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COLLABORATIVE BARGAINING SOLUTION IN TANDEM SUPPLY CHAIN THROUGH COOPERATIVE GAME THEORETICAL APPROACH

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Abstract. There are many studies about negotiation procedures for contract problems in supply chains. Several recent papers have considered a new negotiation procedure for a repurchase contract problem in a supply chain consisting of a manufacturer and a retailer. There, usually, are some wholesalers between a manufacturer and a retailer. Therefore, a supply chain including some wholesalers in addition to a manufacturer and a retailer should be considered. In this study, we call the supply chain in which three or more members are arranged in series the tandem supply chain. We, firstly, address a negotiation problem for a contract about wholesale and repurchase prices in the tandem supply chain in which three members, that is, a manufacturer, a wholesaler and a retailer are arranged in series. The whole contract in the tandem supply chain is composed of two contracts dependent mutually, *i.e.*, the contract between the manufacturer and wholesaler and the contract between the wholesaler and retailer. The collaborative bargaining solution in the tandem supply chain consisting of three members is discussed. This paper, finally, formulates the tandem supply chain including several wholesalers.

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1. Introduction

Recent decision making problems in supply chain management have involved various conditions, circumstances and objectives. Therefore, there are many studies on supply chain management from various perspectives [2, 3, 5, 6, 12, 13, 17, 19, 20, 23, 24]. Design problems of supply chain contracts, in particular, are recognized as one of manifold problems in the supply chain management.

In some supply chain contracts, the repurchase contract that a manufacturer repurchases unsold products of a retailer is especially well known as a useful strategy [9, 22]. The repurchase contract can be interpreted as a procedure that a manufacturer and a retailer share financial risks associated with unsold products [3]. The sharing of risks can promote an increase in the supply of products and can lead to an increased total profit of the supply chain. Simultaneously, the manufacturer and retailer in the supply chain can expect to increase the respective profits by the increased total profit of the supply chain. Hence, the repurchase contract has been investigated by many researchers such as Hafezalkotob and Makui [5] and Wu [23].

Keywords. supply chain contract, collaborative coordination approach, Nash bargaining solution, Hessian matrix .

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On the other hand, Takemoto and Arizono [18] have introduced the concept of a collaborative negotiation procedure between a manufacturer and a retailer as a key concept to resolve a design problem of a contract in a supply chain consisting of two members. In recent years, Tsurui *et al.* [21] have revalidated the effectiveness of the collaborative negotiation procedure introduced by Takemoto and Arizono [18]. Note that these previous studies [18, 21] have dealt with the contract problem in the supply chains composed of a manufacturer and a retailer or retailers.

In actual commerce, it is common that there are some wholesalers dealing with products between a manufacturer and a retailer. In such a case, a negotiation procedure for a contract problem in the supply chain model consisting of the manufacturer, wholesalers and retailer has to be considered. We consider a negotiation problem in such a supply chain that are composed of a manufacturer, a wholesaler (or wholesalers) and a retailer. The supply chain consisting of three or more members arranged serially is called the tandem supply chain in this study. We, firstly, consider a contract problem in a supply chain composed of three supply chain members of a manufacturer, a wholesaler and a retailer as an expanded problem of the collaborative negotiation problem studied by Takemoto and Arizono [18] and Tsurui *et al.* [21]. The contract problem in the tandem supply chain is composed of two contracts between the manufacturer and wholesaler and between the wholesaler and retailer, where those contracts are dependent mutually. Therefore, we have to derive the whole collaborative bargaining solution in the tandem supply chain by considering the mutual-interdependence of the collective bargaining solutions of each of the two contracts. Based on the whole collaborative bargaining solution in the tandem supply chain composed of three members, logistic operation managers in the tandem supply chain can decide the adequate wholesale prices and repurchase prices. Note that the above result in this study benefits as an expansion of the results of previous studies [18, 21].

On one hand, the contract problem in a tandem supply chain consisting of the three members is intrinsically one issue that two mutual-dependent contract problems mentioned above are combined. Accordingly, we have interest also in the procedure of solving this issue as one problem, not in the way of solving it as the problem combining two mutual-dependent contract problems. Hence, instead of obtaining the solution through combining two contract problems between the manufacturer and wholesaler and between the wholesaler and retailer, we attempt to provide the procedure for obtaining the same solution for the contract problem in the tandem supply chain through solving this issue as one problem. In consequence, the whole collaborative bargaining solution in the tandem supply chain consisting of three or more members is derived uniquely.

2. Model setting of tandem supply chain consisting of three members

In this section, to describe the tandem supply chain model, the respective profit functions π^M, π^W and π^R for the manufacturer, wholesaler and retailer are defined.

The symbols and notations for describing the model are shown as follows:

p retail price,
k_w^{MW} wholesale price rate between manufacturer and wholesaler,
k_w^{WR} wholesale price rate between wholesaler and retailer,
k_r^{MW} repurchase price rate between manufacturer and wholesaler,
k_r^{WR} repurchase price rate between wholesaler and retailer,
k_a administration cost rate in retailer,
k_c original cost rate in manufacturer,
k_d disposal cost rate in manufacturer,
q trading quantity,
x quantity of demand,
$f(x)$ probability density function of x,
π^M expected profit function of manufacturer,
π^W expected profit function of wholesaler,
π^R expected profit function of retailer,

π^T total expected profit function in tandem supply chain.

