EQUILIBRIUM DECISIONS AND COORDINATION OF A LOGISTICS SERVICE SUPPLY CHAIN WITH BILATERAL CORPORATE SOCIAL RESPONSIBILITY

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Abstract. This paper establishes centralized decision-making models (CDMM) and decentralized decision-making models (DCDMM) for a two-echelon logistics service supply chain (LSSC) composed of an integrator and a functional provider. This takes into consideration implementation of corporate social responsibility (CSR). The equilibrium decisions are obtained and the impacts of CSR on the price, service level, consumer surplus, profit, and utility of the LSSC are discussed. A newly designed two-part pricing contract ensures implementation of CSR. We demonstrate that (1) under DCDMM, improvement of the functional provider’s CSR has more positive effects on the level of service, market demand, consumer surplus, overall profit, and utility of the LSSC than the same level of CSR for an integrator. (2) An increase of one member’s CSR upgrades the profit of the other members but damages its own profits. Therefore, LSSC members’ final profits will depend on the influence of CSR on both sides. (3) There is an optimal total critical CSR value for an integrator and functional provider. If the sum of both CSR values exceeds this, the profit under CDMM will be less than that under DCDMM, and LSSC members need not implement contract coordination. Finally, a numerical example is used to verify the relevant conclusions.

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1. Introduction

For the past three decades, the Chinese government has been interested in the logistics industry, which plays a basic supporting role in national economic growth [2]. As the logistics industry develops rapidly, various related partners are making full use of supply chain concepts to construct logistics service supply chains (LSSC) and improve their core competitiveness. The core enterprise is the logistics service integrator, which obtains logistics orders from customers and uses its own information system to allocate orders to functional providers [19, 20, 33]. Then, the functional providers complete logistics services for the customers. For example, Cainiao Networks facilitates nationwide delivery in China. It is a logistics integrator that built a logistics service platform. Shentong, Yuantong, and Zhongtong are the supply chain’s functional logistics providers. These companies
cooperate to provide efficient distribution through division of labor and to reduce logistics costs. In this way, they have formed a relatively stable supply chain.

Meanwhile, sustainable development is a common concern for both enterprises and society. More and more enterprises believe that implementing Corporate Social Responsibility (CSR) is critical to achieve sustainable development [9]. CSR means that companies should be concerned with a wider range of stakeholders, such as consumers, employees, and shareholders [6,16].

For example, Alibaba established a poverty alleviation fund and has invested 10 billion yuan in poverty alleviation over the past 10 years in China. While fighting COVID-19 in China in 2020, JD Logistics helped the government deliver millions of relief supplies in a short time with no projected profit. However, lack of CSR is common for logistics nodes in the actual activities of the LSSC [11, 29]. For example, in China, Zhongtong, Shentong, and Yuantong have been reported as frequently collecting secondary charges for township business. Cainiao, the logistics integrator, has also been accused of ignoring the interests of consumers in a business battle with SF Express. As a result, merchants could not determine whether buyers had received the goods, and buyers could not track logistics information during the transportation of goods. There is no doubt that these behaviors lack social responsibility. These actions result in the loss of consumers and the decline of logistics enterprises. Furthermore, it has a serious impact on other LSSC members’ profits and damages the competitiveness of the whole LSSC. Therefore, lack of CSR is not a single enterprise’s internal issue but an issue that damages the interests of all members of the LSSC.

The main reason for this lack of CSR is the fact that the upstream and downstream enterprises in the LSSC focus only on maximizing their own economic interests. It is well known that implementation of CSR is costly. Therefore, enterprises often lack the motivation to implement it [35]. Fortunately, CSR also brings potential benefits to an enterprise due to good publicity and an improved reputation. In this way, CSR exerts a positive externality on other enterprises in the LSSC. However, it is uncertain whether the enterprise of implementing CSR will profit from the change. Therefore, whether an enterprise implements CSR or not depends on whether its profits increase. Regardless of profit, implementation of CSR is required for sustainable development of the LSSC. Finding a way to ensure effective implementation of CSR in the LSSC has become a problem. To address it, the following questions need to be answered:

1. How to determine its equilibrium service level and price for a LSSC implementing CSR?
2. What is the influence of CSR on the equilibrium decision in a LSSC?
3. How can CSR behavior in a LSSC be coordinated?

In the paper, we investigated a two-echelon LSSC with bilateral CSR. That is, the integrator and functional provider of the two-echelon LSSC both implement CSR. A centralized decision-making model (CDMM) and a decentralized decision-making model (DCDMM) were established, and the equilibrium solutions were obtained. At the same time, we discussed the influence of CSR on the equilibrium solutions and utility of supply chains. We included a two-part pricing contract designed to ensure implementation of CSR effectively. This paper makes three primary contributions. First, it has a different focus than prior work on unilateral CSR behavior in supply chains. We considered both the integrator and the functional provider to evaluate CSR behaviors from a holistic perspective and to analyze the influence on the equilibrium decision behavior of the two-echelon LSSC. Second, we looked at how the vertical cooperation decision is affected by the supply chain’s CSR intensity. Third, in order to give the integrator and the functional provider an inherent motivation to undertake CSR, a two-part pricing contract was proposed to ensure that the profits of both parties are improved.

In the rest of the paper, the literature review is presented in Section 2; Section 3 describes the problem and the hypothesis; Section 4 presents the model construction and analysis; Section 5 describes the coordination contract; Section 6 presents a numerical example; and the main conclusions and thoughts on future research are given in Section 7.
2. Literature review

There are three topics that are integral to this paper. The first is CSR in logistics enterprises, the second is management of CSR in supply chains, and the third is coordination of LSSC.

2.1. CSR of logistics enterprises

In the recent 10 years, the CSR of logistics enterprises has attracted the attention of scholars. The research topics mainly were focused on the three aspects: analysis of the core content of CSR, motivation analysis, and impact analysis after implementation of CSR. Duan et al. [8] believed that currently CSR for Chinese logistics enterprises refers to taking responsibility for safe storage and transportation of materials, for the People’s livelihoods, and for rescue and support in emergencies under the unified leadership of the government. Ciliberti et al. [5] discussed the nature of the CSR activities of logistics enterprises based on an empirical analysis of Italian companies. They found that the sustainable responsibility for procurement, transportation, packaging, and reverse logistics fell on the enterprises and were not shared by the local or national economy. Some scholars have focused on the driving factors behind logistics companies’ implementation of CSR. Carter and Jennings [4] believed that the principal driving factors are organizational culture, top-level design, and rules and regulations. Miao et al. [24], on the other hand, proposed that business ethics, consumer pressure, suppliers, competitors, and legal pressure were the main driving forces. The influence of CSR implementation on logistics enterprises have been analyzed by some scholars. Ciliberti et al. [5] proposed that CSR implementation had a positive effect on employee and enterprise. Singh and Verma [31] believed that implementing CSR can improve brand image and brand loyalty, which in turn has a positive impact on brand value. As mentioned above, existing research on the CSR of logistics enterprises is lacking with regard to its impact on the supply chain.

