IMPACTS OF GREEN AND PRESERVATION TECHNOLOGY INVESTMENTS ON A SUSTAINABLE EPQ MODEL DURING COVID-19 PANDEMIC

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Abstract. Carbon and Sulfur dioxides emissions are the key issues of global warming that affects on human health. Emissions cap- and -trade policy is a key mechanism implemented in several countries to reduce the emissions. Nowadays, public gathering is restricted due to the pandemic situation caused by COVID-19. As a result, people are facing huge problems in their regular activities and lifestyle. During the lockdown periods, demands for few merchandises decrease and the deterioration rate increases. Moreover, because of the unavailability of raw materials and labours during the lockdown, shortages occur at the manufacturing company. Keeping these problems in mind, a multi-objective sustainable economic production quantity model is proposed with partially back-ordering shortages, in which the effects of sustainability are investigated. To handle the demand fluctuation throughout the current pandemic, emergency level dependent demand rate is assumed. To reduce greenhouse gases emissions and deterioration rate, investments in green technology and preservation technology efforts are used. The objectives of this study are to maximize the manufacturer’s profit and minimize the greenhouse gases emissions for producing green products. The multi-objective model is solved by utilizing the fuzzy goal programming approach. The mathematical model is illustrated by four numerical examples. The main finding of the work is that under both green and preservation technologies investments, a sustainable model with partially back-ordering shortages and lockdown level dependent demand rate decreases justifiable greenhouse gases emissions and increases the product’s greening level. The results indicate that the system profit is increased by 16.1% by investing in both preservation and green technology. Furthermore, a sensitivity analysis is performed along with some managerial insights for practitioners. Finally, the paper is ended with conclusions and future research tips.

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1. Introduction

In this present situation, coronavirus which is also known as COVID-19 is an important topic of discussion all over the world. It has halted the stride of life and affected the world. Starting from December 2019, it affects
the international market economically. It is predicted that the most current impact of COVID-19 outbreak on the manufacturing firms is higher in comparison with any other previous major outbreaks such as 2003 SARS and 2009 H1N1 (see Ref. [26]). Khalilpourazari and Doulabi [18] proposed an algorithm that forecasts the number of asymptomatic, symptomatic, recovered, life-threatening and death cases during COVID-19 pandemic. Several countries adopted various policies and timings for the lockdown periods. But, their emergency levels mostly followed a similar rule. These levels could affect the market demand negatively for production business managements due to customers’ unavailability in a market and supply shortages of raw materials. On the other hand, some products in the market were saturated during the pandemic due to the unavailability of buyers to buy, which led to economic inconsistency for governments and companies. Moreover, due to this pandemic situation, the income of one class of people in the society has decreased. For these reasons, demand of many products such as commodities associated with transportation, including cooking vegetable oil, chemical products, clothes, and full-fat dairy products etc., has decreased during the current pandemic. Thus, to justify declining demand, the decreasing demand function of emergency level is considered. Due to decreasing in the products’ demand and the lack of workers who manage the storage of goods, the quality of goods is declining. Furthermore, demand is dependent on product’s selling price. Paul et al. [36] studied a retailer model under selling price-dependent demand to maximize the profit. Hence, assumption of variable demand which decreases due to the coronavirus outbreak and selling price, is considered in this present work.

Deterioration is an important numerous attribute that can affect inventory management. Deterioration of goods includes obsoletes, decay, evaporation, and a decline in quality during the holding time in a warehouse. Roy et al. [42] created a two-warehouse probabilistic model for deteriorating products. The deterioration rate was followed an Weibull distribution in their model. Mohammadi and Khalilpourazari [32] considered a single machine arranging problem with linear deterioration rate. Due to the lack of sales during COVID-19 pandemic, finished goods are kept in the warehouse for a long time. As a result, expiry dates of the products are almost reaching. Consequently, the quality of goods spoiled, i.e., deterioration rate increased and it affects the company’s profit negatively by increasing deterioration cost. Hence, reduction of deterioration rate is an important issue to a company during COVID-19 period. The deterioration rate is supposed to be an unbalanced parameter, whereas, investment in preservation technology (PT) can be balanced up to a certain level. PT has been applied in various industries, such as food industries and agriculture, to extend the product’s life cycle, and hence it affects the total profit. Mashud et al. [28] developed a sustainable retailer inventory model by investing in PT and green technology (GT) to control the deterioration rate and greenhouse gases emissions, respectively. A two-echelon location routing problem for green energy efficiency logistics systems was investigated by Tirkolaee et al. [53]. However, the impact of the present COVID-19 pandemic on deterioration rate has not been considered in their model. This work studies this effect under PT investment.

Since the industrial revolution, the manufacturing business has been developed in numerous countries, resulting in factories emitting a large amount of greenhouse gasses, such as carbon dioxide (CO₂), sulphur dioxide (SO₂) gases, etc. These emissions come from the process such as production, storing items in a warehouse, and transportations are the major sources of greenhouse gases. These emissions cause environmental fluctuations such as the greenhouse effect, ozone layer deficiency, and acid rain, which damage the nature and it is ultimately endangered for human health. Hence, to reduce greenhouse gases emissions, many developed countries have begun to discuss related threats and implemented emissions cap- and -trade policy. Under this policy, the most emissions source like as a manufacturing company is subject to a boundary of total emissions. This boundary is called emissions cap. If the produced emissions by the company exceeds the cap then the extra emissions must be purchased at a market price which is called emissions trading price. On the contrary, if emissions do not exceed the cap, the remainder can be sold at the market price. Chen et al. [10] studied a collaborative carbon-reducing investment in a GSC model under carbon cap- and -trade policy. An agreement called the Kyoto Protocol has established for developed countries to grow the energy-efficient tools during the production to restrict greenhouse gases emissions. It has been established in 1997 in Kyoto, Japan, and implemented in 2005. Besides Kyoto Protocol, an investment in GT can be suitable and energy cost-efficient to reduce unwanted emissions. GT is a technology that does not injure or impact on the Earth’s environment through the process
of supply, manufacturing, use and disposal. Electric cars, solar panels and LED light bulbs are a few examples of GT. GT can also be simple, like planting trees. In a vertical farming, GT is used to planting and growing more plants in places with insufficient space or suitable soil. Water-efficient drip irrigation is a GT for farming plants.

A study that has been developed by Pan et al. [33] showed that an investment in GT can effectively reduce carbon emissions in the production system for a sustainable production inventory model. The difference between GT and PT is that GT is used during the production period so that product’s greenness is increased and hence, products become eco-friendly and on the other hand, PT is utilized to hold products for a long time so that products remain same for useful. Hence, inspired by the individual applications of GT and PT investments, the present study pertains to GT and PT in together for diminishing CO$_2$, SO$_2$ emissions and deterioration rate, respectively.

Nowadays, sustainable development (SD) is becoming an warm discussed topic all over the world by academic researchers and practitioners in different areas. Among these areas, production problems are of most significant. A sustainable model gives priority on the balance between three pillars resources; economic, environmental and social. This three pillars of SD are commonly known as triple-bottom-line dimension. Production inventory involves the use of economic, environmental and social resources. So, SD is an important factor to a production factory. Hence, a production factory needs to integrate the sustainability criteria as a core issue into crucial consideration. The main objective of this study consists of the improvement of an EPQ inventory model by adding sustainability criteria within a traditional EPQ model to reduce economic losses and to answer some research questions. The research questions which are raised by studying the above discussions and the questions are:

(i) How the outbreak due to the current pandemic has affected manufacturers’ decisions?
(ii) How much impact does PT investment have on controlling of deterioration rate for a sustainable EPQ model?
(iii) How much impact does GT investment have on greenhouse gases emissions reduction?
(iv) What are the environmental and social criteria for a sustainable production-inventory model?
(v) What are the organizational implications of the proposed sustainable EPQ model?