Note that wholesale prices, repurchase prices, original cost, disposal cost and administration cost are given as $k_w^{MW}p$ and $k_w^{WR}p$, $k_r^{MW}p$ and $k_r^{WR}p$, k_cp, k_dp and k_ap based on the retail price p. Since all prices and costs are given as the product of p and the price rates or cost rates, we can treat the model without loss of generality by considering $p = 1$. In addition, it is reasonable that the following relations are satisfied respectively: $0 < k_c < k_w^{MW} < k_w^{WR} < 1$, $0 < k_r^{MW} < k_w^{MW} < 1$ and $0 < k_r^{WR} < k_w^{WR} < 1$. Also, k_w^{MW} and k_r^{MW} represent the contract parameters for the negotiation between the manufacturer and wholesaler. Similarly, k_w^{WR} and k_r^{WR} mean the contract parameters for the negotiation between the wholesaler and retailer. It is obvious that the relations of $k_w^{MW} < k_w^{WR}$ and $k_w^{WR} + k_a < 1$ should be satisfied from the viewpoint of business. Furthermore, q is decided by these contract parameters under the concept of the newsvendor problem. Namely, k_w^{MW}, k_w^{WR}, k_r^{MW}, k_r^{WR} and q are treated as decision variables of the negotiation problem in the tandem supply chain. By using these symbols, the respective expected profit functions π^M, π^W and π^R are formulated as follows:

$$\pi^M = \left(k_w^{MW} - k_c\right) q - \left(k_r^{MW} + k_d\right) \int_0^q (q - x) f(x)\mathrm{d}x, \tag{2.1}$$

$$\pi^W = \left(k_w^{WR} - k_w^{MW}\right) q - \left(k_r^{WR} - k_r^{MW}\right) \int_0^q (q - x) f(x)\mathrm{d}x, \tag{2.2}$$

$$\pi^R = \left(1 - k_w^{WR} - k_a\right) q - \left(1 - k_r^{WR}\right) \int_0^q (q - x) f(x)\mathrm{d}x. \tag{2.3}$$

Remark that π^M, π^W and π^R are defined by using the respective decision variables (q, k_w^{MW}, k_r^{MW}), $(q, k_w^{MW}, k_r^{MW}, k_w^{WR}, k_r^{WR})$ and (q, k_w^{WR}, k_r^{WR}).

Based on the concept of the newsvendor problem, the retailer decides q as q^R so as to maximize the profit π^R by the following relation:

$$\int_0^{q^R} f(x)\mathrm{d}x = \frac{1 - k_w^{WR} - k_a}{1 - k_r^{WR}}. \tag{2.4}$$

Note that equation (2.4) can be derived from the relations of $d\pi^R/dq = 0$ and $d^2\pi^R/dq^2 < 0$. From equation (2.4), it is reconfirmed that the trading quantity q^R is decided by k_w^{WR} and k_r^{WR}. It means that q^R depends on the contract negotiation between the wholesaler and retailer. On the other hand, it is seen in equation (2.2) that π^W depends on the decision variables k_w^{MW} and k_r^{MW} between the manufacturer and wholesaler in addition to k_w^{WR} and k_r^{WR}. Accordingly, it is easily imagined that q^R depends on also the condition of the contract between the manufacturer and wholesaler. In this manner, for the purpose of deciding $\left(q, k_w^{MW}, k_r^{MW}, k_w^{WR}, k_r^{WR}\right)$, we should address the negotiation problem composed of two contracts between the manufacturer and wholesaler and between the wholesaler and retailer.

3. Solution of negotiation problem in tandem supply chain

In this study, we address how to derive a bargaining solution in the tandem supply chain through a collaborative negotiation procedure based on a cooperative game theoretic approach. In the previous study by Takemoto and Arizono [18], the negotiation process to gain a collaborative bargaining solution for the contract problem of the supply chain consisting of a manufacturer and a retailer has been proposed as the following three steps:

(i) Show such a requirement for contract parameters that both of the manufacturer and retailer are simultaneously motivated to conclude the contract,
(ii) Present a condition of contract parameters achieving both optimality of the manufacturer and retailer,
(iii) Determine a unique combination of contract parameters using the bargaining solution by Nash bargaining theory [10].

According to the steps mentioned above, we consider the contract problem in the tandem supply chain composed of three members. Under the concept of step (i) mentioned above, the following conditions should be satisfied as a minimum requirement:

$$\begin{cases} \pi^M > \pi_0^M, \\ \pi^W > \pi_0^W, \\ \pi^R > \pi_0^R, \end{cases} \tag{3.1}$$

where π_0^M, π_0^W and π_0^R are the reference profits expected under a usual deal in the manufacturer, wholesaler and retailer, respectively. The condition satisfying simultaneously three inequalities in equation (3.1) presents a minimum requirement to conclude the negotiation problem in the tandem supply chain consisting of three members.

As mentioned above, we can consider that the contract problem in the tandem supply chain is composed of two contracts between the manufacturer and wholesaler and between the wholesaler and retailer. Therefore, we consider provisional bargaining solutions for the respective contract problems between two members. Further, by connecting two contract problems, we derive a whole collaborative bargaining solution in the negotiation problem in the tandem supply chain uniquely.

