HIDING OR SHARING? TECHNOLOGY UPGRADE, TECHNOLOGY SPILLOVER AND INFORMATION ASYMMETRY

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Abstract. One giant manufacturer (M1) upgrades a common supplier’s production technology through investments, while the supplier (it), holding the technology spillover information privately, may spill the upgraded technology over to a rival manufacturer (M2). This study examines whether the supplier should share the information of technology spillover with M1. We first find technology spillover hurts not only M1 but also the supplier and M2, when the production cost is high and the investment cost is low at a high level of the real technology spillover degree, no matter whether the supplier shares the technology spillover information or not. As such, it may be unwise for the supplier to implement technology spillover and unprofitable for M2 to take a free ride of technology spillover conditionally. Furthermore, when the supplier can receive more payoffs by spilling the upgraded technology over to M2 under certain conditions, it should share (hide) the technology spillover information, and such sharing (hiding) strategy may create a “win-win-win” outcome for the three players, if the supplier is of low (high)-spillover type and the real degree of technology spillover falls into a high range.

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1. Introduction

Many large-scale manufacturers have invested into upstream suppliers to upgrade their production technology in order that production costs can be reduced and quality of products or services can be improved [32, 34]. For instance, the electronic products maker Apple in 2019 invested $250 million [28] into Corning to support this supplier’s R&D of new mobile phone glasses; Intel invested $4.1 billion in semiconductor machinery maker ASML Holding N.V in 2012, to accelerate R&D of 450-millimeter (mm) wafers technology and extreme ultra-violet (EUV) lithography [13].

With investments from the manufacturer, R&D capabilities and production technology of the supplier would be improved and upgraded. However, the supplier may spill the upgraded technology over to other manufacturers [36] to receive more orders, when it works with multiple manufacturers. For example, Corning, who is also the supplier of smartphone manufacturers Huawei and Samsung, may use the new technology invested by Apple to...

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produce for these Apple’s rivals. When the common supplier offers products with new technology to the other manufacturers who act as free-riders without investments, it indeed serves as an informal channel for knowledge transfer [2], which incurs spillover of the new technology invested by one manufacturer. The effect of technology spillover has received much attention recently in OM field [1, 36], where the existing literature has a well discussion on this issue. However, models in these work assume the information of technology spillover is fully observed by all players. Indeed, technology spillover usually occurs after the supplier’s technology is upgraded, and the manufacturer may have less knowledge on such ex post effect at the stage of deciding to invest in the supplier’s technology. On the other hand, the supplier may hold the technology spillover information as private knowledge and hide this information to the manufacturer. This is because, as the technology spillover arises, the manufacturer’s incentive to invest in the supplier’s technology may be diminished due to the competition, which slows the technology growth of the supplier.

Motivated the above discussion, this study examines whether the supplier should share the technology spillover information with the manufacturer, who upgrades its production technology through investments, by addressing three questions. First, how does technology spillover impact the supplier’s as well as the manufacturers’ payoffs? Second, how does the supplier determine its information strategy (share or hide the technology spillover information) to the manufacturer who conducts the technology upgrade? Third, what are implications of the supplier’s information strategy for the manufacturers?

In this paper, we develop a stylized Stackelberg model in a supply chain comprising two manufacturers and one supplier. At the beginning, the supplier (it) establishes an information strategy for sharing the technology spillover information or not with one manufacturer (she), who intends to upgrade its production technology through investments. Knowing the information strategy, the manufacturer, denoted as M1, determines the upgraded technology level of the supplier. Thereafter, the supplier offers wholesale prices for M1 and the other manufacturer (he), denoted as M2, respectively. Finally, the two manufacturers simultaneously source products from the supplier and resell them to the market in Cournot competition.

We find technology spillover may be adverse to all firms under each information strategy. This study shows that technology spillover is always detrimental to M1, while implementing technology spillover may not be a wise choice for the supplier, and taking a free ride of technology spillover may not be a profitable behavior for M2 either, when the production cost is high and the investment cost is low at a high level of the real technology spillover degree under both of information strategies.

Our work offers the option of information strategy for the supplier, and reveals that strategies of sharing and hiding the technology spillover information may both yield a “win-win-win” outcome for the information observer (M1), the information owner (the supplier) and the free-rider (M2). If the supplier is of type low-spillover, we find M1 and the supplier will be more beneficial under the sharing information strategy than the hiding information strategy, and thus the supplier should share the technology spillover information with M1. In this situation, M2 is better off as well when the real degree of technology spillover falls into a high range. If the supplier is of high-spillover type and the real degree of technology spillover is in a high range, similarly, we find that the supplier should hide the technology spillover information, and such hiding strategy is better off for M1 and M2 as well.

The remainder of this paper is organized as follows. We review related literature in Section 2 and set up the model in Section 3. Our discussion on the case that the information of technology spillover is shared is arranged in Section 4. Section 5 examines the case that the information of technology spillover is hidden. We discuss the choice of information strategy and its implications on the three firms in Section 6. Finally, Section 7 concludes this work. All proofs are related to Appendix A.

2. Literature review

Our study is related to three streams of literature, which are (1) supplier development/improvement, (2) technology spillover and (3) information asymmetry in a supply chain. We review literature based on these streams, and discuss differences of this study from them.
The first stream of literature related to our work focuses on supplier development/improvement. In the OM community, a number of papers examine how to improve the supplier’s capacity (e.g., [4, 11, 29, 33]). Taylor and Plambeck [33] characterize optimal price-only and price-and-quantity promises of the buyer to motivate the supplier to invest in the capacity; while Qi et al. [29], Bai and Sarkis [4] and Fu et al. [11] study the direct investment strategy executed by the buyer to improve the supplier’s capacity or production. In addition, some scholars, such as Hsieh and Liu [14], Yan [40], Lee and Li [17] and Yang and Ouyang [41], discuss the supplier’s quality improvement. Here, Hsieh and Liu [14], Lee and Li [17] and Yang and Ouyang [41] investigate the quality investment and inspection strategies of the supply chain firms, while Yan [40] focuses on the contract efficiency of a decentralized supply chain when the manufacturer improves the product quality. Cost improvement of the supplier also attracts broad attention in the OM field. Bernstein and Kok [5] explore the dynamics of suppliers investments in cost-reduction initiatives over the life cycle of a product under different procurement approaches; Li and Wan [22] and Li [20] examine how a buyer, who sources from multiple suppliers, induces them to exert cost-reduction efforts. Liu et al. [25] investigate the interaction between the issue that the buyer upgrades the supplier’s production technology (cost reduction) through investments and the one that a rival buyer enters the market. Furthermore, supplier improvement as popular approach of mitigating supply risk is used in practice and studied in academic world (e.g., [15, 32, 34, 36]). These studies, except Wang et al. [36], usually compare the strategy of supply process improvement with the traditional policy of multiple-supplier sourcing to find which one is better for buyers. The difference in our study from this stream of literature is that we explore the supplier’s technology upgrade with consideration of technology spillover under asymmetric information.

In recent years, more and more works start to incorporate technology spillover into operation management, and their main concern is the effect of technology spillover on the operation decisions. Wang et al. [36] examine the influence of potential investment spillover on manufacturers’ incentives to improve suppliers reliability. Similarly, Chen et al. [9] consider technology spillover when a manufacturer and a retailer first cooperate to invest green R&D to find the effect of technology spillover on R&D cooperation. In a competing supply chain consisting of an original equipment manufacturer (OEM) and a contract manufacturer (CM), Chen and Chen [8] study how technology spillover, which may occur during the outsourcing from the OEM to the CM, impacts the OEMs sourcing decision. Yoon [44] also explores the spillover effect of a cost-reducing investment made by the supplier, but his focus is to find how that business phenomena affects the supplier’s encroachment into a channel and further the retailer’s response to the encroachment. Additionally, when a firm shares a common supplier with the other one, Agrawal et al. [1] focus on the optimal strategies for the firms to invest in their supplier, while they also examine how the interplay between spillover, competition, and returns from the investment at shared suppliers affect the investment strategy. Differently, some work incorporates quality spillover [27] and service spillover [7, 39] instead of technology spillover into operational decisions. Among these papers, the most relevant to our work is Xue et al. [39]. However, there exists several differences between it and our paper. First, Xue et al. [39] focuses on the impact of spillover effect on quantity decision timing in a dual-channel supply chain under asymmetry of demand information, while our paper aims to find how spillover effect and information structure of technology spillover affect the manufacturers technology investment decision as well as all firms response and performance in a supply chain consisting of two manufacturers and one supplier. Second, the spillover in Xue et al. [39] is that retail service invested by the retailer may better the manufacturers sale, while the spillover in our paper is that the suppliers upgraded technology invested by the manufacturer may benefit the other manufacturer. Third, the service investment level of the retailer is decided by the retailer itself in Xue et al. [39], while the upgraded technology of the supplier is determined by the manufacturer in our paper.

Our paper is also related to the literature on information asymmetry of a supply chain. In this line of research, a substantial amount of work considers information asymmetry under supply chain contract [30]. Cakanyildirim et al. [6], Loffler et al. [26] and Wang et al. [37] discuss how the asymmetric production cost information affects the supply chain contract in a supply chain consisting a buyer and a supplier. With a similar supply chain, Lei et al. [19] examines the optimal linear contract when demand and cost are disrupted simultaneously and the disruption information of both demand and cost is asymmetric. Differently, Xu et al. [38] and Lee and Yang [18] investigate a contract setting problem in a supply chain with a buyer and two suppliers, while the
former is based on the situation that the local supplier’s production cost is unknown to other parties and the later assumes the buyer has superior market information to the suppliers. The other literature group that considers information asymmetry extensively is channel encroachment in a supply chain. Most studies in this literature focus on the supplier’s/manufacturer’s encroachment behavior in the environment where the reseller is much more knowledgeable on demand information than the supplier/manufacturer (e.g., [23, 24, 31, 45]). In view of the demand information asymmetry between the supplier/manufacturer and the reseller, Huang et al. [16] and Zhao and Li [46] make a further discussion on the issue that how to encourage the reseller to share the demand information with the supplier/manufacturer when the upstream firm encroaches into the channel. Additionally, information asymmetry is incorporated into the sourcing strategy of a supply chain in some papers. For instance, Li and Debo [21] study the decision of a manufacturer (the buyer) in selecting between sole- and second-sourcing strategies for a noncommodity component, when the supplier with private cost information invests in capacity; Yang et al. [42] examine a buyers strategic use of a dual-sourcing option when suppliers possess private information about their disruption likelihood. Our work also takes information asymmetry into consideration in a supply chain. In contrast to the extant literature of this stream, we focus on the asymmetric information of technology spillover and discuss whether the supplier should share this information with the manufacturer who makes technology investments in the upstream partner.

3. Model

Our model is engaged in a supply chain comprising two manufacturers and one supplier, and the interactions among the three players are characterized through a single-period Stackelberg game. First, the supplier (it), who knows one manufacturer (she) intends to invest in its production technology, determines whether to share the technology spillover information or not with her. Then, the manufacturer, denoted as M1, upgrades the supplier’s production technology to \( t (t \in [0, 1]) \) level through a direct investment\(^1\), which incurs a reduction of the production cost from \( c > 0 \) to \( (1 - t)c \) for the supplier and the corresponding investment amounts \( \alpha^2 \). Here, \( \alpha \) is the investment cost at the new technology level \( t \), and the cost function with quadratic form has received widespread use in much operations literature such as Tang et al. [32], Yoon [44] and Liu et al. [25].

Thereafter, the supplier provides homogeneous products to the two manufacturers by setting wholesale prices \( w_1 \) and \( w_2 \) for them, respectively. We suppose products offered by the supplier have the same unit production cost \( c \). In response to the wholesale prices, the two manufacturers then make orders \( q_1 \) and \( q_2 \), respectively, from the supplier and resell them to end customers. We assume these products sold to customers by different manufacturers are partially substitutable or differentiated, since, aside from the brand difference, they may be processed further by the two manufacturers variously with negligible costs [25]. The two downstream firms’ competition in the end market is modelled by Cournot game with linear inverse demand function \( p_i = d - q_i - \gamma q_i \), which is adopted commonly in operational field (e.g., [3, 35, 43]). Here, \( p_i \) \( (i = 1, 2) \) and \( d \) \( (d > c) \) denote the market clearing price of manufacturer \( i \) and the market size, respectively, and \( \gamma \in (0, 1) \) measures substitutability of the two manufacturers’ products. Finally, the three players’ payoffs are realized after the market prices are cleared. Figure 1 illustrates the sequence of events.

