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HOW COOPERATIVE ADVERTISING INTERACTS WITH DISTRIBUTIONAL CONTRACTS IN A DUAL-CHANNEL SYSTEM

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Abstract. With the development of E-commerce, an increasing number of online platforms are conducting advertising campaigns to expand their sales. In some situations, the manufacturer is willing to share the advertising cost, while in others it is not. Additionally, recently, many online platforms have started choosing an agency contract, instead of a wholesale contract, to obtain a predetermined proportion of revenue from the manufacturer to make profits. This paper studies a scenario of a manufacturer selling through both a direct channel and a platform channel to investigate the interaction between a manufacturer's cooperative advertising strategy and a platform's distribution contract choice. We develop a stylized model based on game theory to drive the optimal prices and advertising level under different contracts. By using a representative consumer function, we drive the following interesting results. Firstly, under the wholesale contract, a manufacturer prefers cooperating only when the cost-sharing rate is small, but under an agency contract, when the revenue-sharing rate is large, the manufacturer will not choose to cooperate even if the cost-sharing rate is low. Secondly, the platform's profit does not always increase in the revenue-sharing rate. Finally, under some conditions, the platform would prefer that the manufacturer not share the cost. Specifically, when the competition intensity is small and the revenue-share rate is high, the platform would rather choose an agency contract to cover all advertising costs on its own than a wholesale contract where the manufacturer is willing to share advertising costs. We also consider direct channel advertising as well in an extension, and the qualitative results still hold.

Mathematics Subject Classification. 90B06, 90B60, 91A99, 91B24.

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1. Introduction

Currently, as competition intensifies, a growing number of upstream manufacturers seek ways to expand their market demands using methods such as cooperative advertising and adding sales channels. Although a large body of literature focuses on the cooperative advertising determination and the increase of sales channel, little attention has been paid to the interaction between these two. In this work, we bring this problem into focus by asking what the equilibrium condition is considering the two effects. We focus on the condition under which the manufacturer sells through a platform channel in addition to its own direct channel. Additionally, the platform chooses a type of contract to offer the manufacturer, and the manufacturer has the power to decide whether

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to share the advertising cost with the platform. In reality, when the platform advertises the manufacturer's products, a logo is intentionally displayed in the ad to introduce the online platform, or a link or quick response (QR) code is provided to redirect the consumer to the online platform [43]. For example, JD.com appeared in L'Oreal's advertisement, and Tmall appeared in Philips' advertisement. This paper will explain from an aspect when cooperative advertising will occur under an agency contract and a wholesale contract. And how should the platform determine its optimal contracts, considering the cooperative advertising strategy of the manufacturer.

Advertising is a crucial tool for demand creation and market expansion. Major retailers, such as Walmart and Target, frequently advertise products. However, the retailer doing the advertising does not necessarily bear all the costs. Cost sharing has often been implemented in the form of cooperative advertising (*e.g.*, [5,21,22,26,48]). For the manufacturers, obviously, sharing the advertising cost can facilitate the platform's effort and increase the demand of the platform channel, however, doing so would bring additional cost for the manufacturer and cut the demand of his direct channel, which makes cooperative advertising a strategic decision of manufacturers.

Supply chain contracts are usually considered to be effective tools to coordinate all parties' decisions. In light of this, which contract should be offered to manufacturers is also a strategic decision for the platform. Traditionally, the wholesale contract is widely used in this process: the platform purchases the product at a wholesale price and then resells to end consumers at a retail price. When considering wholesale contracts, we have the constraint that the wholesale price must be lower than the direct channel price to avoid speculation, which is required by abundant analytical literature such as Chiang *et al.* [11]. However, in recent years, the agency contract has become prevalent. The agency contract refers to a form in which the platform charges manufacturer a proportion of revenue in return for them regaining the pricing power of products. For instance, when manufacturers' products are sold at JD.COM under an agency contract, JD.COM as a platform charges 7% for clothes and shoes, 6% for luxury items, and 5% for sporting goods [50]. For agency selling, Amazon charges 20% for jewelry and 15% for shoes [32]. By determining the contract strategically, the platform can realize its optimal profit.

We present a model of a supply chain, where the manufacturer ("he") sells through both channels by using a platform ("she") and a direct channel. In practice, many manufacturers distribute through dual channels. For instance, leading electronic product makers, such as Apple and Microsoft, sell their products through third-party platforms as well as through their own direct stores. Apparel makers, such as Nike and Adidas, and beverage and food makers, such as Campbell Soup and Coca-Cola, also adopt both direct and third-party channels [30]. Our point of departure from previous studies is to investigate the interactions between cooperative advertising and contract determination. To focus on the trade-offs between the two processes, we assume in the base model that only the platform will perform advertising. The possibility of direct channel advertising will be studied in the extension.

We first study the optimal cooperation strategies of the manufacturer under a wholesale contract and an agency contract. If a wholesale contract is chosen, the manufacturer will choose to cooperate if and only if the cost-sharing rate is not very large, which is pretty intuitive. Under a wholesale contract, the manufacturer can use higher wholesale price to extort the profit generated by advertising. As such, under a wholesale price the manufacturer has more incentive to implement cooperative advertising. Even so, if the cost-sharing rate is high, the manufacturer would not choose to cooperative due to the higher advertising cost. If an agency contract is chosen, when the revenue-sharing rate is high, the manufacturer would prefer noncooperation even if the cost-sharing rate is infinitely low. The principle behind the manufacturer's behavior is that when the revenue-sharing rate is high enough, the platform will extort more profit generated by advertising.

One may intuitively suspect that cooperative advertising always benefits the retailer due to the reduced advertising costs, but our results show this is not always the case. When taking the form of contracts into account, we find that, in some cases, the platform would prefer the manufacturer not share the cost. When the competition intensity is weak and the revenue-share rate is high, the platform would rather choose an agency contract to cover all advertising costs on its own than a wholesale contract where the manufacturer is willing to share advertising costs. While when the competition intensity is strong and the revenue-share rate is high, the platform will undoubtedly choose the wholesale contract to seek the cost sharing of the manufacturer. The rationale behind the platform's interesting decision hinges on the trade-off between the profit obtained by revenue sharing and the profit obtained by cost sharing. In addition, it is worth noting that under the agency contract, the selling price on the platform is determined by the manufacturer. When market competition intensity is strong and the revenue-share rate is high, the manufacturer will set the selling price on the agent platform very high to attract consumers to buy through the direct channel. In this condition, the demand for the platform drops, and the platform's profit from revenue sharing decreases. At this time, if the platform provides a wholesale contract, the platform can obtain the revenue from the cost sharing of the manufacturer while advertising to increase sales, which causes the wholesale contract to become an optimal choice for the platform. On the contrary, when the market competition intensity is relatively weak, even if the revenue share rate is high, the manufacturer will not significantly increase the price of the platform. At this time, the profit obtained from the revenue share is higher than the revenue obtained from the cost sharing to the platform. At this time, the agency contract will be the best choice for the platform. *This also illustrates another interesting conclusion. When considering the manufacturer's cooperation decisions into account, the platform's profit does not always increase as the revenue sharing rate increases.* Conventional wisdom suggests that the platform would prefer the agency contract more as the revenue-sharing rate rises, because higher rates give her a greater share of the profits. However, this is not the case when the channel competition intensity is intense and the revenue-sharing rate is high.

The rest of this paper is organized as follows. In Section 2, we discuss the literature relevant to our paper. Section 3 specifies the model, and in Section 4, we present the manufacturer's optimal cooperation strategies. In Section 5, we derives the platform's optimal distribution contract. Finally, we relax assumptions made in the base model, so in Section 6, we consider a reverse order of action, direct channel advertising and the scenario where the manufacturer determines the contract type. Section 7 concludes the paper.

2. Literature review

Our work builds upon three streams of the existing literature: (a) literature on cooperative advertising and supply chain; (b) literature related to agency contracts; and (c) literature on the choice of contracts.

Firstly, cooperative advertising has been researched extensively in the literature. The work of Berger [5] is the first study that formulates the mathematical modeling of cooperative advertising, obtaining results which suggest that both the manufacturer and retailer can benefit from cost sharing. Desai [14] studied cooperative advertising for franchises, finding that a franchisor can determine how to spend the advertising fee, and thereby eliminate the free-riding phenomenon. Bergen and John [4] showed that a manufacturer will provide identical co-op plans to ex ante symmetric retailers. Huang and Li [22] studied cooperative advertising between the manufacturer and retailer and showed that cooperative advertising may improve the performance of the supply chain. Zhang *et al.* (2013) studied cooperative advertising with bilateral participation. Yan and Pei [49] studied the effects of cooperative advertising on channel conflict under dual channels. Considering the cooperative search advertising, Cao and Ke [8] found that, different from the traditional cooperative advertising, it may be optimal for a manufacturer to cooperative with just some, rather than all, retailers. Sarkar *et al.* [39] introduced the uncertain cost into the co-op advertising in the centralized supply chain management. Mandal *et al.* [34] studied sustainable inventory management considering advertising and trade credit policies and derived the solution method for the existence of the global optimal solution. Aust and Buscher [3] dealt with vertical cooperative advertising in a manufacturer-retailer channel, considering four different relationships between them. Liu *et al.* [31] examined a cooperative advertising model of two competing manufacturer-retailer supply chains that may differ in market size. They found that the firms performing the advertising would rather bear the costs entirely. Based on that, Yang *et al.* [51] examined service provision in competitive channels whose rationale is similar to cooperative advertising. In our paper, we examine cooperative advertising using a model similar to that in these last two papers. About dual-channel advertising, Pei and Yan [36] focused on the strategic effect of the manufacturer's national advertising on alleviating the channel competition. Karray and Amin [28] found that cooperative advertising may be harmful to retailers or channels, especially when the level of market competition is low and the competition for advertising between retailers is fierce. However, in this paper, the platform can avoid the lose condition by effectively choosing the contract type. Wang *et al.* [43] developed a game theory model to discuss three advertising schemes in a dual-channel supply chain consisting of manufacturers and two competing retailers. Zhang *et al.* [54] examined the impact of manufacturer controls advertising, retailer controls advertising, and cooperation between both parties on the channel encroachment strategy of the manufacturer. Similar to this article, we also study the interaction between the direct channel and distribution channel, but Zhang's study studies the impact of advertising strategies on supplier encroachment, and this paper studies the platform's optimal contract choices considering the manufacturer's cooperation strategies in the dual channels. Forghani *et al.* [15] adopted the rough set theory to study the effect of digital marketing strategies on the buying behavior of customers in online shopping stores in Tehran. There are also many articles that examined supply chain in other domains, such as production and inventory [33], distribution-free [27], remanufacturing [12], reworking [2], green products and social responsibility [37, 38], circular economy [40], Fuzzy demand [6] and prawn fishery (Das *et al.*, [13]).

Secondly, the agency contract, which is similar with the consignment contract with revenue sharing, has attracted little attention in the literature compared with the wholesale contract. Work by Hackett [17] is one of the earliest studies that focus on the consignment contract. Wang *et al.* [44] showed that the performance of a decentralized channel degrades as price elasticity increases under a consignment contract. Yao *et al.* [52] found that a revenue sharing contract may perform better than a price-only contract. Li *et al.* [29] developed a cooperative game model under the agency contract using a Nash bargaining model. Differently, we analyze a Stackelberg game between the manufacturer and retailer. Tan and Carrillo [42] explained how the agency contract can benefit both parties in the digital publishing industry. Research by Hao and Fan ref19a demonstrated that the optimal price of e-books may be higher under an agency contract. Hao *et al.* [19] focused on Apple's app sales under advertising contracts. Tan and Carrillo [42] studied the agency model for the e-book industry and determined that the agency model is superior to other models for distributing digital goods. Shen *et al.* [41] developed a Stackelberg model in which the platform leads by setting a revenue-sharing rate, while the manufacturer chooses to sell through one or two channels.

Finally, the issue of contract choice has attracted much attention in recent years. Abundant literature has explained the strengths and weaknesses of different types of contracts and further argued why the channel members prefer one contract rather than the others. Cachon and Kok [7] considered a case where the manufacturer resorts to one of three contract types – wholesale contract, quantity discount contract, or two-part tariff – to achieve optimal performance. Due to space limitations, we focus on the wholesale contract and agency contract selection. Pan *et al.* [35] verified whether it is beneficial to use a wholesale or consignment contract by comparing various channel power structures. Wang and Shin [45] investigated the wholesale price contract, quality-dependent wholesale price contract, and revenue-sharing contract. Hagiu and Wright [18] illustrated when it is best for intermediaries to function as a marketplace (agency) and when a reseller can achieve maximum profit. Jin *et al.* [25] analyzed the interplay between the contract choice and the decision rights of sales promotion involving the wholesale price contract and agency contract. Abhishek *et al.* [1] investigated when a retailer should use an agency contract instead of the reselling format, considering the cross-channel effect. Geng *et al.* [16] examined the interaction between the manufacturer's add-on strategy and platform's distribution contract choice. Yi *et al.* [53] took the consumers' fairness-seeking behavior into account to investigate the manufacturer's distribution channel selection between direct selling and agent selling. Lu *et al.* [32] concentrated on dynamic advertising by comparing wholesale price contracts with consignment contracts. Ji *et al.* [24] studied how social communications affect upstream product line design when the intermediate platform makes strategic contract choices. Similarly, we pay attention to these two types of contracts, but we focus on the interplay between the contract choice and cooperative advertising, assuming static advertising. There are a lot of literature studying the contract choice under dual channels, such as Pan *et al.* [35], Hagiu and Wright [18], Abhishek *et al.* [1], Yi *et al.* [53].

In summary, we find that there is no paper considers all of the cooperative advertising, dual channel and contract choice in the literature. We take a step in this direction and contribute to the literature on cooperative

TABLE 1. The main differences between this paper and the literature.

Author(s)	Cooperative advertising	Dual channel	Contract choice
Berger [5], Yan and Pei [49], Cao and Ke [8], Wang *et al.* [43]	✓	✓	
Pan *et al.* [35], Hagiu and Wright [18] Abhishek *et al.* [1], Yi *et al.* (2018)		✓	✓
This paper	✓	✓	✓

advertising, dual channels and contract choice. We fill the gap between the practice and the literature. The main differences between this paper and the literature are summarized in Table 1.

3. The model

To study the interaction between cooperative advertising and the platform's contract, we employ the stylized model of a single manufacturer selling an identical product through a direct channel and a single downstream platform. The platform can strategically choose two forms of contract: agency or wholesale contract.

Under a wholesale contract, the platform purchases each product from the manufacturer at a wholesale price w and resell at retail price p_1. While under an agency contract, the manufacturer retains a fraction of α of the revenue, and the platform receives a fraction $1-\alpha$ of the revenue. A significant difference between the wholesale and agency contract is who has the final pricing power on the platform. For brevity, we refer to the manufacturer as "he" and the platform as "she". Since the level of advertising is difficult to change once determined, we allow the platform to first decide her advertising level after the contract and cooperative advertising mode are established. We denote the advertising level as e_p, a higher e_p means a larger market size. The manufacturer has the power to decide whether to share the advertising cost with the platform. The proportion of the advertising cost sharing rate that the manufacturer pays to the platform is η. In reality, η may result endogenously from the balance of power between the manufacturer and platform. The balance of power falls beyond the scope of our model, so we will report how each party's profits vary with η, as was done by Liu *et al.* [31].

