RAIRO-Oper. Res. 56 (2022) 1763–1780
https://doi.org/10.1051/ro/2022074

RAIRO Operations Research
www.rairo-ro.org

OPTIMAL PRICING AND OFFERING REWARD DECISIONS IN A COMPETITIVE CLOSED-LOOP DUAL-CHANNEL SUPPLY CHAIN WITH RECYCLING AND REMANUFACTURING

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Abstract. Recycling of materials has two significant perspectives: it may reduce the waste, and also, it can save raw materials. This study deals with the returned-obsolete products and the fresh items in a closed-loop dual-channel supply chain, where the manufacturer operates the whole production department and sells a percentage of products directly through his online channel, and delivers the rest of them to the retailer at a wholesale price. Additionally, the retailer collects unused items from customers with an appropriate reward to determine whether the customers intend to return the items. Both players screen the condition of the collected materials and then transfer the qualified materials for further use. Our paper formulates a mathematical model to evaluate scenarios such as scenarios manufacturer Stackelberg, Retailer Stackelberg, vertical Nash under the decentralized system, and a centralized system. The study is primarily concerned with finding optimal pricing plans and rewarding the customer analytically under various scenarios. Numerical explorations signify that the manufacturer Stackelberg's scenario is more economical than the retailer Stackelberg and vertical Nash frameworks. The findings illustrate that the higher acceptance ratios of the returned materials benefit all the members and increase the keenness to return. Also, it is important for members to control the price-sensitive parameters within the demand function in order to save their markets. Further, the study suggests that an increase in production cost forces us to collect more returned materials, regardless of whether the increased remanufacturing cost suppresses that collection.

Mathematics Subject Classification. 90B06.

Received June 18, 2021. Accepted May 14, 2022.

1. Introduction

Today's world has a lot of responsibility to give a better planet for our future generations. Recycling of products ([9]) is one most important works for our environment because it may conserve natural resources. The old and obsolete products can be converted to the same new products by properly incorporating appropriate technology. When waste products are recycled, again and again, the demand for raw materials shrinks. To save raw materials and reduce garbage, society has to maintain the *RRR* policy, *i.e.*, reduce, reuse, and recycle.

Keywords. Closed-loop supply chain, Dual channel, Recycling, Remanufacturing, Stackelberg game, Nash game.

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Since the use of electronic pieces of equipment is rapidly growing, the waste of electrical and electronic equipment ([12]) has been increased day after day in the environment, whether the maximum amount of that waste can be recycled for further use. Due to the short life cycle and frequently technological improvement, there are high chances to reform the up-to-date version by incorporating some components and taking engineering advantages ([2]). Recycling can provide two benefits: first, it helps society save raw materials; second, it reduces waste pollution. Different policies should be implemented to enhance recycling. But in reality, the collection rate of obsolete products is not adequate. Over the past years, the proper policy-making for recycling is gaining increased attention from the researcher and practitioners ([8,39]). There must be practical solutions to motivate consumers to return unused or obsolete products so that the rate of returns will increase. Therefore, the industries should have to find out strategies on how to enhance that returned rate. This study will decide the appropriate rewards, which to pay the consumers for returning the products, to optimize the companies' profits adjusting the returned rate.

Again, in the running days, business competition is a conventional matter ([28]). Furthermore, every big company wants to spread its business worldwide. But, if they have to depend only on the traditional markets, they cannot extend it since coordination with existing retail shops is always impossible. Therefore, the company expands the online channels to sell the products, but that channel has faced big competition with the existing retail businesses. In the traditional channel, the customers get the facilities of observing and verifying the products physically. Whereas, the direct channels have to compete with the retailer without those facilities. The advantages of the system are that customers can buy and receive the product from any place. As a result, the popularity of the online channel has been increasing day after day.

Considering all the recent warming factors/challenges regarding the business environment, it is apparent that if a company properly maintains the forward and reverse supply chain (SCn) ([15]), he will get an advantage in the competitive world. It will challenge a company to derive the best strategies in a supply chain along with its reverse procedures. But to expand the business, to reduce the use of fresh materials, to decrease the amount of unused garbage, to save energy, every company has to do more research on RRR policy and different mediums of business.

However, the article must explore the closed-loop supply chain that involves a manufacturer and a retailer, involving these two channels in tandem. The forward chain of the supply chain involves both direct and retail channels. Through the reverse channel, retailers collect obsolete products from their customers in exchange for appropriate incentives, and incentive programs can be used to increase collection rates. In this study, our principal aim is to form a closed-loop supply chain with competitive two channels and then solve the system for different practical scenarios and find the proper strategy such that the reverse cycle would be more useful.

We discuss the main gap in literature and the contribution of the study as follows:

- A variety of closed-loop supply chain models were developed for pricing and collection incentives with remanufacturing. There are extremely few closed-loop dual-channel supply chain models that consider recycling and remanufacturing.
- The past papers considered either a constant collection proportion or a decision variable in the supply chain. However, collection price incentives that respond to the customers' willingness to return would be an innovative concept in a closed-loop dual-channel supply chain.
- Several articles have discussed closed-loop dual-channel supply chains under various scenarios. In closed-loop dual-channel supply chain models, the selling and collection prices are rarely examined together in Stackelberg games, Nash equilibrium games, or coordination contract frameworks.

We propose a closed-loop dual-channel supply chain structure based on recycling and remanufacturing in order to address the research gap above. In this study, we seek answers to the following questions:

- Which game model and scenario will be most beneficial to the players and supply chain?
- How does the acceptance of screening ratios by the players affect equilibrium decisions?
- How do demand parameters affect supply chain profits?
- What is the impact of production and remanufacturing costs on the supply chain and the players?

The remainder of the research article is structured as Section 2 describes a brief literature survey of the related article about our study. Section 3 presents the problem definition of the work along with assumptions and notation. Mathematical modeling of the dual-channel SCn is developed, and different types of decision-making scenarios are analyzed in Section 4. Section 5 illustrates the numerical example and sensitivity analysis with managerial insights. Finally, in Section 6 concludes the model with limitations.

2. Literature review

In this study, three primary research directions are explored: The first focuses on closed-loop supply chains, the second examines dual-channel supply chains, and the third focuses on game-theoretic analyses of supply chains.

2.1. Closed-loop supply chain

Schultmann *et al.* [33] studied a closed-loop SCn ($ClSc$) considering the end-of-life vehicle, where comparison among the different scenarios for collecting secondary material was analyzed. Later, Zhang *et al.* [43] expanded a capacitated production planning problem assuming time-varying setup cost functions under a $ClSc$ with remanufacturing. They used the genetic algorithm heuristic approaches to solve the model. Another $ClSc$ with consumer perceptions related to remanufactured products was developed by [1] considering manipulated price discounts, and brand equity. Hong *et al.* [20] presented a $ClSc$ with a price and advertised sensitive market, and analyzed advantages of the advertizement to enhance business under different model structures. Another study of the $ClSc$ was studied by [42], where pricing and return policy were expanded under several supply contracts . They investigated how the contract scenarios modified the policies decisions to enhance the profit of the entire SCn and its members. Saha *et al.* [27] investigated a $ClSc$ with a reward-driven remanufacturing policy under different modes of collection of old obsolete products, where three coordination mechanisms were presented in collaboration among the members with win-win outcomes conditions. A forward and reverse process in a $ClSc$ with fresh and refurbished items was presented by [41] to differentiate the overall performances under different structures. Sarkar *et al.* [29] expanded a $ClSc$ model with third-party logistics considering the returnable transport items, remanufacturing, and environmental impacts of transportation. A competitive and cooperative $ClSc$ with a manufacturer and independent re-manufacturer was developed by [38], where firm equilibrium decisions and profits in different models investigated under technology licensing and R&D joint venture mechanisms. Heydari *et al.* [18] considered a two-echelon reverse SCn under stochastic remanufacturing capacity where they found the optimal reward paid to the customers. They introduced a revenue-sharing mechanism and showed how it benefitted the members, and improved the performance of the SCn. He *et al.* [17] analyzed the recovery efficiency and customer behavior in a $ClSc$ under the inconvenience-perception in the collection. They also examined mechanisms that became more effective to enhance that efficiency. Another $ClSc$ with third party recycling was developed by [45] considering different remanufacturing roles and technology authorizations, where the degree of consumer willingness to purchase remanufactured products also analyzed. Chen and Akmalul'Ulya [7] developed a green $ClSc$ considering the greening efforts of all members and the government intervention through the reward penalty mechanism. Their model analyzed the return rate and green effort to improve the scenario. Wei *et al.* [37] developed a three-level $ClSc$ for two competitive collectors and investigated how the total collection rate of used products and the channel members' decisions were affected by the integration strategies of the manufacturer. Assarzadegan and Rasti-Barzoki [3] expanded $ClSc$s introducing full and partial Money Back Guarantee for the defective and non-defective returned items from the customers.