2.2. Supply chain CSR

However, CSR management based on supply chains has been extensively discussed. The related research generally regards any supply chain activities that benefit the consumers, such as providing better services and lower prices, as CSR behavior. The objective of decisions made to implement CSR behavior is utility maximization, including working in the interest of consumers. Scholars researched the impact of CSR behaviors implemented by a manufacturer and a retailer on the supply chain’s equilibrium decision-making, and designed a coordination contract correspondingly [26,27]. Raza [30] proposed coordinated contracts for pricing, inventory, and CSR investment decisions for a supply chain which consist of a manufacturer and a retailer. Kong et al. [15] constructed a cost-learning supply chain which consisted of a manufacturer and a retailer. The optimal decision-making of supply chain members was compared and analyzed under the implementation of CSR by the manufacturer and the retailer. Li and Li [17] analyzed a closed-loop supply chain’s decision problem under the centralized decision-making model (CDMM) and the decentralized decision-making model (DCDMM). In the closed-loop supply chain, CSR was exhibited by the manufacturer alone and the retailer alone. The impact of CSR and the optimal CSR exhibiting mode of the closed-loop supply chain were discussed. Long et al. [22] assessed the influence of supply chain members’ CSR behavior on recycling decisions of a closed-loop supply chain and conducted coordination research. Yao et al. [36] researched the influence of CSR behavior on pricing strategies when the retailer and manufacturer were responsible for sales efforts, respectively. Modak et al. [25] analyzed the optimal decision of an implementing CSR, two-echelon, dual-channel supply chain. He et al. [10] researched a travel supply chain composed of an offline travel service provider and an online travel agency. The online travel agency implements CSR. The influence of CSR on supply chain decision-making was assessed.

Most research on the impact of CSR on supply chain equilibrium decision-making or coordination mechanisms is based mainly on product supply chains. However, there is a lack of research on CSR in LSSC. In the LSSC, because customers directly receive both order services from the integrator and logistics services from the functional provider, the quality of logistics services is jointly determined by the integrator and the functional provider. LSSC demand is affected by the service quality level from both the integrator and the function
provider. However, in the product supply chain, customers only receive the manufacturer’s products and perceive their quality. Customer demand is mainly affected by the product quality level provided unilaterally by the manufacturer. Therefore, CSR in the LSSC is also implemented by both integrators and functional providers, and bilateral CSR must be considered in the LSSC. At the same time, existing research is deficient in two areas. One area is insufficient consideration of the profits lost to supply chain members of implementing CSR. Although the decision-making objective of enterprises implementing CSR is to maximize the total utility, the problem of loss of profits caused by implementing CSR has not been completely resolved. Because profit is a necessary condition for enterprises to implement CSR, supply chain members lack internal motivation and cannot guarantee the effectiveness of the implementation. On the other hand, due to the differences in operational management between LSSC and product supply chains [14, 18, 21], there are also differences in the decision process and decision variables. Therefore, the existing research literature cannot provide comprehensive guidance for the implementation of CSR in the increasingly active area of LSSC.

A few scholars have gradually begun to analyze CSR behavior based on LSSC. Tan et al. [32] were the first to investigate the CSR behavior of integrators in a two-echelon LSSC. They analyzed the impacts of CSR behavior on pricing and service decisions and concluded that improvement of CSR behavior was conducive to a service supply chain’s own utility and overall profit and to the utility of the LSSC. However, there were some shortcomings in the conclusions. The analysis was based only on implementation of CSR behavior by a single integrator, without considering implementation of CSR behavior by functional service providers. At the same time, the results only illuminated the impact of CSR on factors like price and utility, without considering the impact on profit. A coordination contract for CSR has not yet been proposed for a LSSC.

2.3. Contract coordination

Since a LSSC is centered on “service”, service costs must be paid to improve service levels. A coordination contract achieves dual optimization of service scores and profits. The coordination of supply chains have been studied by many scholars. Johari et al. [13] considered a pharmaceutical supply chain consisting of two pharma-manufacturers and one pharma-retailer and analyzed the CDMM, DCDMM, and coordinated contract model when pharma-manufacturers implemented CSR. Arcelus et al. [1] found that coordination can increase profits. Zhao et al. [39] discussed the optimal decision of supply chain revenue-sharing (RS) contracts, and further proposed a modified RS contract to coordinate supply chains. Yan and Zheng [34] analyzed the optimal strategies and coordination of supply chains under the DCDMM and CDMM, respectively. Zhang et al. [38] constructed the CDMM, DCDMM and RS contract supply chain model, and the Stackelberg game method was used to solve obtaining the optimal pricing strategy, the demands of lead users, and the contract parameter range of manufacturers and retailers in equilibrium. Hosseini-Motlagh et al. [12] analyzed quality level decisions under three different structural models (decentralized, centralized, and coordinated), and found that under the coordinated model, not only the quantity and quality of the products were improved, but supply chain members also achieved a win–win. Zhong et al. [40] carried out a coordination study of a three-level LSSC consisting of an e-commerce mall, an express company, and terminal distribution service providers. The revenue sharing and cost sharing coordination contracts were designed based on DCDMM and CDMM.

The above studies considered profit under CDMM as the goal when designing a coordination contract. However, simple profit maximization is no longer the decision-making goal when an integrator and functional provider implement CSR behaviors at the same time. We believe that taking the maximum utility of CDMM as the goal will produce some more interesting results than those existing studies on contract coordination.

3. Problem description

A two-echelon LSSC consists of an integrator (I) and a functional provider (S). The integrator accepts a logistics service order demand, $q$, from the customer at a logistics market price, $p$, and then allocates it to the functional provider at the logistics transaction price, $w$. According to Qin et al. [28], the unit logistics operation costs and those paid by the integrator and the functional provider are $c_I$ and $c_S$, respectively, and $p \geq w \geq c_I$, $c_S$. 


