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EXPLOITING PERFORMANCE ANALYSIS OF REDUNDANT SYSTEM (KM+1S)

İNCORPORATING FAULT COVERAGE AND REBOOT DELAY

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Abstract. The present investigation seeks the steady state availability, reliability and mean time to failure of the 1-out-of (k+1): G redundant system. A system having k active units (kM) and one unit (1S) as a warm standby redundancy have considered. The effects of various parameters on reliability measures have been analyzed by deriving two models. Model I is designed as a reliability model and Model II for steady state availability. The assumptions have been made that the detected faults in the redundant system is covered imperfectly. The Markov process, supplementary variable technique, Laplace transformation are adopted to determine the transient behavior of the system. Presented results based on numerical data to demonstrate the practical utilization of the developed models. This study is very helpful for the engineers to design a highly reliable redundant system with high profit in the industry.

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Highlights

- To propose reliability model and steady state availability model of a redundant system (kM+1S) having k active units (kM) and one warm standby unit (1S).
- Discuss some performance measures such as reliability, mean time to failure, availability, reboot probability, recovery probability and failure frequency with 1-out-of (k+1): G system configuration.
- To enhance the system performance, incorporate Fault Coverage and Reboot Delay simultanteously.

1. Introduction

In each and every industry, multi-state systems have been used in practice or we can say that over the world, industry has dependency on the systems for their production, packaging, manufacturing etc., so these systems

Keywords. Markov modelling, warm redundancy, steady state availability, reliability, reboot delay.

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should be reliable. Reliability of the systems is a very big issue in system engineering. As time passes a system have more probability of failure, due to which industry have several types of losses such as loss of production, economic loss, goodwill etc. But a redundant system is more beneficial in context of system reliability. With the high maintenance and good repair facility system can be made highly available to avoid or reduce the losses during production. Although, most of the researchers in field of reliability theory have taken many techniques into consideration to improve the system reliability. Reliability enhancement is a challenging task for the engineers when they called upon to take the decision. According to the researchers, various repair or maintenance facility such as perfect or imperfect coverage factor, with warm/cold/hot redundancy, reboot the system, two types of repair facility, preventive maintenance etc. can be used to achieve high reliability.

In context of redundancy, a number of researchers [3, 8, 20, 25, 29, 32] analyzed the system's reliability with different types of redundancy. Yearout *et al.* [33] explained that how redundancy helps to improve the system reliability either by redesigning or reconfiguration. Cha *et al.* [7] proposed a system with cold, hot and warm redundancy and analyzed the system performance in terms of reliability and availability. They also discussed the reliability allocation problem for the standby system. A detailed explanation of optimal redundant system reliability has been given by Tillman *et al.* [27]. They discussed the various techniques such as integer programming, dynamic programming, geometric programming, sequential unconstrained minimization technique, modified sequential simplex pattern search, Lagrange multipliers and Kuhn-Tucker conditions, generalized Lagrangian function generalized reduced gradient (GRG), heuristic approaches etc. Ram *et al.* [24] modelled a two-unit system in which one is operative and second is in standby under the consideration of waiting time to repair. They analysed the reliability characteristic using supplementary variable techniques and Markov process. Dhillon and Yang [9] also analysed a two-unit system but they consider common-cause failure and human error. El-Said and El-Sherbeny [10] also proposed a two-unit system but focused on the cost analysis incorporating waiting time to repair and two stage of repair of failed unit using regenerative technique. Malhotra and Taneja [17] investigated the availability of the two-unit system with different demand policy using semi-Markov process and regenerative technique. Yuan and Meng [34] discussed the reliability of two-unit warm standby repairable system with one unreliable transfer switch. The working of the system is prior basis but failure of switch lead to the system breakdown. They resulted that the system reliability is sensitive with respect to the working time parameter. Barak *et al.* [5] and Malhotra *et al.* [18] has been discussed about the reliability measures of two-unit cold standby system using the Markov process and regenerative approach. Barak *et al.* introduced the inspection and refreshment facility to analyzed the model while Malhotra *et al.* examined the system with preventive and corrective maintenance. Barak *et al.* [5] described various factors of water supply system by considering this as a two-unit cold standby model. They assumed that the single facility is available for the inspection and repair. For the study of the model, they considered the negative exponential distribution for refreshment and repair while unit failure and server failure are distributed arbitrarily with distinct probability density functions (pdf).

Covering the faults in the repairable system after detection has a great importance in both the performance and economic impact. The concept of coverage has been discussed by number of scientists including Arnold [4]. Coverage factor is defined as the ratio of total detected faults to the faults that can be covered. Many researchers investigated the reliability of various systems using fault coverage technique to improve the performance incorporating various types of failures [1, 2, 16, 21, 30, 31]. Wang *et al.* [28] investigated the availability of two systems with warm standby units under the assumption that the coverage factor of the active unit failure is different from the standby unit failure. The authors have also given the comparative study of the availability of the system in which they compared the analysed availability with Gamma, normal, exponential, uniform and deterministic distribution of failure and repair time but the reliability and mean time to failure of the system can also be analysed. Ram *et al.* [24], Ram and Goyal [22] provided an approach for the improvement of reliability measures of a complex system and modelled by assuming two types of repair policies. The authors studied the effect of the coverage factor on the system reliability and availability. Manglik and Ram [19] investigated the reliability indices of a manufacturing system by introducing three types of failures and incorporated the fault coverage technique. The authors also discussed the expected profit, which is influenced by the service cost and the coverage factor.

Many repairable systems are degraded or not responding due to the age of the system or components or any software error for short period of time in which engineers are not able to detect the faults. At that time, engineers can reboot the system. During the reboot process, the system settings are automatically updated or refreshed and system back in operating condition without repair [11,13,14]. Most of the researchers analysed the multi-state system's reliability using reboot process and considered the coverage probability only for switching failure [15, 26]. Jain [12] proposed a multi component system and discussed some performance indices such as steady state availability, probability of recovery state, probability of rebooting the system and expected number of units for different distribution of repair time. To analyse these measures, they used imperfect fault coverage only for switching, common cause failure and system reboot.