As M1 invests into the supplier’s production technology, the upgraded technology may spill over to the other rival manufacturer (he), who is denoted as M2. That is, the supplier may satisfy M2’s orders by means of the new technology. We let \( \theta q_2 \) be the order quantity satisfied by the upgraded technology, and \( (1 - \theta)q_2 \) be the one satisfied by the original technology. Here, \( 0 \leq \theta \leq 1 \), measuring the degree of technology spillover, can be interpreted as the proportion of M2’s orders adopting new technology (the higher \( \theta \), the more orders producing by the new technology). Note that \( \theta \) is an exogenous variable in this study. Indeed, the extent of the technology spillover may depend on the configurations of the supplier’s production process [36] and its capacity.

\(^1\)Note that the downstream manufacturer determines the upgraded technology level and bears all investment costs in our model. Indeed, the well-funded manufacturer may invest in the impecunious supplier to improve the supply in reality. For example, Apple in 2017 invested $390 million in the supplier Finisar to upgrade the facility that produces vertical-cavity surface-emitting lasers (VCSELS), a key component of iPhone X [10]. In this example, the upgraded facility level may be determined by Apple so that VCSELS can satisfy the technical requirements of iPhone X and the facility upgrade can save investment costs.


If the supplier has only one production line, the full technology spillover may occur, i.e., \( \theta = 1 \). If the supplier has several production lines, some of which are upgraded but with limited capacity, the new technology may partially spill over or completely fail to spill over to M2, i.e., \( 0 < \theta < 1 \) or \( \theta = 0 \).

In this study, technology spillover degree, as private knowledge of the supplier, may not be shared with M1, who only knows the supplier’s types: high technology spillover degree (\( H \)) with the probability \( \beta \) or low technology spillover degree (\( L \)) with the probability \( 1 - \beta \). Specifically, a supplier of type \( j \in \{ H, L \} \) has a technology spillover degree \( \theta_j \), by which \( \theta_H \) and \( \theta_L \) correspond to high and low technology spillover degree, respectively, with \( \theta_H > \theta_L > 0 \). That is,

\[
\theta = \begin{cases} 
\theta_H, & \text{with probability } \beta, \\
\theta_L, & \text{with probability } 1 - \beta.
\end{cases}
\]

(3.1)

Such setup, in line with much literature such as Ha and Tong [12], Cakanyildirim et al. [6] and Li et al. [23], is adopted to make our analysis tractable.

Main notations are summarized in Table 1.

4. TECHNOLOGY UPGRADE UNDER SHARING INFORMATION

We first study the strategy that the supplier shares the technology spillover information with M1, who then knows the true degree of technology spillover. The equilibrium results in this strategy are shown in the following.

4.1. Equilibrium results

When M1 improves the technology level of the supplier under the sharing information strategy of technology spillover, her profit function is given by

\[
\Pi_1(q_1, t; q_2, w_1) = (d - q_1 - \gamma q_2)q_1 - w_1 q_1 - \frac{\alpha t^2}{2},
\]

(4.1)
where the superscript “*" denotes the strategy of sharing technology spillover information. M2, as a rival of M1, does not invest in the supplier’s production technology, and his profit function is given by

$$\Pi_2^s(q_2; q_1, w_2) = (d - q_2 - \gamma q_1)q_2 - w_2q_2. \quad (4.2)$$

Following Cournot competition, the two manufacturers decide their order quantities as

$$q_1^s(t_1, w_1) = \frac{2(d - w_1) - \gamma(d - w_2)}{4 - \gamma^2}, \quad (4.3)$$

$$q_2^s(t_1, w_2) = \frac{2(d - w_2) - \gamma(d - w_1)}{4 - \gamma^2}. \quad (4.4)$$

Given the two manufacturers’ best responses above, the supplier’s profit function can be shown as follow.

$$\Pi_2^s(w_1, w_2; t) = (w_1 - (1 - t)c)q_1^s(w_1, w_2) + (w_2 - (1 - t)c)\theta q_2^s(w_1, w_2) + (w_2 - c)(1 - \theta)q_2^s(w_1, w_2). \quad (4.5)$$

The first term of equation (4.5) is payoff obtained from M1. Note that the production cost is reduced from c to $(1 - t)c$. This is because the production technology level is upgraded to $t$ by M1. The second and third terms in equation (4.5) are profit earned from M2, where the second one is yielded by adopting the new technology and the third one is generated by using the original technology. By maximizing equation (4.5), we can receive the locally optimal wholesale prices for the two manufacturers,

$$w_1^s = \frac{d + (1 - t)c}{2}, \quad (4.6)$$

$$w_2^s = \frac{d + \theta(1 - t)c + (1 - \theta)c}{2}. \quad (4.7)$$

Clearly, both of wholesale prices decline as the supplier’s new production technology $t$. In addition to the upgraded technology, $w_2^s(t)$ depends on the technology spillover degree $\theta$, the higher of which, the much lower wholesale price of M2.

Having the order quantities $q_1^s(w_1, w_2; t)$ and $q_2^s(w_1, w_2; t)$ and the wholesale prices $w_1^s(t)$ and $w_2^s(t)$, M1 can obtain the optimal upgraded technology level by optimizing her profit function equation (4.1). Then,

$$t^* = \frac{(2 - \gamma \theta)(2 - \gamma)(d - c)c}{2(4 - \gamma^2)^2 \alpha - (2 - \gamma \theta)^2 c^2}. \quad (4.8)$$

Because $t^* \in [0, 1]$, we have $\alpha \geq \alpha^*$, where $\alpha^* = \frac{(2 - \gamma \theta)(2 - \gamma)(d - c)c + (2 - \gamma \theta)^2 c^2}{2(4 - \gamma^2)^2}$. The inequality $\alpha \geq \alpha^*$ implies that investing cost in the supplier’s production technology is high enough, which cannot make the setting trivial as well [44].

Given the above results under the sharing information strategy, we can receive the globally optimal decisions, which are denoted as $q_1^s, q_2^s, w_1^s, w_2^s$ and $t^*$, respectively, and the optimal profits of M1, M2 and the supplier, which are denoted as $\Pi_1^s, \Pi_2^s$ and $\Pi_2^s$, respectively.

4.2. The effect of technology spillover

In this section, we examine how technology spillover affects the equilibrium results when the technology spillover information is known to all parties. We first explore the effect of technology spillover on the upgraded technology at the supplier.

Lemma 4.1. The level of upgraded technology at the supplier decreases in the degree of technology spillover under the sharing information strategy.
Intuitively, M1 will upgrade the supplier’s technology to a lower level when the degree of technology spillover becomes higher. This is because M1’s incentive to improve the supplier’s technology will be reduced as the upgraded technology spills over to the rival M2, who fights for market share with M1.

Now, we examine the impact of technology spillover on the supplier’s wholesale prices, and the following lemma shows us the answers.

Lemma 4.2. The effect of technology spillover on the wholesale prices is shown as follows,

(1) the wholesale price for M1 increases in the degree of technology spillover, i.e., \(\partial w_1^* / \partial \theta > 0\);
(2) the effect of technology spillover on the wholesale price for M2 is specified into the following cases,

(i) if \(0 < c \leq \bar{c}\), \(\partial w_2^* / \partial \theta < 0\);
(ii) if \(\bar{c} < c < d\), there exists a \(\bar{\alpha}\) such that \(\partial w_2^* / \partial \theta > 0\) for \(\alpha^s \leq \alpha < \bar{\alpha}\), and \(\partial w_2^* / \partial \theta \leq 0\) for \(\alpha \geq \bar{\alpha}\),

where \(\bar{c} = \frac{(1-\gamma \theta)(2-\gamma)\hat{c}}{(1-\gamma \theta)(2-\gamma+\gamma \theta)\gamma \theta} \), \(\bar{\alpha} = \frac{(2-\gamma \theta)^2 \hat{c}^2}{2(1-\gamma \theta)(4-\gamma \theta)^2}\).

Item (1) of the above lemma shows us that the wholesale price provided for M1 always increases as the upgraded technology spills over to M2. The reason for such a result is that a greater degree of technology spillover will strongly reduce M1’s incentive to upgrade the supplier’s technology and then cause a less reduction of the production cost, which induces the supplier to set a higher wholesale price in response to the change of production cost.

Note that the effect of technology spillover on M2’s wholesale price depends on the production cost and the investment cost. When the production cost of the supplier is low or the production cost is high and the investment cost is high as well, the wholesale price of M2 decreases in the degree of technology spillover. This is because more orders of M2 will be produced at a lower cost as the degree of technology spillover becomes greater. However, when the production cost is high but the investment cost is low, M2’s wholesale price increases as the new technology spills over to him with an increasing degree. Indeed, M1 would like to upgrade the supplier’s technology to a higher level given a high production cost and a low investment cost so that the double marginal effect has a great alleviation, if there is no technology spillover. Nevertheless, on such cost conditions with technology spillover, M1 will upgrade the supplier’s technology with a very low level to prevent the rival benefitting much more from technology spillover, which consequently brings a higher wholesale price for M2.

Next, we discuss how technology spillover affects the two manufacturers’ order quantities, and we have the following results.

Proposition 4.3. The effect of technology spillover on the order quantities is shown as follows,

(1) the order quantity of M1 decreases in the degree of technology spillover, i.e., \(\partial q_1^* / \partial \theta < 0\);
(2) the effect of technology spillover on the order quantity of M2 is specified into the following cases,

(i) if \(0 \leq \theta < \frac{2}{\gamma}\), \(\partial q_2^* / \partial \theta > 0\);
(ii) if \(\frac{2}{\gamma} \leq \theta \leq 1\), there exists a \(\hat{c}\) such that \(\partial q_2^* / \partial \theta > 0\) when \(0 < c < \hat{c}\) and the results depend on \(\alpha\) when \(\hat{c} \leq c < d\), that is, \(\partial q_2^* / \partial \theta < 0\) when \(\alpha^s \leq \alpha < \hat{\alpha}\) and \(\partial q_2^* / \partial \theta \geq 0\) when \(\alpha \geq \hat{\alpha}\),

where \(\hat{c} = \frac{(4-4\gamma \theta+\gamma \theta^2)(2-\gamma)\hat{c}}{2(4-4\gamma \theta+\gamma \theta^2)(2-\gamma)}\), \(\hat{\alpha} = \frac{(4-\gamma \theta)^2 (2-\gamma \theta)^2 \hat{c}^2}{2(4-4\gamma \theta+\gamma \theta^2)(1-\gamma \theta)^2}\).
becomes higher. However, if the production cost goes beyond \( \hat{c} \) and the investment cost is lower than \( \hat{\alpha} \), M2 will reduce his order quantity when the upgraded technology spills over to him along with a greater degree. This is because, as shown in Lemma 4.2, the supplier would raise the wholesale price for M2 as \( \theta \) increases under the similar conditions of the production cost and the investment cost.

In what follows, we examine the impact of technology spillover on the three firms’ profits under the sharing information strategy. The proposition below shows us the results.

**Proposition 4.4.** The effect of technology spillover on the three firms’ profits is shown as follows,

1. the profit of M1 decreases in the degree of technology spillover, i.e., \( \partial \Pi_1^{**}/\partial \theta < 0 \);
2. the effect of technology spillover on M2’s profit is similar to that on his order quantity;
3. the effect of technology spillover on the supplier’s profit is specified into the following cases,
   - if \( 0 \leq \theta < \min\{1, \frac{2-\gamma}{2\gamma}\} \), there exists a \( \tilde{c} \) such that \( \partial \Pi_S^{**}/\partial \theta > 0 \) when \( 0 < c < \tilde{c} \) and the results depend on \( \alpha \) when \( \tilde{c} \leq c < \hat{c} \), that is, \( \partial \Pi_S^{**}/\partial \theta > 0 \) when \( \alpha^s < \alpha < \hat{\alpha} \) and \( \partial \Pi_S^{**}/\partial \theta \geq 0 \) when \( \alpha \geq \hat{\alpha} \);
   - if \( \min\{1, \frac{2-\gamma}{2\gamma}\} \leq \theta \leq 1 \), \( \partial \Pi_S^{**}/\partial \theta < 0 \),

where \( \tilde{c} \) and \( \hat{\alpha} \) are shown in Appendix A.

Intuitively, the profit of M1 will decrease as the upgraded technology spills over to her rival\(^2\), because, as shown in Lemma 4.2 and Proposition 4.3, the supplier provides a higher wholesale price, and M1 makes less orders. This result is illustrated in Figure 2, where descending curves are plotted under different investment costs, when the production cost is low (\( c = 30 \)) and high (\( c = 80 \)), respectively. Note that \( \alpha \) is set as values of 1400, 1600 and 1800 instead of 400, 600 and 800 in Figure 2b, because we need to guarantee \( \alpha \geq \alpha^s \) holds.