$w \geq c_S$. Service is a core issue during the operation of the supply chain. Both the integrator and functional provider provide services to customers. The main services include the integrator allocating orders reasonably and dealing with customers’ problems in a timely way. The functional provider continues to improve the on-time delivery rate of goods, the package integrity rate, and other logistics services. According to Zhu et al. [41], the more logistics services a company offers, the more market demand there will be for its services. On one hand, logistics market demand is negatively correlated with market price, $p$. On the other hand, it is positively correlated with integrator service level $e_I$ and functional provider service level $e_S$. Thus, the logistics service demand $q$ is as shown in equation (3.1).

$$q = a - bp + \lambda_I e_I + \lambda_S e_S$$

(3.1)

where $b$ is the customer’s sensitivity coefficient to the market price, $\lambda_I$ is the customer’s sensitivity factor to the integrator’s service level, and $\lambda_S$ is the customer’s sensitivity factor to the functional provider’s service level. Referring to the study of Dan [7], when supply chain members improve their own service levels, they will pay a certain amount of logistics service costs, specifically $\frac{k_i(e_i - e_0)^2}{2}$, $i = \{I, S\}$, in which $k_i$ represents the service improvement cost coefficient and $e_0$ represents the basic service level. Only when each member’s service level exceeds the basic service level will service improvement costs be incurred. For simplifying the calculation process, $e_0 = 0$ is assumed. The profit function of the integrator, the functional provider, and the whole LSSC are given by equations (3.2)–(3.4).

$$\pi_I = (p - w - c_I)(a - bp + \lambda_I e_I + \lambda_S e_S) - \frac{k_I e_I^2}{2}$$

(3.2)

$$\pi_S = (w - c_S)(a - bp + \lambda_I e_I + \lambda_S e_S) - \frac{k_S e_S^2}{2}$$

(3.3)

$$\pi_{SC} = (p - c_I - c_S)(a - bp + \lambda_I e_I + \lambda_S e_S) - \frac{k_I e_I^2}{2} - \frac{k_S e_S^2}{2}.$$ 

(3.4)

Assuming that both the integrator and the functional provider implement CSR behavior, Panda and Modak [27] predicted that CSR can be interpreted as a company transferring part of its own revenue to increase consumer surplus. Consumer surplus is expressed by equation (3.5), which shows the difference between the maximum price that consumers are willing to pay and the market price that they actually pay.

$$CS = \int_{p_{min}}^{p_{max}} q \, dp = \int_{\frac{a + \lambda_I e_I + \lambda_S e_S}{b}}^{\frac{a + \lambda_I e_I + \lambda_S e_S + \lambda_I e_I + \lambda_S e_S + a}{b}} (-bp + \lambda_I e_I + \lambda_S e_S + a) \, dp$$

$$= \frac{q^2}{2b} = \frac{(-bp + \lambda_I e_I + \lambda_S e_S + a)^2}{2b}.$$ 

(3.5)

It can be seen from the consumer surplus equation that the CSR fulfilled by the LSSC is jointly affected by its price and service level. The degree of CSR is determined by the proportion of consumer surplus considered by the LSSC. We use $t_I$, $t_S$, and $t_S + t_I \in (0, 1)$ to represent the degree of CSR behavior implemented by the integrator, the functional provider, and the whole LSSC. $t_I, t_S, t_S + t_I = 0$ indicates that each member of the LSSC only pursues its own profit and the decision-making goal is to maximize economic profit. $t_I, t_S, t_S + t_I \neq 0$ indicates that each member of the LSSC makes concessions on profit, and the decision-making goal is to maximize the sum of economic profit and some consumer surplus.

By combining equations (3.2)–(3.5), the utility function of the integrator, the functional provider, and the whole LSSC can be obtained, as shown in equations (3.6)–(3.8), respectively.

$$V_I = \pi_I + t_I CS$$

(3.6)

$$V_S = \pi_S + t_S CS$$

(3.7)

$$V_{SC} = \pi_{SC} + (t_I + t_S)CS.$$ 

(3.8)
4. Model construction and analysis

In the section, the equilibrium solutions for both CDMM and DCDMM were calculated. Then, the influence of the CSR degree of the integrator and functional provider on the equilibrium solution was discussed. Comparing the results for the two models allows us to design a two-part pricing contract to coordinate the profits of both parties.

4.1. The centralized decision-making model

The optimal decision can be gained under the CDMM for a LSSC similar to a general supply chain. The integrator manages the whole LSSC chain with the goal of maximizing utility and takes into consideration the interests of the consumers.

According to equations (3.4) and (3.8), the total utility of the LSSC can be obtained as shown in equation (4.1). C represents the CDMM.

\[ V^C_{SC} = \pi^C_{SC} + (t_I + t_S)CS. \]  

**Proposition 4.1.** In a LSSC where both integrator and functional provider implement CSR behavior, under the condition that the centralized decision model satisfies \( bk_1k_Ist_I - bk_1k IST_S + 2bk_Ik_S - k_I\lambda_S^2 - k_S\lambda_I^2 > 0 \) and \( 2k_SK_I(1-t_I-t_S)b - k_S\lambda_I^2 - k_I\lambda_S^2 > 0 \), the optimal market price is \( P^C = \frac{A_2k_I + \lambda_S^2(k_S + c_I)}{A_1} \) and the optimal service levels of the integrator and functional provider are, respectively, \( e^C_I = \frac{k_S\lambda_I A_S}{A_1} \) and \( e^C_S = \frac{k_I\lambda_S A_S}{A_3} \). Here,

\[
\begin{align*}
A_1 &= -bk_1k_Ist_I - bk_1k_Ist_S + 2bk_Ik_S - k_I\lambda_S^2 - k_S\lambda_I^2 \\
A_2 &= -ak_Sst_I - ak_Sst_S + bc_Ik_S + bc_Sk_S - c_I\lambda_S^2 - c_S\lambda_I^2 + ak_S \\
A_3 &= -bc_I - bc_S + a.
\end{align*}
\]

Based on Proposition 4.1, the optimal market demand is \( q^C = \frac{k_I A_S b k_S}{2A_1} \), the optimal consumer surplus is \( CS^C = \frac{k_I A_S b k_S}{2A_1} \), and the maximum profit and utility of the whole LSSC are \( \pi^C_{SC} = \frac{k_I A_S b k_S}{2A_1} \) and \( V^C_{SC} = \frac{k_I((c_I - c_S)b + a)^2k_S}{2A_1} \), respectively.

**Proof.** See Appendix A.

**Corollary 4.2.** \( \frac{\partial q^C}{\partial t_I} > 0, \frac{\partial q^C}{\partial t_S} > 0, \frac{\partial q^C}{\partial c_I} > 0, \frac{\partial q^C}{\partial c_S} > 0, \frac{\partial P^C}{\partial t_I} < 0, \frac{\partial P^C}{\partial t_S} < 0. \)

Corollary 4.2 shows that as the integrator and functional provider’s degrees of CSR increase, service levels will rise and market prices will fall. This is because in a LSSC, “service” is the most important issue for market competition. When the LSSC makes consumer interests a higher priority in its decision-making, it will provide customers with higher-quality services and will also make concessions to consumers with regard to market prices.