At first, the contract problem between the wholesaler and retailer is considered. Note that the contract parameters which relate to the contract between the wholesaler and retailer are $\left(q, k_w^{WR}, k_r^{WR}\right)$, and the other parameters are treated as boundary conditions. Therefore, the following relations are derived based on equations (2.1)–(3.1):

$$k_w^{MW} > \frac{S\left(q^R\right)}{q^R} k_r^{MW} + k_c + \frac{S\left(q^R\right)}{q^R} k_d + \frac{\pi_0^M}{q^R}, \tag{3.2}$$

$$k_w^{MW} < \frac{S\left(q^R\right)}{q^R} k_r^{MW} + k_w^{WR} - \frac{S\left(q^R\right)}{q^R} k_r^{WR} - \frac{\pi_0^W}{q^R}, \tag{3.3}$$

$$k_w^{WR} < -\frac{S\left(q^R\right)}{q^R} \left(1 - k_r^{WR}\right) + 1 - k_a - \frac{\pi_0^R}{q^R}, \tag{3.4}$$

where $S\left(q^R\right)$ denotes the expected quantity of unsold products as follows:

$$S\left(q^R\right) = \int_0^{q^R} \left(q^R - x\right) f(x)\mathrm{d}x. \tag{3.5}$$

From equations (3.2) and (3.3), we obtain the following relation between k_w^{WR} and k_r^{WR}:

$$k_w^{WR} > \frac{S\left(q^R\right)}{q^R} \left(k_r^{WR} + k_d\right) + \frac{\pi_0^W + \pi_0^M}{q^R} + k_c. \tag{3.6}$$

As mentioned previously, q is given as q^R which satisfies equation (2.4). On the other hand, based on equation (2.2), the desirable trading quantity for the wholesaler q^W is given by the following relation:

$$\int_0^{q^W} f(x)\mathrm{d}x = \frac{k_w^{WR} - k_w^{MW}}{k_r^{WR} - k_r^{MW}}. \tag{3.7}$$

Note that equation (3.7) can be derived from the relations of $d\pi^W/dq = 0$ and $d^2\pi^W/dq^2 < 0$. Under the condition in step (ii), the relation $q^R = q^W (\equiv q)$ should be satisfied in order to conclude the contract between the wholesaler and retailer. Takemoto and Arizono [18] have named such a negotiation procedure the collaborative coordination approach in the negotiation problem. Accordingly, based on equations (2.4) and (3.7) and the relation of $q^R = q^W (\equiv q)$, we can obtain the following relation:

$$k_w^{WR} = \frac{1 - k_a - k_w^{MW}}{1 - k_r^{MW}} k_r^{WR} + \frac{k_w^{MW} - (1 - k_a)\, k_r^{MW}}{1 - k_r^{MW}}. \tag{3.8}$$

Furthermore, from step (iii), the evaluation function in the contract problem between the wholesaler and retailer can be defined as follows:

$$T_{WR}\left(q, k_r^{WR}\right) = \left(\pi^W - \pi_0^W\right)\left(\pi^R - \pi_0^R\right), \tag{3.9}$$

where $T_{WR}\left(q, k_r^{WR}\right)$ has been called the Nash product (Nash, 1950) in the negotiation problem between the wholesaler and retailer. The notation of $T_{WR}\left(q, k_r^{WR}\right)$ indicates that the Nash product in equation (3.9) is defined as the function of $\left(q, k_r^{WR}\right)$. Through the relation of equation (2.4), we can see that q and k_w^{WR} are one-to-one correspondence for given k_r^{WR}. Hence, it can be also interpreted that the Nash product in equation (3.9) is the function of $\left(k_w^{WR}, k_r^{WR}\right)$.

The provisional bargaining solution in the contract problem between the wholesaler and retailer is derived as $\left(q, k_w^{WR}, k_r^{WR}\right)$ maximizing $T_{WR}\left(q, k_r^{WR}\right)$, if we have $\left(q, k_w^{WR}, k_r^{WR}\right)$ satisfying the first-order conditions for the Nash product $T_{WR}\left(q, k_r^{WR}\right)$ as a unique solution.

Proposition 3.1. *The Nash product $T_{WR}\left(q, k_r^{WR}\right)$ has uniquely the maximum value, if we have $\left(q, k_w^{WR}, k_r^{WR}\right)$ satisfying the first-order conditions for the Nash product $T_{WR}\left(q, k_r^{WR}\right)$ as a unique solution.*

Proof. Through the partial differentiation of $T_{WR}\left(q, k_r^{WR}\right)$ with respect to q and k_r^{WR}, we obtain the first-order conditions as

$$\frac{\partial T_{WR}\left(q, k_r^{WR}\right)}{\partial q} = \frac{\partial \pi^W}{\partial q}\left(\pi^R - \pi_0^R\right) + \frac{\partial \pi^R}{\partial q}\left(\pi^W - \pi_0^W\right) = 0, \tag{3.10}$$

$$\frac{\partial T_{WR}\left(q, k_r^{WR}\right)}{\partial k_r^{WR}} = \frac{\partial \pi^W}{\partial k_r^{WR}}\left(\pi^R - \pi_0^R\right) + \frac{\partial \pi^R}{\partial k_r^{WR}}\left(\pi^W - \pi_0^W\right) = 0. \tag{3.11}$$

In equation (3.10), because of $q = q^R = q^W$, we have known

$$\frac{\partial \pi^W}{\partial q} = \frac{\partial \pi^R}{\partial q} = 0. \tag{3.12}$$

Hence, it is obvious that

$$\frac{\partial \pi^W}{\partial q} + \frac{\partial \pi^R}{\partial q} = 0. \tag{3.13}$$