2.2. Dual-channel supply chain

Mondal *et al.* [26] studied a dual-channel $ClSc$ with green products where both channels could collect the used products for remanufacturing. They also investigated the model under different Stackelberg and Nash games and discussed the best profitable scenarios. A dual-channel SCn with a monopoly manufacturer, a retailer, and the heterogeneous consumers was presented by [40]. Studying the effect of retail costs, they showed that it had an enormous impact on the structures for different parties. Multiple channel structures for the manufacturer in a dual-channel $ClSc$ under government subsidy of remanufactured products were analyzed by [16]. They found that higher subsidy was not always benefitted to the environment, but it was profitable for the consumers and the whole SCn. Wang *et al.* [36] explored two $ClSc$ models with dual-collecting channels and discussed the channel decisions for varying consumer behavior corresponding to different collection procedures. A $ClSc$ with two types of returned products was introduced by [44] in which they derived the product quality and price under a dual-channel structure.

2.3. Game-theoretic analyses of supply chains

After that, Hong *et al.* [19] considered a reverse channel in a $ClSc$ with different collection procedures of used products using the Stackelberg and Nash bargaining game. They explored the hybrid cannel for the members, and also discussed the effectiveness reverse channel structure. Ma *et al.* [23] expanded $ClSc$s with the price of anarchy with a manufacturer and retailer and analyzed different collection alternatives of used products under the Stackelberg game model. Their model discussed the push-pull systems and also found a better collection option. Giri and Dey [14] studied another $ClSc$ introducing a back-up supplier due to the uncertainty of the collection of used products, and then developed different game-theoretic models to adjust the optimal strategies under different situations. After that, Taleizadeh *et al.* [34] extended a SCn considering the factors carbon emission reduction, return policy, and quality improvement effort, and designed two-hybrid remanufacturing scenarios under a technology license. They investigated the relations among the three factors, such that which framework would give better performance. Mondal and Giri [25] studied a green SCn with two members and investigated the green innovation, marketing effort, and collection rate of used products under a centralized and three decentralized scenarios. Liu *et al.* [22] explored the centralized, decentralized, and channel coordination scenarios in a SCn with the manufacturer's returns handling strategies under demand uncertainty environment.

In addition, many supply chain models ([4–6, 10, 11, 13, 21, 30–32]) have been studied to examine different strategies for decision-making.

The contribution of the study, compared to the existing literature, are shown in Table 1. Studying dual-channel under $ClSc$ with remanufacturing is a new aspect of research within a competitive environment. Generally, we observe that willingness of consumers to return an item was almost not recognized previously. A model with the stated factors will be formulated and then optimal solution under different co-operative (centralized) and non-cooperative (DS_{MS}, DS_{RS}, DS_{VN}) will be analyzed such that players can survive in the real competitive world. Table 1 illustrates that investigation of scenarios DS_{RS} and DS_{VN} along with other popular frameworks will provide new direction rather than previous studies.

3. PROBLEM DEFINITION

This model deals with a closed-loop dual-channel SCn (CDSC) with a manufacturer (MR) and a retailer (RR). The MR produces new items with fresh raw-materials and remanufactures the recycled materials into new products to satisfy the market demand. The RR offers suitable value to the customers to return the old product. He also tries to increase the willingness to return the products by improving the level of reward. The RR first screens the items and then sells the standard material to the MR and demolishes the others. The manufacturer also inspects the materials and then transfers the recycled materials to the production house to refurbish it into new products and disposes of the rest. Since all the remanufactured products are insufficient to satisfy the total demand, fresh new items are also needed to serve it. The MR sells the products through two channels: his direct channel and RR's traditional channel. The two channels compete in the system with selling price (SPr).

The main factor of the model that enhances the SCn is to study the forward and reverse SCn systems with returned products under centralized and different types of decentralized cases. The quantity of the items can be increased by given suitable rewards to the consumers, which is a decision parameter. In the decentralized study, the MR maximizes his profit by optimizing the direct channel SPr and wholesale price, whether the

TABLE 1. A comparison table among the present article and related previous articles.

Article	Supply chain		Model description						Decision variables	
	$ClSc$	Dual channel	Competition	Remanufacturing	$CWtR$	DS_{MS} $^{(a)}$	DS_{RS} $^{(b)}$	DS_{VN} $^{(c)}$	prices	ORRC $^{(d)}$
Heydari *et al.* [18]	×	×	×	✓	✓	×	×	×	×	✓
Taleizadeh *et al.* [34]	✓	×	×	✓	×	✓	×	×	✓	×
Zhao *et al.* [45]	✓	×	✓	✓	×	✓	×	×	✓	×
He *et al.* [16]	✓	✓	✓	✓	×	×	×	×	✓	×
Giri and Dey [14]	✓	✓	×	×	×	✓	×	×	✓	×
Chen and Akmalul'Ulya [7]	✓	×	✓	✓	×	✓	×	×	✓	×
Wang *et al.* [36]	✓	×	✓	×	×	✓	×	×	✓	×
Wei *et al.* [37]	✓	×	✓	✓	×	✓	×	×	✓	×
Assarzadegan and Rasti-Barzoki [3]	✓	×	×	✓	×	✓	×	×	✓	×
Liu *et al.* [22]	✓	×	×	✓	×	✓	×	×	×	×
Zhang *et al.* [44]	✓	✓	✓	✓	×	×	×	×	✓	×
Mondal and Giri [25]	✓	×	×	✓	×	✓	×	×	✓	×
This work	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Notes. $^{(a)}$Manufacturer Stackelberg. $^{(b)}$Retailer Stackelberg. $^{(c)}$Vertical Nash. $^{(d)}$Reward offered by the RR to the customers for returned items.

RR explorations the traditional track SPr and the reward amount of returned items. In this paper, we find the most suitable situations for the individual member and the whole system. We also investigate whether the customers' willingness to return ($CWtR$) will be maximum.

We use the following notation of the parameters and variables to describe the model:

Parameters	
D_1	Customer demand of the MR in direct channel (unit \ unit time).
D_2	Customer demand in traditional channel (unit \ unit time).
$\alpha_1, \alpha_2, \beta$	price sensitivity of the customer's demand (> 0).
θ	The acceptance of screening ratio of returned item at the RR site.
ϕ	The acceptance of screening ratio of returned item at the MR site.
C_R	Remanufacturing cost per item (\$ \ unit).
C_P	Production cost per item (\$ \ unit).
SC_R	Screening cost at the RR site per item (\$\ unit).
SC_M	Screening cost at the MR site per item (\$\ unit).
w_r	Fees of the returned items, which are qualified the RR standard criteria, paid to the RR by the MR (\$\ unit).
$p_{\max}$	Maximum reward offered by the RR by which all products purchased by customers will be returned as obsolete products at the end of their use (\$).
C_d	Disposal cost per unit low standard returned product for both the members (\$\ unit).
Variables	
p_1	Selling price for the MR for one unit of finished product in direct channel (\$\ unit).
p_2	Selling price for the RR for one unit of finished product in traditional channel (\$\ unit).
w_p	Wholesale price for the MR for one unit of finished product (\$\ unit).
p_r	Reward offered by the RR to the customers for the one unit returned product (\$\ unit).

