to be made before receiving the order
t	The length of time during which the buyer will make the prepayments (time)
π	Backordered cost per unit per time period (\$/unit-time)
π'	Lost sale cost per unit (\$/unit)
h	Inventory holding cost per unit per time period (\$/unit-time)
$L_{\max}$	Upper limit of delivery time in uniform distribution (time)
$L_{\min}$	Lower limit of delivery time in uniform distribution (time)
I_c	Capital cost rate per unit time (\$/time)
$f(l)$	Probability distribution function (PDF) of L
$F(l)$	Cumulative distribution function of L, $\overline{F}(l) = 1 - F(l)$
λ	Arrival rate in exponential PDF

(b) Independent Decision variables

R	The replenish-up-to level (unit)

(c) Other variables

$\overline{B}$	Expected backordered quantity per cycle (unit)
$\overline{L}$	Expected lost sale quantity per cycle (unit)
$\overline{I}$	Expected inventory per cycle (unit)
Q	Expected number of units replenished per cycle (unit)
l_R	Amount of time that the inventory equals zero, $l_R = R/D$ (time)
ECP	Expected cyclic profit (\$)
ER	Expected revenue per cycle (\$)
EHC	Expected cyclic holding cost (\$)
EPC	Expected cyclic purchasing cost (\$)
EBC	Expected cyclic backordering cost (\$)
ELC	Expected cyclic lost sale cost (\$)
CCC	Cyclic capital cost (\$)
(*)	Indicate the optimal value

2.1. Profit function

To determine the decision variable of the buyer and calculate the expected profit in each cycle, ECP, all the terms in the profit function need to be modeled. The expected revenue, the expected purchasing cost per cycle, the expected holding cost per cycle, the expected shortage cost both lost sale and backordering and finally expected capital cost from the prepayment are the components of profit function should be derived separately. The profit function is given as follows.

$$\begin{aligned} \text{ECP} &= \text{ER} - \text{EPC} - \text{EHC} - \text{EBC} - \text{ELC} - \text{CCC} \\ &= PQ - CQ - h\overline{I} - \pi\overline{B} - \pi'\overline{L} - \text{CCC}. \end{aligned} \tag{2.1}$$

Since the manufacturer's lead time is a stochastic variable, two possible cases may occur. In the first case, the time between replenishments is less than the time required for the inventory level to reach zero, so at the time of delivery of last order the inventory level is positive as shown in Figure 1. In the second case, the time-period between replenishments is greater than the amount of time required for the inventory level to reach zero and the inventory diagram of this case is shown in Figure 2. In both figures n equal installments, represent the

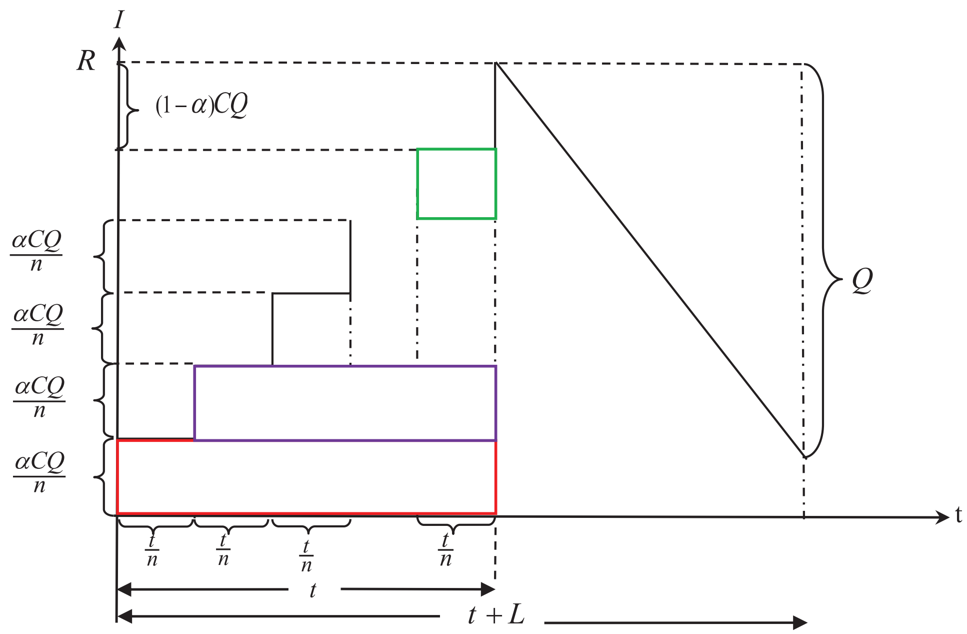

FIGURE 1. Invested capital for inventory under prepayments and stochastic periodic length with no shortages.

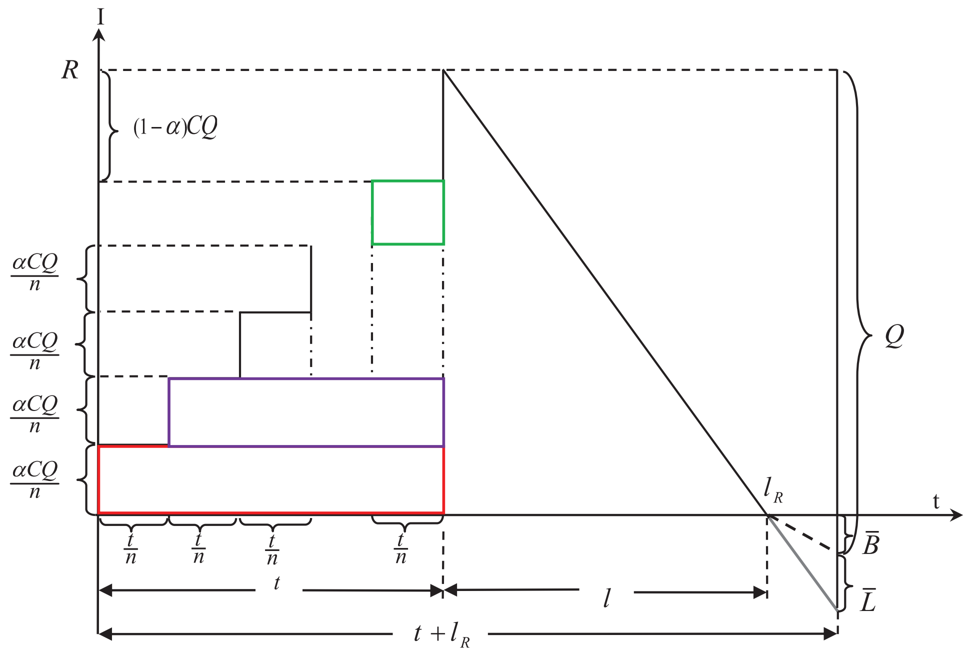

FIGURE 2. Invested capital for inventory under prepayments and stochastic periodic length, when there are partial backordering shortages.