In our notation, the index i $(i = 1, 2)$ identifies the platform channel $(i = 1)$ and the direct channel $(i = 2)$, and D_i represents the demand for the product sold in channel i. The retail prices are p_i, and the wholesale price is w under a wholesale contract. When under an agency contract, the manufacturer decides the retail price of the product while the platform takes a predetermined cut α from each sale. The remaining $1-\alpha$ revenue is passed to the manufacturer. In typical business practice, the manufacturer can keep the majority of the revenue, so we assume that $\alpha \leq \frac{3}{4}$ to retain realism in our conclusions [10, 46, 47]. For instance, when manufacturers' products are sold at JD.COM under an agency contract, JD.COM as a platform charges 7% for clothes and shoes, 6% for luxury items, and 5% for sporting goods [50]. A_i is channel i's base demand with zero price and no advertising advertising is conducted. To enable a fair comparison among the various structures, we assume $A_1 = A_2 = 1$. To describe the problem more clearly, we assume that the manufacturer does not perform advertising for the direct channel in our base model. Therefore, with the impact of the advertising, the new demand of the platform channel becomes $\xi_1 = A_1(1 + e_p) = 1 + e_p$. In the following, we use ξ_i $(i = 1, 2)$ to denote the new demand of the platform channel and the direct channel (we conduct the analysis of manufacturer advertising on the direct channel in the extension).

The function representing the cost of advertising effort is $C(e_p) = \lambda e_p^2$. The quadratic form reflects the increasing marginal cost of effort, and using it is consistent with Yang *et al.* [51], Chen *et al.* [9], and Desai [14]. For simplicity in our base model, we normalize λ to 1.

To obtain the demand functions in different channel structures, we adopt the elegant framework established by Ingene and Parry ([23], Chapt. 11) and employ a similar utility function for a representative consumer as follows.

$$U = \sum_{i=1,2} (\xi_i D_i - D_i^2/2) - \theta D_1 D_2 - \sum_{i=1,2} p_i D_i \tag{3.1}$$

where θ represents the channel substitutability, and the impact of advertising is embedded in ξ_i and $\xi_i = 1 + e_p$. D_i represents the demand in channel i. By solving the first order conditions $\frac{\partial U}{\partial D_1} = 0$ and $\frac{\partial U}{\partial D_2} = 0$ simultaneously, we get the demand functions, that is $D_1 = \frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}$ and $D_2 = \frac{1-\theta(1+e_p)-p_2+\theta p_1}{1-\theta^2}$. Considering that there exists plentiful differences between the direct and platform channels in service provision, consumers' preferences, and other factors. We assume that the channel substitutability is not very large; we restrict it to $\theta \in (0, \frac{1}{2}]$. The reason for this assumption is that when θ is large, that is, the intensity of competition between channels is large, the direct channel demand may degenerate to zero due to the the role of advertising. This paper does not consider the effect of advertising on encroaching direct channel's sales. Thus, we excludes this situation in this article, so that the platform and the direct channel all have positive sales.

The production costs and supply chain operational costs are normalized to zero. We use π_r (π_a) to denote platform's profit under wholesale (agency) contract and similarly π_{mr} (π_{ma}) is the manufacturer's profit. Thus, the platform and manufacturer profits are given, respectively, by

$$\pi_r = (p_1 - w)D_1 - (1 - \mathbf{1}_m \eta)e_p^2 \tag{3.2}$$

$$\pi_a = \alpha p_1 D_1 - (1 - \mathbf{1}_m \eta)e_p^2 \tag{3.3}$$

$$\pi_{mr} = wD_1 + p_2 D_2 - \mathbf{1}_m \eta e_p^2 \tag{3.4}$$

$$\pi_{ma} = (1 - \alpha)p_1 D_1 + p_2 D_2 - \mathbf{1}_m \eta e_p^2. \tag{3.5}$$

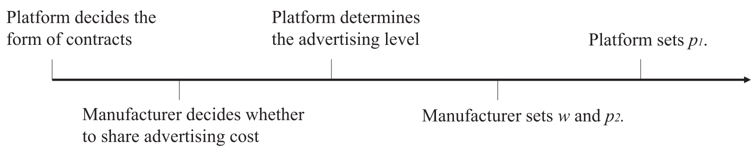
Here, $\mathbf{1}_m = 0$ or $\mathbf{1}_m = 1$ is the indicator of whether the manufacturer funds the advertising activity.

We compare scenarios with and without cooperative advertising under wholesale and agency contracts. Here, the platform decides the form of the contract and the manufacturer decides whether to share advertising costs with the platform. Both parties make decisions with the goal of maximizing profits, which generates interaction between them. The structure of the game is as follows. Stage 1: The platform decides the form of the contract offered to the manufacturer. Stage 2: The manufacturer decides whether to share advertising costs with the platform. Stage 3: The platform determines her advertising level. Stage 4: The manufacturer simultaneously sets his wholesale and direct channel prices under a wholesale contract, or the platform price and direct channel price under an agency contract. Stage 5: The platform sets her price p_1 under a wholesale contract. For each game, we characterize the perfect equilibrium for the games. In the extension, we also investigate the sequence in which the manufacturer first announces his decision on cooperative advertising before the contract is established. Decisions about platform advertising and the manufacturer's cost sharing proceed as shown in Figure 1.

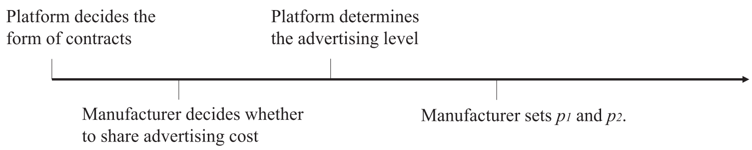
The following sections will examine four different scenarios. In each scenario, each party seeks to independently maximize its profit. We will obtain and analyze the perfect equilibrium outcomes. Table 2 summarizes the notations in this study.

4. Manufacturer's optimal cooperation strategies

The platform providing wholesale and agency contracts can generate four Scenarios: wholesale contract without cooperative advertising (WN), wholesale contract with cooperative advertising (WC), agency contract without cooperative advertising (AN) or agency contract with cooperative advertising (AC). We identify each scenario with a two-character string in which the first character depicts the form of contract ("W" for wholesale, "A" for agency), and the second character describes whether the manufacturer conducts cooperation ("C" for cooperation, "N" for noncooperation). Specifically, $\mathbf{1}_m = 1$ for WC and AC; otherwise $\mathbf{1}_m = 0$. For all four scenarios, all feasible domains of the specific cases are detailed in the Appendix A. We present only the main results here.



(a)



(b)

FIGURE 1. Time sequence. (a) Under wholesale contract. (b) Under agency contract.

TABLE 2. Model variables and parameters.

Notation	Definition
i	The platform channel ($i = 1$) or The direct channel ($i = 2$)
p_i	The selling price of channel i
D_i	The demand of channel i
w	The wholesale price
e_p	Platform advertising level
e_m	Direct channel advertising level
θ	Channel competition intensity
α	The manufacturer's revenue-sharing rate under an agency contract
η	The manufacturer's cost-sharing rate
π_a, π_{ma}	The agent's and manufacturer's profit under an agency contract
π_r, π_{mr}	The reseller's and manufacturer's profit under a wholesale contract

Consistent with the extant literature [41,42], the platform in the supply chain has greater market power than upstream manufacturer in determining channel contracts. Abhishek *et al.* [1] also claimed that: we give e-tailers the power to make this decision because the online platform has a large customer base and a wide reach. This gives them a powerful ability to determine the sales format they want to use. Therefore, in this paper, we assume that the contract type is determined by the platform.

We demonstrate that if the platform provides a wholesale contract, the manufacturer will choose the cooperation strategy if and only if the cost-sharing rate is low, which is very intuitive. Otherwise, noncooperation is the equilibrium. Additionally, when the manufacturer adopts cooperative advertising under a wholesale contract, it is a "win-win" strategy for both parties. If the platform signs an agency contract with the manufacturer, the platform will always prefer Scenario AC once this case is feasible. For the manufacturer, however, the condition is more complicated, depending on the combination of θ, α, and η. By representing the conclusions graphically, we see that the manufacturer's optimal choice between cooperation and noncooperation changes as the revenue-sharing rate increases. When the revenue-sharing rate is relatively high, the manufacturer will not choose to cooperate even if the cost sharing rate is extremely small.

TABLE 3. Optimal solutions in scenarios WN and WC.

	Scenario WN	Scenario WC
e_p^*	$\frac{(1-\theta)(3+\theta)(5+3\theta)}{119+\theta(66-\theta(137+16\theta(6+\theta)))}$	$\frac{(1-\theta)(3+\theta)(5+3\theta)}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}$
p_1^*	$\frac{2(1+\theta)(49-\theta(39+2\theta(12-\theta(2+\theta))))}{119+\theta(66-\theta(137+16\theta(6+\theta)))}$	$\frac{2(1+\theta)(49-39\theta-24\theta^2+4\theta^3+2\theta^4-2\eta(3+\theta)^2(3-4\theta+\theta^2))}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}$
$p_2^*=w^*$	$\frac{3+\theta+\frac{(1-\theta)(3+\theta)(5+3\theta)}{119+\theta(66-\theta(137+16\theta(6+\theta)))}}{6+2\theta}$	$\frac{3+\theta-\frac{(1-\theta)(3+\theta)(5+3\theta)}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}}{6+2\theta}$
D_1^*	$\frac{4(1-\theta)(3+\theta)^2}{119+\theta(66-\theta(137+16\theta(6+\theta)))}$	$\frac{4(1-\eta)(1-\theta)(3+\theta)^2}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}$
D_2^*	$\frac{(3+\theta)(19-\theta(1+2\theta)(7+2\theta))}{119+\theta(66-\theta(137+16\theta(6+\theta)))}$	$\frac{(3+\theta)(19-4\eta(1-\theta)(2+\theta)(3+\theta)-\theta(1+2\theta)(7+2\theta))}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}$

4.1. Manufacturer's cooperation strategies under wholesale contract

We now analyze the scenarios in which the platform provides a wholesale contract. The profit functions of the platform and manufacturer in Scenarios WC and WN are as follows.

In WN, the profit functions of the two parties are:

$$\pi_{mr} = w\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) + p_2\left(\frac{1-\theta(1+e_p)-p_2+\theta p_1}{1-\theta^2}\right)$$
$$\pi_r = (p_1-w)\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) - e_p^2.$$

In WC, the profit functions of the two parties are:

$$\pi_{mr} = w\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) + p_2\left(\frac{1-\theta(1+e_p)-p_2+\theta p_1}{1-\theta^2}\right) - \eta e_p^2$$
$$\pi_r = (p_1-w)\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) - (1-\eta)e_p^2.$$

By tedious calculations as detailed in the Appendix A, we obtain the main results about the equilibrium wholesale price, retail prices, advertising level, demands, and the feasible domain. These results are summarized in Lemma 4.1. Because the equilibrium profit results are very complicated, we also put their derivation in the Appendix A. It is noted that, when calculating the feasible domains of WC and WN, we use the constraint that the wholesale price must not be higher than the direct channel price, which is also required in Chiang [11]. By calculation, we find that the wholesale price is equal to the direct channel's price at equilibrium in both cases WN and WC.

Lemma 4.1. *In case WC, the equilibrium solutions' feasible domain is* $0 < \eta \leq \frac{1}{64}\left(\frac{1513+12690\theta-14445\theta^2-17210\theta^3-5760\theta^4-640\theta^5}{(3+\theta)^3(1-\theta^2)}\right) - \sqrt{\frac{(5+3\theta)^2(3001+25330\theta-28690\theta^2-34330\theta^3-11520\theta^4-1280\theta^5)}{(1-\theta)(1+\theta)^2(3+\theta)^6}}$. *While case WN is always feasible. The equilibriums are shown in Table 3.*

Clearly, the common feasible domain of WN and WC is the domain of case WC. Table 2 gives the equilibrium advertising level, prices, and demand. In case WC, both the retail prices and wholesale price increase with the cost-sharing rate. Given that $0 < \theta \leq \frac{1}{2}$, we find that $\frac{\partial p_1^*}{\partial\eta} = \frac{4(1-\theta)^2(1+\theta)(3+\theta)^2(5+3\theta)(7+3\theta)}{(119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137-16\theta(6+\theta))))^2} > \frac{\partial w^*}{\partial\eta} = \frac{8(1-\theta)^2(1+\theta)(3+\theta)^2(5+3\theta)}{(119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137-16\theta(6+\theta))))^2}$. This result shows that the retail price increases more quickly than the wholesale price as η increases, that is, the double marginalization is worsened as the cost-sharing rate grows. Define $U = \frac{\partial p_1^*}{\partial\eta} - \frac{\partial w^*}{\partial\eta}$. We know that U decreases with $\theta \in (0, 0.174]$ when $0 < \eta < \frac{433-16370\theta-43260\theta^2-31300\theta^3-5230\theta^4+6070\theta^5+3360\theta^6+480\theta^7}{16(3-2\theta-\theta^2)^2(7+13\theta+9\theta^2+3\theta^3)}$; thus, the double marginalization is reduced as the

TABLE 4. Comparison of variables under wholesale contract.

Advertising level	$e_p^*(\mathrm{WC}) > e_p^*(\mathrm{WN})$
Wholesale price (direct channel price)	$w^*(\mathrm{WC}) > w^*(\mathrm{WN})(p_2^*(\mathrm{WC}) > p_2^*(\mathrm{WN}))$
Retail prices	$p_1^*(\mathrm{WC}) > p_1^*(\mathrm{WN})$
Demand	$D_1^*(\mathrm{WC}) > D_1^*(\mathrm{WN}), D_2^*(\mathrm{WC}) < D_2^*(\mathrm{WN})$
Net surplus	$p_1^* - w^*(\mathrm{WC}) > p_1^* - w^*(\mathrm{WN})$

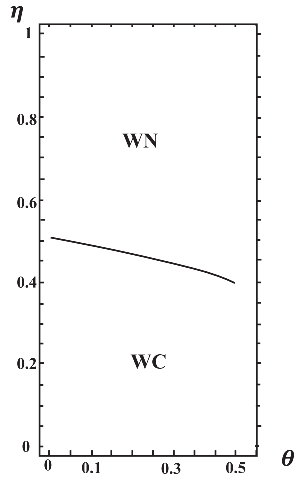


FIGURE 2. Manufacturer's cooperation strategies under wholesale contract.

horizontal competition intensifies when competition intensity and cost-sharing rate are relatively low. Otherwise, the double marginalization worsens.

