SHORT LIFETIME PRODUCT SUPPLY CHAIN COORDINATION AND SOCIAL BENEFIT CONSIDERING CANNIBALIZATION EFFECT AND MARKET SEGMENTATION

TAHEREH HASHEMI©, EBRAHIM TEIMOURY*© AND FARNAZ BARZINPOUR

Abstract. Short lifetime product retailers often face the challenge of cannibalization between new and old products, which can negatively impact their profitability. They attempt to influence consumers’ choices through price differentiation, resulting in internal competition regarding products’ age and price. The pricing decisions affect market demand, sales volume, and as a result, the whole supply chain (SC) profit. This paper coordinates inventory and pricing decisions in a short lifetime product supply chain (SLPSC), considering the cannibalization effect. The investigated SLPSC includes a supplier and a retailer operating in a segmented market. Firstly, the optimal decisions of the SLPSC members are obtained under decentralized and centralized decision-making structures. Then, a new coordination contract named wholesale price and double compensation (WPDC) is designed to motivate the SC members to shift from the decentralized structure to the centralized one. The findings indicate that the coordinated model creates more economic profitability for the whole SLPSC than the decentralized one. Furthermore, the proposed WPDC contract is more beneficial for the SLPSC from a social viewpoint, as it increases consumer surplus. The results also demonstrate that when consumers are more sensitive to the product’s freshness, a price differentiation policy is more profitable than the same pricing.

Mathematics Subject Classification. 91A65, 91B24, 91B42.

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1. Introduction

Pricing and inventory management of short lifetime products are regarded as challenging decisions for grocery retailers. Perishable products such as bread, dairy, meat, fruits, and vegetables have a very limited lifetime and decreasing utility over time [1]. Product lifetime affects the consumer’s buying tendency. Therefore, the retailers must provide new products for consumers; otherwise, they may lose a significant portion of their sales [27]. They have to order new products before the shelves are empty. As a result, many shelves hold the same type of product, but with various expiry dates. In this case, a challenge arises as consumers have different valuations for the products of different remaining lifetime, which can lead to internal competition according to the product age and price [19].

Keywords. Channel coordination, short lifetime product supply chain, cannibalization effect, consumer surplus, market segmentation.

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To solve the problem, the retailer can implement a markdown pricing policy by offering a lower price on the old products. The price differentiation is attractive for some consumers, especially those who only look for discounted products, which results in increasing the market demand share. However, setting too low prices on old products may lead to the cannibalization of new ones [5]. The term “cannibalization” refers to an internal competition between products that partially substitute each other, which causes a decrease in sales volume, sales revenue, and market share [8]. Hence, the coexistence of new and old products on the shelves results in challenges in the retailer’s pricing and inventory decisions.

The pricing and inventory decisions made by the retailers have an effect on the sales volume of the whole supply chain (SC) and therefore impact the profitability of the upstream SC members [12]. In addition, the retailers’ decisions are influenced by the terms of the contract with suppliers. Hence, an individual decision-making approach in which each SC member makes its decisions by optimizing its own profit may not be a proper method. Therefore, it is essential to coordinate the inventory replenishment and pricing decisions in the short lifetime product supply chain (SLPSC), to enhance the whole profitability and performance of the SC and its members.

In this paper, we address the coordination of pricing and inventory replenishment decisions in an SLPSC context considering the internal competition between new and old units. Coordination of pricing and inventory decisions in the SLPSC has been studied by some researchers such as Hou and Liu [18], Moon et al. [29], and Mohammadi et al. [28]. However, combining the challenge of cannibalization between new and old products with coordination models in the SLPSC has rarely been studied, which calls for further investigation. Moreover, coordinating the members’ decisions affects the economic, social, and environmental performances of the SC [17]. However, the majority of the studies in the related literature have focused on the economic benefit of coordination in the SLPSC. To the best of our knowledge, very few studies have analyzed social benefit such as consumer surplus in the SLPSC coordination models, which is studied in the current paper. This research aims to answer the following questions:

1. How does the simultaneous sale of new and old units influence the inventory and pricing decisions in the SLPSC context?
2. Which pricing policy should be applied in the case of product cannibalization: price differentiation policy or the same pricing? What does this choice depend on?
3. How can the short lifetime product SC be coordinated by wholesale price and double compensation (WPDC) contract?
4. How can coordinating the pricing and inventory decisions impact on the SLPSC performance from economic aspect? How do the optimal values of the decision variables change under different decision-making approaches?
5. From the social perspective, how can coordinating the decision variables impact on the consumer satisfaction?

To answer the research queries, in this paper, a short lifetime product SC including a supplier and a retailer operating in a segmented market is considered. The market is made up of two segments: freshness-oriented and discount-oriented consumers. The supplier, as a Stackelberg game leader, determines the wholesale and buy-back prices while the retailer reacts as a follower and decides on the old product price and the order quantity. Firstly, the optimal values of decision variables are determined under a decentralized structure in which each SC member maximizes its own profit without considering the SLPSC as a whole. Then, a centralized model is utilized to optimize the SLPSC decisions from the total SC perspective. Finally, a novel coordination mechanism named wholesale price and double compensation (WPDC) contract is designed to motivate the SC members to shift from the decentralized structure to the centralized one.

The current research contributes to the literature on SLPSC coordination by investigating the coordination of pricing and inventory decisions considering the cannibalization between new and old products. This study is the first attempt to address the coordination of SLPSC members operating in a segmented market. Moreover, to extend the incentive mechanisms in the SLPSC coordination literature, a new coordination mechanism named wholesale price and double compensation (WPDC) contract is proposed to achieve a win-win situation for SLPSC
members. To the best of our knowledge, this is the first research that examines the effects of cannibalization on the social and economic aspects of the SC. To this end, the social performance of the SLPSC is examined in terms of consumer surplus as an index of consumer satisfaction.

In the following, a review of related research is provided in Section 2. Section 3 describes the investigated problem. In Section 4, mathematical models are developed under three decision-making approaches: (1) decentralized, (2) centralized, and (3) coordinated approach. Section 5 presents some numerical examples and sensitivity analysis and provides managerial insights achieved from the research. Finally, the conclusion and potential future researches are proposed in Section 6.

2. Literature review

There are two major research areas that closely relate to this study: (1) cannibalization effects in multiple-aged inventories, and (2) coordination models of short lifetime product SCs. In the following, we review the most important studies in these two areas.

The first area mainly focuses on the pricing and inventory control of perishable products of different ages. Ferguson and Koenigsberg [11] studied the challenge of cannibalization between old and new items and developed a two-period inventory and pricing model based on consumers’ utility functions. Herbon et al. [16] investigated a stochastic deteriorating inventory system with heterogeneous consumers who have different utilities regarding price and freshness. Li et al. [22] studied the performance of different methods for solving a multi-period pricing and ordering problem under the cannibalization between new and out-of-season units. Sainathan [39] considered dynamic demand substitution between new and old units. Chintapalli [5] studied a joint inventory control and pricing model for perishable products with a two-period lifetime under a myopic policy. Fan et al. [9] considered a dynamic pricing strategy in accordance with real-time freshness and analyzed the impacts of the inventory level and freshness of the old units on the replenishment decision. Qiao et al. [35] studied a joint ordering and markdown pricing problem for perishable products with a multi-period lifetime, where stochastic demand depends on price and freshness. Moreover, some researchers proved that when the market demand is deterministic, price differentiation is not profitable for the perishable product retailers [14, 15, 24].

All the above papers merely focus on the cannibalization of multiple-aged perishable products in a retail store, while actually the influences of price differentiation policy, such as changes in market demand, extend to the performance of the upstream SC members. Different from the previous studies, we address the coordination of pricing and inventory decisions in an SLPSC context considering the cannibalization between new and old products. Moreover, for the first time, we examine the effects of cannibalization on the social and economic aspects of the SLPSC.

The second area of research is related to the coordination models of short lifetime products SC. In the related literature, different issues such as logistics outsourcing [4], freshness-keeping efforts [3, 10, 55], cost and loss disruptions [44], and preservation technology investment [28] have been addressed. Furthermore, the risk of unsold products at the end of selling season is studied by some researchers. Wu et al. [46] proposed a fresh-product SC coordination model with inventory-risk sharing and price-discount contract. In another study, Su et al. [41] developed a coordination model by taking subsidies for unsold fresh products into account. Beheshti et al. [2] studied food waste recycling in a closed loop supply chain considering rental facilities. They designed a quantity flexibility contract with standard and expedited lead time to coordinate the channel. Nematollahi et al. [32] developed a fair profit sharing strategy for supply chain coordination considering minimum remaining shelf life required by the retailer. Table 1 compares the main related studies in the area of SLPSC coordination models.

