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DYNAMIC PRICING IN A TWO-ECHELON STOCHASTIC SUPPLY CHAIN FOR PERISHABLE PRODUCTS

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Abstract. Supply chain management of perishable products has to use some mechanisms to control the product waste amount. Dynamic pricing and cooperation of the chain members are some mechanisms which mitigate the waste amount. This paper studies the dynamic pricing problem of a perishable product supply chain with one manufacturer, one retailer, and two periods: production and selling periods. The problem considers price markdown policy to manage the total quality-dependent stochastic demand: dividing the selling period into two different terms and offering two selling prices. This paper analyzes the problem heuristically *via* Stackelberg and cooperation games. Obtained results demonstrate that the cooperation scenario allocates the maximum profits to the chain members and customers due to the least selling prices. Also, in the Stackelberg cases, both members gain higher profits under the manufacturer-led Stackelberg game; however, the retailer-led Stackelberg game represents lower selling prices and the greatest price markdowns which is profitable to customers.

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1. Introduction

In many industries, raw materials, intermediate goods and final products are perishable [1] such as vegetables, fruit, meat, blood, medicine and dairy products, which have a small shelf life [10]. They may get ruined during the storage period and transportation process due to pressure, damage, dryness, evaporation, decay, and spoilage. Approximately 10% of food products such as fresh products spoil before they are sold to customers [10]. This issue imposes several constraints on the supply chain different processes such as procurement, production planning, inventory management and distribution. For instance, in the yoghurt industry, perishability exists in every stage of the supply chain from the procurement process to the distribution stage. Because it happens in different situations, perishability has a rather fuzzy definition. Identifying a general perishable supply chain is difficult [1]. Perishability is defined as damage, spoilage, decay, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of product that leads to decreasing usefulness from the original one [41].

Perishable products' supply chain management is considered both crucial and challenging. It manages the risk that the product quality decreases as time passes by Bai *et al.* [8]. Subsequently, a definite expiration date known as the product shelf life is printed on the package. In order to track the real quality of perishable products dynamically, some advanced identifications and sensory technologies such as Ratio Frequency Identification

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Technology (RFID) and Time-Temperature Indicator (TTI) have been invented, and all of which leads to manage the waste amount of the supply chain effectively. In reality, the lower the products' quality is, the lower the demand will be. The literature has represented price discount policies and dynamic pricing approach to manage the demand and design a scheme against the product waste. Under these strategies, the product's price is adjusted according to its quality changes. This fact increases the demand due to the price elasticity effects; hence, the products will be sold during their shelf life [37].

Generally, in perishable products supply chain literature, the pricing problem in a supply chain is considered under different game scenarios: cooperation and non-cooperation scenarios. In the non-cooperation games, each supply chain member decides individually with the goal of maximizing its own profit. However, in the cooperation game, the chain members decide cooperatively with the aim of maximizing the chain profit. In fact, the cooperation game is the other strategy used in the literature to intensify the revenue of selling the products and mitigate the extra costs, especially holding and lost sales costs. This method causes the supply chain decisions to be made cooperatively, and thus, the double margins of the profit are omitted. Deciding cooperatively and centrally leads the supply chain to achieve maximum efficiency, which intensifies the members' profits [45].

Related literature considered a retailer that faces a quality and price dependent demand or a supply chain whose demand depends on the product's price and quality. They utilized a dynamic pricing approach with the aim of profit maximization [23, 37]. This study extends [37] and considers a perishable product supply chain with a stochastic demand, which depends on the products' price and quality, over two periods. In this research, the dynamic pricing approach is used under the three scenarios of the manufacturer/retailer-led Stackelberg and cooperation games to address the following key questions: (i) Which game can yield the best performance? (ii) How the cooperation game profit can be divided between the members? (iii) Which game offers the greatest price markdowns in the second selling term to customers?

This paper considers a supply chain consisting of one retailer and one manufacturer that produces perishable products like medicine and dairy products. The products can be deteriorated during every stage of the chain. The problem has two production and selling periods. During the selling period, demand is stochastic. It depends on the price and quality of the product. In the production period, the retailer orders the selling period's requirements to the manufacturer considering the selling period's demand, and then, the manufacturer produces the received orders. The products are sold by the retailer during the selling period which is divided into two different terms; in the second selling term, the products are sold at a lower price. In order to provide an appropriate holding condition for perishable products, the retailer stores the intended products using the manufacturer's refrigerator used during the production period in order to keep the raw materials (*e.g.*, unpasteurized milk). Due to offering the products at two different prices in the selling period, the study develops a dynamic pricing mechanism. Subsequently, it solves the proposed problem using the manufacturer-led Stackelberg, retailer-led Stackelberg and cooperation games. The outcomes prove that the Nash bargaining model divides the cooperation game profit between the chain members such that they maximize their profits. Because the cooperation game offers the least selling prices, it is the most beneficial game to customers. The manufacturer-led Stackelberg game is more profitable to the chain members because it offers a higher selling price whereas the retailer-led Stackelberg scenario is more beneficial to customers. Because it yields the lower selling prices and the greatest price markdowns. Key contributions of this study are as follows: (1) This study analyzes the dynamic pricing problem of a two-echelon supply chain of perishable products under the stochastic demand. (2) The manufacturer/retailer-led Stackelberg and cooperation games are surveyed and optimal solutions are obtained upon heuristic methods. (3) The Nash bargaining model is developed to divide the cooperation game profit between the chain members.

The remainder of the study is organized as follows: Section 2 reviews the related literature. Section 3 represents notations and assumptions and then develops demand functions and mathematical models of the retailer and manufacturer. Section 4 analyzes the Stackelberg and cooperation game scenarios. Subsequently, it yields the numerical example. Section 5 discusses the sensitivity analysis of the problem. Additionally, it presents some managerial insights. Section 6 concludes the study.

2. Literature review

Three streams of the literature relevant to this study are represented: product perishability, pricing in the supply chain of perishable/deteriorating products and discount policy.

2.1. Product perishability

Product perishability is a topic that has received substantial coverage in the field of operations management. These products have limited shelf lives. Perishable products have either fixed [1, 11, 18, 20, 24, 26, 43] and random lifetimes [3–5, 16, 31, 36, 44]. Fixed lifetime of these products is determined beforehand, and the effects of deteriorating factors are considered when fixing it. Therefore, the utility of these products decreases over their lifetimes until the products perish completely and have no value to customers, *e.g.*, milk, yogurt, and blood in inventory. Random lifetime products have no determined lifetime. The lifetime of these products is considered a random variable, and it has a given probability distribution function. Fruits and electronic products can be considered as examples of these products. These streams of existing research assume that the demand declines as the product ages. The last decade confronts a rapid pace of technological developments, and customers are willing to use innovative products including high technology products (laptops, smartphones, software products, etc.) [15].