In this paper, authors analysed some performance indices for the designed system (kM+1S) using coverage factor for any type of detected faults either switching failure, software error, component failure etc. In order to enhance the system performance, only to include redundancy or reboot methodology is not sufficient due to their own limitations. But in world of industry 4.0, there are big challenges. So, in this study, authors determined the reliability measures by considering fault coverage, reboot delay and warm redundancy simultaneously and also examined the effects of various parameters on it.

2. System description

The repairable system proposed in this work, consists of k main units (kM) and one standby unit (1S) as a warm redundancy and follows 1-out-of (k+1): G policy. The redundant system avoids the system failure due to waiting time to repair. When the system is in active or operational mode, repairman is always available and aim to detect the faults when failures occur partially or fully during the process of switching or component failure. When active unit fails then standby unit takes the charge immediately and failed unit sent for the repair, and still system is in good working condition. But during this process, the probability of switching failure exists. When the system has any error either switching failure (α) or unit failure (λ), repairman firstly seek to cover the detected faults. The detected faults can be covered perfectly or imperfectly with probability c and $(1-c)$ respectively. If the detected faults covered perfectly then it goes for the recovery otherwise it goes for reboot with mean time θ and β respectively. The system failed completely when all the units have been failed. The system has been divided into two parts as Model 1 for reliability analysis and Model II for availability analysis (for example as shown in Figs. 1 and 2). The status of the system (kM+1S) states are given below

States i	Status
k+2	Active
k	Recovery
k	Reboot

Furthermore, following assumptions have been taken for the considered system.

(1) Initially, the system is in good working condition.
(2) Detected faults are covered perfectly with probability c or imperfectly with probability $(1-c)$.
(3) Switch is changed perfectly or imperfectly, that means system can be failed or degraded due to switching.
(4) No vacation is allowed that means repairmen is available 24x7.
(5) Failure and recovery rate follows exponential distribution and repair time μ is distributed generally.
(6) After repair, system works like a new one.

3. System modelling

As mention above, the system is divided into two models: Model I (For Reliability), Model II (For Availability).

3.1. Model I (For Reliability)

To predict the behavior of reliability of the proposed system, reliability model possesses following difference differential equations by applying Markov process in the form Laplace transformation. The Laplace transform is used to solve differential equations by converting it into simple algebraic expressions that makes it easier. Also, being an altered and efficient substitute to variation of parameters and undetermined coefficients, the Laplace transformation is significantly beneficial for input expressions that are piecewise-defined, periodic or impulsive.

The Laplace transform of a function $f(t)$ is defined as

$$\int_0^\infty f(t)e^{-st}\,\mathrm{d}t \quad \text{for} \quad 0 \le t<\infty.$$

The Laplace Transform is a generalized Fourier Transform, since it allows one to obtain the function transform that have no Fourier Transform. It is used to solve initial value problem.

$$(s+k\lambda+\alpha)\overline{P}_0(s) = \mu\overline{P}_1(s)+1 \tag{3.1}$$

$$(s+k\lambda+\mu)\overline{P}_1(s) = \alpha\overline{P}_0(s)+\mu\overline{P}_2(s)+\theta\overline{P}_{R1}(s) \tag{3.2}$$

$$(s+(k-i)\lambda+\mu)\overline{P}_{i+1}(s) = \mu\overline{P}_{i+2}(s)+\theta\overline{P}_{R(i+1)}(s); \qquad i=1,\,2,\ldots,k-2 \tag{3.3}$$

$$(s+\lambda+\mu)\overline{P}_k(s) = \theta\overline{P}_{Rk}(s) \tag{3.4}$$

$$s\overline{P}_{k+1}(s) = \lambda\overline{P}_k(s). \tag{3.5}$$

Recovery states

$$(s+\theta)\overline{P}_{R1}(s) = k\lambda c\overline{P}_0(s) \tag{3.6}$$

$$(s+\theta)\overline{P}_{Ri+2}(s) = (k-i)\lambda c\overline{P}_{i+1}(s); \qquad i=0,\,1,\,2,\ldots,k-2. \tag{3.7}$$

Reboot states

$$s\overline{P}_{\mathrm{RB1}}(s) = k(1-c)\lambda\overline{P}_0(s) \tag{3.8}$$

$$s\overline{P}_{\mathrm{RB}i+2}(s) = (k-i)(1-c)\lambda\overline{P}_{i+1}(s); \qquad i=0,\,1,\,2,\,\ldots,\,k-2. \tag{3.9}$$

3.2. Model II (Availability)

To examined the steady state availability of the considered system, following equations have been possessed for Model II

$$(k\lambda+\alpha)P_0 = \mu P_1 \tag{3.10}$$

$$(k\lambda+\mu)P_1 = \theta P_{R1}+\beta P_{\mathrm{RB1}}+\mu P_2+\alpha P_0 \tag{3.11}$$

$$((k-i)\lambda+\mu)P_{i+1} = \theta P_{Ri+1}+\beta P_{\mathrm{RB}i+1}+\mu P_{i+2}; \qquad i=1,\,2,\ldots,k-1 \tag{3.12}$$

$$\mu P_{k+1} = \lambda P_k. \tag{3.13}$$

Recovery states

$$\theta P_{R1} = k\lambda cP_0 \tag{3.14}$$

$$\theta P_{Ri+2} = (k-i)\lambda cP_{i+1}; \qquad i=0,\,1,\,2,\ldots,k-2. \tag{3.15}$$

Reboot states

$$\beta P_{\mathrm{RB1}} = k(1-c)\lambda P_0 \tag{3.16}$$

$$\beta P_{\mathrm{RB}i+2} = (k-i)(1-c)\lambda P_{i+1}; \qquad i=0,\,1,\,2,\ldots,k-2. \tag{3.17}$$