Indeed, receiving the equilibrium results, we can get \( \Pi_2^{**} = (q_2^{**})^2 \), thus the effect of technology spillover on M2’s profit is similar to that on his order quantity. Figure 3 confirms this result with fours subgraphs. In Figures 3a and 3b, where \( 0 \leq \theta \leq \gamma/2 \), M2’s profit increases in the degree of technology spillover under different investment costs, given a low production cost (\( c = 30 \)) and a high production cost (\( c = 80 \)), respectively. Note that a high investment cost is favorable to M2, as shown in Figures 3a and 3b. This is because M1 would reduce technology investments into the supplier and receive less payoff at a high investment cost, under which M2, as

\(^2\)A natural question based on this result is how M1 designs or develops mechanism(s) to avoid technology spillover or reduce the negative effects. This issue, as an important extension in future, is not examined in this study, because it seems to deviate from our focus that whether the supplier shares the technology information with M1 or not. On the other hand, M1 still upgrades the supplier’s technology even though there exists technology spillover \([25]\), and thus it is justified to examine the supplier’s information strategy of technology spillover to some extent.
M1’s rival, can obtain a good profit when the degree of technology spillover is small. When $\gamma/2 \leq \theta \leq 1$, M2’s payoff still increases in $\theta$ given a production cost ($c = 30$) less than $\hat{c}$, as shown in Figure 3c. However, if the production cost ($c = 90$) goes beyond $\hat{c}$, M2’s payoff will decrease in $\theta$ when the investment cost is less than $\hat{\alpha}$, as illustrated by the dark curve with rounds in Figure 3d. In contrast, provided the investment cost is greater than $\hat{\alpha}$, say $\alpha = 1200$, $\alpha = 1400$, M2’s payoff will increase in $\theta$.

Item (3) in Proposition 4.4 shows how technology spillover affects the supplier’s payoff, which is specified into two cases in terms of $\theta$. When $\theta$ falls into a small range, i.e., $\theta \in [0, \min(1, 2\gamma/\gamma)]$, the effect of technology spillover on the supplier’s payoff relies on the production cost and the investment cost. If the production cost is low (see Fig. 4a) or it is high while the investment is high as well (see Fig. 4c), the supplier’s profit increases in the degree of technology spillover. We know that there are two direct outcomes incurred by technology spillover: one is that the double marginal effect between the supplier and M2 is alleviated as the new technology is used to satisfy M2’s orders, the other is that the level of upgraded technology at the supplier is reduced as result of the dulled investment incentive of M1. Under the condition of a low production cost, M1’s willingness to upgrade the supplier’s technology is mild, since the double marginal problem between them is not significant. Therefore, the influence of technology spillover on the level of upgraded technology is weak, and then the first outcome

3In fact, $\alpha = 1000$ is greater than $\hat{\alpha}$ when $\theta$ is close to $\gamma/2 = 0.4$, thus the curve increases at the beginning.

4Note that the curves decrease in Figure 4c when $\theta$ is close to $(2 - \gamma)/2\gamma = 0.75$. This is because the given values of $\alpha$ become less than $\hat{\alpha}$ as $\theta$ trends to $(2 - \gamma)/2\gamma$. 
caused by technology spillover is dominated over the second one. Consequently, the supplier’s payoff will go up as the degree of technology spillover increases. Such result arises as well given a high production cost and a high investment cost, because the high investment cost will moderate M1’s willingness to upgrade technology despite the significant double marginal problem with a high production cost. If the production cost is high and the investment cost is low (see Fig. 4b), however, the supplier’s payoff decreases as the degree of technology spillover becomes higher, which indicates the supplier is hurt by technology spillover. In fact, the high production cost and the low investment cost may drive the manufacturer to upgrade the technology to a higher level with much more willingness. Nevertheless, such willingness may be mitigated by technology spillover, resulting in a huge loss for the supplier.

When \( \theta \) locates in a high range, \( i.e., \min\{1, \frac{2-\gamma}{2\gamma}\} \leq \theta \leq 1 \), the supplier’s profit always decreases in \( \theta \). This is because the high degree of technology spillover will strongly dull M1’s investment incentive of technology upgrade, enabling the supplier to suffer a huge loss, no matter what the other conditions are. This result is illustrated by Figure 4d, where, given three different values of the investment cost, three decreasing curves in \( \theta \) are plotted.

In short, this section discusses the effect of technology spillover on the three players when the supplier shares the technology spillover information with M1. Intuitively, technology spillover would hurt M1 as the upgraded technology benefits her rival. However, spilling the upgraded technology to M2 may not be a wise choice for the
supplier, and taking a free ride of the upgraded technology may not be a profitable behavior for M2. This is because the two firms may receive a less payoff when technology spillover occurs, as shown in Proposition 4.4.

5. Technology upgrade under hiding information

In this section, we examine the strategy of hiding information, where the supplier does not share the degree of technology spillover with M1 when she upgrades its production technology through investments. Equilibrium results are shown first in the following.

5.1. Equilibrium results

Although M1 is uninformed about technology spillover, she can form the belief on the supplier’s type by observing the supplier’s wholesale price\(^5\). However, after given the wholesale price \(w_1\) and the wholesale price \(w_2\), M1 and M2 determine their order quantities under Cournot competition as follows,

\[
q_1^{a1}(w_1, w_2) = \frac{2(d - w_1) - \gamma(d - w_2)}{4 - \gamma^2}, \tag{5.1}
\]

\[
q_2^{a1}(w_1, w_2) = \frac{2(d - w_2) - \gamma(d - w_1)}{4 - \gamma^2}, \tag{5.2}
\]

by optimizing equations (4.1) and (4.2), respectively, where the superscript “a” refers to the strategy of hiding information. Note that \(q_i^{a1}(w_1, w_2)\) is not dependent on technology spillover directly. Hence, M1’s response function is unique, no matter what wholesale price, which can reflect the degree of technology spillover, is observed by her.

In anticipation of M1’s reaction as well as M2’s, the supplier, who knows the true technology spillover \(\theta_j\), decides optimal wholesale price for M1 as well as the one for M2 by maximizing

\[
\Pi^*_S(w_1, w_2; t) = (w_1 - (1 - t)c)q_1^{a1}(w_1, w_2) + (w_2 - (1 - t)c)\theta_jq_2^{a1}(w_1, w_2) + (w_2 - c)(1 - \theta_j)q_2^{a1}(w_1, w_2). \tag{5.3}
\]

Since M1 has a unique reaction function (so does M2 indeed), the supplier’s profit function only has one formulation, as shown by equation (5.3). Then, the locally optimal solutions of equation (5.3) are

\[
w_1^{a1}(t) = \frac{d + (1 - t)c}{2}, \tag{5.4}
\]

\[
w_2^{a1}(t) = \frac{d + \theta_j(1 - t)c + (1 - \theta_j)c}{2}. \tag{5.5}
\]

Receiving the order quantities \(q_1^{a1}(w_1, w_2)\) and \(q_2^{a1}(w_1, w_2)\) and the wholesale prices \(w_1^{a1}(t)\) and \(w_2^{a1}(t)\), we begin to examine M1’s decision of technology upgrade. Since M1 cannot observe the true degree of technology spillover, she chooses the upgraded technology level \(t\) as the solution to maximize the following function,

\[
\Pi^*_1(t) = (1 - \beta)[d - q_1^{a1}(w_1, w_2 | \theta_L) - \gamma q_2^{a1}(w_1, w_2 | \theta_L) - w_1^{a1}(t | \theta_L)]q_1^{a1}(w_1, w_2 | \theta_L)

+ \beta[d - q_1^{a1}(w_1, w_2 | \theta_H) - \gamma q_2^{a1}(w_1, w_2 | \theta_H) - w_1^{a1}(t | \theta_H)]q_1^{a1}(w_1, w_2 | \theta_H) - \alpha \frac{t^2}{2}. \tag{5.6}
\]

In equilibrium, the optimal upgraded technology level is

\[
t^{a*} = \frac{[(1 - \beta)(2 - \gamma \theta_L) + \beta(2 - \gamma \theta_H)](2 - \gamma)(d - c)c}{2(4 - \gamma^2)\alpha - [(1 - \beta)(2 - \gamma \theta_L)^2 + \beta(2 - \gamma \theta_H)^2]c^2}. \tag{5.7}
\]

Since \(t^{a*} \in [0, 1]\), we have \(\alpha > \alpha^a\), where \(\alpha^a = \frac{[(1 - \beta)(2 - \gamma \theta_L) + \beta(2 - \gamma \theta_H)](2 - \gamma)(d - c)c + [(1 - \beta)(2 - \gamma \theta_L)^2 + \beta(2 - \gamma \theta_H)^2]c^2}{2(4 - \gamma^2)\alpha - [(1 - \beta)(2 - \gamma \theta_L)^2 + \beta(2 - \gamma \theta_H)^2]c^2}\).

With the equilibrium results above, we can get the globally optimal decisions, which are denoted as \(q_1^{a*}, q_2^{a*}, w_1^{a*}, w_2^{a*}\) and \(t^{a*}\), respectively, and the three players’ optimal profits, denoted as \(\Pi^*_1, \Pi^*_2\), and \(\Pi^*_S\), respectively.

\(^5\)We do not give the formulation of M1’s belief, because, as we will show later, equilibrium results in this study do not depend on M1’s belief.
5.2. The effect of technology spillover

From now on, we start to discuss how technology spillover influences the optimal results obtained in the previous section. First, the effect of technology spillover on the upgraded technology level is shown in the following.

Lemma 5.1. The level of upgraded technology at the supplier decreases in the degree of technology spillover under the hiding information strategy, i.e., $\frac{\partial q^*_M}{\partial \theta} < 0$, $\frac{\partial q^*_M}{\partial \theta} < 0$.

Even though M1 does not know the exact information on technology spillover, similar to the strategy of sharing information, she will choose a lower level of the upgraded technology as well when the supplier spills the new technology over to M2, as shown in Lemma 5.1.

The following lemma shows us the effect of technology spillover on the optimal wholesale prices in strategy that the technology spillover information is incomplete for M1.

Lemma 5.2. Under the hiding information strategy, the effect of technology spillover on the wholesale prices is shown as follows,

(1) the wholesale price for M1 increases in the degree of technology spillover, i.e., $\frac{\partial w_1^a}{\partial \theta_j} > 0$, $j \in \{L, H\}$;
(2) the effect of technology spillover on the wholesale price for M2 is specified into the following cases,
   (i) if $0 < c < \tilde{c}_j$, $\frac{\partial w_2^a}{\partial \theta_j} < 0$;
   (ii) if $\tilde{c}_j < c < d$, there exists a $\tilde{\alpha}_j$ such that $\frac{\partial w_2^a}{\partial \theta_j} > 0$ for $\alpha < \tilde{\alpha}_j$ and $\frac{\partial w_2^a}{\partial \theta_j} < 0$ for $\alpha > \tilde{\alpha}_j$;
   (iii) $\frac{\partial w_2^a}{\partial \theta_j} > 0$ always holds;

where $-j = H$ if $j = L$, and $-j = L$ otherwise, and $\tilde{c}_j$ and $\tilde{\alpha}_j$ are shown in Appendix A.

Under the hiding information strategy, the wholesale price of M1, as shown in item (1) of Lemma 5.2, climbs up with the increase of the technology spillover degree, no matter what type of the supplier is. This is because M1 will choose a lower level of the upgraded technology, which cannot dull the double marginal effect too much, as the new technology spills over to M2 over to 5.1.

Item (2) of Lemma 5.2 states the effect of technology spillover on the wholesale price of M2, and the result depends on the production cost $c$ and the investment cost $\alpha$. If either the production cost is low or it is high while the investment cost is high, M2’s wholesale price decreases in the real degree of technology spillover $\theta_j$. However, given the condition that the production cost is high but the investment cost is low, the wholesale price $w_2^a$ ascends in $\theta_j$. Notice that the effect of the real degree of technology spillover on M2’s wholesale price is similar to that of the sharing information strategy, by comparing Lemma 5.2 with Lemma 4.2. Then, the corresponding explanations on Lemma 4.2 are applicable to these results in Lemma 5.2. Moreover, Lemma 5.2 shows M2’s wholesale price increases in the technology spillover degree $\theta_j$ when the real degree of technology spillover is $\theta_j$. The reason incurring this result can be found directly through equation (5.5), where $w_2^a$ decreases in the upgraded technology level, which falls off as $\theta_j$ becomes higher (please see Lem. 5.1).