The following main assumptions are adopted to develop the model:

- The RR collects the used products from the customers by the suitable rewards and the collection rate can be increased by offering higher rewards.
- The customers' returned tendency for obsolete products is increased if the value of the products goes towards the maximum reward by which all products purchased by customers will be returned after their use. The fraction of these two rewards is defined as the $CWtR$ ([18]). The expression of the customers willingness to return is as follows:
$$W_c = \begin{cases} \frac{p_r}{p_{\max}}, & 0 < p_r < p_{\max} \\ 1, & p_r \geq p_{\max} \end{cases}$$
- The RR inspects the products and then sells the standard materials at his site to the MR. The MR also screens them again and differentiates the recycled materials for remanufacturing.
- The MR produces the fresh products along with remanufactured products and sells them in the market by two channels: his direct channel and the RR's traditional channel.
- The customers' demand is considered to depend linearly on the prices of the products where the increment of their SPr has a negative impact, and the rival's price has a positive influence on it. We assumed that the own-price has more dominating power than competitors, and the high price for the direct channel has a more adverse effect on the demand as the customers purchase the products traditionally from the market. Therefore, the demand functions for the two channels are:
  Direct channel: $D_1 = a_1 - \alpha_1 p_1 + \beta p_2$,
  Retail channel: $D_2 = a_2 - \alpha_2 p_2 + \beta p_1$,
  where $a_1, a_2 (> 0)$ are the base market demand, α_1, $\alpha_2 (> 0)$ measure the impact of SPr on it, $\beta (> 0)$ assesses the effect of rival's price on the demand, and the parameters satisfy the relations $\alpha_1 > \alpha_2 > \beta$ ([36,44]).

The next section discusses the centralized and decentralized scenarios and explains how pricing and reward strategies should be optimized to maximize profits. In the decentralized case, we derive the optimal solutions for the Vertical Nash, MR Stackelberg, and Retailer Stackelberg model ([24,35]).

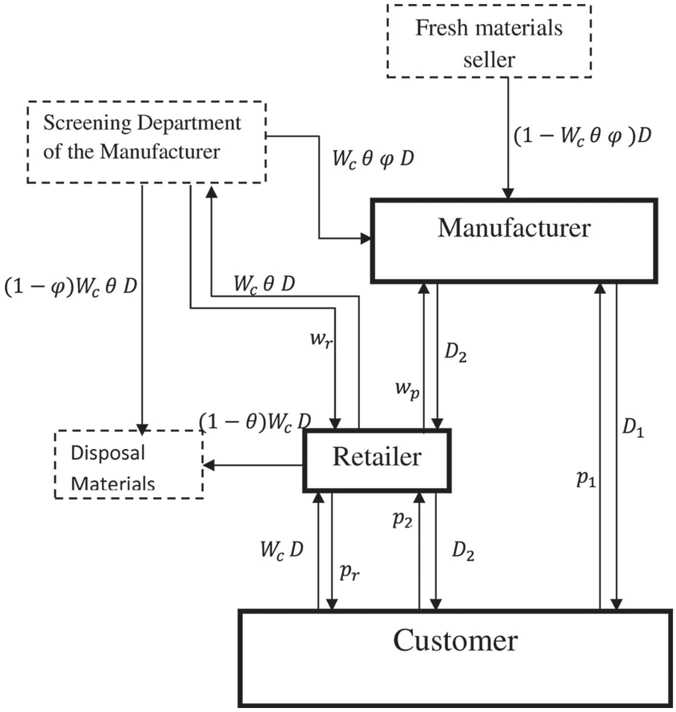


Figure 1. Closed-loop dual-channel SCn.

4. Mathematical modeling

Here, the total market demand of the customer is $D = D_1 + D_2$ for the combined products of fresh and remanufactured in which the MR directly sells D_1 through the online channel, and the RR fulfils the rest of the demand. We assume that the maximum availability of the useless old product is also D. Adjusting the $CWtR$ parameter, the RR can collect W_c fraction of the total available obsolete products from the customers. We consider that θ fraction of returned products satisfies the RRs' standard level. Then, he supplies the products to the MR at a worth of w_r per unit. The MR also screens the products at a cost SC_M per unit, and ϕ fraction of products are passed for remanufacturing. Both members dispose of the low standard materials at a cost C_d per item. The manufacture raises the sale revenue $p_1D_1 + w_pD_2$ from the RR and the direct channel market, whether the RR collects the revenue p_2D_2 from the market and $w_r\theta W_cD$ from the MR for returned materials. The RR pays the rewards p_rW_cD to the customers and spends cost SC_R per unit for the inspection of the returned products. The MR pays C_P per unit for the production and expends C_R per unit for remanufacturing. We draw Figure 1 to describe model graphically. We summarize the players' earned revenue and associated costs in Table 2 and Table 3.

Therefore, the profit function of the MR and RR are respectively

$$MP = p_1D_1 + w_pD_2 - \theta W_cD(w_r + SC_M) - C_R\phi\theta W_cD - C_PD(1-\phi\theta W_c) - C_d(1-\phi)\theta W_cD \tag{4.1}$$

$$RP = (p_2 - w_p)D_2 + \theta W_cD(w_r - \frac{p_r + SC_R}{\theta}) - C_d(1-\theta)W_cD. \tag{4.2}$$

In this model, different types of decision-making scenarios have been analyzed for both the member. Here, we first formulate the centralized and decentralized cases and then derive the optimal strategies under the forward and reverse SCn systems.

TABLE 2. Manufacturer's earned revenue with several cost functions.

Revenue/cost functions	Expressions
Earned revenue	$p_1D_1 + w_pD_2$
Production cost for new items	$C_PD(1-\phi\theta W_c)$
Cost of the returned items paid to retailer	θW_cDw_r
Screening cost	θW_cDSC_M
Disposal cost for low standard recycled product	$C_d(1-\phi)\theta W_cD$

TABLE 3. Retailer's earned revenue with several cost functions.

Revenue/cost functions	Expressions
Earned revenue	$p_2D_2 + \theta W_cDw_r$
Cost of new items paid to manufacturer	w_pD_2
Cost of the returned items paid to customers	p_rW_cD
Screening cost	W_cDSC_R
Disposal cost for low standard recycled product	$C_d(1-\theta)W_cD$

4.1. Centralized Scenario (CS)

Here, the members of the chain make the optimal strategies in such a way that the profit of the whole SCn will be maximum. Here, the total profit of the system is the combined profits (4.1) and (4.2) of the MR and RR, *i.e.*, $TP = MP + RP$. Objective for this case is to find optimal SPrs p_1, p_2 and reward p_r such that total profit will be maximum.

Proposition 4.1. *Under CS, there exist optimal solution*
$p_1^* = \frac{4p_{\max}(a_1\alpha_2+a_2\beta)\theta+A_1\left(4C_Pp_{\max}\theta-A_3^2\right)}{8A_1p_{\max}\theta}$, $p_2^* = \frac{4p_{\max}(a_2\alpha_1+a_1\beta)\theta+A_1\left(4C_Pp_{\max}\theta-A_3^2\right)}{8A_1p_{\max}\theta}$ *and* $p_r^* = \frac{A_3}{2\theta}$,
where $A_1 = \left(\alpha_1\alpha_2-\beta^2\right)$, $A_2 = (\alpha_1+\alpha_2-2\beta)$, *and* $A_3 = (-C_d - SC_R - SC_M\theta + (C_d + C_P - C_R)\theta\phi)$.