2.2. Pricing in the supply chain of the perishable/deteriorating products

Pricing is one of the operational decisions in supply chain of perishable/deteriorating items. Xiao and Xu [38] regarded a deteriorating product supply chain with one supplier and one retailer under VMI approach, and they proposed a Stackelberg game model. They investigated the coordination problem of the price and service level decisions using a generalized revenue sharing contract. Bai *et al.* [7] considered a two-echelon supply chain system for the deteriorating items where shortages are not allowed. They coordinated the selling price and promotional effort cost decisions using the revenue sharing as well as the revenue and cost sharing contracts. Zhang *et al.* [46] investigated a supply chain model for the deteriorating products and proposed a revenue sharing and cooperative investment contract to coordinate the price and preservation technology investment heuristically. Bai *et al.* [9] analyzed the pricing and coordination problem of the deteriorating items supply chain under the carbon cap-and-trade regulation. They gained the coordination conditions of the manufacturer-led Stackelberg problem using the revenue and promotional cost sharing as well as two-part tariff contracts. Maihami *et al.* [32] developed a pricing and inventory control model for a two-stage supply chain of the deteriorating items using the manufacturer-led Stackelberg game.

He *et al.* [21] analyzed the pricing and inventory decisions of a two-stage dual-channel supply chain system under the manufacturer-led Stackelberg model. They studied the coordination conditions of the chain decisions using the revenue sharing as well as two-part tariff contracts. Huang *et al.* [22] examined the pricing, inventory, and production reliability decisions of a two-stage supply chain under the retailer-led Stackelberg game. They coordinated the chain using cooperative investment plus revenue sharing contract. Yang *et al.* [42] considered two competitive supply chains under the carbon cap-and-trade scheme in the vertical direction (manufacturer-led Stackelberg game) and horizontal direction (Nash game) scenarios. They coordinated the pricing and carbon emissions rate decisions using the revenue sharing contract. Tiwari *et al.* [35] studied a two-echelon supply chain model for the deteriorating items considering the warehouse capacity limitation assumption. They analyzed the problem under three unintegrated and integrated policies utilizing heuristic methods. Bai *et al.* [10] considered the pricing problem of a supply chain with deteriorating items under the carbon cap-and-trade rule and VMI system. They gained the coordination condition by the revenue sharing contract. Maihami *et al.* [30] considered the inventory control and pricing issues in a three-echelon supply chain for the deteriorating items, and analyzed one integrated and three unintegrated games using heuristic algorithms. Mahmoodi [29] analyzed the joint pricing and replenishment issues of a supply chain of the deteriorating products, and solved the manufacturer-led Stackelberg model exactly and heuristically. Lou *et al.* [28] considered a two-echelon supply chain pricing and ordering decisions using the Stackelberg game and the cooperation scenario. Yan *et al.* [39] considered the pricing and coordination problem of a dual-channel supply chain under the demand disruption using the revenue sharing contract. Fu *et al.* [17] analyzed the quality and pricing decisions in a supply chain including two manufacturers and one retailer under four game scenarios considering two situations *i.e.*, the manufacturer-dominant as well as the retailer-dominant cases. Giallombardo *et al.* [19] tried to integrate the harvesting, storage, and distribution activities of an agricultural company dealing with perishable products. Subsequently, they proposed a horizontal collaboration between heterogeneous agricultural companies for the distribution phase to gain cost savings.

Other researches have analyzed the pricing problem dynamically. Chen and Wei [14] examined the dynamic joint decisions of replenishment, price, and revenue sharing allocation in a vertical decentralized supply chain of the deteriorating goods over the multi-period planning horizon and under single-manufacturer Stackelberg game. They coordinated the chain using three agreements: price-only method, revenue sharing mechanism, as well as revenue sharing plus linear rebate and side payment contract. Jia and Hu [23] studied a combined problem of pricing and ordering for a perishable product supply chain over a finite time horizon. Assuming that the lifetime of the product was two periods and the demand in each period was random and time-sensitive, the problem was solved using the manufacturer-led Stackelberg game. Bai *et al.* [8] analyzed a supply chain model for the deteriorating items over the finite planning horizon considering a multi-factor dependent demand. They coordinated the selling price and promotional effort cost using the revenue sharing as well as the revised revenue sharing contracts. Buratto *et al.* [13] investigated the pricing and the quality investment issues of a dynamic supply chain under the manufacturer-led Stackelberg game, and they analyzed the chain coordination using two mechanisms: a cooperative advertising program and a price discount approach. Lou *et al.* [27] considered a perishable product supply chain under the price and utility dependent demand over the finite time horizon, and they analyzed the Stackelberg and the cooperation scenarios.

2.3. Discount policy

A price discount policy is a method that encourages the sale of perishable products. Thangam [33] investigated the discounting and lot-sizing policies in a perishable product supply chain under an advanced payment. Buhayenko *et al.* [12] considered the problem of supply chain coordination using temporary price discount. They assumed that the supplier decides how much discount should be introduced in each period to every customer not only to maximize its profit but also to give to customers the incentive to order in the desired periods. They introduced a variable neighborhood search algorithm as a solution method. Azadi *et al.* [6] considered the pricing and replenishment problems of a perishable products supply chain consisting of two suppliers and one retailer. They utilized the benders decomposition approach as a solution method.

Table 1 summarizes some relevant articles below.

3. Mathematical model

This study analyzes a supply chain with one retailer and one manufacturer that produces perishable products. The assumptions of the problem are defined as follows:

(1) The problem has two production and selling periods. In the production period, after the retailer orders its requirements regarding the selling period's total stochastic demand, the manufacturer produces the products at production cost c_p per unit product, and sells them to the retailer at wholesale price w per unit product. The retailer sells the products to customers in the selling period.

(2) As Figure 1 demonstrates, the selling period covers a time period $[0, T]$. It is assumed that, during the selling period and at time T_m, the retailer reorders the products to the manufacturer to replenish them for the next selling period, and the products are delivered to the retailer after the end of the selling period. On the other side, total demand of the selling period is assumed to be the product's quality as well as price-dependent and stochastic. Therefore, the intended total demand decreases over time. To control the demand decrease and product waste, and in order to sell the products during the selling period and before the replenishment, the retailer applies a price markdown to the unsold products at time T_m. As a result, the selling period is divided

TABLE 1. Summary of the recent related literature.

