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OPTIMAL PRODUCTION STRATEGIES OF COMPETITIVE FIRMS CONSIDERING PRODUCT INNOVATION

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Abstract. Consumer preference for product innovation/functionality has become increasingly diverse, therefore firms produce products with distinct versions/generations to satisfy consumers. This paper investigates the decision-making problem for multiple competitive firms considering consumers' diversified preferences for product functionality. This paper develops an optimization model, in which the profit maximizing firms need to determine the production quantities of their products with different versions. Due to our model's computational complexity, it motivates us to adopt variational inequalities theory, which is applied to convert an original model into a new variational inequality problem. On this basis, the existence and uniqueness of an equilibrium solution are proved, and a high-efficient Euler algorithm is proposed. A case study focused on the 5G smartphone market is conducted. Numerical results show that firms may obtain more profits by producing products with newer versions if the consumer preference for product functionality is high. However, if the preference level of consumer is under a certain level, it is not necessarily beneficial for firms who launch new versions of their products to the demand market. In addition, when the competition in market becomes intensive (due to new entrants), giving up the production of previous-version products may be more conducive to existing firms.

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1. Introduction

Many consumers are willing to pay a higher price to purchase the newest version of products. In general, product with a new version is technologically superior to an old version [49]. An increase of consumers believes that the reasons why they buy new products are more functionalities and additional attributes, especially regarding electronics industry and auto industry [20]. According to a survey from Counterpoint Technology Market Research, a majority of consumers indicated positive interest in purchasing a new-generation 5G smartphone and about 25% of them are willing to spend 20% more to upgrade to a 5G smartphone compared with a 4G model [32]. For another instance, Oppo, one of the world's leading smart device innovators, launched the model of Find X for around $699 in 2018. Its latest models, Find X2-series, armed with four cameras, Super VOOC

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2.0 Flash Charge technology for resilient battery life and other attracting extras has a starting price of $1200. In the first quarter of 2020, however, the sale of Oppo registered a growth of 67% year-on-year [1].

From the perspective of manufacturers, staying ahead of the technology curve is critical for a company to remain competitive and work towards the future. The increasing competitiveness and the constantly accelerating pace of technological progress require that firms allocate increasing resources to Research and Development (R&D), and thus accelerate and diversify their technological abilities [28]. Many firms invest in R&D because they want to survive and grow by developing new products and services [16,59]. According to CRS Report, total global R&D expenditures have more than tripled in current dollars, from $676 billion in 2000 to $2.0 trillion in 2018 [5]. As a global leader in R&D investments, United States spent $581.4 billion in R&D in 2018, which represents over 2.7% of America's GDP and 71% of it is from private business R&D investments [39]. Samsung, Alphabet, Volkswagen, Microsoft and Huawei rounded out the top five of companies with the highest R&D spending in 2020 [4].

Even though the newest-generation products attract consumers and might generate a fairly large demand, meanwhile, firms also continue to produce previous versions [23]. One of the primary reasons is that consumers have different preferences for product functionalities and prices. The diversified preferences lead to an increase of overall market demands while influencing the resource allocation, production and pricing strategy among each model of product. For instance, according to the consumer preference analysis, Nuna, a premium baby gear brand, found more modern parents favor a stroller with compact folded size for a smoother and safer ride. After launching the newest model of stroller TAVO Next with an innovative new buckle, upgraded material to make it lighter and more compact when folded in December 2021, Nuna has discontinued its previous model, TAVO. The Nuna TAVO Next costs $100 more than the TAVO because of its added features and more premium look satisfying more consumers' needs. Another example, when Apple released its iPhone X model, which is the expensive model at that time, the sales volume is modest in most of the emerging markets, in which older-version iPhone models keep more popular. Therefore, when firms provide products with a variety of versions, determining production quantity and price of each version considering consumer preference is significant and complex in practice. Motivated by the interesting phenomena, we try to address and solve the following research questions:

(1) For an innovative firm, what is the optimal production quantity of each model of product under the competitive market?
(2) For an innovative firm, what is the optimal pricing strategy of each model of product under the competitive market?
(3) What is the impact of consumer preference level for product functionality on firms' decisions?
(4) How does the market competition influence firms' optimal decisions?

In the paper, we consider a finite number of firms competing non-cooperatively in a market. Each firm maximizes its own profit by determining the production quantity of each version of products based on the other rivals' optimal strategies. Specifically, we propose a profit optimization model to depict this problem. The Nash equilibrium conditions are next formulated as a variational inequality (VI) problem, and the existence and uniqueness of the equilibrium are proved. Furthermore, we solve the VI problem by an Euler algorithm and a case study focused on the smartphone market is conducted. We find even if the competition is increasingly fiercer, some firms may obtain more profits due to the change of consumer preference for product functionality. However, when the consumer preference level is below a certain value, it is not always beneficial for firms who launch new versions of their products to satisfy the demand market because of their cost structures. Moreover, if a new firm enters the market, existing firms who choose to give up the production of products with old versions may reduce the loss of their profits.

The remainder of the paper is organized as follows. Section 2 outlines related literature. Section 3 develops the optimization model for multiple competitive firms. We formulate our model as a variational inequality problem and propose an algorithm to solve the variational inequality problem in Section 4. Following a case study focused on the 5G smartphone market (Sect. 5), we conclude the paper in Section 6.

2. Literature review

This study is related to the emerging field of product innovation and improvement; for relevant examples, please refer to Qi *et al.* [38], Pal and Sarkar [34] and Song *et al.* [43]. Product innovation refers to the creation of new concepts, and aims to satisfy customer demands [36]. Product innovation has significantly positive driving-force effect and plays an important role in manufacturing improvement [50]. In car manufacturing, product innovation drives the evolution of product efficiency in Spain [12]. In the smart electronic industry, areas for innovative leather are emerging as coverings for smart electronic devices due to its thermal insulation function by some mobile phone manufacturers [55]. Grtzmann *et al.* [13] indicated that there is an important tool that internet technologies can support Brazilian firms to update products functionality to satisfy consumer preference. In the medical industry, Enrique *et al.* [8] studied the impacts of innovation and competition on medical products. The two factors strongly entice the medicine firms to develop the advanced therapy medicinal products. Ganuza *et al.* [10] analyzed the reason why medical firms have directed their research and development at small improvements of existing drugs instead of the pursuit of significant innovations. Ganuza *et al.* reported that a small improvement can cause an increase of profit. Guo *et al.* [14] respectively analyzed how new firms and established firms develop their products by using the data of 211 firms. Their results showed that the new firms and the established firms should take the advantage of their features to foster their own product innovation and improvement. Liu *et al.* [23] considered that many innovating firms provide new generations of products to attract consumers' repeat purchasing. Hong *et al.* [17] considered a green product as an innovative product and characterized its diffusion process by using Bass model. Mandal and Pal [27] considered that a manufacturer invests in green technology to reduce carbon emissions during production process. Although these works have studied why and how firms make progress in product improving, our study proposes that firms should make an innovation and improvement in products with an existing version, and thus launch products with a new version to encourage more consumers' purchasing. Different from those literature, the purpose of our work is to investigate production decisions of multiple competitive innovating firms who provide products with different versions.