To answer the above-mentioned questions, this research paper is developed on a multi-objective sustainable EPQ model by considering demand disruptions during the present pandemic outbreak with the following assumptions:

(1) The manufacturer tries to reduce the product’s deterioration rate by using PT to hold products for a longer time.
(2) Demand for a product depends on the product’s selling price, greening level and emergency level of lock down during COVID-19 pandemic situation.
(3) CO$_2$ emissions produced by the manufacturer in the operational activities such as setup, production, deterioration and holding of inventories; SO$_2$ emissions produced by setup and production of products.
(4) The manufacturing company pays fines for amount of greenhouse gases emissions that exceed the associated cap.
(5) Due to over emissions of SO$_2$ and the effect of this emissions on workers health, the company pays an allowance to workers yearly.
(6) During the current pandemic situation, the company has an additional cost for purchasing sanitizers, masks to protect the workers from the pandemic.
(7) GT investment is used to reduce total emissions.
(8) Shortages are allowed and those are partially back-ordered.

The proposed model of this paper is on multi-objective EPQ problem in which objective functions are conflicting to each other. There does not usually exist an optimal solution that would simultaneously satisfy all the objective functions. Therefore, it is desirable to seek suitable compromise solution for such problem. Different approaches have been proposed for handling with multi-objective optimization problems. One of the most applicable approach is goal programming (GP) approach. A traditional GP model assumes that the decision maker
(DM) is able to determine a precise aspiration level for each of the objective functions. However, in most real-life problems, the aspiration levels are not known clearly. In this context, the fuzzy goal programming approach is chosen to find the compromise (optimal) solution of the proposed multi-objective sustainable model and illustrated some numerical experiments in this study. However, by studying the importance of above discussed parameters in an inventory system, the proposed study contributes:

1. In providing a clear way for a manufacturer, how to preserve perishable products in a balanced way and giving an idea about the sources of greenhouses gases emissions in a production system.
2. To confer an inkling about the nature of product’s demand during the COVID pandemic period.
3. The study combines deterioration, shortages and greenness, and includes environmental and social aspects into the economic aspect in the developed sustainable multi-objective EPQ model.
4. To reveal how a manufacturer could invest in PT and GT for optimizing deterioration and greenhouse gases emissions rate.
5. To suggest the fact that how much GT, PT, and carbon cap- and -trade policy altogether may significantly curb the greenhouse gases which are emitted due to holding, deterioration, production of products and during setup of the production process. Furthermore, how much these technologies are profitable to a manufacturer in economic aspect.
6. To explore the effects of backorder case on the production industry.

The rest part of the paper is set out as follows: In Section 2, some existing literatures on sustainable EPQ inventory model are reviewed. Section 3 considers the proposed model notations, assumptions and product’s demand function. Section 4 contains problem description. The mathematical formulation is developed in Section 5, which includes economic, environmental and social aspects. The solving approach, numerical illustrations, and sensitivity analysis are discussed in Sections 6, 7, and 8, respectively. Managerial insights are included in Section 9; and finally, the conclusions and recommendations for future study are presented in Section 10.

2. Previous work

Herein, the previous works related to this study are divided into three major parts. One is the discussion on inventory management with PT investment for reducing deterioration rate of products during COVID-19 pandemic situation, the second is the discussion about GT investment for reducing greenhouse gases emissions, and the last one is the study of sustainable inventory model. The related previous works have been explained thereafter.

2.1. Inventory model for deteriorating products

Deterioration rate is a common phenomenon of products that affects a manufacturer’s inventory. It curtails the quantity as well as the quality of the products. Vegetables, fruits, oils, fish, etc. are examples of decaying products. Pervin et al. [40] developed an integrated vendor-buyer model with PT for deteriorating products. In that model, an inspection policy has been discussed to separate spoilage products. When a supply chain uses vendor-management inventory to tackle deteriorating goods then significant amount of CO2 emits as examined in the model generated by Bai et al. [5]. They assumed that demand depends on green level and retailers’ selling price. Banerjee and Agrawal [6] applied an integrated approach to dynamic change in the value aspect of an inventory in terms of loss to deterioration and freshness. Freshness and price dependent demand have been studied in their investigation. Sivashankari and Ramachandran [47] studied an inventory model for deteriorating products with time-dependent demand. A stock-dependent demand has been considered in the model demonstrated by Pervin et al. [39]. Aghighi et al. [1] studied an inventory location-routing problem for perishable products under stochastic environment. They have solved their formulated problem by genetic algorithm and GAMS software. An economic ordered quantity (EOQ) model has been investigated by Indrajitsingha et al. [16] with non-instantaneous deteriorating products during the present COVID-19 pandemic. In addition, they have seen the effect of partially-backlogging shortages. For highly deteriorate products like, ice cream, dairy
products, investment in PT can play a substantial role for preserving products. A manufacturing inventory model with collaborative PT investment under carbon policy has been investigated by Shen et al. [45]. Mashud et al. [28] used PT to curb deterioration rate for a sustainable inventory model. A mathematical inventory model for decaying items has been presented by Kumar et al. [25] under PT with the present COVID-19 epidemic environment. Meanwhile, a manufacturing supply chain model for high demand products during the current COVID-19 pandemic has been proposed by Paul and Chowdhury [35]. They assumed that demands of sanitizer and masks have increased during the pandemic.

None of the above researchers has paid enough attention to the emergency level-dependent demand for deteriorating products. But, it is seen that emergency levels, i.e., levels of lockdown during COVID-19 pandemic have a negative effect on the demand rate of some deteriorating products, such as oil, snacks, various chemical products excluding medicines, and full-fat dairy products, etc. The present investigation fills this research gap by assuming emergency level-dependent demand during the present pandemic.

2.2. Inventory model with GT investment

Global warming could be a great threat to the world. It affects nature, human fitness, and well-being. CO$_2$ and SO$_2$ dioxide emissions play a vital role in heating the Earth. An enormous portion of intellectual researchers has additionally regarded the topics of GT and CO$_2$ emissions reduction. Datta [12] believed that GT investment is an efficient tool for reducing emissions during a production model. The capital investment amount has been considered as a decision variable in that investigation. An eco-friendly green inventory model has been studied by Saxena et al. [43] under fuzzy logic. GT investment has been applied in that paper for saving the surroundings from pollution. Under an assumption that greenhouse gases are generated due to the production and transportation of product, Sim and Jung [46] planned to design a sustainable supply chain model. A relation between investment in GT and CO$_2$ concern profits has been taken into their research. Xia and Niu [55] designed a supply chain model with environmental responsibility under asymmetric information where GT is applied for reducing carbon emissions.

One can see that few works have paid attention to SO$_2$ gas emissions. Wherever it is well-known that a significant quantity of SO$_2$ gas emitted because of producing a product, and this emission is very harmful to human health by the assorted some diseases like as skin problem and asthma, etc. Hence, this study is developed a sustainable EPQ model for a clean product bearing in mind that SO$_2$ gas emissions together with CO$_2$ emissions emit throughout production time. In this study, it is considered that a company pays a health allowance to workers due to over limit of emissions, in addition to the cost for providing sanitizers, masks to workers. Hence, the proposed model is designed under GT investment for reducing the each mentioned greenhouse gases emissions.