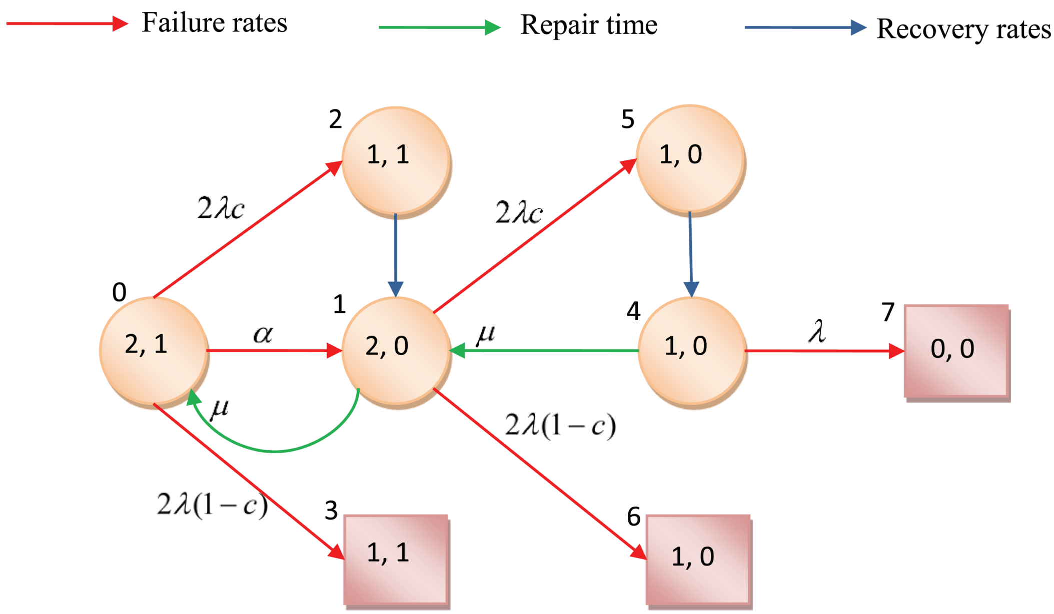

FIGURE 1. Reliability model of the (2, 1) redundant system.

3.3. Case study

For better understanding of these models, authors take an example of a system which consists of two main units (2M) and one standby unit (1S) as a warm redundancy.

3.3.1. Reliability model

To examine the reliability of the system as shown in Figure 1, following differential equations have been possessed

$$(s+2\lambda+\alpha)\overline{P}_0(s) = \mu\overline{P}_1(s)+1 \tag{3.18}$$
$$(s+2\lambda+\mu)\overline{P}_1(s) = \alpha\overline{P}_0(s)+\mu\overline{P}_4(s)+\theta\overline{P}_2(s) \tag{3.19}$$
$$(s+\theta)\overline{P}_2(s) = 2\lambda c\overline{P}_0(s) \tag{3.20}$$
$$s\overline{P}_3(s) = 2(1-c)\lambda\overline{P}_0(s) \tag{3.21}$$
$$(s+\lambda+\mu)\overline{P}_4(s) = \theta\overline{P}_5(s) \tag{3.22}$$
$$(s+\theta)\overline{P}_5(s) = 2\lambda c\overline{P}_1(s) \tag{3.23}$$
$$s\overline{P}_6(s) = 2(1-c)\lambda\overline{P}_1(s) \tag{3.24}$$
$$s\overline{P}_7(s) = \lambda\overline{P}_4(s). \tag{3.25}$$

State probabilities for the states notified in Figure 1 are derived as

$$\overline{P}_0(s) = \frac{1}{s+2\lambda+\alpha-\mu A_1} \tag{3.26}$$
$$\overline{P}_1(s) = \frac{\alpha+A_1\theta}{K_1(s)} \tag{3.27}$$
$$\overline{P}_2(s) = A_1\overline{P}_0(s) \tag{3.28}$$

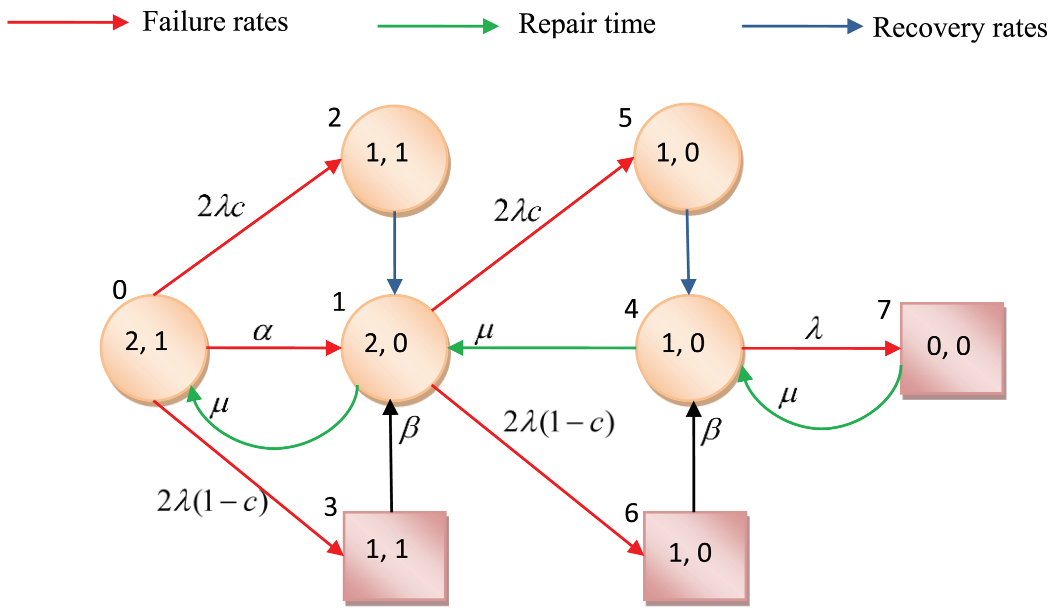

FIGURE 2. Availability model of the $(2, 1)$ redundant system.