Year	Article	Demand function		Levels of SC		Game Theory				Mathematical model		Collaboration mechanism		Solution approach		Price discount
		D	S	2	3	NS	RS	MS	CP	ST	DY	CP	CT	E	H	
2011	Jia and Hu [23]	*		*				*			*			*		*
2012	Thangam [33]	*												*		*
2012	Chen and Wei [14]	*		*				*			*		*		*	
2013	Xiao and Xu [38]	*		*				*		*			*	*		
2015	Bai *et al.* [7]	*		*				*		*			*	*		
2015	Zhang *et al.* [46]	*		*				*		*			*		*	
2016	Bai *et al.* [8]	*		*				*		*			*	*		
2017	Bai *et al.* [9]	*		*				*		*			*	*		
2017	Maihami *et al.* [32]		*	*				*		*					*	
2018	He *et al.* [21]	*		*				*	*	*		*			*	
2018	Huang *et al.* [22]	*		*			*		*	*		*	*		*	
2018	Tiwari [35]	*		*		*	*	*	*	*		*			*	
2018	Buhayneko *et al.* [12]		*	*				*			*				*	*
2019	Bai *et al.* [10]	*		*				*		*			*	*		
2019	Azadi *et al.* [6]		*											*		*
2019	Maihami *et al.* [30]		*		*	*	*	*	*	*		*		*	*	
2019	Buratto *et al.* [13]		*	*				*			*		*	*		*
2020	Mahmoodi *et al.* [29]	*		*				*						*	*	
2020	Lou *et al.* [27]		*	*				*	*		*	*		*		
2021	Yan *et al.* [39]	*		*				*		*			*	*		
2021	Fu *et al.* [17]	*		*			*	*		*	*			*		
Present paper			*	*				*	*		*	*		*	*	

Notes. * = Covered, D = Deterministic, S = Stochastic, 2 = Two-level, 3 = Three-level, NS = Nash, RS = Retailer Stackelberg, MS = Manufacturer-Stackelberg, CP = Cooperation, CT = Contract design, E = Exact, H = Heuristic.

into two distinct terms. That is why, in the production period, the retailer determines the requirements of each selling term separately and orders the total amount to the manufacturer.

(3) It is supposed that since the manufacturer keeps its raw materials (for instance unpasteurized milk) by a refrigerator, it can store the retailer's total order by the intended refrigerator. Therefore, the retailer pays the holding cost to the manufacturer and keeps these products appropriately. Thus, the problem considers the integrated holding system.
(4) The product replenishment is ignored in the problem modelling and this problem considers one cycle of the production and selling periods.
(5) The inventory level of the retailer at the start of the selling period ($t = 0$) equals its total order level which is calculated and ordered to the manufacturer in the production period. The unsold products in the first selling term are transferred to the second selling term.
(6) The unsold products at the end of the second selling term are supposed to have no salvage value.
(7) Backlogged orders and lost sales do not exist in the problem.
(8) Purchase and holding costs of the raw materials of a product are considered over calculating the production cost.

The retailer should determine the selling prices and total order level, and the manufacturer should decide the wholesale price.

The notations of the problem are as follows:

Indexes

r	Retailer's index
m	Manufacturer's index
i	Selling term index ($i = 1, 2$)

t	Time (shelf life) index ($t = 0, \ldots, T$)

Parameters

α	Elasticity effect coefficient of the price on the demand
λ	Breakdown rate
q_0	Product's first quality
B	Product's quality effect coefficient on the demand
T_m	Time of the selling period at which the retailer reorders to the manufacturer and the price markdown occurs
$f(d_t)$	Demand of time t
p_t	Time-dependent selling price
c_p	Production cost of unit product
i_1	Inventory amount of the retailer in the first selling term
Q_i	Order level of the retailer for the selling term i
Q	Total order level of the retailer
D_0	Potential market demand
D_i	Deterministic demand of the retailer in the selling term i
ED_i	Total demand of the retailer in the selling term i
$\mathrm{ED}_{\mathrm{Total}}$	Total demand of the selling period
ε_i	Stochastic element of the demand in the selling term i
ε_{it}	Stochastic element of the tth time in the selling term i
h_m	Holding cost of the unit product per unit shelf life in the centralized scenario (and also holding cost of the manufacturer)
h_r	Holding cost of the unit product per unit shelf life in the decentralized scenario which the retailer pays to the manufacturer
$U(e, f)$	Uniform distribution function of the stochastic element of the demand in the first selling term
$U(ue, uf)$	Uniform distribution function of the stochastic element of the demand in the second selling term

Profit functions

Π_m	Manufacturer's profit
Π_r	Retailer's profit
Π_c	Supply chain total profit

Decision variables

p_1	Selling price of the retailer in the first selling term
P_2	Selling price of the retailer in the second selling term
z	Stocking factor of the retailer
w	Wholesale price of the manufacturer

Figure 1 shows the supply chain of the problem.

3.1. Stochastic demand functions

The quality prediction of a perishable product is a complex issue. As the freshness level of perishable products such as fruits and vegetables is the most important quality factor [34], a constant quality deterioration factor is defined which is obtained by the observation or experiment.

The deterministic demand at time t is calculated in the following [23, 37]

$$f(d_t) = D_0 - \alpha p(t) + \beta q_0 e^{-\lambda t} \tag{3.1}$$

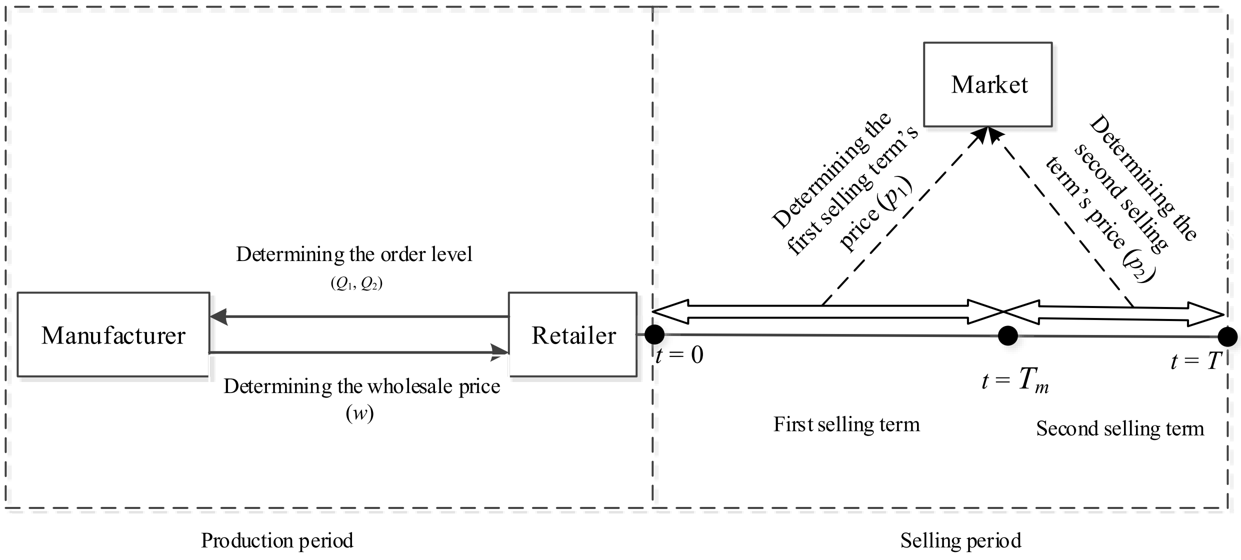


FIGURE 1. The structure of the intended supply chain.

where $p(t)$ is the time-dependent selling price which is affected by the product's quality. The product's quality decreases exponentially as time passes by. The total demand function at time t which is composed of the deterministic demand function and the stochastic element is as:

$$Ed_t = f(d_t) + \varepsilon_t. \tag{3.2}$$

Therefore, the total demand over a period of time can be written as follows:

$$\mathrm{ED}(T) = \int_0^T (f(d_t))\,\mathrm{d}t + \sum_0^T \varepsilon_t \tag{3.3}$$

where, $\mathrm{ED}(T)$ is the total demand during the selling period $[0, T]$.