Proposition 5.3. Under the hiding information strategy, the effect of technology spillover on the order quantities is shown as follows,

(1) the order quantity of M1 decreases in the degree of technology spillover, i.e., $\frac{\partial q_1^L}{\partial \theta_j} < 0$, $j \in \{L, H\}$;
(2) the effect of technology spillover on the order quantity of M2 is specified into the following cases,
   (a) when $j = L$
      (i) if $\theta_L < \min\{\frac{c}{L}, \theta_H\}$, $\frac{\partial q_2^L}{\partial \theta_L} > 0$; $\frac{\partial q_2^H}{\partial \theta_H} > 0$;
      (ii) if $\min\{\frac{c}{L}, \theta_H\} \leq \theta_L < \theta_H$, there exists a $\tilde{\alpha}_L$ such that $\frac{\partial q_2^L}{\partial \theta_L} > 0$ when $0 < c < \tilde{c}_L$ and the results depend on $\alpha$ when $\tilde{c}_L \leq c < d$, that is, $\frac{\partial q_2^L}{\partial \theta_L} < 0$ when $\alpha < \tilde{\alpha}_L$ and $\frac{\partial q_2^L}{\partial \theta_L} > 0$ when $\alpha \geq \tilde{\alpha}_L$;
   (b) when $j = H$
      (i) if $\theta_L \leq \theta_H < \max\{\frac{c}{H}, \theta_L\}$, $\frac{\partial q_2^H}{\partial \theta_H} > 0$; $\frac{\partial q_2^L}{\partial \theta_L} > 0$;
(ii) if $\max\{\frac{c}{2}, \theta_H\} \leq \theta_H \leq 1$, there exists a $\hat{c}_H$ such that $\partial q_2^{\ast}/\partial \theta_H > 0$ when $0 < c < \hat{c}_H$ and the results depend on $\alpha$ when $\hat{c}_H \leq c < d$, that is, $\partial q_2^{\ast}/\partial \theta_L < 0$ when $\alpha^a \leq \alpha < \hat{\alpha}_L$ and $\partial q_2^{\ast}/\partial \theta_H \geq 0$ when $\alpha \geq \hat{\alpha}_H$; $\partial q_2^{\ast}/\partial \theta_L < 0$;

where $\hat{c}_L$, $\hat{\alpha}_L$, $\hat{c}_H$ and $\hat{\alpha}_H$ are shown in Appendix A.

Item (1) of Proposition 5.3 characterizes the impact of technology spillover on M1’s order quantity when the technology spillover information is withheld, following the result that her order quantity decreases in the degree of technology spillover, no matter what type of the supplier is. The reason incurring such results is similar to that of the information sharing strategy, in which the upgraded technology level goes down and thus the wholesale price goes up as M1’s incentive to upgrade the supplier’s technology is eased by the technology spillover effect.

Item (2) of Proposition 5.3 establishes the influence of technology spillover on M2’s order quantity, and the result depends on the range that $\theta_j$ locates into. Taking $j = L$ as an example, if the real degree of technology spillover $\theta_L$ is in a small range, i.e., $0 \leq \theta_L < \min\{\frac{c}{2}, \theta_H\}$, the order quantity of M2 increases in $\theta_L$ as well as $\theta_H$.

This is because the upgraded technology, which reduces the double marginal effect, is adopted in the production of M2’s orders. If the real degree of technology spillover $\theta_L$ falls into a high range, that is, $\theta_L \in [\min\{\frac{c}{2}, \theta_H\}, \theta_H)$, the effect of $\theta_L$ on $q_2^{\ast}$ relies on the production cost and the investment cost. When the production cost is lower than the threshold $\hat{c}_L$ or it is greater than $\hat{c}_L$, while the investment cost is greater than the threshold $\hat{\alpha}_L$, the order quantity of M2 increases in the low technology spillover degree $\theta_L$. When the production cost goes beyond the threshold $\hat{c}_L$, but the investment cost is below the threshold $\hat{\alpha}_L$, in contrast, $q_2^{\ast}$ decreases in $\theta_L$. As shown in (i) and (ii) of Lemma 5.2 when $j = L$, the effect of technology spillover on the M2’s wholesale price has two different outcomes, and then we can receive two different outcomes on M2’s order quantity correspondingly. In addition, $q_2^{\ast}$ always decreases in the high technology spillover degree $\theta_H$, since the wholesale price $w_2^{\ast}$ increases in it, as shown in (iii) of Lemma 5.2 when $j = L$. In a similar way, we can discuss the effect of technology spillover on M2’s order quantity when $j = H$.

In the following, we examine how technology spillover impacts the three firms’ profits under the hiding information strategy.

**Proposition 5.4.** The effect of technology spillover on the three firms’ profits is shown as follows,

1. the profit of M1 decreases in the degree of technology spillover, i.e., $\partial \Pi_1^{\ast}/\partial \theta_j < 0$, $j \in \{L, H\}$;
2. the effect of technology spillover on M2’s profit is similar to that on his order quantity;
3. the effect of technology spillover on the supplier’s profit is specified into the following cases,
   (a) when $j = L$
   (i) if $0 \leq \theta_L < \min\{\theta_H, \theta_L^U\}$, there exists a $\tilde{c}_L$ such that $\partial \Pi_2^{\ast}/\partial \theta_L > 0$ when $0 < c < \tilde{c}_L$ and the results depend on $\alpha$ when $\tilde{c}_L \leq c < d$, that is, $\partial \Pi_2^{\ast}/\partial \theta_L < 0$ when $\alpha^a \leq \alpha < \tilde{\alpha}_L$ and $\partial \Pi_2^{\ast}/\partial \theta_L \geq 0$ when $\alpha \geq \tilde{\alpha}_L$;
   (ii) if $\min\{\theta_H, \theta_L^U\} \leq \theta_L < \theta_H$, $\partial \Pi_2^{\ast}/\partial \theta_L \leq 0$;
   (iii) $\partial \Pi_2^{\ast}/\partial \theta_H < 0$ always holds;
   (b) when $j = H$
   (i) if $\theta_L < \theta_H < \max\{\theta_L, \theta_H^U\}$, there exists a $\tilde{c}_H$ such that $\partial \Pi_2^{\ast}/\partial \theta_H > 0$ when $0 < c < \tilde{c}_H$ and the results depend on $\alpha$ when $\tilde{c}_H \leq c < d$, that is, $\partial \Pi_2^{\ast}/\partial \theta_H < 0$ when $\alpha^a \leq \alpha < \tilde{\alpha}_H$ and $\partial \Pi_2^{\ast}/\partial \theta_H \geq 0$ when $\alpha \geq \tilde{\alpha}_H$;
   (ii) if $\max\{\theta_L, \theta_H^U\} \leq \theta_H \leq 1$, $\partial \Pi_2^{\ast}/\partial \theta_H \leq 0$;
   (iii) $\partial \Pi_2^{\ast}/\partial \theta_L < 0$ always holds;

where $\theta_L^U = \min\{\frac{(1-\beta)(2-\gamma)+\beta(2-\gamma\theta_L)}{2(1-\beta)}, 1\}$, $\theta_H^U = \min\{\frac{\beta(2-\gamma)(1-\beta)(2-\gamma\theta_L)}{2\gamma(1-\beta)}, 1\}$, and $\tilde{c}_L$, $\tilde{\alpha}_L$, $\tilde{c}_H$ and $\tilde{\alpha}_H$ are shown in Appendix A.

Item (1) of Proposition 5.4 shows that M1 is always hurt by the technology spillover effect, no matter whether the supplier is of high-spillover type or low-spillover type. This result is caused by the fact that technology
spillover alleviates M1’s incentive to upgrade the supplier’s technology and aggravates the double marginal problem relatively. Figure 5 illustrates this result with three different values of the investment cost.

Like the strategy of sharing information, we find the effect of technology spillover on M2’s payoff is similar to that on his order quantity in the strategy of hiding information, as shown in item (2) of Proposition 5.3. Indeed, M2’s payoff $\Pi_{a^*}^2$ equals $(q_{a^*}^2)^2$, i.e., $\Pi_{a^*}^2 = (q_{a^*}^2)^2$, thus the result in item (2) of Proposition 5.4 holds.

What we are more concerned about is the effect of the real technology spillover, and we here only illustrate the impact of $\theta_L$ on M2’s payoff through Figure 6 when the supplier is of low-spillover type. Figure 6a shows that M2’s payoff increases in $\theta_L$ given different values of the investment cost when $\theta_L < \min\{\gamma/2, \theta_H\}$, which is consistent with the theoretical finding (please see the result (i) in item (2) of Prop. 5.3 when $j = L$). We use Figures 6b–6d to illustrate the result (ii) in item (2) of Proposition 5.3 when $j = L$. Figure 6b presents similar trend of curves to Figure 6a, and it confirms the finding when $\theta_L \geq \min\{\gamma/2, \theta_H\}$ and $\tilde{c}_L < c < \hat{c}_L$. When $c > \hat{c}_L$ holds under the condition $\theta_L \geq \min\{\gamma/2, \theta_H\}$, M2’s payoff decreases in $\theta_L$ if $\alpha < \tilde{\alpha}_L$, as shown in Figure 6c, while it increases in $\theta_L$ if $\alpha \geq \alpha_L$, as shown in Figure 6d.

Note that the effect of the real technology spillover degree on the supplier’s payoff under the two information strategies is alike after comparing item (3) of Proposition 5.4 with item (3) of Proposition 4.4. Therefore, the explanation on item (3) of Proposition 4.4 is appropriate for the results of part (i) and (ii) in item (3) of Proposition 5.4 when $j = L$ or $j = H$. Our previous analysis has already shown the real technology spillover degree has analogous influence on the optimal decisions (wholesale prices and order quantities), regardless of the spillover information is shared or hidden. It is nature to receive similar results on how the real technology spillover degree affects the supplier’s payoff in both sharing and hiding information strategies. We use Figure 7 to illustrate this result in four cases, which correspond to four subfigures. In the case that $0 \leq \theta_L < \min\{\theta_H, \theta_L^U\}$ and $0 < c < \tilde{c}_L$, Figure 7a displays that the supplier’s payoff always increases in $\theta_L$, whereas it decreases in $\theta_L$, as shown in Figure 7b, in the case that $0 \leq \theta_L < \min\{\theta_H, \theta_L^U\}$, $\tilde{c}_L \leq c < d$ and $\alpha^U < \alpha < \tilde{\alpha}_L$. When $\alpha$ becomes greater than $\tilde{\alpha}_L$, and meanwhile the inequalities $0 \leq \theta_L < \min\{\theta_H, \theta_L^U\}$ and $\tilde{c}_L \leq c < d$ hold, the supplier will be better off as $\theta_L$ increases, as shown in Figure 7c. Furthermore, Figure 7d illustrates the result in the case that $\min\{\theta_H, \theta_L^U\} \leq \theta_L < \theta_H$, where the supplier is worse off as $\theta_L$ increases.

Item (3) of Proposition 5.4 also exhibits that the supplier’s payoff decreases in the technology spillover degree $\theta_H$ (or $\theta_L$) when the real technology spillover value is $\theta_L$ (or $\theta_H$). The reason for this result is that the supplier

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6 Notice that M2’s payoff increases in the investment cost, as shown in Figure 6a. This is because of the competition and the small degree of technology spillover.

7 Note that the curves increase at the beginning in Figure 6c. This is because $\tilde{\alpha}_L$ is less than the given values of the investment cost at the starting points.
receives less orders from the two manufacturers, as shown in Proposition 5.3, when the possible technology spillover degree becomes much larger.

Summarily, we mainly examine the effect of technology spillover on the three players’ payoffs, when the supplier hides the technology spillover information to M1. In this scenario, no matter whether the supplier is of high-spillover type or low-spillover type, technology spillover always hurts M1, while it does not always benefit the supplier and M2. In other words, implementing technology spillover may not be a sensible choice for the supplier, and employing the spillovered technology may not be a beneficial behavior for M2. This finding is similar to that in the scenario of sharing technology spillover information.

6. Option and implications of information strategies

In this section, we explore how the supplier makes a choice between sharing and hiding information strategies and how information strategies impact the two manufacturers’ decisions as well as their payoffs.

6.1. Option of information strategies for the supplier

We first discuss the supplier’s option on the information strategies by comparing its optimal payoffs between sharing information and hiding information. Recall that the supplier decides wholesale prices for the two man-

Figure 6. The impact of $\theta_L$ on $\Pi_2^*$. 

receives less orders from the two manufacturers, as shown in Proposition 5.3, when the possible technology spillover degree becomes much larger.