2.3. Sustainable and multi-objective inventory model

Traditional inventory management has not been studied on environmental and social impacts with economical aspects. Whereas, inventory management with sustainability analyzes the environmental impacts, i.e., effects of CO$_2$, NO$_2$ and SO$_2$ gases emissions on human health. Zadjafar and Gholamian [57] have assessed the effect of SO$_2$ to develop a socially and environmentally responsible production model in their work. A health allowance for SO$_2$ gas inhaler has been taken into account in that paper. Many researchers such as Pathak et al. [34] and Yadav and Khanna [56] have applied carbon tax policy to reduce CO$_2$ emissions in their proposed model. They believed that a price-sensitive demand is a better realistic assumption because products’ prices have a crucial influence on the customers’ getting choice. Taleizadeh et al. [49] presented a green supply chain with varied coordination contracts. In their model, it is said that demand has been affected by the greening level which reduces CO$_2$ emissions. Khalilpourazari and Pasandideh [19] designed emergency flood banishment strategies using robust optimization. Kocmanová et al. [24] deemed that sustainable investment is an investment approach compliance with social, environmental, and corporate governance standards. Tirkolaee et al. [52] developed a multi-objective optimization problem for the reliable pollution-routing problem and solved the problem using
pareto-based algorithm. Traditional inventory models concerned numerous choice that plan to optimize a lot sizes of materials by increasing or decreasing the total annual profit or cost of the system. However, to reduce various environmental damages a production company should take decisions by seeing the environmental impact along with economic aspects. Hence, Battini et al. [8] created a sustainable EOQ model by adding the environmental aspect to the annual average cost function. Under carbon-cap and -trade policy, Pan et al. [33] recommended a manufacturing inventory model in which the vendor and buyer agreed to co-invest funds for reducing CO$_2$ emissions. Furthermore, to minimize greenhouse gases emissions and maximize inventory management profit simultaneously, many researchers have focused on multi-objective sustainable models. A multi-objective supply chain problem has been designed by Jian et al. [17] by assuming selling price and green level dependent demand. Main objectives of the model were maximizing profit and minimization of CO$_2$ emissions. Furthermore, Chen and Andresen [9] promoted a sustainable supply chain problem, where three objective functions have been discussed. The first objective was to acquire economic sustainability, the second one was to minimize CO$_2$ emissions, and the third objective included the rephrased injury/illness incident rate. Later, Sazvar et al. [44] exhibited a new replenishment policy in a coordinated supply chain by developing a linear multi-objective mathematical model. Babaeinesami et al. [4] and Alinezhad et al. [2] developed a closed-loop supply chain problem by considering three, objective functions; one is maximization of the total profit, second is the minimization of total carbon footprint cost and third is the minimization of the maximum demand back log rate. To solve multi-objective optimization problems, various methods such as multi-objective grey wolf optimizer and multi-objective water cycle algorithm (Khalilpourazari et al. [22]) have been proposed. Fuzzy goal programming approaches have been adopted by Choudhary and Shankar [11] and Tirkolaee and Aydin [51] to solve a multi-objective optimization programming model. 

By studying above previous research works, the research lacuna can be turned out and described as follows: Most of the researchers have concentrated on air pollution especially CO$_2$ emissions in a sustainable EPQ model. Whereas, another greenhouse gas emissions such as SO$_2$ has not yet been discussed seriously. To our best knowledge, there is no preceding work on sustainable EPQ models where the health concern stipend has been calculated for manufacturing workers to protect them from COVID-19 pandemic. Moreover, none of the above researchers has discussed the issue of products’ shortages for a sustainable inventory model. To fill these research gaps, the present study focuses on developing a multi-objective sustainable EPQ model by assuming the following assumptions.

1. There is an interdependency between environmental and social aspects.
2. Demand for product is dependent on the selling price and green level of the product. Demand is also dependent on lockdown’s level during on going pandemic.
3. Health allowance for workers due to over limit of SO$_2$ emission and protect from COVID-19 pandemic is taken into account.
4. Shortages are allowed and partially back-ordered.

Let us refer to Table 1 to see the main contributions of the present work examined by comparing several previous works and present work.

3. Notations and Assumptions

The present inventory model consists of some specific notations and assumptions, which are made to develop the model.

3.1. Notations

The notations are divided into the parts of parameters, decision variables and other functions, which are as follows:
### Table 1. Contributions of various authors related to the present work.

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**Notes.** SC: Supply chain model, PD: price-dependent, GD: Greening level-dependent.

**Parameters**

- **D:** Variable demand rate for a product per unit time
- **D\(_0\):** Market potential demand rate per unit time
- **α\(_1\):** Consumer sensitivity to selling price
- **α\(_2\):** Consumer sensitivity to greening level
- **P:** Production rate for a product per unit time
- **λ\(_0\):** Deterioration rate without PT per product per unit time
- **λ:** Deterioration rate per product per unit time after an investment in PT
- **p\(_c\):** Unit production cost
- **B:** Setup cost per cycle
- **h:** Cost of holding a unit of inventory per unit time
- **δ:** Fraction of product shortages that is back-ordered
- **d\(_c\):** Deteriorating cost per unit deteriorated product
- **b\(_c\):** Back ordering cost per unit back-ordered product
- **l\(_c\):** Lost sale cost per unit product
- **W\(_1\):** Total number of workers for producing products
- **W\(_2\):** Total number of workers for packaging products
- **f\(_1\):** Each worker’s production rate
- **f\(_2\):** Each worker’s packaging rate
- **H\(_a\):** Health allowance per worker for extra SO\(_2\) emissions from the associated cap
- **m\(_c\):** Cost for masks and sanitizers per worker during COVID-19 pandemic
Production time with backorder

$T_2 - T_1$ is the production time when positive inventory builds up and depletes due to demand and deterioration

**Emission parameters**

- $\eta$: GT investment per unit greening level per unit time
- $K$: Total GT investment cost per unit time where $K = \eta g^2$
- $c_{fp}$: Fixed amount of CO$_2$ emissions per production run due to setup
- $c_{vp}$: CO$_2$ emissions rate per unit product during production process
- $c_{vh}$: CO$_2$ emissions rate per unit product per unit time for storing in warehouse
- $c_{vd}$: CO$_2$ emissions per unit deteriorate product per unit time
- $s_{fp}$: Fixed amount of SO$_2$ emissions per production run due to setup
- $s_{vp}$: SO$_2$ emissions rate per unit product during production process
- $\rho_s$: SO$_2$ emissions cap
- $\rho_c$: CO$_2$ emissions cap
- $TX_c$: Unit CO$_2$ emissions trading price
- $TX_s$: Unit SO$_2$ emissions trading price

**Decision variables**

- $s_p$: Selling price per unit product
- $g$: Greening level of product
- $\beta$: PT investment cost per unit time
- $T_3$: $T_3 - T_2$ is time period when inventory depletes due to demand and deterioration
- $T$: Cycle time

**Other functions**

- $I(t)$: Inventory level at time $t$
- TSR: Total sales revenue per cycle
- PC: Total production cost per cycle
- HC: Total holding cost per cycle
- DC: Total deteriorating cost per cycle
- BC: Total back-ordering cost per cycle
- LSC: Total lost sale cost per cycle
- PTC: Total PT investments per cycle
- GIC: Total GT investment cost per cycle
- TGE: Total reduced greenhouse gases emissions per cycle
- TEC: Total emissions cost per cycle
- WHA: Total health allowance per cycle
- AC: Total additional cost for purchasing masks and sanitizers
- TPF: Total profit per unit time

### 3.2. Assumptions

The following assumptions are used to formulate the proposed model.