As this study assumes two selling terms, a price markdown occurs at time T_m, and therefore, the first selling term includes the time period $[0, T_m]$ and the second selling term covers the time range $[T_m, T]$. It is worth mentioning that in this study, t is the shelf life of the product. Now, the total demand functions of the two selling terms which are composed of the deterministic demand function (D_i) and a stochastic element (ε_i) are calculated as:

$$\mathrm{ED}_1 = (D_0 - \alpha p_1)T_m + \beta q\left(1 - e^{-\lambda T_m}\right)/\lambda + \varepsilon_1, \varepsilon_1 = \sum_0^{T_m} \varepsilon_{1t} \tag{3.4}$$

$$\mathrm{ED}_2 = (D_0 - \alpha p_2)(T - T_m) + \beta q\left(1 - e^{-\lambda T_m} - e^{-\lambda T}\right)/\lambda + \varepsilon_2, \varepsilon_2 = \sum_{T_m+1}^{T} \varepsilon_{2t}. \tag{3.5}$$

As equations (3.4) and (3.5) shows, in the deterministic demand functions, the first and second expressions present the effects of the price elasticity and the product's quality on the potential market demand.

It is assumed that the distribution functions of ε_{1t} $(t = 0, \ldots, T_m)$ are identical. Their distribution and cumulative functions are $y(\cdot)$ and $Y(\cdot)$. Due to the identical distribution functions, sum of ε_{1t} is shown as ε_1.

Similar to the first selling term, the distribution and cumulative functions of ε_{2t} $(t = T_m, \ldots, T)$ of the second selling term are the same and they are shown as $k(\cdot)$ and $K(\cdot)$.

Total demand of the selling period is as:

$$\mathrm{ED}_{\mathrm{total}} = \mathrm{ED}_1 + \mathrm{ED}_2. \tag{3.6}$$

3.2. Retailer's profit function

Dynamic pricing strategy is used to stimulate the demand in the second selling term, wherein the product's quality is lower. Therefore, in the first and second selling terms, the selling prices are p_1 and p_2 per unit product such that $p_1 \geq p_2$. Thus, the retailer orders its total requirement, which is the sum of the first and second selling terms' order levels $(Q = Q_1 + Q_2)$, to the manufacturer in the production period. The selling period encompasses a time range $[0, T]$ and the inventory of the retailer at the beginning of the selling period $(t = 0)$ is Q. As the selling period consists of two terms and the problem is dynamic, the unsold products in the first selling term are transferred to the second selling term. Therefore, the first selling term's inventory can be formulated as:

$$I_1 = (Q - \mathrm{ED}_1)^+. \tag{3.7}$$

Now, the second selling term's inventory is:

$$I_2 = (I_1 - \mathrm{ED}_2)^+. \tag{3.8}$$

Regarding the assumption that the manufacturer uses the refrigerator in the production period to keep its raw materials in a proper condition, and in the selling period, the retailer can hold its orders by this refrigerator, the integrated holding system is considered. According to this holding system type, the retailer pays the holding cost h_r per unit product and per unit shelf life to the manufacturer. The holding costs of the retailer in both selling terms which are paid to the manufacturer are as:

$$\mathrm{TOH}_{r1} = h_r\left[\int_{t=0}^{t=T_m} (Q - \mathrm{ED}_1(t))^+ \,\mathrm{d}t\right] = h_r\big[QT_m - (D_0 - \alpha p_1)T_m - \beta q\big(1 - e^{-\lambda T_m}\big)/\lambda - \varepsilon_1\big] \tag{3.9}$$

$$\mathrm{TOH}_{r2} = h_r\left[\int_{T_m}^{T} (I_1 - \mathrm{ED}_2(t))^+ \,\mathrm{d}t\right] = h_r\big[I_1(T - T_m) - \big((D_0 - \alpha p_2)(T - T_m) - \beta q\big(1 - e^{-\lambda T_m} - e^{-\lambda T}\big)/\lambda$$

$$- \ (T - T_m)\varepsilon_2)]p(I_1 \geq \mathrm{ED}_2) \tag{3.10}$$

$$\mathrm{TOH}_r = h_r[QT_m - \mathrm{ED}_1 + (I_1(T - T_m) - \mathrm{ED}_2)p(I_1 \geq \mathrm{ED}_2)]. \tag{3.11}$$

According to the problem definition, the selling period has two terms and in the production period, the retailer orders Q_1 and Q_2 units of the products for the first and second selling terms. Despite the fact that the demand of the first selling term is stochastic and the retailer is willing to sell its products during the first selling term, the inventory at the end of the first selling term is positive most probably and an amount of the products are transferred to the second selling term. Hence, the inventory at the end of the first selling term is assumed to be positive always, and the probability of negative inventory of the retailer at the end of the first selling term is neglected.

Regarding the stochastic demands of the selling terms, the retailer's profit can be yielded as:

$$\mathrm{Max}(\Pi_r) = p_1\mathrm{ED}_1 + p_2 \min(I_1, \mathrm{ED}_2) - wQ - \mathrm{TOH}_r. \tag{3.12}$$

In equation (3.12), the first and second expressions indicate the revenue of selling the products in the first and second selling terms. The third expression presents the purchase cost of the products from the manufacturer. The fourth expression calculates the holding cost of the retailer.

To simplify equation (3.12), a stocking factor as $z_i = Q_i - \mathrm{ED}_i$, is defined, and afterwards, the stocking factors of the first and second selling terms are assumed to be equal ($z_1 = z_2$). Subsequently, the total order level (Q) can be displayed as:

$$Q = Q_1 + Q_2 = D_1 + D_2 + 2z. \tag{3.13}$$

In order to simplify the problem, the stochastic element of the second selling term's demand is assumed to be a multiple of that of the first selling term ($\varepsilon_1 = u\varepsilon_2$). Since ε_1 is composed of ε_{1t}, ($t = 1, \ldots, T_m$), therefore, ε_{1t} must follow a distribution function such that the distribution function of ε_1 follows a known and specific one, and additionally, the distribution functions of ε_{1t} and ε_{2t} are common and they have a rational relationship. Therefore, uniform, normal, exponential and Gama can be used as the distribution function of ε_{1t}.