In case WN, we have that $|\frac{\partial p_1^*}{\partial\theta}| = \frac{4(1022+\theta(784-\theta(2092+\theta(2636-\theta(225+\theta(1566+\theta(859+16\theta(12+\theta))))))))}{(119+\theta(66-\theta(137+16\theta(6+\theta))))^2} > |\frac{\partial w^*}{\partial\theta}| = \frac{4(71-\theta(2+\theta)(41+4\theta(10-3\theta(2+\theta))))}{(119+\theta(66-\theta(137+16\theta(6+\theta))))^2}$, which reveals that the double marginalization is worsened as the channel competition intensifies. By comparing the equilibrium results between the two cases in their common feasible region, we obtain the major relationships shown in Table 4.

Table 4 shows that under the wholesale contract, e_p, p_1, p_2, w, D_1, and $p_1 - w$ are all larger in WC than in WN, whereas D_2 is smaller. This phenomenon echoes the conventional wisdom that cooperation can increase the advertising level, further increase the total demand, and push up both prices. For the platform, we can easily find that both the price p_1 and demand D_1 are bigger in WC. The combination of increased price and demand undoubtedly lead to higher profit. Nevertheless, the case is somewhat complicated for the manufacturer. In the following, we will summarize the optimal choices for the two parties. Note that all our conclusions are based on the common feasible region of WN and WC.

Theorem 4.2. *Under a wholesale contract, cooperative advertising is the platform's optimal choice. For the manufacturer, cooperative advertising is the optimal choice only when the cost-sharing rate is low; otherwise, noncooperation is the equilibrium strategy. The specific interval in which cooperation dominates is* $0 < \eta \leq \eta_1$. *(The concrete value of* η_1 *can be found in the Appendix A).*

Theorem 4.2 shows that cooperation is the dominant equilibrium strategy for the platform under a wholesale contract, which can be verified by the above analysis. Figure 2 further illustrates Theorem 4.2. For the manufacturer, when the cost-sharing rate is low, cooperation incurs an extra cost, which pushes up wholesale prices

TABLE 5. Optimal solutions in scenarios AN and AC.

	Scenario AN	Scenario AC
e_p^*	$\frac{\alpha(8-4\theta-\tilde{\alpha}(2(2-\theta)\theta^2+\alpha(8-\theta(4+\theta(4-\theta)))))}{2S}$	$\frac{\alpha(4(2-\theta)-\tilde{\alpha}(2(2-\theta)\theta^2+\alpha(8-\theta(4+(4-\theta)\theta))))}{2R}$
p_1^*	$\frac{16\bar{\alpha}^2+6\bar{\alpha}\alpha\theta-4(4-\alpha)\tilde{\alpha}\bar{\alpha}\theta^2-\tilde{\alpha}\alpha(3-2\alpha)\theta^3+2\tilde{\alpha}^3\theta^4}{2S}$	$\frac{16\bar{\alpha}^2\bar{\eta}+2\alpha\theta\bar{\alpha}(3-4\eta)-4\theta^2(4-\alpha)\tilde{\alpha}\bar{\alpha}\bar{\eta}+\alpha\theta^3\tilde{\alpha}(3-2\alpha\bar{\eta}-4\eta)+2\theta^4\tilde{\alpha}^3\bar{\eta}}{2R}$
p_2^*	$\frac{\bar{\alpha}(4(4-\alpha)\bar{\alpha}-8\bar{\alpha}\alpha\theta-(4-3\alpha)(8-5\alpha)\theta^2+2\tilde{\alpha}^2\alpha\theta^3+2\tilde{\alpha}^3\theta^4)}{2S}$	$\frac{\bar{\alpha}\left(2-\tilde{\alpha}\theta^2-\alpha\theta\left(1+\frac{\alpha(4(2-\theta)-\tilde{\alpha}(2(2-\theta)\theta^2+\alpha(8-\theta(4-(4-\theta)\theta))))}{2R}\right)\right)}{4\bar{\alpha}-\tilde{\alpha}^2\theta^2}$
D_1^*	$\frac{8\bar{\alpha}^2-\bar{\alpha}(8-5\alpha)\theta-2\tilde{\alpha}^2\bar{\alpha}\theta^2+\tilde{\alpha}^3\theta^3}{S}$	$\frac{8\bar{\alpha}^2\bar{\eta}-\bar{\alpha}(8-5\alpha-4\tilde{\alpha}\eta)\theta-2\tilde{\alpha}^2\bar{\alpha}\bar{\eta}\theta^2+\tilde{\alpha}^3\bar{\eta}\theta^3}{R}$
D_2^*	$\frac{\bar{\alpha}(4(4-\alpha)\bar{\alpha}-8\tilde{\alpha}\bar{\alpha}\theta-\tilde{\alpha}(8-5\alpha)\theta^2+2\tilde{\alpha}^3\theta^3)}{2S}$	$\frac{\bar{\alpha}(4\bar{\alpha}(4-\alpha-4\eta)-8\tilde{\alpha}\bar{\alpha}\bar{\eta}\theta-\tilde{\alpha}(8-5\alpha-4\tilde{\alpha}\eta)\theta^2+2\tilde{\alpha}^3\bar{\eta}\theta^3)}{2R}$

$S = 4(4-\alpha)(1-\alpha)^2 + (2-\alpha)^4\theta^4 - 2(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2$, $R = 4(1-\alpha)^2(4-\alpha-4\eta) - 2(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + (2-\alpha)^4(1-\eta)\theta^4$, $\bar{\alpha} = 1-\alpha$, $\bar{\eta} = 1-\eta$, $\tilde{\alpha} = 2-\alpha$.

and, consequently, retail prices. In other words, if a wholesale price contract is chosen, the manufacturer can use higher wholesale price to extort the profit generated by advertising. As such, under a wholesale price the manufacturer has more incentive to implement cooperative advertising. Even so, if the cost-sharing rate is high, the manufacturer would not choose to cooperative due to the higher advertising cost. Overall, in this condition, cooperative advertising yields more profits for the manufacturer. This conclusion is intuitive.

4.2. Manufacturer's cooperation strategies under agency contract

The agency contract has a significant difference from the wholesale contract in pricing power. That is to say, different from the wholesale contract, the manufacturer yields the pricing power and assigns a fraction of revenue to the platform under an agency contract. Therefore, there is no double marginalization under this type of contract.

In this part, we analyze the scenario in which the platform provides an agency contract. The profit functions of the platform and manufacturer in Scenarios AC and AN are respectively given by

In AN, the profit functions of the two parties are:

$$\pi_{mr} = (1-\alpha)p_1\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) + p_2\left(\frac{1-\theta(1+e_p)-p_2+\theta p_1}{1-\theta^2}\right)$$
$$\pi_a = \alpha p_1\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) - e_p^2.$$

In AC, the profit functions of the two parties are:

$$\pi_{mr} = (1-\alpha)p_1\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) + p_2\left(\frac{1-\theta(1+e_p)-p_2+\theta p_1}{1-\theta^2}\right) - \eta e_p^2$$
$$\pi_a = \alpha p_1\left(\frac{1+e_p-\theta-p_1+\theta p_2}{1-\theta^2}\right) - (1-\eta)e_p^2.$$

Lemma 4.3. *In case AC, the equilibrium solutions' feasible domain is* $0 < \eta \leq \frac{1}{8}\left(\frac{Z}{(2-3\alpha+\alpha^2)(1-\theta)(4-4\alpha-4\theta^2+4\alpha\theta^2-\alpha^2\theta^2)^3} - \sqrt{\frac{Y}{(2-3\alpha+\alpha^2)^2(1-\theta)^2(4-4\alpha-4\theta^2+4\alpha\theta^2-\alpha^2\theta^2)^6}}\right)$. *While case AN is always feasible. The equilibriums are shown in Table 5. (The concrete values of Z and Y are shown in the Appendix A).*

As we did in the WC and WN cases, by comparing the equilibrium results between the AN and AC cases in their common feasible region, we find the major relationships, which are shown in Table 6. Table 6 shows that under an agency contract, e_p, p_1, and D_1 are all larger in AC, whereas p_2 and D_2 are smaller.

TABLE 6. Comparison of variables under agency contract.

Advertising level	$e_p^*(\mathrm{AC}) > e_p^*(\mathrm{AN})$
Direct channel price	$p_2^*(\mathrm{AC}) < p_2^*(\mathrm{AN}))$
Retail prices	$p_1^*(\mathrm{AC}) > p_1^*(\mathrm{AN})$
Demand	$D_1^*(\mathrm{AC}) > D_1^*(\mathrm{AN}), D_2^*(\mathrm{AC}) < D_2^*(\mathrm{AN})$

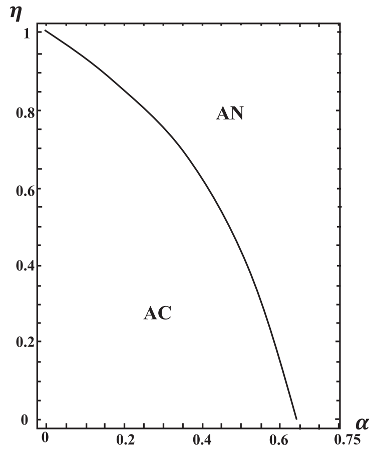


FIGURE 3. Comparison of the manufacturer's profit in AC and AN.

Theorem 4.4. *Under an agency contract, the platform always prefers scenario AC if AC is feasible, while the manufacturer will choose between cooperation and noncooperation as the revenue-sharing rate increases. The specific intervals of cooperation are:* $0 < \alpha \leq \alpha_0$ *and* $0 < \eta < \eta_0$*. Otherwise, the manufacturer will not choose to cooperate. (The concrete values of* α_0 *and* η_0 *can be found in the Appendix A).*

Due to the complexity of the form of α_0, η_0 and to express the condition more clearly, we represent this conclusion graphically. Here, we fix θ at 0.1. Figure 3 vividly exhibits Theorem 4.4.

As indicated in Figure 3, we find that the manufacturer will choose between cooperation and noncooperation as the revenue-sharing rate increases. At first, when the revenue-sharing rate is low, the manufacturer will agree to cooperate only when the cost-sharing rate is not very high, which is intuitive. As the revenue-sharing rate increases, a region emerges where the manufacturer will choose noncooperation even if the cost-sharing rate is infinitely low. This is because the platform will extort more profit generated by advertising. The disadvantage of a higher revenue-sharing rate overshadowing the benefit of higher demand. Most of the revenue from advertising is transferred to the platform, and this, combined with the reduced demand for the direct channel, means cooperation is not an optimal strategy for the manufacturer.

5. Platform's optimal distribution contract

To concentrate on the impact of cost-sharing rate (η) and the revenue-sharing rate (α) on the platform preferences, we fix θ at various values and repeat our main text analysis of the four cases. This technique is similar to that of Yang *et al.* [51]. Here, we fix $\theta = 0.1, 0.5$ in our analyses. The two values of θ can comprehensively explain the equilibrium choices of the platform and how the equilibrium varies with competition intensity. In this section, we will combine Figures 2 and 3 to compare the profit of the platform under the agency and the wholesale contract in the intersection area. We put the specific comparison process in the Appendix A. Due to

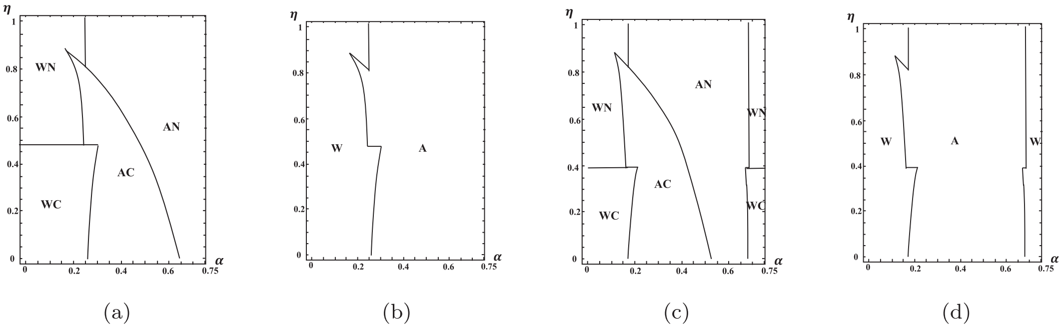


FIGURE 4. Platform's optimal distribution contract when θ is set. (a) $\theta = 0.1$. (b) $\theta = 0.1$. (c) $\theta = 0.5$. (d) $\theta = 0.5$.

the computational complexity, we resort to a graphical solution *via* contour plotting, as illustrated in Figure 4. As before, we use "A" and "W" to denote the agency and wholesale contract, respectively.

Theorem 5.1. *As illustrated in Figure 4, the following hold.*

(1) When the channel competition intensity is small, the platform will offer a wholesale contract if and only if the revenue-sharing rate is low; otherwise, an agency contract is provided.
(2) When the channel competition intensity is large (θ is close to the maximum $\frac{1}{2}$), the platform will offer a wholesale contract when the revenue-sharing rate is either low (near zero) or high (near $\frac{3}{4}$).
(3) Assume that the channel competition intensity neither small nor large, and that it is increasing. In this case, when the revenue-sharing rate high, the platform is more inclined to wholesale contracts, and when the revenue-sharing rate is low, the platform is more inclined to agency contracts.

Theorem 5.1 and Figure 4 indicate that the platform's optimal contract selection is mainly influenced by the channel competition intensity and revenue-sharing rate.

Comparing Figures 4b and 4d, we find that the possibility that the platform offers an agency contract is declining. Namely, with the channel competition intensity increasing, when the revenue-sharing rate is relatively high, the platform is more inclined to wholesale contracts; when the revenue-sharing rate is relatively low, the platform is more inclined to the agency contract.

First, we pay attention to the condition where the market competition intensity is relatively strong. In this condition, the platform will provide an agency contract only when the revenue-sharing rate (α) in the intermediate range. On the contrary, when α is either high or low, a wholesale contract will be provided. When the revenue sharing rate is relatively low, it is very intuitive why the platform is reluctant to adopt agency contracts even if the manufacturer is willing to share the advertising cost. This is due to the trade-off between the revenue-sharing rate and the cost-sharing rate. Recall that the manufacturer is willing to share advertising costs only when the cost-sharing rate is very low. Therefore, the platform's revenue from advertising costs is much lower than the revenue lost due to the lower revenue-sharing rate, which leads to the platform is reluctant to provide agency contracts. Additionally, when the revenue-sharing rate is relatively high, as indicated in Theorem 4.4 and Figure 3, under the agency contract, the manufacturer intends to choose not to share the advertising cost due to the platform will extort more profit generated by advertising. Therefore, when the revenue-sharing rate is high in this case, the platform should compare her profits in Scenarios AN and WC. In this situation, the benefit from high revenue-sharing rate is lower than the profit from cost sharing, so the platform will choose a wholesale contract. Recollect that the pricing power is completely controlled by the manufacturer under an agency contract. Thus, when the market is very competitive, the manufacturer will set the price of the platform high to attract more consumers to buy through the direct channel.