An increasing body of literature has considered consumer preference and its impact on supply chain operation management [24, 35, 41, 54, 59]. Due to frequent changes in consumer preference, Wu and Lai [48] found that more and more firms continuously introduce new products to meet the desires of consumers. Tong *et al.* [46] showed that consumers are willing to pay higher price to purchase higher version for low-carbon products, and found that consumer preference for the product is the important source, which significantly influences the supply chain decisions. Han and Liu [15] indicated that with the increase of consumer preferences for high-quality products, manufacturers tend to produce more high-quality products. In a closed-loop supply chain, it is necessary for remanufacturers to completely understand consumers' preferences since the preferences strongly affect the remanufacturers' optimal strategies [62]. Chien *et al.* [3] found that firms can realize consumers' preferences based on the visual function and indicated that user experience is an important factor for the product update to capture user attention. Yu and Nagurney [57] used variational inequality theory to solve a network-based supply chain problem considering different consumer demands. Yu *et al.* [58] developed a supply chain system model considering the preferences of consumers for offline and online selling channels. Based on the discussion of consumers' demand preferences, product innovation design features were classified by Kano model, then Yang *et al.* [53] proposed Non-dominated Sorting Genetic Algorithm to solve the model. Yenipazarli [56] investigated the impacts of consumers' preferences on the incentives of environmental research and development of firms, based on a two-stage duopoly model. Yan *et al.* [52] considered consumer convenience preferences and developed a channel decision model. However, our paper mainly analyzes the relationship between consumers' preferences and product innovation. Specifically, this paper studies how consumer preference entices firms to update their products with more functionalities and additional attributes. Different from these works, our paper builds an optimization model to discuss the influence of the consumer preference on product improving decisions, and then applies the theory of variational inequality to solve it.

To compare the novelty of our paper with respect to existing literature, a table is provided as follows (Tab. 1):

TABLE 1. The difference between our paper and existing literature.

	Product innovation	Consumer preference	Production decisions	Competition	Variational inequality
Hao *et al.* [16]	√	√			
Liu and Nagurney [21]			√	√	√
Zhao *et al.* [62]		√	√		
Pal *et al.* [35]				√	
Yu *et al.* [59]		√	√	√	√
Mandal and Pal [27]	√		√	√	
Han and Liu [15]		√	√	√	√
Yu *et al.* [60]	√				
Ma *et al.* [25]		√	√	√	√
Our paper	√	√	√	√	√

To summarize, the amount of relevant literature that studies both product innovation and the effects of consumer preferences on production decisions is scarce. This motivates us to generate this paper. Therefore, in contrast to the above-mentioned studies, the main knowledge gaps can be offered as follows. First, differing from previous models on product innovation, in this paper, a wider range of functionality levels (versions) of products based on the reality is investigated. Relevant literature only considered scenarios where there was one version of product (or a kind of new product). Second, this paper considers the purchase preferences of consumers into multi-firms' production decisions, and discusses how both consumer preference and competition affect innovating firms' decisions for products with different versions, which has not been studied in previous publications. On this basis, management insights drawn from a case study focused on the 5G smartphone market provide practical suggestions for firms. Third, variational inequality theory is used to develop an algorithm to solve our complex optimization model, and optimal strategies are gained efficiently. Furthermore, the proposed model, as well as the Euler algorithm, can be generalized to solve operation management competition problems according to any feature levels besides product functionality levels. Therefore, our model has extensive impacts on the ongoing competitive world. Specifically, in addition to production strategies, the model can also support capacity allocation and marketing strategies in service industries.

3. MODEL FORMULATION

In this study, m firms are considered, and they compete non-cooperatively in a market. Firms produce products in a variety of versions with different levels of functionality and sell them to consumers. Each firm's decision-making problem is to maximize its own profit by determining the optimal production quantities of products in each version. Consumers' preferences for product functionality greatly affect firms' production decisions in practice. Steenis *et al.* [44] also discussed the impact of consumers' preference on optimal strategies. In addition, product differentiation by consumers is allowed, due to brand related concerns and product version differences associated with a particular firm. A growing number of consumers are willing to pay a higher price to purchase a newer version of products. Therefore, firms would have motivation to update their products although it might mean a large amount of expenditures in R&D. Table 2 provides a summary of notation.

For a particular firm j, product version s_j is exogenous and a positive integer, where $s_j = 1, 2, \ldots, h_j$; h_j represents the latest version that firm j can produce, *e.g.*, M1 is 1 and M9 is h for Millet. The price $p_j^{s_j}$ is assumed to depend on the all firms' production quantities, firm j's product version and consumer preference level.

$$p_j^{s_j} = p_j^{s_j}(q, s_j, \varphi), \quad s_j = 1, ..., h_j; \quad j = 1, ..., m, \tag{3.1}$$

TABLE 2. Summary of notation.

Type	Notation	Definition
Parameter	φ	random consumer preference level for product functionality
	α	expected value of φ
	s_j	product version of firm j
	h_j	the latest version that firm j can produce
Decision variable	$q_j^{s_j}$	firm j's production quantity of products with version s_j
Function	$y_j^{s_j}$	a binary variable, 1 if $s_j = h_j$, 0 if $s_j < h_j$
	$p_j^{s_j}$	price of products with version s_j of firm j
	$c_j^{s_j}$	total operation cost of products with version s_j of firm j
	$f_j^{s_j}$	research and development (R&D) cost of firm j
	Z_j	profit of firm j

where $p_j^{s_j}$ is considered to be continuous, continuously differentiable and monotone decreasing with respect to $q_j^{s_j}$; $s_j = 1, 2, \ldots, h_j$; $j = 1, \ldots, m$ [18, 37]. We group the production quantity $q_j^{s_j}$ into $\sum_{j=1}^m h_j$-dimensional vector $q = (q_1^1, \ldots, q_1^{h_1}, \ldots, q_i^1, \ldots, q_i^{h_i}, \ldots, q_m^1, \ldots, q_m^{h_m})^T$. Moreover, $p_j^{s_j}$ correlates positively to product version s_j. In addition, the price is linearly related to the consumer preference level for product functionality φ. For instance, the price of P30-Pro exceeds that of P30 for Huawei. At the same time, consumers are willing to pay a higher price to purchase P30-Pro because their expectation value (due to functionality improvement and additional attributes) is larger. We assume φ is a random factor, and its expected value $E(\varphi) = \alpha$. In fact, φ is the rate of price increase per functional improvement, which means consumers are willing to pay higher prices for products with higher functionality level. The upper bound of is unique to each product, and the lower bound is 0 [22].