**Corollary 4.3.** \( \frac{\partial q^C}{\partial t_I} < 0, \frac{\partial q^C}{\partial t_S} > 0, \frac{\partial q^C}{\partial c_I} > 0, \frac{\partial q^C}{\partial c_S} < 0, \frac{\partial P^C}{\partial t_I} < 0, \frac{\partial P^C}{\partial t_S} > 0, \frac{\partial P^C}{\partial c_I} > 0, \frac{\partial P^C}{\partial c_S} > 0. \)

Corollary 4.3 shows that as the integrator’s and functional provider’s degrees of CSR increase, market demand, consumer benefits, and the total utility of the LSSC increase, but the overall LSSC profit decreases. Thus, there is a loss of profit in the LSSC under CDMM, meaning that there is no inherent motivation to implement CSR for members of the LSSC.
4.2. The decentralized decision-making model

Because both the integrator and functional provider implement CSR behaviors under the DCDMM, both elements aim to maximize utility, including the interests of consumers. The integrator and the functional provider play a Stackelberg game in which the integrator is the leader and the functional provider is the follower.

Let $m$ be the unit profit of the integrator. This specific decision sequence is: First, the integrator decides its own unit profit and service level; then the functional provider determines its own transaction price and service level. Substituting $p = w + m + c_I$ into equations (3.6) and (3.7), the utility functions of the integrator and functional provider can be found in equations (4.2) and (4.3), respectively. $D$ represents the DCDMM.

$$V_l^D = m(a - b(w + m + c_I) + \lambda_Ie_I + \lambda_SE_S) - \frac{k_Ie_I^2}{2} + \frac{t_I(a - b(w + m + c_I) + \lambda_Ie_I + \lambda_SE_S)^2}{2b} \tag{4.2}$$

$$V_S^D = (w - c_S)(a - b(w + m + c_I) + \lambda_Ie_I + \lambda_SE_S) - \frac{k_Se_S^2}{2b} + \frac{ts(a - b(w + m + c_I) + \lambda_Ie_I + \lambda_SE_S)^2}{2b} \tag{4.3}$$

**Proposition 4.4.** In a LSSC where both the integrator and functional provider implement CSR behavior, when the decentralized decision model satisfies the conditions $-btks + 2bks - \lambda_S^2 > 0$ and $-b_kk_St - 2b_kk_kStS + 4b_kk_kS - 2k_I\lambda_S^2 - k_S\lambda_I^2 > 0$, the optimal market and transaction prices are $p^D > \frac{k_s0k}{A_4}$ and $w^D > \frac{A_6}{A_4}$, respectively. The optimal market demand is $q^D > \frac{bsA_kk_k}{A_4}$, and the optimal service levels of the integrator and functional provider are $e^D > \frac{A_6k\lambda_S}{A_4}$ and $e^D > \frac{A_6k\lambda_I}{A_4}$, respectively. Here,

$$A_4 = -b_kk_kSt - 2b_kk_kStS + 4b_kk_kS - 2k_I\lambda_S^2 - k_S\lambda_I^2$$

$$A_5 = -bcskst - bcskstS + bcskstS - akstS + 3bcskS - bcsk - 2cs\lambda_S^2 + akS$$

$$A_6 = akk_kst + 2akk_kstS + cskk_kS^2 + cskk_kS\lambda_I^2 + \lambda_S^2 cskk_kI + kSk\lambda_I^2 cI - 3akk_kS.$$  

Based on Proposition 4.4, the optimal consumer surplus is $CS^D = \frac{k_s0k}{8A_4^2}$ and the optimal profit and utility of the integrator are $\pi^D_I = \frac{ksA_sA_kk_kk}{2A_4}$ and $V^D_I = \frac{k_sA_kk_kk}{2A_4}$, respectively. The optimal profit and utility of the functional provider are $\pi^D_S = \frac{k_s(-2b_kk_kS + aS - \lambda_S^2)A_kk_kk}{2A_4^2}$ and $V^D_S = \frac{k_s(2S - \lambda_S^2)k_kk_kk}{2A_4^2}$, respectively, and the optimal profit and utility of the whole LSSC are $\pi^D_{SC} = \frac{k_sA_sA_kk_kk}{2A_4^2}$ and $V^D_{SC} = \frac{k_sA_sA_kk_kk}{2A_4^2}$, respectively. Here,

$$A_7 = 2S - (t_I + t_S - 2)b + k_S\lambda_I^2 + 2k_I\lambda_S^2$$

$$A_8 = -2b_kk_kSt - 4b_kk_kStS + 6b_kk_kS - 3k_I\lambda_S^2 - k_S\lambda_I^2$$

$$A_9 = -bk_kk_kst - 3bk_kk_kstS + 6bk_kk_kS - 3k_I\lambda_S^2 - k_S\lambda_I^2.$$  

**Proof.** See Appendix A. \hfill \Box

**Corollary 4.5.** $\frac{\partial e^D_I}{\partial t_I} > 0, \frac{\partial e^D_S}{\partial t_S} > 0, \frac{\partial e^D_I}{\partial t_I} > 0, \frac{\partial e^D_S}{\partial t_S} > 0, \frac{\partial p^D_I}{\partial t_I} < 0, \frac{\partial p^D_S}{\partial t_S} < 0; \frac{\partial e^D_I}{\partial t_S} > 0, \frac{\partial e^D_S}{\partial t_S} > 0, \frac{\partial CS^D}{\partial t_I} > 0, \frac{\partial CS^D}{\partial t_S} > 0; \frac{\partial e^D_I}{\partial t_I} > 0, \frac{\partial e^D_S}{\partial t_S} > 0, \frac{\partial V^D_I}{\partial t_I} > 0, \frac{\partial V^D_S}{\partial t_S} > 0, \frac{\partial V^D_I}{\partial t_I} > 0, \frac{\partial V^D_S}{\partial t_S} > 0.$

Corollary 4.5 shows that increasing degrees of CSR for the integrator and the functional provider will increase service levels and decrease market prices. The reason is that when the integrator and functional provider prioritize consumer interests in decision-making, their service levels to customers improve and they also make price concessions to consumers. In addition, with a higher degree of integrator CSR, the transaction price will rise, which is beneficial to the functional provider. Meanwhile, the functional provider’s increases in CSR will cause the transaction price to decline, which is beneficial to the integrator. Therefore, subjects who implement CSR behavior will make concessions on the transaction price, a behavior that actually satisfies the subject’s own interests.