Proof. Under CS, the profit TP can be optimized using the classical optimization approach.
The first step of this method is to solve the necessary conditions $\frac{\partial TP}{\partial p_1} = 0$, $\frac{\partial TP}{\partial p_2} = 0$ $\frac{\partial TP}{\partial p_r} = 0$ for maximization and then get one solution
$p_1 = \frac{4p_{\max}(a_1\alpha_2+a_2\beta)\theta+A_1\left(4C_Pp_{\max}\theta-A_3^2\right)}{8A_1p_{\max}\theta}$, $p_2 = \frac{4p_{\max}(a_2\alpha_1+a_1\beta)\theta+A_1\left(4C_Pp_{\max}\theta-A_3^2\right)}{8A_1p_{\max}\theta}$ and $p_r = \frac{A_3}{2\theta}$,
where $A_1 = \left(\alpha_1\alpha_2-\beta^2\right)$, $A_2 = (\alpha_1+\alpha_2-2\beta)$, and $A_3 = (-C_d - SC_R - SC_M\theta + (C_d + C_P - C_R)\theta\phi)$.
The above solution will be optimum if the Hessian matrix of the profit function is negative definite at the (p_1^*, p_2^*, p_r^*).
The Hessian matrix of the profit function is given by

$$H_{TP} = \begin{pmatrix} -2\alpha_1 & 2\beta & \frac{(\alpha_1-\beta)(2\theta p_r-A_3)}{p_{\max}} \\ 2\beta & -2\alpha_2 & \frac{(\alpha_2-\beta)(2\theta p_r-A_3)}{p_{\max}} \\ \frac{(\alpha_1-\beta)(2\theta p_r-A_3)}{p_{\max}} & \frac{(\alpha_2-\beta)(2\theta p_r-A_3)}{p_{\max}} & -\frac{2D\theta}{p_{\max}} \end{pmatrix}.$$

The matrix H_{TP} is negative definite since

1. $H_{TP}(1,1) = -2\alpha_1 < 0$.
2. $\begin{vmatrix} -2\alpha_1 & 2\beta \\ 2\beta & -2\alpha_2 \end{vmatrix} = 4(\alpha_1\alpha_2 - \beta^2) > 0$ as relation among the parameters is $\alpha_1 > \alpha_2 > \beta$.

3. Determinant of $H_{TP} = \left[p_r - \left(\frac{A_3}{2\theta} - \frac{\sqrt{A_2 p_{\max} D\theta}}{A_2\theta}\right)\right]\left[p_r - \left(\frac{A_3}{2\theta} + \frac{\sqrt{A_2 p_{\max} D\theta}}{A_2\theta}\right)\right] < 0$ at (p_1^*, p_2^*, p_r^*) as $\left(\frac{A_3}{2\theta} - \frac{\sqrt{A_2 p_{\max} D\theta}}{A_2\theta}\right) < p_r^* = \frac{A_3}{2\theta} < \left(\frac{A_3}{2\theta} + \frac{\sqrt{A_2 p_{\max} D\theta}}{A_2\theta}\right)$.

Therefore, the optimal solution of the scenario is (p_1^*, p_2^*, p_r^*).
Hence the proof. □

4.2. Decentralized Scenario (DS)

Under the DS, the members of the chain are competitor each other, and each of them takes the optimal decision on one's own. Here we study three DSs: MR Stackelberg, Retailer Stackelberg, and Vertical Nash cases.

4.2.1. Manufacturer Stackelberg (DS_{MS})

In the DS_{MS}, the power of the decision-making of the MR is more than the RR. As a result, the MR decides his strategy after observing the decision of the RR. Here, the RR first settles upon the optimal strategies on p_2 and p_r on his profit (4.2) for a given p_1. After that, the MR regulates the decision on p_1 to optimize his profit (4.1) using the RR's decisions.

Proposition 4.2. *Under DS_{MS}, the RR's optimal solution is*
$p_2^{DS_{MS}} = \frac{1}{8\alpha_2}\left(4(a_2 + p_1(\beta + \alpha_2\lambda)) - \frac{(\alpha_2-\beta)B_2^2}{p_{\max}\theta}\right)$, *and* $p_r^{DS_{MS}} = \frac{B_2}{2\theta}$,
where $B_1 = C_d + SC_M + w_r - (C_d - C_R + C_P)\phi$ *and* $B_2 = (C_d + w_r)\theta - (C_d + SC_R)$.

Proof. The necessary conditions to maximize the RR's profit are $\frac{\partial RP}{\partial p_2} = 0$ and $\frac{\partial RP}{\partial p_r} = 0$. Solving the equations, we get a solution $p_2^{DS_{MS}} = \frac{1}{8\alpha_2}\left(4(a_2 + p_1(\beta + \alpha_2\lambda)) - \frac{(\alpha_2-\beta)B_2^2}{p_{\max}\theta}\right)$, and $p_r^{DS_{MS}} = \frac{B_2}{2\theta}$, where $B_1 = C_d + SC_M + w_r - (C_d - C_R + C_P)\phi$ and $B_2 = (C_d + w_r)\theta - (C_d + SC_R)$.
The Hessian matrix of the RR's profit is

$$H_{RP} = \begin{pmatrix} -2\alpha_1 & \frac{(\alpha_2-\beta)(C_d+SC_R-(C_d-2p_r+w_r)\theta)}{p_{\max}} \\ \frac{(\alpha_2-\beta)(C_d+SC_R-(C_d-2p_r+w_r)\theta)}{p_{\max}} & -\frac{2d\theta}{p_{\max}} \end{pmatrix}.$$

The determinant value of the matrix is
$Det(H_{RP}) = -\frac{4(\alpha_2-\beta)^2\theta^2}{p_{\max}^2}\left[p_r - \left(\frac{B_2}{2\theta} - \frac{2\sqrt{p_{\max}\,\alpha_2\,d\,\theta}}{2(\alpha_2-\beta)\theta}\right)\right]\left[p_r - \left(\frac{B_2}{2\theta} + \frac{2\sqrt{p_{\max}\,\alpha_2\,d\,\theta}}{2(\alpha_2-\beta)\theta}\right)\right]$.
It is clear that $Det(H_{RP}) > 0$ for all $p_r \in \left[\left(\frac{B_2}{2\theta} - \frac{2\sqrt{p_{\max}\,\alpha_2\,d\,\theta}}{2(\alpha_2-\beta)\theta}\right), \left(\frac{B_2}{2\theta} + \frac{2\sqrt{p_{\max}\,\alpha_2\,d\,\theta}}{2(\alpha_2-\beta)\theta}\right)\right]$.
Since, $p_r^{DS_{MS}} = \frac{B_2}{2\theta}$ and $\frac{\partial^2 RP}{\partial p_2^2} = -2\alpha_1 < 0$, therefore, the solution of the RR is optimal.
Hence the proof. □

Proposition 4.3. *Under DS_{MS}, the MR's optimal solution is*
$p_1^{DS_{MS}} =$
$\frac{\alpha_2}{8(\beta^2-\alpha_2(2\alpha_1+\lambda(\alpha_2\lambda-2\beta)))}\left(8a_1 + \frac{4a_2(\beta+\alpha_2\lambda)}{\alpha_2} + \frac{(\alpha_2-\beta)B_2^2(\alpha_2\lambda-\beta)}{\alpha_2\theta p_{\max}} + \frac{2(2\alpha_1\alpha_2-\beta(\alpha_2+\beta)+\alpha_2(\alpha_2-\beta)\lambda)(2C_P p_{\max}+B_1B_2)}{\alpha_2 p_{\max}}\right)$.

Proof. In this scenario, observing the decision of the RR, MR optimizes his profit. Therefore, using the results $p_2^{DS_{MS}} = \frac{1}{8\alpha_2}\left(4(a_2 + p_1(\beta + \alpha_2\lambda)) - \frac{(\alpha_2-\beta)B_2^2}{p_{\max}\theta}\right)$, and $p_r^{DS_{MS}} = \frac{B_2}{2\theta}$, the profit of the MR will be $MP^{DS_{MS}}(p_1) = MP(p_1, p_2^{DS_{MS}}, p_r^{DS_{MS}})$.
Solving the necessary condition $\frac{d\,MP^{DS_{MS}}(p_1)}{d\,p_1} = 0$ to optimize that profit, we get
$p_1^{DS_{MS}} =$
$\frac{\alpha_2}{8(\beta^2-\alpha_2(2\alpha_1+\lambda(\alpha_2\lambda-2\beta)))}\left(8a_1 + \frac{4a_2(\beta+\alpha_2\lambda)}{\alpha_2} + \frac{(\alpha_2-\beta)B_2^2(\alpha_2\lambda-\beta)}{\alpha_2\theta p_{\max}} + \frac{2(2\alpha_1\alpha_2-\beta(\alpha_2+\beta)+\alpha_2(\alpha_2-\beta)\lambda)(2C_P p_{\max}+B_1B_2)}{\alpha_2 p_{\max}}\right)$.
Again $\frac{d^2\,MP^{DS_{MS}}(p_1)}{d\,p_1^2} = -(2\alpha_1 + \lambda^2\alpha_2) + \beta\left(\frac{\beta}{\alpha_2} + 2\lambda\right) < 0$ since the assumptions on the parameters are $\alpha_1 >$

$\alpha_2 > \beta$ and $\lambda < 1$.
Therefore, the result is optimum.
Hence the proof. □

4.2.2. Retailer Stackelberg (DS_{RS})

This scenario is exactly opposite decision-making power of (DS_{MS}), *i.e.*, looking on the MR's optimal strategy, the RR finds his optimal decision.