According to Table 1, only the study done by Hashemi et al. [12] has addressed the competition between multiple-aged products in the fresh-products SC context. They presented an incentive mechanism to coordinate the pricing and inventory decisions in the fresh-product SC. However, they only examined the economic performance of the SC, assuming the implementation of a price differentiation policy in the retailing sector. Different from Hashemi et al. [12], we analyze the cannibalization phenomenon in the SLPSC coordination problem from
<table>
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<td>Xiao et al. [47]</td>
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<tr>
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</tr>
<tr>
<td>Cai et al. [4]</td>
<td>1 producer, 1 3PL provider, 1 distributor</td>
<td>Price, Time</td>
<td>Stochastic</td>
<td>Inventory, pricing, Transportation fee</td>
<td></td>
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<td>Su et al. [31]</td>
<td>1 supplier, 1 retailer</td>
<td>Price, Time</td>
<td>Stochastic</td>
<td>Inventory, pricing, Transportation time</td>
<td></td>
<td></td>
<td>Wholesale Price Discount &amp; Unsaleable Produce Subsidy &amp; Cost Compensating</td>
</tr>
<tr>
<td>Wu et al. [46]</td>
<td>1 distributor, 1 3PL provider, 1 retailer</td>
<td>Price, Logistics service level</td>
<td>Stochastic</td>
<td>Inventory, pricing, logistics service level</td>
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<td>1. Revenue-and-service-cost-sharing 2. Price-discount and inventory-risk sharing</td>
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<tr>
<td>Hou and Liu [18]</td>
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<td>Price, Quality</td>
<td>Stochastic</td>
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<td>Zheng et al. [55]</td>
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<tr>
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<tr>
<td>Wang and Chen [45]</td>
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<td>–</td>
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<td>Pricing, Inventory</td>
<td></td>
<td></td>
<td>Call option contract</td>
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<td>Moon et al. [29]</td>
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<td>Price, Freshness keeping effort</td>
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<td></td>
<td></td>
<td>1. Revenue-and-investment-cost-sharing 2. Incremental quantity discount</td>
</tr>
<tr>
<td>Mohammadi et al. [26]</td>
<td>1 supplier, 1 buyer</td>
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<td>Stochastic</td>
<td>Inventory, Pricing, Preservation technology investment</td>
<td></td>
<td></td>
<td>Revenue-and-preservation-technology-investment-sharing</td>
</tr>
<tr>
<td>Yang and Tang [51]</td>
<td>1 supplier, 1 retailer</td>
<td>Price, Freshness level</td>
<td>Deterministic</td>
<td>Pricing, Freshness-keeping effort</td>
<td>✓</td>
<td></td>
<td>Two-part tariff</td>
</tr>
<tr>
<td>Zheng et al. [56]</td>
<td>1 supplier, N retailer</td>
<td>–</td>
<td>Deterministic</td>
<td>Pricing, Inventory</td>
<td></td>
<td></td>
<td>Quantity discount</td>
</tr>
<tr>
<td>Ma et al. [25]</td>
<td>1 supplier, 1 3PL provider, 1 retailer</td>
<td>Price, Freshness keeping effort</td>
<td>Stochastic</td>
<td>Inventory, Pricing, Freshness-keeping effort, Freshness-keeping price</td>
<td></td>
<td></td>
<td>Revenue and cost-sharing</td>
</tr>
<tr>
<td>Hashemi et al. [12]</td>
<td>1 supplier, 1 retailer</td>
<td>Price, Freshness level</td>
<td>Stochastic</td>
<td>Inventory, Pricing</td>
<td>✓</td>
<td></td>
<td>Return policy with revenue-and-cost-sharing</td>
</tr>
<tr>
<td>Yan et al. [50]</td>
<td>1 manufacturer, 1 retailer, strategic customers</td>
<td>Freshness level</td>
<td>Stochastic</td>
<td>Inventory, Pricing</td>
<td></td>
<td></td>
<td>Revenue-sharing</td>
</tr>
<tr>
<td>Qiu et al. [36]</td>
<td>1 3PL, 1 retailer</td>
<td>Price, Logistics service level, Freshness</td>
<td>Stochastic</td>
<td>Pricing, Logistics service level</td>
<td></td>
<td></td>
<td>Revenue-and-services-cost-sharing</td>
</tr>
<tr>
<td>Feng et al. [10]</td>
<td>1 supplier, 1 retailer</td>
<td>Price, Freshness</td>
<td>Stochastic</td>
<td>Pricing, Freshness-keeping effort</td>
<td></td>
<td></td>
<td>Cost-sharing and compensation strategy</td>
</tr>
<tr>
<td>Nematoollahi et al. [31]</td>
<td>2 farmers, 1 Agri. enterprise</td>
<td>Price</td>
<td>Deterministic</td>
<td>Pricing</td>
<td>Production quantities</td>
<td>✓</td>
<td>Synergistic trilateral contract farming</td>
</tr>
<tr>
<td>Xu et al. [48]</td>
<td>1 supplier, 1 retailer</td>
<td>Price, Time</td>
<td>Deterministic</td>
<td>Pricing</td>
<td></td>
<td></td>
<td>Consignment revenue-sharing contract</td>
</tr>
<tr>
<td>This paper</td>
<td>1 supplier, 1 retailer</td>
<td>Price, Freshness level</td>
<td>Stochastic</td>
<td>Inventory, Pricing</td>
<td>✓</td>
<td>✓</td>
<td>Wholesale price and double compensation</td>
</tr>
</tbody>
</table>
economic and social viewpoints and determine optimal pricing policy, choosing from price differentiation policy and the same pricing.

In the literature on the SLPSC coordination models, various coordination contracts such as quantity discount [56], two-part tariff [13,51], wholesale price discount [4], revenue-and/or-cost-sharing [25,36,47,49,50,53,55,57], and call option contracts [45] have been proposed to motivate the SC members to participate in the coordinated structure. In the real world, for some types of short lifetime products (e.g., pasteurized dairy, packed fresh meat, and seafood), the supplier may offer a buyback contract to decrease the overstocking risk that retailers face [23, 27]. In this paper, a novel coordination contract based on return policy is proposed to enhance the whole SLPSC performance. This incentive mechanism is named wholesale price and double compensation (WPDC) contract.

According to Table 1, there has been less attention on the cannibalization between new and old products in the SLPSC coordination context. Moreover, this study is the first attempt to address the coordination of SLPSC members operating in a segmented market. In the current research, we design a new incentive mechanism to coordinate the SLPSC members and examine the cannibalization effects on the social and economic performances of the SLPSC.

3. Problem definition

This paper investigates a two-level SC comprising a supplier and a retailer with one type of perishable product with a short and fixed shelf life. The supplier’s capacity is assumed to be unlimited. The retailer uses a periodic review inventory system over an infinite planning horizon. The supplier visits the retailer in constant intervals to receive his order and deliver it instantaneously. Because of the short lifetime of products, the review period is short and predetermined by the supplier. The market demand is stochastic, and consumers are sensitive to the product price and freshness.

Each product has a two-period lifetime, and it is classified into new and old. Demand uncertainty may result in unsold new units at the end of each period, which are carried over to the next period and sold as old products. There is a difference in consumer valuation between new and old products. A freshness factor \( \delta \in [0, 1] \) is used to show the consumer valuation for the old products. Of note, if \( \delta = 1 \), consumers are indifferent about which product to buy.

The retailer uses a price differentiation policy to influence consumers’ preferences. Hence, the new and old items compete on freshness and price to attract consumers. The market is made up of two segments: freshness-oriented and discount-oriented consumers. It is assumed that none of the discount-oriented consumers are interested in buying the new product because the retail price is higher than the consumer’s willingness to pay. On the other hand, freshness-oriented consumers choose between new and old products based on the freshness level and the relative affordability of prices. The price differentiation strategy leads to the migration of some freshness-oriented consumers from buying the new product to the old one; in other words, the lower-priced old items could cannibalize the sales of new items.

The price-dependent demand functions for the new and old items are modeled in an additive fashion. It is assumed that the demand function of the new products has an independent random component, showing that the demand reacts to other agents, in addition to prices and freshness degree. Such demand function is popular in the related literature (e.g., [34,38]). Accordingly, the demand function for the new products during each period can be expressed as:

\[
\xi_n = a_1 - k_1 p_n - \frac{\delta}{1 - \delta} (p_n - p_o) + \tilde{\epsilon}_0
\]  

where \( p_o \) and \( p_n \) denote the retail price for the old and new items, and \( \tilde{\epsilon}_0 \in [A_0 \cdot C_0] \) is the random component of demand. The parameter \( a_1 \) denotes the potential size of market demand, and \( k_1 \) represents the price sensitivity of freshness-oriented consumers for the new products.

Moreover, a parameter \( \beta \) is used to represent a minimum discount requested by the discount-oriented consumers for buying an old product; in other words, they buy the old products if \( p_o \leq (1 - \beta) p_n \). The parameter \( \beta \)
is assumed to be a function of the freshness factor $\delta$, implying the fact that the minimum discount requested by the discount-oriented consumers depends on the consumer valuation for the old products. The lower the value of the old unit from the consumers’ viewpoint, the more discount they demand.

The demand function for the old items is as follows:

$$D_o = \begin{cases} a_2 - k_2p_o + \frac{\delta}{1-\delta} (p_n - p_o) & \text{if } p_o \leq (1 - \beta)p_n , \\ \frac{\delta}{1-\delta} (p_n - p_o) & \text{otherwise} \end{cases}$$

(3.2)

where $(1 - \beta)p_n$ denotes the discount-oriented consumers’ maximum willingness to pay; the parameter $a_2$ denotes the potential size of market demand for the old items, and $k_2$ represents the price sensitivity of discount-oriented consumers for the old products. It is assumed that $k_2 > k_1$, implying that the market demand is more negatively affected by any marginal increase in the retail price of the old products than the new ones, which mainly depends on the value drop of the old products.

The expression of $\frac{\delta}{1-\delta} (p_n - p_o)$ indicates the cannibalization effect and depends on two factors: the cannibalization amplitude, $\frac{\delta}{1-\delta}$, and the price difference [37]. The cannibalization amplitude indicates the marginal decrease (increase) in the sale of new (old) products due to the price differentiation, and it can be defined as a function of the freshness factor $\delta$, as implied in references [11, 26, 39]. In this paper, following Zhang and Zhang [54], the cannibalization amplitude is defined as $\frac{\delta}{1-\delta}$. Accordingly, when the parameter $\delta$ is near to one, the cannibalization amplitude is high. In this case, the consumers’ sensitivity to the freshness is low; therefore, a little discount on the old units results in more migration of consumers from buying the new products to the old ones.