Additionally, the following relation is given by equations (2.2) and (2.3):

$$\frac{\partial \pi^R}{\partial k_r^{WR}} = -\frac{\partial \pi^W}{\partial k_r^{WR}}. \tag{3.14}$$

By using equations (3.11) and (3.14), we obtain the following relation:

$$\pi^R - \pi_0^R = \pi^W - \pi_0^W\ (> 0). \tag{3.15}$$

The relation in equation (3.15) indicates that the increment of the wholesaler's profit is equivalent to the increment of the retailer's profit. On one hand, it is found from equation (3.13) that the summation of the profits of the wholesaler and retailer is maximized in the bargaining solution between the wholesaler and retailer. As a result, it is found that the first-order conditions for the Nash product $T_{WR}\left(q, k_r^{WR}\right)$ with respect to q and k_r^{WR} provide the condition that the increment of the wholesaler's profit is equivalent to the increment of the retailer's profit and the summation of the profits of the wholesaler and retailer is maximized by the bargaining solution between the wholesaler and retailer.

Further, we have the Hessian matrix for $T_{WR}\left(q, k_r^{WR}\right)$ with respect to $\left(q, k_r^{WR}\right)$ as follows:

$$H_{WR} = \begin{bmatrix} \dfrac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q^2} & \dfrac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q \partial k_r^{WR}} \\ \dfrac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial k_r^{WR} \partial q} & \dfrac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial \left(k_r^{WR}\right)^2} \end{bmatrix}. \tag{3.16}$$

The determinant of H_{WR} is defined as follows:

$$\det H_{WR} = \frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q^2} \frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial \left(k_r^{WR}\right)^2} - \left(\frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q \partial k_r^{WR}}\right)^2. \tag{3.17}$$

Based on equations (3.9) and (3.15), we have

$$\frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q^2} = \left(\pi^W - \pi_0^W\right)\left(\frac{\partial^2 \pi^R}{\partial q^2} + \frac{\partial^2 \pi^W}{\partial q^2}\right) + 2\frac{\partial \pi^R}{\partial q}\frac{\partial \pi^W}{\partial q}, \tag{3.18}$$

$$\frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial \left(k_r^{WR}\right)^2} = \left(\pi^W - \pi_0^W\right)\left(\frac{\partial^2 \pi^R}{\partial \left(k_r^{WR}\right)^2} + \frac{\partial^2 \pi^W}{\partial \left(k_r^{WR}\right)^2}\right) + 2\frac{\partial \pi^R}{\partial k_r^{WR}}\frac{\partial \pi^W}{\partial k_r^{WR}}, \tag{3.19}$$

$$\frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q \partial k_r^{WR}} = \left(\pi^W - \pi_0^W\right)\left(\frac{\partial^2 \pi^R}{\partial q \partial k_r^{WR}} + \frac{\partial^2 \pi^W}{\partial q \partial k_r^{WR}}\right) + \frac{\partial \pi^R}{\partial q}\frac{\partial \pi^W}{\partial k_r^{WR}} + \frac{\partial \pi^R}{\partial k_r^{WR}}\frac{\partial \pi^W}{\partial q}. \tag{3.20}$$

Based on equations (2.2), (2.3), (3.12) and (3.18), the following relation can be derived:

$$\frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q^2} = -\left(\pi^W - \pi_0^W\right)\left(1 - k_r^{MW}\right) f(q) < 0. \tag{3.21}$$

Also, equations (2.2), (2.3) and (3.19) provide the following relation:

$$\frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial \left(k_r^{WR}\right)^2} = -2\left(\int_0^q (q - x) f(x) \mathrm{d}x\right)^2 < 0. \tag{3.22}$$

Finally, from equations (2.2), (2.3), (3.12) and (3.20), we have

$$\frac{\partial^2 T_{WR}\left(q, k_r^{WR}\right)}{\partial q \partial k_r^{WR}} = \left(\pi^W - \pi_0^W\right)\left(\int_0^q f(x)\mathrm{d}x - \int_0^q f(x)\mathrm{d}x\right) = 0. \tag{3.23}$$

Therefore, we can understand that the determinant $\det H_{WR}$ in equation (3.17) is given as a positive value. Simultaneously, since $\partial^2 T_{WR}\left(q, k_r^{WR}\right)/\partial q^2$ is a negative value, the Nash product $T_{WR}\left(q, k_r^{WR}\right)$ has uniquely the maximum value at $\left(q, k_w^{WR}, k_r^{WR}\right)$, when $\left(q, k_w^{WR}, k_r^{WR}\right)$ is the unique solution satisfying the first-order conditions for the Nash product $T_{WR}\left(q, k_r^{WR}\right)$.

Q.E.D.