Summarily, we mainly examine the effect of technology spillover on the three players’ payoffs, when the supplier hides the technology spillover information to M1. In this scenario, no matter whether the supplier is of high-spillover type or low-spillover type, technology spillover always hurts M1, while it does not always benefit the supplier and M2. In other words, implementing technology spillover may not be a sensible choice for the supplier, and employing the spillovered technology may not be a beneficial behavior for M2. This finding is similar to that in the scenario of sharing technology spillover information.

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Manufacturers to receive the optimal payoff. These optimal decisions as well as payoffs under the two information strategies are compared in the following proposition.

**Proposition 6.1.** The comparison of the supplier’s optimal results is specified into the following cases,

1. If \( j = L \), \( \Pi_s^* > \Pi_S^* \), \( w_1^* < w_1^a \) and \( w_2^* < w_2^a \) hold;
2. If \( j = H \), \( \Pi_S^* < \Pi_S^* \), \( w_1^* > w_1^a \) and \( w_2^* > w_2^a \) hold.

Proposition 6.1 shows that the option of information strategy and the effect of information strategies depend on the supplier’s type. If the supplier is a low-spillover type, i.e., \( j = L \), the supplier obtains a higher payoff and the two manufacturers receive less wholesale prices by sharing information than hiding information. Therefore, the supplier should share the technology spillover information with M1. If the supplier is a high-spillover type, i.e., \( j = H \), the results are inverse. Then, it is better for the supplier to withhold the technology spillover information. The reason incurring two kinds of results is that the upgraded technology levels are discrepant for different types of the supplier. When \( j = L \) (\( j = H \)), the upgraded technology level is much higher (lower) under the strategy of sharing information, and thus the double marginal effect will be mitigated significantly (insignificantly). As a result, lower (higher) wholesale prices are offered by the supplier, who receives more (less) benefits, through sharing the technology spillover information.
We focus far more on the difference of the supplier’s payoffs under the two information strategies, and thus illustrate this result through Figure 8, in which Figure 8a verifies the difference when $j = L$ and Figure 8b verifies the difference when $j = H$, numerically.

### 6.2. Implications of information strategies on M1

We next discuss the impact of information strategies on M1 by comparing her optimal decisions of the upgraded technology level and the order quantity and her payoffs under the sharing and hiding information strategies. The next proposition characterizes the difference of M1’s optimal results between the two information strategies.

**Proposition 6.2.** The comparison of M1’s optimal results is specified into the following cases,

1. if $j = L$, $t^s > t^a$, $q^s_1 > q^a_1$ and $\Pi^s_1 > \Pi^a_1$ hold;
2. if $j = H$, $t^s < t^a$, $q^s_1 < q^a_1$ and $\Pi^s_1 < \Pi^a_1$ hold.

Proposition 6.2 suggests that the difference of M1’s optimal results under the two information strategies depends on the type of the supplier. When the supplier is of low-spillover type, i.e., $j = L$, M1 upgrades the supplier’s technology to a higher level (please refer to Fig. 9a) and makes more orders under the sharing information strategy than the hiding information strategy. As a result, M1 receives much more profit under the strategy of sharing information than the one of hiding information (please refer to Fig. 10a).

When the supplier is a type with high-spillover, on the contrary, the upgraded technology level is lower (please refer to Fig. 9b) and meanwhile M1’s order is less by sharing information than hiding information. More importantly, M1’s profit obtained under the strategy of sharing information is not greater than that under the strategy of hiding information any more (please refer to Fig. 10b). If the supplier is of high-spillover type that $j = H$, M1 may undervalue the actual degree of technology spillover and thus have a weak reaction to technology spillover by hiding the technology spillover information. As a consequence, the upgraded technology level may be reduced moderately, and then more orders are made and more profits are obtained, comparing with the results when the technology spillover information is disclosed to all parties.

### 6.3. Implications of information strategies on M2

M2 has only one decision in the game, where he makes order from the supplier. As a player, M2’s order decision will be impacted as well by the supplier’s information strategies. We present implications of information strategies on his optimal results through the following proposition.
Figure 9. The difference of upgraded technology level under two information strategies.

Figure 10. The difference of M1’s payoffs under two information strategies.

Proposition 6.3. The comparison of M2’s optimal results is specified into the following cases,

1. when $j = L$,
   (a) if $0 \leq \theta_L < \min\{\gamma \frac{3}{2}, \theta_H\}$, $q_2^{**} < q_2^{*}$ and $\Pi_2^{**} < \Pi_2^{*}$ hold;
   (b) if $\min\{\frac{3}{2}, \theta_H\} \leq \theta_L < \theta_H$, $q_2^{**} > q_2^{*}$ and $\Pi_2^{**} > \Pi_2^{*}$ hold;

2. when $j = H$,
   (a) if $\theta_L < \theta_H \leq \max\{\frac{3}{2}, \theta_L\}$, $q_2^{**} > q_2^{*}$ and $\Pi_2^{**} > \Pi_2^{*}$ hold;
   (b) if $\max\{\frac{3}{2}, \theta_L\} < \theta_H \leq 1$, $q_2^{**} < q_2^{*}$ and $\Pi_2^{**} < \Pi_2^{*}$ hold.

Proposition 6.3 suggests that the implication of information strategies on M2 depends on not only the type of the supplier but also the range of technology spillover degree. For the supplier of low-spillover type, if $\theta_L$ falls into a low range, i.e., $0 \leq \theta_L < \min\{\frac{3}{2}, \theta_H\}$, M2 makes less orders and receives less profit under the strategy of sharing information than the one of hiding information. As shown in Proposition 6.2, M1 upgrades the supplier’s technology to a higher level and makes more orders if the technology spillover information is shared, when $j = L$. Nevertheless, because of the lower $\theta_L$, M2 cannot enjoy too much dividend of upgraded technology, and he is less competitive than M1 in the market. As a result, M2’s order quantities under the strategy of sharing information are smaller than these under the strategy of hiding information. However, if the
technology spillover degree $\theta_L$ becomes higher, that is, $\min\{\frac{\gamma}{2}, \theta_H\} \leq \theta_L < \theta_H$, the new technology benefits M2 much more, which can enhance his competitiveness. The much higher of the upgraded technology level, the more competitive for M2. Therefore, more orders are made by M2 under the strategy of sharing information than that of hiding information. Recall that $\Pi_s^* = (q_s^*)^2$ and $\Pi_a^* = (q_a^*)^2$. Then, we can get $\Pi_s^* < \Pi_a^*$ when $0 \leq \theta_L < \min\{\frac{\gamma}{2}, \theta_H\}$ and $\Pi_s^* > \Pi_a^*$ when $\min\{\frac{\gamma}{2}, \theta_H\} \leq \theta_L < \theta_H$. When $j = H$, the explanation for the results can be provided in a similar way to the above.

Figure 11 illustrates the results above by plotting the difference of M2’s payoffs under two information strategies. The difference when $j = L$ is confirmed by Figure 11a, in which $\Pi_s^* - \Pi_a^* > 0$ holds if $0 \leq \theta_L < \min\{\frac{\gamma}{2}, \theta_H\}$ and $\Pi_s^* - \Pi_a^* < 0$ holds if $\min\{\frac{\gamma}{2}, \theta_H\} \leq \theta_L < \theta_H$. Obviously, these curves echo the results well. Figure 11b confirms the difference of M2’s payoffs when $j = H$ in the same way.

Recall that the implications of information strategies on the supplier and M1, as shown in Proposition 6.2 and Proposition 6.1, respectively. Proposition 6.1 shows the supplier receives much more payoff through sharing information than hiding information when $j = L$. Then, the supplier should share technology spillover information with M1. Under the same condition that $j = L$, M1 obtains much more profit in the sharing information strategy than the hiding information strategy, as shown in Proposition 6.2. Moreover, if $j = L$ as well as the technology spillover degree falls into a high range, according to Proposition 6.3, M2 is better off as well under the strategy of sharing information. Therefore, the strategy of sharing technology spillover information may create a “win-win-win” outcome for the information owner (the supplier), the information observer (M1) and the free-rider (M2) under the conditions that $j = L$ with a high range of the technology spillover degree. Similarly, we find that the strategy of hiding technology spillover information is better off for all players as well, when $j = H$ and the technology spillover degree is in a high range. In a word, both sharing and hiding technology spillover information can generate a “triple-win” outcome for the three parties conditionally.

**7. Conclusion and discussion**

In a context where one manufacturer (M1) upgrades the supplier’s production technology through direct investments, while the supplier may spill the upgraded technology over to the rival manufacturer (M2) and withhold the technology spillover information privately, this article focuses on the issue that whether the supplier...
should share the technology spillover information with M1. Based on the model that catches on this issue, we present some novel implications and insights as follows.

We first examine the effect of technology spillover on all firms, and find technology spillover not only harms M1 but also hurts the supplier and M2 probably, no matter whether the supplier shares the technology spillover information or not with M1. Thus, spilling the upgraded technology over to M2 may not be a wise choice for the supplier, and taking a free ride of technology spillover may not be a profitable behavior for M2. This outcome takes place if the real degree of technology spillover falls into a high range as well as the production cost is high and the investment is low under each information strategies.

Moreover, this study presents answers for the issue that how the supplier should determine its information strategy, and further reveals that sharing and hiding strategies of the technology spillover information may both create a “triple-win” outcome for the information owner (the supplier), the information observer (M1) and the free-rider (M2). When the supplier is of low-spillover type as well as the real degree of technology spillover falls into a high range, the three firms get worse off under the strategy of sharing information, and such sharing strategy is better off for all players. If the supplier is of high-spillover type and the real degree of technology spillover is in a high range, the three firms get worse off under the strategy of sharing information than the one of hiding information. Thus, the supplier should share the technology spillover information with M1, and such sharing strategy is better off for all players. If the supplier is of high-spillover type and the real degree of technology spillover is in a high range, the three firms get worse off under the strategy of sharing information, and thus the supplier should hide the technology spillover information to M1, which is favorable to M1 and M2 as well.

In future, in addition to the extension mentioned in §4.2, two possible extensions can be made based on this work. One is to extend single supplier to multi suppliers and consider the competition in upstream market. In addition, we can extend this model by assuming products are heterogeneous in quality or production cost, since in reality products that the supplier offers to the two manufacturers may be variant.

**APPENDIX A. PROOFS**

We put the proofs of the propositions, lemmas, and so on into Appendix A.

**Proof of Lemma 4.1.** Taking partial derivative of \( t^{**} \) w.r.t. \( \theta \), we have
\[
\frac{\partial t^{**}}{\partial \theta} = \frac{2(4-\gamma^2)^2\alpha + (2-\gamma)\gamma^2c^2}{2(4-\gamma^2)^2\alpha - (2-\gamma)\gamma^2c^2} (2-\gamma)(d-c)c < 0. 
\]
Thus, the result holds.

**Proof of Lemma 4.2.** (1) It is straightforward to get \( \partial w_2^*/\partial \theta > 0 \) from equations (4.6) and (4.8).

(2) Taking partial derivative of \( w_2^*/\partial \theta \), we have
\[
\frac{\partial w_2^*/\partial \theta} = \frac{(2-\gamma)(d-c)c^2}{2} \left[ \frac{-4(1-\gamma\theta)(4-\gamma^2)^2\alpha + (2-\gamma)\gamma^2c^2}{2(4-\gamma^2)^2\alpha - (2-\gamma)\gamma^2c^2} \right].
\] (A.1)
Let \( M^* = -4(1-\gamma\theta)(4-\gamma^2)^2\alpha + (2-\gamma)\gamma^2c^2 \), and it is a decreasing function of \( \alpha \). Since \( \alpha \geq \alpha^* \), we can receive \( M^*(\alpha^*) = 2[-(1-\gamma\theta)(2-\gamma)d + ((1-\gamma\theta)(2-\gamma) + (2-\gamma)\gamma\theta)c](2-\gamma)c \). If \( 0 < c < \pi \), \( M^*(\alpha^*) < 0 \) holds, and thus \( \frac{\partial w_2^*/\partial \theta} < 0 \) holds, where \( \bar{c} = \frac{(1-\gamma\theta)(2-\gamma)d}{(1-\gamma\theta)(2-\gamma) + (2-\gamma)\gamma\theta} \). If \( \bar{c} < c < d \), there exists a \( \bar{\alpha} \) such that \( M^* > 0 \) holds when \( \alpha^* \leq \alpha < \bar{\alpha} \), under which \( M^*(\alpha^*) > 0 \) holds, and \( M^* \leq 0 \) holds when \( \alpha \geq \bar{\alpha} \), under which \( M^*(\alpha^*) \leq 0 \). We can get \( \bar{\alpha} = \frac{(2-\gamma)\gamma^2c^2}{2(1-\gamma\theta)(4-\gamma^2)} \) through \( M^* = 0 \).