1. A sustainable EPQ model is developed for a single manufacturer and a product.
2. The manufacturer maintains product’s deterioration rate. For this reason, the manufacturer uses PT investment to hold products for a longer time, i.e., to reduce deterioration rate. The reduced deterioration rate per unit product is $\lambda(\beta) (0 < \lambda(\beta) < 1)$, where $\lambda(\beta) = \lambda_0 e^{-\psi\beta}$. Here, $\lambda_0$ is the deterioration rate without investment in PT, $\beta$ denotes the PT investment and $\psi (\psi > 0)$ is the sensitive parameter of investment (motivated by Kumar et al. [25] and Mashud et al. [28]). This PT investment function is compatible with various industries.
Environmental and social impacts are added to the economic aspect.

Demand for a product depends on the product’s selling price $[14, 41]$, greening level, as in Jian et al. $[17]$. Based on the model developed by Alkahtani et al. $[3]$, this study also assumes that demand is dependent on emergency level $(\omega)$ of lockdown due to COVID-19 pandemic situation.

Production rate depends on the demand rate, i.e., $P = \alpha_4D$, where $\alpha_4 \geq 1$.

CO$_2$ emissions are produced by the manufacturer in the operational activities such as setup, production, holding $[14]$ and deterioration of products; SO$_2$ are emissions produced by setup and production of products.

Emissions cap- and -trade policy is applied to curb emissions.

Due to over emissions of SO$_2$ and the effect of the emissions on workers health, the company pays an allowance to workers yearly.

During the current pandemic situation, the company pays an additional cost for sanitizers, masks to protect the workers from the pandemic.

GT investment is used to reduce total emissions and reduced emissions rate is $\theta$ $(0 < \theta < 1)$, it is a function of GT investment cost $K = \eta g^2$, $\eta > 0$ $[12]$. The function $\theta$ is defined as: $\theta = \theta_m(1-e^{-\gamma K}) = \theta_m(1-e^{-\gamma \eta g^2})$. Here, $\theta$ becomes zero when greening level $g = 0$. In addition, if $g \to \infty$ then $\theta \to \theta_m$, where $\theta_m$ is the maximum reduced unit emissions as a result of utilizing GT investment and $\gamma$ is the efficiency level.

Shortages are allowed and those are partially back-ordered, as in Barman et al. $[7]$, Khalilpourazari et al. $[21]$, Escalona et al. $[13]$ and Khalilpourazari et al. $[20]$. Back-ordered demands are to be satisfied at the next cycle.

### 4. Problem description

This study considers a real-life sustainable production model. A manufacturer produces and sells one type of deteriorating product to maximize its profit and to preserve the environment. Nowadays, environmental pollution affects the life of all living things very badly. The greenhouse gases CO$_2$ and SO$_2$ are most harmful for human being. CO$_2$ and SO$_2$ are mostly emitted due to production, deterioration and holding of products. Hence, the manufacturer utilizes GT to reduce greenhouse gas emissions during production and increase products’ greenness. In addition, to prevent deterioration, the manufacturer holds products using suitable PT such as refrigerator and hence, the manufacturer invests an extra amount in GT and PT. Carbon cap- and -trade policy is an effective tool for curbing greenhouse gases emissions, so the cap- and -trade policy is applied along with GT and PT. The manufacturer gains an automatic advantage by selling green products because many customers prefer products that address environmental concerns compared to traditional goods. Thus, products’ demand depends on the selling price and greenness of products. Furthermore, during the COVID-19 pandemic, products’ demand depends on the extreme emergency level of lockdown. The effect of Coronavirus on human health is very painful. As a result, there is a shortage of workers in the production house. Hence, products shortages occur and are partially back-ordered. Figure 1 represents the proposed EPQ model. The developed sustainable multi-objective EPQ model includes environmental and social aspects into an economic aspect.

To formulate the inventory model, we need to explain the following component of the model.

#### 4.1. Demand function

COVID-19 pandemic generated an emergency environment for production and supply of finished commodities. This emergency environment adversely affected inventory management socially and economically. At first, it was seen that various governments forced to reduce the social physical contact among people and gradually closed public activities. Ultimately this reached an extreme emergency level of lockdown. This lockdown affected the movements of people, production, transportation, and supply of products. Moreover, due to this pandemic situation, the income of one class of people in the society has decreased. For these reasons, many products’ demands have also decreased.

Several countries adopted various policies and timings for the lockdown periods. But, their emergency levels mostly followed a similar rule. Observing the local, national and international lockdown situation, it is evident
that there are approximately six different levels of emergency for controlling the pandemic based on the real-time consequences. The proposed six emergency levels are given in Table 2. These levels affected the market demand negatively for production business managements due to customers’ unavailability in a market and supply shortages of raw materials. On the other hand, some products in the market were saturated during the pandemic due to the unavailability of buyers to buy, which led to economic inconsistency for governments and companies. Thus, to justify declining demand, the decreasing demand function of emergency level is considered.

Furthermore, market demand is affected negatively by the product’s selling price and positively affected by product’s greening level. Hence, the variable demand depending on emergency level \( \omega \) due to COVID-19 pandemic, product’s selling price \( s_p \), and greening level \( g \) is given as follows:

\[
D = D_0 + \alpha_1 g - \alpha_2 s_p - \alpha_3 \omega, \quad \text{where} \quad D_0, \alpha_1, \alpha_2, \alpha_3 > 0. 
\]

Here, \( D_0 \) is the potential market demand, \( \alpha_1 \) is the coefficient of customer preference for low greenhouse gas emissions, \( \alpha_2 \) is customer sensitivity to the selling price, and \( \alpha_3 \) is the lowest possible market demand and scaling factor due to pandemic.

### 5. Mathematical Formulation

Figure 1 represents the stock level for the proposed EPQ model of degradation items with permissible shortages. The manufacturer starts the cycle at time \( t = 0 \) at \( P \) production rate per unit time. Production runs during the first period \([0, T_1]\) to meet the previous cycle’s back-ordered demands as well as arrival demands. So the line under the horizontal axis shows that there is no inventory in stock where it needs to be delivered to customers. During the period \([T_1, T_2]\) inventory grows with production rate \( P \) and declines due to demands and deterioration. After the time \( T_2 \), inventory gradually decreases according to the demands and even deterioration upto the time \( T_3 \). From \( T_3 \) to \( T \) there is no physical stock causes shortages occur and these shortages will be partially back-ordered to the next cycle.