Replacing equation (3.13) with Q, equation (3.12) is rewritten as follows:

$$\mathrm{Max}(\Pi_r) = p_1\mathrm{ED}_1 + p_2(D_2 + \min(2z, \varepsilon_1 + \varepsilon_2)) - wQ - \mathrm{TOH}_r. \tag{3.14}$$

3.3. Manufacturer's profit function

The manufacturer receives the retailer's total order in the production period, and subsequently, it launches producing perishable products at production cost c_p per unit product and sells them to the retailer at wholesale price w per unit product. In the selling period, the manufacturer also holds the intended order at holding cost h_r per unit product and per unit shelf life. It is worth noting that the holding cost for the manufacturer is $h_m(h_m < h_r)$ per unit product and per unit shelf life. Broadly speaking, the manufacturer efforts to maximize its profit function by optimizing the wholesale price as:

$$\mathrm{Max}(\Pi_m) = (w - c_p)Q + (h_r - h_m)[QT_m - \mathrm{ED}_1 + (I_1(T - T_m) - \mathrm{ED}_2)p(I_1 \geq \mathrm{ED}_2)]. \tag{3.15}$$

In equation (3.15), the first expression displays the manufacturer's profit, which is obtained by selling the products to the retailer. The second expression computes the profit of holding the retailer's total order over the selling period.

4. Analysis of the problem under different power scenarios

Now, the developed problem in the previous section is analyzed under the three games: the manufacturer/retailer-led Stackelberg and the cooperation games. The results are represented below.

4.1. Manufacturer-led Stackelberg game

In this section, the manufacturer-led Stackelberg game is considered, which has two stages: the manufacturer determines the value of w in the first stage, and the retailer decides p_1, p_2, and z in the second stage regarding the value of w. The utilized solution method is the backward induction. Equation (3.14) indicates that the power of z is three and this profit function is not quadratic. Therefore, proving the concavity of equation (3.14) and obtaining the best responses of the variables are difficult. Hence, a heuristics method is needed to obtain the values of the decision variables. To solve the problem, we expand min $(\varepsilon_1 + \varepsilon_2, 2z)$ of equation (3.14). Hence, the distribution functions of ε_1 and ε_2 should be determined. That is why, the uniform distribution function is supposed [2]. It is assumed that the distribution functions of the first and second selling terms are $U(e, f)$ and $U(ue, uf)$. Due to the fact that these two distributions must include z, the following condition is defined as:

$$\begin{cases} z \in [ue, f], u > 1 \\ z \in [e, uf], u < 1. \end{cases} \tag{4.1}$$

In this problem, it is supposed that the value of u is lower than 1.

Since the uniform distribution function is used for ε_1 and ε_2, the distribution function of $\varepsilon_{1+}\varepsilon_2$ is $U((u+1)e, (u+1)f)$.

4.1.1. Heuristic method

The basis of the method is that an initial value for w is generated at first. Then, in every iteration, after generating w as the variable of the first stage in a definite range randomly, in the second stage, some values for z are generated randomly, and then, for every value of z, the amounts of p_1 and p_2 are calculated using equations (4.2) and (4.3). The algorithm repeats iteratively unless the differences between the present objective amounts of both members and their corresponding objective values, which have been obtained in the last previous iteration, are less than ε. The steps are detailed as follows:

Step 1. Randomly generate a value for w_0, which should be greater than c_p, and set $n = 1$. Then, set e as the value of z and calculate the amounts of p_1 and p_2 using equations (4.2) and (4.3) and obtain the amount of the retailer's objective function.
Giving the value of z, equation (3.14) is concave regarding p_1 and p_2 and the values of p_1 and p_2 in equation (3.14) can be obtained using equations (4.2) and (4.3) as follows:

$$p_1 = \frac{D_0 T_m + \beta q\left(1 - e^{-\lambda T_m}\right)/\lambda - w\frac{\partial D_1}{\partial p_1}}{-2\frac{\partial D_1}{\partial p_1}} \tag{4.2}$$

$$p_2 = \frac{\begin{array}{c} D_0(T - T_m) + \beta q\left(1 - e^{-\lambda T_m} - e^{-\lambda T}\right)/\lambda + \min(2z, \varepsilon_1 + \varepsilon_2) \\ -(w + h_r T_m - h_r(1 + T - T_m)p(2z \geq \varepsilon_1 + \varepsilon_2))\frac{\partial D_2}{\partial p_2} \end{array}}{-2\frac{\partial D_2}{\partial p_2}}. \tag{4.3}$$

Appendix A represents the proof.

Step 2. Randomly generate an amount for w in the range $(w^{n-1},\ w^{n-1} + \Delta_w)$.
In this step, w^{n-1} is called the upper bound of the previous range, and n is called the numerator of the iterations. Thus, $n - 1$ and n are the previous and present iterations.

Step 3. In the range $(z_0,\ z_0 + \Delta z)$, randomly generate a number for z. Then, add Δz to each bound of the range and choose a random number for z again. Repeat this process until the upper bound of the intended range reaches uf and set $b = 1$.
Regarding equation (4.1), the lower bound of z is e and its upper bound is uf. The number of z generated during step (3) is shown by B and its value is the same as the iterations of this step.

Step 4. For $z_b(b = 1, \ldots, B)$ and the value of w obtained from step (2), compute the optimum values of p_1 and p_2 using equations (4.2), (4.3), and obtain the amount of the retailer's objective function. Calculate the difference between the retailer's objective function gained in the last previous iteration and its present objective function. If the difference is lower than ε, the amounts of p_1, p_2, and z are the best-obtained solutions, and go to step (5), else, go to step (6).

Step 5. For w, p_1, p_2, and z of step (4), calculate the manufacturer's objective function, and then, compute the difference between the present objective function with the intended profit obtained in the last previous iteration. If the computed difference is less than ε, stop. The values of w, p_1, p_2, and z, which are pointed at the beginning of this step, are the obtained Stackelberg point. Else go to step (6).

Step 6. If $b < B$, set $b = b + 1$ and go back to step (4). If $b = B$, add Δ_w to every bound of w in step (2), set $n = n + 1$, and go back to step (2).

4.2. Retailer-led Stackelberg game

In this game, the retailer is the leader of the game, and the manufacturer is the follower. The retailer determines its decision variables and subsequently, the manufacturer decides the wholesale price. According to the backward approach, after obtaining the manufacturer's decision variable, the retailer's decision variables are obtained. Xie and Neyret [40] developed a static pricing mathematical model and under the retailer-led Stackelberg game, they considered a constraint which said that the manufacturer's margin could not be greater than that of the retailer. The margin of the manufacturer was assumed to be the same as that of the retailer.