prepayments which should be paid during t, before receiving the goods. In this problem, the cyclic capital cost, CCC, will be:

$$\begin{aligned}\text{CCC} &= \left(I_c\frac{\alpha CQ}{n}\times n\times\frac{t}{n}\right)+\left(I_c\frac{\alpha CQ}{n}\times(n-1)\times\frac{t}{n}\right)+\ldots+\left(I_c\frac{\alpha CQ}{n}\times[n-(n-2)]\times\frac{t}{n}\right)\\ &\quad+\left(I_c\frac{\alpha CQ}{n}\times[n-(n-1)]\times\frac{t}{n}\right)\\ &=\left(I_c\frac{\alpha CQ}{n}\times\frac{t}{n}\right)[n+(n-1)+\ldots+2+1]=\alpha I_c Ct\frac{(n+1)}{2n}Q. \end{aligned}\tag{2.2}$$

This cost term is the same as what Taleizadeh *et al.* [33] derived for an EOQ based model.

Also in the profit function, the expected number of units replenished per cycle, Q, is:

$$Q=\int_{L_{\min}}^{l_R} DL\, f_L(l)\,\mathrm{d}l+\int_{l_R}^{L_{\max}}(R+\gamma(DL-R))f_L(l)\,\mathrm{d}l.\tag{2.3}$$

Because if the visitor arrives within $[L_{\min}, l_R]$ the order quantity is DL and if arrives within $[l_R, L_{\max}]$ the order quantity will be $R+\gamma(DL-R)$. Since the lead-time is random variable, the expected value of the order quantity is what presented in equation (2.3). Also, the average on hand inventory for the first and second cases are $RL-\frac{DL^2}{2}$ and $\frac{R^2}{2D}$, respectively. So the expected inventory per cycle is:

$$\overline{I}=\int_{L_{\min}}^{l_R}\left(RL-\frac{DL^2}{2}\right)f_L(l)\,\mathrm{d}l+\int_{l_R}^{L_{\max}}\frac{R^2}{2D}f_L(l)\,\mathrm{d}l.\tag{2.4}$$

Surely the shortage occurs only when the visitor arrives within $[l_R, L_{\max}]$ and in this situation the shortage quantity is $(DL-R)$ and γ percent of the shortage is backordered. Moreover, the expected backordered, $\overline{B}$ and the lost sale quantities per cycle, $\overline{L}$ are derived as below, respectively (also, see Fig. 2).

$$\overline{B}=\gamma\int_{l_R}^{L_{\max}}(DL-R)f_L(l)\,\mathrm{d}l\tag{2.5}$$

and

$$\overline{L}=(1-\gamma)\int_{l_R}^{L_{\max}}(DL-R)f_L(l)\,\mathrm{d}l.\tag{2.6}$$

Finally the expected profit function, ECP, is given by

$$\begin{aligned}\text{ECP} &= \left(P-C-\frac{I_cCt\alpha(n+1)}{2n}\right)\left[\int_{L_{\min}}^{l_R} DL\, f_L(l)\,\mathrm{d}l+\int_{l_R}^{L_{\max}}(R+\gamma(DL-R))f_L(l)\,\mathrm{d}l\right]\\ &\quad-h\left[\int_{L_{\min}}^{l_R}\left(RL-\frac{DL^2}{2}\right)f_L(l)\,\mathrm{d}l+\int_{l_R}^{L_{\max}}\frac{R^2}{2D}f_L(l)\,\mathrm{d}l\right]\\ &\quad-\pi\left[\gamma\int_{l_R}^{L_{\max}}(DL-R)f_L(l)\,\mathrm{d}l\right]-\pi'\left[(1-\gamma)\int_{l_R}^{L_{\max}}(DL-R)f_L(l)\,\mathrm{d}l\right].\end{aligned}\tag{2.7}$$

Since the profit function shown in equation (2.7) includes only one variable, which is the maximum inventory level, R, and optimal value needs to be determined such that the profit function is maximized, it is needed to show that the profit function is concave with respect to R. To prove the concavity of the profit function with respect to R, the first and the second derivatives of profit function need to be developed as

$$\frac{\mathrm{d}\,\mathrm{ECP}}{\mathrm{d}\,R} = \left(P - C - \frac{I_c C t\alpha(n+1)}{2n}\right)(1-\gamma)\overline{F}(l_{R^*}) + \left[\gamma\pi + (1-\gamma)\pi' - h\frac{R^*}{D}\right]\overline{F}(l_{R^*}) - hE(L|L \leq l_{R^*}) \quad (2.8)$$

$$\frac{\mathrm{d}^2\mathrm{ECP}}{\mathrm{d}\,R^2} = -\frac{1}{D}\left\{\left[2\left(P - C - \frac{I_c C t\alpha(n+1)}{2n}\right)(1-\gamma) + \gamma P\right](1-\gamma)f(l_R) + \frac{h}{D}\overline{F}(l_{R^*})\right\}. \quad (2.9)$$

In equation (2.9), when $l_R < L_{\min}$, $f_l(l_R)$ is zero and $\overline{F}_l(l_R)$ is positive, so $\mathrm{d}^2\mathrm{ECP}/\mathrm{d}\,R^2$ is negative. Moreover when $L_{\min} \leq l_R \leq L_{\max}$, both $f_l(l_R)$ and $\overline{F}_l(l_R)$ are positive and the second derivative is positive too; but when $L_{\max} < l_R$, both $f_l(l_R)$ and $\overline{F}_l(l_R)$ are zero and $\mathrm{d}^2\mathrm{ECP}/\mathrm{d}\,R^2$ approaches to zero too. Therefore, the profit function over $(-\infty, L_{\max}]$ is convex which means $l_R = R/D$ should be less than $L_{\max}$ and an upper bound is obtained for the maximum level of inventory level as $R \leq DL_{\max}$. Then, because of the concavity of profit function, setting the first derivative of the profit function with respect to R equal to zero, the optimal replenish-up-to level will be

$$\overline{F}(l_{R^*}) = \frac{hE(L|L \leq l_{R^*})}{\left[\left(P - C - \frac{I_c C t\alpha(n+1)}{2n}\right)(1-\gamma) + \gamma\pi + (1-\gamma)\pi'\right] - h\frac{R^*}{D}}. \quad (2.10)$$

In the next subsection the profit function and optimal solution for two special cases, when lead time follows uniform and exponential probability distribution, are derived.