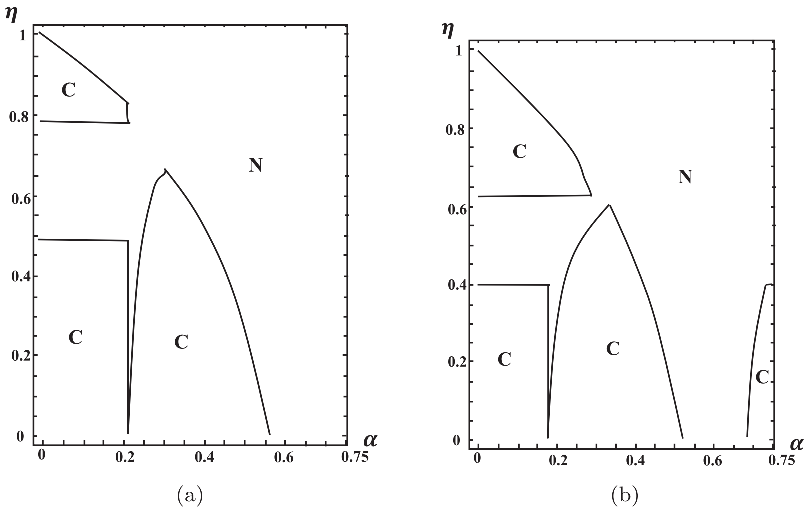


FIGURE 5. Manufacturer's optimal cooperation strategy. (a) $\theta = 0.1$. (b) $\theta = 0.5$.

Next we analyze the situation when the market competition intensity is relatively small. Unlike when the market competition intensity is relatively large, when the revenue-sharing rate is relatively high, a counterintuitive phenomenon occurs: the platform will definitely choose an agency contract in this situation. When competition is weakened, even if the platform's revenue-sharing rate is high, the manufacturer will not significantly increase the price of the agent platform. At this time, for the agent platform, even if the manufacturer does not share the advertising costs, the agent contract is also beneficial.

Another interesting phenomenon is that as the revenue-sharing rate increases, the platform's profit does not always increase. If we do not consider the cooperative advertising strategy of the manufacturer, for the platform, it must be that the higher the revenue-sharing rate, the more inclined the platform is to agency contracts. However, when we consider the interaction between the platform's contract choice and the manufacturer's cooperative advertising decisions, the willingness to choose agency contracts is not simply enhanced with the increase in the revenue-sharing rate. This also illustrates the interaction between contract selection and cooperative advertising from another perspective.

6. Extension

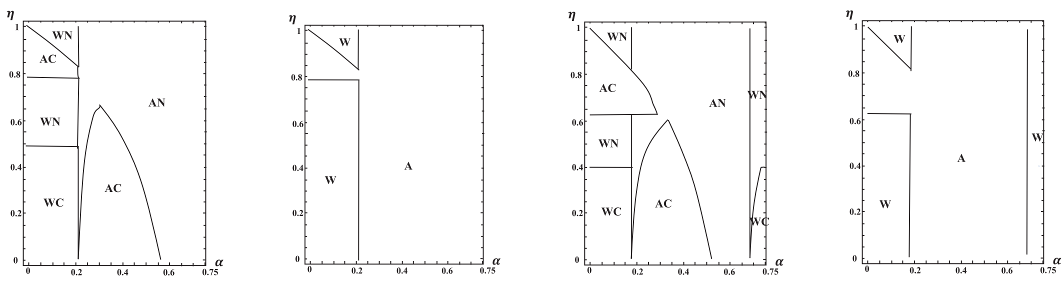
6.1. Manufacturer determines whether to cooperate first

In this subsection, we consider that in reality there may be situations where the manufacturer decides the strategy of cooperative advertising first and then the platform decides the appropriate contract. The rest of the order below is the same as above. As we did in the base model, we first focus on the choice of the manufacturer. In the following, we first characterize the optimal cooperation decisions of the manufacturer and then derives the platform's optimal contract.

6.1.1. Manufacturer's optimal cooperation strategy

Again, we resort to a graphical solution *via* contour plotting, as illustrated in Figure 5. We use "C" and "W" to denote the manufacturer's decisions regarding cooperation or noncooperation.

Theorem 6.1. *As illustrated in Figure 5, the following hold.*



(a) (b) (c) (d)

FIGURE 6. Platform's optimal distribution contract. (a) $\theta = 0.1$. (b) $\theta = 0.1$. (c) $\theta = 0.5$. (d) $\theta = 0.5$.

The manufacturer will adopt cooperation strategy when the revenue-sharing rate is low and the cost-sharing rate is either high or low. Moreover, when the channel competition intensity is large, the manufacturer will also choose to cooperate when the revenue-sharing rate is high and the cost-sharing rate is low.

Consistent with the above analysis, under a wholesale contract, the manufacturer will choose to cooperate if and only if the cost-sharing rate is low. We find that, when the cost-sharing rate is high, the manufacturer will choose cooperation only when the revenue-sharing rate is low. And this condition is under an agency contract. This region does not appear in the previous action sequence, which makes it interesting. We will explain this phenomenon in the following part.

6.1.2. Platform's optimal distribution contract

Finally, we pay attention to the platform's choices, given the manufacturer's cooperation strategy. As above, we resort to a graphical solution *via* contour plotting, as illustrated in Figure 6.

Theorem 6.2. *As illustrated in Figure 6, the following hold.*

(1) When the market competition intensity is large, the platform will provide an agency contract when (1) the revenue-sharing rate is intermediate; (2) the revenue-sharing rate is low and the cost-sharing rate is high. Otherwise, a wholesale contract will be provided.
(2) When the market competition intensity is small, the platform will provide an agency contract when (1) the revenue-sharing rate is high; (2) the revenue-sharing rate is low and the cost-sharing rate is high. Otherwise, a wholesale contract will be provided.

Comparing the results of Figures 4 and 6, we see that there are similarities and differences in the final equilibriums between the two sequences.

The main similarity is that the trend of platform selection of wholesale contract is consistent with the previous sequence: when the competition intensity is small, wholesale contract is selected only when the revenue-sharing rate is low; when the competition intensity is large, the wholesale contract is chosen when either the revenue-sharing rate is high or low.

The main difference is that when cost-sharing rate is high and the revenue-sharing rate is low, if the platform moves first, she will elect a wholesale contract and the final equilibrium is WN, whereas, if the manufacturer moves first, he will elect cooperation and the final equilibrium is AC. This is the famous theory of "first-mover advantage" in game theory.

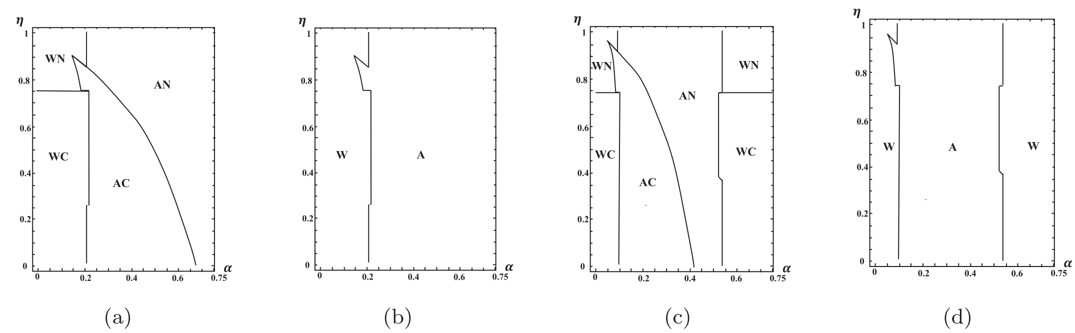


FIGURE 7. Platform's optimal distribution contract. (a) $\theta = 0.1$. (b) $\theta = 0.1$. (c) $\theta = 0.5$. (d) $\theta = 0.5$.

6.2. Manufacturer advertises on the direct channel

This subsection examines the prospect of the manufacturer considering co-advertising on platforms as well as his own direct channel, doing so to explore the impact of direct channel advertising strategy on the final equilibrium results. In this subsection, we assume the manufacturer advertises first before the platform's advertising. Extending the base model, we assume the direct channel's advertising level is e_m. Therefore, with the impact of the advertising, the new demands of the two channels become $\alpha_1 = A_1(1 + e_p) = 1 + e_p$ and $\alpha_2 = A_2(1 + e_m) = 1 + e_m$.

For brevity, we assume the advertising cost coefficient of the manufacturer and the platform is the same. Therefore, the function representing the cost of advertising are $C(e_p) = e_p^2$ and $C(e_m) = e_m^2$. Thus, the platform and manufacturer profits are given, respectively, by

$$\pi_r = (p_1 - w)D_1 - (1 - \mathbf{1}_m \eta)e_p^2 \tag{6.1}$$

$$\pi_a = \alpha p_1 D_1 - (1 - \mathbf{1}_m \eta)e_p^2 \tag{6.2}$$

$$\pi_{mr} = wD_1 + p_2 D_2 - \mathbf{1}_m \eta e_p^2 - e_m^2 \tag{6.3}$$

$$\pi_{ma} = (1 - \alpha)p_1 D_1 + p_2 D_2 - \mathbf{1}_m \eta e_p^2 - e_m^2. \tag{6.4}$$

The calculation process, which follows that of Section 5, is omitted here for brevity. There is a clear difference of the direct channel's advertising, which shows that when the manufacturer advertises on the direct channel, he can effectively coordinate the effect of the platform's advertising by designing his own advertising level on the direct channel. As such, we further fix $\theta = 0.1, 0.5$ in our analysis.

6.2.1. Platform takes first action

We first focus on the sequence that the platform first determines the contract, and subsequently, the manufacturer announces whether to cooperate. Figure 7 depicts the platform's optimal distribution contract.

Theorem 6.3. *When the manufacturer advertises on the direct channel and the platform declares the contract type first, the variation trends of platform's contract choice with the revenue-sharing rate and competition intensity are consistent with the case when there is no direct advertising.*

Clearly, the base model's results are still valid, which indicates the robustness of our main results. By comparing Figures 4 and 7, as the channel competition intensity increases, the possibility of the platform providing an agency contract becomes less and less. This is because the direct channel's advertising can soften the double marginalization that occurs under a wholesale contract. Therefore, as competition intensifies, compared with the case of no direct channel advertising, the wholesale contract becomes more attractive.

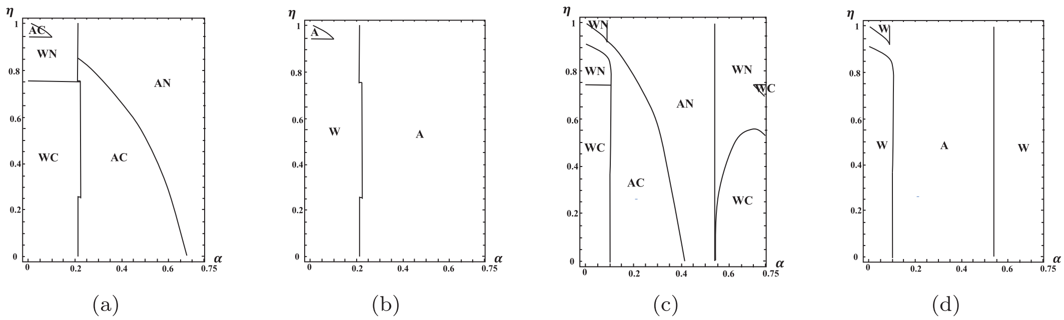


FIGURE 8. Manufacturer's optimal cooperation strategy. (a) $\theta = 0.1$. (b) $\theta = 0.1$. (c) $\theta = 0.5$. (d) $\theta = 0.5$.

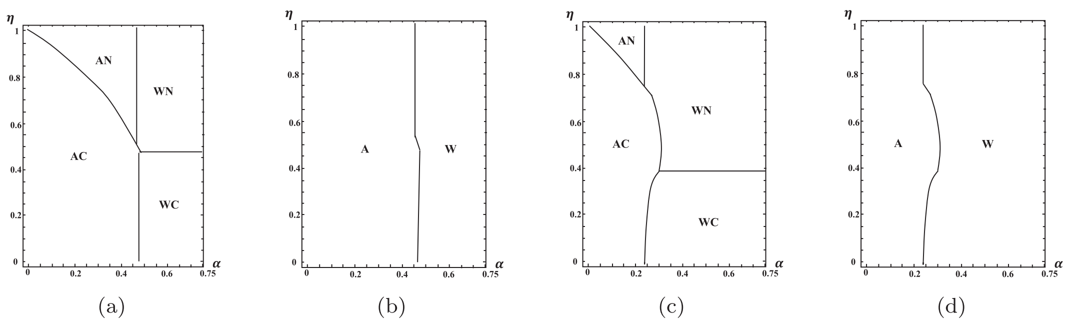


FIGURE 9. Manufacturer's optimal distribution contract selection. (a) $\theta = 0.1$. (b) $\theta = 0.1$. (c) $\theta = 0.5$. (d) $\theta = 0.5$.

6.2.2. Manufacturer takes first action

Secondly, we focus on the sequence that the manufacturer first determines the cooperation strategy and, subsequently, the platform announces the type of contract. Figure 8 depicts the platform's optimal choice of contracts.

Theorem 6.4. *When the manufacturer advertises on the direct channel and announces his cooperation strategies first, the variation trend of the platform's contract choice with revenue-sharing rate and competition intensity is consistent with when there is no direct advertising.*

6.3. A powerful manufacturer

In the above we have studied a common situation: a powerful platform determines the contract type. But in practice, when the platform faces a powerful manufacturer, she may offer two types of contract, and the manufacturer may choose one of them to distribute the products. For example, JD offers both agency channel and reselling channel for some suppliers, and Apple only chooses the reselling format to distribute its iPhone. Therefore, in this subsection, we consider the condition where a powerful manufacturer decides on the type of contract.

First, following Section 4, we get the manufacturer's cooperation strategies under the two contracts. Then, we compare the optimal profits of the manufacturer under these two contracts. Figure 9 depicts the manufacturer's contract selection.

Theorem 6.5. *As illustrated in Figure 9, the following hold.*

The contract selection of the manufacturer is mainly affected by the revenue-sharing rate and the intensity of channel competition. The manufacturer will choose an agency contract when the revenue sharing rate is small; otherwise, a wholesale contract is selected. As the intensity of competition increases, the possibility of the manufacturer choosing an agency contract is declining.

The conclusions is this subsection are pretty straightforward. When the manufacturer decides which contract to choose and whether to cooperate, the platform is completely a passive receiver. At this time, the two parties have lost the mutual checks and balances studied previously, and the manufacturer only aims to maximize his own profits.

7. Conclusions

This paper evaluates the interaction between a platform's contract selection and a manufacturer's decision on cooperative advertising, doing so considering varying action sequences. We first focus on the sequence when the platform makes the first move by specifying the type of contract she offers (her optimal choice). Our results show that when the platform offers a wholesale contract, the manufacturer will select cooperative advertising if and only if the cost-sharing rate is not high, which is a straightforward idea. However, when the platform offers an agency contract, the manufacturer prefers cooperation when the revenue-sharing rate is low and noncooperation when that rate is high. Further, we consider the platform' optimal contract selection. We find that when considering the interplay between them, some interesting phenomena appear. Conventional wisdom suggests that the platform would prefer the manufacturer share advertising costs for them. However, when considering the interaction between the contract choice and cooperative advertising, in some cases, the platform would prefer that the manufacturer not share the cost, especially when the revenue-sharing rate is high and the competition intensity is small. This is because the trade-off between the payoff from the revenue sharing and the cost sharing. At this time, even if the manufacturer is willing to share the advertising costs under the wholesale contract, the platform would prefer to choose the agency contract to bear all the advertising costs. The other interesting conclusion is that as the revenue-sharing rate increases, the platform's profit does not always increase. When the market competition intensity is large, the platform will abandon the agency contract and choose a wholesale contract when the revenue sharing rate is high, considering that under the agency contract, the manufacturer will not share advertising costs.