**Proof of Proposition 4.3.** Substituting equations (4.6)–(4.8) into (4.3) and (4.4) can generate
\[
q_1^{**} = \frac{(2-\gamma)(d-c) + (2-\gamma\theta)t^{**}c}{2(4-\gamma^2)} = \frac{(2-\gamma)(4-\gamma^2)(d-c)\alpha}{2(4-\gamma^2)^2\alpha - (2-\gamma)\gamma^2c^2},
\] (A.2)
and
\[
q_2^{**} = \frac{(2-\gamma)(d-c) + (2\theta-\gamma)t^{**}c}{2(4-\gamma^2)} = \frac{(2-\gamma)(d-c)}{2(4-\gamma^2)} \left[ \frac{2(4-\gamma^2)^2\alpha - (1-\theta)(2+\gamma)(2-\gamma)c^2}{2(4-\gamma^2)^2\alpha - (2-\gamma)\gamma^2c^2} \right].
\] (A.3)
(1) It is straightforward to get \( \partial q_1^{ss}/\partial \theta < 0 \) from equation (A.2).

(2) Taking partial derivative of \( q_2^{ss} \) w.r.t. \( \theta \), we have

\[
\frac{\partial q_2^{ss}}{\partial \theta} = \frac{(2 - \gamma)(d - c)c^2}{2(4 - \gamma^2)} \left[ \frac{2(4 - 4\gamma\theta + \gamma^2)(4 - \gamma^2)\alpha - (4 - \gamma^2)(2 - \gamma\theta)^2c^2}{[2(4 - \gamma^2)^2\alpha - (2 - \gamma\theta)^2c^2]^2} \right].
\]  

(A.4)

Let \( N^* = 2(4 - 4\gamma\theta + \gamma^2)(4 - \gamma^2)^2\alpha - (4 - \gamma^2)(2 - \gamma\theta)^2c^2 \), and it increases in \( \alpha \). Since \( \alpha \geq \alpha^* \), we can receive their optimal profits as follows.

\[
\begin{align*}
\Pi_1^{ss} &= \frac{(2 - \gamma)^2(d - c)^2\alpha}{2(4 - \gamma^2)^2\alpha - (2 - \gamma\theta)^2c^2}, \\
\Pi_2^{ss} &= \frac{(q_2^{ss})^2}{(2 - \gamma)^2}, \\
\Pi_S^{ss} &= \frac{(d - c + t^{ss}c - \gamma(d - c + \theta t^{ss}c)(d - c + t^{ss}c) + (d - c + \theta t^{ss}c)^2)}{2(4 - \gamma^2)} \\
&= \frac{[(1 + A)^2 - \gamma(1 + \theta A)(1 + A) + (1 + \theta A)^2](d - c)^2}{2(4 - \gamma^2)},
\end{align*}
\]

where \( A = \frac{(2 - \gamma\theta)(2 - \gamma)^2c^2}{2(4 - \gamma^2)^2\alpha - (2 - \gamma\theta)^2c^2} \).

Proof of Proposition 4.4. Substituting \( q_1^{ss}, q_2^{ss}, w_1^{ss}, w_2^{ss} \) and \( t^{ss} \) into the three players' profit function, we can receive their optimal profits as follows.

(1) It is straightforward to find \( \Pi_1^{ss} \) decreases in \( \alpha \), i.e., \( \partial \Pi_1^{ss}/\partial \theta < 0 \) from equation (A.5).

(2) The result is straightforward according to equation (A.6).

(3) Taking partial derivative of \( \Pi_S^{ss} \) w.r.t. \( \theta \), we have

\[
\frac{\partial \Pi_S^{ss}}{\partial \theta} = \frac{(2 - \gamma)^2(d - c)^2Z^*}{2(4 - \gamma^2)^2\alpha - (2 - \gamma\theta)^2c^2},
\]

where \( Z^* = 4(4 - \gamma)^3(2 - \gamma)(2 - \gamma - 2\gamma\theta)\alpha^2 - 2(4 - \gamma^2)^2(2 - \gamma\theta)(2 - \gamma\theta + 2(1 - \theta)(1 - \gamma\theta))c^2\alpha + 2(2 - \gamma\theta)^3(1 - \theta)c^4 \), and it is a function on \( \alpha \).

If \( 2 - \gamma - 2\gamma\theta \leq 0 \), i.e., min\{1, \( \frac{2 - \gamma}{2\gamma} \} \leq \theta \leq 1 \), \( Z^* \) is a decreasing function for \( \alpha > 0 \). Since \( \alpha \geq \alpha^* \), we have

\[
Z^*(\alpha^*) = (2 - \gamma\theta)^3\left[ \frac{(2 - \gamma - 2\gamma\theta)(2 - \gamma)(d - c) + (2 - \gamma\theta)c^2}{2 + \gamma} - [2 - \gamma + 2(1 - \theta)(1 - \gamma\theta))(2 - \gamma)(d - c)c - [2(1 - \gamma\theta) + \gamma\theta(2\theta - 1))(2 - \gamma\theta)c^2]ight] < 0.
\]

Thus, \( Z^* < 0 \) holds for \( \alpha \geq \alpha^* \), and then \( \partial \Pi_S^{ss}/\partial \theta < 0 \) holds.

If \( 2 - \gamma - 2\gamma\theta > 0 \), i.e., \( 0 \leq \theta < \min\{1, \frac{2 - \gamma}{2\gamma} \} \), we have the following discussion.

The discriminant of \( Z^* \) is given by
\[ \Delta Z = 4(4 - \gamma^2)^3(2 - \gamma)(2 - \gamma \theta)^2c^4[(2 + \gamma)((2 - \gamma \theta)^2 + 4(1 - \theta)^2(1 - \gamma \theta)^2) - 4(2 - \gamma \theta)(1 - \theta)((2 - \gamma)(1 - \gamma \theta) - 2\gamma)], \]

where \( \Delta Z > 0 \) since \((2 - \gamma)(1 - \gamma \theta) - 2\gamma < 0.\)

We rewrite \( Z^*(\alpha^*) \) as
\[
Z^*(\alpha^*) = \frac{(2 - \gamma \theta)^2c^2}{2 + \gamma} \left[ -4\gamma(1 - \gamma \theta + \theta^2)(2 - \gamma \theta)c^2 + (2 - \gamma)[(1 - \gamma \theta)((2 - \gamma)(1 + 2\theta) - 4\gamma) - (3\gamma + 2)](d - c)c + (2 - \gamma)^2(2 - \gamma - 2\gamma \theta)(d - c)^2, \right.

which is quadratic function of \( c. \) The discriminant of \( z^* \) is given by \( \Delta_c = (2 - \gamma)^2c^2[((1 - \gamma \theta)((2 - \gamma)(1 + 2\theta) - 4\gamma) - (3\gamma + 2))^2 + 16\gamma(1 - \gamma \theta + \theta^2)(2 - \gamma \theta)(2 - \gamma - 2\gamma \theta)] > 0. \) If \( c = 0, \) we have \( z^* > 0; \) if \( c = d, \) we have \( z^* < 0. \) Therefore, there exists a \( \tilde{c} \) such that \( z^* > 0 \) when \( 0 < c < \tilde{c} \) and \( z^* < 0 \) when \( \tilde{c} < c < d. \) When \( 0 < c < \tilde{c}, \) we can get \( Z^*(\alpha^*) > 0, \) thus \( \partial \Pi_{2L}^* / \partial \theta > 0 \) holds. When \( \tilde{c} < c < a, Z^*(\alpha^*) \leq 0, \) thus there exists a \( \tilde{\alpha} \) such that \( \partial \Pi_{2L}^* / \partial \theta < 0 \) when \( \alpha < \tilde{\alpha} \) and \( \partial \Pi_{2L}^* / \partial \theta \geq 0 \) when \( \alpha \geq \tilde{\alpha}. \) Here \( \tilde{\alpha} \) can be obtained through
\[
Z^* = 0, \text{ and } \tilde{c} \text{ can be obtained through } z^* = 0. \text{ Then, we have } \tilde{\alpha} = \frac{2(4 - \gamma)^2(2 - \gamma \theta)((1 - \gamma \theta)(2 - \gamma \theta) + 2(1 - \theta)(1 - \gamma \theta) + \sqrt{\Delta_c})c^2}{8(1 - \gamma^2)((2 - \gamma - 2\gamma \theta))^2}, \]

where \( \Delta_c = \Delta Z/c^4 \) and \( \Delta_c = \Delta Z/d^2. \)

**Proof of Lemma 5.1.** Taking partial derivative of \( t^* \) w.r.t. \( \theta_L \) and \( \theta_H, \) respectively, we have
\[
\frac{\partial t^*}{\partial \theta_L} = \left[ \frac{2(4 - \gamma^2)^2\alpha - (1 - \beta)(2 - \gamma \theta L)^2 + \beta(2 - \gamma \theta H)^2c^2}{2(1 - \beta)(2 - \gamma \theta L)((1 - \beta)(2 - \gamma \theta L) + \beta(2 - \gamma \theta H))c^2} \right] - \frac{2\gamma(1 - \beta)(1 - \beta)(2 - \gamma \theta L) + \beta(2 - \gamma \theta H)}{(2 - \gamma)(d - c)c} < 0, \]
\[
\frac{\partial t^*}{\partial \theta_H} = \left[ \frac{2(4 - \gamma^2)^2\alpha - (1 - \beta)(2 - \gamma \theta H)^2 + \beta(2 - \gamma \theta L)^2c^2}{2(1 - \beta)(2 - \gamma \theta H)((1 - \beta)(2 - \gamma \theta L) + \beta(2 - \gamma \theta H))c^2} \right] - \frac{2\beta(2 - \gamma \theta H)((1 - \beta)(2 - \gamma \theta L) + \beta(2 - \gamma \theta H))c^2}{(2 - \gamma)(d - c)c} < 0. \]

**Proof of Lemma 5.2.** (1) It is straightforward to get \( \partial w_{2L}^*/\partial \theta > 0 \) and \( \partial w_{1H}^*/\partial \theta > 0. \) from equation (5.4).
(2) Equation (5.5) can be rewritten as \( w_{2L}^* = \frac{d+c}{2} - \theta_t/c^4. \)

If \( j = L, \) we have \( \frac{\partial w_{2L}^*}{\partial \theta_L} = -\frac{\alpha\theta_t}{2} \cdot \frac{\partial w_{2L}^*}{\partial \theta_H} > 0 \) and
\[
\frac{\partial w_{2L}^*}{\partial \theta_L} = -\frac{(2 - \gamma)(d - c)c^2}{2} \left[ \frac{2(1 - \beta)(1 - \gamma \theta L) + \beta(2 - \gamma \theta H)}{\Lambda^2} - 2\gamma \theta L((1 - \beta)(2 - \gamma \theta H)(1 - \gamma \theta L))c^2 \right], \]

where \( \Lambda = 2(4 - \gamma^2)^2\alpha - ((1 - \beta)(2 - \gamma \theta L)^2 + \beta(2 - \gamma \theta H)^2)c^2. \) Here we let
\[
G = \left[ 2(1 - \beta)(1 - \gamma \theta L) + \beta(2 - \gamma \theta H)\Lambda - 2\gamma \theta L((1 - \beta)(2 - \gamma \theta H))c^2 \right], \]

and it is an increasing function of \( \alpha. \) Since \( \alpha \geq \alpha^*, \) we have \( G(\alpha^*) = [(1 - \beta)(2 - \gamma \theta L) + \beta(2 - \gamma \theta H)]G_0c, \)

where \( G_0 = [2(1 - \beta)(1 - \gamma \theta L) + \beta(2 - \gamma \theta H)](2 - \gamma)d - [2(1 - \beta)(1 - \gamma \theta L) + \beta(2 - \gamma)(2 - \gamma \theta)]^c. \)
Define \( \bar{c}_L = \frac{[2(1-\beta)(1-\gamma \theta_L) + \beta(2-\gamma \theta_H)](2-\gamma)c}{(1-\beta)(1-\gamma \theta_L) + \beta(2-\gamma \theta_H)}, \) which satisfies \( G_0 = 0. \) If \( G_0 \geq 0, \) i.e., \( 0 \leq c \leq \bar{c}_L, \) we have \( G(\alpha^*) \geq 0, \) and thus \( \bar{G} \geq 0 \) for \( \alpha \geq \alpha^*. \) In this situation, \( \partial w_1^*/\partial \theta_L \leq 0 \) holds. If \( G_0 < 0, \) i.e., \( \bar{c}_L < c < d, \) we have \( G(\alpha^*) < 0. \) Therefore, there exists a \( \bar{\alpha}_L \) such that \( G < 0 \) and thus \( \partial w_2^*/\partial \theta_L > 0 \) when \( \alpha^* \leq \alpha \leq \bar{\alpha}_L, \) and \( G \geq 0 \) and thus \( \partial w_2^*/\partial \theta_L \leq 0 \) when \( \alpha \geq \bar{\alpha}_L. \) Here, \( \bar{\alpha}_L \) can be obtained through \( G = 0, \) and we have

\[
\bar{\alpha}_L = \frac{2(2-\beta)(1-\gamma \theta) + \beta(2-\gamma \theta_H))(4-\gamma^2)^2}{[(2(1-\beta)(1-\gamma \theta) + \beta(2-\gamma \theta_H))\mu_2 + 2\gamma \theta_L(1-\beta)(2-\gamma \theta_L)\mu_1]c^2},
\]

where \( \mu_1 = (1-\beta)(2-\gamma \theta_L) + \beta(2-\gamma \theta_H), \) \( \mu_2 = (1-\beta)(2-\gamma \theta_L)^2 + (2-\gamma \theta_H)^2. \)