Proposition 4.4. *Under* DS_{RS}*, the* MR*'s optimal solution is*
$p_1^{DS_{RS}} = \frac{B_1 p_r(\alpha_1-\beta)\theta+p_{\max}(a_1+a_2\lambda+C_P(\alpha_1-\beta)-p_2(\alpha_2\lambda-\beta))}{2p_{\max}(\alpha_1-\beta\lambda)}$.

Proof. The MR find the decision on his profit function (4.1) by solving the necessary condition $\frac{d\,MP}{d\,p_1} = 0$ for optimization.
The solution of the equation is $p_1^{DS_{RS}} = \frac{B_1 p_r(\alpha_1-\beta)\theta+p_{\max}(a_1+a_2\lambda+C_P(\alpha_1-\beta)-p_2(\alpha_2\lambda-\beta))}{2p_{\max}(\alpha_1-\beta\lambda)}$.
Again, we get $\frac{d^2\,MP}{d\,p_1^2} = -(2\alpha_1 - \beta\lambda) < 0$ for all p_1.
Therefore, the solution is optimal as the profit function is concave.
Hence the proof. □

Proposition 4.5. *Under* DS_{RS}*, the* RR*'s optimal solution is* $p_2 = p_2^{DS_{RS}}$ *and* $p_r = p_r^{DS_{RS}}$.

Proof. Putting the value $p_1 = p_1^{DS_{RS}}$ of the MR's decision parameter, the profit function of the RR will be $RP^{DS_{RS}}(p_2, p_r) = RP(p_1^{DS_{RS}}, p_2, p_r)$.
Solving the conditions $\frac{\partial RP^{DS_{RS}}(p_2,p_r)}{\partial p_2} = 0$ and $\frac{\partial RP^{DS_{RS}}(p_2,p_r)}{\partial p_r} = 0$ of optimality of the the profit function $RP^{DS_{RS}}(p_2, p_r)$, we get $p_2 = p_2^{DS_{RS}}$ and $p_r = p_r^{DS_{RS}}$ (since the exact expression is too lengthy, so we do not write the full form of the solution).
This solution will be optimal if the Hessian matrix of the profit function $RP^{DS_{RS}}(p_2, p_r)$ is negative definite at the solution point. □

4.2.3. Vertical Nash (DS_{VN})

In the scenario, the members of the SCn have the equal decision-making power to settle down their respective strategies, *i.e.*, the MR takes the decision on p_1 optimizing the profit (4.1), and in the same time, the RR settles down p_2 and p_r to maximize his profit (4.2).
Let us consider
$B_1 = C_d + SC_M + w_r - (C_d - C_R + C_P)\phi$, $B_2 = C_d\theta + w_r\theta - (C_d + SC_R)$,
$B_3 = a_1 + C_P\alpha_1 - CP\beta$, and $B_4 = B_1B_2(\alpha_1 - \beta)\theta$.

Proposition 4.6. *Under* DS_{VM}*, Nash equilibrium solution is*
$p_1^{DS_{VM}} = \frac{4\alpha_2 B_4+B_2^2(\alpha_2-\beta)(\alpha_2\lambda-\beta)+4p_{\max}\theta(a_2\beta+\alpha_2(2B_3+a_2\lambda))}{4p_{\max}\theta(4\alpha_2(\alpha_1-\beta\lambda)-\beta^2+\alpha_2^2\lambda^2)}$,
$p_2^{DS_{VM}} = \frac{B_4(\beta+\alpha_2\lambda)-B_2^2(\alpha_2-\beta)(\alpha_1-\beta\lambda)+2p_{\max}\theta(B_3(\beta+\alpha_2\lambda)+a_2(2\alpha_1-\beta\lambda+\alpha_2\lambda^2))}{2p_{\max}\theta(4\alpha_2(\alpha_1-\beta\lambda)-\beta^2+\alpha_2^2\lambda^2)}$, *and* $p_r^{DS_{VM}} = \frac{B_2}{2\theta}$.

Proof. The first-order condition for a Nash equilibrium solution for the MR is $\frac{d\,MP}{d\,p_1} = 0$.
Since, $\frac{d^2\,MP}{d\,p_1^2} = -(2\alpha_1 - \beta\lambda) < 0$ for all p_1, the profit function of the MR is strictly concave in p_1.
Again, the first-order conditions for a Nash equilibrium solution for the RR are $\frac{\partial\,RP}{\partial\,p_2} = 0$ and $\frac{\partial\,RP}{\partial\,p_r} = 0$.
From Proposition 4.2, we can say that the profit function of the RR is strictly concave in p_2 and for all $p_r \in \left[\left(\frac{B_2}{2\theta} - \frac{2\sqrt{p_{\max}\,\alpha_2\,d\,\theta}}{2(\alpha_2-\beta)\theta}\right), \left(\frac{B_2}{2\theta} + \frac{2\sqrt{p_{\max}\,\alpha_2\,d\,\theta}}{2(\alpha_2-\beta)\theta}\right)\right]$.
Solving the first-order conditions $\frac{d\,MP}{d\,p_1} = 0$, $\frac{d\,RP}{d\,p_2} = 0$ and $\frac{d\,RP}{d\,p_r} = 0$, we obtain the equilibrium solution
$p_1^{DS_{VM}} = \frac{4\alpha_2 B_4+B_2^2(\alpha_2-\beta)(\alpha_2\lambda-\beta)+4p_{\max}\theta(a_2\beta+\alpha_2(2B_3+a_2\lambda))}{4p_{\max}\theta(4\alpha_2(\alpha_1-\beta\lambda)-\beta^2+\alpha_2^2\lambda^2)}$,

TABLE 4. Value of the parameters.

Demand parameters	Cost parameters ($ / unit)	Other parameters
$a_1 = 1000$, $a_2 = 1050$, $\alpha_1 = 15$, $\alpha_2 = 12.5$, $\beta = 5$	$p_{\max} = 10$, $SC_M = 2.5$, $SC_R = 2$, $C_R = 10$, $C_P = 35$, $C_d = 1$, $w_r = 15$	$\theta = 0.85$, $\phi = 0.9$, $\lambda = 0.80$

TABLE 5. Eigenvalues of the Hessian matrices of profit functions.

Scenarios	Profit function of	Decision variable	Eigenvalue
CS	Supply chain	p_1, p_2, p_r	−131.725, −37.8078, −17.1922
DS_{VN}	Manufacturer	p_1	−22
	Retailer	p_2, p_r	−103.058, −25
DS_{MS}	Manufacturer	p_1	−28
	Retailer	p_2, p_r	−110.916, −25
DS_{RS}	Manufacturer	p_1	−22
	Retailer	p_2, p_r	−102.649, −32.1101

TABLE 6. Optimal results under different scenarios.

Optimal result	CS	DS_{VN}	DS_{MS}	DS_{RS}
p_1 ($ / unit)	68.91	78.36	75.17	78.02
p_2 ($ / unit)	78.14	88.02	86.11	89.26
p_r ($ / unit)	8.69	6.235	6.235	6.529
W_c ($ / unit)	0.868	0.623	0.623	**0.653**
MP ($)	23319.8	22508.00	**22650.30**	22398.00
RP ($)	11779.9	10655.70	**11233.20**	10686.00
TP ($)	**35099.7**	33163.70	**33883.60**	33084.00

$p_2^{DS_{VM}} = \frac{B_4(\beta+\alpha_2\lambda)-B_2^2(\alpha_2-\beta)(\alpha_1-\beta\lambda)+2p_{\max}\theta(B_3(\beta+\alpha_2\lambda)+a_2(2\alpha_1-\beta\lambda+\alpha_2\lambda^2))}{2p_{\max}\theta(4\alpha_2(\alpha_1-\beta\lambda)-\beta^2+\alpha_2^2\lambda^2)}$, and $p_r^{DS_{VM}} = \frac{B_2}{2\theta}$.
In the scenario, the MR and RR achieve maximum profit at the equilibrium solution, since the concavity conditions for the respective profit functions are satisfied.
Hence the proof. $\square$

5. NUMERICAL EXAMPLE

The model behavior is evaluated numerically using an example, and then sensitivity analysis is performed in order to validate the model. Table 4 shows experimental data sets that satisfy all the assumptions of the model. Numerical solutions are performed with Mathematica 11.1.1 software.