The following differential equations represent the inventory levels during \([0, T_1]\), \([T_1, T_2]\), \([T_2, T_3]\), and \([T_3, T]\), respectively.

\[
\frac{dI_1(t)}{dt} = P - \delta D - D, \quad 0 \leq t \leq T_1 \quad \text{with} \quad I_1(T_1) = 0, \tag{5.1}
\]

\[
\frac{dI_2(t)}{dt} + \lambda(\beta)I_2(t) = P - D, \quad T_1 \leq t \leq T_2 \quad \text{with} \quad I_2(T_1) = 0, \tag{5.2}
\]
By solving equations (5.1), (5.2), (5.3), and (5.4) with their respective boundary conditions, the following inventory levels occur.

\[ I_1(t) = -(P - \delta D - D)(T_1 - t), \quad 0 \leq t \leq T_1, \]  
\[ I_2(t) = \frac{P - D}{\lambda(\beta)} \left( 1 - e^{\lambda(\beta)(T_1 - t)} \right), \quad T_1 \leq t \leq T_2, \]  
\[ I_3(t) = \frac{D}{\lambda(\beta)} \left( e^{\lambda(\beta)(T_3 - t)} - 1 \right), \quad T_2 \leq t \leq T_3, \]  
and \[ I_4(t) = -\delta D(t - T_3), \quad T_3 \leq t \leq T. \]  

Moreover, from Figure 1, it is seen that \( I_1(T_1) = I_2(T_1) \). From this condition it is obtained that \( T_2 = \frac{1}{\lambda(\beta)} \ln \left( \frac{(P - D)e^{\lambda(\beta)T_1} + De^{\lambda(\beta)T_3}}{P - \delta D} \right) \). As maximum back-order level = \( I_1(0) = I_4(T_3) \), so the relation between \( T_1 \) and \( T_3 \) can be shown as \( T_1 = \frac{\delta D}{P - \delta D - D}(T - T_3) \).

### 5.1. Economical aspect of the proposed EPQ model

In this section, economical factors of the model are discussed. The total profit per unit time with respect to only economic criteria includes sales revenue, setup cost, production cost, holding cost, deteriorating cost, back-ordered cost, and lost sale cost, as well as investment in PT for reducing deterioration rate. These components are calculated subsequently.

(a) Total sales revenue per cycle is \( \text{TSR} = s_p(D + \delta D)T_1 + s_pD(T_3 - T_1) \).
(b) Total production cost per cycle is \( PC = p_e PT_2 \) and setup cost per cycle is \( B \).

(c) **Holding cost:** total amount of inventories for holding is the area under \( T_2 - T_1 \) and \( T_3 - T_2 \) in Figure 1. Thus, amount of holding inventories is

\[
HI = \int_{T_1}^{T_2} I_2(t)dt + \int_{T_2}^{T_3} I_3(t)dt \\
= \frac{P - D}{\lambda(\beta)} \left( T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right) \\
+ \frac{D}{\lambda(\beta)} \left( T_2 - T_3 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right),
\]

where \( I_2(t) \) and \( I_3(t) \) are given in equations (5.6) and (5.7), respectively. Therefore, total holding cost of the system per cycle is

\[
HC = hHI \\
= h \left[ \frac{P - D}{\lambda(\beta)} \left( T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right) \right] \\
+ h \left[ \frac{D}{\lambda(\beta)} \left( T_2 - T_3 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right) \right].
\]

(d) **Deteriorating cost:** due to COVID-19 pandemic, customer’s demand is decreasing and hence, deterioration rate is increasing. To reduce the deterioration rate, PT is applied. The reduced deterioration rate after investment in PT is \( \lambda(\beta) \). Thus, total deteriorating cost per cycle is

\[
DC = d_e \lambda(\beta) P(T_2 - T_1).
\]

(e) **Back-ordered cost:** due to increase of deterioration rate and unavailability of labours during COVID-19 pandemic, shortages occur and those are partially back-ordered at rate \( \delta \). Hence, total back-ordered cost per cycle is

\[
BC = -b_c \left[ \int_{0}^{T_1} I_1(t)dt + \int_{T_3}^{T} I_4(t)dt \right] \\
= b_c \left[ (P - D - \delta D) \frac{T_1^2}{2} + \delta D \left( \frac{T^2}{2} + \frac{T_3^2}{2} - T_3 T \right) \right],
\]

where \( I_1(t) \) and \( I_4(t) \) are given in equations (5.5) and (5.8), respectively.

(f) **Lost sale cost:** due to shortages, many customers are not interested to wait for buying their products, which may cause a loss in profit. Hence, the lost sale cost is \( LSC = l_c (1 - \delta) D(T - T_3) \).

(g) The PT investment cost per cycle is \( PTC = \beta T \).

Now, the total profit per unit time with respect to only economic aspect is:

\[
TPF(s_p, g, \beta, T, T_3) = \frac{1}{T} (TSR - B - PC - LSC - DC - BC - HC - PTC).
\]

5.2. Environmental impact on the EPQ model

In this Section, \( \text{CO}_2 \) and \( \text{SO}_2 \) emissions due to production and holding of inventory are discussed. Here, costs for \( \text{CO}_2 \) and \( \text{SO}_2 \) emissions are calculated, which have a negative effect on the total profit of the proposed EPQ model. Based on our assumptions, \( \text{CO}_2 \) emissions are produced due to the setup, production and holding of
products, whereas, SO$_2$ emissions occurred only through the setup of the system and production of products. Thus, the total amount of emissions per cycle is:

$$\text{Total CO}_2 \text{ emissions} + \text{Total SO}_2 \text{ emissions}$$

$$= c_{fp} + c_{vp} PT_2 + c_{vd}\lambda(\beta) P(T_2 - T_1) + c_{vh}\frac{P}{\lambda(\beta)} \left(T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right) + c_{eh}\frac{D}{\lambda(\beta)} \left(T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} \right) + s_{fp} + s_{vp} PT_2.$$  

For detail calculation one can see Appendix A.

**GT investment in emissions reduction**

By investing an amount $K = \eta g^2$ in GT, the manufacturer reduces the per unit greenhouse gases emissions by $\theta$ percentage, as previously implemented by Mashud *et al.* [28] and Datta [12]. The reduced percentage can be shown as $\theta = \theta_m(1 - e^{-\gamma K}) = \theta_m(1 - e^{-\gamma \eta g^2})$. Here, $\theta$ becomes zero when greening level $g = 0$. In addition, if $g \to \infty$ then $\theta \to \theta_m$, where $\theta_m$ is the maximum reduced unit emissions as a result of utilizing GT investment and $\gamma$ is the efficiency level. So, the cost of investment in GT, total reduced amount of emissions and emissions fine are stated subsequently.

(i) The GT investment cost per cycle is

$$\text{GIC} = KT = \eta g^2 T.$$  

(ii) Total reduced emissions from the manufacturing company is

$$\text{TGE} = [c_{fp} + c_{vp} PT_2 + s_{vp} PT_2 + c_{vd}\lambda(\beta) P(T_2 - T_1)](1 - \theta)$$

$$+ c_{vh}\frac{D}{\lambda(\beta)} \left(T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} \right)(1 - \theta)$$

$$+ c_{eh}\frac{P}{\lambda(\beta)} \left(T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right)(1 - \theta).$$

Detail calculation is given in Appendix B.