Our extended analysis looks at other conditions, including another action sequence in which the manufacturer declares whether he will cooperate before the contract type is selected, the case where the manufacturer advertises on the direct channel and a powerful manufacturer. Considering the sequence in which the manufacturer acts first, our results reveal that the manufacturer will agree to cooperative advertising when (1) the revenue-sharing rate is low, and the cost-sharing rate is either high or low under any competition intensity; or (2) the revenus-sharing rate is high, and the cost-sharing rate is low under strong competition intensity. Moreover, when the manufacturer chooses cooperation, the platform will provide an agency contract when (1) the cost-sharing rate is low, and the revenue-sharing rate is intermediate; or (2) the cost-sharing rate is high, and the revenue-sharing rate is low. Otherwise, a wholesale contract will be provided. If the manufacturer chooses noncooperation, the platform will provide an agency contract if and only if the revenue-sharing rate is intermediate. Otherwise, the platform will offer a wholesale contract. Furthermore, when taking direct channel advertising into account, we find that the qualitative results are the same as when there is no direct advertising no matter what the sequence. When the manufacturer has more power than the platform, the decisions of cooperative advertising and contract selection are all determined by the manufacturer, the interaction between the two disappears, and the platform is completely a passive receiver. The manufacturer will choose an agency contract only when the revenue-sharing rate is small.

Our results offer some managerial insights to better understand the interaction between contract selection and cooperative advertising determination in practice. We find that the manufacturer should carefully consider the contract choice of the platform when deciding the cost-sharing rate of cooperative advertising. Ignoring the choice of the contract choice may cause certain losses to the manufacturer. At the same time, when the platform chooses a contract, it needs to carefully consider the manufacturer's cost-sharing decision of cooperative advertising to optimize its own profits. Additionally, the platform would prefer that the manufacturer not share the cost under some conditions. Therefore, the platform should carefully balance the relationship between channel contract and manufacturer's cost-sharing rate when making decisions. There are limitations to our research which need further study. Firstly, the cost-sharing rate is exogenously given in our study. While when this rate be examined endogenously in future, it may yield different conclusions. Secondly, we only consider static cooperative advertising, and some other interesting conclusions may emerge when considering dynamic cooperative advertising.

Appendix A.

The concrete values of η_0, η_1 and α_0 are

$\eta_0 = \frac{Z}{Y}$, where $Z = 128(4-\alpha)(1-\alpha)^6(8-(12-\alpha)\alpha) - 64(4-\alpha)(1-\alpha)^6(16-(14-\alpha)\alpha)\theta - 64(2-\alpha)(1-\alpha)^5(160-\alpha(352-\alpha(188-23\alpha)))\theta^2 + 16(2-\alpha)(1-\alpha)^4(640-\alpha(1632-\alpha(1420-\alpha(486-49\alpha))))\theta^3 + 32(2-\alpha)^2(1-\alpha)^4(320-\alpha(848-\alpha(660-\alpha(173-8\alpha))))\theta^4 - 8(2-\alpha)^2(1-\alpha)^3(1280-\alpha(3808-\alpha(4124-\alpha(1994-\alpha(397-17\alpha)))))\theta^5 - 16(2-\alpha)^3(1-\alpha)^3(320-\alpha(992-\alpha(1012-\alpha(425-(65-\alpha)\alpha))))\theta^6 + 4(2-\alpha)^4(1-\alpha)^2(640-\alpha(1856-\alpha(1898-\alpha(832-\alpha(137-2\alpha)))))\theta^7 + 8(2-\alpha)^6(1-\alpha)^2(10-3\alpha)(4-\alpha(9-4\alpha))\theta^8 - 2(2-\alpha)^7(1-\alpha)(8-5\alpha)(10-\alpha(17-5\alpha))\theta^9 - 4(2-\alpha)^9(1-\alpha)(2-(4-\alpha)\alpha)\theta^{10} + (2-\alpha)^{10}(4-\alpha(7-2\alpha))\theta^{11}$,
$Y = (1-\alpha)(4(1-\alpha)-(2-\alpha)^2\theta^2)^3(8(8-\alpha)(1-\alpha)^3 - 4(1-\alpha)^2(16-(11-\alpha)\alpha)\theta - 4(2-\alpha)(1-\alpha)^2(16-9\alpha)\theta^2 + (2-\alpha)^2(1-\alpha)(32-19\alpha)\theta^3 + 4(2-\alpha)^4(1-\alpha)\theta^4 - 2(2-\alpha)^5\theta^5)$.

$$\eta_1 = \frac{106\,267 + 127\,601\theta - 195\,762\theta^2 - 324\,318\theta^3 - 20\,249\theta^4 + 177\,565\theta^5 + 121\,248\theta^6 + 35\,296\theta^7 + 4864\theta^8 + 256\theta^9}{209\,952 + 322\,272\theta - 266\,976\theta^2 - 682\,272\theta^3 - 212\,128\theta^4 + 268\,192\theta^5 + 253\,792\theta^6 + 90\,784\theta^7 + 15\,360\theta^8 + 1024\theta^9}.$$

α_0 is the second root of $256 - 256\theta - 768\theta^2 + 768\theta^3 + 768\theta^4 - 768\theta^5 - 256\theta^6 + 256\theta^7 + (-1408 + 1248\theta + 4224\theta^2 - 3712\theta^3 - 4224\theta^4 + 3680\theta^5 + 1408\theta^6 - 1216\theta^7)\alpha + (3104 - 2448\theta - 9408\theta^2 + 7328\theta^3 + 9504\theta^4 - 7312\theta^5 - 3200\theta^6 + 2432\theta^7)\alpha^2 + (-3456 + 2432\theta + 10\,848\theta^2 - 7560\theta^3 - 11\,360\theta^4 + 7832\theta^5 + 3968\theta^6 - 2704\theta^7)\alpha^3 + (1984 - 1248\theta - 6816\theta^2 + 4300\theta^3 + 7800\theta^4 - 4900\theta^5 - 2960\theta^6 + 1840\theta^7)\alpha^4 + (-512 + 288\theta + 2208\theta^2 - 1280\theta^3 - 3080\theta^4 + 1796\theta^5 + 1368\theta^6 - 788\theta^7)\alpha^5 + (32 - 16\theta - 288\theta^2 + 156\theta^3 + 648\theta^4 - 358\theta^5 - 384\theta^6 + 208\theta^7)\alpha^6 + (-56\theta^4 + 30\theta^5 + 60\theta^6 - 31\theta^7)\alpha^7 + (-4\theta^6 + 2\theta^7)\alpha^8 = 0$.

Proof of Lemma 4.1

Case WN

Due to the constraint that the wholesale price must not be higher than the direct channel price. We can easily verify that there is no interior solution, which is to say $w = p_2$ in this case.

We solve by reverse induction. More specifically, we first compute the platform's best-response price, then substitute it into the manufacturer's profit function, and solve the manufacturer's first-order conditions for direct channel price (wholesale price). Finally substituting all the above variables into the platform's profit function and solving her advertising levels.

Given e_p, $w = p_2$ and θ, platform's profit is concave with respect to p_1 because $\frac{\partial^2\pi_r}{\partial p_1^2} = -\frac{2}{1-\theta^2} < 0$. The best response price function can be obtained by solving from the first-order condition.

$$p_1 = \frac{1 + e_p + p_2 - (1-p_2)\theta}{2}.$$

Then, substituting p_1 into the manufacturer's profit function. We find that manufacturer's profit is concave with respect to p_2 because $\frac{\partial^2\pi_{mr}}{\partial p_2^2} = -\frac{3+\theta}{1+\theta} < 0$. So we can get the best response direct channel price $p_2 = \frac{3+e_p+\theta}{6+2\theta}$.

Substituting all the variables into the platform's profit, we get

$$\pi_r(e_p) = \frac{(1-\theta)^2(3+\theta)^2 + 2e_p(1-\theta)(3+\theta)(5+3\theta) - e_p^2(119+\theta(66-\theta(137+16\theta(6+\theta))))}{16(3+\theta)^2(1-\theta^2)}.$$

The second derivative of e_p is $\frac{\partial^2 \pi_r}{\partial e_p^2} = -\frac{119+\theta(66-\theta(137+16\theta(6+\theta)))}{8(3+\theta)^2(1-\theta^2)}$. It is concave when $119 + \theta(66 - \theta(137 + 16\theta(6 + \theta))) > 0$. Obviously, when $0 < \theta \leq \frac{1}{2}$, this function is always satisfied.

So, we can obtain the optimal e_p^*. In summary, the unique equilibrium for WN is:

$$\begin{aligned}
e_p^* &= \frac{(1-\theta)(3+\theta)(5+3\theta)}{119+\theta(66-\theta(137+16\theta(6+\theta)))} \\
p_1^* &= \frac{2(1+\theta)(49-\theta(39+2\theta(12-\theta(2+\theta))))}{119+\theta(66-\theta(137+16\theta(6+\theta)))} \\
p_2^* = w^* &= \frac{3+\theta+\frac{(1-\theta)(3+\theta)(5+3\theta)}{119+\theta(66-\theta(137+16\theta(6+\theta)))}}{6+2\theta} \\
D_1^* &= \frac{4(1-\theta)(3+\theta)^2}{119+\theta(66-\theta(137+16\theta(6+\theta)))} \\
D_2^* &= \frac{(3+\theta)(19-\theta(1+2\theta)(7+2\theta))}{119+\theta(66-\theta(137+16\theta(6+\theta)))} \\
\pi_r^* &= \frac{(1-\theta)^2(3+\theta)^2}{119+\theta(66-\theta(137+16\theta(6+\theta)))} \\
\pi_{mr}^* &= \frac{2(\theta^2+4\theta+3)(31-15\theta-20\theta^2+4\theta^3)^2}{(119+\theta(66-\theta(137+16\theta(6+\theta))))^2}.
\end{aligned}$$

We can easily find that when $\theta \in (0, \frac{1}{2}]$, all prices and demand are positive.

Case WC

Similarly, we can easily verify that there is also no interior solution, which is to say $w = p_2$ in this case. We solve by reverse induction.

Given e_p, $w = p_2$, η and θ, platform's profit is concave with respect to p_1 because $\frac{\partial^2 \pi_r}{\partial p_1^2} = -\frac{2}{1-\theta^2} < 0$. The best response price function can be obtained by solving from the first-order condition.

$$p_1 = \frac{1+e_p+p_2-(1-p_2)\theta}{2}.$$

Then, substituting p_1 into the manufacturer's profit function. We find that manufacturer's profit is concave with respect to p_2 because $\frac{\partial^2 \pi_{mr}}{\partial p_2^2} = -\frac{3+\theta}{1+\theta} < 0$. So we can get the best response direct channel price $p_2 = \frac{3+e_p+\theta}{6+2\theta}$. Substituting all the variables into the platform's profit, we get

$$\pi_r(e_p) = \frac{(3-2\theta-\theta^2)^2 + 2e_p(15-\theta-11\theta^2-3\theta^3) - e_p^2(119+66\theta-137\theta^2-96\theta^3-16\theta^4-16\eta(3+\theta)^2(1-\theta^2))}{16(3+\theta)^2(1-\theta^2)}.$$

The second derivative of e_p is $\frac{\partial^2 \pi_r}{\partial e_p^2} = -\frac{119+66\theta-137\theta^2-96\theta^3-16\theta^4-16\eta(3+\theta)^2(1-\theta^2)}{8(3+\theta)^2(1-\theta^2)}$. It is concave when $119 + 66\theta - 137\theta^2 - 96\theta^3 - 16\theta^4 - 16\eta(3+\theta)^2(1-\theta^2) > 0$, which is equivalent to $0 < \eta < \frac{119+66\theta-137\theta^2-96\theta^3-16\theta^4}{144+96\theta-128\theta^2-96\theta^3-16\theta^4}$ and $\theta \in (0, \frac{1}{2}]$.

So, we can obtain the optimal e_p^*. In summary, the unique equilibrium for case WC is:

$$\begin{aligned}
e_p^* &= \frac{(1-\theta)(3+\theta)(5+3\theta)}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))} \\
p_1^* &= \frac{2(1+\theta)(49-39\theta-24\theta^2+4\theta^3+2\theta^4-2\eta(3+\theta)^2(3-4\theta+\theta^2))}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))} \\
p_2^* = w^* &= \frac{3+\theta-\frac{(1-\theta)(3+\theta)(5+3\theta)}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}}{6+2\theta}
\end{aligned}$$

$$\begin{aligned}
D_1^* &= \frac{4(1-\eta)(1-\theta)(3+\theta)^2}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}\\
D_2^* &= \frac{(3+\theta)(19-4\eta(1-\theta)(2+\theta)(3+\theta)-\theta(1+2\theta)(7+2\theta))}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}\\
\pi_r^* &= \frac{(1-\eta)(1-\theta)^2(3+\theta)^2}{119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta)))}\\
\pi_{mr}^* &= (3+\theta)\left(\frac{\begin{array}{c}32\eta^2(1-\theta)^2(1+\theta)(3+\theta)^4+2(1+\theta)(31-15\theta-20\theta^2-4\theta^3)^2\\ -\eta(4539+781\theta-8386\theta^2-3542\theta^3+3159\theta^4+2681\theta^5+704\theta^6+64\theta^7)\end{array}}{(119-16\eta(3+\theta)^2(1-\theta^2)+\theta(66-\theta(137+16\theta(6+\theta))))^2}\right).
\end{aligned}$$

For the demand of the direct channel and both parties' profits to remain nonnegative requires $0 < \eta \leq \frac{1}{64}\left(\frac{1513+1269\theta-14445\theta^2-17121\theta^3-576\theta^4-64\theta^5}{(3+\theta)^3(1-\theta^2)}\right) - \sqrt{\frac{(5+3\theta)^2(3001+25333\theta-28690\theta^2-34333\theta^3-1152\theta^4-128\theta^5)}{(1-\theta)(1+\theta)^2(3+\theta)^6}}$ and the feasible domain for θ is $(0, \frac{1}{2}]$.

Proof of Lemma 4.3

Case AN

Given e_p, α and θ, the Hessian matrix of the manufacturer's profit is $H = \begin{bmatrix} -\frac{2(1-\alpha)}{1-\theta^2} & \frac{\theta(2-\alpha)}{1-\theta^2} \\ \frac{\theta(2-\alpha)}{1-\theta^2} & -\frac{2}{1-\theta^2}\end{bmatrix}$.