To test the concavity of the profit function, the eigenvalues of the Hessian matrix are calculated for each scenario. All eigenvalues of the Hessian vectors are negative, as shown in Table 5. Profit functions exhibit a unique solution in all scenarios. All the optimal results are represented in Table 6.

The numerical results indicate that the CS scenario is best for total SCn profit, and DC_{RS} framework is worst for it. The players will be benefitted by considering the DC_{MS} scenario, whether the cases DC_{RS} and DC_{VN} will give less profit for the MR and the RR, respectively.

Table 7. Effect of the parameters on the decision variables.

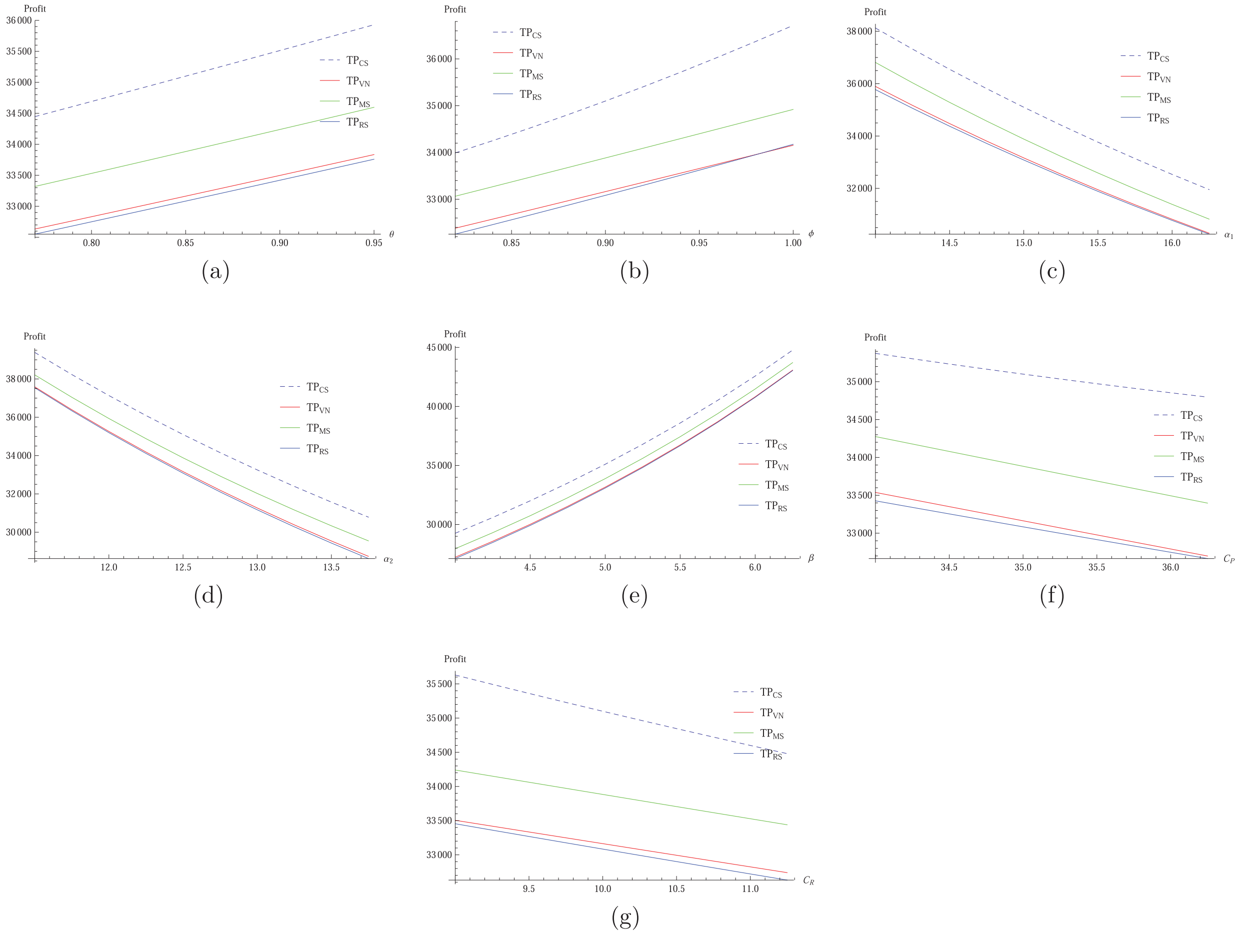
Parameter		Manufacturer's SPr				Retailer's SPr				Retailer's offered reward			
	Test value	p_1^{CS}	$p_1^{DS_{VM}}$	$p_1^{DS_{MS}}$	$p_1^{DS_{RS}}$	p_2^{CS}	$p_2^{DS_{VM}}$	$p_2^{DS_{MS}}$	$p_2^{DS_{RS}}$	p_r^{CS}	$p_r^{DS_{VM}}$	$p_r^{DS_{MS}}$	$p_r^{DS_{RS}}$
θ	0.77	69.33	78.46	75.31	78.14	78.56	88.23	86.34	89.42	8.502	6.052	6.052	6.335
	0.81	69.12	78.41	75.24	78.08	78.35	88.13	86.23	89.34	8.598	6.148	6.148	6.436
	0.85	68.91	78.36	75.17	78.02	78.14	88.02	86.11	89.26	8.685	6.235	6.235	6.529
	0.89	68.70	78.31	75.10	77.97	77.93	87.92	86.00	89.18	8.765	6.315	6.315	6.613
	0.93	68.48	78.26	75.04	77.91	77.71	87.82	85.88	89.11	8.837	6.387	6.387	6.691
ϕ	0.82	69.63	78.80	75.74	78.50	78.86	88.29	86.45	89.54	7.645	6.235	6.235	6.406
	0.86	69.28	78.58	75.46	78.26	78.51	88.16	86.28	89.40	8.165	6.235	6.235	6.468
	0.90	68.91	78.36	75.17	78.02	78.14	88.02	86.11	89.26	8.685	6.235	6.235	6.529
	0.94	68.51	78.14	74.89	77.78	77.74	87.89	85.94	89.12	9.205	6.235	6.235	6.589
	0.98	68.10	77.92	74.60	77.53	77.33	87.76	85.77	88.97	9.725	6.235	6.235	6.649
α_1	14.0	73.46	83.76	79.71	83.40	79.96	91.27	88.83	92.54	8.685	6.235	6.235	6.506
	14.5	71.09	80.95	77.36	80.60	79.01	89.58	87.42	90.83	8.685	6.235	6.235	6.518
	15.0	68.91	78.36	75.17	78.02	78.14	88.02	86.11	89.26	8.685	6.235	6.235	6.529
	15.5	66.89	75.97	73.14	75.65	77.33	86.59	84.89	87.81	8.685	6.235	6.235	6.54
	16.0	65.01	73.76	71.24	73.44	76.58	85.26	83.75	86.47	8.685	6.235	6.235	6.551
α_2	11.5	71.07	80.36	77.48	80.09	84.63	94.33	92.55	95.47	8.685	6.235	6.235	6.515
	12.0	69.94	79.33	76.28	79.02	81.23	91.04	89.19	92.24	8.685	6.235	6.235	6.523
	12.5	68.91	78.36	75.17	78.02	78.14	88.02	86.11	89.26	8.685	6.235	6.235	6.529
	13.0	67.97	77.44	74.13	77.07	75.32	85.24	83.27	86.51	8.685	6.235	6.235	6.534
	13.5	67.11	76.57	73.14	76.18	72.74	82.66	80.65	83.95	8.685	6.235	6.235	6.538
β	4.0	62.98	72.37	69.44	71.91	71.87	81.41	79.76	82.94	8.685	6.235	6.235	6.597
	4.5	65.79	75.22	72.15	74.83	74.83	84.57	82.79	85.96	8.685	6.235	6.235	6.562
	5.0	68.91	78.36	75.17	78.02	78.14	88.02	86.11	89.26	8.685	6.235	6.235	6.529
	5.5	72.40	81.83	78.55	81.54	81.86	91.81	89.78	92.90	8.685	6.235	6.235	6.497
	6.0	76.34	85.67	82.36	85.45	86.08	95.97	93.85	96.92	8.685	6.235	6.235	6.467
C_P	34.0	68.73	78.15	74.90	77.83	77.96	87.90	85.95	89.15	8.235	6.235	6.235	6.474
	34.5	68.82	78.25	75.04	77.93	78.05	87.96	86.03	89.21	8.460	6.235	6.235	6.501
	35.0	68.91	78.36	75.17	78.02	78.14	88.02	86.11	89.26	8.685	6.235	6.235	6.529
	35.5	68.99	78.46	75.31	78.12	78.22	88.09	86.19	89.32	8.910	6.235	6.235	6.556
	36.0	69.07	78.57	75.44	78.21	78.30	88.15	86.27	89.38	9.135	6.235	6.235	6.584
C_R	9.0	68.57	78.17	74.92	77.81	77.80	87.91	85.96	89.14	9.135	6.235	6.235	6.581
	9.5	68.74	78.26	75.05	77.92	77.97	87.97	86.04	89.20	8.910	6.235	6.235	6.555
	10.0	68.91	78.36	75.17	78.02	78.14	88.02	86.11	89.26	8.685	6.235	6.235	6.529
	10.5	69.07	78.45	75.29	78.13	78.30	88.08	86.19	89.32	8.460	6.235	6.235	6.502
	11.0	69.23	78.55	75.42	78.23	78.46	88.14	86.26	89.38	8.235	6.235	6.235	6.476