$$\overline{P}_3(s) = A_3\overline{P}_0(s) \tag{3.29}$$

$$\overline{P}_4(s) = A_1A_2\left(\frac{\alpha + A_1\theta}{K_1(s)}\right)\overline{P}_0(s) \tag{3.30}$$

$$\overline{P}_5(s) = A_1\left(\frac{\alpha + A_1\theta}{K_1(s)}\right)\overline{P}_0(s) \tag{3.31}$$

$$\overline{P}_6(s) = A_3\left(\frac{\alpha + A_1\theta}{K_1(s)}\right)\overline{P}_0(s) \tag{3.32}$$

$$\overline{P}_7(s) = \frac{\lambda}{s}A_1A_2\left(\frac{\alpha + A_1\theta}{K_1(s)}\right)\overline{P}_0(s) \tag{3.33}$$

where,

$$A_1 = \frac{2\lambda c}{s+\theta}, \quad A_2 = \frac{\theta}{s+\lambda+\mu}, \quad A_3 = \frac{2(1-c)\lambda}{s}, \quad K_1(s) = s + 2\lambda + \mu - A_1A_2\mu.$$

3.3.2. Availability model

The following differential equations have been derived to analyze the steady state availability of the designed system as shown in Figure 2

$$(2\lambda + \alpha)P_0 = \mu P_1 \tag{3.34}$$

$$(2\lambda + \mu)P_1 = \theta P_2 + \beta P_3 + \mu P_4 + \alpha P_0 \tag{3.35}$$

$$\theta P_2 = 2\lambda c P_0 \tag{3.36}$$

$$\beta P_3 = 2(1-c)\lambda P_0 \tag{3.37}$$

$$(\lambda + \mu)P_4 = \theta P_5 + \beta P_6 + \mu P_7 \tag{3.38}$$

$$\theta P_5 = 2\lambda c P_1 \tag{3.39}$$

TABLE 1. Reliability of the system.

	t	$c = 0.7$	$c = 0.8$	$c = 0.9$
$\theta = 0.3$	0	0.99567	0.99684	0.99771
	1	0.93940	0.95849	0.97739
	2	0.88883	0.92352	0.95874
	3	0.83903	0.88820	0.93952
	4	0.78713	0.85031	0.91838
	5	0.73142	0.80844	0.89440
	6	0.67067	0.76151	0.86683
	7	0.60397	0.70854	0.83493
	8	0.53045	0.64861	0.79792
	9	0.44931	0.58069	0.75495
	10	0.35969	0.50370	0.70504
$\theta = 0.6$	0	1.00000	0.84427	0.94763
	1	0.99234	0.80040	0.92130
	2	0.98368	0.76293	0.89970
	3	0.97832	0.72775	0.87958
	4	0.94130	0.69323	0.85963
	5	0.89533	0.65871	0.83931
	6	0.85008	0.62392	0.81841
	7	0.80541	0.58877	0.79682
	8	0.76128	0.55321	0.77450
	9	0.71767	0.51722	0.75140
	10	0.67457	0.48081	0.72749

$$\beta P_6 = 2(1-c)\lambda P_1 \tag{3.40}$$

$$\mu P_7 = \lambda P_4. \tag{3.41}$$

State probabilities of the states notified in Figure 2 are expressed as

$$P_0 = \frac{1}{K_2} \tag{3.42}$$

$$P_i = \frac{B_i}{K_2}; \qquad i = 1,\ 2,\ 3 \tag{3.43}$$

$$P_4 = \frac{2B_1B_4}{K_2} \tag{3.44}$$

$$P_5 = \frac{B_1B_2}{K_2} \tag{3.45}$$

$$P_6 = \frac{B_1B_3}{K_2} \tag{3.46}$$

$$P_7 = \frac{2B_1B_4^2}{K_2} \tag{3.47}$$

where,

$$B_1 = \frac{2\lambda+\alpha}{\mu}, \quad B_2 = \frac{2\lambda c}{\theta}, \quad B_3 = \frac{2\lambda(1-c)}{\beta}, \quad B_4 = \frac{\lambda}{\mu},$$
$$K_2 = (1+B_1+B_2)(1+B_3) + 2B_3B_4(1+B_4).$$