At the end of each period, the cost of preparing the leftover new products for sale in the next period consists of the normal inventory holding cost, special storage, packaging, and re-tagging discounted products that poses extra work for personnel. Furthermore, the retailer may encounter unsold old products. In order to prevent the negative environmental effects of the expired products, the supplier offers a return policy to the retailer at a specified buyback price, $b$, per unit. The supplier salvages the expired returned products at a value $g$ per unit and sends them to other industries for recovery. Indeed, the short lifetime of products and sanitary concerns prevent the expired product from being reused in the forward SC.

In the investigated SLPSC, the supplier decides on the wholesale and buyback prices that impact the retailer’s decisions. Moreover, the retailer’s decisions on the old product price and the order quantity influence the supplier’s profit by affecting SC sales volume. Due to the dominant power of the supplier, the interaction between SC members is considered as a Stackelberg game in which the supplier acts as a leader, and the retailer reacts as a follower. However, in the decentralized decision-making, the SC members optimize their personal decisions while ignoring the whole SC. Therefore, the decentralized structure may not be a proper decision-making method. In the centralized approach, all decision variables are optimized from the whole SLPSC viewpoint, which can also enhance the environmental and social performances of SC, but it may reduce the SC members’ profit. Therefore, a coordination mechanism should be used to encourage the SC members to join in the centralized structure. In the following, several basic assumptions are considered before modeling:

- The retailer’s decisions on the old product price and the order quantity are assumed to be constant in different periods. In other words, they are independent of the initial inventory of the old units at the beginning of each period. This is a simple and practical policy to apply, which often prevails at supermarkets where the short lifetime products such as meat, vegetables, bakery, and dairy products are sold. This price-constancy is termed as markdown pricing stickiness [5].
- The selling price of new units is assumed to be fixed and predetermined. This is quite common in industries such as dairy products, foods, etc.
- The random component $\tilde{\xi}_0$ is a random variable distributed uniformly in the range $[A_0, C_0]$ with Probability density function $f(x)$ and Cumulative distribution function $F(x)$. We define the new random variable $\xi_1 = \tilde{\xi}_0 - A_0$, that $\xi_1 \in [0, B_0]$ where $B_0 = C_0 - A_0$. The demand function of the new products can be written as $\xi_n(p_o) = D_n(p_o) + \xi_1$ where $D_n(p_o) = a_1 + A_0 - k_1p_n - \frac{\delta}{1-\delta} (p_n - p_o)$. 


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− No substitution occurs between new and old units during the shortage of the one preferred by the consumer. Note that stock-out results in lost sales with a negligible cost.

The following notations are used in this study.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_m )</td>
<td>Production cost per unit product</td>
</tr>
<tr>
<td>( \xi_0 )</td>
<td>Random component of demand that ranges in ([A_0, C_0]) with Probability density function ( f(x) ) and Cumulative distribution function ( F(x) )</td>
</tr>
<tr>
<td>( \xi_n(p_o) )</td>
<td>Stochastic demand for the new product during a period</td>
</tr>
<tr>
<td>( D_o(p_o) )</td>
<td>Deterministic demand for the old product during a period</td>
</tr>
<tr>
<td>( a_1, a_2 )</td>
<td>Potential size of market demand for new and old items</td>
</tr>
<tr>
<td>( k_1, k_2 )</td>
<td>Consumer’s sensitivity to the prices of new and old items</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Freshness factor</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Minimum discount requested by the discount-oriented consumers for buying an old product</td>
</tr>
<tr>
<td>( g )</td>
<td>Salvage value</td>
</tr>
<tr>
<td>( p_n )</td>
<td>Retail price for a new product</td>
</tr>
<tr>
<td>( h )</td>
<td>Holding cost per remaining new product at the end of each period</td>
</tr>
<tr>
<td>( \Pi^N_M )</td>
<td>Total profit of ( M ) in ( N ) decision-making model, where the suffixes ( R; S; SC ) are used to represent the retailer, supplier, and whole SC, respectively, and the prefixes ( dc; c; co ) denote the decentralized, centralized, and coordinated approaches</td>
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</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_o )</td>
<td>Retail price for an old product</td>
</tr>
<tr>
<td>( q )</td>
<td>Order quantity</td>
</tr>
<tr>
<td>( z )</td>
<td>Stocking factor</td>
</tr>
<tr>
<td>( w )</td>
<td>Wholesale price</td>
</tr>
<tr>
<td>( b )</td>
<td>Buy back price</td>
</tr>
</tbody>
</table>

3.1. Social benefit

Although the retailer may consider the cannibalization effect as a threat for his business, the price differentiation can increase the total demand by attracting consumers who only seek to purchase discounted products. Therefore, the implied cannibalization threat can be converted to a business opportunity for increasing the market share. Moreover, there is an opportunity to achieve social benefits. In this study, for quantitative analysis, the social benefit is considered in the form of consumer surplus, which is defined as the difference between the consumer’s willingness to pay and the market price. This can be calculated as the area under the demand curve above the market price [6]. When demand is uncertain, some consumers may not find the product available; therefore, the consumer surplus is computed based on the stock-out probability. The expected consumer surplus (ECS) in the segmented market can be calculated based on the study conducted by Cohen et al. [7]. Note that the first-come-first-served logic with the consumers’ random arrival is considered.

**Proposition 3.1.** In the segmented market, the expected consumer surplus of new and old products can be computed as follows, respectively:

\[
ECS_{new} = \int_{0}^{B_0} CS_{new}^\max(\xi_1) \frac{\min(\xi(\xi_1), q)}{\xi(\xi_1)} f(\xi_1) d\xi_1 \tag{3.3}
\]

\[
ECS_{old} = \int_{l}^{\infty} CS_{old}^\max(\min([D_{o1} + D_{o2}], s), s) f(s) ds \tag{3.4}
\]

where \( D_{o1} = \frac{\delta}{1-\delta}(p_n - p_o) \) is the freshness-oriented consumers’ demand for the old units, and \( D_{o2} = a_2 - k_2p_o \) is the discount-oriented consumers’ demand for them. Moreover, the random variable \( s \) represents the number...
of new products remaining at the end of each period, and

\[
CS_{\text{new}}^{\text{max}}(\varepsilon_1) = \frac{(D_n + \varepsilon_1)^2}{2\left(k_1 + \frac{\delta}{(1-\delta)}\right)} \tag{3.5}
\]

\[
CS_{\text{old}}^{\text{max}} = \frac{D_2(1-\delta)}{2\delta} + (1-\beta)p_n\left(a_2 - \frac{k_2(1-\beta)p_n}{2}\right) + \frac{D_2^2 - a_2^2}{2k_2}. \tag{3.6}
\]

Proof. See Appendix A. \qed

4. Mathematical modeling

In this section, the investigated SLPSC is modeled under various decision-making approaches.

4.1. Decentralized decision-making

In the decentralized structure, each member aims to optimize the individual objective function [33]. In this section, the interaction between the retailer and the supplier is considered as a Stackelberg game. Firstly, the supplier, as a leader, decides on the wholesale and buyback prices by predicting the retailer’s responses; then, the retailer, as a follower, determines the selling price of old units and the order quantity of the new ones according to the supplier’s announced decisions. Thus, a backward induction can be used to find the optimal solution.

We consider a myopic policy that is a generic followed approach for solving complex dynamic problems such as scheduling, pricing, and inventory problems [22]. In this policy, the optimal solutions only maximize the profit of the current period regardless of the future. It has been proved, in the literature, that the myopic policy is optimal for some stationary problems in which variables and parameters are independent of time (e.g., [5,43]). In the study conducted by Chintapalli [5], the optimality of the myopic policy was proved for an inventory and markdown pricing problem of perishable products with a two-period lifetime in an infinite planning horizon. Accordingly, the retailer’s decision variables are determined so as to optimize the profit of the current period. Under the myopic policy, the retailer’s profit function in the decentralized approach can be formulated as:

\[
\Pi_{R}^{\text{dec}}(q,p_o) = \begin{cases} 
 p_n\xi_n - wq + p_oD_o(p_o) + b(q - \xi_n - D_o(p_o)) - h(q - \xi_n) & \xi_n \leq q - D_o(p_o) \\
 p_n\xi_n - wq + p_o(q - \xi_n) - h(q - \xi_n) & q - D_o(p_o) < \xi_n \leq q \\
 (p_n - w)q & q < \xi_n.
\end{cases} \tag{4.1}
\]

The profit function (4.1) consists of the sales revenue of the new and old units, ordering cost, the buyback revenue of expired units, and inventory holding cost. The retailer’s expected profit function can be written by defining the stocking factor of new products as \(z = q - D_o(p_o)\) [34]. Given that \(\xi_n = D_n(p_o) + \varepsilon_1\), we have:

\[
E[\Pi_{R}^{\text{dec}}(z,p_o)] = p_n\left(\int_0^z (D_n(p_o) + x) f_{\xi_1}(x) \, dx + \int_{z}^{B_0} (D_n(p_o)+z) f_{\xi_1}(x) \, dx\right) \\
+ p_o\left(\int_0^{z-D_o(p_o)} D_o(p_o) f_{\xi_1}(x) \, dx + \int_{z-D_o(p_o)}^{z} (z-x) f_{\xi_1}(x) \, dx\right) \\
- w(z + D_n(p_o)) + b\int_0^{z-D_o(p_o)} (z-D_o(p_o) - x) f_{\xi_1}(x) \, dx \\
- h\int_0^{z} (z-x) f_{\xi_1}(x) \, dx. \tag{4.2}
\]

By simplifying (4.2), the mathematical model for the retailer can be formulated as:

\[
\text{Max } E\left[\Pi_{R}^{\text{dec}}(z,p_o)\right] = -w(z + D_n(p_o)) + p_n(z + D_n(p_o)) - (p_n + h)\int_0^{z} F(x) \, dx
\]
The optimal profit of the retailer can be obtained by substituting \( p \) function. Hence, comparing the feasible stationary points as well as the boundary and break-points of the retailer’s profit

In the decentralized approach, the optimal price of the old item \( p \) can be achieved by comparing two pricing policies: price differentiation versus the same pricing. Constraint (4.5) shows the non-negativity restriction on the stocking factor.