**Corollary 4.6.** $\frac{\partial e^D_I}{\partial t_S} > 0, \frac{\partial e^D_S}{\partial t_I} > 0, \frac{\partial V^D_I}{\partial t_I} > 0, \frac{\partial V^D_S}{\partial t_S} > 0, \frac{\partial V^D_I}{\partial t_I} > 0, \frac{\partial V^D_S}{\partial t_S} > 0; \frac{\partial e^D_I}{\partial t_S} > 0, \frac{\partial e^D_S}{\partial t_I} > 0, \frac{\partial V^D_I}{\partial t_I} > 0, \frac{\partial V^D_S}{\partial t_S} > 0, \frac{\partial V^D_I}{\partial t_I} > 0, \frac{\partial V^D_S}{\partial t_S} > 0.$
Corollary 4.6 shows that with increasing integrator and functional provider CSR, the utility of the integrator and functional provider, the consumer surplus, and the profit and utility of the LSSC all increase. CSR behavior has a beneficial impact on consumer interests and on the entire LSSC. This proves the necessity of implementing CSR for the LSSC.

**Corollary 4.7.** 1 \( \frac{\partial \pi_I^*}{\partial t_I} < 0 \), \( \frac{\partial \pi_S^*}{\partial t_S} > 0 \) 2 \( \frac{\partial \pi_F^*}{\partial t_I} > 0 \), when \( t_S < \frac{t_f}{2} + \frac{\lambda_S^2}{2kIb} \), \( \frac{\partial \pi_S^*}{\partial t_S} > 0 \); when \( t_S > \frac{t_f}{2} + \frac{\lambda_S^2}{2kIb} \), \( \frac{\partial \pi_S^*}{\partial t_S} < 0 \).

Corollary 4.7 shows that 1 as the integrator’s CSR degree increases, its profit decreases. However, as the functional provider’s CSR degree increases, the integrator’s profit increases. The final impact on the profit of integrator depends on the degree of impact of the CSR behavior. 2 As the integrator’s CSR degree increases, the profit of the functional provider increases. When satisfying \( t_S < \frac{t_f}{2} + \frac{\lambda_S^2}{2kIb} \), an increased degree of functional provider CSR will increase the functional provider’s profit. Thus, the impact on the functional provider’s profit will eventually increase. When \( t_S > \frac{t_f}{2} + \frac{\lambda_S^2}{2kIb} \) is satisfied, as the functional provider’s CSR degree increases, the functional provider’s profit decreases, and the final impact on the functional provider’s profit also depends on the degree of influence of the CSR behavior. For example, the beneficial effect of the integrator’s CSR behavior is greater than the negative effect of the functional provider’s CSR behavior, and the functional provider’s profits will increase.

Furthermore, increased integrator CSR behavior will increase of the functional provider’s profit, and increased functional provider CSR behavior will increase the integrator’s profit. That is, an increase of one member’s CSR will upgrade the profit of the other members, but its own profit will be damaged. Therefore, the member’s final profit will depend on the mutual influence of both parties’ CSR behavior.

**Corollary 4.8.** \( \frac{\partial V^{D^*}}{\partial t_I} < \frac{\partial V^{D^*}}{\partial t_S} \); \( \frac{\partial V^{D^*}}{\partial t_I} < \frac{\partial V^{D^*}}{\partial t_S} \); \( \frac{\partial V^{D^*}}{\partial t_I} < \frac{\partial V^{D^*}}{\partial t_S} \); \( \frac{\partial V^{D^*}}{\partial t_I} < \frac{\partial V^{D^*}}{\partial t_S} \); \( \frac{\partial V^{D^*}}{\partial t_I} < \frac{\partial V^{D^*}}{\partial t_S} \).

Corollary 4.8 shows that a one-unit CSR increase has different effects on an integrator and a functional provider. For the functional provider, it brings about a more significant drop in the market price, a greater increase in the service level, and more market demand volume, consumer surplus, and overall supply chain profit and utility. Therefore, because the functional provider is a crucial link in providing logistics services to customers, its CSR will have a greater positive effect on the whole LSSC and its stakeholders. Integrators, as the core enterprises, should carefully guide the CSR behavior of functional providers.

### 4.3. Comparison of equilibrium results

By comparing the of the two models’ equilibrium results, Corollaries 4.9 and 4.10 can be obtained.

**Corollary 4.9.** \( e^{C^*}_f > e^{D^*}_f, e^{C^*}_S > e^{D^*}_S, q^{C^*} > q^{D^*}, p^{C^*} < p^{D^*}, C^{SC^*} > CS^{D^*}, V^{C^*}_{SC} > V^{D^*}_{SC} \).

Corollary 4.9 shows that because of the double marginal effect, compared with the DCDMM, the CDMM has the lower market price and the higher service score, the higher market demand, the higher overall utility of the LSSC, and the higher consumer surplus. However, the entire LSSC’s profit under the CDMM is not necessarily higher than that under the DCDMM; the entire LSSC’s profit under the CDMM is closely related to the degree of CSR.

**Corollary 4.10.** When \( t_I + t_S \leq t^* \), \( \pi^{C^*}_{SC} > \pi^{D^*}_{SC} \); when \( t_I + t_S \geq t^* \), \( \pi^{C^*}_{SC} < \pi^{D^*}_{SC} \).

\[
t^* = \max(f(t_S)) = \frac{-bt_I k_I S - 4bk_I k_S + 2k_I \lambda_S^2 + k_S \lambda_I^2}{2bk_I k_S} + \frac{\sqrt{(\lambda_I^2 - 4bk_I \lambda_S^2 + b^2k_I^2(t_S^2 - 4t_I S + 8))k_S^2 + 2(\lambda_I^2 + bk_I(t_S - 4))\lambda_S^2 k_I k_S + 2\lambda_S^4 k_I^4}}{2bk_I k_S}.
\]
Corollary 4.10 shows that when the sum of the CSR levels of the integrator and the functional provider is less than the critical value \( t^* \), CDMM yields the greater total profit for the LSSC than DCDMM. Conversely, the entire LSSC’s profit under CDMM is less than that under DCDMM. This is mainly because some of profits of the LSSC are transferred to consumers as the degree of the total CSR increases under CDMM. However, under DCDMM, as the degree of one member’s CSR increases, its own profit decreases while the LSSC’s profit increases. Therefore, with CDMM, when the sum of the CSR levels is too high, the LSSC’s profit will be less than that with DCDMM. At that point, the LSSC members will not participate in contract coordination because there is no way to improve their profits. When the total integrator and functional provider CSR levels are low, the profit of the LSSC under CDMM is higher than that under DCDMM. Therefore, coordination is feasible.