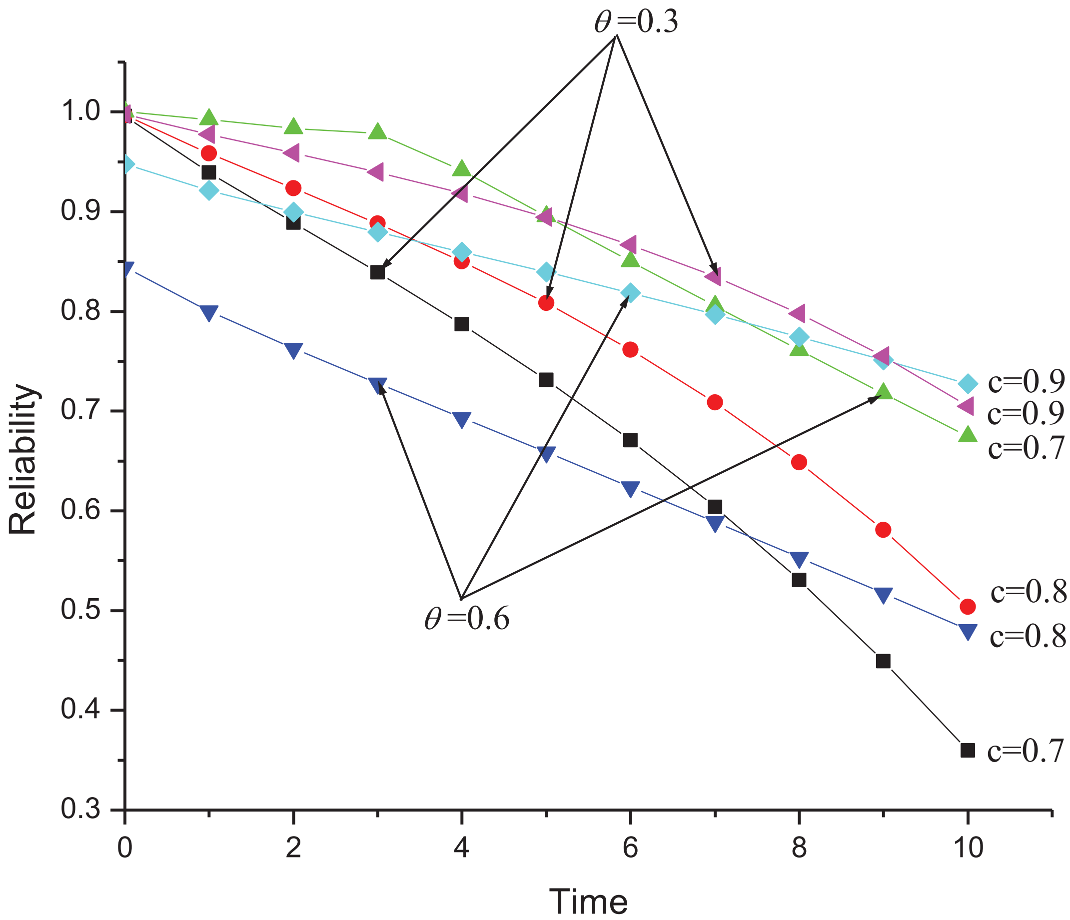

FIGURE 3. Reliability behavior w.r.t. various parameters.

4. Performance indices and numerical analysis

To explore system behavior, some performance indices, reliability, mean time to failure (MTTF), steady state availability and some other measures such as recovery state probability, reboot state probability, failure frequency have been investigated.

4.1. Reliability study

Reliability is defined as the probability of the upstate of the system. In Figure 1, reboot state 3 and 6, and state 7 are the failed state of the system at or before time t. So, reliability of the designed system can be expressed as

$$R(t) = 1 - \sum_{i=3,6,7} P_i(t) \tag{4.1}$$

where, $P_i(t)$ is the inverse Laplace transformation of $\overline{P}_0(s)$ for $t \geq 0$. By setting the value of various parameter as $\lambda = 0.1$, $\mu = 1$, $\alpha = 0.05$ and scrutinize the effects of coverage factor, recovery rate ant failure time as calculated in Table 1 and demonstrated graphically in Figure 3.

TABLE 2. MTTF of the system.

	μ	$c = 0.7$	$c = 0.8$	$c = 0.9$
$\theta = 0.3$	1	11.98148	13.83340	15.93105
	2	11.23314	12.72721	14.19635
	3	16.42064	20.81900	26.65990
	4	42.14022	99.08373	896.42505
	5	568.41470	428.91072	29.05124
	6	172.46737	23.34520	5.41660
	7	26.69539	7.51126	2.18467
	8	10.58847	3.73689	1.19424
	9	5.76852	2.27767	0.76803
	10	3.69200	1.56041	0.54557
$\theta = 0.6$	1	366.66100	262.35797	16.22311
	2	2.44589	1.01712	0.35516
	3	0.66894	0.32165	0.12332
	4	0.33748	0.17118	0.06821
	5	0.21450	0.11290	0.04562
	6	0.15360	0.08170	0.03376
	7	0.11818	0.06369	0.02266
	8	0.09537	0.05190	0.02183
	9	0.07960	0.04364	0.01847
	10	0.06811	0.03757	0.01598

4.2. MTTF study

Let MTTF be the mean time to failure of the system. The MTTF of the system can be deliberated as

$$\mathrm{MTTF} = \int_0^\infty R(t)\,\mathrm{d}t. \tag{4.2}$$

Using the reliability function as computed in Section 4.1 and putting the value of various parameters as $\lambda = 0.1$, $\alpha = 0.05$, examine the effects of various parameters on MTTF as given in Table 2 and depict its behaviour as demonstrated in Figure 4.

4.3. Availability study

The steady state availability of the designed system is the long run availability. It is the probability of the system in operational stage and under maintenance at any random time. It can be expressed as

$$\begin{aligned} A(\infty) &= P_0 + P_1 + P_2 + P_4 + P_5 \\ &= \frac{1 + B_1 + B_2 + 2B_1B_4 + B_1B_2}{K_2}. \end{aligned} \tag{4.3}$$

By setting the value of various parameter as $\lambda = 0.1$, $\beta = 1$, $\alpha = 0.05$ and the steady state availability as calculated in Table 3. The comparative study of availability with respect to coverage factor and recovery rate is demonstrated graphically in Figure 5 and also scrutinize the effects of recovery rate and reboot time as shown in Figure 6. In Figure 6, b stands for reboot delay, and r stands for recovery rate.