Due to $\frac{\partial^2\pi_{ma}}{\partial p_1^2} = -\frac{2(1-\alpha)}{1-\theta^2} < 0$, $\frac{\partial^2\pi_{ma}}{\partial p_2^2} = -\frac{2}{1-\theta^2} < 0$ and $\frac{\partial^2\pi_{ma}}{\partial p_1^2}\frac{\partial^2\pi_{ma}}{\partial p_2^2} - \frac{\partial^2\pi_{ma}}{\partial p_1 p_2}\frac{\partial^2\pi_{ma}}{\partial p_2 p_1} = \frac{4(1-\alpha)-(2-\alpha)^2\theta^2}{(1-\theta^2)^2}$. The corresponding Hessian matrix is always negative definite when $0 < \alpha \leq \frac{3}{4}$.

Then, the best response prices function can be obtained by solving from the first-order condition.

$$\begin{aligned}
p_1 &= \frac{2(1+e_p)(1-\alpha)+\alpha\theta-(1+e_p)(2-\alpha)\theta^2}{4(1-\alpha)-(2-\alpha)^2\theta^2}\\
p_2 &= \frac{(1-\alpha)(2-(1+e_p)\alpha\theta-(2-\alpha)\theta^2)}{4(1-\alpha)-(2-\alpha)^2\theta^2}.
\end{aligned}$$

Substituting p_1 and p_2 into the platform's profit, we get

$$\pi_a(e_p) = \frac{\alpha(2(1+e_p)(1-\alpha)-(2-\alpha)\theta)(2(1+e_p)(1-\alpha)+\alpha\theta-(1+e_p)(2-\alpha)\theta^2)}{(4(1-\alpha)-(2-\alpha)^2\theta^2)^2} - e_p^2.$$

The second derivative of e_p is $\frac{\partial^2\pi_a}{\partial e_p^2} = \frac{4(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2-8(4-\alpha)(1-\alpha)^2-2(2-\alpha)^4\theta^4}{(4(1-\alpha)-(2-\alpha)^2\theta^2)^2}$. By simple calculation, we find that $4(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2 - 8(4-\alpha)(1-\alpha)^2 - 2(2-\alpha)^4\theta^4 < 0$ when $0 < \alpha \leq \frac{3}{4}$.

So, we can obtain the optimal e_p^*. In summary, the unique equilibrium for AN is:

$$\begin{aligned}
e_p^* &= \frac{\alpha(8-4\theta-(2-\alpha)(2(2-\theta)\theta^2+\alpha(8-\theta(4+\theta(4-\theta)))))}{8(4-\alpha)(1-\alpha)^2+2(2-\alpha)^4\theta^4-4(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2}\\
p_1^* &= \frac{16(1-\alpha)^2+6(1-\alpha)\alpha\theta-4(4-\alpha)(2-\alpha)(1-\alpha)\theta^2-(2-\alpha)\alpha(3-2\alpha)\theta^3+2(2-\alpha)^3\theta^4}{8(4-\alpha)(1-\alpha)^2+2(2-\alpha)^4\theta^4-4(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2}\\
p_2^* &= \frac{(1-\alpha)(4(4-\alpha)(1-\alpha)-8(1-\alpha)\alpha\theta-(4-3\alpha)(8-5\alpha)\theta^2+2(2-\alpha)^2\alpha\theta^3+2(2-\alpha)^3\theta^4)}{8(4-\alpha)(1-\alpha)^2+2(2-\alpha)^4\theta^4-4(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2}\\
D_1^* &= \frac{8(1-\alpha)^2-(1-\alpha)(8-5\alpha)\theta-2(2-\alpha)^2(1-\alpha)\theta^2+(2-\alpha)^3\theta^3}{4(4-\alpha)(1-\alpha)^2+(2-\alpha)^4\theta^4-2(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2}\\
D_2^* &= \frac{(1-\alpha)(4(4-\alpha)(1-\alpha)-8(2-\alpha)(1-\alpha)\theta-(2-\alpha)(8-5\alpha)\theta^2+2(2-\alpha)^3\theta^3)}{8(4-\alpha)(1-\alpha)^2+2(2-\alpha)^4\theta^4-4(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2}
\end{aligned}$$

$$\pi_a^* = \frac{\alpha(16(1-\theta)(1-\alpha)^2 - (16-(17-4\alpha)\alpha)\theta^2 + 4(2-\alpha)^2\theta^3)}{16(4-\alpha)(1-\alpha)^2 + 4(2-\alpha)^4\theta^4 - 8(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2}$$

$$\pi_{ma}^* = (1-\alpha)\left(\frac{\begin{array}{c}16(1-\alpha)^3(32-(24-\alpha)\alpha) - 64(1-\alpha)^3(8-5\alpha)\theta - 4(1-\alpha)^2(384-\alpha(656-\alpha(367-68\alpha)))\theta^2\\ +32(12-7\alpha)(2-3\alpha+\alpha^2)^2\theta^3 + (2-\alpha)^2(1-\alpha)(384-\alpha(640-\alpha(339-56\alpha)))\theta^4\\ -4(2-\alpha)^4(1-\alpha)(24-13\alpha)\theta^5 - 2(2-\alpha)^5(8-\alpha(9-2\alpha))\theta^6 - 4(2-\alpha)^7\theta^7\end{array}}{4(4(4-\alpha)(1-\alpha)^2 + (2-\alpha)^4\theta^4 - 2(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2)^2}\right).$$

It is easy to find that all prices and demand are positive.

Case AC

Given e_p, α, η and θ, the Hessian matrix of the manufacturer's profit is $H = \begin{bmatrix} -\frac{2(1-\alpha)}{1-\theta^2} & \frac{\theta(2-\alpha)}{1-\theta^2} \\ \frac{\theta(2-\alpha)}{1-\theta^2} & -\frac{2}{1-\theta^2} \end{bmatrix}$.

Due to $\frac{\partial^2\pi_{ma}}{\partial p_1^2} = -\frac{2(1-\alpha)}{1-\theta^2} < 0$, $\frac{\partial^2\pi_{ma}}{\partial p_2^2} = -\frac{2}{1-\theta^2} < 0$ and $\frac{\partial^2\pi_{ma}}{\partial p_1^2}\frac{\partial^2\pi_{ma}}{\partial p_2^2} - \frac{\partial^2\pi_{ma}}{\partial p_1 p_2}\frac{\partial^2\pi_{ma}}{\partial p_2 p_1} = \frac{4(1-\alpha)-(2-\alpha)^2\theta^2}{(1-\theta^2)^2} > 0$. The corresponding Hessian matrix is negative definite.

Then, the best response prices functions can be obtained by solving from the first-order condition.

$$p_1 = \frac{2(1+e_p)(1-\alpha) + \alpha\theta - (1+e_p)(2-\alpha)\theta^2}{4(1-\alpha) - (2-\alpha)^2\theta^2}$$
$$p_2 = \frac{(1-\alpha)(2-(1+e_p)\alpha\theta - (2-\alpha)\theta^2)}{4(1-\alpha) - (2-\alpha)^2\theta^2}.$$

Substituting p_1 and p_2 into the platform's profit, we get

$$\pi_a(e_p) = \frac{\alpha(2(1+e_p)(1-\alpha) - (2-\alpha)\theta)(2(1+e_p)(1-\alpha) + \alpha\theta - (1+e_p)(2-\alpha)\theta^2)}{(4(1-\alpha) - (2-\alpha)^2\theta^2)^2} - (1-\eta)e_p^2.$$

The second derivative of e_p is $\frac{\partial^2\pi_a}{\partial e_p^2} = 2\left(-1+\eta+\frac{2\alpha(1-\alpha)(2(1-\alpha)-(2-\alpha)\theta^2)}{(4(1-\alpha)-(2-\alpha)^2\theta^2)^2}\right)$. By tedious calculation, the constraint which ensure $\frac{\partial^2\pi_a}{\partial e_p^2} < 0$ is $0 < \eta < \frac{4(4-\alpha)(1-\alpha)^2 - 2(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2 + (2-\alpha)^4\theta^4}{(4(1-\alpha)-(2-\alpha)^2\theta^2)^2}$.

So, we can obtain the optimal e_p^*. In summary, the unique equilibrium for case AC is:

$$e_p^* = \frac{\alpha(4(2-\theta) - (2-\alpha)(2(2-\theta)\theta^2 + \alpha(8-\theta(4+(4-\theta)\theta))))}{8(1-\alpha)^2(4-\alpha-4\eta) - 4(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + 2(2-\alpha)^4(1-\eta)\theta^4}$$

$$p_1^* = \frac{\begin{array}{c}16(1-\alpha)^2(1-\eta) + 2\alpha\theta(1-\alpha)(3-4\eta) - 4\theta^2(4-\alpha)(2-\alpha)(1-\alpha)(1-\eta)\\ +\alpha\theta^3(2-\alpha)(3-2\alpha(1-\eta)-4\eta) + 2\theta^4(2-\alpha)^3(1-\eta)\end{array}}{8(1-\alpha)^2(4-\alpha-4\eta) - 4(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + 2(2-\alpha)^4(1-\eta)\theta^4}$$

$$p_2^* = \frac{(1-\alpha)\left(2-(2-\alpha)\theta^2 - \alpha\theta\left(1+\frac{\alpha(4(2-\theta)-(2-\alpha)(2(2-\theta)\theta^2+\alpha(8-\theta(4-(4-\theta)\theta))))}{8(1-\alpha)^2(4-\alpha-4\eta)-4(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2+2(2-\alpha)^4(1-\eta)\theta^4}\right)\right)}{4(1-\alpha) - (2-\alpha)^2\theta^2}$$

$$D_1^* = \frac{8(1-\alpha)^2(1-\eta) - (1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta - 2(2-\alpha)^2(1-\alpha)(1-\eta)\theta^2 + (2-\alpha)^3(1-\eta)\theta^3}{4(1-\alpha)^2(4-\alpha-4\eta) - 2(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + (2-\alpha)^4(1-\eta)\theta^4}$$

$$D_2^* = \frac{(1-\alpha)(4(1-\alpha)(4-\alpha-4\eta) - 8(2-\alpha)(1-\alpha)(1-\eta)\theta - (2-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + 2(2-\alpha)^3(1-\eta)\theta^3)}{8(1-\alpha)^2(4-\alpha-4\eta) - 4(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + 2(2-\alpha)^4(1-\eta)\theta^4}$$

$$\pi_a^* = \frac{\alpha(16(1-\alpha)^2(1-\eta)(1-\theta) - (16-17\alpha+4\alpha^2-4(2-\alpha)^2\eta)\theta^2 + 4(2-\alpha)^2(1-\eta)\theta^3)}{16(1-\alpha)^2(4-\alpha-4\eta) - 8(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + 4(2-\alpha)^4(1-\eta)\theta^4}$$

$$\pi_{ma}^{*} = \frac{\begin{array}{c}16(1-\alpha)^4(32(1-\eta)^2 - 8\alpha(1-\eta)(3-2\eta) + \alpha^2(1-4\eta)) - 64(1-\alpha)^4(8-5\alpha-(16-(9-\alpha)\alpha)\eta + 4(2-\alpha)\eta^2)\theta \\ +4(1-\alpha)^3(\alpha(656-\alpha(367-68\alpha)) - 4\alpha(308-\alpha(169-31\alpha))\eta - 48(2-\alpha)^3\eta^2 - 384(1-2\eta))\theta^2 \\ -16(2-\alpha)(1-\alpha)^2(-2(2-\alpha)(1-\alpha)(12-7\alpha) + (96-\alpha(196-(130-29\alpha)\alpha))\eta - 12(2-\alpha)^2(1-\alpha)\eta^2)\theta^3 \\ +(2-\alpha)^2(1-\alpha)^2(384-640\alpha+339\alpha^2-56\alpha^3-8(3-\alpha)(32-\alpha(40-13\alpha))\eta + 48(2-\alpha)^3\eta^2)\theta^4 \\ +4(2-\alpha)^3(1-\alpha)(-48+\alpha(98-\alpha(63-13\alpha)) + 96\eta - \alpha(194-25(5-\alpha)\alpha)\eta - 12(2-\alpha)^2(1-\alpha)\eta^2)\theta^5 \\ +(2-\alpha)^4(-2(2-\alpha)(1-\alpha)(8-\alpha(9-2\alpha)) + (64-\alpha(164-\alpha(149-58\alpha+8\alpha^2)))\eta - 4(2-\alpha)^3(1-\alpha)\eta^2)\theta^6 \\ +4(2-\alpha)^7(1-\alpha)(1-\eta)^2\theta^7\end{array}}{4(4(1-\alpha)^2(4-\alpha-4\eta) - 2(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2 + (2-\alpha)^4(1-\eta)\theta^4)^2}.$$

For prices, demand and both parties' profits to remain nonnegative requires $0 < \eta \leq \frac{1}{8}\left(\frac{Z}{(2-3\alpha+\alpha^2)(1-\theta)(4-4\alpha-4\theta^2+4\alpha\theta^2-\alpha^2\theta^2)^3} - \sqrt{\frac{Y}{(2-3\alpha+\alpha^2)^2(1-\theta)^2(4-4\alpha-4\theta^2+4\alpha\theta^2-\alpha^2\theta^2)^6}}\right)$.

Where $Z = -8\alpha^8(1-\theta)\theta^6 + 1024(1-\theta)^4(1+\theta)^3 - 64\alpha(1-\theta)^3(1+\theta)^2(74+\theta-72\theta^2) - 2\alpha^7\theta^4(52-50\theta-61\theta^2+60\theta^3) + 16\alpha^2(1-\theta^2)^2(548-532\theta-573\theta^2+560\theta^3) - 32\alpha^3(1-\theta)^2(256+268\theta-547\theta^2-568\theta^3+299\theta^4+308\theta^5) - 4\alpha^5(224-208\theta-1048\theta^2+984\theta^3+1590\theta^4-1519\theta^5-751\theta^6+728\theta^7) + \alpha^6(64-64\theta-496\theta^2+464\theta^3+1256\theta^4-1200\theta^5-805\theta^6+784\theta^7) + 8\alpha^4(496-464\theta-1816\theta^2+1722\theta^3+2191\theta^4-2102\theta^5-867\theta^6+840\theta^7)$, $Y = \alpha^2(\alpha^2(8-4\theta-4\theta^2+\theta^3) - 4\alpha(4-2\theta-3\theta^2+\theta^3) + 4(2-\theta-2\theta^2+\theta^3))^2(-4\theta^8\theta^6(3-4\theta) + 256(7-8\theta)(1-\theta^2)^3 - 128\alpha(1-\theta)^3(1+\theta)^2(64-9\theta-72\theta^2) - 4\alpha^7\theta^4(40-50\theta-47\theta^2+60\theta^3) + 16\alpha^2(1-\theta^2)^2(932-1060\theta-975\theta^2+1120\theta^3) - 32\alpha^3(1-\theta)^2(424+368\theta-1066\theta^2-925\theta^3+701\theta^4+616\theta^5) - 16\alpha^5(80-88\theta-412\theta^2+476\theta^3+639\theta^4-756\theta^5-303\theta^6+364\theta^7) + \alpha^6(64-64\theta-752\theta^2+880\theta^3+1976\theta^4-2392\theta^5-1271\theta^6+1568\theta^7) + 8\alpha^4(784-880\theta-2948\theta^2+3382\theta^3+3595\theta^4-4186\theta^5-1427\theta^6+1680\theta^7))$.