5. Contract coordination model

In this section, the design of a two-part pricing contract to coordinate the LSSC was described [3,37]. Because consumers mainly perceive functional providers’ services in a LSSC, the service level of the functional provider has an important influence on consumers’ purchase decisions. In order to motivate a functional provider to improve its service level, the integrator must make concessions on the transaction price. This will inevitably harm the profits of the integrator. Therefore the loss needs to be compensated after the completion of the logistics service. Adopting a two-part pricing contract is a reasonable choice. The main implementation mechanism of the proposed contract is for the integrator to first accept the functional provider’s higher transaction price \( w^H \). After the logistics service is completed, the functional provider transfers part of the fixed fee subsidy \( F \) to the integrator. The utility function of the integrator and the functional provider are shown in equations (5.1) and (5.2), respectively. The coordination contract model is represented by \( H \).

\[
V_i^H = m(a - b(w + m + c_I) + \lambda_I e_I + \lambda_S e_S) - \frac{k_I e_I^2}{2} + \frac{t_I(a - b(w + m + c_I) + \lambda_I e_I + \lambda_S e_S)^2}{2b} + F \tag{5.1}
\]

\[
V_S^H = (w - c_S)(a - b(w + m + c_I) + \lambda_I e_I + \lambda_S e_S) - \frac{k_S e_S^2}{2} + \frac{t_S(a - b(w + m + c_I) + \lambda_I e_I + \lambda_S e_S)^2}{2b} - F \tag{5.2}
\]

**Proposition 5.1.** In a LSSC where both integrator and functional provider implement CSR behaviors, when \( t_I + t_S \leq t^* \), supply chain coordination can be achieved by drafting two pricing contracts. The optimal market price is \( P^{H^*} = \frac{\lambda_k k_I + \lambda_S k_S(c_S + e_S)}{A_I} \), the optimal service levels of the integrator and functional provider are \( e_I^{H^*} = \frac{k_S \lambda_S A_S}{A_I} \) and \( e_S^{H^*} = \frac{k_I \lambda_I A_I}{A_S} \), respectively, and the optimal transaction price is \( w^{H^*} = \frac{(t_I + t_S)k_S + (c_S + e_S)}{A_S} \), \( p^{H^*} \leq w^{H^*} \). The two-part pricing contract can be implemented between supply chain members when the fixed fee subsidy \( F \) satisfies \( F \leq F \).

Based on Proposition 5.1, the profit and utility of the integrator are \( \pi_i^{H^*} = \frac{-\lambda^2 k_I k_S(2k_I t_I b + \lambda_I^2)}{2A_I^2} + F \) and \( V_i^{H^*} = \frac{-\lambda^2 k_I k_S^2}{2A_I^2} + F \); the profit and utility of the functional provider are \( V_S^{H^*} = \frac{-\lambda^2 (k_S (t_S + t_I - 2b) + \lambda_S^2) k_I k_S}{2A_S^2} - F \); and the optimal consumer surplus is \( CS^{H^*} = \frac{k_I \lambda^2 A_S^2}{2A_I^2} \), and the maximum profit and utility of the whole LSSC are \( \pi_{SC}^{H^*} = \frac{k_I \lambda^2 A_S^2 k_S}{2A_I^2} \) and \( V_{SC}^{H^*} = \frac{k_I ((-c_I - e_S)b + a)^2 k_S}{2A_I^2} \).

**Proof.** See Appendix A. \( \Box \)

In order to encourage both the integrator and the functional provider to participate in contract coordination, their profits must at least be greater than the profits under the DCDMM. That is, \( \pi_i^{H^*} \geq \pi_i^D \) and \( \pi_S^{H^*} \geq \pi_S^D \).
Based on this, the change interval of the fixed fee subsidy can be obtained as follows:

\[
F = -\frac{A_2^2k_1k_S^2(2k_1t_Ib + \lambda_I^2)}{2A_1^2} + \frac{k_SA_3^2A_7k_I}{2A_4^2},
\]

\[
\bar{F} = -\frac{(k_S(t_S - 1)b + 1/2\lambda_S^2)k_S A_3^2}{A_1^2} - \frac{k_S(-2bk_St_S + 2bk_S - \lambda_S^2)A_3^2k_I^2}{2A_4^2}.
\]

Proposition 5.1 shows that the total utility and profit of the LSSC can match those in the CDMM with the two-part pricing contract in place. Implementing the contract will make the integrator’s market price less than or equal to the sum of the transaction price and the unit logistics operation cost. When undertaking CSR behavior, the integrator will make concessions on pricing. The functional provider needs to transfer the fixed subsidy fee to the integrator. The closer \(F\) is to \(\bar{F}\), the more the integrator benefits. The closer \(F\) is to \(\bar{F}\), the more the functional provider benefits.

6. Numerical Analysis

The numerical analysis in this section refers to the research [23]. We used Maple software to compare the supply chain equilibrium solutions from different decision-making models: \(a = 100, b = 5, k_I = 7, \lambda_I = 3, \lambda_S = 4, k_S = 6, c_I = 7, \) and \(c_S = 6\). According to Proposition 4.1, only when \(t_I + t_S \leq 0.67\) it is possible to ensure that the LSSC members’ profits and the overall profits are positive.

It can be seen from Figures 1 to 4 that with improvement of integrator and functional provider CSR behavior, market prices fall, service levels and market demands increase, and the effect of the CDMM is better than that of the DCDMM.

It can be seen from Figure 5 that in the CDMM, improving the integrator and functional provider CSR behavior benefits to their own utility.

Figure 6 shows that as the integrator’s CSR behavior increases, profit decreases. In contrast, as the functional provider’s CSR behavior increases, the integrator’s profit increases. If the positive effect of the functional provider’s CSR behavior is greater than the negative effect of the integrator’s CSR behavior, the integrator’s profit will eventually be greater than it would have been without implementing CSR behavior. For example, when \(t_S = 0.3\) and \(t_I = 0.1\), the integrator’s profit is much greater than the profit when \(t_S = 0\) and \(t_I = 0\), and there is an inherent motivation to implement CSR behavior.

When the integrator’s CSR behavior increases, the functional provider’s profit increases. When the functional provider’s CSR behavior increases and \(t_S < \frac{t_I}{2} + \frac{9}{70}\), its profit increases; when \(t_S > \frac{t_I}{2} + \frac{9}{70}\), its profit decreases. For example, when \(t_I = 0.2\), if \(t_S < 0.23\), the functional provider’s profit increases (see Fig. 6); if \(t_S > 0.23\), its profit decreases. Furthermore, it is evident that when \(t_S < \frac{t_I}{2} + \frac{9}{70}\), the functional provider’s profit is greater
Figure 2. The influence of \((t_I, t_S)\) on market demand.

Figure 3. The influence of \((t_I, t_S)\) on integrator’ service level.

Figure 4. The influence of \((t_I, t_S)\) on functional provider’ service level.
than it would be without CSR behavior. When \( t_S < \frac{t_I^2}{2} + \frac{9}{40} \), if the positive impact of the integrator’s CSR behavior is greater than the negative impact of the functional provider’s CSR behavior, the functional provider’s profit is greater than it would be without implementing CSR behavior. For example (see Fig. 6), when \( t_S = 0.1 \) and \( t_I = 0.2 \), the profit of the functional provider is much greater than that produced by \( t_S = 0 \) and \( t_I = 0 \), and the functional provider has an inherent motivation to implement CSR behavior.