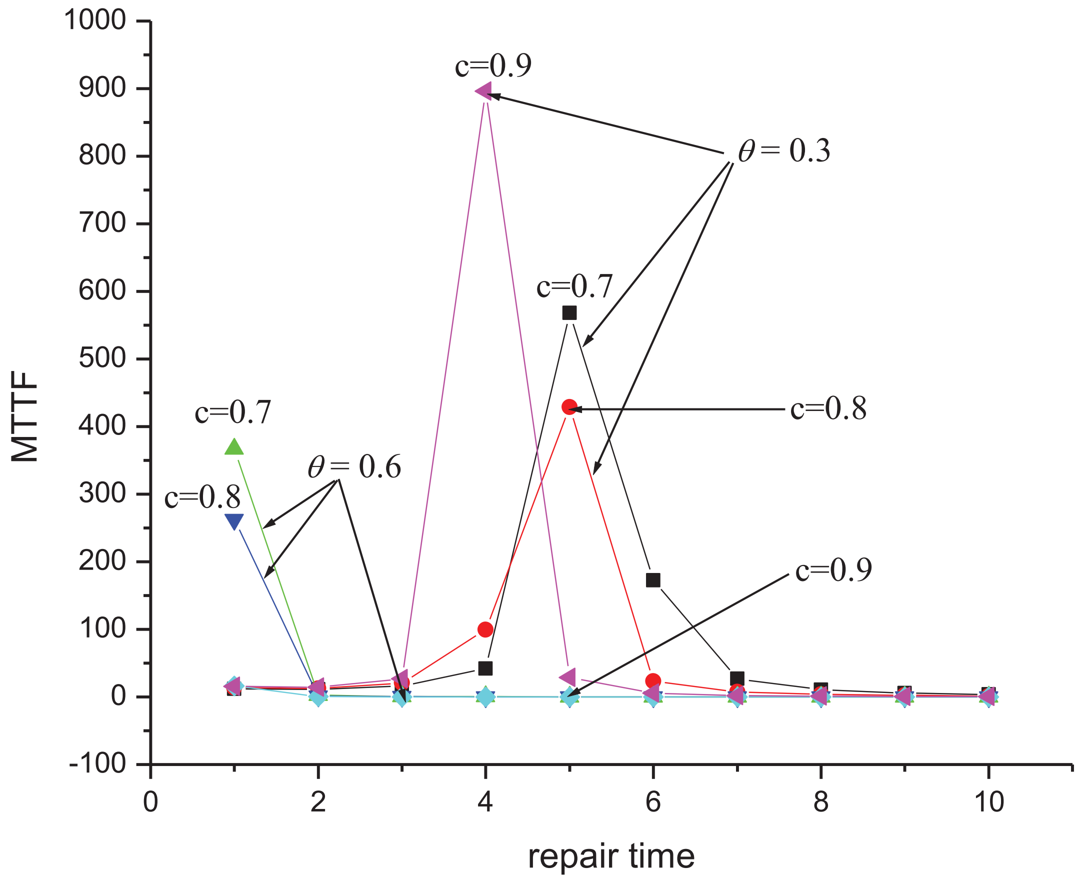

FIGURE 4. Graph of MTTF w.r.t. various parameters.

Some other performance indices

– **Recovery state probability**

The probability that the system is in recovery state, is computed by the Equation given as

$$\begin{aligned} P_{\mathrm{RC}} &= P_2 + P_5 \\ &= \frac{B_2}{K_2}(1 + B_1). \end{aligned} \tag{4.4}$$

– **Reboot state probability**

The probability that the system is in reboot state, is computed by the Equation given as

$$\begin{aligned} P_{\mathrm{RB}} &= P_3 + P_6 \\ &= \frac{B_3}{K_2}(1 + B_1). \end{aligned} \tag{4.5}$$

– **Failure frequency**

Failure frequency of the system after long run is described as

$$F_F = (2\lambda + \alpha)P_0 + 2\lambda P_1 + \lambda P_4. \tag{4.6}$$

TABLE 3. Steady state availability of the system.

	μ	$c = 0.7$	$c = 0.8$	$c = 0.9$
$\theta = 0.3$	1	0.95925	0.97279	0.98558
	2	0.96063	0.97441	0.98740
	3	0.96072	0.97455	0.98759
	4	0.96073	0.97457	0.98763
	5	0.96072	0.97458	0.98764
	6	0.96072	0.97458	0.98765
	7	0.96071	0.97458	0.98765
	8	0.96071	0.97458	0.98765
	9	0.96071	0.97458	0.98765
	10	0.96071	0.97458	0.98765
$\theta = 0.6$	1	0.95214	0.96742	0.98240
	2	0.95360	0.96924	0.98456
	3	0.95366	0.96938	0.98478
	4	0.95366	0.96940	0.98483
	5	0.95365	0.96940	0.98484
	6	0.95364	0.96940	0.98485
	7	0.95363	0.96940	0.98485
	8	0.95363	0.96940	0.98485
	9	0.95362	0.96940	0.98485
	10	0.95362	0.96940	0.98485

The effects of recovery rate, reboot delay and coverage factor on failure frequnency of the system are explained by the graphs shown in Figure 7. In Figure 7, b stands for reboot delay, c stands for coverage factor and r stands for recovery rate.

5. Result discussion

Reliability applications in industries faced many issues based on complexity in design. Various technologies are used to analyze the reliability characteristics that addresses some issues fixed by expertise and try to enhance the system performance. The computation tractability of performance indices are explored to validate and utilize the obtained results by numerical experiments considered for this study. In this section, the effects of various parameters on some performance measures are determined and consider the default value of parameters as $\lambda = 0.1$, $\beta = 1$, $\alpha = 0.05$, $c = 0.7, 0.8, 0.9$, $\theta = 0.3, 0.6$ and consider the time span 1 to 10 with equal intervals.

Table 1 and corresponding Figure 3 shows the trend of reliability of the system with respect to failure time. From the critical study of graphs presented in Figure 3 enlightens that the system's reliability decreases as failure time increases. The recovery rate and coverage factor both have the significant effect on reliability. One can see that, when recovery rate is 0.3, the reliability of the system increases as coverage factor increases with the increment in time. When recovery rate 60%, and coverage factor c equals to 0.7 give the high reliability between 0 and 5 units of time while between 5 and 9 units of failure time, system is highly reliable at recovery rate 30% and coverage factor 0.9 after that system is highly reliable when recovery rate 60% and coverage factor 0.9. When recovery rate 0.6 and coverage factor 0.8, system is least reliable. From the overall study of Figure 3, one can conclude that the balance between coverage factor and recovery rate is required because at high recovery rate with high coverage factor, system reliability decreases initially. Therefore, simulation results provide that system achieve best reliability with high recovery rate and low coverage factor otherwise high coverage factor and low recovery rate.