In our problem, the price markdown is happened, and the retailer's margin of a number of products is $(p_1 - w)$ and that of an unknown amount of them is $(p_2 - w)$. As the margin of the retailer for any product should not be lower than that of the manufacturer, the value of w is defined as follows:

$$p_2 - w \geq w - c_p \Rightarrow w \leq \frac{p_2 + c_p}{2} \Rightarrow w = \frac{p_2 + c_p}{2} + \Delta, \Delta > 1. \tag{4.4}$$

Due to the fact that equation (3.15) is increasing by w, the amount of w equals $((p_2 + c_p)/2) + \Delta$. The reason for adding Δ, which is a small number, is that the amount of w for some products is $((p_1 + c_p)/2)$, thus, the actual value of w is within the range $[((p_2 + c_p)/2), ((p_1 + c_p)/2)]$. Δ eliminates the gap between the actual value of w and $((p_2 + c_p)/2)$.

In this section, the heuristic method, which was described in Section 4.1.1, is used. The algorithm generates initial values for p_1, p_2 and z at first. Then, in every iteration, after generating p_1 and p_2, some values for z are generated randomly, and then, for every value of z, equation (4.4) calculates w. The algorithm repeats iteratively unless the differences between the present objective amounts of both members and their corresponding objective values, which have been obtained in the last previous iteration, are less than ε. The steps were represented in Section 4.1.1.

4.3. Cooperation scenario

In this scenario, the chain decisions are made cooperatively to maximize its profit. This game has one stage, and the chain profit function is sum of the objective functions of the manufacturer and retailer as follows:

$$\mathrm{Max}(\Pi_C) = p_1 \mathrm{ED}_1 + p_2 \min(I_1, \mathrm{ED}_2) - c_p Q - h_m[QT_m - \mathrm{ED}_1 + (I_1(T - T_m) - \mathrm{ED}_2)p(I_1 \geq \mathrm{ED}_2)]. \tag{4.5}$$

The decision variables of equation (4.5) are p_1, p_2, and z. The holding cost of the unit product per unit shelf life is h_m. Similar to equation (3.14), the power of z in equation (4.5) is three and proving the concavity of equation (4.5) regarding the decision variables is difficult. Therefore, a heuristics method is developed below with the goal of representing the best obtained values of the three variables.

4.3.1. Heuristic method

The steps of the heuristic method are represented in the following.

Step 1. Set e as the value of z and calculate the amounts of p_1 and p_2 using equations (4.6) and (4.7), obtain the amount of the chain objective function and set $b = 1$.
Giving the value of z, the values of p_1 and p_2 in equation (4.5) can be obtained using equations (4.6) and (4.7) as follows:

$$p_1 = \frac{D_0 T_m + \beta q\left(1 - e^{-\lambda T_m}\right)/\lambda - c_p \frac{\partial D_1}{\partial p_1}}{-2\frac{\partial D_1}{\partial p_1}} \tag{4.6}$$

$$p_2 = \frac{\begin{array}{c} D_0(T - T_m) + \beta q\left(1 - e^{-\lambda T_m} - e^{-\lambda T}\right)/\lambda + \min(2z, \varepsilon_1 + \varepsilon_2) \\ -(c_p + h_r T_m - h_r(1 + T - T_m)p(2z \geq \varepsilon_1 + \varepsilon_2))\frac{\partial D_2}{\partial p_2} \end{array}}{-2\frac{\partial D_2}{\partial p_2}}. \tag{4.7}$$

Step 2. In the range $(z_0,\ z_0 + \Delta z)$, randomly generate a number for z. Then, add Δz to each bound of the range and choose a random number for$z$ again. Repeat this process until the upper bound of the intended range reaches *uf*.
The lower bound of z is e and its upper bound is *uf*. The number of z generated during step (2) is shown by B and its value is the same as the iterations of this step.

TABLE 2. Data of the numerical example.

$T = 10$	$c_p = 30$	$t = 8$	$h_r = 1$	$\varepsilon_{1t} = U(0.5, 10)$	$\varepsilon_1 = U(5, 50)$
$B = 0.5$	$q_0 = 30$	$d_0 = 65$	$\lambda = 0.0916$	$\varepsilon_{2t} = 0.8 \times U(0.5, 10)$	$\varepsilon_2 = 0.8 \times U(5, 50)$
$a = 3$	$h_m = 0.5$	$\alpha = 0.4$			

TABLE 3. Results obtained from solving the retailer/manufacturer-led Stackelberg and cooperation game scenarios using the numerical example.

Model	p_1	p_2	z	w	Π_r^*	Π_m^*	Π_r/Π_{CR}	Π_m/Π_{CM}	Π_T
Retailer-led	123.14	116.42	38.7	75.71	–	–	3805.47	13 502.6	17 308.1
Manufacturer-led	150.7	143.97	39.4	90.1	–	–	4964.92	16 049.64	21 014.56
Cooperation	113.72	111.29	17.46	–	4964.92	16 049.64	17 261.8	6177.08	23 438.88

Step 3. For $z_b(b = 1, \ldots, B)$, compute the optimum values of p_1 and p_2 using equations (4.6) and (4.7), and obtain the amount of the chain objective function. Calculate the difference between the chain objective function gained in the last previous iteration and its present profit. If the difference is lower than ε, the amounts of p_1, p_2, and z are the best-obtained solutions, and stop. Else, set $b = b + 1$ and go back to step (3). If $b = B$, the algorithm cannot obtain a solution and restart the algorithm.

4.3.2. Nash bargaining model

As mentioned previously, the cooperation game has one stage in which only the supply chain profit is calculated. Therefore, a bargaining method is required to divide the obtained profit among the members. We use the Nash equilibrium bargaining model [25]. Assuming that, the total profit of the cooperation scenario is denoted by Π_c^*, and Π_r^*, Π_m^* represents the maximum profits of the retailer and manufacturer which are obtained in the Stackelberg scenarios, if the total profits of the manufacturer and retailer in the cooperation case are given by Π_{CM}, Π_{CR}, then Π_{CM}, Π_{CR} are represented as:

$$\Pi_{\text{CR}}^* = \frac{1}{2}(\Pi_C^* - \Pi_m^* + \Pi_r^*) \tag{4.8}$$

$$\Pi_{\text{CM}}^* = \frac{1}{2}(\Pi_C^* + \Pi_m^* - \Pi_r^*). \tag{4.9}$$

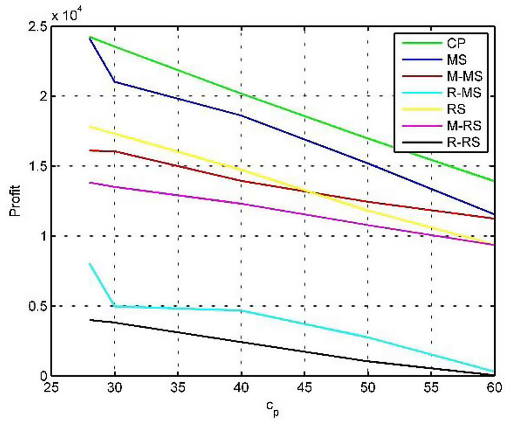
4.4. Numerical example

Two mathematical models are solved by a numerical example. Table 2 indicates the numerical example's data that are generated referring to the relevant literature [2,37]. It is worth noting that Wang and Li [37] gathered the numerical example's data from a grocery retail chain in the UK.