From the process in Proof of Proposition 3.1, it is found that the bargaining solution $\left(q^{WR\dagger}, k_w^{WR\dagger}, k_r^{WR\dagger}\right)$ between the wholesaler and retailer satisfies the following relations:

$$\pi^R\Big|_{q=q^{WR\dagger},\, k_r^{WR}=k_r^{WR\dagger}} - \pi_0^R = \pi^W\Big|_{q=q^{WR\dagger},\, k_r^{WR}=k_r^{WR\dagger}} - \pi_0^W, \tag{3.24}$$

$$\left\{\frac{\partial \pi^R}{\partial q} + \frac{\partial \pi^W}{\partial q}\right\}\Bigg|_{q=q^{WR\dagger},\, k_r^{WR}=k_r^{WR\dagger}} = 0. \tag{3.25}$$

Next, we consider the negotiation problem between the manufacturer and wholesaler. The contract parameters are given by $\left(q, k_w^{MW}, k_r^{MW}\right)$. Similar to the case of the contract between the wholesaler and retailer, we derive the following relation between k_w^{MW} and k_r^{MW} based on equations (3.3) and (3.4):

$$k_w^{MW} < \frac{S\left(q^R\right)}{q^R} k_r^{MW} - \frac{S\left(q^R\right)}{q^R} - \frac{\pi_0^W + \pi_0^R}{q^R} + 1 - k_a. \tag{3.26}$$

On the other hand, from maximizing the expected profit π^M, the desirable trading quantity for the manufacturer q^M is given as follows:

$$\int_0^{q^M} f(x)\mathrm{d}x = \frac{k_w^{MW} - k_c}{k_r^{MW} + k_d}. \tag{3.27}$$

Note that equation (3.27) can be derived from the relations of $d\pi^M/dq = 0$ and $d^2\pi^M/dq^2 < 0$. Through the collaborative coordination approach represented as step (ii), the relation of $q^M = q^W (\equiv q)$ should be satisfied for the bargaining solution between the manufacturer and wholesaler. Accordingly, the relation between k_w^{MW} and k_r^{MW} in the provisional bargaining solution between the manufacturer and wholesaler is derived as follows:

$$k_w^{MW} = \frac{k_w^{WR} - k_c}{k_r^{WR} + k_d} k_r^{MW} + \frac{k_w^{WR} k_d + k_r^{WR} k_c}{k_r^{WR} + k_d}. \tag{3.28}$$

Further, the Nash product [10] as the evaluation function in the contract between the manufacturer and wholesaler can be defined as

$$T_{MW}\left(q, k_r^{MW}\right) = \left(\pi^M - \pi_0^M\right)\left(\pi^W - \pi_0^W\right). \tag{3.29}$$

□

In the same manner for Proposition 3.1, Proposition 3.2 can be considered as follows.

Proposition 3.2. *The Nash product* $T_{MW}\left(q, k_r^{MW}\right)$ *has uniquely the maximum value, if we have* $\left(q, k_w^{MW}, k_r^{MW}\right)$ *satisfying the first-order conditions for the Nash product* $T_{WR}\left(q, k_r^{MW}\right)$ *as a unique solution.*

The proof procedure for Proposition 3.2 can be represented by the same way as the proof procedure for Proposition 3.1 except for the difference of symbols. Hence, the proof for Proposition 3.2 has been omitted.

Based on Proposition 3.2, the bargaining solution $\left(q^{MW\dagger}, k_w^{MW\dagger}, k_r^{MW\dagger}\right)$ between the manufacturer and wholesaler can be derived by the following relations:

$$\pi^M\Big|_{q=q^{MW\dagger},\, k_r^{MW}=k_r^{MW\dagger}} - \pi_0^M = \pi^W\Big|_{q=q^{MW\dagger},\, k_r^{MW}=k_r^{MW\dagger}} - \pi_0^W, \tag{3.30}$$

$$\left\{\frac{\partial \pi^M}{\partial q} + \frac{\partial \pi^W}{\partial q}\right\}\Bigg|_{q=q^{MW\dagger},\, k_r^{MW}=k_r^{MW\dagger}} = 0. \tag{3.31}$$

Finally, because the trading quantity should be unique in the tandem supply chain, we have the relation of $q = q^M = q^W = q^R$. Furthermore, from the results derived previously, the following relations for the whole bargaining solution $\left(q, k_w^{MW}, k_r^{MW}, k_w^{WR}, k_r^{WR}\right)$ in the negotiation problem of the tandem supply chain can be obtained

$$\frac{\partial \pi^M}{\partial q} = \frac{\partial \pi^W}{\partial q} = \frac{\partial \pi^R}{\partial q} = 0, \tag{3.32}$$

$$\frac{\partial \pi^M}{\partial q} + \frac{\partial \pi^W}{\partial q} + \frac{\partial \pi^R}{\partial q} = 0, \tag{3.33}$$

$$\pi^M - \pi_0^M = \pi^W - \pi_0^W = \pi^R - \pi_0^R. \tag{3.34}$$

From the relation of $q = q^M = q^W = q^R$ and equations (3.32)–(3.34), the whole collaborative bargaining solution $\left(q^*, k_w^{MW*}, k_r^{MW*}, k_w^{WR*}, k_r^{WR*}\right)$ in the negotiation problem of the tandem supply chain can be derived.

Equation (3.33) can derive the following relation for evaluating q^* in the whole collaborative bargaining solution:

$$\int_0^{q^*} f(x)\mathrm{d}x = \frac{1 - k_a - k_c}{1 + k_d}. \tag{3.35}$$

Next, based on the relation $q^R = q^M$, we obtain

$$\frac{1 - k_w^{WR} - k_a}{1 - k_r^{WR}} = \frac{k_w^{MW} - k_c}{k_r^{MW} + k_d}. \tag{3.36}$$

By solving the simultaneous equations of equations (3.8), (3.28) and (3.36), we obtain the following relations about k_w^{MW} and k_w^{WR}:

$$k_w^{MW} = \frac{1 - k_a - k_c}{1 + k_d} k_r^{MW} + \frac{k_c + k_d\,(1 - k_a)}{1 + k_d}, \tag{3.37}$$

$$k_w^{WR} = \frac{1 - k_a - k_c}{1 + k_d} k_r^{WR} + \frac{k_c + k_d\,(1 - k_a)}{1 + k_d}. \tag{3.38}$$