Graphical representation of MTTF analysis of designed system as arranged in Table 2 is given in Figure 4. In this figure, the effects of recovery rate and coverage factor on system's MTTF are predicted. From the critical

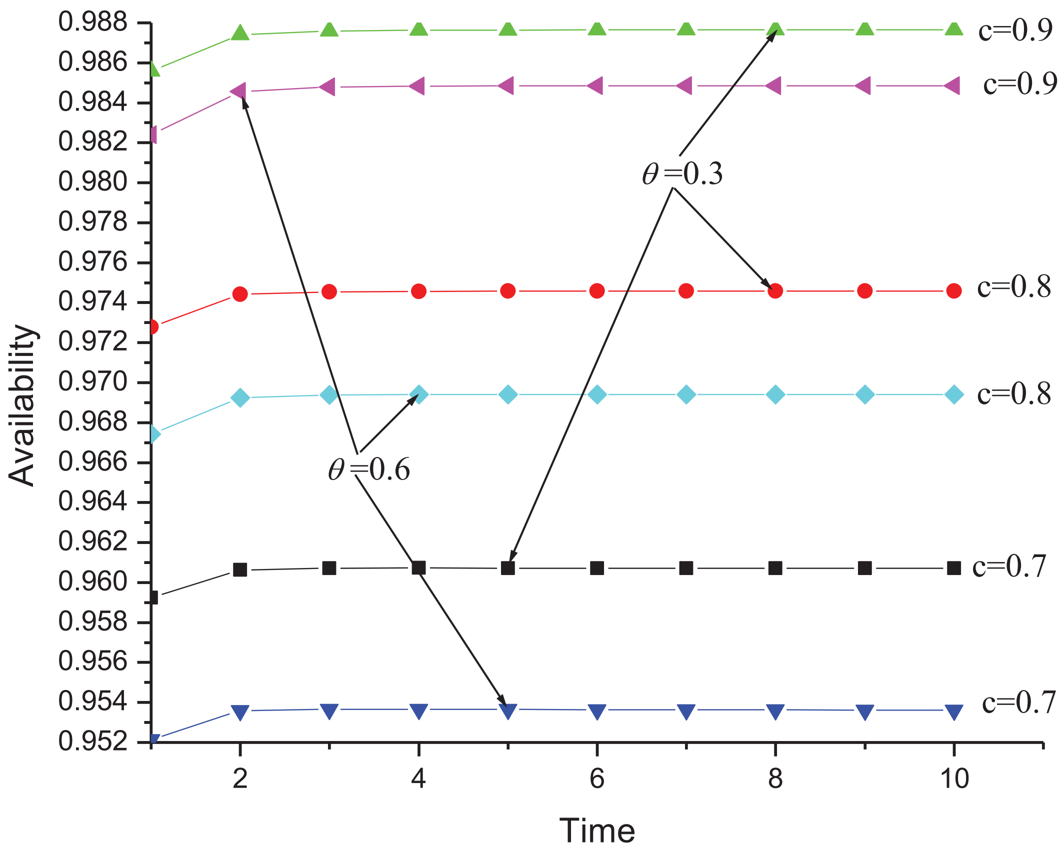

Figure 5. Availability of the system.

study of the graphs, one can see that at recovery rate 30%, the MTTF of the system first decreases very slowly as time increases but after a short period of time it will increase straightly and again after a fix period, it will decrease rapidly and after a short period of time, it will be almost constant. At recovery rate 60%, and coverage factor 0.8 and 0.7, MTTF of the system decreases straightly during 0 to 2 units of time, and after that, it will be constant. While at recovery rate 60% and coverage factor 0.9, MTTF of the system decreases very slightly initially and after some time it will be constant. As retried above, balance between recovery rate and coverage factor is necessary. Take these factors in reciprocal order *i.e.* if one is high then other will be low.

The effects of recovery rate and coverage factor on system's availability are predicted by the authors in Table 3, and are demonstrated graphically in Figure 5. From the critical examination of Figure 5, one can see that the availability of the system increases straightly for first short period and after that it is constant. For any value of recovery rate and coverage factor, the availability of the designed system has same discipline. As coverage factor increases, the system's availability decreases while as recovery rate increases, system's availability decreases at fix value of coverage factor. From the examination of Figure 6, one obtained that the system availability increases smoothly with the increment in reboot time and after a fix time, it will be constant. The system is highly available when coverage factor $c = 0.9$ and recovery rate is 30%. At coverage factor $c = 0.7$ and recovery rate 60%, the system is least available.

To predict the frequency in system failure, ones obtained Figure 7 in which the effects of various parameters such as reboot delay, recovery rate and coverage factor are analyzed. From the critical study of graphs, it is

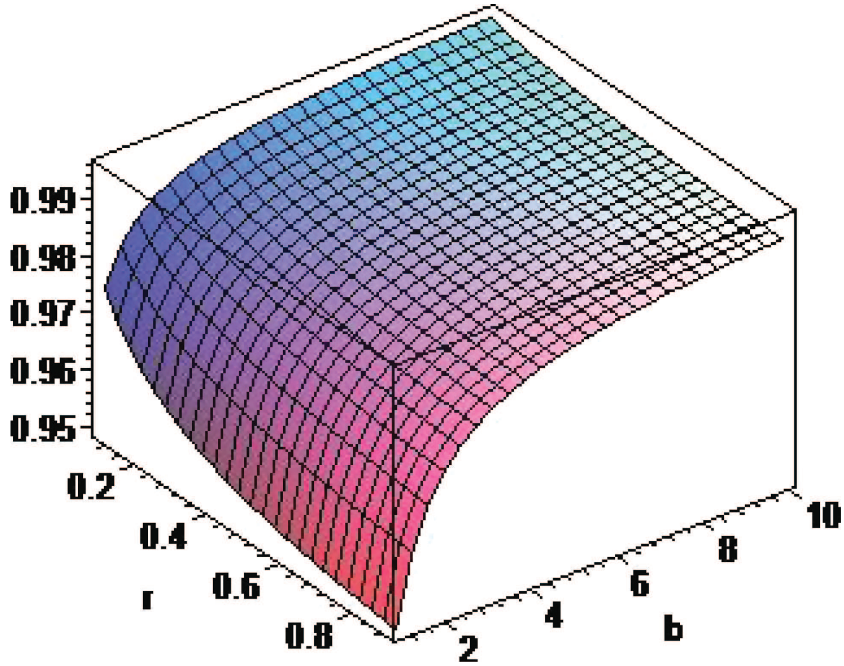

FIGURE 6. Effects of recovery rate and reboot time.