**Proposition 4.1.** In the decentralized approach, the retailer’s objective function is concave with respect to \( z \) for a given \( p \).

Proof. See Appendix B.

Considering the uniformly distributed random component of demand, we have:

\[
E[\Pi^\text{dec}_R(z, p)] = (p_n - w)(z + D_n(p_o)) - \left( \frac{p_n + h}{2B_o} \right) z^2 + \frac{p_o}{2B_o} \left( z^2 - (z - D_o(p_o))^2 \right) + \frac{b}{2B_o}(z - D_o(p_o))^2. \tag{4.6}
\]

The function (4.6) has exactly one maximum point \( z_{\text{dec}}(p_o) \), which can be obtained by \( \partial E(\Pi^\text{dec}_R(p)) / \partial z = 0 \).

\[
z_{\text{dec}}(p_o) = \frac{B_o(p_n - w) + D_o(p_o)(p_o - b)}{p_n + h - b}. \tag{4.7}
\]

By substituting \( z_{\text{dec}}(p_o) \) into equation (4.6), the retailer’s profit function, \( E(\Pi^\text{dec}_R(p_o)) \), is obtained as a piecewise polynomial function with a breakpoint at \( p_o = (1 - \beta)p_n \) (see Appendix C).

**Proposition 4.2.** In the decentralized approach, the optimal price of the old item \( p^\text{dec}_o \) can be achieved by comparing the feasible stationary points as well as the boundary and break-points of the retailer’s profit function. Hence, \( p^\text{dec}_o = \arg \max \{ \Pi^\text{dec}_R(b), \Pi^\text{dec}_R(p_o1), \Pi^\text{dec}_R(p_o2), \ldots, \Pi^\text{dec}_R(p_o), \Pi^\text{dec}_R((1 - \beta)p_n), \Pi^\text{dec}_R((1 - \beta)p_n + \varepsilon) \}, \) in which \( p_o1, p_o2, \ldots, p_ok \) is the set of feasible solutions of the equation \( \frac{\partial E(\Pi^\text{dec}_R(p_o))}{\partial p_o} = 0 \), and \( \Pi^\text{dec}_R(1 - \beta)p_n + \varepsilon = \lim_{p_o \rightarrow 1 - \beta} \Pi^\text{dec}_R(p_o) \).

Proof. See Appendix D.

By substituting \( p^\text{dec}_o \) into (4.7), the optimal stocking factor is:

\[
z_{\text{dec}} = \frac{B_o(p_n - w) + D_o(p^\text{dec}_o)(p^\text{dec}_o - b)}{p_n + h - b}. \tag{4.8}
\]

The optimal profit of the retailer can be obtained by substituting \( p^\text{dec}_o \) and \( z_{\text{dec}} \) into (4.6):

\[
E(\Pi^\text{dec}_R) = (p_n - w)(z_{\text{dec}} + D_n(p^\text{dec}_o)) - \left( \frac{p_n + h}{2B_o} \right) z_{\text{dec}}^2 + \frac{p_o}{2B_o} \left( z_{\text{dec}}^2 - (z_{\text{dec}} - D_o(p^\text{dec}_o))^2 \right) + \frac{b}{2B_o}(z_{\text{dec}} - D_o(p^\text{dec}_o))^2. \tag{4.9}
\]

In the decentralized approach, the supplier determines the wholesale and buyback prices by taking into account the retailer’s response. Therefore, the supplier’s profit function considering the retailer’s optimal decisions can be written as:

\[
\text{Max } E(\Pi^\text{dec}_S(w, b)) = (w - c_n)(z_{\text{dec}} + D_n(p^\text{dec}_o)) + (g - b) \int_{0}^{z_{\text{dec}} - D_o(p^\text{dec}_o)} F(x) \, dx. \tag{4.10}
\]
By assuming the uniform distribution for the random component of demand, the mathematical model for optimizing the supplier’s profit is formulated as:

\[
\text{Max } E(\Pi_S^{\text{dec}}(w, b)) = (w - c_m)(z_{\text{dec}} + D_n(p_o^{\text{dec}})) + \frac{(g - b)}{2B_0}(z_{\text{dec}} - D_o(p_o^{\text{dec}}))^2
\]  

(4.11)

Subject to:

\[
(z_{\text{dec}}, p_o^{\text{dec}}) \in \arg \max E(\Pi_R^{\text{dec}}(z, p_o)) = -w(z + D_n(p_o)) + p_n z + p_n D_n(p_o) - \left(p_n + h\right)z^2 + \frac{p_n}{2B_0}(z^2 - (z - D_o(p_o))^2) + \frac{b}{2B_0}(z - D_o(p_o))^2
\]  

(4.12)

\[c_m < w \leq (1 - \theta)p_n\]  

(4.13)

\[0 \leq b \leq w.\]  

(4.14)

In the investigated problem, the retailer seeks at least $100\%$ percent profit margins on new products. Thus, the wholesale price must be lower than $(1 - \theta)p_n$. Moreover, to obtain positive profit, the supplier must set the wholesale price greater than the production cost and also the buy-back price (Constraints (4.13) and (4.14)).

The following algorithm is designed to determine the optimal values of the supplier’s decisions:

**Search procedure**

Step 1. Set the lowest feasible value to $b$.

Step 2. Set $w = \max\{c_m, b\}$.

Step 3. Calculate $p_o^{\text{dec}}$ using Proposition 4.2.

Step 4. Calculate $z_{\text{dec}}$ from equation (4.8).

Step 5. Evaluate and save the supplier’s profit from equation (4.11) for $(w, b, z_{\text{dec}}, p_o^{\text{dec}})$.

Step 6. If $w < (1 - \theta)p_n$, then set $w = w + \varepsilon$ (where $\varepsilon$ is small) and go to step 3.

Step 7. If $b < (1 - \theta)p_n$, then set $b = b + \varepsilon$ and go to step 2.

Step 8. The values of $(w, b)$ with the greatest supplier’s profit are optimal.

### 4.2. Centralized decision-making

In the centralized structure, one SC manager determines all decision variables to optimize the whole SLPSC profit, which is written as the sum of the retailer’s and supplier’s profit functions. The mathematical model for the centralized approach is as follows:

\[
E[\Pi_{SC}^{\text{cen}}(z, p_o)] = p_n z + p_n D_n(p_o) - (p_n + h) \int_0^z F(x) \, dx + p_o \int_{z - D_o(p_o)}^z F(x) \, dx - c_m(z + D_n(p_o)) + g \int_0^{z - D_o(p_o)} F(x) \, dx
\]  

(4.15)

Subject to:

\[g \leq p_o \leq p_n\]  

(4.16)

\[z \geq 0.\]  

(4.17)

The profit function (4.15) consists of the expected revenue from the sold new and old products, inventory holding cost, production cost, and the salvage revenue of expired returned products. The allowable range of price changes for the old products is showed in Constraint (4.16).
Proposition 4.3. The SLPSC profit function is concave with regard to \( z \) for a given \( p_o \).

**Proof.** Similar to the proof procedure of Proposition 4.1.

The function (4.15) can be rewritten by considering the uniformly distributed random component of demand as follows:

\[
E[\Pi_{\text{cen}}^\text{SC}(z, p_o)] = -c_m(z + D_n(p_o)) + p_n z + p_n D_n(p_o) - \left( \frac{p_n + h}{2B_0} \right) z^2 + \frac{p_o}{2B_0} \left( z^2 - (z - D_o(p_o))^2 \right) + \frac{g}{2B_0} (z - D_o(p_o))^2.
\]

The optimal value of the stocking factor in the centralized model can be calculated by \( \partial E(\Pi_{\text{cen}}^\text{SC})/\partial z = 0 \):

\[
z_{\text{cen}}(p_o) = \frac{B_0(p_n - c_m) + D_o(p_o)(p_o - g)}{p_n + h - g}.
\]

By substituting equation (4.19) into equation (4.18), the whole SC profit function is obtained as a piecewise polynomial function with a breakpoint at \( p_o = (1 - \beta)p_n \) (see Appendix E). Then, the \( p_o^\text{cen} \) can be determined through the following proposition.

**Proposition 4.4.** In the centralized approach, the optimal price of the old item, \( p_o^\text{cen} \) can be achieved by comparing the feasible stationary points as well as the boundary and break-points of the SC profit function. Hence, \( p_o^\text{cen} = \arg\max \{ \Pi_{\text{cen}}^\text{SC}(g), \Pi_{\text{cen}}^\text{SC}(p_o1), \Pi_{\text{cen}}^\text{SC}(p_o2), \ldots, \Pi_{\text{cen}}^\text{SC}(p_0k), \Pi_{\text{cen}}^\text{SC}(p_n), \Pi_{\text{cen}}^\text{SC}((1 - \beta)p_n), \Pi_{\text{cen}}^\text{SC}((1 - \beta)p_n + \varepsilon) \} \), in which \( \{ p_o1, p_o2, \ldots, p_0k \} \) is the set of feasible solutions of the equation \( \frac{\partial \Pi_{\text{cen}}^\text{SC}(p_o)}{\partial p_o} = 0 \), and \( \Pi_{\text{cen}}^\text{SC}((1 - \beta)p_n + \varepsilon) = \lim_{p_o \to (1 - \beta)p_n} \Pi_{\text{cen}}^\text{SC}(p_o) \).