2.2. Special cases

Two specific probability distribution functions are used for the random variable. First, a continuous uniform distribution function is used followed by an exponential probability distribution function in the subsequent part.

2.2.1. Uniformly distributed lead time

If the lead-time follows a uniform probability distribution, then the expected profit function can be rewritten as below.

$$\begin{aligned}\mathrm{ECP}(R) = {} & \frac{\left(P - C - \frac{I_c C t\alpha(n+1)}{2n}\right)}{2(L_{\max} - L_{\min})}\left[(\gamma - 1)\frac{R^2}{D} - DL_{\min}^2 + (2(1-\gamma) - 2\gamma)L_{\max}R + \gamma DL_{\max}^2\right] \\ & - \frac{(\pi'(1-\gamma) + \pi\gamma)}{(L_{\max} - L_{\min})} \times \left[\frac{DL_{\max}^2}{2} + \frac{R^2}{2D} - RL_{\max}\right] \\ & - \frac{h}{(L_{\max} - L_{\min})}\left[-\frac{R^3}{6D^2} - \frac{RL_{\min}^2}{2} + \frac{DL_{\min}^3}{6} + \frac{L_{\max}R^2}{2D}\right]. \end{aligned} \quad (2.11)$$

In this case, when lead-time follows uniform probability distribution,

$$\overline{F}(l_{R^*}) = \frac{DL_{\max} - R^*}{D(L_{\max} - L_{\min})} \quad (2.12)$$

$$E(L|L \leq l_{R^*}) = \frac{1}{2}\frac{R^{*2} - L_{\min}^2 D^2}{D^2(L_{\max} - L_{\min})}. \quad (2.13)$$

Then replacing $E(L|L \leq l_{R^*})$ and $\overline{F}(l_{R^*})$ shown in equations (2.12) and (2.13), in equation (2.10) two possible solutions for R is derived as shown in equation (2.14).

$$R^*_{1,2} = \frac{D}{h}\left[\begin{array}{l} hL_{\max} + \left(P - C - \frac{I_c C t\alpha(n+1)}{2n}\right)(1-\gamma) + \gamma\pi + (1-\gamma)\pi' \\ \pm\sqrt{h^2(L^2_{\max} - L^2_{\min}) + \left(\left(P - C - \frac{I_c C t\alpha(n+1)}{2n}\right)(1-\gamma) + \gamma\pi + (1-\gamma)\pi'\right)^2}\end{array}\right]. \tag{2.14}$$

Since $\frac{D}{h}\left[\begin{array}{l} hL_{\max} + (P - C - (I_c C t\alpha(n+1)/2n))(1-\gamma) + \gamma\pi + (1-\gamma)\pi' \\ +\sqrt{h^2(L^2_{\max} - L^2_{\min}) + ((P - C - (I_c C t\alpha(n+1)/2n))(1-\gamma) + \gamma\pi + (1-\gamma)\pi')^2}\end{array}\right]$ does not satisfy constraint $R \leq DL_{\max}$, so the optimal solution is;

$$R^* = \frac{D}{h}\left[\begin{array}{l} hL_{\max} + (P - C - (I_c C t\alpha(n+1)/2n))(1-\gamma) + \gamma\pi + (1-\gamma)\pi' \\ -\sqrt{h^2(L^2_{\max} - L^2_{\min}) + ((P - C - I_c C t\alpha(n+1)/2n))(1-\gamma) + \gamma\pi + (1-\gamma)\pi')^2}\end{array}\right] \tag{2.15}$$

and

$$Q = \frac{1}{(L_{\max} - L_{\min})}\left[\frac{D\left(\alpha L^2_{\max} - L^2_{\min}\right)}{2} + (1-\alpha)RL_{\max} - \frac{(1-\alpha)R^2}{2D}\right]. \tag{2.16}$$

2.2.2. Exponentially distributed lead time

If the lead time follows an exponential distribution, then the expected profit function can be rewritten as

$$\begin{aligned} \mathrm{ECP}(R) = {} & \frac{(2n(P-C) - I_c C\,t\alpha(n+1))}{2n\lambda^2}\left[\left(1 - e^{-\lambda l_R}(\lambda l_R + 1)\right)\lambda D + (\gamma D + \lambda R)e^{-\lambda l_R}\right] \\ & - \frac{h}{\lambda}\left[R - Re^{-\lambda l_R}(\lambda l_R + 1) - \frac{D}{2\lambda}\left(2 - e^{-\lambda l_R}\left(\lambda^2 l_R^2 + 2\lambda l_R + 2\right)\right)\right] \\ & + \frac{hR^2}{2D}e^{-\lambda l_R} - \frac{1}{\lambda^2}(\gamma\pi + (1-\gamma)\pi')\left(2De^{-\lambda l_R}\right). \end{aligned} \tag{2.17}$$

In this case, when lead-time follows an exponential probability distribution, we have

$$\overline{F}(l_{R^*}) = e^{-l^*_R \lambda} \tag{2.18}$$

$$E(L|L \leq l_{R^*}) = \frac{-e^{-l_{R^*}\lambda}(R^*\lambda + D) + D}{D\lambda}. \tag{2.19}$$

Then replacing $E(L|L \leq l_{R^*})$ and $\overline{F}(l_{R^*})$ in equation (2.10) with the expressions in equations (2.18) and (2.19), the optimal solution for R is derived as shown below.

$$R^* = \frac{\left[\ln\left(\left[\left(P - C - \frac{I_c C t\alpha(n+1)}{2n}\right)(1-\gamma) + \gamma\pi + (1-\gamma)\pi'\right]\lambda + h\right) - \ln(h)\right]D}{\lambda} \tag{2.20}$$

and

$$Q^* = \frac{D}{\lambda}\left[1 - (1-\alpha)e^{-\lambda l_R}\right]. \tag{2.21}$$

3. Numerical examples and sensitivity analysis

Consider a situation in which the visitor calls the customer with a phone to receive the order from him. The lead time is stochastic and follows a uniform or exponential distribution. Also, visitor wants customer to pay α percent of purchasing cost in advance during several payments because he wants to be sure that customer needs the goods, and he will not cancel the order. When the customer ordered, with stochastic lead time, the visitor comes with the goods to deliver them to customer or to check the quantity of goods that he must deliver the customer. In this situation customer wants to know how much he should purchase to maximize his profit. This

TABLE 2. Parametric values and the optimal solutions for uniformly distributed lead time.