Cost structure is an important factor in supply chain management [33,41]. Mohammed *et al.* [29] developed an optimization model to decrease the total cost of a multi-period supply chain in a closed-loop system. Dumrongsiri *et al.* [7] reported that two marginal costs affect the equilibrium strategy of dual channels. In our paper, firms face different cost structures for different versions of products.

In general, the total operational cost $c_j^{s_j}$ (including production, processing, storage, and distribution cost) depends on the production quantity and product version, *i.e.*,

$$c_j^{s_j} = c_j^{s_j}(q, s_j), \quad s_j = 1, ..., h_j; \quad j = 1, ..., m, \tag{3.2}$$

where $c_j^{s_j}$ is assumed to be convex, and continuously differentiable with respect to $q_j^{s_j}$; $s_j = 1, 2, \ldots, h_j$; $j = 1, \ldots, m$ [58], and correlates positively to version s_j. The reason is that the production of per unit of product with higher version may require more expensive materials and production operations.

The research and development (R&D) cost $f_j^{s_j}$ is a fixed cost for firm j who is willing to produce products with version s_j. In practice, the R&D cost is only used to those products with the newest version, and there is not R&D cost for products with the old versions. Hence, $y_j^{s_j}$ is a binary variable,

$$y_j^{s_j} = \begin{cases} 1, & \text{if} \quad s_j = h_j, \\ 0, & \text{if} \quad s_j < h_j. \end{cases} \tag{3.3}$$

Firm j's problem can be expressed as follows:

$$\text{Maximize } Z_j = E\left[\sum_{s_j=1}^{h_j} \left[p_j^{s_j}(q, s_j, \varphi) \cdot q_j^{s_j} - c_j^{s_j}(q, s_j) - y_j^{s_j} \cdot f_j^{s_j}\right]\right]. \tag{3.4}$$

In the optimization problem (3.4), there are $\sum_{j=1}^{m} h_j$ decision variables represented with the vector $q = (q_1^1, \ldots, q_1^{h_1}, \ldots, q_i^1, \ldots, q_i^{h_i}, \ldots, q_m^1, \ldots, q_m^{h_m})^T$. The first term of equation (3.4) is the expected revenue obtained by selling products. The second term denotes the total operational cost and the third term represents the R&D cost. In the optimization model, firm j needs to determine the production quantities of products with different versions for profit maximization.

4. Solution approach and algorithm

In this section, the theory of variational inequality is used to transform the optimization problem (3.4) to a variational inequality problem, and the Nash equilibrium among multiple firms is derived. On this basis, an Euler method is proposed to solve the variational inequality problem.

4.1. Solution approach

The optimization model (3.4) is fairly complex due to including $\sum_{j=1}^{m} h_j$-dimensional decision variables. Hence, the variational inequality theory is adopted to solve it. The theory of variational inequality applied in solving equilibrium strategy was addressed in Nagurney *et al.* [31] and Liu and Nagurney [21]. Wakolbinger and Cruz [47] applied the theory to analyze the relationship between strategic information acquisition and supply chain disruption risk. Zhu *et al.* [63] also solved a multi-class network equilibrium problem using the variational inequality theory in a tradable credit scheme system. Moreover, Yu *et al.* [58] extended the applications of variational inequality theory to other fields. Ma *et al.* [25] built an integrated model for competition between supply chains with heterogeneous customers and then transform it as a multinomial logit based variational inequality problem.

The variational inequality theory was applied here to solve the optimization problem by defining:

$$Z = Z(Q), \tag{4.1}$$

where Z denotes the m-dimensional vector of all firms' profits, and Q is the vector of all firms' production quantities.

In this paper, each firm j decides its optimal production number of products for each version, given the optimal strategies of the other firms. According to equation (3.4), the optimal strategies of all firms can be described as a variational inequality problem. Consequently, the equilibrium strategies of firms can be obtained by solving the variational inequality problem of production quantities for products of each version in each firm $Q^* \in G = \prod_{j=1}^{m} G_j$ is determined for which m firms are in a state of equilibrium, according to Definition 4.1:

Definition 4.1. The supply chain equilibrium state is one where each firm satisfies:

$$Z_j(Q_j^*, \widetilde{Q_j^*}) \geq Z_j(Q_j, \widetilde{Q_j^*}), \quad \forall Q_j \in G_j, \tag{4.2}$$

where Q_j denotes the vector of production quantities associated with firm j; $j = 1, ..., m$, $\widetilde{Q_j^*} \equiv (Q_1^*, ..., Q_{j-1}^*, Q_{j+1}^*, ..., Q_m^*)$ and $G_j \equiv \{Q_j | Q_j \in R_+^{h_j}\}$.

Definition 4.1 expresses that each firm seeks to maximize its own profit by determining its production quantities in a non-cooperative manner until an equilibrium is established [30]. Following Zhang [61], and Yu and Nagurney [57], the variational inequality of the equilibrium satisfies Definition 4.1. The following theorem is then obtained:

Theorem 4.2. *For each firm j; $j = 1, ..., m$, the profit function $G_j(Q)$ is concave, and is continuously differentiable with respect to Q_j. Such, based on Definition 4.1, $Q_j \in G$ is a Nash equilibrium if (and only if) it satisfies variational inequality (4.3):*

$$-\sum_{j=1}^{m} \langle \nabla_{Q_j} Z_j(Q^*)^T, Q_j - Q_j^* \rangle \geq 0, \quad \forall Q \in G, \tag{4.3}$$

where $\nabla_{Q_j} Z_j(Q)$ *means the gradient of* $Z_j(Q)$ *and* $\langle \cdot, \cdot \rangle$ *indicates the inner product in the* h_j*-dimensional Euclidean space. The equilibrium production quantities of each firm are defined as* $q^* \in G_{j1}$*, where* $G_{j1} \equiv \{q_j | q_j \in R_+^{h_j}\}$*. For our optimization model, the following variational inequality (4.4) can be derived based on variational inequality (4.3). The vector* $q^* \in G_1$ *can be determined when satisfying:*

$$\sum_{j=1}^{m} \sum_{s_j=1}^{h_j} \left[-p_j^{s_j}(q^*, s_j, \alpha) - \frac{\partial p_j^{s_j}(q^*, s_j, \alpha)}{\partial q_j^{s_j}} q_j^{s_j^*} + \frac{\partial c_j^{s_j}(q^*, s_j)}{\partial q_j^{s_j}} \right] \times (q_j^{s_j} - q_j^{s_j^*}) \geq 0, \quad \forall q \in G_1, \tag{4.4}$$

where $G_1 \equiv \left\{ q | q \in R_+^{\sum_{j=1}^{m} h_j} \right\}$.