(iii) **Emissions cap- and -trade policy:** the manufacturer is subject to the $\rho_c$ and $\rho_s$ caps total CO$_2$ and SO$_2$ emissions, respectively, under the emissions cap- and -trade policy. If CO$_2$ and SO$_2$ emissions exceed respective boundaries, products outside the boundaries must be purchased at the respective trading prices TX$_c$ and TX$_s$. On the contrary, if emissions do not exceed their respective boundaries then the remaining CO$_2$ and SO$_2$ emissions can be sold at the trading prices TX$_c$ and TX$_s$, respectively. Assuming greenhouse gases emissions allowances can be bought and sold in the market, the total emissions fine or subsidy is stated as:

$$\text{TEC} = \text{Total CO}_2 \text{ emissions allowances} + \text{Total SO}_2 \text{ emissions allowances}$$

$$= (c_{fp} + c_{vp} PT_2 + c_{vd}\lambda(\beta) P(T_2 - T_1)) (1 - \theta) TX_c$$

$$+ c_{vh}\frac{P}{\lambda(\beta)} \left(T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right)(1 - \theta) TX_c$$

$$+ c_{eh}\frac{D}{\lambda(\beta)} \left(T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} \right)(1 - \theta) TX_c - \rho_c TX_c$$

$$+ [(s_{fp} + s_{vp} PT_2)(1 - \theta) - \rho_s] TX_s.$$  

(5.12)

The detail calculation is referred to Appendix C.
5.3. Social impact

The largest source of SO\textsubscript{2} in the atmosphere is the burning of fossil fuels by power plants, \textit{i.e.}, during production process and other industrial facilities. SO\textsubscript{2} gas can affect both human health and environment. Over limit of SO\textsubscript{2} emissions can harm the human respiratory system. Hence, in addition to environmental emissions fines, the manufacturing company pays an extra money as health allowance to each worker due to over limit SO\textsubscript{2} emissions, as discussed in Zadjafar and Gholamian \cite{57}. Therefore, the total health allowance per cycle is given by:

\[
\text{WHA} = H_0 W_1 [(s_{fp} + s_{vp}PT_2)(1 - \theta) - \rho_s].
\]  

(5.13)

It should be noted that number of workers \(W_1\) per cycle is calculated from the division of total production by each worker’s production rate, \textit{i.e.}, \(W_1 = \frac{PT_2}{f_1}\), where \(f_1\) is the production rate per worker per unit time.

Furthermore, since Coronavirus is affecting human health badly, so the manufacturing company should take a special care about the workers’ health by maintaining social distance and providing enough amount of masks and sanitizers. For this reason, the company needs an additional cost per cycle and it is depicted as:

\[
\text{AC} = m_w(W_1 + W_2).
\]  

(5.14)

Here, \(W_2\) is the number of labours for packaging finished goods per cycle.

5.4. Sustainable EPQ model

Now, by implementing the environmental criteria (Eqs. (5.10) and (5.12)) and the social aspects (Eqs. (5.13) and (5.14)) to the profit function (Eq. (5.9)) of the EPQ model, the total profit of the sustainable EPQ model is obtained as follows:

\[
\text{TPF} = \frac{1}{T} (\text{TSR} - B - PC - \text{LSC} - \text{DC} - \text{BC} - \text{HC} - \text{PTC})
\]

\[ - \frac{1}{T} (\text{GIC} + \text{TEC}) - \frac{1}{T} (\text{WHA} + \text{AC})
\]

\[= \frac{1}{T} \left[ s_p(D + \delta D)T_1 + s_p D(T_3 - T_1) - B - p_c PT_2 - l_c(1 - \delta) D(T - T_3) \right]
\]

\[- \frac{1}{T} \left[ \frac{\lambda(\beta)}{\lambda(\beta)} \left( T_2 - T_1 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right) + \frac{D}{\lambda(\beta)} \left( T_1 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} \right) \right]
\]

\[- \frac{1}{T} h \left[ \frac{P}{\lambda(\beta)} \left( T_3 + \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} \right) - \frac{\beta T}{T} \right] - \frac{1}{T} \left[ \eta g^2 T + (c_{fp} + c_{vp} PT_2)(1 - \theta)T X_c \right]
\]

\[- \frac{1}{T} \left[ \frac{D}{\lambda(\beta)} \left( T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3 - T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2 - T_1)}}{\lambda(\beta)} \right) c v_h (1 - \theta) T X_c \right]
\]

\[- \frac{1}{T} \left[ s_{vp} PT_2(1 - \theta) T X_s + H_a W_1 [(s_{fp} + s_{vp} PT_2)(1 - \theta) - \rho_s] + m_w (W_1 + W_2) \right]
\]

\[+ \frac{1}{T} \left( \rho_e T X_c + \rho_s T X_s \right).
\]  

(5.15)

Finally, the multi-objective sustainable EPQ model derived as follows:

\[
\text{maximize} \quad Z_1 = \text{TPF}
\]  

(5.16)
minimize \( Z_2 = \text{TGE} \) \hspace{1cm} (5.17)

subject to \[
T_2 = \frac{1}{\lambda(\beta)} \ln \left( \frac{(P-D)e^{\lambda(\beta)T_1} + De^{\lambda(\beta)T_3}}{P} \right),
\] \hspace{1cm} (5.18)

\[
T_1 = \frac{\delta D}{P - \delta D - D} (T - T_3),
\] \hspace{1cm} (5.19)

\[
T > 0, \ 0 \leq g \leq 1, \ s_p > p_c, \text{ and } \beta > 0.
\] \hspace{1cm} (5.20)

Here \( Z_1 \) is the total profit function given in equation (5.15) and \( Z_2 \) is the total reduced greenhouse gases emissions function given in equation (5.11).

We solve the designed multi-objective EPQ model by using weighted max–min fuzzy GP approach.

6. Solution methodology

6.1. Fuzzy GP approach

GP approach is the most widely used approach in multi-objective decision making problem. In GP, we assign a target level for achievement and pre-specified priority of DM on achieving the target for each objective function. The role of GP is to minimize the unwanted deviations between the achievement of goals and their aspiration levels. But, in real-life situation, a DM does not has exact and complete information about the target of each objective function. For this reason, there occurs an uncertain atmosphere. The fuzzy set theory is one of the best way to handle uncertainty. In fuzzy GP approach, the individual optimum values of each objective function are used as target values. Zimmermann [58] first applied the fuzzy optimization concept to solve multi-objective programming problem. Midya et al. [30] applied the fuzzy programming technique to solved their proposed fuzzy multiple objective fractional optimization transportation problem. Tirkolaee and Aydin [51] applied fuzzy weighted GP approach to solve multi-objective linear programming problem. The advantage of weighted max–min fuzzy GP is that no target values have to be specified by a DM. Based on the weighted max–min fuzzy GP approach, the crisp single objective function for the proposed multi-objective EPQ model is stated as:

\[
\text{maximize} \quad \alpha \quad \hspace{1cm} (6.1)
\]

subject to \[
w_1 \alpha \leq \mu_{Z_1}(x),
\] \hspace{1cm} (6.2)

\[
w_2 \alpha \leq \mu_{Z_2}(x),
\] \hspace{1cm} (6.3)

\[
T_2 = \frac{1}{\lambda(\beta)} \ln \left( \frac{(P-D)e^{\lambda(\beta)T_1} + De^{\lambda(\beta)T_3}}{P} \right),
\] \hspace{1cm} (6.4)

\[
T_1 = \frac{\delta D}{P - \delta D - D} (T - T_3),
\] \hspace{1cm} (6.5)

\[
T > 0, \ 0 \leq g \leq 1, \ s_p > p_c, \text{ and } \beta > 0.
\] \hspace{1cm} (6.6)