Specific parameters						Results		
α	γ	n	D	P	$[L_{\min}, L_{\max}]$	Q^*	R^*	ECP(R^*)
0.3	0.4	3	100	70	$[0.05, 0.1]$	7.4988	9.8573	61.94
0.3	0.8	3	120	70	$[0.05, 0.1]$	8.9994	11.8075	74.34
0.3	0.4	5	140	80	$[0.05, 0.1]$	10.4992	13.8635	193.21
0.5	0.8	5	160	80	$[0.05, 0.1]$	11.9988	15.6866	210.47
0.5	0.4	7	180	90	$[0.05, 0.1]$	13.4994	17.8637	373.13
0.5	0.8	7	200	90	$[0.05, 0.1]$	14.9990	19.6765	414.62
0.7	0.4	9	220	100	$[0.2, 0.3]$	54.9888	65.0933	1998.90
0.7	0.8	9	240	100	$[0.2, 0.3]$	59.9793	69.7717	2181.40
0.7	0.4	11	260	120	$[0.2, 0.3]$	64.9932	77.2327	3665.50
0.9	0.8	11	280	120	$[0.2, 0.3]$	69.9854	81.9782	3893.20
0.9	0.4	13	300	140	$[0.2, 0.3]$	74.9952	89.3044	5673.70
0.9	0.8	13	320	140	$[0.2, 0.3]$	79.9890	94.1235	6052.70

TABLE 3. Parametric values and the optimal solutions for exponential lead time.

Specific parameters						Results		
α	γ	n	D	P	λ	Q^*	R^*	ECP(R^*)
0.3	0.4	3	100	70	5	19.1511	52.9745	105.40
0.3	0.8	3	120	70	5	23.4715	51.9528	120.38
0.3	0.4	5	140	80	5	27.1692	84.1901	388.10
0.5	0.8	5	160	80	5	31.3944	75.4506	386.70
0.5	0.4	7	180	90	5	35.1617	116.9656	796.46
0.5	0.8	7	200	90	10	19.8346	63.7082	452.26
0.7	0.4	9	220	100	10	21.7859	90.6688	724.35
0.7	0.8	9	240	100	10	23.8278	79.8698	742.79
0.7	0.4	11	260	120	10	25.8180	115.7225	1359.80
0.9	0.8	11	280	120	10	27.8426	100.0148	1365.00

problem is common in drugstores when they want to make an order for medicines or for stores that want to purchase dairy products. With these assumptions, several problems are created to show how much the customer should purchase to maximize his profit. To show the applicability of the proposed model a real problem as reflected from experiences is defined and several examples for both cases (PDFs) are solved and the results are shown in Tables 2 and 3 for the first and second cases, respectively. For the example of both cases, a drugstore wants to make an order and purchase a valuable medicine, the general data for the drugstore in this problem are $C = 60\,\$/\text{unit}$, $h = 5\,\$/\text{unit/year}$, $\pi = 5\,\$/\text{unit/year}$, $\pi' = 10\,\$/\text{unit}$, $I_c = 20\,\%$, $t = 0.6\,\text{year}$. Specific data are shown in Tables 2 and 3 for both cases of uniform and exponential distributions, respectively.

3.1. Example 1: Uniformly distributed lead time

An example is framed by considering uniform distribution for the visitor's arrival time. The specific data for this case and the related results are presented in Table 2. The effect of specific parameters are reflected on the results of Q^*, R^* and ECP(R^*) corresponding to the system parameters α, γ, n, D, P and the distribution range.

TABLE 4. The sensitivity analysis in Example 1 (Uniform PDF).

Parameter	Percentage of change in parameters	Value			Percentage of change in value		
		Q^*	R^*	ECP(R^*)	Q^*	R^*	ECP(R^*)
D	+75	26.2470	34.58	1002.90	+75	+75	+75
	+50	22.4974	29.64	859.65	+50	+50	+50
	+25	18.7478	24.70	716.38	+25	+25	+25
	0	14.9983	19.76	573.10	0	0	0
	−25	11.2487	14.82	429.83	−25	−25	−25
	−50	7.4991	9.88	286.55	−50	−50	−50
	−75	3.7496	4.94	143.28	−75	−75	−75
P	+75	14.9997	19.90	1698.10	+0.01	0.709	+196.30
	+50	14.9996	19.88	1323.10	+0.01	0.607	+130.87
	+25	14.9992	19.84	948.010	+0.01	0.405	+65.41
	0	14.9983	19.76	573.10	0	0	0
	−25	14.9937	19.54	198.19	−0.03	−1.11	−65.42
	−50	14.1636	14.72	−174.94	−5.57	−25.50	−130.52
	−75	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
α	+75	14.9991	19.76	559.72	+0.01	0	−2.33
	+50	14.9988	19.76	564.22	0	0	−1.55
	+25	14.9986	19.76	571.57	0	0	−0.27
	0	14.9983	19.76	573.10	0	0	0
	−25	14.9980	19.76	577.57	0	0	+0.78
	−50	14.9977	19.76	582.07	0	0	+1.57
	−75	14.9974	19.76	586.42	−0.01	0	+2.32
N	+75	14.9983	19.76	573.82	0	0	+0.13
	+50	14.9983	19.76	573.67	0	0	+0.1
	+25	14.9983	19.76	573.37	0	0	+0.05
	0	14.9983	19.76	573.10	0	0	0
	−25	14.9983	19.76	572.62	0	0	−0.08
	−50	14.9983	19.76	571.27	0	0	−0.32
	−75	14.9983	19.76	568.42	0	0	−0.82
π	+75	14.9984	19.77	573.10	0	0	0
	+50	14.9984	19.77	573.10	0	0	0
	+25	14.9983	19.76	573.10	0	0	0
	0	14.9983	19.76	573.10	0	0	0
	−25	14.9983	19.76	573.10	0	0	0
	−50	14.9981	19.75	573.10	0	0	0
	−75	14.9981	19.75	573.10	0	0	0

3.2. Example 2: Exponentially distributed lead time

We propose another example by considering exponential distribution for the visitor's arrival time which is basically the lead time. The specific data for this case and the related results are presented in Table 3.

To show the effect of changing the value of some parameters D, α, n, P and π on the amount of replenish-up-to level and expected cyclic profit, some problems are solved and the results of Example 1 (Uniform PDF) are shown in Table 4 and those of Example 2 (Exponential PDF) are shown in Table 5.

The general data for this problem are $C = 60\,\$/\text{unit}$, $h = 5\,\$/\text{unit/year}$, $\pi' = 10\,\$/\text{unit}$, $I_c = 20\%$, and $t = 0.6\,\text{year}$. The default amount of data to analyze their effect in this part are $P = 100\,\$/\text{unit}$, $D = 100\,\text{unit/year}$, $\alpha = 0.3$, $n = 10$, $\pi = 5\,\$/\text{unit/year}$, $\gamma = 0.4$ and in Example 1, the lead time is between $[0.1, 0.2]$ year and in Example 2, $\lambda = 10\,\text{arrivals/year}$.