If \( j = H, \) the results can be obtained in a similar way, and \( \bar{c}_H \) and \( \bar{\alpha}_H \) is shown as follows.

\[
\bar{c}_H = \frac{[(1-\beta)(2-\gamma \theta_L) + \beta(1-\gamma \theta)](2-\gamma)d}{[(1-\beta)(2-\gamma \theta_L) + \beta(1-\gamma \theta)](2-\gamma) + 2\beta \gamma \theta_H(2-\gamma \theta_H)},
\]

\[
\bar{\alpha}_H = \frac{2[(1-\beta)(2-\gamma \theta_L) + \beta(1-\gamma \theta)](4-\gamma^2)}{[(2(1-\beta)(2-\gamma \theta_L) + \beta(1-\gamma \theta))\mu_2 + 2\beta \gamma \theta_H(2-\gamma \theta_H)\mu_1]c^2}.
\]

\[
\square
\]

**Proof of Proposition 5.3.** With a combination of \( q_1^*, q_2^*, w_1^*, w_2^* \) and \( t^*, \) we have

\[
q_1^* = \frac{(2-\gamma)(d-c) + (2-\gamma \theta) d^* c}{2(4-\gamma^2)} \tag{A.9}
\]

and

\[
q_2^* = \frac{(2-\gamma)(d-c) + (2\theta_j - \gamma) d^* c}{2(4-\gamma^2)} \tag{A.10}
\]

(1) It is straightforward to get \( \partial q_1^*/\partial \theta_L < 0 \) and \( \partial q_1^*/\partial \theta_H < 0 \) for \( j = L \) or \( j = H \) from equation (A.9).

(2) If \( j = L, \) we have \( \partial q_2^*/\partial \theta_H = \frac{(2\theta_j - \gamma)c}{(4-\gamma^2)^2} \partial q_1^* / \partial \theta_L \) and \( \partial q_2^*/\partial \theta_L = \frac{c}{(4-\gamma^2)^2} \left[ 2 t^* + (2\theta_j - \gamma) \partial q_1^* / \partial \theta_L \right]. \)

When \( 2\theta_L - \gamma < 0, \) i.e., \( 0 \leq \theta_L < \frac{\gamma}{2}, \) \( \partial q_2^*/\partial \theta_H > 0 \) and \( \partial q_2^*/\partial \theta_L > 0 \) hold. Since \( \theta_L < \theta_H, \) the inequalities \( 0 \leq \theta_L < \frac{\gamma}{2} \) should be written as \( 0 \leq \theta_L < \min\{\frac{\gamma}{2}, \theta_H\}. \)

When \( 2\theta_L - \gamma \geq 0, \) i.e., \( \frac{\gamma}{2} \leq \theta_L < 1, \) \( \partial q_2^*/\partial \theta_H \leq 0 \) holds, and we have the following discussion about \( \partial q_2^*/\partial \theta_L. \)

\[ \partial q_2^*/\partial \theta_L \]

\[
\partial q_2^*/\partial \theta_L = \frac{(2-\gamma)(d-c)c}{2(4-\gamma^2)} \left[ \frac{2 \mu_1 - (1-\beta)(2-\gamma \theta_L) \gamma}{\Lambda} - \frac{2(1-\beta)(2\theta_L - \gamma)(2-\gamma \theta_L) \mu_1 c^2}{\Lambda^2} \right].
\]

Let \( J = [2 \mu_1 - (1-\beta)(2-\gamma \theta_L) \gamma] \Lambda - 2(1-\beta)(2\theta_L - \gamma)(2-\gamma \theta_L) \mu_1 c^2, \) which is an increasing function of \( \alpha. \)

Since \( \alpha \geq \alpha^*, \) we have

\[
J(\alpha^*) = [(1-\beta)(4(1-\gamma \theta_L) + \gamma^2) + 2\beta(2-\gamma \theta_H))(2-\gamma)(d-c) - 2(1-\beta)(2\theta_L - \gamma)(2-\gamma \theta_L) \gamma c] \mu_1 c.
\]

Since \( \frac{\gamma}{2} \leq \theta_L < 1, \) there exists a \( \hat{c}_L, \) such that \( J(\alpha^*) > 0 \) when \( 0 < c < \hat{c}_L \) and \( J(\alpha^*) \leq 0 \) when \( \hat{c}_L \leq c < d, \) where \( \hat{c}_L \) can be obtained through \( J(\alpha^*) = 0 \) and it is expressed as

\[
\hat{c}_L = \frac{(2-\gamma)(1-\beta)(4(1-\gamma \theta_L) + \gamma^2) + 2\beta(2-\gamma \theta_H)d}{(2-\gamma)(1-\beta)(4(1-\gamma \theta_L) + \gamma^2) + 2\beta(2-\gamma \theta_H)}.
\]
When \( J(\alpha^*) > 0 \), \( J > 0 \) holds and thus \( \partial q_2^*/\partial \theta_L > 0 \) holds, since \( J \) increases in \( \alpha \). When \( J(\alpha^*) \leq 0 \), there exists a \( \hat{\alpha}_L \) such that \( J < 0 \) holds and thus \( \partial q_2^*/\partial \theta_L < 0 \) holds when \( \alpha^* \leq \alpha < \hat{\alpha}_L \), and \( J \geq 0 \) and thus \( \partial q_2^*/\partial \theta_L \geq 0 \) holds when \( \alpha \geq \hat{\alpha}_L \), where \( \hat{\alpha}_L \) can be obtained through \( J = 0 \) and it is expressed as

\[
\hat{\alpha}_L = \frac{[(1 - \beta)(4(1 - \gamma \theta_L) + \gamma^2) + 2\beta(2(2 - \gamma \theta_H))\mu_2 c^2 + 2(1 - \beta)(2(2L - \gamma)(2 - \gamma \theta_L)\mu_1 \gamma c^2}{2(1 - \beta)(4(1 - \gamma \theta_L) + \gamma^2) + 2\beta(2 - \gamma \theta_H))(4 - \gamma^2)^2}.
\]

Here, the inequalities \( \frac{\gamma}{2} \leq \theta_L \leq 1 \) should be written as minimum \( \{\frac{\gamma}{2}, \theta_H\} < \theta_L < \theta_H \), since \( \theta_L < \theta_H \).

If \( j = H \), the results can be received in a similar way. \( \hat{c}_H \) and \( \hat{\alpha}_H \) are shown as follows.

\[
\hat{c}_H = \frac{(2 - \gamma)[2(1 - \beta)(2 - \gamma \theta_L) + \beta(4(1 - \gamma \theta_H) + \gamma^2)]}{(2 - \gamma)[2(1 - \beta)(2 - \gamma \theta_L) + \beta(4(1 - \gamma \theta_H) + \gamma^2)] + 2\beta(2(2L - \gamma)(2 - \gamma \theta_H) + (2 - \gamma \theta_H)\gamma},
\]

\[
\hat{\alpha}_H = \frac{2(1 - \beta)(2 - \gamma \theta_L) + \beta(4(1 - \gamma \theta_H) + \gamma^2)]\mu_2 c^2 + 2\beta(2(2L - \gamma)(2 - \gamma \theta_H)\gamma \mu_1 c^2}{2(2(2L - \gamma)(2 - \gamma \theta_L) + \beta(4(1 - \gamma \theta_H) + \gamma^2)))(4 - \gamma^2)^2}.
\]

\( \Box \)

**Proof of Proposition 5.4.** Substituting \( q_1^*, q_2^*, w_1^*, w_2^* \) and \( t^* \) into the three players’ profit function, we can receive their optimal profits as follows.

\[
\pi_1^* = \frac{(2 - \gamma)^2(d - c)^2}{4(4 - \gamma^2)^2}, \quad \pi_2^* = \frac{2(4 - \gamma^2)^2\alpha - (\theta_H - \theta_L)^2(1 - \beta)\gamma^2c^2}{2(4 - \gamma^2)^2\alpha - [(1 - \beta)(2 - \gamma \theta_L)^2 + \beta(2 - \gamma \theta_H)^2]c^2}, \quad \pi_3^* = \frac{(q_2^*)^2}{2(4 - \gamma^2)^2}.
\]

(1) Taking partial derivative of \( \pi_1^* \) w.r.t. \( \theta_L \) and \( \theta_H \) can generate the result.

(2) According to equation (A.12), the result is straightforward.

(3) Before proving, we define \( \phi_{L1} = (2 - \gamma \theta_L)(1 + \theta_L) + \theta_L^2 - \gamma \theta_L + 1 \) and \( \phi_{L2} = (2 - \gamma \theta_L)(\theta_L^2 - \gamma \theta_L + 1) \).

If \( j = L \), we have

\[
\frac{\partial \pi_2^*}{\partial \theta_H} = \frac{c}{2(4 - \gamma^2)}\left[(2 - \gamma)(1 + \theta_H)(d - c) + 2(1 - \gamma \theta_L + \theta_L^2)\alpha c\right] \frac{\partial t^*}{\partial \theta_H} > 0
\]

\[
\frac{\partial \pi_3^*}{\partial \theta_L} = \frac{(2 - \gamma)^2(d - c)^2 c^2}{2(4 - \gamma^2)[2(4 - \gamma^2)^2 - \mu_2 c^2]^3} \left[4(\mu_1 - (1 - \beta)\gamma(1 + \theta_L))(4 - \gamma^2)^4\alpha^2 \right.
\]

\[
+ 2\left[2((1 - \beta)\gamma(1 + \theta_L) - \mu_1)\mu_2 - (2(1 - \beta)\gamma \phi_{L1} - (2\theta_L - \gamma)\mu_1)\mu_1 \right)(4 - \gamma^2)^2\gamma^2c^2
\]

\[
+ \left[ - ((2\theta_L - \gamma)\mu_2 + 4(1 - \beta)\gamma \phi_{L2})\mu_2 + 2(1 - \beta)\gamma \phi_{L1}\mu_1 \mu_2 + (\mu_1 - (1 - \beta)\gamma(1 + \theta_L))\mu_2^2 c^4
\]

We let

\[
V_L = 4(\mu_1 - (1 - \beta)\gamma(1 + \theta_L))(4 - \gamma^2)^4\alpha^2
\]

\[
+ 2\left[2((1 - \beta)\gamma(1 + \theta_L) - \mu_1)\mu_2 - (2(1 - \beta)\gamma \phi_{L1} - (2\theta_L - \gamma)\mu_1)\mu_1 \right)(4 - \gamma^2)^2\gamma^2c^2
\]

\[
+ \left[ - ((2\theta_L - \gamma)\mu_2 + 4(1 - \beta)\gamma \phi_{L2})\mu_2 + 2(1 - \beta)\gamma \phi_{L1}\mu_1 \mu_2 + (\mu_1 - (1 - \beta)\gamma(1 + \theta_L))\mu_2^2 c^4
\]

and \( V_L \) is a function of \( \alpha \). If \( \mu_1 - (1 - \beta)\gamma(1 + \theta_L) < 0 \), i.e., \( 0 \leq \theta_L < \min\{1, \frac{(1 - \beta)(2 - \gamma) + \beta(2 - \gamma)\mu_1}{2\gamma(1 - \beta)}\} \), \( V_L \) is a decreasing function for \( \alpha > 0 \), since \( 2((1 - \beta)\gamma(1 + \theta_L) - \mu_1)\mu_2 - (2(1 - \beta)\gamma \phi_{L1} - (2\theta_L - \gamma)\mu_1)\mu_1 < 0 \). Since
\(\alpha > \tilde{\alpha}^a\), we have
\[
V_L(\tilde{\alpha}^a) = \left[ \mu_1 - (1 - \beta)\gamma(1 + \theta_L) \right] \left[ (2 - \gamma)\mu_1 dc + (\mu_2 - (2 - \gamma)\mu_1)c^2 \right]^2 \\
+ 2 \left[ 2(1 - \beta)\gamma(1 + \theta_L) - \mu_2 - (2 - \gamma)\gamma\phi_L1 - (2\theta_L - \gamma)\mu_1 \right] \\
\times \left[ (2 - \gamma)\mu_1 dc + (\mu_2 - (2 - \gamma)\mu_1)c^2 \right] \\
+ \left[ -((2\theta_L - \gamma)\mu_2 + 4(1 - \beta)\gamma\phi_L2)\mu_1^2 + 2(1 - \beta)\gamma\phi_L1\mu_2 + (\mu_1 - (1 - \beta)\gamma(1 + \theta_L))\mu_2^2 \right] c^4 \\
< 0.
\]

Let \(\theta_L^U = \min \left\{ 1, \frac{(1 - \beta)(2 - \gamma) + (2 - \gamma)\theta_L}{2\gamma(1 - \beta)} \right\}\). and \(\theta_L < \theta_H\), the inequalities \(0 \leq \theta_L < \min \left\{ 1, \frac{(1 - \beta)(2 - \gamma) + (2 - \gamma)\theta_L}{2\gamma(1 - \beta)} \right\}\) should be written as \(0 \leq \theta_L < \min\{\theta_H, \theta_L^U\}\).