Proof. Variational inequality (4.3) can follow from Gabay and Moulin [9]. For the optimization model (3.4), equation (4.5) is gained,

$$\nabla_{Q_j} Z_j(Q) = \left[\frac{\partial Z_j}{\partial q_j^{s_j}}; j = 1, ..., m; s_j = 1, ..., h_j \right]. \tag{4.5}$$

According to the price function (3.1), we assume that the consumer preference level for product functionality φ is a random factor, and $E(\varphi) = \alpha$. Thus, on the basis of equation (3.4), we can obtain the following equation:

$$Z_j = \sum_{s_j=1}^{h_j} \left[p_j^{s_j}(q, s_j, \alpha) \cdot q_j^{s_j} - c_j^{s_j}(q, s_j) - y_j^{s_j} \cdot f_j^{s_j} \right] \tag{4.6}$$

For each firm j ($j = 1, ..., m$), we obtain

$$\begin{aligned} \frac{\partial Z_j}{\partial q_j^{s_j}} &= \frac{\partial \sum_{s_j=1}^{h_j} \left[p_j^{s_j}(q, s_j, \alpha) \cdot q_j^{s_j} - c_j^{s_j}(q, s_j) - y_j^{s_j} \cdot f_j^{s_j} \right]}{\partial q_j^{s_j}}, \\ & \sum_{s_j=1}^{h_j} \left[p_j^{s_j}(q, s_j, \alpha) + \frac{\partial p_j^{s_j}(q, s_j, \alpha)}{\partial q_j^{s_j}} q_j^{s_j} - \frac{\partial c_j^{s_j}(q, s_j)}{\partial q_j^{s_j}} \right]. \end{aligned} \tag{4.7}$$

For each firm, multiplying the expression in (4.7) by the term $\left(q_j^{s_j} - q_j^{s_j^*}\right)$ and a minus sign based on the definition of , then we obtain each firm's variational inequality (4.8).

$$\sum_{s_j=1}^{h_j} \left[-p_j^{s_j}(q^*, s_j, \alpha) - \frac{\partial p_j^{s_j}(q^*, s_j, \alpha)}{\partial q_j^{s_j}} q_j^{s_j^*} + \frac{\partial c_j^{s_j}(q^*, s_j)}{\partial q_j^{s_j}} \right] \times (q_j^{s_j} - q_j^{s_j^*}) \geq 0, \quad \forall q_j \in G_{j1}, \tag{4.8}$$

where $G_{j1} \equiv \left\{ q_j | q_j \in R_+^{\sum_{j=1}^{m} h_j} \right\}$. Furthermore, m firms compete in this study, and thus there are m variational inequalities. When m variational inequalities are simultaneously achieved, a Nash equilibrium among multiple firms can be obtained. Therefore, referring to variational inequality properties, the variational inequality problem (4.4) requires to be solved by using the definition of G_1 and summing over all j. □

To present the expressions conveniently in the following sections, the standard form of variational inequality (4.4) is given. The vector $Q^* \in G^0$ can be determined when meeting

$$\langle B(Q^*)^T, Q - Q^* \rangle \geq 0, \quad \forall Q \in G^0. \tag{4.9}$$

Letting $Q \equiv q$, we have

$$B(Q) \equiv \left[-p_j^{s_j}(q, s_j, \alpha) - \frac{\partial p_j^{s_j}(q, s_j, \alpha)}{\partial q_j^{s_j}} q_j^{s_j} + \frac{\partial c_j^{s_j}(q, s_j)}{\partial q_j^{s_j}}; j = 1, ..., m; s_j = 1, ..., h_j \right], \tag{4.10}$$

and $G^0 \equiv G_1$. The existence and uniqueness of the Equilibrium solution is given in the following two theorems.

Next, the existence and uniqueness of the Equilibrium solution are given in Theorems 4.3 and 4.4, respectively.

Theorem 4.3. *(Existence): There must be a $v > 0$, which leads to that the variational inequality (4.4) exists at least one Equilibrium solution. Therefore, one Equilibrium solution in G_v^0 to variational inequality (4.9) is admitted with:*

$$q^v \leq v. \tag{4.11}$$

Proof. Since the demand in the market is finite, the following formula is obtained,

$$G_v^0 \equiv \{q | 0 \leq q \leq v\}, \tag{4.12}$$

which indicates firm j's production quantity $q_j^{s_j}$ of products with version s_j must be bounded ($j = 1, ..., m$; $s_j = 1, ..., h_j$). In the formula (4.12), $v > 0$ and $q \leq v$ means $q_j^{s_j} \leq v$ for all j ($j = 1, ..., m$) and all s_j ($s_j = 1, ..., h_j$). In addition, G_v^0 can be gained and be a bounded, closed and convex subset of G. In our study, G_v^0 is compact and B is continuous, then the following variational inequality

$$\langle B(Q^v)^T, Q - Q^v \rangle \geq 0, \quad \forall Q \in G_v^0, \tag{4.13}$$

has at least one Equilibrium solution $q^v \in C_v^0$. □

Theorem 4.4 (Uniqueness)**.** *The formula $B(Q)$ that appears in variational inequality (4.9) is strictly monotone on $G^0 \equiv G_1$. Then, on the basis of the presented model, the Equilibrium solution by solving variational inequality (4.4) is unique.*

Proof. In this study, the price $p_j^{s_j}$ is considered to be continuous, continuously differentiable and monotone decreasing with respect to $q_j^{s_j}$; $s_j = 1, ..., h_j; j = 1, ..., m$, and the cost $c_j^{s_j}$ is assumed to be convex, and continuously differentiable with respect to $q_j^{s_j}$; $s_j = 1, ..., h_j; j = 1, ..., m$ [58]. As a result, we obtain

$$\left\langle (B(Q^1) - B(Q^2))^T, Q^1 - Q^2 \right\rangle \geq 0, \quad \forall Q^1, Q^2 \in G^0, Q^1 \neq Q^2. \tag{4.14}$$

The formula (4.14) shows that $B(Q)$ is strictly monotone on $G^0 \equiv G_1$, and indicates the Equilibrium solution must be unique. □

According to Theorems 4.3 and 4.4, the equilibrium production quantities of products with different versions exist uniquely.

4.2. Algorithm

In this section, an Euler method is proposed to solve variational inequality (4.4). When a solution exists uniquely (Thms. 4.3 and 4.4), the algorithm converges to the optimal solution of (4.4). The algorithm has been extensively studied in the existing literature [2, 45].

Therefore, the computational framework is provided by using the Euler method at an iteration τ, which is presented as follows:

$$Q^{\tau+1} = I_{G^0}(Q^\tau - \lambda_\tau B(Q^\tau)), \tag{4.15}$$

where I_{G^0} represents the projection on the set G^0 and B denotes the function that enters variational inequality (4.9). To guarantee the convergence, the sequence $\{\lambda_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} \lambda_\tau = \infty$, $\lambda_\tau > 0$, $\lambda_\tau \to 0$, as $\tau \to \infty$. When a given convergence tolerance is no less than the spatial distance between adjacent flows, the optimal solution will be obtained.