Here, \( x \) is the vector \((s_p, g, T, T_3, \beta)\) and \( \alpha (0 \leq \alpha \leq 1) \) is the degree up to which the aspiration level of the decision-maker is met. Also, \( \alpha = \min\{\mu_{Z_1}(x), \mu_{Z_2}(x)\} \). Constraints (6.2) and (6.3) are the constraints due to defuzzification, and constraints (6.4) to (6.6) are the multi-objective model’s constraints. In addition, \( w_1 \) and \( w_2 \) are the respective weight values of the objective functions \( Z_1 \) and \( Z_2 \), are assigned by the DM (here manufacturer). Moreover, \( \mu_{Z_1}(x) \) and \( \mu_{Z_2}(x) \) are the membership functions for the maximization goal objective function \( Z_1 \) and minimization goal objective function \( Z_2 \), respectively. These membership functions are defined as:

\[
\mu_{Z_1}(x) = \begin{cases} 
0, & \text{if } Z_1(x) \leq Z_1^-, \\
\frac{Z_1(x) - Z_1^-}{Z_1^+ - Z_1^-}, & \text{if } Z_1^- \leq Z_1(x) \leq Z_1^+, \\
1, & \text{if } Z_1(x) \geq Z_1^+,
\end{cases}
\]
and

$$\mu_{Z_2}(x) = \begin{cases} 
0, & \text{if } Z_2(x) \geq Z_2^+, \\
\frac{Z_2^- - Z_2(x)}{Z_2^- - Z_2^+}, & \text{if } Z_2^- \leq Z_2(x) \leq Z_2^+, \\
1, & \text{if } Z_2(x) \leq Z_2^-.
\end{cases}$$

Here, $Z_1^+$ and $Z_1^-$ are the values by solving the proposed multi-objective optimization problem as a single objective non-linear programming problem taking only one objective function at a time and ignoring the other objective function. Consequently, $Z_1^-$ is the minimum value (worst solution) of the objective function $Z_1(x)$ and $Z_2^+$ is the maximum value (worst value) of the objective function $Z_2(x)$. The membership functions $\mu_{Z_1}(x)$ and $\mu_{Z_2}(x)$ are shown in Figure 2.

There are many existing methods for solving a multi-objective problem in literature. However, fuzzy GP approach is used to solve the proposed model because

(1) it is a more reflexive procedure that is relatively easy to understand and utilize,

(2) it allows a DM to find accurately an acceptable better solution of each fuzzy goal in the sense that the objective values are sufficiently closer to their aspiration levels. Hence, it is easily applicable in real-life situations.

6.2. Solution algorithm

This section proposes an algorithm that finds the optimal solutions for decision variables using fuzzy GP approach which is discussed in earlier section. The steps to solve the proposed model are as follows:

**Step 1.** Start

**Step 2.** Declare and assign the values of all known parameters of the model.

**Step 3.** Calculate the individual goals $Z_1^+$ and $Z_2^-$ of the objective functions $Z_1$ and $Z_2$ which are presented in equations (5.16) and (5.17), respectively, by optimizing $Z_1$ and $Z_2$ separately along with the constraints (6.4)–(6.6). The expressions of the functions $Z_1$ and $Z_2$ are lengthy and complex. Hence, we determine the required goals of the functions using Wolfram Mathematica 12 software.

**Step 4.** Determine the worst value $Z_1^-$ of $Z_1$ by putting the values of $s_p, g, T, T_3$ and $\beta$ (obtained by minimizing $Z_2$) in equation (5.16). After that, determine the worst value $Z_2^+$ of the functions $Z_2$ by substituting the values of $s_p, g, T, T_3$ and $\beta$ (obtained by maximizing $Z_1$) in equation (5.17).
Step 5. Calculate the membership functions $\mu_{Z_1}(x)$ and $\mu_{Z_2}(x)$.

Step 6. Put the expressions of the function $\mu_{Z_1}(x)$ and $\mu_{Z_2}(x)$ into the constraints (6.2) and (6.3).

Step 7. Now, determine the maximum value of the aspiration level $\alpha$ along with the constraints (6.2)–(6.6) using a suitable software like Wolfram Mathematica 12 software. By determining the value of $\alpha$, the optimal values of $Z_1$ and $Z_2$, and also the optimal values of decision variables $s_p, g, T, T_3$ and $\beta$ are obtained.

Step 8. Stop

7. Numerical experiments

A case study is presented in this Section of a greenhouse farm (Fig. 3) of India. In this green farm, many green deteriorating products like fruits, vegetables, vegetable oil and flowers are produced and sold. As these crops have the highest probability of deteriorating over time, DM uses PT such as refrigeration to reduce deterioration rate for these crops, and DM also incorporates GT like water-efficient drip irrigation to reduce CO$_2$ and SO$_2$ emissions. These greenhouse gases are emitted during setup of the farm house, spraying fertiliser and giving water using various machines into the farm, holding products, etc. We cannot collect actual data regarding this greenhouse farm, so we consider hypothetical data for numerical examples based on the real scenario.

To illustrate the proposed model and solution procedure, several sustainable EPQ models have examined with reliable data.

Example 7.1. Partially back-ordering model with PT and GT investment.

Here, a numerical experiment is taken into account for an EPQ model that has investment in both PT and GT. The required parametric values are as follows: $D_0 = 1200$ units/year, $\alpha_1 = 20$, $\alpha_2 = 4$, $\alpha_3 = 8$, $\alpha_4 = 4$, $\lambda_0 = 0.2$ units/year, $\psi = 0.5$, $p_c = $10/unit/year, $B = $500/setup, $h = $0.3/unit/year, $\delta = 0.6$ units/year, $d_c = $2/unit/year, $b_c = $2.5/unit/year, $l_c = $3/unit/year, $\eta = 50$, $\theta_m = 0.02$, $\gamma = 0.6$, $c_{fp} = 20$ kg/year, $c_{vp} = 0.5$ kg/unit product, $c_{vh} = 2$ kg/unit product, $s_{fp} = 15$ kg/year, $s_{vp} = 0.3$ kg/unit product, $\rho_c = 100$ kg/year, $\rho_s = 50$ kg/year, $T_Xc = $3/unit/year, $T_Xs = $2/unit/year, $f_1 = 100$ units/worker/year, $f_2 = 150$ units/worker/year, $H_a = $0.5/worker/year, and $m_c = $1/worker/year.

Optimum results for Experiment 7.1

The weighted max–min fuzzy GP model (referred to in Sect. 6) is utilized for solving the multi-objective sustainable EPQ model. By optimizing both objective functions individually in the proposed model as explained

![Figure 3. A greenhouse farm (https://www.agrifarming.in/greenhouse-agriculture).](https://www.agrifarming.in/greenhouse-agriculture)
Table 3. Data set for membership functions.

<table>
<thead>
<tr>
<th></th>
<th>$\mu = 1$</th>
<th>$\mu = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total profit $Z_1$</td>
<td>61 495.7</td>
<td>50 262</td>
</tr>
<tr>
<td>Total emissions $Z_2$</td>
<td>1078.8</td>
<td>3079.96</td>
</tr>
</tbody>
</table>

Figure 4. Membership functions for total profit and emissions for Experiment 7.1.