5.1. Sensitivity analysis and managerial insights

5.1.1. *Analysis of the acceptance of screening ratio θ and ϕ*

Figures 2a, 2b, 3a, 3b, 4a, and 4b show that profits are increasing with the increments of the θ and ϕ. Table 7 also exhibits that the SPrs of the members are decreasing when the ratios increase. But, the reward to the customer for the returned materials increases. In all the four cases (CS, DS_{VM}, DS_{MS}, and DS_{RS}), the changes in the values are in a similar pattern. We find that the effect on profits and decision parameters for the variation of ϕ is more than θ because the manufacture will lose or gain more than the RR for that variation.

Managerial insights. The acceptance ratios (θ, ϕ) of the returned material have an immense impact on the profits as it has great potential to diminish the production cost. Therefore, the ratios' increasing rate reduces the use of new raw materials and expands the benefit of the recyclability. The physical significance of the increasing ratios is that the returned materials for further use are satisfactory. This incident diminishes the cost of total disposal and increases remanufactured products. Since initiate production cost is much higher than the remanufacturing, the total cost of the MR is reducing with the higher values of θ and ϕ). Less production cost and high quality valued returned materials are two highly effective decision-making incidents for expanding their business. Consequently, the sellers should reduce their prices to increase market demand.



(a) (b) (c)

(d) (e) (f)

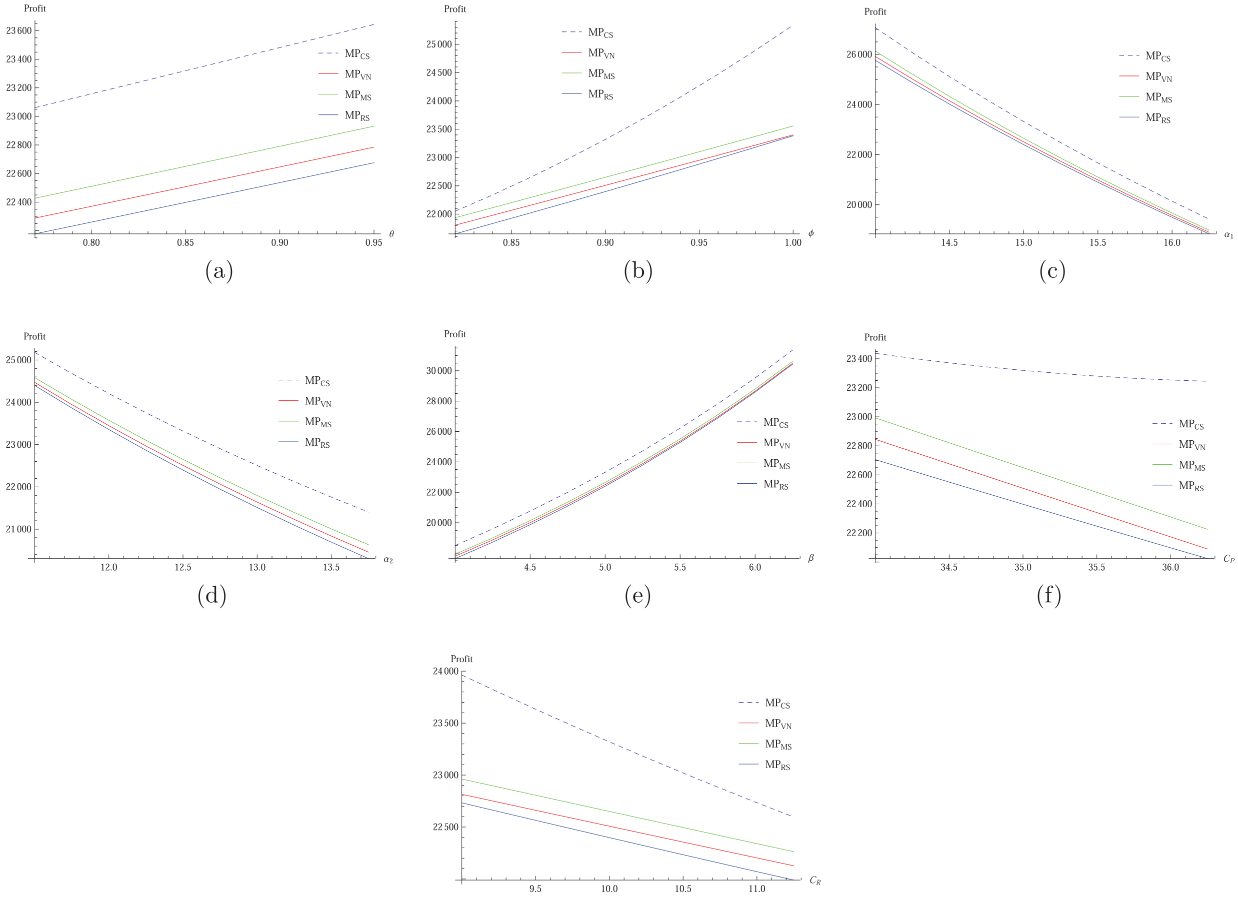
(g)

FIGURE 2. Effect of the parameters on the total Profit of the SCn. (a) Total profit vs θ. (b) Total profit vs ϕ. (c) Total profit vs α_1. (d) Total profit vs α_2. (e) Total profit vs β. (f) Total profit vs C_P. (g) Total profit vs C_R.

5.1.2. Analysis of the price sensitivity parameters α_1, α_2 and β

The impact of the price sensitivity parameters α_1, α_2, and β on the profits are shown in Figures 2c, 2d, 2e, 3c, 3d, 3e, 4c, 4d, and 4e. In all the four scenarios, we find that increasing α_1 decreases all profits except the RR, whether all decrease with increasing α_2 and decreasing β. Again, Table 7 shows that when α_1 and α_2 increase, all the SPrs fall off, but they boost with bigger β. Here, p_r has no such change with the variation of the parameters except only DC_{RS} case. The high value of α_1 must reduce the SPr of the MR since his demand is directly affected by the change. But, that situation helps the RR to increase his return. The effect of the variation of α_2 falls on both profits and price since, besides the RR, the MR also suffers indirectly for the RR's reduced demand.

Managerial insights. Since demand is a very significant component of the business and the variation of α_1, α_2, and β have a direct impact on it, the organization should very much care about those parameters. The physical interpretation of α_1 and α_2 is the proportion ratio of losing demand with the increment of their respective SPr, whether β signifies their market growth ratio with the increase of the rival's market price. If α_1 and α_2 increase, the players must reduce their SPr to adjust their respective demands because no one wants to lose their market.