If \(\mu_1 - (1 - \beta)\gamma(1 + \theta_L) \geq 0\), i.e., \(\min \left\{ 1, \frac{(1 - \beta)(2 - \gamma) + (2 - \gamma)\theta_L}{2\gamma(1 - \beta)} \right\} \leq \theta_L \leq 1\), we have the following discussion. The discriminant of \(V_L\) is given by
\[
\Delta_{VL} = \left[ 2(1 - \beta)\gamma\phi_L1 - (2\theta_L - \gamma)\mu_1 \right]^2 + 16(1 - \beta)\gamma(\mu_1 - (1 - \beta)\gamma(1 + \theta_L))\phi_L2 \times 4(4 - \gamma^2)c^4 \mu_1^2 > 0.
\]

We rewrite \(V_L(\tilde{\alpha}^a)\) as
\[
V_L(\tilde{\alpha}^a) = \left[ 2(2 - \gamma)\mu_1 + (1 - \beta)\gamma(2\theta_L - \gamma)(2(2 - \gamma)\theta_L) - (2 - \gamma)) \right] (1 - \theta_L)c^2 \\
+ \left[ ((2\theta_L - \gamma) - 2(2 - \gamma))\mu_1 - 2(1 - \beta)\gamma(\gamma(1 - \theta_L^2) + \theta_L^2 - \gamma\theta_L + 1) \right] (2 - \gamma)dc \\
+ (\mu_1 - (1 - \beta)\gamma(1 + \theta_L))(2 - \gamma)\gamma^2 d^2 \mu_1^2 c^2.
\]

Let
\[
v_I = \left[ 2(2 - \gamma)\mu_1 + (1 - \beta)\gamma(2\theta_L - \gamma)(2(2 - \gamma)\theta_L) - (2 - \gamma)) \right] (1 - \theta_L)c^2 \\
- \left[ (2(2 - \gamma) - (2\theta_L - \gamma))\mu_1 + 2(1 - \beta)\gamma(\gamma(1 - \theta_L^2) + \theta_L^2 - \gamma\theta_L + 1) \right] (2 - \gamma)dc \\
+ (\mu_1 - (1 - \beta)\gamma(1 + \theta_L))(2 - \gamma)\gamma^2 d^2 c,
\]
and \(v_I\) is a quadratic function of \(c\). The discriminant of \(v_I\) is given by
\[
\Delta_{vl} = \left[ (2\theta_L - \gamma)^2 \mu_1^2 + 4(1 - \beta)(1 - \theta_L)\gamma(6 - 2\theta_L - \gamma)(1 - \gamma)\theta_L + 8 + 2\gamma^2 - 7\gamma\gamma\mu_1 \\
+ (\gamma^2 - 4\gamma\theta_L + 3)\theta_L^2 + (6 - \theta_L(1 + \gamma)^2)(1 - \theta_L^2)\theta_L + 1 \right] (2 - \gamma)\gamma^2 d^2 \\
> 0.
\]

When \(c = 0\), we have \(v_I > 0\), and when \(c = d\), we have \(v_I < 0\). Therefore, there exists a \(\bar{c}\) such that \(v_I > 0\) when \(0 < c < \bar{c}\) and \(v_I \leq 0\) when \(\bar{c} \leq c < d\). When \(0 < c < \bar{c}\), we can get \(V_L(\tilde{\alpha}^a) > 0\), and thus \(\partial^2 \Pi_s^U / \partial \theta_L > 0\) holds. When \(\bar{c} \leq c < d\), \(V_L(\tilde{\alpha}^a) \leq 0\) holds, thus there exists a \(\tilde{\alpha}_L\) such that \(\partial^2 \Pi_s^U / \partial \theta_L < 0\) when \(\bar{\alpha}^a < \alpha < \tilde{\alpha}_L\) and \(\partial^2 \Pi_s^U / \partial \theta_L \geq 0\) when \(\alpha \geq \tilde{\alpha}_L\). Here, \(\tilde{\alpha}_L\) can be obtained through \(V_L = 0\) and \(\tilde{\alpha}_L\) can be obtained \(v_I = 0\). Then we have
\[
\tilde{\alpha}_L = \frac{-2 \left[ (2(2 - \gamma) - (2\theta_L - \gamma))\mu_1 + 2(1 - \beta)\gamma(\gamma(1 - \theta_L^2) + \theta_L^2 - \gamma\theta_L + 1) \right] (2 - \gamma)\gamma^2 d - \sqrt{\Delta_{vl}}}{2 \left[ 2(2 - \gamma)\mu_1 + (1 - \beta)\gamma(2\theta_L - \gamma)(2(2 - \gamma)\theta_L) - (2 - \gamma)) \right] (1 - \theta_L)}.
\]

Since \(\theta_L < \theta_H\), the inequalities \(0 \leq \theta_L < \min \left\{ 1, \frac{(1 - \beta)(2 - \gamma) + (2 - \gamma)\theta_L}{2\gamma(1 - \beta)} \right\}\) should be written as \(\min\{\theta_H, \theta_L^U\} \leq \theta_L < \theta_H\).
If \( j = H \), the results can be received in a similar way. \( \tilde{\alpha}_H \) and \( \tilde{c}_H \) are shown as follows.

\[
\tilde{\alpha}_H = -2\left[2(\beta(1 + \theta_H) - \mu_1)\right] \frac{(2\beta(1 + \theta_H) - \mu_1)\right] (4 - \gamma^2)^2\alpha^2 + \sqrt{\Delta_{VH}}}{8(\mu_1 - \beta(1 + \theta_H))(4 - \gamma^2)},
\]

\[
\tilde{c}_H = \frac{\left[2(2\beta(1 + \theta_H) - \mu_1) + 2\beta(1 + \theta_H)\right] (2\beta(1 + \theta_H) - \mu_1)\right] (4 - \gamma^2)^2\alpha^2 + \sqrt{\Delta_{VH}}}{2(2\beta(1 + \theta_H) - \mu_1)}.
\]

Proof of Proposition 6.1. From equations (4.6), (4.7), (5.4) and (5.5), we find the wholesale prices of the two manufacturers decrease in the upgraded technology level \( t \).

\[\text{From equations (A.2) and (A.9), we find the optimal order quantity in two structures increases in the upgraded technology level} \]

\[
\Pi_{1*} - \Pi_{1*} = \frac{\left[2(\beta(1 + \theta_H) - \mu_1)\right] (4 - \gamma^2)^2\alpha^2 + \sqrt{\Delta_{VH}}}{2(4 - \gamma^2)^2\alpha - (2 - \gamma \theta_L)c^2}.
\]

\[\text{Let } K_L = -2(\mu_1 + (2 - \gamma \theta_L))(4 - \gamma^2)^2\alpha + (1 - \beta)(\theta_H - \theta_L)(2 - \gamma \theta_L)c^2, \text{ which is a decreasing function of } \alpha. \text{ We can find } \alpha^* > \alpha^* \text{ when } j = L, \text{ thus } \alpha \geq \alpha^* \text{ holds in this case. Then, we have } K_L(\alpha^*) = -[\mu_1 + (2 - \gamma \theta_L)](2 - \gamma)(d - c)c + [(\theta_H - \theta_L)\gamma - 2(2 - \gamma \theta_L)](2 - \gamma \theta_L)c < 0. \text{ Thus, } \Pi_{1*} - \Pi_{1*} < 0 \text{ holds.}
\]

From equations (A.2) and (A.9), we find the optimal order quantity in two structures increases in the upgraded technology level \( t \). Therefore, \( q_{1*} < q_{1*} \) holds when \( t^* < t^* \).

(2) If \( j = H \), we have

\[
\Pi_{1*} - \Pi_{1*} = \frac{\left[2(\beta(1 + \theta_H) - \mu_1)\right] (4 - \gamma^2)^2\alpha^2 + \sqrt{\Delta_{VH}}}{2(4 - \gamma^2)^2\alpha - (2 - \gamma \theta_L)c^2}.
\]

\[
\text{Let } K_H = -2(\mu_1 + (2 - \gamma \theta_H))(4 - \gamma^2)^2\alpha + (1 - \beta)(\theta_H - \theta_L)(2 - \gamma \theta_L)c^2, \text{ which is a decreasing function of } \alpha. \text{ We can find } \alpha^* > \alpha^* \text{ when } j = L, \text{ thus } \alpha \geq \alpha^* \text{ holds in this case. Then, we have } K_H(\alpha^*) = -[\mu_1 + (2 - \gamma \theta_H)](2 - \gamma)(d - c)c + [(\theta_H - \theta_L)\gamma - 2(2 - \gamma \theta_L)](2 - \gamma \theta_L)c < 0. \text{ Thus, } \Pi_{1*} - \Pi_{1*} < 0 \text{ holds.}
\]

From equations (A.2) and (A.9), we find the optimal order quantity in two structures increases in the upgraded technology level \( t \). Therefore, \( q_{1*} < q_{1*} \) holds when \( t^* < t^* \).
Similarly, we can get \( q_1^* > q_1^* \) holds when \( t^* > t^* \).

\( \square \)

**Proof of Proposition 6.3.** Taking partial derivative of \( q_2^* \) and \( q_3^* \) w.r.t. \( t \), we have \( \frac{\partial q_2^*}{\partial t} = \frac{(2\gamma - \gamma)c}{2(4 - \gamma)} \), \( \frac{\partial q_3^*}{\partial t} = \frac{(2\gamma - \gamma)c}{2(4 - \gamma)} \).

1. When \( j = L \), if \( 2\theta_L - \gamma < 0 \), i.e., \( 0 < \theta_L < \frac{\gamma}{2} \), we have \( \frac{\partial q_2^*}{\partial t} < 0 \), \( \frac{\partial q_3^*}{\partial t} < 0 \). Then, \( t^* > t^* \) can lead to \( q_2^* < q_2^* \). If \( 2\theta_L - \gamma \geq 0 \), i.e., \( \frac{\gamma}{2} \leq \theta_L < 1 \), we have \( \frac{\partial q_2^*}{\partial t} \geq 0 \), \( \frac{\partial q_3^*}{\partial t} \geq 0 \). Then, \( t^* > t^* \) can lead to \( q_2^* > q_2^* \). Since \( \Pi_2^* = (q_2^*)^2 \) and \( \Pi_3^* = (q_3^*)^2 \), \( t^* < t^* \) can lead to \( \Pi_2^* > \Pi_3^* \) as well in this case. Therefore, the results can be obtained.

Here, the inequalities \( 0 < \theta_L < \frac{\gamma}{2} \) and \( \frac{\gamma}{2} \leq \theta_L < 1 \) should be written as \( 0 < \theta_L < \min\{\frac{\gamma}{2}, \theta_H\} \) and \( \min\{\frac{\gamma}{2}, \theta_H\} \leq \theta_L < \theta_H \), respectively, since \( \theta_L < \theta_H \).

2. When \( j = H \), we can get the results in a similar way.

\( \square \)

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