Further, from equations (3.37), (3.38) and the relation $\pi^W - \pi_0^W = \pi^R - \pi_0^R$ in equation (3.34), we obtain the relation between k_r^{MW} and k_r^{WR} as follows:

$$k_r^{WR} = \frac{1}{2}\left(1 + k_r^{MW}\right) - \frac{\left(\pi_0^R - \pi_0^W\right)}{2\int_0^{q^*} x f(x)\mathrm{d}x}. \tag{3.39}$$

Similarly, we can derive the following relation between k_r^{MW} and k_r^{WR} based on equations (3.37), (3.38) and the relation $\pi^M - \pi_0^M = \pi^W - \pi_0^W$ in equation (3.34):

$$k_r^{WR} = 2k_r^{MW} + k_d + \frac{\left(\pi_0^W - \pi_0^M\right)}{\int_0^{q^*} x f(x)\mathrm{d}x}. \tag{3.40}$$

Finally, by using equations (3.37)–(3.40), we can derive the whole collaborative bargaining solution $\left(k_w^{MW*}, k_r^{MW*}, k_w^{WR*}, k_r^{WR*}\right)$ as follows:

$$k_r^{MW*} = \frac{1-2k_d}{3} - \frac{\left(\pi_0^R - \pi_0^M\right) + \left(\pi_0^W - \pi_0^M\right)}{3\int_0^{q^*} xf(x)\mathrm{d}x}, \tag{3.41}$$

$$k_r^{WR*} = \frac{2-k_d}{3} - \frac{\left(\pi_0^R - \pi_0^W\right) + \left(\pi_0^R - \pi_0^M\right)}{3\int_0^{q^*} xf(x)\mathrm{d}x}, \tag{3.42}$$

$$k_w^{MW*} = \frac{1-k_a+2k_c}{3} - \frac{1-k_a-k_c}{1+k_d}\frac{\left(\pi_0^R - \pi_0^M\right) + \left(\pi_0^W - \pi_0^M\right)}{3\int_0^{q^*} xf(x)\mathrm{d}x}, \tag{3.43}$$

$$k_w^{WR*} = \frac{2-2k_a+k_c}{3} - \frac{1-k_a-k_c}{1+k_d}\frac{\left(\pi_0^R - \pi_0^W\right) + \left(\pi_0^R - \pi_0^M\right)}{3\int_0^{q^*} xf(x)\mathrm{d}x}. \tag{3.44}$$

It can be seen that equations (3.41)–(3.44) provide the unique solution. In this case, it is obvious that q is uniquely determined based on equation (3.36). Hence, through Propositions 3.1 and 3.2, $\left(k_w^{MW*}, k_r^{MW*}, k_w^{WR*}, k_r^{WR*}\right)$ maximizes the Nash products $T_{WR}\left(q, k_r^{WR}\right)$ and $T_{MW}\left(q, k_r^{MW}\right)$, respectively. As a consequence, we can see that the whole collaborative bargaining solution $\left(k_w^{MW*}, k_r^{MW*}, k_w^{WR*}, k_r^{WR*}\right)$ is provided as equations (3.41)–(3.44).

4. Consideration in solution through simultaneous negotiation on round-table

In the argument for deriving the whole collaborative bargaining solution mentioned in Section 3, the negotiation problem has been dealt with by combining two contracts between the manufacturer and wholesaler and between the wholesaler and retailer. In this section, we check up on a negotiation for deriving the whole collaborative bargaining solution in the tandem supply chain through the simultaneous negotiation on a round-table by the manufacturer, wholesaler, and retailer.

It is obvious that the relation of $q = q^M = q^W = q^R$ should be satisfied in the simultaneous negotiation on the same table together. Further, the relations in equations (3.8) and (3.28) are required for the whole collaborative bargaining solution in the tandem supply chain consisting of three members. After all, we can see that it is required to decide only two contract parameters k_r^{MW} and k_r^{WR} in this negotiation problem. For this purpose, we represent an extended Nash product in the negotiation problem by three members as follows:

$$T_{MWR}\left(k_r^{MW}, k_r^{WR}\right) = \left(\pi^M - \pi_0^M\right)\left(\pi^W - \pi_0^W\right)\left(\pi^R - \pi_0^R\right). \tag{4.1}$$

From the extended Nash product $T_{MWR}\left(k_r^{MW}, k_r^{WR}\right)$, it can be found to derive the following relations:

$$\begin{cases} \dfrac{\partial T_{MWR}\left(k_r^{MW}, k_r^{WR}\right)}{\partial k_r^{MW}} = 0, \\ \dfrac{\partial T_{MWR}\left(k_r^{MW}, k_r^{WR}\right)}{\partial k_r^{WR}} = 0. \end{cases} \tag{4.2}$$

From equation (4.2), we can obtain the relations of equations (3.32)–(3.34). Therefore, we can again obtain the unique solution $\left(k_w^{MW*}, k_r^{MW*}, k_w^{WR*}, k_r^{WR*}\right)$ under the first-order condition of $T_{MWR}\left(k_r^{MW}, k_r^{WR}\right)$.