**Proof.** Similar to the proof of Proposition 4.2.

By substituting \( p_o^\text{cen} \) into equation (4.19), the optimal stocking factor is:

\[
z_{\text{cen}} = \frac{B_0(p_n - c_m) + D_o(p_o^\text{cen})(p_o^\text{cen} - g)}{p_n + h - g}.
\]

Since \( p_o^\text{cen} \) and \( z_{\text{cen}} \) are globally optimized, therefore

\[
\Pi_{\text{cen}}^\text{SC}(z_{\text{cen}}, p_o^\text{cen}) \geq \Pi_{\text{cen}}^\text{SC}(z_{\text{dec}}, p_o^\text{dec}).
\]

In other words, the centralized approach results in the best performance of the whole SLPSC. However, the centralized solution may reduce each member’s profitability compared to the decentralized approach. As a result, the SC members may not accept to join the centralized decision-making model. Hence, to encourage the SC members to take part in the centralized approach, a new incentive scheme is proposed in the following.

### 4.3. Coordination model and incentive scheme

Although the centralized decision-making model is more profitable for the whole SLPSC compared to the decentralized one, the members’ profitability may decrease [20]. In the following, a novel coordination contract is designed to optimize the whole SLPSC profit and ensure both members’ participation. Accordingly, an incentive scheme named wholesale price and double compensation (WPDC) contract is proposed. In this contract, the supplier sets a wholesale price \( w_r \) and offers the retailer double compensation for the unsold products. In this regard, a sharing rate \( \varphi \in [0, 1] \) is used to coordinate the SC. In each period, the retailer may miss the opportunity to sell a number of new products at price \( p_n \). These products are offered as old units with a lower price \( p_o \) in the next period. Moreover, the retailer pays the holding and preparing costs of the leftover new units at the end of each period. Hence, the retailer loses \( (p_n - p_o + h) \) for each unsold new product. In the proposed
WPDC contract, the supplier shares some loss of the leftover new products by providing compensation to the retailer as $s_1 = \varphi(p_n - p_o + h)$ per unit. Moreover, the retailer may encounter unsold old products, in which case the retailer loses $p_o$ per unit, and the supplier pays compensation of $\varphi p_o$ per unit. The supplier salvages the expired products at a value $g$ per unit and renders them to other industries for recovery. In the WPDC contract, the supplier shares a proportion $(1 - \varphi)$ of the salvage revenue with the retailer. Therefore, the supplier pays $s_2 = \varphi p_o + (1 - \varphi)g$ per unit of the returned product to the retailer.

Accordingly, under the WPDC contract, the retailer’s expected profit function can be written as:

$$E[\Pi^\text{Co}_R(z, p_o)] = -w_r(z + D_n(p_o)) + p_n(z + D_n(p_o)) - \left(\frac{p_n + (1 - \varphi)h - \varphi(p_n - p_o)}{2B_0}\right)z^2 + \frac{p_o}{2B_0}(z^2 - (z - D_o(p_o))^2) + \left(\frac{\varphi p_o + (1 - \varphi)g}{2B_0}\right)(z - D_o(p_o))^2. \quad (4.22)$$

Similarly, the supplier’s expected profit under the coordinated contract is:

$$E[\Pi^\text{Co}_S(w_r, \varphi)] = (w_r - c_m)(z + D_n(p_o)) - \left(\frac{\varphi(p_n - p_o + h)}{2B_0}\right)z^2 - \left(\frac{\varphi(p_o - g)}{2B_0}\right)(z - D_o(p_o))^2. \quad (4.23)$$

Calculating $\partial E(\Pi^\text{Co}_R)/\partial z = 0$, the optimal value of the stocking factor in the coordinated model is obtained as

$$z_{\text{Co}} = \frac{B_0(p_n - w_r) + D_o(p^\text{Co}_o)((1 - \varphi)(p^\text{Co}_o - g))}{(1 - \varphi)(p_n + h - g)}. \quad (4.24)$$

Incorporating $z_{\text{Co}} = z_{\text{cen}}$ and $p^\text{Co}_o = p^\text{cen}_o$ into equation (4.24), we have:

$$\frac{B_0(p_n - c_m) + D_o(p^\text{Co}_o)(p^\text{cen}_o - g)}{p_n + h - g} = \frac{B_0(p_n - w_r) + D_o(p^\text{cen}_o)((1 - \varphi)(p^\text{cen}_o - g))}{(1 - \varphi)(p_n + h - g)}. \quad (4.25)$$

By simplifying equation (4.25), the optimal wholesale price is:

$$w^\text{Co}_r = (1 - \varphi)c_m + \varphi p_n. \quad (4.26)$$

Accordingly, the supplier’s revenue directly depends on the parameter $\varphi$. In other words, the proposed WPDC contract acts similar to a revenue-and-cost-sharing contract, except that WPDC also shares the lost opportunity cost (as compensation). Considering the rational decisions of the SC members, both retailer and supplier accept to join in the coordination mechanism if and only if their expected profits under the proposed coordination mechanism are more than the decentralized structure. In other words, the proposed contract must satisfy the conditions $E(\Pi^\text{Co}_R(z_{\text{cen}}, p^\text{cen}_o)) \geq E(\Pi^\text{dec}_R(z_{\text{dec}}, p^\text{dec}_o))$ and $E(\Pi^\text{Co}_S(w_r)) \geq E(\Pi^\text{dec}_S(w^*, b^*))$. Accordingly, the maximum and minimum admissible values of sharing rate can be achieved through equations (4.9), (4.11), (4.22), (4.23), and (4.26) as follows:

$$\varphi_{\text{max}} = 1 - \frac{(p_n - w^*)(z_{\text{dec}} + D_n(p^\text{dec}_o)) - \left(\frac{p_n + h}{2B_0}\right)z_{\text{dec}}^2 + \frac{p^\text{cen}_o}{2B_0}(z_{\text{dec}} - (z_{\text{dec}} - D_o(p^\text{dec}_o))^2) + \frac{g}{2B_0}(z_{\text{dec}} - D_o(p^\text{dec}_o))^2}{(p_n - c_m)(z_{\text{cen}} + D_n(p^\text{cen}_o)) - \left(\frac{p_n + h}{2B_0}\right)z_{\text{cen}}^2 + \frac{p^\text{cen}_o}{2B_0}(z_{\text{cen}} - (z_{\text{cen}} - D_o(p^\text{cen}_o))^2) + \frac{g}{2B_0}(z_{\text{cen}} - D_o(p^\text{cen}_o))^2}. \quad (4.27)$$

$$\varphi_{\text{min}} = \frac{(w^* - c_m)(z_{\text{dec}} + D_n(p^\text{dec}_o)) + \left(\frac{g - b^*}{2B_0}\right)(z_{\text{dec}} - D_o(p^\text{dec}_o))^2}{(p_n - c_m)(z_{\text{cen}} + D_n(p^\text{cen}_o)) - \left(\frac{p_n + h}{2B_0}\right)z_{\text{cen}}^2 + \frac{p^\text{cen}_o}{2B_0}(z_{\text{cen}} - (z_{\text{cen}} - D_o(p^\text{cen}_o))^2) + \frac{g}{2B_0}(z_{\text{cen}} - D_o(p^\text{cen}_o))^2}. \quad (4.28)$$

Hence, the proposed incentive mechanism can reach perfect coordination of the SC when $\varphi$ belongs to $[\varphi_{\text{min}}, \varphi_{\text{max}}]$. Regarding the equation (4.26), the sharing rate $\varphi$ demonstrates the proportion that the supplier obtains of the whole SC profit. Accordingly, at the upper bound $\varphi_{\text{max}}$, all increased profits resulted from the coordination contract are gained by the supplier. On the contrary, at the lower bound $\varphi_{\text{min}}$, all increased profits are achieved by the retailer. The practical value of $\varphi$ is determined based on the bargaining power of the SC members.
Table 2. Data for five test problems.

<table>
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<th>Test problems</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$\delta$</th>
<th>$g$</th>
<th>$p_n$</th>
<th>$h$</th>
<th>$A_0, C_0$</th>
<th>$c_m$</th>
<th>$\theta$</th>
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<td>6</td>
<td>0.6</td>
<td>5</td>
<td>55</td>
<td>4</td>
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<td>20</td>
<td>0.15</td>
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<td>5</td>
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<td>2</td>
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<td>7</td>
<td>0.2</td>
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<td>4</td>
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<td>90</td>
<td>0.15</td>
</tr>
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5. Numerical examples and discussions

In this section, the performance of the proposed models is investigated through five test problems provided in Table 2. The parameter values are randomly set. The discount parameter, $\beta$, is assumed to be a function of the freshness factor $\delta$. Therefore, the lower the value of the old unit from the consumers’ viewpoint, the more discount they demand. In this study, it is assumed that $\beta = (1 - \delta)/2$.

The results of different decision-making approaches for the five test problems are illustrated in Table 3. As can be seen in Table 3, the whole SC profit and the retailer’s order quantity in the centralized model are higher than those in the decentralized one. Further, the old product price in the centralized model is lower than that of the decentralized one. Thus, the retailer’s profit may reduce, and therefore the retailer may not agree with the centralized decision-making structure without a proper incentive scheme.

Furthermore, according to Table 3, the optimal values of decision variables, including the old product price, the order quantity, and the profitability of the whole SC in the coordinated model are equal to those of the centralized model. However, in the coordinated approach, the wholesale price and the profitability of each SC member depend on the sharing rate $\phi$. Table 3 provides the feasible ranges of the sharing rate in the coordinated approach for five test problems. In these ranges, the proposed WPDC contract can be agreed by both retailer and supplier because their profits under the coordination contract are more than the decentralized approach. Note that by increasing the sharing rate $\phi$, the wholesale price increases, and thereby the members’ profit changes. The variations of both retailer’s and supplier’s profit against $\phi$ in TP#1 are shown in Figure 1.