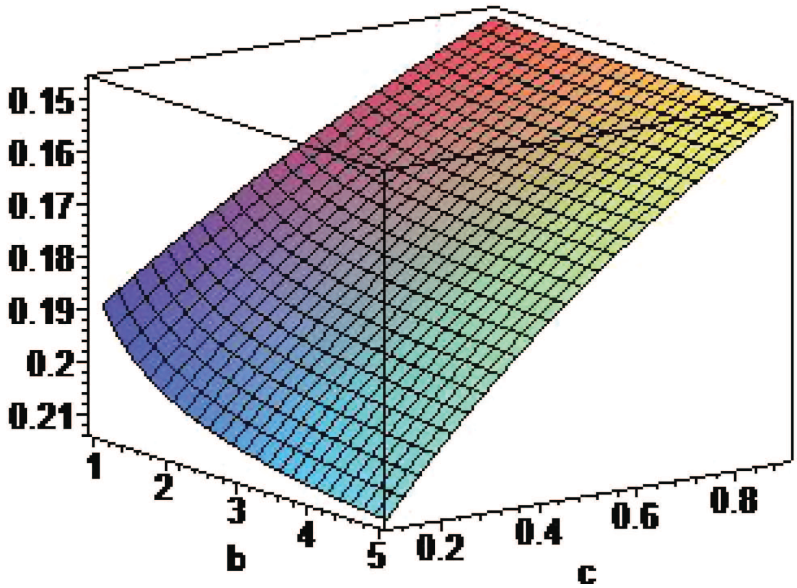

FIGURE 7. (a) Effects of coverage factor and reboot delay. (b) Effects of coverage factor and recovery rate.

concluded that failure frequency increases as recovery rate and reboot time increases but it decreases when coverage factor increases.

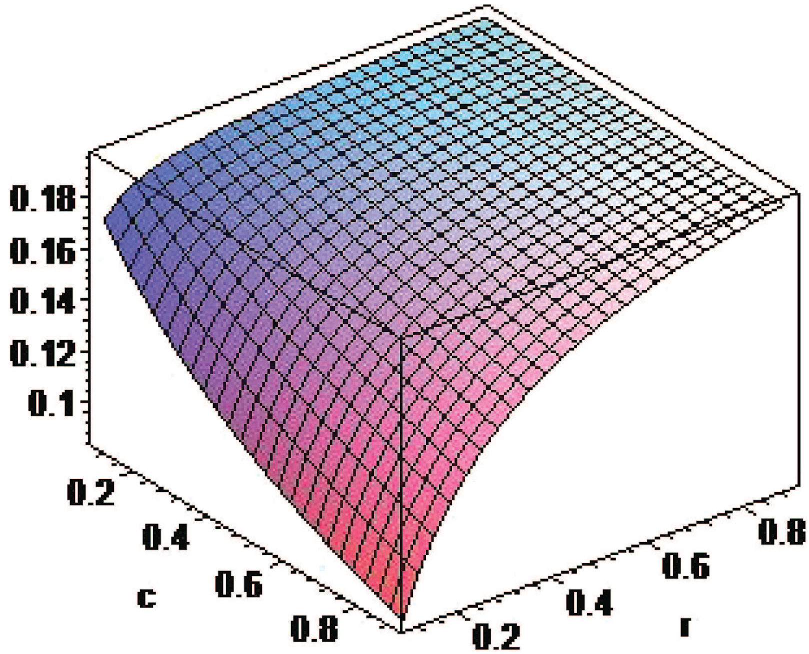

FIGURE 7. contined.

6. CONCLUSION

This research focused on some performance measures such as reliability, mean time to failure, availability, reboot probability, recovery probability and failure frequency of the 1-out-of (k+1): G system. Although, in the field of reliability, many relevant issues include: (1) Jain and Kumar [13], Barak *et al.* [6] obtained the reliability measures only for two-unit system (one main unit and one standby unit). (2) Imperfect coverage only for switching failure [14, 34].

Performance measures have great impact on the durability of the system in the industry and effect economically also. To predict the better analysis of these measures, authors discussed the effects of various parameters on these as retried. Recovery rate and fault coverage technique have the foremost concern in system reliability and maintenance.

Through the overall study of designed system, authors explained the research by considering a system which consists of two active units and 1 standby unit (2M+1S), authors have concluded that the reliability of the system decreases as time increases; and steady state availability of the system increases during initial period and after that, it will be constant. In general, reliability and availability of the system decreases as time increases but from this research work, we obtained that using fault coverage techniques and various maintenance concept such as redundancy, recovery and reboot simultaneously, reliability and availability of the system increases. Subsequently, it is concluded that the correct balance in between recovery rate and coverage factor is very much important and it should be monitored carefully during the system operation while both the parameters help to enhance the system performance. To get the more clarification about these factors, further we discussed the failure frequency which provides us a significant result that to achieve the better availability of the system that helps to increase the coverage factor.

Every industry wants highly reliable system and good maintenance approach that can reduce the failure frequency to attain the optimal production with optimal cost. These results are very advantageous for the system designers and engineers to support the industry in order to achieve their aim. The custom of this work enables the application of reliability theory with a high level of assurance. The present study can be sustained further to explore the mean time to repair, maintenance cost of various approaches used for reliability enhancement and expected profit of the overall system. Markov decision process can be used to take the decision about the optimal maintenance facility and components selection in future study.

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