(a) (b) (c)

(d) (e) (f)

(g)

FIGURE 3. Effect of the parameters on the manufacturer's Profit of the SCn. (a) Manufacturer profit vs θ. (b) Manufacturer profit vs ϕ. (c) Manufacturer profit vs α_1. (d) Manufacturer profit vs α_2. (e) Manufacturer profit vs β. (f) Manufacturer profit vs C_P. (g) Manufacturer profit vs C_R.

In the model, the MR's profit is more affected than the RR because he supplies total demand to the customers through his channel and indirectly to the traditional channel. Again, all profits will increase when β increases since it has a positive impact on the members. Therefore, members should make goodwill with the customers and their opponents to save or enlarge the market since their business reputation can control the variation of α_1, α_2, and β for their benefits.

5.1.3. *Analysis of the production and remanufacturing cost C_P and C_R*

Figures 2f, 2g, 3f, 3g, 4f, and 4g show that the profits of the members and total SCn decrease with the increment of C_P and C_R except for the RR's in the CS case. In those situations, the decision variables p_1 and p_2 for all the scenarios increase with C_P and C_R. But the variable p_r changes oppositely with C_P and C_R in the cases CS and DC_{RS}. Increasing production and remanufactured cost always enhance the MR's expenditure. As a result, he must increase the wholesale and SPr to reduce his loss, and the high wholesale price enhances the RR's price. Again, a higher value of C_P must enforce to collect more returned materials, whether the greater

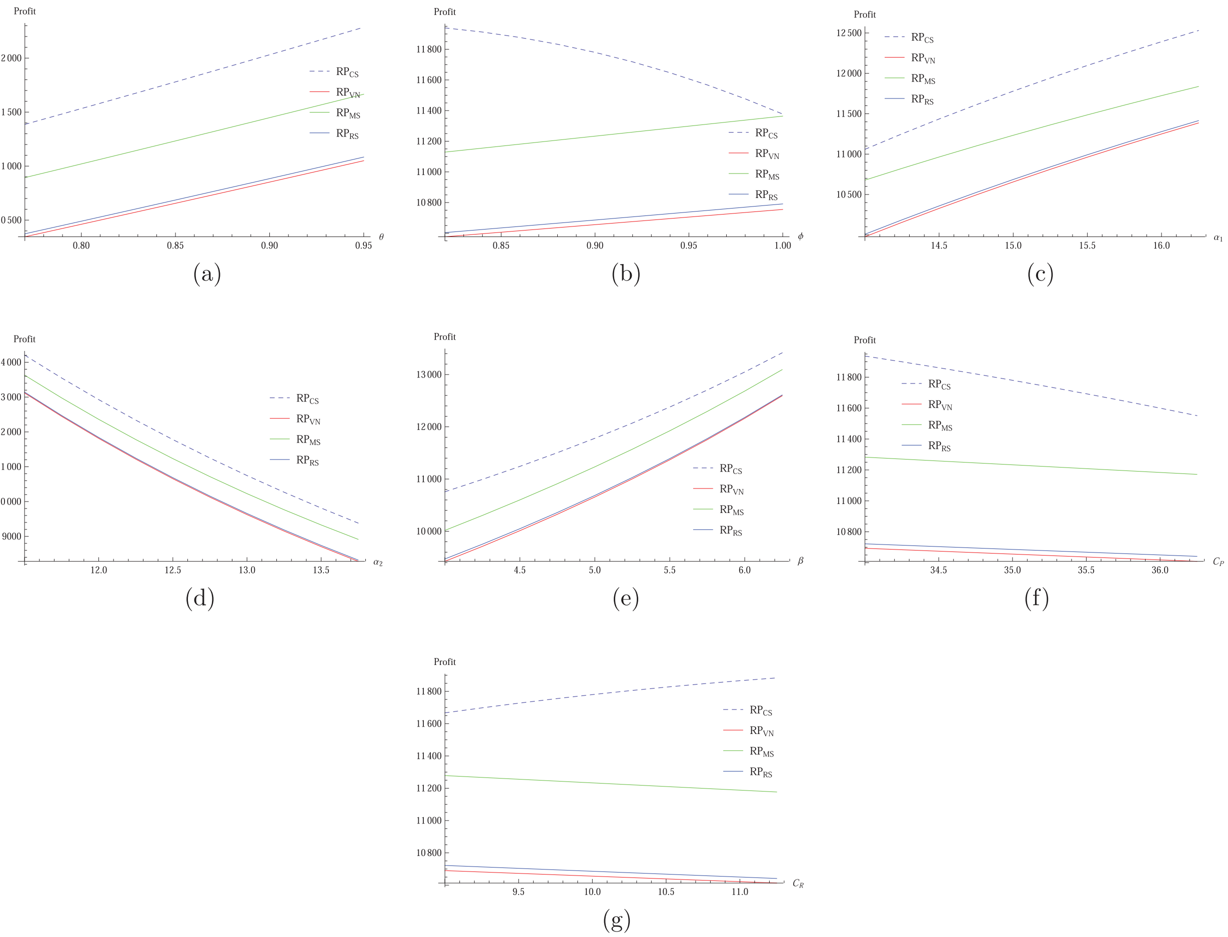


Figure 4. Effect of the parameters on the retailer's Profit of the SCn. (a) Retailer profit vs θ. (b) Retailer profit vs ϕ. (c) Retailer profit vs α_1. (d) Retailer profit vs α_2. (e) Retailer profit vs β. (f) Retailer profit vs C_P. (g) Retailer profit vs C_R.

C_R suppresses that collection. The RR has gotten more benefit for both the situations since both the cases directly link with the MR's cost.

Managerial insights. C_P and C_R are two main cost components of the MR, and the value of C_P is much higher than C_R. These two can influence the SPr of the products with their increased value. Since increased own channel price always decreases the market demand, the member should synthesize policy to reduce the total cost. Analyzing the model, when C_P increases, the members should pay attention to enhance the $CWtR$ such that they can reduce the total cost by re-manufacturing more recycled products. Again, if C_R increases, the player should control the returned items' collection and maintain their production process from the fresh and recycled materials such that the total cost will be less.

6. Conclusion

The research work has exhibited a forward and reverse SCn system with collected returned-obsolete materials for remanufacturing and the manufactured products for sale. Managing the challenges of the formulation of

CDSC, the two-ways SCn has been expanded with a MR and RR. Here, the $CWtR$ items, which can explore at an optimum level by varying the rewards for the products offered by the RR, set the products' collection rate. Furthermore, the returned items, which qualify the standard level for both the members, are remanufactured, and the rest are demolished. To satisfy the total market demand, completing all remanufacturing, the MR produces the remaining products by using fresh raw-materials. We have considered two competitive channels to sell the products, where the MR handles the direct channel, and the RR sells through the traditional channel. A CDSC under three decentralized situations, DS_{MS}, DS_{RS}, DS_{VN}, and one centralized system was investigated, and the optimal analytical solution was found for each case. The impact of screening ratios, price sensitivity parameters, and production and remanufacturing cost parameters on profits and decision variables has been extensively examined. The results indicate that the DS_{MS} model will be the best choice, and DS_{VN} will be the worst choice within the decentralized scenarios for the members respective.

The presented article has some limitations. The model has been formulated for one product, one manufacturer, and one retailer. Therefore, this can be extended to omitting those restrictions. This study considers deterministic linear demand functions. Future studies may include more demand functions by incorporating various random distributions. Introducing some other factors than retailer's offered reward, which can increase the returned rate, will be an extension of the model. Advertising, service effort, etc., are not considered in the paper. These could also be included to expand the market demand. In addition to these policies, a warranty and discount policy may be included in future work to increase customer satisfaction. The study assumes that all channel members are risk-neutral, so it could be extended to consider cases where the manufacturer or retailer is a risk-averse decision maker.

Acknowledgements. The author would like to express their gratitude to the editors and referees for their valuable suggestions and corrections to enhance the clarity of the present article. The author also sincerely acknowledge the support received from DST FIST programme (File No.SR/FST/MSII/2017/10(C)).

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