Figure 1 indicates that by increasing the sharing rate $\phi$, the supplier’s profit in the coordinated model increases while the retailer’s profit decreases. According to Figure 1, when the value of $\phi$ is greater than almost 0.72, the supplier’s profit in the coordinated model is greater than that in the decentralized one. Moreover, when $\phi$ is higher than 0.77, the retailer’s profit in the coordinated model is less than that in the decentralized one. Therefore, if $\phi \in [0.72, 0.77]$, then the proposed coordination mechanism is acceptable to both SC members, and the overall SC profit of the centralized model is obtained.

Furthermore, the expected consumer surplus for the five test problems under the various approaches is given in Table 4. Accordingly, the proposed coordination approach can improve the social performance of the short lifetime product SC compared to the decentralized approach.

5.1. Sensitivity analysis

In this sub-section, sensitivity analyses on some key parameters are done to better show the performance of different decision-making approaches and provide some managerial insights for the short lifetime product SC in practice. To carry out the sensitivity analyses, TP#1 is selected. Firstly, the effect of the freshness factor $\delta$ on decision-making results is studied in the decentralized and coordinated approaches, assuming the other parameters remain unchanged. Recall that the parameter $\delta$ represents the consumers’ valuation for the old product. By increasing $\delta$, the consumers’ sensitivity to the product freshness decreases, and the sensitivity to the price difference between products (i.e., cannibalization amplitude) increases.
Table 3. Results of five test problems under different decision-making approaches.

<table>
<thead>
<tr>
<th></th>
<th>TP#1</th>
<th>TP#2</th>
<th>TP#3</th>
<th>TP#4</th>
<th>TP#5</th>
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<tr>
<td>$p_{o}^{dec}$</td>
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<td>14</td>
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<tr>
<td>$q_{o}^{dec}$</td>
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<tr>
<td>$w^{*}$</td>
<td>46</td>
<td>16</td>
<td>90</td>
<td>63</td>
<td>126</td>
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<tr>
<td>$b^{*}$</td>
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<td>0</td>
<td>34</td>
<td>114</td>
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<tr>
<td>$\Pi_{R}^{dec}$</td>
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<tr>
<td>$\Pi_{S}^{dec}$</td>
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<td>Centralized decision making</td>
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<td>13.9</td>
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<td>$q_{cen}$</td>
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</tr>
<tr>
<td>$\Pi_{SC}^{cen}$</td>
<td>22233</td>
<td>3247</td>
<td>41146</td>
<td>7302</td>
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</tr>
<tr>
<td>Coordinated approach</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$p_{o}^{co}$</td>
<td>38.5</td>
<td>13.9</td>
<td>74</td>
<td>57.3</td>
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<td>$q_{co}$</td>
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<td>$\Pi_{R}^{co}$</td>
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<td>974</td>
<td>8229</td>
<td>1825</td>
<td>41436</td>
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<tr>
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<td>2273</td>
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<td>3247</td>
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<td>7302</td>
<td>109041</td>
</tr>
<tr>
<td>$\varphi$</td>
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<td>0.8</td>
<td>0.75</td>
<td>0.62</td>
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<tr>
<td>$\varphi_{max}$</td>
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<td>0.76</td>
<td>0.63</td>
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<tr>
<td>$\varphi_{min}$</td>
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<td>0.67</td>
<td>0.77</td>
<td>0.705</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Figure 1. The variations of each SC members’ profit with respect to sharing rate $\varphi$. 
Table 4. Social results of the proposed models.

<table>
<thead>
<tr>
<th></th>
<th>TP#1</th>
<th>TP#2</th>
<th>TP#3</th>
<th>TP#4</th>
<th>TP#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decentralized approach</td>
<td>ECS</td>
<td>65 096</td>
<td>5836</td>
<td>101 390</td>
<td>7035</td>
</tr>
<tr>
<td>Coordinated (centralized) approach</td>
<td>ECS</td>
<td>77 903</td>
<td>6538</td>
<td>113 650</td>
<td>7874</td>
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</tbody>
</table>

Figure 2. Effect of freshness factor on the cannibalization effect.

Figure 2 indicates variations in the cannibalization effect by increasing the freshness factor $\delta$ under two decision-making approaches. Regarding the equation (3.1), the cannibalization effect depends on the cannibalization amplitude, $\frac{\delta}{(1-\delta)}$, and the price difference between new and old products. From Figure 2, by increasing the parameter $\delta$, the cannibalization effect increases in both models. Moreover, the cannibalization effect in the coordinated model is more than that in the decentralized one. Figure 2 shows that the cannibalization effect in the decentralized model is zero when $\delta$ is higher than 0.66; this threshold value for the coordinated model is 0.87. A cannibalization effect of zero represents setting the same price for all products. This means changing the policy from price differentiation to the same pricing. For analyzing the results, the effect of increasing freshness factor $\delta$ on the optimal price of the old units is studied (see Fig. 3).

Based on Figure 3, when the freshness factor $\delta$ increases, the optimal price of the old product increases in both models. However, the old product price under the coordinated model is less than that in the decentralized one. Therefore, the price difference between products and, as a result, the cannibalization effect in the proposed coordinated model are greater than the decentralized one. Moreover, the points of changing pricing policy in both models can be seen in Figure 3.

Based on Figure 4, by increasing the freshness factor $\delta$, the profitability of the whole SLPSC decreases in both models. Therefore, it can be concluded that the cannibalization effect harms the SC profitability. Moreover, in both models, when the parameter $\delta$ reaches the threshold values, the pricing policy changes from price differentiation to the same pricing policy in which the SC profitability is independent of the freshness factor $\delta$. Accordingly, when the consumers are more sensitive to freshness, the price differentiation policy is more profitable. The results of the current research are in accordance with the study conducted by Herbon et al. [16]. They investigated the impact of price differentiation policy in a deteriorating inventory system and concluded that the retailer’s profit from this policy for freshness-oriented customers is more than price-
oriented and indifference-oriented customers. However, in this paper, we study the problem in the SC context with coordination mechanisms. According to Figure 4, the proposed coordinated model is more profitable for the whole SLPSC in comparison with the decentralized one, especially when consumers are more sensitive to freshness.

As can be seen in Figure 4, the threshold value of $\delta$ for policy change in the coordinated model is greater than that in the decentralized one. In addition, as previously mentioned, the cannibalization effect in the coordinated model is higher than the decentralized one. Therefore, in comparison with the decentralized approach, the coordination model is more successful in turning the cannibalization threat into a business opportunity for increasing the whole SLPSC profit.

Figure 5 indicates changes in the expected sales volume (SV) by increasing the freshness factor $\delta$. From Figure 5, as $\delta$ increases, the expected SV decreases under the price differentiation policy in both decentralized
and coordinated approaches. Although the policy change results in a sudden increase in the expected SV in the decentralized model, the proposed coordination approach creates more expected SV under different freshness factor $\delta$ compared to the decentralized one. Note that higher SV in the coordinated approach results in greater earnings for the whole SLPSC. Moreover, from a social viewpoint, increasing SV can enhance the consumption of short lifetime products such as fresh foods needed to keep a healthy diet.

Furthermore, the expected CS variations by increasing $\delta$ under the coordinated and decentralized models are depicted in Figure 6. As shown, the expected CS decreases in both models by increasing the freshness factor $\delta$. Moreover, the effects of pricing policy change in both approaches can be seen in Figure 6. Accordingly, when $\delta$ is less than 0.66, and the price differentiation is determined as the optimal policy in both models, the expected CS in the coordinated model is higher than the decentralized one. This condition also occurs when $\delta$ is higher than 0.87, and the same price policy is optimal in both models. However, when $\delta \in [0.66, 0.87)$, the same price policy in the decentralized model results in slightly more expected CS than the price differentiation policy in the coordinated model. This is because of an upward jump in the expected CS that occurs after the policy change in the decentralized approach. Based on Figures 5 and 6, it can be concluded that the proposed coordination mechanism is beneficial for the SLPSC in terms of social aspects, especially at higher sensitivity to the product freshness.

The price sensitivity coefficients, $k_1$ and $k_2$, indicate consumers’ sensitivity to the retail prices of new and old units, respectively. Figures 7 and 8 show that how the whole SC profit would be influenced by changing the price sensitivity coefficients in various approaches. Based on these figures, the whole SLPSC profit decreases by increasing the price sensitivity coefficients in both the coordinated and decentralized approaches. Moreover, the coordinated approach results in more profitability for the whole SC than the decentralized one.

According to Figure 8, by increasing the parameter $k_2$, the pricing policy changes from price differentiation to the same price in both approaches. Therefore, the same pricing policy could be of high benefit to the SLPSC as the discount-oriented consumers are more sensitive to the old product price. The threshold values of the parameter $k_2$ at which the policy change occurs are shown in Figure 8. This value for the coordinated approach is more than the decentralized one. Hence, the proposed coordinated model is more resistant to policy change than the decentralized model.

5.2. Discussion

In what follows, the effect of freshness factor $\delta$ on the old product price, cannibalization effect, sales volume and overall SLPSC profit is analytically discussed. The freshness factor $\delta$ represents the consumers’ valuation
for the old product. By increasing $\delta$, the consumers’ sensitivity to the product freshness decreases. Note that the parameter $\delta$ indicates a product feature or consumer preferences. For instance, in a market with consumers who have low daily consumption rate, they have a greater tendency to purchase the fresher products, in which case the parameter $\delta$ is relatively small.

From the equation (3.1), the expression of $\frac{\delta}{(1-\delta)}(p_n - p_o)$ indicates the cannibalization effect which directly depends on the cannibalization amplitude, $\frac{\delta}{(1-\delta)}$, and the price difference between new and old products. By increasing the parameter $\delta$, the cannibalization effect increases which harms the SC members’ profitability [34]. In such a situation, the proposed mathematical models set a higher price for the old products to reduce the price difference and to mitigate the cannibalization threat. Therefore, in a market with consumers who have low sensitivity to the product’s freshness, the price differentiation is limited. Finally, when the parameter $\delta$ is high enough, the pricing policy changes from price differentiation to the same pricing policy.
Moreover, by increasing the old product price, the product demand decreases, which results in increasing the risk of unsold products at the end of each period. Thus, the retailer reduces the order quantity. This result is also obtained from the proposed mathematical models and the equations (4.7) and (4.19), in which the stocking factor directly depends on the demand function of the old product. Decreasing the sales volume will reduce the profitability of the whole SLPSC and its members. Therefore, when the parameter $\delta$ increases, the whole SC profit decreases.