The Euler method is applied to the supply chain, including improving products with different versions of variational inequality (4.4). Using fixed point theory, the following explicit formula is applied:

$$(q_j^{s_j})^{\tau+1} = \max\left\{0, (q_j^{s_j})^{\tau} + \lambda_\tau \left(p_j^{s_j}(q, s_j, \alpha) + \frac{\partial p_j^{s_j}(q, s_j, \alpha)}{\partial q_j^{s_j}} q_j^{s_j} - \frac{\partial c_j^{s_j}(q, s_j)}{\partial q_j^{s_j}}\right)\right\}, \quad \forall j; \forall s_j; j = 1, ..., m; s_j = 0, ..., h_j. \tag{4.16}$$

At the same time, equation (4.16) converges if a given convergence tolerance is no less than the spatial distance between the successive production quantities of products with different versions.

5. Case study

A case study for the 5G smartphone market is presented to validate and illustrate the developed optimization model. This case study is motivated based on a real market scenario.

With the release of 5G connectivity, 5G technology holds the potential to revolutionize the way mobile networks function. According to a study conducted by GSMA, 5G is expected to account for 15% of the global mobile industry by 2025, with a predicted 1.4 billion devices operating on a 5G network [26]. Many firms, such as Samsung, Huawei and Millet have launched their 5G models of smartphones.

Samsung and Huawei, two of the most popular makers and pioneers of 5G phones, announced their debut 5G phones, Samsung S10 5G and Huawei Mate 20 X 5G, respectively in 2019. We now can see even more models/generations of 5G phones since then. Samsung has several models under its belt, including the newly announced Galaxy Z Flip 5G, the Galaxy S20 and the more budget-friendly Galaxy A71 5G [19]. Huawei released Mate S30 5G and P40 5G. As 71% of mobile service providers claimed that they have either already been in substantial progress of deploying 5G networks or will do so [42]. More smartphones providers then unveiled their own 5G phones. Millet, the sixth largest mobile phone firm in the world successively launched the model of M10 5G. To save costs and obtain more profits, the M8 and other old generations are discontinued. As the most popular mobile phone firm in the world, Apple also offered its first-ever 5G-capable iPhone, iPhone 12 line. With the launch of 5G iPhone 12, Apple surpassed Samsung and Huawei to lead the 5G-smartphone market in the fourth quarter of 2020 [11].

In this section, three cases have been conducted to analyze firms' production decisions of smartphones with different versions. Case 1 presents a base situation, where two existing firms are considered (Firm 1 is as one firm who produces smartphones with two versions (one original version and one new version), and Firm 2 is the other firm who produces smartphones with one original version). With the increase of consumer preference for product functionality in Case 2, Firm 1 and Firm 2 both add a new version. In Case 3, the competition is further strengthened due to the entrance of a new firm (Firm 3), and two situations are discussed based on Firm 1's production decisions. For the three cases, each firm maximizes its own profit by determining the production quantities of smartphones in each level of model under oligopolistic competition.

The price and operational cost functions are provided for the three cases. First, the price function is given by:

$$p_j^{s_j} = \beta_j^{s_j} + s_j\alpha - \sum_{j=1}^{m}\sum_{s_j=1}^{h_j} \mu_j^{s_j} q_j^{s_j}, \quad \beta_j^{s_j} > 0, \quad \mu_j^{s_j} > 0. \tag{5.1}$$

It is assumed that the price of firms for smartphones with version correlates negatively with the production quantity of firms for smartphones at each version [37,57]. The price is positively correlated to both the version and consumer preference for product functionality [51].

The total operational cost function is:

$$c_j^{s_j} = \eta_j^{s_j}(q_j^{s_j})^2 + \theta_j^{s_j} s_j^2 q_j^{s_j}, \quad \eta_j^{s_j} > 0, \quad \theta_j^{s_j} > 0. \tag{5.2}$$

TABLE 3. Optimal results for Firm 1 and Firm 2 in Case 1.

	Firm 1		Firm 2
Version	1	2	1
Production quantity	1.5632	3.3498	2.2827
Price	1.9555	4.3401	2.9307
Cost	0.3694	6.1941	0.8536
Profit	2.6874	8.3444	5.8365
Total profit	11.0318		5.8365

The cost of firms for smartphones with version s_j is convex on the production quantities of smartphones with version s_j [6]. If the firms decide to produce smartphones with a higher level of version, they will have a higher operational cost. Therefore, the cost increases over the version level s_j.

Applying the proposed Euler algorithm in Section 4, we solve the firms' production decision problem in the following cases. For the computational purpose, the convergence tolerance is set to 10^{-6} and the sequence $a_\tau = 0.1\left(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ...\right)$ for the cases. In addition, the algorithm is initialized by setting the production quantity for smartphones with each version equal to 2.5 [57]. The results of the three cases are analyzed based on relative trends and numbers, and the specific units are not shown.

5.1. Case 1 (A base case)

A base situation is first considered in Case 1. In this case, consumers' average preference level for product functionality (version) is assumed to be 2, *i.e.*, $\alpha = 2$ [22]. Given the preference level of consumers, Firm 1, who has invested in R&D and released a new model with the version level of 2, has two models of smartphones, *i.e.*, Versions 1 and Version 2. Lacking the ability of R&D investment, Firm 2 has only one model of smartphone with the version level 1. Both firms' R&D costs are $f_1^1 = 0$, $f_1^2 = 4$ and $f_2^1 = 0$. There is no R&D cost for smartphones with Version 1 of the two firms, which have been paid in the past. The price functions of products in Firms 1 and 2 are

$$p_1^1 = 4 + 1 \times 2 - q_1^1 - 0.4q_1^2 - 0.5q_2^1;$$
$$p_1^2 = 5 + 2 \times 2 - q_1^2 - 0.4q_1^1 - 0.3q_2^1;$$
$$p_2^1 = 5 + 1 \times 2 - q_2^1 - 0.5q_1^1 - 0.3q_1^2.$$

The corresponding total operational cost functions are

$$c_1^1 = 0.1(q_1^1)^2 + 0.08 \times 1^2 \times q_1^1;$$
$$c_1^2 = 0.1(q_1^2)^2 + 0.08 \times 2^2 \times q_1^2;$$
$$c_2^1 = 0.12(q_2^1)^2 + 0.1 \times 1^2 \times q_2^1.$$

Both firms' optimal production quantities and profits are computed by the proposed algorithm and shown in Table 3.