Prove for Theorem 4.2

In the following, we divide two regions to investigate this problem, which can be shown in Figure A.1. The boundary between the two regions is $\eta = \frac{1}{64}\left(\frac{1513+12669\theta-14445\theta^2-17221\theta^3-576\theta^4-64\theta^5}{(3+\theta)^3(1-\theta^2)}\right) - \sqrt{\frac{(5+3\theta)^2(3001+25333\theta-28669\theta^2-34333\theta^3-11152\theta^4-128\theta^5)}{(1-\theta)(1+\theta)^2(3+\theta)^6}}$. In region (1), we will compare the equilibrium profits of cases WC and WN; in region (2), only case WN scenario is feasible, so the noncooperation strategy is the unique equilibrium.

Firstly, by comparing the platform's profits under region (1), we can easily get that case WC always outperforms case WN. Therefore, when cooperation strategy is feasible, the platform constantly prefers the manufacturer cooperate. Next, we pay attention to the manufacturer's optimal response. In region (1), by comparing the manufacturer's profits, we have that if and only if $\eta_1 < \eta \leq \frac{1}{64}\left(\frac{1513+12669\theta-14445\theta^2-17221\theta^3-576\theta^4-64\theta^5}{(3+\theta)^3(1-\theta^2)}\right) - \sqrt{\frac{(5+3\theta)^2(3001+25333\theta-28669\theta^2-34333\theta^3-11152\theta^4-128\theta^5)}{(1-\theta)(1+\theta)^2(3+\theta)^6}}$, case WN dominates case WC. Otherwise, case WC dominates case WN.

Prove for Section 5

In this situation, we will combine Figures 2 and 3 to compare the profit of the platform under the agency contract and wholesale contract in the intersection area. We take $\theta = 0.1$ for example to explain the selection process of the optimal distribution contract of the platform. Figure A.2a is the comparison of the platform's choice and Figure A.2b is the platform's optimal distribution contract.

Prove for Section 6.2

Case WN

We first pay attention to the interior solution when $p_2 > w$.

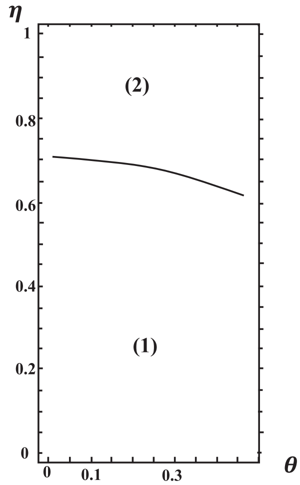
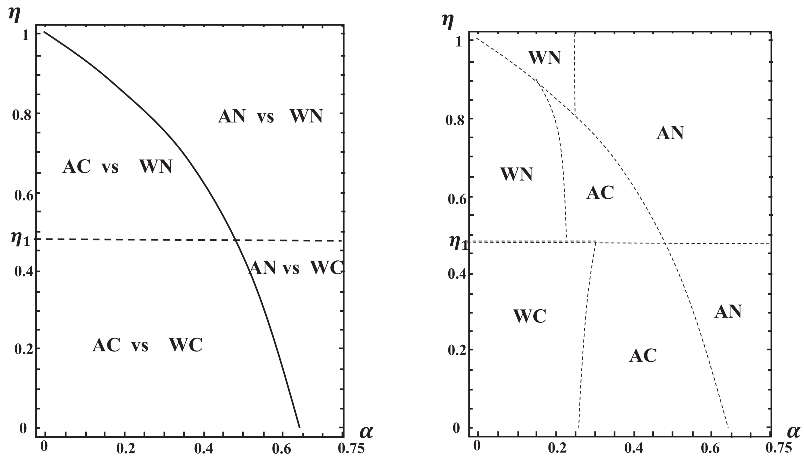


FIGURE A.1. Manufacturer's choice under wholesale contract.



(a) (b)

FIGURE A.2. Comparison of the platform's optimal distribution contract. (a) $\alpha = 0.1$. (b) $\alpha = 0.1$.

Given e_p, e_m, w, p_2 and θ, the platform's profit is concave with respect to p_1 because $\frac{\partial^2 \pi_r}{\partial p_1^2} = -\frac{2}{1-\theta^2} < 0$. The best response price function can be obtained by solving from the first-order condition.

$$p_1 = \frac{1 + e_p + w - (1 + e_m - p_2)\theta}{2}.$$

Then, substituting p_1 into manufacturer's profit function. And the manufacturer determines the wholesale price and direct channel price simultaneously. We find that the Hessian matrix of the manufacturer's profit function is $H = \begin{bmatrix} -\frac{1}{1-\theta^2} & \frac{\theta}{1-\theta^2} \\ \frac{\theta}{1-\theta^2} & -\frac{2-\theta^2}{1-\theta^2} \end{bmatrix}$, which is negative definite. So we can get the best response wholesale price and direct channel price $w = \frac{1+e_p}{2}$ and $p_2 = \frac{1+em}{2}$. To ensure $w \le p_2$, we have $e_p \le e_m$.

Substituting all the variables into the platform's profit, we get

$$\pi_r(e_p) = \frac{2e_p(1-\theta-e_m\theta)+(1-\theta-e_m\theta)^2-e_p^2(15-16\theta^2)}{16(1-\theta^2)}.$$

The second derivative of e_p is $\frac{\partial^2\pi_r}{\partial e_p^2} = -\frac{15-16\theta^2}{8(1-\theta^2)}$. It is concave when $15-16\theta^2>0$, which is always true when $\theta\in(0,\frac{1}{2}]$. So, we can obtain the best response advertising level $e_p = \frac{1-(1+e_m)\theta}{15-16\theta^2}$. Recalling the constraint that $w\le p_2$, which needs $e_p\le e_m$, we have the constraint of $e_m > \frac{1}{15+16\theta}$.

In the following, we pay attention to the manufacturer's response of e_m. Substituting all the variables into the manufacturer's profit and the second derivative of e_m is $\frac{\partial^2\pi_{mr}}{\partial e_m^2} = -\frac{675-224\theta^2(7-4\theta^2)}{2(15-16\theta^2)^2}$. It is concave when $\theta\in(0,\frac{1}{2}]$. Therefore, the optimal advertising level is $e_m^* = \frac{225-32\theta(4+\theta(11-4\theta(1+\theta)))}{675-224\theta^2(7-4\theta^2)}$, we can easily find that $e_m \ge \frac{1}{15+16\theta}$ is always satisfied. Therefore, there is only interior solution in case WN.

In summary, the unique interior equilibrium for case WN is:

$$\begin{aligned}
e_p^* &= \frac{(3-4\theta)(15-16\theta^2)}{675-224\theta^2(7-4\theta^2)}\\
w^* &= \frac{360-30\theta-808\theta^2+32\theta^3+448\theta^4}{675-224\theta^2(7-4\theta^2)}\\
p_1^* &= \frac{540-270\theta-1180\theta^2+528\theta^3+640\theta^4-256\theta^5}{675-224\theta^2(7-4\theta^2)}\\
p_2^* &= \frac{1}{2}\left(1+\frac{225-32\theta(4+\theta(11-4\theta(1+\theta)))}{675-224\theta^2(7-4\theta^2)}\right)\\
D_1^* &= \frac{4(3-4\theta)(15-16\theta^2)}{675-224\theta^2(7-4\theta^2)}\\
D_2^* &= \frac{450-4\theta(61+4\theta(5-4\theta)(9-4\theta))}{675-224\theta^2(7-4\theta^2)}\\
\pi_r^* &= \frac{(3-4\theta)^2(15-16\theta^2)^3}{(675-224\theta^2(7-4\theta^2))^2}\\
\pi_{mr}^* &= \frac{321-64\theta(4+\theta(7-2\theta(2+\theta)))}{675-224\theta^2(7-4\theta^2)}.
\end{aligned}$$

Both channel's demand are nonnegative.

Case WC

We first pay attention to the interior solution where $p_2 > w$.

Given e_p, e_m, w, p_2, η and θ, platform's profit is concave with respect to p_1 because $\frac{\partial^2\pi_r}{\partial p_1^2} = -\frac{2}{1-\theta^2} < 0$. The best response price function can be obtained by solving from the first-order condition.

$$p_1 = \frac{1+e_p+w-(1+e_m-p_2)\theta}{2}.$$

Then, substituting p_1 into manufacturer's profit function. And the manufacturer determines the wholesale price and direct channel's price simultaneously. We find that the Hessian matrix of the manufacturer's profit function is $H = \begin{bmatrix} -\frac{1}{1-\theta^2} & \frac{\theta}{1-\theta^2} \\ \frac{\theta}{1-\theta^2} & -\frac{2-\theta^2}{1-\theta^2}\end{bmatrix}$, which is negative definite. So we can get the best response wholesale price and direct channel price $w = \frac{1+e_p}{2}$ and $p_2 = \frac{1+e_m}{2}$. To ensure $w < p_2$, we have $e_p < e_m$.

Substituting all the variables into the platform's profit, we get

$$\pi_r(e_p) = \frac{2e_p(1-\theta-e_m\theta+(1-\theta-e_m\theta)^2-e_p^2(15-16\eta-16(1-\eta)\theta^2))}{16(1-\theta^2)}.$$

The second derivative of e_p is $\frac{\partial^2\pi_r}{\partial e_p^2} = -\frac{15-16\eta-16(1-\eta)\theta^2}{8(1-\theta^2)}$. It is concave when $15-16\eta-16(1-\eta)\theta^2>0$, which is when $0<\eta<\frac{15-16\theta^2}{16-16\theta^2}$.

So, we can obtain the best response advertising level $e_p = \frac{1-(1+e_m)\theta}{15-16\eta-16(1-\eta)\theta^2}$. Recalling the constraint that $w<p_2$ which needs $e_p<e_m$, we have the constraint of e_m is $e_m > \frac{1}{15+16\theta-16\eta(1+\theta)}$.

In the following, we pay attention to the manufacturer's response of e_m. Substituting all the variables into the manufacturer's profit and the second derivative of e_m is $\frac{\partial^2\pi_{mr}}{\partial e_m^2} = -\frac{3(15-16\eta)^2-4(392-\eta(809-416\eta))\theta^2+896(1-\eta)^2\theta^4}{2(15-16\eta-16(1-\eta)\theta^2)^2}$. It is concave when $3(15-16\eta)^2-4(392-\eta(809-416\eta))\theta^2+896(1-\eta)^2\theta^4>0$. Combining the above constraints, we get $0<\eta<\frac{15-16\theta^2}{16-16\theta^2}$.

Therefore, the best response advertising level of direct channel is

$$e_m^* = \frac{225+128\eta^2(1-\theta)^2(1+\theta)(2+\theta)-32\theta(4+\theta(11-4\theta(1+\theta)))-4\eta(1-\eta)(120+\theta(55-64\theta(2+\theta)))}{3(15-16\eta)^2-4(392-\eta(809-416\eta))\theta^2+896(1-\eta)^2\theta^4}.$$

To ensure $e_m>e_r$, we update the constraints as

(1) when $\theta\in(0,0.446], 0<\eta<\frac{360-809\theta^2+448\theta^4-\theta\sqrt{1521\theta^2-896\theta^4-624}}{64(6-13\theta^2+7\theta^4)}$ or $\frac{360-809\theta^2+448\theta^4+\theta\sqrt{1521\theta^2-896\theta^4-624}}{64(6-13\theta^2+7\theta^4)}<\eta<\frac{15-16\theta^2}{16-16\theta^2}$;

(2) when $\theta\in(0.446,\frac{1}{2}], 0<\eta<\frac{15-16\theta^2}{16-16\theta^2}$.

In summary, the unique interior equilibriums for case WC is:

$$e_p^* = \frac{(3-4\theta)(15-16\theta^2-16\eta(1-\theta^2))}{675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4)},$$

$$e_m^* = \frac{225+128\eta^2(1-\theta)^2(1+\theta)(2+\theta)-32\theta(4+\theta(11-4\theta(1+\theta)))-4\eta(1-\eta)(120+\theta(55-64\theta(2+\theta)))}{3(15-16\eta)^2-4(392-\eta(809-416\eta))\theta^2+896(1-\eta)^2\theta^4},$$

$$p_1^* = \frac{\begin{array}{l}2(270-135\theta-590\theta^2+264\theta^3+320\theta^4-128\theta^5-\eta(558-264\theta-1199\theta^2+520\theta^3+640\theta^4-256\theta^5)\\ \quad +32\eta^2(9-4\theta-19\theta^2+8\theta^3+10\theta^4-4\theta^5))\end{array}}{675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4)},$$

$$p_2^* = \frac{1}{2}\left(1+\frac{225+128\eta^2(1-\theta)^2(1+\theta)(2+\theta)-32\theta(4+\theta(11-4\theta(1+\theta)))-4\eta(1-\theta)(120+\theta(55-64\theta(2+\theta)))}{675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4)}\right),$$

$$w^* = \frac{360-30\theta-808\theta^2+32\theta^3+448\theta^4+64\eta^2(6-13\theta^2+7\theta^4)-2\eta(372-16\theta-821\theta^2+16\theta^3+448\theta^4)}{675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4)},$$

$$D_1^* = \frac{4(1-\eta)(3-4\theta)(15-16\eta-16(1-\eta)\theta^2)}{675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4)},$$

$$D_2^* = \frac{450+256\eta^2(1-\theta)^2(1+\theta)(2+\theta)-4\theta(61+4\theta(5-4\theta)(9+4\theta))-2\eta(480-\theta(251+8\theta(93-32\theta(1+\theta))))}{675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4)},$$

$$\pi_r^* = \frac{(1-\eta)(3-4\theta)^2(15-16\eta-16(1-\eta)\theta^2)^3}{(675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4))^2},$$

$$\pi_{mr}^* = \frac{321+32\eta^2(1-\theta)(1+\theta)(11-4\theta(2+\theta))-64\theta(4+\theta(7-2\theta(2+\theta)))-\eta(675-4\theta(130+\theta(231-64\theta(2+\theta))))}{675-1568\theta^2+896\theta^4+128\eta^2(6-13\theta^2+7\theta^4)-4\eta(360-809\theta^2+448\theta^4)}.$$

For both channel's demand and profits to remain nonnegative require the following constraints:

(1) when $\theta \in (0, 0.379], 0 < \eta \leq \frac{108+59\theta-112\theta^2-64\theta^3-\sqrt{144-184\theta-487\theta^2+224\theta^3+384\theta^4}}{64(2+\theta-2\theta^2-\theta^3)}$

(2) when $\theta \in (0.379, 0.446), 0 < \eta \leq \frac{108+59\theta-112\theta^2-64\theta^3-\sqrt{144-184\theta-487\theta^2+224\theta^3+384\theta^4}}{64(2+\theta-2\theta^2-\theta^3)}$ or $\frac{108+59\theta-112\theta^2-64\theta^3+\sqrt{144-184\theta-487\theta^2+224\theta^3+384\theta^4}}{64(2+\theta-2\theta^2-\theta^3)} \leq \eta$ $< \frac{675-520\theta-924\theta^2+512\theta^3+256\theta^4-\sqrt{3657-12\,848\theta+7960\theta^2+12\,736\theta^3-13\,662\theta^4+2048\theta^6}}{64(11-8\theta-15\theta^2+8\theta^3+4\theta^4)}$.