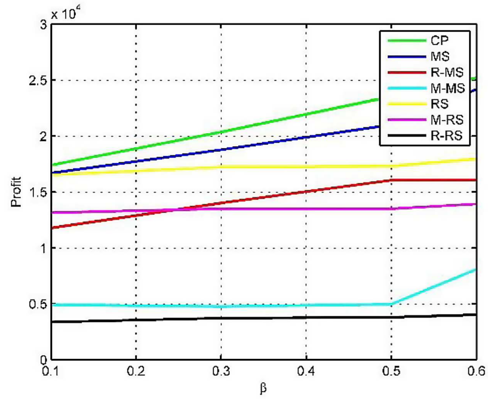
The problems of the three scenarios are solved by the algorithms shown in Sections 4.1.1 and 4.3.1 using Matlab 2012, and the level of ε is considered 15. Obtained results are yielded in Table 3.

In Table 3, the selling prices and the stocking factor in the cooperation game are the least, which shows the best performance of the cooperation scenario. Comparing the Stackelberg cases, it is clarified that when the manufacturer is the leader of the game, both the retailer and manufacturer gain more profits. The reason is that the selling and wholesale prices in the retailer-led Stackelberg game are lower than those of the manufacturer-led Stackelberg case.

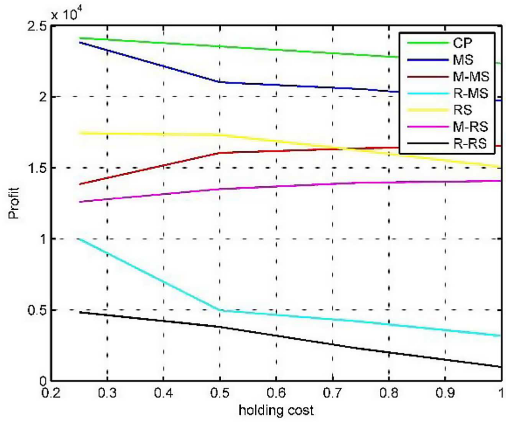
Defining Price Markdown as $\text{PM} = (p_1 - p_2)/p_1$, Table 3 indicates that in the manufacturer-led and retailer-led Stackelberg scenarios, the price markdowns are 4% and 5%; however, the cooperation game represents the price markdown 2% approximately. Therefore, not only the cooperation game yields the least prices, which leads to the greatest demands, but also it represents the least price markdown which is beneficial to the chain.



A



B



C

FIGURE 2. Effects of A: parameter c_p, B: parameter β, C: parameters h_m and $(h_r - h_m)$ on the profit functions of the cooperation game scenario (CP), manufacturer-led Stackelberg scenario (MS), Retailer (R), Manufacturer (M).

Table 3 shows that the profits of both members in the cooperation scenario are the best among the games. This result persuades both members to cooperate. Some cooperation contracts can be developed referring to the literature, which is remained as a future research opportunity. It should be noticed that in the cooperation game, the profits of the retailer and manufacturer are obtained using the Nash bargaining model which was discussed in Section 4.3.2.

5. Sensitivity analysis

In this section, the sensitivity of the objective functions regarding some parameters (c_p, β, (h_r, h_m)) is investigated. It should be related that due to some complexities of the problem, not only the mentioned parameters are considered over small ranges but also analyzing the sensitivity of some parameters like α is proved to be impossible. The results are represented below.

TABLE 4. Chain members' obtained profits of the experiments in the retailer/manufacturer-led Stackelberg and cooperation scenarios.

		Cooperation game						Manufacturer-led Stackelberg scenario						Retailer-led Stackelberg scenario					
No.		Π_T	Π_m	Π_r	p_1	p_2	PM	Π_T	Π_m	Π_r	p_1	p_2	PM	Π_T	Π_m	Π_r	p_1	p_2	PM
c_p																			
1	28	24 229.32	16 141.34	8087.98	111.88	110.42	1%	24 134.7	16 094	8040.64	143.2	138.25	3%	17 809.64	13 824.17	3985.47	123.2	116.2	6%
2	30	23 438.9	17 261.8	6177.08	113.72	111.29	2%	21 014.6	16 049.6	4964.92	150.7	143.97	4%	17 308.1	13 502.6	3805.47	123.14	116.42	5%
3	40	20 172.27	14 717.7	5454.57	118.71	116.57	2%	18 601.2	13 932.1	4669	150.75	144.95	4%	14 715.82	12 303.85	2412.96	123.6	116.9	5%
4	50	16 959.14	13 328.91	3630.22	124.47	121.71	2%	15 183.4	12 441.1	2742.36	155.43	149.11	4%	11 813.18	10 776	1037.17	127.44	120	6%
5	60	13 904.14	12 420.67	1483.47	130.26	126.86	3%	11 530.1	11 233.7	296.45	161.87	154.84	4%	9382.12	9345.47	36.65	128	118	8%
Holding cost																			
6	0.25	24 134.63	13 991.97	10 142.6	111.59	110.66	1%	23 828.4	13 838.9	9989.56	138.75	134.86	3%	17 442.41	12 605.29	4837.12	122.33	115.09	6%
7	0.5	23 438.9	17 261.8	6177.08	113.72	111.29	2%	21 014.6	16 049.6	4964.92	150.7	143.97	4%	17 308.1	13 502.6	3805.47	123.14	116.42	5%
8	0.75	22 941.4	17 569.15	5372.25	114.43	112.22	2%	20 543.4	16 370.1	4173.23	152.27	145.75	4%	16 215.81	13 953.7	2262.11	129.02	122.46	5%
9	1	22 343.97	17 859.94	4484.03	115.86	113	2%	19 723.8	16 549.9	3173.95	155.02	147.63	5%	15 057.21	14 075.65	981.55	134.14	127	5%
B																			
10	0.1	17 397.41	12 132.48	5264.93	103.53	100.86	3%	16 693.5	11 780.5	4912.97	131.42	125.29	5%	16 525.83	13 149.89	3375.94	123.1	116.4	5%
11	0.3	20 360.71	14 806.74	5553.97	108.26	106.15	2%	18 780.2	14 016.5	4763.7	141.16	135.17	4%	17 226.72	13 502.37	3724.25	121.17	114.06	6%
12	0.5	23 438.9	17 261.8	6177.08	113.72	111.29	2%	21 014.6	16 049.6	4964.92	150.7	143.97	4%	17 308.1	13 502.6	3805.47	123.14	116.42	5%
13	0.6	25 207.6	16 608.55	8599.04	115.39	114.1	1%	24 180.7	16 095.1	8085.59	147.86	143.2	3%	17 957.64	13 930.76	4026.88	120	113	6%

(1) The sensitivity analysis of the objective functions concerning c_p is conducted within [28, 60] in both the Stackelberg and cooperation games. The obtained results in Figure 2A demonstrate that all games' profits decrease as c_p increases. Figure 2A also declares that both members' profits are reduced with the positive changes of c_p in both Stackelberg games. This result is expected since the increase in the production cost increases the total cost and the selling prices, and decreases the demands.