According to Table 3, adding a new version (*i.e.* Version 2) of smartphones and providing smartphones with two versions generates a high profit (11.0318) for Firm 1. Specifically, Version 2 contributes 75.6% of Firm 1's profit and takes up 46.6% of the overall market demand. The investment in breakthrough innovation does pay off since the release of Version 2 meets preference of many consumers. However, even though the average consumer preference is 2, the Version-1 items still have considerably large market share 53.4% (including 21.7% from Firm 1 and 31.7% from Firm 2). That's because some consumers cannot afford Version 2 for the price of 4.3401, which is more than double of Version 1's price in Firm 1. Also because of production capacity limitation on Version 1 of Firm 1, Firm 2 smartphone obtains the profit of 5.8365 from Version-1 product, which is more than that of Version 1 from Firm 1.

TABLE 4. Optimal results for Firm 1 and Firm 2 in Case 2.

	Firm 1			Firm 2	
Version	1	2	3	1	2
Production quantity	0.7878	2.1948	4.6009	1.2852	2.6214
Price	1.0251	2.9535	6.2414	1.6938	3.6506
Cost	0.1251	1.1840	9.9295	0.3267	5.3731
Profit	0.6824	5.2983	18.7866	1.8501	4.1963
Total profit		24.7673		6.0464	

5.2. Case 2 (The increasing consumer preference level)

Due to the development of improvements and innovation levels in technology, a growing number of consumers pay higher prices for high-tech products, with higher expectations in terms of functionality level [20, 32]. To study the influence of consumer preferences on firms' production strategies, we consider the average consumer preference level for product function increases, *i.e.*, $\alpha = 3$ in Case 2. Firm 1 and Firm 2 both realize the increase of consumer preference level and therefore add a new version of smartphones (*i.e.*, Versions 1, 2 and 3 for Firm 1; Versions 1 and 2 for Firm 2). For smartphones with the new version, the two firms both need to invest, which increases costs. Hence, it can be assumed that the R&D costs of both firms are $f_1^1 = 0$, $f_1^2 = 0$, $f_1^3 = 4.5$, $f_2^1 = 0$ and $f_2^2 = 3.5$. Here, we consider that there is not R&D cost for smartphones with existing versions. The price functions of both firms are

$$\begin{aligned}
p_1^1 &= 2.5 + 1 \times 3 - q_1^1 - 0.4q_1^2 - 0.3q_1^3 - 0.5q_2^1 - 0.3q_2^2;\\
p_1^2 &= 3 + 2 \times 3 - q_1^2 - 0.4q_1^2 - 0.4q_1^3 - 0.3q_2^1 - 0.5q_2^2;\\
p_1^3 &= 4 + 3 \times 3 - q_1^3 - 0.3q_1^1 - 0.4q_1^2 - 0.2q_2^1 - 0.3q_2^2;\\
p_2^1 &= 3 + 1 \times 3 - q_2^1 - 0.5q_1^1 - 0.3q_1^2 - 0.2q_1^3 - 0.4q_2^2;\\
p_2^2 &= 3.5 + 2 \times 3 - q_2^2 - 0.3q_1^1 - 0.5q_2^1 - 0.3q_1^3 - 0.4q_2^1.
\end{aligned}$$

The total operational cost functions of both firms are:

$$\begin{aligned}
c_1^1 &= 0.1(q_1^1)^2 + 0.08 \times 1^2 \times q_1^1;\\
c_1^2 &= 0.1(q_1^2)^2 + 0.08 \times 2^2 \times q_1^2;\\
c_1^3 &= 0.1(q_1^3)^2 + 0.08 \times 3^2 \times q_1^3;\\
c_2^1 &= 0.12(q_2^1)^2 + 0.1 \times 1^2 \times q_2^1;\\
c_2^2 &= 0.12(q_2^2)^2 + 0.1 \times 2^2 \times q_2^2.
\end{aligned}$$

In this case, the optimal solutions of Firm 1 and Firm 2 are presented in Table 4.

The increasing consumer preference for product functionality can incentivize firms to add higher versions of products, and thus they may obtain more profits, although the competition in the market is increasingly more intensive. Moreover, with the increase of consumers' preferences, more consumers are willing to pay higher price for products with higher versions. Therefore, when the preference level of consumers reaches a certain level, the production of products with new version can be more profitable for firms.

Table 4 shows that Firm 1 produces smartphones with three versions (1, 2 and 3). As consumer preference level increases and smartphones with Version 3 leads to a higher cost, with its price (6.2414) higher than that of smartphones with Versions 1 and 2 (1.0251 and 2.9535). Moreover, the production quantity of smartphone Version 3 also greatly exceeds that of smartphone Versions 1 and 2 due to the increasing consumer preference level. As a result, Firm 1 earns a very high profit of 24.7673. For Firm 2, although it only produces smartphones with two versions, we find that the prices and production quantities of smartphone Versions 1 and 2 for Firm

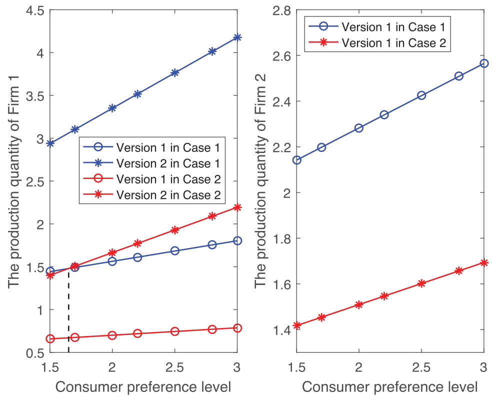

FIGURE 1. Impacts of α on both firms' optimal production quantities for Cases 1 and 2.

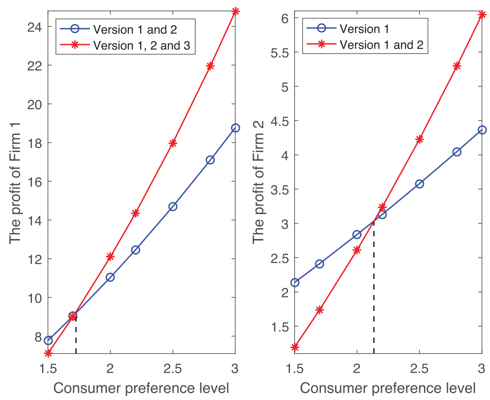

FIGURE 2. Impacts of α on both firms' profits for Cases 1 and 2.

2 both exceed those of smartphone Versions 1 and 2 for Firm 1. In other words, for smartphones with the same version, more consumers choose to purchase smartphone from Firm 2 with higher prices, compared with Firm 1.