(3) when $\theta \in [0.446, \frac{1}{2}], 0 < \eta < \frac{675-520\theta-924\theta^2+512\theta^3+256\theta^4-\sqrt{3657-12\,848\theta+7960\theta^2+12\,736\theta^3-13\,662\theta^4+2048\theta^6}}{64(11-8\theta-15\theta^2+8\theta^3+4\theta^4)}$.

Next, we focus on the boundary solution when $p_2 = w$, which is also means $e_m = e_p$, thus $e_m^* = e_p^* = \frac{1}{15+16\theta-16\eta(1+\theta)}$. In summary, the unique boundary equilibrium for case WC is: $e_m^* = e_p^* = \frac{1}{15+16\theta-16\eta(1+\theta)}$, $w^* = p_2^* = \frac{1}{2}(1 + \frac{1}{15+16\theta-16\eta(1+\theta)})$, $p_1^* = \frac{4(1-\eta)(3-\theta)(1+\theta)}{15+16\theta-16\eta(1+\theta)}$, $D_1^* = \frac{4(1-\eta)}{15+16\theta-16\eta(1+\theta)}$, $D_2^* = \frac{4(1-\eta)(2+\theta)}{15+16\theta-16\eta(1+\theta)}$, $\pi_r^* = \frac{(1-\eta)(15-16\eta-16(1-\eta)\theta^2)}{(15+16\theta-16\eta(1+\theta))^2}$, $\pi_{mr}^* = \frac{95+32\eta^2(1+\theta)(3+\theta)+32\theta(4+\theta)-\eta(193+64\theta(4+\theta))}{(15+16\theta-16\eta(1+\theta))^2}$.

For both channel's demand to remain nonnegative require $0 < \eta < \frac{193+256\theta+64\theta^2-\sqrt{769+1024\theta+256\theta^2}}{64(3+4\theta+\theta^2)}$. It is well known that the internal solution must be superior to the boundary solution if the internal solution is feasible. Therefore, we exclude the interval of the internal solution and will get the pure interval of the boundary solution.

Case AN

Given e_p, e_m, α and θ, the Hessian matrix of the manufacturer's profit is $H = \begin{bmatrix} -\frac{2(1-\alpha)}{1-\theta^2} & \frac{\theta(2-\alpha)}{1-\theta^2} \\ \frac{\theta(2-\alpha)}{1-\theta^2} & -\frac{2}{1-\theta^2} \end{bmatrix}$.

Due to $\frac{\partial^2\pi_{ma}}{\partial p_1^2} = -\frac{2(1-\alpha)}{1-\theta^2} < 0$, $\frac{\partial^2\pi_{ma}}{\partial p_2^2} = -\frac{2}{1-\theta^2} < 0$ and $\frac{\partial^2\pi_{ma}}{\partial p_1^2}\frac{\partial^2\pi_{ma}}{\partial p_2^2} - \frac{\partial^2\pi_{ma}}{\partial p_1 p_2}\frac{\partial^2\pi_{ma}}{\partial p_2 p_1} = \frac{4(1-\alpha)-(2-\alpha)^2\theta^2}{(1-\theta^2)^2}$. The corresponding Hessian matrix is always negative definite when $0 < \alpha \leq \frac{3}{4}$. Then, the best response prices function can be obtained by solving from the first-order condition.

$$p_1 = \frac{2(1+e_p)(1-\alpha) + (1+e_m)\alpha\theta - (1+e_p)(2-\alpha)\theta^2}{4(1-\alpha) - (2-\alpha)^2\theta^2}$$
$$p_2 = \frac{(1-\alpha)(2(1+e_m) - (1+e_p)\alpha\theta - (1+e_m)(2-\alpha)\theta^2)}{4(1-\alpha) - (2-\alpha)^2\theta^2}.$$

Substituting p_1 and p_2 into the platform's profit, and the second derivative of e_p is $\frac{\partial^2\pi_a}{\partial e_p^2} = -2 - \frac{4(1-\alpha)\alpha(2-2\alpha-(2-\alpha)\theta^2)}{(4(1-\alpha)-(2-\alpha)^2\theta^2)^2}$. By simple calculation, we find that $\frac{\partial^2\pi_a}{\partial e_p^2} < 0$. So, we can obtain the best response advertising level $e_p = \frac{\alpha(4(2-\theta-\theta e_m)-(2-\alpha)(8\alpha-4(1+e_m)\alpha\theta+4(1-\alpha)\theta^2-(1+e_m)(2-\alpha)\theta^3))}{8(1-\alpha)^2(4-\alpha-4\eta)-4(2-\alpha)(1-\alpha)(8-5\alpha-4(2-\alpha)\eta)\theta^2+2(2-\alpha)^4(1-\eta)\theta^4}$. Substituting all the prices and advertising level into manufacturer's profit, and solve the second derivative of e_m can we have

$$\frac{\partial^2\pi_{ma}}{\partial e_m^2} = \frac{\begin{array}{c}-48(4-\alpha)^2(1-\alpha)^4 + 4(1-\alpha)^3(8-5\alpha)(104-\alpha(77-12\alpha))\theta^2 \\ -(2-\alpha)^2(1-\alpha)^2(1344-\alpha(1664-\alpha(557-24\alpha)))\theta^4 \\ +2(2-\alpha)^5(1-\alpha)(60-37\alpha)\theta^6 - 4(2-\alpha)^8\theta^8\end{array}}{2(4(4-\alpha)(1-\alpha)^2 - 2(2-\alpha)(1-\alpha)(8-5\alpha)\theta^2 + (2-\alpha)^4\theta^4)^2}.$$

By tedious calculation, we find that $\frac{\partial^2\pi_{ma}}{\partial e_m^2} < 0$ is always satisfied. So, we can get the optimal e_m^*. In summary, the unique interior equilibrium for case AN is:

$$e_m^* = (1-\alpha)\frac{\begin{array}{c}16(4-\alpha)^2(1-\alpha)^3 - 32(1-\alpha)^3(8-5\alpha)\theta - 4(1-\alpha)^2(8-5\alpha)(24-\alpha(19-4\alpha))\theta^2\\ +16(12-7\alpha)(2-3\alpha+\alpha^2)^2\theta^3 + (2-\alpha)^2(1-\alpha)(192-\alpha(256-\alpha(99-8\alpha)))\theta^4\\ -2(2-\alpha)^4(1-\alpha)(24-13\alpha)\theta^5 - 2(2-\alpha)^5(4-3\alpha)\theta^6 + 2(2-\alpha)^7\theta^7\end{array}}{F}$$

$$e_r^* = \alpha\frac{\begin{array}{c}48(4-\alpha)(1-\alpha)^4 - 32(4-\alpha)(1-\alpha)^4\theta - 4(1-\alpha)^3(152-\alpha(149-36\alpha))\theta^2\\ +8(2-\alpha)(1-\alpha)^2(24-\alpha(32-11\alpha))\theta^3 + (2-\alpha)^2(1-\alpha)^2(160-3\alpha(39-4\alpha))\theta^4\\ -4(4-\alpha)(2-\alpha)^3(1-\alpha)(3-2\alpha)\theta^5 - 7(2-\alpha)^5(1-\alpha)\theta^6 + 2(2-\alpha)^6\theta^7\end{array}}{F}$$

$$p_1^* = \frac{\begin{array}{c}96(4-\alpha)(1-\alpha)^4 + 48(4-\alpha)(1-\alpha)^3\alpha\theta - 8(1-\alpha)^3(208-\alpha(214-3(20-\alpha)\alpha))\theta^2\\ -16(2-\alpha)(1-\alpha)^2\alpha(18-(13-\alpha)\alpha)\theta^3 + 16(2-\alpha)^2(1-\alpha)^2(42-\alpha(35-6\alpha))\theta^4\\ +4(2-\alpha)^2(1-\alpha)\alpha(36-\alpha(43-13\alpha))\theta^5 - (2-\alpha)^4(1-\alpha)(120-\alpha(79-6\alpha))\theta^6\\ -2(2-\alpha)^5\alpha(3-2\alpha)\theta^7 + 4(2-\alpha)^7\theta^8\end{array}}{F}$$

$$p_2^* = (1-\alpha)\frac{\begin{array}{c}32(4-\alpha)^2(1-\alpha)^3 - 16(1-\alpha)^3(8+\alpha(7-3\alpha))\theta - 8(8-5\alpha)^2(4-\alpha)(1-\alpha)^2\theta^2\\ +4(2-\alpha)(1-\alpha)^2(48+\alpha(20-\alpha(34-3\alpha)))\theta^3\\ -4(2-\alpha)(1-\alpha)(4-3\alpha)(96-\alpha(120-\alpha(41-2\alpha)))\theta^4 - (2-\alpha)^3(1-\alpha)(48+\alpha(22-29\alpha))\theta^5\\ -2(2-\alpha)^4(8-5\alpha)(8-7\alpha)\theta^6 + 2(2-\alpha)^6(1+\alpha)\theta^7 + 4(2-\alpha)^7\theta^8\end{array}}{F}$$

$$D_1^* = 2\frac{\begin{array}{c}48(4-\alpha)(1-\alpha)^4 - 8(4-\alpha)(1-\alpha)^3(8-5\alpha)\theta - 12(2-\alpha)(1-\alpha)^3(24-(16-\alpha)\alpha)\theta^2\\ +4(2-\alpha)(1-\alpha)^2\left(96-\alpha(120-\alpha(41-2\alpha))\right)\theta^3 + 6(2-\alpha)^3(1-\alpha)^2(12-7\alpha)\theta^4\\ -6(2-\alpha)^4(1-\alpha)(8-5\alpha)\theta^5 - 3(2-\alpha)^6(1-\alpha)\theta^6 + 2(2-\alpha)^7\theta^7\end{array}}{F}$$

$$D_2^* = (1-\alpha)\frac{\begin{array}{c}32(4-\alpha)^2(1-\alpha)^3 - 16(1-\alpha)^3(32-\alpha(23-3\alpha))\theta - 24(4-\alpha)(2-\alpha)(1-\alpha)^2(8-5\alpha)\theta^2\\ +4(2-\alpha)^2(1-\alpha)^2(96-\alpha(62-3\alpha))\theta^3 + 4(2-\alpha)^2(1-\alpha)(96-\alpha(120-\alpha(41-2\alpha)))\theta^4\\ -(2-\alpha)^4(1-\alpha)(96-55\alpha)\theta^5 - 2(2-\alpha)^5(8-5\alpha)\theta^6 + 4(2-\alpha)^7\theta^7\end{array}}{F}$$

$$\pi_{ma}^* = (1-\alpha)\frac{\begin{array}{c}16(1-\alpha)^3(28-(20-\alpha)\alpha) - 64(1-\alpha)^3(8-5\alpha)\theta - 4(1-\alpha)^2(336-\alpha(560-\alpha(307-56\alpha)))\theta^2\\ +32(12-7\alpha)(2-3\alpha+\alpha^2)^2\theta^3 + (2-\alpha)^2(1-\alpha)(336-\alpha(544-\alpha(279-44\alpha)))\theta^4\\ -4(2-\alpha)^4(1-\alpha)(24-13\alpha)\theta^5 - (2-\alpha)^5(14-3(5-\alpha)\alpha)\theta^6 + 4(2-\alpha)^7\theta^7\end{array}}{F}$$

$$\pi_a^* = \alpha\frac{\begin{array}{c}-\alpha(1-\theta^2)(48(4-\alpha)(1-\alpha)^4 - 32(4-\alpha)(1-\alpha)^4\theta - 4(1-\alpha)^3(152-\alpha(149-36\alpha))\theta^2\\ +8(2-\alpha)(1-\alpha)^2(24-\alpha(32-11\alpha))\theta^3 + (2-\alpha)^2(1-\alpha)^2(160-3\alpha(39-4\alpha))\theta^4\\ -4(4-\alpha)(2-\alpha)^3(1-\alpha)(3-2\alpha)\theta^5 - 7(2-\alpha)^5(1-\alpha)\theta^6 + 2(2-\alpha)^6\theta^7)^2\\ +2(1-\theta^2)(48(4-\alpha)(1-\alpha)^4 - 8(4-\alpha)(1-\alpha)^3(8-5\alpha)\theta\\ -12(2-\alpha)(1-\alpha)^3(24-(16-\alpha)\alpha)\theta^2 + 4(2-\alpha)(1-\alpha)^2(96-\alpha(120-\alpha(41-2\alpha)))\theta^3\\ +6(2-\alpha)^3(1-\alpha)^2(12-7\alpha)\theta^4 - 6(2-\alpha)^4(1-\alpha)(8-5\alpha)\theta^5\\ -3(2-\alpha)^6(1-\alpha)\theta^6 + 2(2-\alpha)^7\theta^7)(96(4-\alpha)(1-\alpha)^4 + 48(4-\alpha)(1-\alpha)^3\alpha\theta\\ -8(1-\alpha)^3(208-\alpha(214-3(20-\alpha)\alpha))\theta^2 - 16(2-\alpha)(1-\alpha)^2\alpha(18-(13-\alpha)\alpha)\theta^3\\ +16(2-\alpha)^2(1-\alpha)^2(42-\alpha(35-6\alpha))\theta^4 + 4(2-\alpha)^2(1-\alpha)\alpha(36-\alpha(43-13\alpha))\theta^5\\ -(2-\alpha)^4(1-\alpha)(120-\alpha(79-6\alpha))\theta^6 - 2(2-\alpha)^5\alpha(3-2\alpha)\theta^7 + 4(2-\alpha)^7\theta^8)\end{array}}{(1-\theta^2)F^2}.$$

Where $F = 48(4-\alpha)^2(1-\alpha)^4 - 4(1-\alpha)^3(8-5\alpha)(104-\alpha(77-12\alpha))\theta^2 + (2-\alpha)^2(1-\alpha)^2(1344-\alpha(1664-\alpha(557-24\alpha)))\theta^4 - 2(2-\alpha)^5(1-\alpha)(60-37\alpha)\theta^6 + 4(2-\alpha)^8\theta^8$.

To keep all the prices and demand positive needs (1) when $\theta \in (0, 0.371]$ and $\alpha \in (0, \frac{3}{4}]$; (2) when $\theta \in (0.371, \frac{1}{2}]$ and $\alpha \in (0, \alpha_1]$, where α_1 is the second root of $-12+16\theta+12\theta^2-16\theta^3+(24-26\theta-24\theta^2+24\theta^3)\alpha+(-12+10\theta+15\theta^2-12\theta^3)\alpha^2+(-3\theta^2+2\theta^3)\alpha^3=0$.

The case AC is totally similar to the above calculations and is extremely tedious, which we omitted here and resort to the numerical calculation directly.

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