(2) The sensitivity analysis of the objective functions regarding β is considered within $[0.1, 0.6]$. As Figure 2B indicates, all objective functions' values change positively with the changes of β. As the value of β increases, the level of the demands, and hence, the chain members' profits increase.

(3) The third parameter which is taken into analysis is the holding cost. In the cooperation scenario, the range $[0.25, 1]$ is considered for h_m. However, in the Stackelberg cases, the difference of both members' holding costs $(h_r - h_m)$ is taken into consideration, and the range $[0.25, 1]$ is regarded. Figure 2C demonstrates that the profits of the decentralized and cooperation cases are reduced as the values of h_m and $(h_r - h_m)$ increase due to increasing the total cost. In both Stackelberg cases, as the value of $(h_r - h_m)$ is added, the manufacturer's profit increases and the retailer's profit is reduced due to using the integrated holding system. Keeping the inventory of the retailer by the manufacturer is a revenue resource for it. As the amount of $(h_r - h_m)$ increases, the manufacturer obtains more revenue from holding the inventory for the retailer.

The experiments' outcomes are represented in Table 4. The analytical results are obtained as follows:

(1) The cooperation scenario assigns the greatest profits to the members. Therefore, it is advised to them to have a cooperation relationship. Also, they can design some contracts to achieve this goal. Comparing two Stackelberg games reveals that except for experiment (10), the manufacturer-led Stackelberg game gives more profits to both members.

(2) From customers' point of view, excepting experiment (4), the cooperation scenario is the best game due to offering lower selling prices in both selling terms. The manufacturer-led Stackelberg game is the worst scenario for customers.

(3) The price markdowns of the cooperation scenario are the least in all experiments. Also, the retailer-led Stackelberg case represents the higher price markdowns comparing with the manufacturer-led Stackelberg scenario.

5.1. Managerial insights

This paper represents some managerial insights as follows:

(1) This study suggests the cooperation to the chain members. Under this collaboration, minimum selling prices and price markdowns, and maximum demand levels are yielded which results that both members' profits are the best comparing with the non-cooperation cases. It is obvious that customers gain the greatest profit under the cooperation case due to the least prices.
(2) If the supply chain members have to operate under the non-cooperation scenario, this study suggests that the manufacturer should be the leader of the game. Since under this case, both members achieve more profits. However, this game is detrimental to customers. The retailer-led Stackelberg game offers higher price markdowns and lower selling prices to them.
(3) It is advised that the parameter changes should be conducted carefully. Otherwise, the analytical results give some infeasible solutions possibly, due to the problem complexities.

6. Conclusion

This study considered the pricing problem of a stochastic supply chain system consisting of one manufacturer and one retailer with two periods: production and selling periods. In the production period, the retailer ordered its requirements to the manufacturer regarding the selling period's quality-dependent stochastic demand. After producing the products in the production period by the manufacturer, the retailer offered them to customers in the selling period. It was assumed that during the selling period and at time T_m, the retailer reordered the products to the manufacturer to replenish them for the next selling period, and received them after the end of the present selling period. Due to the perishable nature of the products and in order to sell them before replenishment, the retailer utilized the price markdown to manage the waste amount by stimulating the demand; therefore, the selling period was separated into two terms. In the selling period, the retailer stored its products using the manufacturer's refrigerator which was used during the production period in order to store the raw materials. Therefore, the holding system was integrated.

The problem was solved under the manufacturer/retailer-led Stackelberg and cooperation scenarios. The Nash equilibrium bargaining model was utilized to divide the cooperation game profit between the members. The obtained results showed that the cooperation case was the most beneficial game to both chain members and customers. The obtained results persuaded the members to cooperate using some contracts. The manufacturer-led Stackelberg game was more profitable to the chain members, however, the retailer-led Stackelberg scenario was more beneficial to customers. Also the retailer-led Stackelberg game offered the greatest price markdowns.

The difficulty and limitation of the problem was that considering a distinct distribution function for each stochastic element of the two selling periods' demand functions intensified the complexity of the problem. To solve the problem under three games, the study assumed that both stocking factors were equal and the uniform distribution functions of the stochastic elements of demand functions were multiple of each other. Considering the different stochastic elements and distinct stocking factors and offering some analytical methods to solve the problem can be considered as a future research opportunity. Some other future research chances regarding the intended problem are: developing the problem and analyzing the replenishment cycle of the chain, solving the problem under other game scenarios such as Nash game and coordinating the problem by some contracts like revenue-sharing and two-part tariff mechanisms.

Appendix A.

As mentioned previously, equation (3.14) has three decision variables: p_1, z, and p_2. Since the power of z in equation (3.14) is three, the concavity of equation (3.14) is difficult to be proved, and the algorithm assigns a value for z. Now, equation (3.14) has two decision variables p_1 and p_2 and their value can be obtained as follows:

$$\frac{\partial^2 \Pi_C}{\partial p_1^2} = -2\alpha T_m = \delta_1 \leq 0 \tag{A.1}$$

$$\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2} = \frac{\partial^2 \Pi_C}{\partial p_2 \partial p_1} = 0 \tag{A.2}$$

$$\frac{\partial^2 \Pi_C}{\partial p_2^2} = -2\alpha(T - T_m) = \delta_2 \leq 0. \tag{A.3}$$

Derivatives of the demand function regarding p_1 and p_2 are as:

$$\frac{\partial D_1}{\partial p_1} = -\alpha T_m, \frac{\partial D_2}{\partial p_2} = -\alpha(T - T_m).$$

Now, the determinant of the Hessian matrix and the relevant sub-determinants are computed as:

$$\frac{\partial^2 \Pi_C}{\partial p^2} = \delta_1 \leq 0 \tag{A.4}$$

$$\begin{vmatrix} \frac{\partial^2 \Pi_C}{\partial p_1^2} & \frac{\partial^2 \Pi_C}{\partial p_2 \partial p_1} \\ \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi_C}{\partial p_2^2} \end{vmatrix} = \frac{\partial^2 \Pi_C}{\partial p_1^2}\frac{\partial^2 \Pi_C}{\partial p_2^2} - \frac{\partial^2 \Pi_C}{\partial p_2 \partial p_1}\frac{\partial^2 \Pi_C}{\partial p_2 \partial p_1} = \delta_1 \times \delta_2 - 0 \geq 0. \tag{A.5}$$

Equations (A.4) and (A.5) proves the concavity of equation (3.14) regarding the selling prices.

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