Furthermore, we can represent that the determinant det H_{MWR} of the Hessian matrix for $T_{MWR}\left(k_r^{MW}, k_r^{WR}\right)$ with respect to $\left(k_r^{MW}, k_r^{WR}\right)$ is positive and the extended Nash product $T_{MWR}\left(k_r^{MW}, k_r^{WR}\right)$ has a unique maximum value. In consequence, by employing the extended Nash product $T_{MWR}\left(k_r^{MW}, k_r^{WR}\right)$, we can obtain the whole collaborative bargaining solution $\left(q^*, k_w^{MW*}, k_r^{MW*}, k_w^{WR*}, k_r^{WR*}\right)$ through the simultaneous negotiation on a round-table.

From the consideration above, we have seen that the result by combining the two contracts between the manufacturer and wholesaler and between the wholesaler and retailer is correspondent to the result by bargaining among three members on a round-table. Utilizing this implication, we can consider the negotiation problem in the tandem supply chain consisting of the manufacturer, several wholesalers, and retailer.

5. Expansion of negotiation problem in tandem supply chain into n members

In Section 4, the extended Nash product in the negotiation problem by three members has been given as equation (4.1). In this section, we consider the extended Nash product in the negotiation problem in the tandem supply chain consisting of n members. In this case, each member in the tandem supply chain consisting of n members is represented by index j. Here, note that indices $j = 1$ and $j = n$ indicate the retailer and the manufacturer, respectively. Further, the intermediate wholesalers from the manufacturer to the retailer are presented as indices $j = n-1, \cdots, 2$.

Suppose that the repurchase contract between the adjacent members, *e.g.*, j and $j-1$, would be concluded individually. In such a case, under the adjusted three steps for the negotiation process to gain a collaborative bargaining solution for the contract problem of the tandem supply chain consisting of n members, an extended Nash product can be defined as:

$$T^{(n)} = \prod_{j=1}^{n} (\pi^{(j)} - \pi_0^{(j)}), \tag{5.1}$$

where denote the expected profit function and the reference profit as $\pi^{(j)}$ and $\pi_0^{(j)}$ for member j, respectively. In addition, the wholesale price rate and the repurchase price rate between members j and $j-1$ are presented as $k_w^{(j,\,j-1)}$ and $k_r^{(j,\,j-1)}$. Under these notations, we can derive $\pi^{(1)}$, $\pi^{(n)}$ and $\pi^{(j)}$, $j = 2, \cdots, n-1$, according to equations (2.3), (2.1) and (2.2), respectively.

The extended Nash product $T^{(n)}$ in equation (5.1) can be dealt with as the function of $q, k_r^{(n,\,n-1)}, \cdots, k_r^{(2,\,1)}$, because of the condition in the adjusted step (ii). Hence, through the partial differentiation of $T^{(n)}$ with respect to $q, k_r^{(n,\,n-1)}, \cdots, k_r^{(2,\,1)}$, we obtain the first-order conditions as

$$\frac{\partial T^{(n)}}{\partial q} = \frac{\partial \pi^{(\ell)}}{\partial q} \prod_{\substack{j=1 \\ j \neq \ell}}^{n} (\pi^{(j)} - \pi_0^{(j)}) = 0, \tag{5.2}$$

$$\begin{aligned} \frac{\partial T^{(n)}}{\partial k_r^{(\ell,\,\ell-1)}} = \frac{\partial \pi^{(\ell)}}{\partial k_r^{(\ell,\,\ell-1)}} \prod_{\substack{j=1 \\ j \neq \ell}}^{n} (\pi^{(j)} - \pi_0^{(j)}) \\ + \frac{\partial \pi^{(\ell-1)}}{\partial k_r^{(\ell,\,\ell-1)}} \prod_{\substack{j=1 \\ j \neq \ell-1}}^{n} (\pi^{(j)} - \pi_0^{(j)}) = 0. \end{aligned} \tag{5.3}$$

From equations (5.2), (5.3) and the conditions in the adjusted steps (i) and (ii), we obtain the following relations:

$$\frac{\partial \pi^{(j)}}{\partial k_r^{(j,\,j-1)}} = -\frac{\partial \pi^{(j-1)}}{\partial k_r^{(j,\,j-1)}} \tag{5.4}$$

$$\begin{aligned}\pi^{(n)} - \pi_0^{(n)} &= \pi^{(n-1)} - \pi_0^{(n-1)} = \cdots \\ &= \pi^{(1)} - \pi_0^{(1)} > 0,\end{aligned} \tag{5.5}$$

$$\frac{\partial \pi^{(n)}}{\partial q} + \frac{\partial \pi^{(n-1)}}{\partial q} + \cdots + \frac{\partial \pi^{(1)}}{\partial q} = 0. \tag{5.6}$$

It is obvious that the relation in equation (5.6) should be consistent with the following relation:

$$\frac{\partial \pi^{(n)}}{\partial q} = \frac{\partial \pi^{(n-1)}}{\partial q} = \cdots = \frac{\partial \pi^{(1)}}{\partial q} = 0, \tag{5.7}$$

Therefore, we can obtain the whole collaborative bargaining solution that is given as a condition to maximize the expected profit of the entire supply chain by maximizing the expected profit of each member j, as with the tandem supply chains consisting of three members. In addition, equation (5.5) shows the property that the increment of expected profit over the reference profit is equal for all members. This property is also corresponding to the property in the collaborative bargaining solution for the tandem supply chains consisting of three members. Hence, it can be concluded that the extended Nash product $T^{(n)}$ in equation (5.1) is adequate as the evaluation function for deriving the collaborative bargaining solution for the tandem supply chain consisting of n members.