Figures 7 and 8 show that as the total integrator and functional provider CSR increases, the overall utility of the LSSC and consumer surplus increases; therefore, the value of CDMM is greater than that of DCDMM.

Figure 9 shows that the total profit of the LSSC declines under the CDMM and rises under the DCDMM with improvements in the CSR scores for the integrator and functional provider. When \( t_I + t_S > 0.38758 \), the total profit of the LSSC under the CDMM is smaller than that under the DCDMM. However, in order to improve the LSSC’s total utility and profit and to enable the LSSC members to coordinate, it is necessary to satisfy \( t_I + t_S < 0.38758 \). For example, when \( t_I = 0.1 \) and \( t_S = 0.1 \), the total profit of the supply chain under the CDMM is greater than that under the DCDMM.

Next, we verify the validity of the two-part pricing contract. When \( t_I = 0.1 \) and \( t_S = 0.1 \), the equilibrium results for the CDMM, the DCDMM, and the two-part pricing contract are followed as Table 1.

Table 1 reveals that

(1) After participating in the two-part pricing contract, no matter how much of the fixed payment the integrator receives, the integrator and functional provider service levels (3.49 and 5.43), market price (18.29), market demand (40.72), consumer surplus (165.81), total profit (84.49), and the utility of the supply chain (142.52)
Figure 7. The influence of \((t_I, t_S)\) on consumer surplus.

Figure 8. The influence of \((t_I, t_S)\) on the total utility of a supply chain.

Figure 9. The influence of \((t_I, t_S)\) on overall profit of a supply chain.
Table 1. Equilibrium solution of the LSSC under different decision mode models.

<table>
<thead>
<tr>
<th>Variable</th>
<th>DCDMM</th>
<th>CDMM</th>
<th>Two-part pricing contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^*$</td>
<td>–</td>
<td>104.33</td>
<td>105.67</td>
</tr>
<tr>
<td>$e^*_I$</td>
<td>1.96</td>
<td>3.49</td>
<td>3.49</td>
</tr>
<tr>
<td>$e^*_S$</td>
<td>1.26</td>
<td>5.43</td>
<td>5.43</td>
</tr>
<tr>
<td>$q^*$</td>
<td>14.73</td>
<td>40.72</td>
<td>40.72</td>
</tr>
<tr>
<td>$\pi^*_I$</td>
<td>51.55</td>
<td>–</td>
<td>61.40</td>
</tr>
<tr>
<td>$\pi^*_S$</td>
<td>27.48</td>
<td>–</td>
<td>47.92</td>
</tr>
<tr>
<td>$V^*_I$</td>
<td>29.65</td>
<td>–</td>
<td>59.94</td>
</tr>
<tr>
<td>$V^*_S$</td>
<td>17.97</td>
<td>54.91</td>
<td>54.91</td>
</tr>
<tr>
<td>CS*</td>
<td>76.86</td>
<td>84.94</td>
<td>84.94</td>
</tr>
<tr>
<td>$V^*_SC$</td>
<td>81.20</td>
<td>142.52</td>
<td>142.52</td>
</tr>
</tbody>
</table>

are all equal to those obtained by centralized decision-making. The transaction price under coordination (12.24) is far greater than that under decentralized decision-making (8.65). Evidently, the integrator makes concessions on the transaction price under contract coordination. Therefore, the functional provider needs to transfer a certain fixed fee subsidy to the integrator.

(2) When the integrator obtains the smallest optimal fixed fee subsidy (104.33), it obtains at least the profit (49.34) under the DCDMM. When the integrator obtains the largest optimal fixed fee subsidy (124.77), it gains the greatest profit improvement (69.82).

(3) With constantly increasing fixed fee subsidies (104.33, 105.67, 110.89, 115.67, 120.78, 122.34), the functional provider’s profit continues to decrease (47.92, 46.58, 41.36, 36.58, 31.47, 27.48). It is at least equal to the profit under the DCDMM (27.48).

7. Conclusions and management insights

This paper is based on a two-echelon LSSC composed of an integrator and a functional provider. The impact of the integrator’s and functional provider’s CSR behaviors on pricing and service decisions, the profit and utility of all parties, and the implementation of contract coordination was discussed. Our study led to some important conclusions and management implications.

7.1. Main conclusions

The paper makes three main contributions to LSSC literature. First, an increase in the degree of one party’s CSR will increase the other party’s profit but will bring about a reduction in its own profit. The final profit of the integrator and the functional provider depend on the interaction of bilateral CSR.

Secondly, when the functional provider increases its degree of CSR, there is a positive impact on the service level, market price, market demand, and consumer surplus, as well as the total profit and utility of the LSSC. The effect is greater than that produced by the same degree of integrator CSR.

Finally, compared with CDMM, DCDMM results in lower service levels, lower market demand, lower consumer surplus, and lower overall utility of the LSSC. Therefore, these values can be increased. However, when the integrator and the functional provider CSR scores exceed a certain critical value, the total profit of the LSSC under CDMM will be less than that under DCDMM. The double marginal effect that exists in the traditional supply chain disappears, and the LSSC members will not participate in contract coordination. So, CSR must be within a certain critical range for the contract coordination design to be feasible. Within this range, LSSC coordination can be achieved by implementing a two-part pricing contract. The overall utility and profit of the
LSSC can thus reach the levels achieved by CDMM, and the profits of the integrator and the functional provider can be improved as well.

7.2. Insights for managers and researchers

This study offers several insights for enterprise's managers. First, the implementation of CSR in the LSSC is at the cost of profits. The greater the degree of CSR implementation, the greater the utility and consumer surplus, and the smaller the profit of the implementer. This conclusion suggests that decision makers in the LSSC need to balance between profits and CSR. Second, it is easier for the functional provider to undertake CSR than the integrator; therefore, the integrator needs to guide the provider's CSR behavior. Integrators can formulate certain measures to encourage logistics service providers to actively undertake CSR. Third, when the degree of total CSR of the LSSC is lower, the integrator must coordinate through a certain contract such as a two-part pricing contract to obtain a higher total LSSC profit. Otherwise, there is no need for coordination, and the decision making can be decentralized.

This paper offers researchers two main theoretical aspects for consideration of CSR in LSSCs. The implementation of bilateral social responsibility in a LSSC results in the loss of one's own interests and the profit of the other party. This has the characteristic of a free ride. The question becomes how to allocate CSR fairly between the integrator and the functional provider. Additionally, this study considers a LSSC composed of an integrator and several similar functional providers with competitive relationships. Therefore, the multilateral CSR aspects of a LSSC need to be considered.