5.3. Managerial insights

The important findings of this research can be noted as follows:

- Product cannibalization can occur as a result of the simultaneous sale of products of different ages. The cannibalization effect harms the SC members’ profitability. The findings indicate that, in the coordinated approach, the optimal price of the old product is lower, and the cannibalization effect is more than the decentralized one. However, the proposed coordinated approach is more profitable for the whole SC in comparison with the decentralized one, especially when consumers are more sensitive to the product freshness. This is because in the coordinated model, the retailer increases the order quantity, which results in increasing the expected sales volume. Therefore, the proposed coordination mechanism is more successful in turning the cannibalization threat into a business opportunity for increasing the whole SLPSC profit.

- From a social viewpoint, the coordinated model provides acceptable social performance in terms of consumer surplus. Moreover, the lower price of the old products leads to increased satisfaction of consumers, especially low-income families. The sensitivity analyses demonstrate the effectiveness of the proposed coordination mechanism to achieve social benefit, particularly in markets with high sensitivity to the product freshness.

- The results show that the expected sales volume in the coordinated structure is more than that in the decentralized one. From a social viewpoint, increasing SV can enhance the consumption of short lifetime products such as fresh foods needed to maintain a healthy diet. It is especially effective where the old items keep their nutritional quality at an appropriate level. For example, for some fresh products, such as pasteurized milk, the old items stored in a proper environmental condition have almost the same nutritional quality as the new ones (see [30, 40, 52]).

- The sensitivity analyses show that when consumers are more sensitive to the product freshness, the price differentiation policy is more profitable for the whole SLPSC in comparison with the same pricing policy. By decreasing the consumers’ sensitivity to the product freshness, the cannibalization effect increases, which results in changing the pricing policy. In practice, some perishable products such as fresh meat, seafood, and
poultry, have a high quality risk when approaching their expiration dates, and therefore consumers have a greater tendency to purchase the fresher products. Moreover, consumers with a lower household consumption rate are more sensitive to the remaining life of the product [42]. In these cases, the price differentiation policy creates more profitability than the same pricing policy.

- Based on the sensitivity analyses, as the price sensitivity coefficients increase, the whole SC profitability decreases in both approaches. That is because an increase in the price sensitivity coefficients reduces the amount of demand, which results in decreasing the expected sales volume and the SC profit. However, the proposed coordinated approach under the price differentiation policy is more profitable than the decentralized one, especially where the price sensitivity of discount-oriented consumers is low. In practice, demand for necessary fresh foods such as bread is relatively little affected by price [21]. Therefore, the coordination mechanism with differentiated pricing is more beneficial and effective for necessary fresh foods in comparison with the decentralized approach.

6. Conclusion

Short lifetime product retailers often encounter the coexistence of new and old products on the shelves, which results in the product cannibalization. Pricing and inventory decisions made by the retailer impact sales volume, sales revenue, market share, and as a result, the SC members’ profitability. Therefore, coordination of inventory replenishment and pricing decisions in the SLPSC can enhance the whole profitability and performance of the SC and its members. In this paper, a novel coordination mechanism in a two-level short lifetime product SC is designed to improve the SC members’ profitability considering the simultaneous sale of new and old items in the retail store. The investigated SLPSC comprises a supplier and a retailer operating in a market made up of two segments: freshness-oriented and discount-oriented consumers. The stochastic demand function depends on the product price and freshness. The supplier determines the wholesale and buy-back prices while the retailer decides on the old product price and the order quantity. To the best of our knowledge, such an issue has not been researched in the SLPSC literature. Moreover, this is the first research that examines the effects of cannibalization on the social and economic aspects of the SC.

Firstly, the optimal values of decision variables are determined under a decentralized structure in which each SC member maximizes its own profit without considering the SLPSC as a whole. Therefore, the decentralized decision-making approach may result in a reduction in the profit of the whole SC. Then, a centralized model is utilized to optimize the SLPSC decisions from the total SC perspective. Finally, a coordination model is developed to motivate the SC members to shift from the decentralized structure to the centralized one. Accordingly, a new incentive scheme named wholesale price and double compensation (WPDC) contract is designed to implement the centralized solution. Furthermore, some sensitivity analyses are conducted with respect to the key parameters in the various decision-making approaches.

The findings indicate that the cannibalization effect harms the SC members’ profitability. However, in comparison with the decentralized structure, the proposed coordination mechanism is more successful in turning the cannibalization threat into a business opportunity for increasing the whole SLPSC profit. The coordinated model creates more economic profitability for the whole SC, especially when consumers are more sensitive to the product freshness. Furthermore, the old product price in the coordinated model is less than that in the decentralized one. The lower price leads to increased satisfaction of consumers, especially low-income families. Therefore, the proposed WPDC contract is more beneficial for the SLPSC from a social viewpoint since it can increase the consumer surplus and the sales volume of fresh foods needed to keep a healthy diet. Moreover, when consumers are more sensitive to the freshness, the price differentiation policy is more profitable for the whole SC compared to the same pricing.

Future research directions

To extend the proposed models, demand uncertainty for the old product and substitution between new and old units during stock-outs can be addressed as future researches. The paper can also be extended by relaxing
the assumption of price constancy which complicates the problem and makes the old product price in each period depends on the initial stock level of the period. Moreover, the models may be developed by considering the supplier’s promotional efforts and advertising. Besides, coordinating the investigated SLPSC by applying other coordination contracts such as sales rebate may be an interesting issue for future studies.

APPENDIX A. PROOF OF PROPOSITION 3.1

The expected CS for the new product is calculated based on the formulation presented by Cohen et al. [7]. For each realization of demand uncertainty \( \varepsilon_1 \), the maximum amount of potential CS for the new product is

\[
CS_{\text{new}}^{\max}(\varepsilon_1) = \int_{p_{n}^{\min}}^{p_{n}^{\max}(\varepsilon_1)} \xi(p_n, \varepsilon_1) \, dp_n = \int_{p_{n}^{\min}}^{p_{n}^{\max}(\varepsilon_1)} \left( a_1 + A_0 - k_1 p_n - \delta \frac{(1 - \delta)}{(1 - \delta)} (p_n - p_o) + \varepsilon_1 \right) \, dp_n
\]

\[
= \frac{(D_n + \varepsilon_1)^2}{2 \left( k_1 + \frac{\delta}{(1 - \delta)} \right)}
\]

where \( p_{n}^{\min} = \frac{a_1 + A_0 + \frac{\delta}{(1 - \delta)} p_o - D_n}{k_1 + \frac{\delta}{(1 - \delta)}} \) represents the market price of a new product and \( p_{n}^{\max}(\varepsilon_1) = \frac{a_1 + A_0 + \frac{\delta}{(1 - \delta)} p_o + \varepsilon_1}{k_1 + \frac{\delta}{(1 - \delta)}} \) denotes the consumers’ maximum willingness to pay. By assuming the first-come-first-served logic with the consumers’ random arrival, the proportion of served consumers is given by \( \min(\xi(\varepsilon_1), q)/\xi(\varepsilon_1) \). Therefore, the expected consumer surplus for the new products is calculated as:

\[
ECS_{\text{new}} = \int_0^{B_0} CS_{\text{new}}^{\max}(\varepsilon_1) \frac{\min(\xi(\varepsilon_1), q)}{\xi(\varepsilon_1)} f(\varepsilon_1) \, d\varepsilon_1
\]

where \( \varepsilon_1 \) is the random component of demand that ranges in \([0, B_0]\). For the old products, the maximum amount of potential CS is calculated as:

\[
CS_{\text{old}}^{\max} = \int_{p_{o1}^{\min}}^{p_{o1}^{\max}} D_{o1}(p_o) \, dp_o + \int_{p_{o2}^{\min}}^{p_{o2}^{\max}} D_{o2}(p_o) \, dp_o
\]

\[
= \int_{(\gamma p_n - D_{o1})/\gamma}^{p_n} \left( \frac{\delta}{(1 - \delta)} (p_n - p_o) \right) \, dp_o + \int_{(a_2 - D_{o2})/k_2}^{(1 - \beta)p_n} (a_2 - k_2 p_o) \, dp_o
\]

\[
= \frac{D_{o1}^2(1 - \delta)}{2k} + (1 - \beta)p_n \left( a_2 - \frac{k_2(1 - \beta)p_n}{2} \right)
\]

Note that \( p_{o1}^{\max} = p_n \) and \( p_{o2}^{\max} = (1 - \beta)p_n \) indicate maximum willingness to pay of freshness-oriented and discount-oriented consumers, respectively, for an old product. \( p_{o1}^{\min} = (\gamma p_n - D_{o1})/\gamma \) and \( p_{o2}^{\min} = (a_2 - D_{o2})/k_2 \) denote the old product price, where \( \gamma = \frac{\delta}{(1 - \delta)} \).