TABLE 5. The sensitivity analysis in Example 2 (Exponential PDF).

Parameter	Percentage of change in parameters	Value			Percentage of change in value		
		Q^*	R^*	ECP(R^*)	Q^*	R^*	ECP(R^*)
D	+75	17.3348	72.66	603.24	+75	+75	+75
	+50	14.8584	62.28	517.06	+50	+50	+50
	+25	12.3820	51.90	430.88	+25	+25	+25
	0	9.9056	41.52	344.71	0	0	0
	−25	7.4292	31.14	258.53	−25	−25	−25
	−50	4.9526	20.76	172.35	−50	−50	−50
	−75	2.4764	10.38	86.18	−75	−75	−75
P	+75	9.9609	50.34	1077.90	+0.56	+21.24	+212.7
	+50	9.9515	48.17	831.75	+0.46	+16.01	+141.29
	+25	9.9359	45.39	586.91	+0.31	+9.32	+70.26
	0	9.9056	41.52	344.71	0	0	0
	−25	9.8213	35.14	109.81	−0.85	−15.37	−68.14
	−50	8.3217	12.74	−50.93	−15.99	−69.32	−114.78
	−75	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
α	+75	9.9520	41.35	336.15	+0.47	−0.41	−2.48
	+50	9.9364	41.41	339.03	+0.31	−0.27	−1.65
	+25	9.9212	41.50	343.73	+0.16	−0.05	−0.28
	0	9.9056	41.52	344.71	0	0	0
	−25	9.8905	41.58	347.57	−0.15	+0.15	+0.83
	−50	9.8755	41.63	350.44	−0.3	+0.27	+1.66
	−75	9.8608	41.69	353.24	−0.45	+0.41	+2.48
n	+75	9.9057	41.53	345.17	0	+0.02	+0.13
	+50	9.9057	41.53	345.08	0	+0.02	+0.11
	+25	9.9057	41.53	344.87	0	+0.02	+0.05
	0	9.9056	41.52	344.71	0	0	0
	−25	9.9056	41.52	344.41	0	0	−0.09
	−50	9.9054	41.50	343.63	0	−0.05	−0.31
	−75	9.9050	41.46	341.71	0	−0.15	−0.87
π	+75	9.9098	41.98	345.46	+0.04	+1.11	+0.22
	+50	9.9085	41.83	345.22	+0.03	+0.75	+0.15
	+25	9.9071	41.68	344.97	+0.02	+0.39	+0.08
	0	9.9056	41.52	344.71	0	0	0
	−25	9.9041	41.36	344.43	−0.02	−0.39	−0.08
	−50	9.9025	41.20	344.16	−0.03	−0.77	−0.16
	−75	9.9010	41.04	343.88	−0.05	−1.16	−0.24

According to Tables 4 and 5, when the demand for goods or services increases, the replenish-up-to level and the expected cyclic profit increase too, and the percentage of changes is the same in D, R^* and ECP(R^*). When the selling price increases, it can increase both R^* and ECP(R^*) when it decreases to less than purchasing price, profit is not gained because the selling price is less than the purchasing price and the maximum inventory level and order quantity both becomes negative which can be interpreted as an infeasible solution.

The effect of the fraction of purchase cost that paid as multiple prepayments is not as much as D and P and as α increases, R^* and ECP(R^*) decrease but it is very little on the R^* in Example 1. As the number of prepayments to be made before receiving the order increases, R^* and ECP(R^*) increase, but it has a little effect on them and in Example 1 its effect on R^* is near zero. As Backordered cost per unit per time period increases, in Example 1 it almost has no effect on R^* and ECP(R^*), but in Example 2 it increases the value

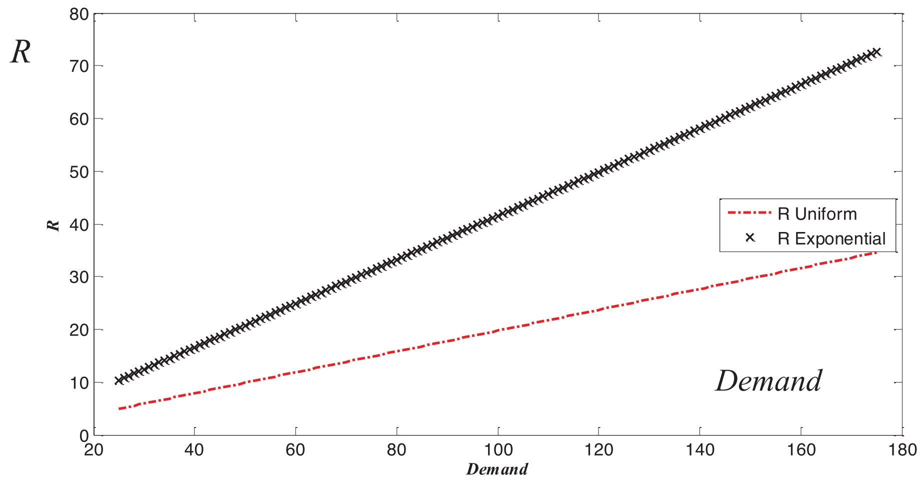

FIGURE 3. Effect of Demand on Replenishment-up-to level.

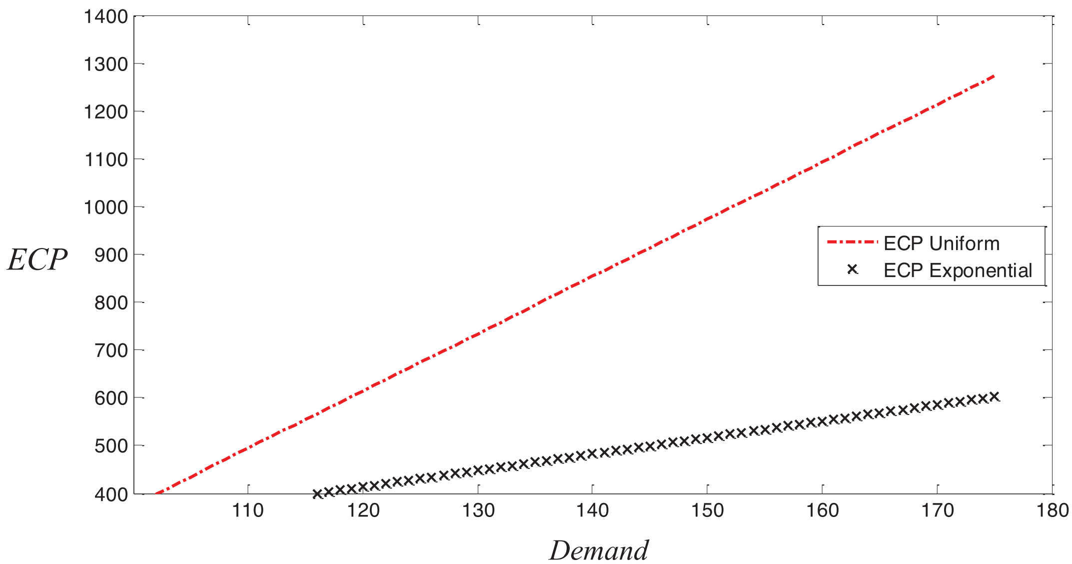

FIGURE 4. Effect of Demand on ECP.

of R^* and $\mathrm{ECP}(R^*)$. Also the changes of variables with respect to the parameter's changes are shown in Figures 3–10.