One possibility for future research in this area is investigation of optimal allocation of CSR in a LSSC. At the same time, the multilateral CSR of a LSSC can also be used to guide further research.

APPENDIX A.

A.1. Proof of Proposition 4.1

Solving the Hessian matrix

\[
H_1 = \begin{bmatrix}
\frac{\partial^2 V_{SC}}{\partial p \partial e_I} & \frac{\partial^2 V_{SC}}{\partial p \partial e_S} & \frac{\partial^2 V_{SC}}{\partial e_I \partial e_S} \\
\frac{\partial^2 V_{SC}}{\partial p \partial e_I} & \frac{\partial^2 V_{SC}}{\partial e_I^2} & \frac{\partial^2 V_{SC}}{\partial e_I \partial e_S} \\
\frac{\partial^2 V_{SC}}{\partial e_S \partial e_I} & \frac{\partial^2 V_{SC}}{\partial e_I \partial e_S} & \frac{\partial^2 V_{SC}}{\partial e_S^2}
\end{bmatrix} = \begin{bmatrix}
(t_I + t_S - 2)b & -(t_I + t_S)\lambda_I + \lambda_S & -(t_I + t_S)\lambda_S + \lambda_S \\
-(t_I + t_S)\lambda_I + \lambda_I & -k_I + \frac{(t_I + t_S)\lambda_I^2}{b} & \frac{(t_I + t_S)\lambda_I}{b} \\
-(t_I + t_S)\lambda_S + \lambda_S & \frac{(t_I + t_S)\lambda_S \lambda_I}{b} & -k_S + \frac{(t_I + t_S)\lambda_S^2}{b}
\end{bmatrix}
\]

of \( V_{SC}^C(p, e_S, e_I) \) and when the conditions \(-bt_1k_S + 2bk_S - \lambda_S^2 > 0 \) and \( bt_1k_S - 2bk_1k_S + k_I\lambda_S^2 + k_S\lambda_I^2 < 0 \) hold, we can obtain the first-order principal minor \((t_I + t_S - 2)b < 0\), the second-order principal minor \(-bt_1k_S + 2bk_S - \lambda_S^2 > 0\), and the third-order principal minor \(b(t_I + t_S)k_1k_S - 2bk_1k_S + k_I\lambda_S^2 + k_S\lambda_I^2 < 0\). Hence, \( H_1 \) is negative definite, and \( V_{SC}^C(p, e_S, e_I) \) has a unique optimal solution. By solving the system of equations \( \frac{\partial V_{SC}^C}{\partial p} = 0 \), \( \frac{\partial V_{SC}^C}{\partial e_I} = 0 \), and \( \frac{\partial V_{SC}^C}{\partial e_S} = 0 \), the equilibrium values of the LSSC decision variables \( p^C^*, e_I^C^*, e_S^C^* \), hence, the values of \( q^C^*, \pi_{SC}^C^*, C^C^*, V_{SC}^C^* \), can be obtained. The proof is completed.

A.2. Proof of Proposition 4.4

First, find the Hessian matrix \( H_2 = \begin{bmatrix}
(t_S - 2)b & -t_S\lambda_S + \lambda_S \\
-t_S\lambda_S + \lambda_S & -k_S + \frac{t_S\lambda_S^2}{b}
\end{bmatrix} \) of the functional quotient \( V_{SC}^D(w, e_S) \), and when the condition \(-bt_1k_S + 2bk_S - \lambda_S^2 > 0 \) holds, the first-order principal minor is less than zero, and the second-order principal minor is greater than zero is obtained. Then \( H_2 \) is negative definite. Therefore, under the above condition, the equilibrium values of the LSSC decision variables \( w^*, e_S^* \) can be obtained from the solution
of $\frac{\partial V^D}{\partial w} = 0$ and $\frac{\partial V^D}{\partial e_S} = 0$. Substituting $w^*$ and $e_S^*$ into the utility function of the integrator and solving for the Hessian matrix $V^D_I(m, e_I)$ of integrator, we obtain

$$H_7 = \begin{bmatrix}
2((t_S+1/2t_I-2)k_Sb+\lambda^2)^2k_S & -\lambda b((t_S+1/2t_I-2)k_Sb+\lambda^2)k_S \\
\lambda b((t_S+1/2t_I-2)k_Sb+\lambda^2)k_S & -b((t_S-2)k_S+\lambda^2)k_S
\end{bmatrix}.$$  

When $-bk_1k_SI - 2bk_1k_SI + 4bk_1k_SI - 2k_1\lambda^2 - k_SI_2 > 0$ holds, we can obtain the first-order principal minor $2((t_S+1/2t_I-2)k_Sb+\lambda^2)^2k_S < 0$, the second-order principal minor $-2b^2((k_S(t_S+1/2t_I-2)b+1/2\lambda^2)k_S+\lambda^2)k_S > 0$, and then $H_3$ is negative definite. Hence, under the above conditions, the equilibrium values of the LSSC decision variables $m^D^*, e_I^D^*$ can be obtained from the solution of $\frac{\partial V^D}{\partial m} = 0$ and $\frac{\partial V^D}{\partial e_I} = 0$. Substituting $m^D^*, e_I^D^*$ into $w^*, e_S^*$, we can get $w^D^*, e_S^D^*$. According to $p = w + m + c_I$, we can obtain $p^D^*$, and hence $q^D^*, \pi^D^I, V^D_I, \pi^D_S, V^D_S, \pi^D_C, CS^D, V^D_{SC}$. The proof is completed.

A.3. Proof of Proposition 5.1

If this condition $-bt_SI + 2bk_SI - \lambda^2 > 0$ holds when solving for the Hessian matrix $H_2 = \begin{bmatrix}
(t_S-2)b & -t_SI^2 + \lambda_S^2 \\
-t_SI^2 + \lambda_S^2 & -k_S + t_SI^2k_S^2
\end{bmatrix}$ of functional quotient $V^D_I(w, e_S)$, we can determine that the first-order principal minor is less than zero, the second-order principal minor is greater than zero. Hence, $H_2$ is negative definite. Under the above conditions, the equilibrium values of the LSSC decision variables $w^*$ and $e_S^*$ can be obtained from the solution of $\frac{\partial V^D}{\partial w} = 0$ and $\frac{\partial V^D}{\partial e_S} = 0$. In order to make the total profit and utility of the LSSC fit the CDMM, $e_S^H^* = e_S^C^*$ and $e_I^H^* = e_I^C^*$ must be satisfied, and the logistics transaction price after contract coordination can be obtained is written as $w^H^*$. Hence, all the values $p^H^*, q^H^*, \pi^H^I, V^H_I, \pi^H_S, V^H_S, CS^H, \pi^H_C,$ and $V^H_{SC}$ can be obtained.

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References