The number of new products remaining at the end of each period is given by the random variable \( s = q - \xi_n \) that ranges in the interval \([\max\{0, (z - B_0)\}, z]\). Therefore, The expression of \( \min[(D_{o1} + D_{o2}), s]/(D_{o1} + D_{o2}) \) represents the proportion of consumers who find the old products available for purchase. The expected consumer surplus of the old products can be computed as:

\[
ECS_{\text{old}} = \int_l^z CS_{\text{old}}^{\max}[D_{o1} + D_{o2}, s] \frac{f(s)}{D_{o1} + D_{o2}} \, ds
\]

where \( l = \max\{0, (z - B_0)\} \).
APPENDIX B. PROOF OF PROPOSITION 4.1

We fix \( p_o \) and calculate the first and second-order derivatives of the retailer’s objective function with regard to \( z \):

\[
\frac{\partial E(\Pi_R^{dec})}{\partial z} = -w + p_n + p_o[F(z) - F(z - D_o)] - (p_n + h) F(z) + b \cdot F(z - D_o)
\]

\[
\frac{\partial^2 E(\Pi_R^{dec})}{\partial z^2} = (p_o - p_n - h) f(z) + (b - p_o) f(z - D_o).
\]

It can be observed that the second-order derivative is always negative because \( b < p_o \leq p_n \); hence, \( E(\Pi_R^{dec}) \) is a concave function with regard to \( z \) for a given \( p_o \).

APPENDIX C.

In the decentralized approach, by substituting \( z_{dec}(p_o) \) into equation (4.6), the retailer’s profit function is obtained as:

\[
\Pi_R^{dec}(p_o) = \begin{cases} 
F_1 + U(F_2 p_o - F_3 p_o^2 + F_4 p_o^3 - F_5 p_o^4), & p_o \leq (1 - \beta)p_n \\
E_1 + U(E_2 p_o - E_3 p_o^2 + E_4 p_o^3 - \gamma^2 p_o^4), & p_o > (1 - \beta)p_n 
\end{cases}
\]

where \( \gamma = \frac{\delta}{1 - \delta} \), and

\[
F_1 = -\frac{1}{(2B_0(b - h - p_n))} \left[ 2bw A_0 B_0 - 2hw A_0 B_0 + w^2 B_0^2 + 2a_1 B_0 (b - h - p_n)(w - p_n) + 2hw\gamma B_0 p_n \\
- 2b A_0 B_0 p_n + 2h A_0 B_0 p_n - 2w A_0 B_0 p_n - 2hw k_1 B_0 p_n - 2bw k_1 B_0 p_n - 2w B_0^2 p_n + bh\gamma^2 p_n^2 - 2h\gamma B_0 p_n^2 \\
+ 2w\gamma B_0 p_n^2 + 2A_0 B_0 p_n^2 + 2bk_1 B_0 p_n^2 - 2hk_1 B_0 p_n^2 + 2wk_1 B_0 p_n^2 + B_0^2 p_n^2 + b\gamma^2 p_n^3 - 2\gamma B_0 p_n^3 - 2k_1 B_0 p_n^3 \\
+ a_2^2 h + a_2^2 p_n + 2a_2 b (B_0 (w - p_n) + h\gamma p_n + \gamma p_n^2) \right]
\]

\[
F_2 = (\gamma p_n(2bh\gamma + 2bk_2(h + p_n) + 2p_n(3b + h + p_n))) + 2B_0(w - p_n)(bk_2 + (h + 2p_n)) + a_2^2(b + h + p_n) \\
+ 2a_2 b(h\gamma + hkn + k_n) + 2a_2 (B_0 (w - p_n + h\gamma p_n + \gamma p_n^2))
\]

\[
F_3 = ((\gamma + k_2)(bh + hkn + k_n) + (\gamma + k_2)(3b + 2h\gamma + 2bk_2 - 2B_0 p_n) + \gamma(3\gamma + 2k_2) p_n^2 + a_2^2 \\
+ 2a_2 b(h\gamma + hkn + 2\gamma p_n + k_n))
\]

\[
F_4 = (\gamma + k_2)(bk_2(b + h + p_n) + (b + h + 3p_n)) + 2a_2(\gamma + k_2)
\]

\[
F_5 = (\gamma + k_2)^2
\]

\[
U = \frac{1}{2B_0(b - h - p_n)}
\]

\[
E_1 = -\frac{1}{(2B_0(b - h - p_n))} \left[ w^2 B_0^2 + 2a_1 B_0 (b - h - p_n)(w - p_n) + 2A_0 B_0 (b - h - p_n)(w - p_n) + 2hw\gamma B_0 p_n \\
- 2bwk_2 B_0 p_n + 2hwk_2 B_0 p_n - 2w B_0^2 p_n + bh\gamma^2 p_n^2 - 2h\gamma B_0 p_n^2 + 2w\gamma B_0 p_n^2 + 2bk_2 B_0 p_n^2 - 2hk_2 B_0 p_n^2 \\
+ 2wk_2 B_0 p_n^2 + B_0^2 p_n^2 + b\gamma^2 p_n^3 - 2\gamma B_0 p_n^3 - 2k_2 B_0 p_n^3 \right]
\]

\[
E_2 = \gamma(2B_0(w - p_n)(h + 2p_n) + \gamma p_n(2bh + p_n(3b + h + p_n)))
\]

\[
E_3 = \gamma(2B_0(w - p_n) + (bh + p_n(3b + h + 3p_n)))
\]

\[
E_4 = \gamma^2(b + h + 3p_n).
\]

APPENDIX D. PROOF OF PROPOSITION 4.2

The retailer’s profit function, \( E(\Pi_R^{dec}(p_o)) \), is a piecewise polynomial function, left-continuous at the breakpoint \( p_o = (1 - \beta)p_n \), and bounded in the range of \( [b, p_n] \) (see Appendix C). Since a polynomial function is
continuous and differentiable in its domain, the optimal value of $p_o^{\text{disc}}$ can be obtained by comparing the feasible stationary points in each piecewise part of the function $E(\Pi_{R}^{\text{disc}}(p_o))$ as well as the boundaries of each piecewise region.

**Appendix E.**

In the centralized approach, by substituting $z_{cen}(p_o)$ into equation (4.18), the whole SC profit function is obtained as:

$$
\Pi_{cen}^{\text{SC}}(p_o) = \begin{cases} 
M_1 + G(M_2p_o - M_3p_o^2 + M_4p_o^3 - M_5p_o^4), & p_o \leq (1 - \beta)p_n \\
N_1 + G(N_2p_o - N_3p_o^2 + N_4p_o^3 - \gamma^2p_o^4), & p_o > (1 - \beta)p_n 
\end{cases} 
$$

(E.1)

where $\gamma = \frac{\delta}{1 - \beta}$, and

$$
M_1 = -\frac{1}{(2B_0(g - h - p_n))} \left[ 2gc_mB_0B_0 - 2hc_mB_0 + c_m^2B_0^2 + 2a_1B_0(g - h - p_n)(c_m - p_n) + 2hc_m\gamma B_0p_n \\
- 2gA_0B_0p_n + 2hA_0B_0p_n - 2c_mA_0B_0p_n - 2gc_mA_0B_0p_n + 2hc_mA_0B_0p_n - 2c_mB_0^2B_0p_n + gh\gamma^2p_n^2 \\
- 2\gamma B_0^2a_2 + 2c_mA_0B_0p_n^2 + 2A_0B_0^2p_n^2 + 2ghB_0B_0^2p_n^2 - 2\gamma B_0^2p_n^2 + 2c_mA_0B_0p_n^2 + B_0^2B_0^2 + g^2\gamma^2p_n^3 \\
- 2\gamma B_0p_n^3 - 2k_1B_0p_n^3 + a_2^2gh + a_2^2gB_0(c_m - p_n) + h\gamma p_n + \gamma^2p_n^2 \right] 
$$

(E.2)

$$
M_2 = (\gamma p_n(2gh + 2k_2(h + p_n) + \gamma p_n(3g + h + p_n)) + 2B_0(c_m - p_n)(gk_2 + \gamma(h + 2p_n)) + a_2^2(g + h + p_n) \\
+ 2a_2g(h\gamma + hk_2 + 2\gamma p_n + k_2p_n) + 2a_2(B_0(c_m - p_n) + h\gamma p_n + \gamma^2p_n^2) 
$$

(E.3)

$$
M_3 = ((\gamma + k_2)(gh + k_2) + 2c_mB_0) + (\gamma + k_2)(3g\gamma + 2h\gamma + gk_2 - 2B_0)p_n + \gamma(3\gamma + k_2)p_n^2 \\
+ a_2^2 + 2a_2g(\gamma + k_2) + 2a_2(h\gamma + hk_2 + 2\gamma p_n + k_2p_n) 
$$

(E.4)

$$
M_4 = ((\gamma + k_2)(k_2(g + h + p_n) + \gamma(g + h + 3p_n)) + 2a_2(\gamma + k_2)) 
$$

(E.5)

$$
G = \frac{1}{2B_0(g - h - p_n)} 
$$

(E.6)

$$
N_1 = -\frac{1}{(2B_0(g - h - p_n))} \left[ (c_m^2B_0^2 + 2a_1B_0(g - h - p_n)(c_m - p_n) + 2A_0B_0(g - h - p_n)(c_m - p_n) \\
+ 2hc_mB_0p_n - 2gc_mB_0p_n + 2hc_mB_0p_n - 2c_mB_0^2p_n + gh\gamma^2p_n^2 - 2h\beta B_0p_n^2 + 2c_m\gamma B_0p_n^2 \\
+ 2ghB_0p_n^2 - 2h\beta B_0p_n^2 + 2c_mk_2B_0p_n^2 + B_0^2p_n^2 + g^2\gamma^2p_n^3 - 2\gamma B_0p_n^3 - 2k_2B_0p_n^3 \right] 
$$

(E.7)

$$
N_2 = \gamma(2B_0(c_m - p_n)(h + 2p_n) + \gamma p_n(2gh + p_n(3g + h + p_n))) 
$$

(E.8)

$$
N_3 = \gamma(2B_0(c_m - p_n) + \gamma(gh + p_n(3g + h + 2p_n))) 
$$

(E.9)

$$
N_4 = \gamma^2(g + h + 3p_n). 
$$

(E.10)

**References**


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