We can consider from Figure 3 that when the demand for the goods increases, the replenishment-up-to level increase too in both examples, and the incremental rise in replenishment-up-to level also increases proportionately with the demand value.

In Figure 4, when the demand increases and it causes the replenish-up-to level to increase, the ECP increase too in both examples and their diversion increases with demand as well.

When the selling price increases, the replenish-up-to level increases too as can be seen it in Figure 5. As it is expected, the selling price should be more than purchasing price, but when it is lower than the purchasing price, the profit function is not concave anymore and it has an unexpected effect on replenishment-up-to level

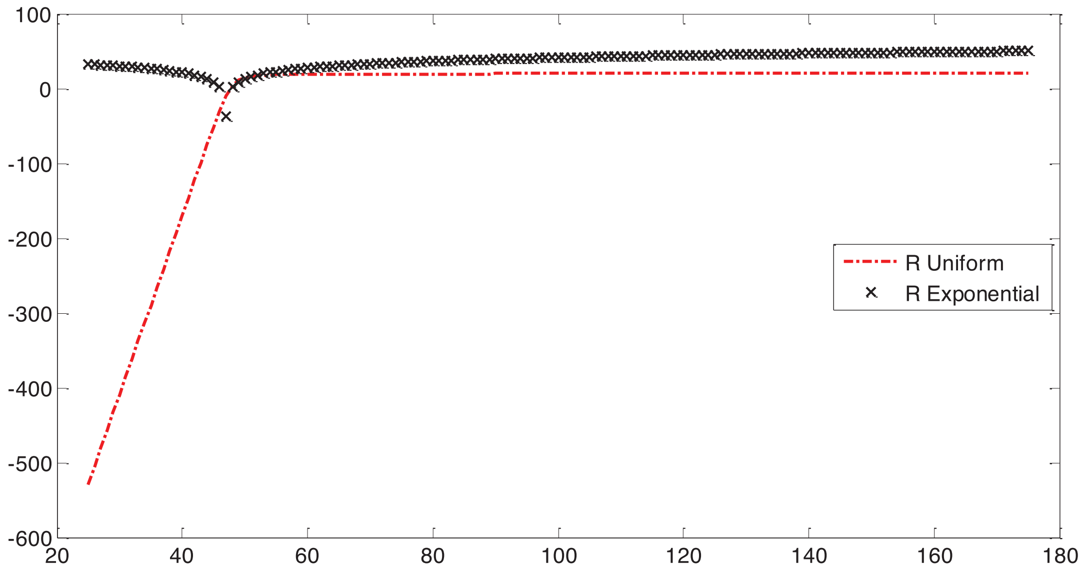

FIGURE 5. Effect of Selling Price on Replenishment-up-to level.

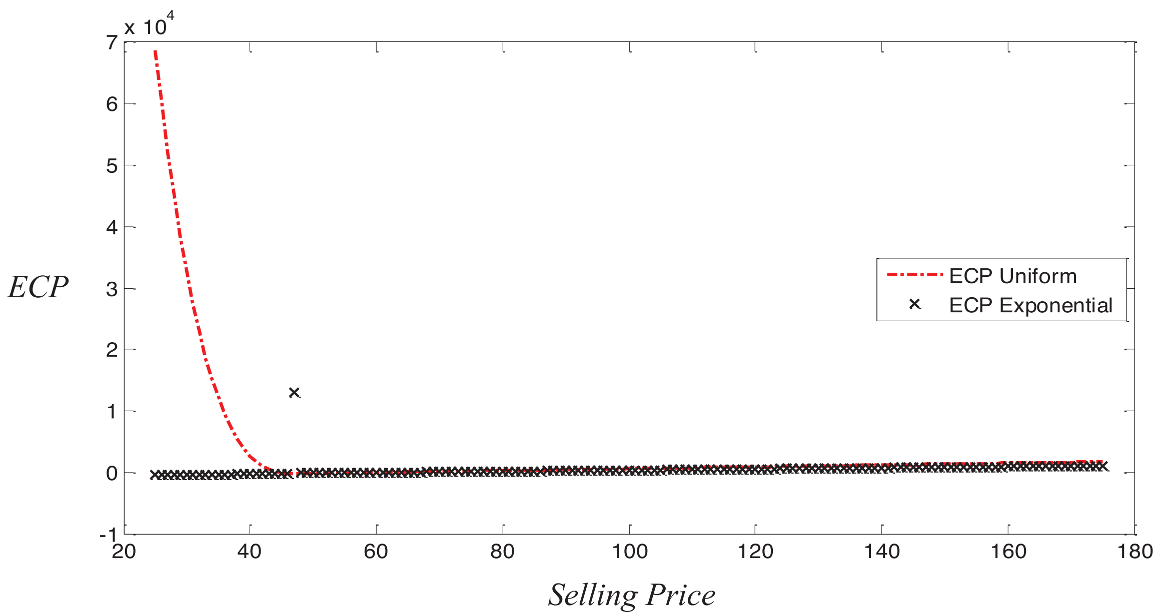

FIGURE 6. Effect of Selling Price on ECP.