On the other hand, there are several studies about cooperative games [8, 11] considering cooperation among members. In the cooperative game, any cooperation among members, such as between not adjacent members, and among three or more members can be considered. In contrast, the contract problem in the tandem supply chain has the premise that the adjacent members such as the manufacturer and wholesaler, and the wholesaler and retailer just have a business transaction. This concept is in general in business deals. Therefore, remark that the extended Nash products in Okada [11] and in this study are derived from different concepts.

6. Conclusions

In this study, as the extension of the existing problem of deriving the collaborative bargaining solution between the manufacturer and the retailer, we have addressed the problem of the collaborative bargaining solution in the tandem supply chain consisting of the manufacturer, wholesalers, and retailer. We have first considered the collaborative negotiation procedure for concluding the contract in the tandem supply chain consisting of the manufacturer, wholesaler, and retailer. The negotiation problem in the tandem supply chain consisting of the manufacturer, wholesaler, and retailer has been built up by combining two contracts between the manufacturer and wholesaler and between the wholesaler and retailer. Then, individual contracts between two members in the tandem supply chain have been solved with the condition in the remaining contract as boundary conditions. Further, by connecting the provisional solutions of two contracts, the whole bargaining solution in the tandem supply chain has been successfully derived. Specifically, the whole bargaining solution in the tandem supply chain consisting of the manufacturer, wholesaler, and retailer has been presented by five explicit functions for each decision variables. Based on the whole collaborative bargaining solution in the tandem supply chain consisting of three members, logistic operation managers in the tandem supply chain can decide the adequate wholesale prices and repurchase prices. Note that the above result in this study benefits as an expansion of the results of previous studies [18, 21].

In addition, we could check up on a negotiation for deriving the whole bargaining solution in the tandem supply chain through the simultaneous negotiation on a round-table by the manufacturer, wholesaler, and retailer. In such a consideration, we have presented the extended Nash product for obtaining the whole collaborative bargaining solution through the simultaneous negotiation on a round-table. The bargaining problem among three members on a round-table has been formulated as the extended Nash bargaining problem. Based on this extended Nash bargaining problem, the whole collaborative bargaining solution on a round-table in the tandem supply chain consisting of three members has been derived. As a result, we have seen that the result by combining the two contracts between the manufacturer and wholesaler and between the wholesaler and retailer is correspondent to the result by bargaining among three members on a round-table.

Furthermore, utilizing this implication, we have considered the collaborative negotiation procedure for concluding the contract in the tandem supply chain consisting of the manufacturer, several wholesalers, and retailer. As a consequence, we have confirmed that a series of agreements between adjacent members have been equivalent to the agreement in the negotiation of whole members. Remark again that the whole collaborative bargaining solution in the tandem supply chain consisting of three or more members can help logistic operation managers to decide the adequate wholesale prices and repurchase prices.

The results of this study can be also applied to negotiation problems in the tandem supply chain composed of four or more members. The expansion of the model into such a situation has been considered. As the result, we have shown the whole collaborative bargaining solution in the tandem supply chain consisting of n members.

By the way, Ernez-Gahbiche *et al.* [4] have investigated a supply chain consisting of two suppliers and a customer. Their research examines a decentralized system in which each member optimizes only their own profits without considering the profit of the entire system. The member's policy in Ernez-Gahbiche *et al.* [4] is different from the member's policy in our study. However, the cooperative contract problem in the supply chain consisting of two suppliers and a customer might be one of interesting subjects. Furthermore, Kato *et al.* [7] have proposed the concept of new bargaining solution in consideration of the power balance between a manufacturer and a retailer. The influence of power balance in the tandem supply chain consisting of three members is also an interesting problem. We would like to consider also the influence of power balance in the tandem supply chain as one of further issue.

In this study, we have mainly considered about the collaborative bargaining solution of a negotiation problem in the tandem supply chain consisting of the manufacturer, wholesaler and retailer. Further, utilizing the implication obtained by solving this problem, we have considered about the collaborative bargaining solution of an extended negotiation problem in the tandem supply chain consisting of the manufacturer, several wholesalers and retailer. Furthermore, there are many other subjects for supply chains besides the collaborative bargaining solution of a negotiation problem in the supply chain. Roy *et al.* [14] have studied a optimal retail pricing problem in a two-echelon supply chain comprising of one manufacturer and two competing retailers with sales price dependent demand and random arrival of customers.

As mentioned above, demands of products are influenced by sales price. On one hand, the demands of products influence carbon emission. Hence, the decision of sales price is also related with corporate social responsibility index. Sara [16,17] has considered a problem about price contest between green and non green producer from the viewpoint of corporate social responsible firms. The problems about optimal retail pricing or green supply chain are also important subjects. We would like to also investigate the collaborative solution of retail pricing under the influence of retail pricing and the bargaining solution between the green supply chain and the non-green supply chain.

Further, Barman *et al.* [1] and Saha *et al.* [15] have examined on the role of government through a means of the government subsidy and tax policy for green supply chain management. The role of government through a means of the government subsidy and tax policy for green supply chain management has not been considered in this study. The design and operation of the green supply chain is considered as an important issue at present from the viewpoint of sustainable development goals (SDGs). Hence, we would like to also examine the mathematical formulation and the collaborative solution for the contract problem in the green supply chain as the tandem supply chain. In consequence, the problems mentioned above would be addressed as future subjects.

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