The results of Tables 3 and 4 are compared, which shows that it is important for the two firms to change their production decisions, and thus their profits both greatly increase. Specifically, Firm 1's profit increases by 124.51%, and Firm 2 has a profit increase of 113.16%. The reason is that with the increase of consumer preference level, a large number of consumers prefer smartphone latest-version items of each firm. In addition, it is noted that the competition has been strengthened and occurs among five models of smartphones (Versions 1, 2 and 3 of Firm 1, and Versions 1 and 2 of Firm 2). The two firms both suffer a loss of sales of smartphone old versions although their prices also decrease. However, the advantage of producing smartphones with new version can greatly offset the decrease of profits due to the production of smartphones with old versions because consumers pay high attention to the functionality of smartphones. Figures 1 and 2 respectively show the effects of consumer preference level on the two firms' optimal production quantities and their profits.

From Figure 1, with the increase of consumer preference for version level (changing from 1.5 to 3), when the two firms both add a new version of smartphones, the production quantities of smartphone previous versions all decrease. That means fewer consumers are willing to purchase smartphones with old versions.

TABLE 5. Optimal results for Firm 1, Firm 2 and Firm 3 in the first situation in Case 3.

	Firm 1			Firm 2		Firm 3
Version	1	2	3	1	2	1
Production quantity	0.7291	2.0501	4.3110	1.1575	2.4097	2.0057
Price	0.9547	2.7800	5.8935	1.5354	3.3882	2.3763
Cost	0.1115	1.0763	9.4624	0.2765	5.1607	4.4221
Profit	0.5846	4.6230	15.9447	1.5007	3.0039	0.3440
Total profit		21.1523		4.5046		0.3440

According to Figure 2, we find that it is not necessarily beneficial for the two firms who launch a new version smartphone with the increasing consumer preference. When the consumer preference is at a low level, it brings a decrease of profit for the two firms when choosing to produce smartphones with new version. To be specific, for Firm 1, there exists a critical point $\alpha = 1.725$. Specifically, when $1.5 \leq \alpha < 1.725$, it will obtain more profit by producing smartphone Versions 1 and 2, compared with that by producing smartphone Versions 1, 2 and 3. When $1.725 \leq \alpha \leq 3$, it is more profitable for Firm 1 by adding Version 3. Similarly, a critical point also exists for Firm 2, *i.e.*, $\alpha = 1.65$, below which Firm 2 gets more profit by producing smartphones with Version 1 while above which it is more beneficial for Firm 2 if it adds Version 2 smartphones.

5.3. Case 3 (A new entrant)

Some potential firms who have high technological levels and good cost structures may enter the smartphone industry to earn some profits. To discuss the effects of a new entrant (Firm 3) on production decisions and profits of the existing firms (Firm 1 and Firm 2), the average consumer preference level for product function is assumed to be the same with that in Case 2 (*i.e.* $\alpha = 3$). In this case, we consider two situations. In the first situation, Firm 1 still produces smartphones with three versions and Firm 2 produces two versions. Firm 3, as a new entrant, produces smartphones with one version. In the second situation, Firm 1 will give up the production of smartphones with Version 1, and the other settings keep the same.

First, we consider the first situation. The R&D costs of the three firms are $f_1^1 = 0$, $f_1^2 = 0$, $f_1^3 = 4.5$, $f_2^1 = 0$, $f_2^2 = 3.5$ and $f_3^1 = 4$. The price functions of the three firms are

$$\begin{aligned}
p_1^1 &= 2.5 + 1 \times 3 - q_1^1 - 0.4q_1^2 - 0.3q_1^3 - 0.5q_2^1 - 0.3q_2^2 - 0.2q_3^1;\\
p_1^2 &= 3 + 2 \times 3 - q_1^2 - 0.4q_1^2 - 0.4q_1^3 - 0.3q_2^1 - 0.5q_2^2 - 0.3q_3^1;\\
p_1^3 &= 4 + 3 \times 3 - q_1^3 - 0.3q_1^1 - 0.4q_1^2 - 0.2q_2^1 - 0.3q_2^2 - 0.3q_3^1;\\
p_2^1 &= 3 + 1 \times 3 - q_2^1 - 0.5q_1^1 - 0.3q_1^2 - 0.2q_1^3 - 0.4q_2^2 - 0.25q_3^1;\\
p_2^2 &= 3.5 + 2 \times 3 - q_2^2 - 0.3q_1^1 - 0.5q_2^1 - 0.3q_1^3 - 0.4q_2^1 - 0.35q_3^1;\\
p_3^1 &= 5 + 1 \times 3 - q_3^1 - 0.2q_1^1 - 0.3q_2^1 - 0.4q_1^3 - 0.25q_2^1 - 0.35q_2^2.
\end{aligned}$$

The operation cost functions of Firm 1 and Firm 2 are the same with that in Case 2, and the operation cost functions of Firm 3 are shown as follows.

$$c_3^1 = 0.08(q_3^1)^2 + 0.05 \times 1^2 \times q_3^1.$$

The optimal results for Firm 1, Firm 2 and Firm 3 are presented in Table 5.

The competition is more intensive due to the entrance of a new firm. The existing firms in market will suffer a loss of profits because their market shares encounter shrinkage and the prices of products decrease. According to Table 5, Firm 1 produces smartphones with Versions 1, 2, and 3. The production quantity of Version 2 of Firm 1 (2.0501) is higher than that of Version 1 (0.7291), but lower than that of Version 3 (4.3110). Similarly,

TABLE 6. Optimal results for Firm 1, Firm 2 and Firm 3 in the second situation in Case 3.

	Firm 1		Firm 2		Firm 3
Version	2	3	1	2	1
Production quantity	2.1408	4.3721	1.2929	2.4514	2.0268
Price	2.8887	5.9668	1.7031	3.4400	2.4009
Cost	1.1434	9.5594	0.3299	5.2017	4.4300
Profit	5.0408	16.5282	1.8721	3.2312	0.4361
Total profit	21.5690		5.1033		0.4361

for Firm 2, as the increasing consumer preference for function, the price and production quantity of Version 2 are both higher than those of Version 1. It is noted that Firm 3 begins to produce smartphones with a low version. It needs to pay some cost to research and develop its products, and thus it earns the least profit among the three firms.

By comparing Tables 4 and 5, the results indicate that both Firm 1 and Firm 2 suffer a loss of profits when Firm 3 enters the market. Firm 1's profit decreases by 14.60%, and Firm 2 has a profit decrease of 25.50%. We can find that with the increasing competition, the prices and optimal production quantities of smartphones with each version of old firms all decrease. For example, the production quantity of Version 3 of Firm 1 decreases by 6.30%, and the corresponding price also decreases by 5.57%. Due to the entrance of Firm 3, some consumers prefer its smartphones compared with that of existing firms. As a result, it is beneficial for Firm 3 who chooses to enter the market.