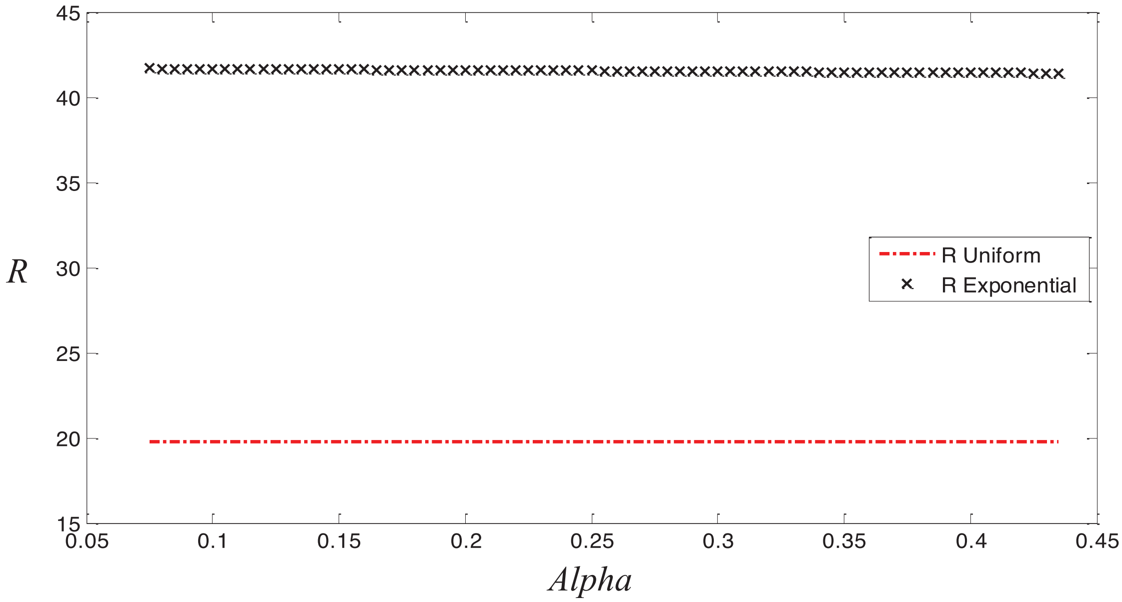

FIGURE 7. Effect of α on Replenishment-up-to level.

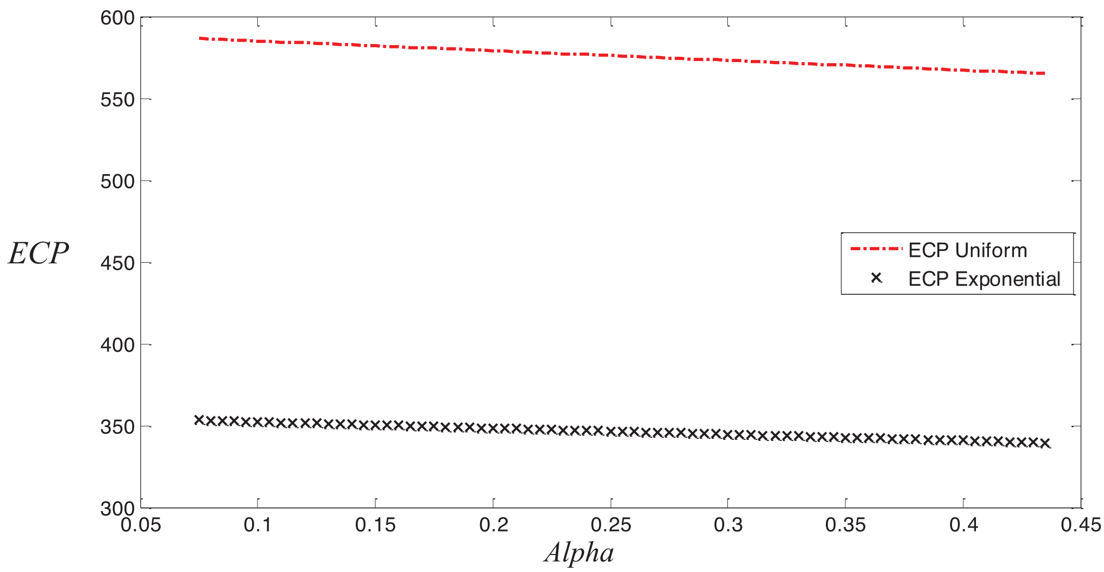

FIGURE 8. Effect of α on ECP.

for some quantities in both examples yielding negative values for R and Q (see drops in Fig. 5). When the selling price increases, it increases the expected cyclic profit too, and as it is explained for Figure 5, when the replenishment-up-to level behaves strange, ECP behaves strange too because of their relationship.

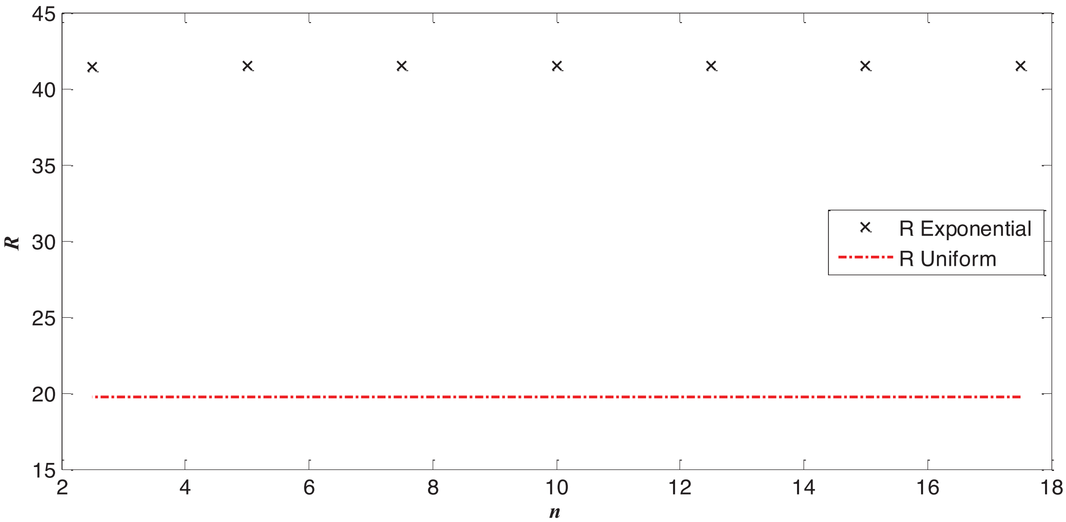

FIGURE 9. Effect of n on replenishment-up-to level.

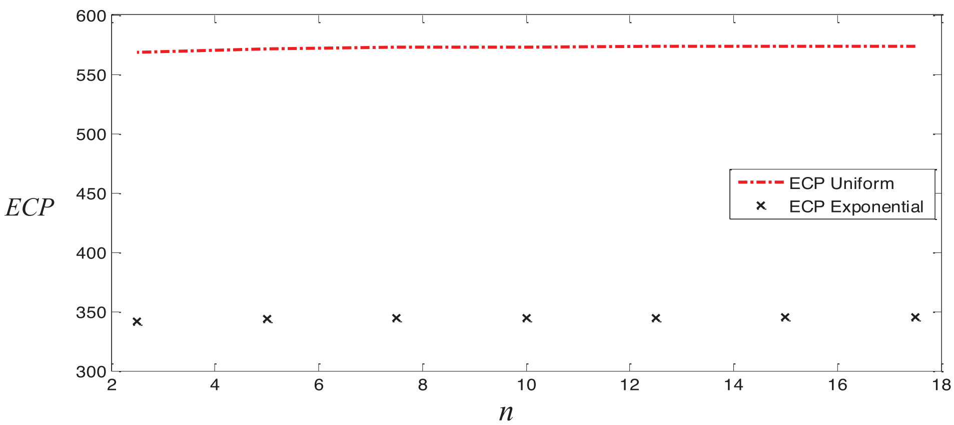

FIGURE 10. Effect of n on ECP.