Tables 3 and 5 are also compared. As consumers have more preferences for higher functionality, Firm 1 and Firm 2 both realize the importance of product innovation, and Firm 3 sees the advantage of smartphone industry and chooses to enter the market with its own offering. On this basis, the external competition among firms becomes more intensive, in the meanwhile, the internal competition within a firm, who has more than one offering, is also increasing. However, for existing firms (Firm 1 and Firm 2), their profits have a big increase (Firm 1's profit increases by 91.47%, and 58.81% for Firm 2). This is because, with the increase of consumer preference for high functionality, more consumers are willing to purchase smartphones with the newest versions at a high price. Even if the demands of smartphones with old versions for Firm 1 and Firm 2 decrease, this can be greatly offset by the advantage from new-version products.

We next consider the second situation where Firm 1 produces Versions 2 and 3, Firm 2 produces Versions 1 and 2, and Firm 3 produces Version 1. The operation cost functions and the R&D costs of the three firms are the same as those in the first situation. The price functions of the three firms are

$$\begin{aligned}
p_1^2 &= 3 + 2 \times 3 - q_1^2 - 0.4q_1^3 - 0.3q_2^1 - 0.5q_2^2 - 0.3q_3^1;\\
p_1^3 &= 4 + 3 \times 3 - q_1^3 - 0.4q_1^2 - 0.2q_2^1 - 0.3q_2^2 - 0.3q_3^1;\\
p_2^1 &= 3 + 1 \times 3 - q_2^1 - 0.3q_1^2 - 0.2q_1^3 - 0.4q_2^2 - 0.25q_3^1;\\
p_2^2 &= 3.5 + 2 \times 3 - q_2^2 - 0.5q_2^1 - 0.3q_1^3 - 0.4q_2^1 - 0.35q_3^1;\\
p_3^1 &= 5 + 1 \times 3 - q_3^1 - 0.3q_2^1 - 0.4q_1^3 - 0.25q_2^1 - 0.35q_2^2.
\end{aligned}$$

The optimal results for Firms 1, 2 and 3 are shown in Table 6.

With the intensifying competition and increasing preference level of consumers for product functionality, it may be more advantageous for the existing firms who choose to give up the production of products with old versions, which can also lead to more profits for other firms.

By comparing results from Tables 5 and 6, we see that the production adjustment of Firm 1 (Firm 1 gives up production of Version 1) generates its profit by 21.569. Moreover, other competitive firms' profits also increase (Firm 2's profit increases to 5.1033, and Firm 3's profit increases to 0.4361). As the increase of their preference

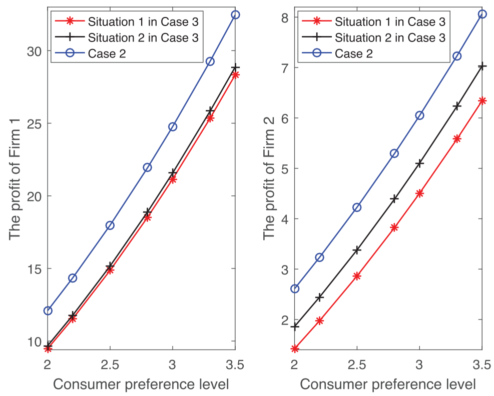

FIGURE 3. Impacts of α on the existing firms' profits in different scenarios.

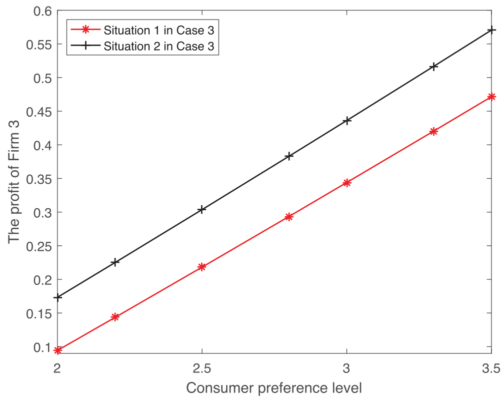

FIGURE 4. Impacts of α on the new firm's profit for different situations in Case 3.

for functionality, fewer consumers are willing to purchase Firm 1's smartphones with low version. Therefore, it is much more beneficial for Firm 1, who has the most models of smartphones, giving up the production of lowest version, and enlarging the production quantities of smartphones with Versions 2 and 3. As a result, only the other two firms (Firm 2 and Firm 3) continue to produce Version-1 smartphones. They can get more profits because there are still some demands of Version 1 for lower prices. Figure 3 shows the change of the profit of the existing firms (Firm 1 and Firm 2) with the increase of consumer preference in different scenarios. Figure 4 presents the effect of consumer preference on the profit of the new firm (Firm 3) in different scenarios.

Figure 3 shows profits of the firms will increase in each scenario as the average consumer preference level increases. Moreover, without the new entrant (Firm 3), the existing firms (Firm 1 and Firm 2) can always obtain high profits when consumer preference increases. If Firm 3 enters the market, Firm 1 and Firm 2 will suffer a loss of profits. However, if Firm 1 chooses to change its production decisions by giving up the production of lowest-version products, both Firm 1 and Firm 2 can reduce the loss of their profits.

From Figure 4, Firm 3's profit will increase with the increasing consumer preference level in the above-mentioned two situations. In addition, when Firm 1 only produces smartphones with Version 2 and 3, Firm 3's profit is higher than that in the situation where Firm 1 does not change its production decisions.

6. Conclusions

With the increasing preference level of consumers for product functionality, more and more consumers are willing to pay a higher price for purchasing the newest version of products. In this paper, the optimal production decisions are studied for firms under consideration of consumer preference for product functionality. On this basis, an optimization model is built, in which the firms need to determine the production quantities of their products with different versions. The optimization model is formulated as a variational inequality problem and solved by an Euler algorithm. Finally, a case study is motivated based on the reality, which is focused on the 5G smartphone market. On this basis, relative managerial insights are derived, which are summarized as follows.

First, the increasing consumer preference for product functionality can incentivize firms to produce products with new version to obtain high profits. Therefore, the production of products with new version can bring an increase of competitive advantage for firms. Second, it is not always beneficial for the firms who add a new version of their products. When consumer preference is at a low level, it may bring a decrease of profits for the firms who produce products with new version. However, when the preference level of consumers reaches a certain level, the production of products with new version can be more profitable for firms. Third, if the competition is strengthened due to the new entrant, the existing firms in market will suffer a loss of profits due to the shrinkage of their market shares. However, under the increasing competition and consumer preference, it may be more beneficial for the existing firms who give up the production of products with old versions.

To summarize, the proposed algorithm and management insights drawn from this study provide practical suggestions for operations managers. However, several possible improvements to this model can be made. First, valuation bias of consumers can be incorporated. After purchasing products with a newer version, the difference between consumers' expected product functionality and the actual one may exist. Second, this work does not explicitly analyze the inventory decisions and after-market decisions. When consumers are sufficiently strategic, firms need to consider their inventory decisions in practice. All these paths serve as directions for future research.

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