When the percentage of purchasing cost that must be prepaid in multiple prepayments increases, it has a little negative effect of replenishment-up-to level in both examples. As it is clear in Figure 8, when the percentage of purchasing cost that must be paid in advance increases, the expected cyclic profit decreases in both examples.

In Figure 9, when n increases the replenishment-up-to level in uniform case remains fixed but in exponential case the replenishment-up-to level a little increase. In Figure 10, when n increases, the expected cyclic profit increases in both examples. When the backordered cost per unit increases, it increases the replenishment-up-to level too, but this effect is more in exponential case than uniform case as it is clear in Figure 9. In Figures 11 and 12, when π increase, it increases the amount of replenishment-up-to level, and it causes the ECP to increase. This effect is little in this example and in exponential case is more than uniform case level, and it causes the ECP to increase. This effect is little in this example and more in the exponential case than in the uniform case

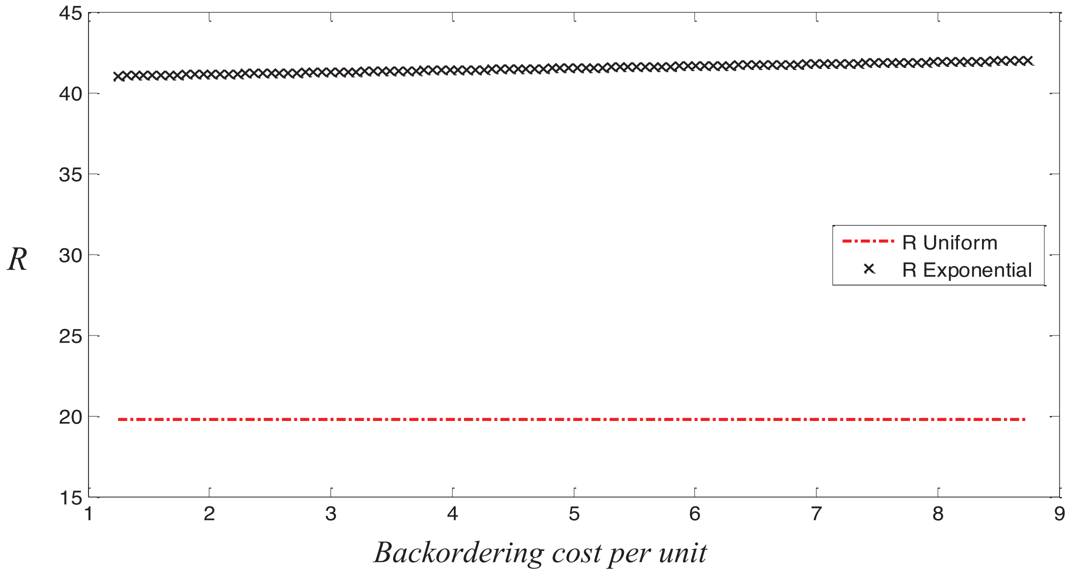

FIGURE 11. Effect of π on replenishment-up-to level.

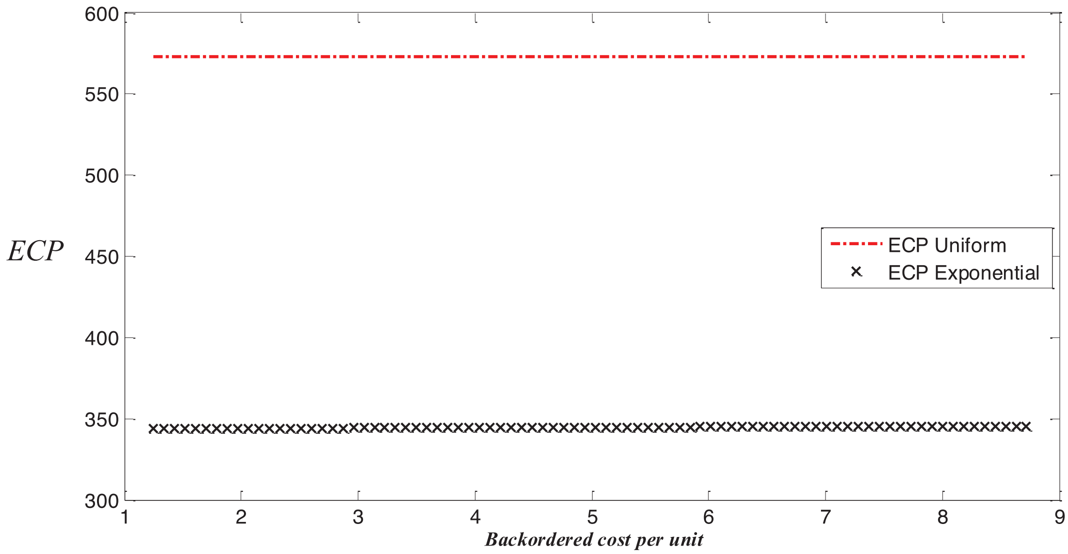

FIGURE 12. Effect of π on ECP.

4. Conclusion

In this paper, an optimal control model is developed for a purchasing system with stochastic delivery time, partial prepayment and partial backordering. It is considered that the buyer must pay a fraction of the purchasing cost as prepayment during several payments. A mathematical model under general probability distribution function is developed and the concavity of profit function is proved, and global optimal value of replenishment level is derived. Two special cases with uniform and exponential PDFs are exemplified to show the applicability of the proposed model. To analyze the effects of several parameters like demand, selling price, and percentage of purchasing cost that must be paid in advance or backordered were observed on the replenishment-up-to level and the expected cyclic profit, and it was realized that the demand and selling price have a more significant effect on problem decision especially on replenishment-up-to-level and expected cyclic profit than other parameters. Both customers and suppliers can benefit from these results to gain more profit and invest to prove some parameters that have more effect on their business. Future works may be directed toward developing the model for deteriorating products or considering delayed payment policy in transaction process to trade off the conflicting situation between sellers and buyers.

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