Table 4. Solutions for Experiment 7.1 ($\delta = 0.6$).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price ($s^*_p$)</td>
<td>$85.57$</td>
</tr>
<tr>
<td>Production time ($T^*_2$)</td>
<td>0.3452 years</td>
</tr>
<tr>
<td>Cycle time ($T^*$)</td>
<td>2.0412 years</td>
</tr>
<tr>
<td>Greening level ($g^*$)</td>
<td>0.64</td>
</tr>
<tr>
<td>PT investment ($\beta^*$)</td>
<td>$25.41$</td>
</tr>
<tr>
<td>Total profit ($TPF^*$)</td>
<td>$61 481.1$</td>
</tr>
<tr>
<td>Total emissions ($TGE^*$)</td>
<td>1081.84 kg</td>
</tr>
</tbody>
</table>

in equations (5.16) and (5.17), ideal objective values are obtained with the help of Wolfram Mathematica 12 software. In the weighted max–min GP model, objectives’ ideal values are treated as goals or targets for corresponding two objective functions. The membership functions $\mu_{Z_1}(x)$ and $\mu_{Z_2}(x)$ are adopted for fuzzifying the two objective functions $Z_1$ and $Z_2$ and these membership functions are shown in Figure 4. The upper and lower bounds for the respective objective functions are given in Table 3.

The optimal solutions achieved from the weighted max–min fuzzy GP model have depicted in Table 4. Furthermore, the 3D graph shows the concavity of total profit $TPF$ with respect to $\beta$ and $g$ in Figure 5. Meanwhile, in Figure 6, it is established that total manufacturer’s profit is concave concerning the greening level $g$.

If our proposed model is applied to solve the numerical experiment in Datta [12] who did not study the effects of greening level and COVID-19 pandemic on demand, then the obtained optimal profit is $TPF^* = 10 537.812$. In addition, the optimal profit which is derived by using the model of Datta [12] is $8781.51$. So, our proposed model concludes that the effect of the greening level on demand can increase the profit of a company.
Example 7.2. *Fully back-ordering model with PT and GT investment.*

Here, an experiment is chosen where shortages products are fully back-ordered, i.e., \( \delta = 1 \) and the other remaining parameters remain the same as in Experiment 7.1.

By using the weighted max–min fuzzy GP model, obtained optimal solutions have shown in Table 5. Figure 9 shows the concavity of the total profit function with respect to \( \beta \) and \( g \) for Experiment 7.2. Furthermore, the concavity of the profit with respect to greening level \( g \) is shown in Figure 8.

Example 7.3. *Partially back-ordering model without PT investment.*

This experiment is assumed when there is no investment in PT, i.e., \( \beta = 0 \) and values of the other parameters are the same as in Experiment 7.1. Using the same solution procedure of Experiments 7.1 and 7.2, the optimal results are derived. The optimal results are listed in Table 6.

Example 7.4. *Partially back-ordered case without GT investment.*
Table 5. Solutions for Experiment 7.2 ($\delta = 1$).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price ($s_p^*$)</td>
<td>$91,977</td>
</tr>
<tr>
<td>Cycle time ($T^*$)</td>
<td>4.334 years</td>
</tr>
<tr>
<td>Greening level ($g^*$)</td>
<td>0.81</td>
</tr>
<tr>
<td>PT investment ($\beta^*$)</td>
<td>$35,698.3</td>
</tr>
<tr>
<td>Total profit (TPF*)</td>
<td>$77,869</td>
</tr>
<tr>
<td>Total emissions (TGE*)</td>
<td>1147.42 kg</td>
</tr>
</tbody>
</table>

Figure 7. Concavity of TPF with respect to $g$ for Experiment 7.2.

Figure 8. Concavity of TPF with respect $\beta$ and $g$ for Experiment 7.2.

Table 6. Optimal solutions for Experiment 7.3 ($\beta = 0$).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price ($s_p^*$)</td>
<td>$182,159.7</td>
</tr>
<tr>
<td>Cycle time ($T^*$)</td>
<td>4.767 years</td>
</tr>
<tr>
<td>Greening level ($g^*$)</td>
<td>0.467</td>
</tr>
<tr>
<td>Total profit (TPF*)</td>
<td>$59,885.8</td>
</tr>
<tr>
<td>Total emissions (TGE*)</td>
<td>1132.5 kg</td>
</tr>
</tbody>
</table>
For this case, it is considered that $\eta = 0$, i.e., $\theta = 0$, and the remaining parameters’ values are the same as in Experiment 7.1. To find optimal solutions, one can apply the same solution procedure, weighted max–min fuzzy GP model as in Experiment 7.1. The obtained solutions have shown in Table 7.

### Table 7. Solutions for Experiment 7.4 ($\eta = 0$).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price ($s^*_p$)</td>
<td>$125.612$</td>
</tr>
<tr>
<td>Cycle time ($T^*$)</td>
<td>2.55 years</td>
</tr>
<tr>
<td>Greening level ($g^*$)</td>
<td>0.45</td>
</tr>
<tr>
<td>PT investment ($\beta^*$)</td>
<td>$28.41$</td>
</tr>
<tr>
<td>Total profit ($\text{TPF}^*$)</td>
<td>$60,200.3$</td>
</tr>
<tr>
<td>Total emissions ($\text{TGE}^*$)</td>
<td>1096.05 kg</td>
</tr>
</tbody>
</table>

7.1. Results and discussion

The outcomes of the numerical experiments are discussed in this section. Tables 4 and 7 show that the optimum total profit for the sustainable EPQ model with GT investment has the highest value when compared to the model without GT investment. Furthermore, the overall CO$_2$ and SO$_2$ emissions for a sustainable EPQ with GT investment are lower than for a sustainable model without GT investment. From Tables 4 and 5, the total profit for sustainable EPQ model with GT investment has a better result in full back-ordering case by investing high PT capital than a partially back-ordering case. Moreover, the total emissions in full back-ordering case are greater than that of partially back-ordering case. For the case of full back-ordering, as $\delta = 1$, all orders can be preserved and back-ordered. Tables 4 and 6 show that the sustainable EPQ partially back-ordering model with investment in GT has a maximum profit by investing capital in PT compared to without investing in PT. From Tables 4–7, it is clear that the cycle time $T^*$ and PT investment $\beta^*$ for the sustainable EPQ model with the investment in GT, are lower than the other models. The minimum total CO$_2$ and SO$_2$ emissions have occurred for the sustainable EPQ model with GT and PT under partially back-ordering case. From Figure 9, it is noted that investments in both GT and PT would be beneficial than the other cases as it increases the total profit. The total profit in the case of PT and GT investment is 16.1% more than in the case of neither PT investment nor GT investment. Furthermore, the sustainable model for full back-ordering case with investment in GT and PT having the highest greening level compared to partially back-ordering case. But, in many cases, full back-ordering is impossible. Finally, the sustainable EPQ partially back-ordering model with PT and GT investments is a viable EPQ model that considers economic, environmental, and social factors. As a result, the sensitivity analysis for a sustainable EPQ model with PT and GT investments must be discussed under partially back-ordered rate.
If carbon cap- and -trade policy is not applied but GT is applied in the proposed EOQ model then total greenhouse gases emissions $TGE = 1085$ kg and total profit $TPF = $60202.5. Furthermore, when carbon cap- and -trade policy is utilized without GT investment then $TGE = 1096.05$ kg and $TPE = $600200.3. The results have shown that GT is better than carbon cap- and -trade policy for diminishing greenhouse gases emissions to save the environment. But, there is no significant difference between GT and carbon cap- and -trade policy to the profit of the model because an investment is needed to use GT which is subtracted from the total profit. So, to save the environment and to increase the total profit of a manufacturer, GT and carbon cap- and -trade policy should be applied simultaneously.

The superiorities of the developed model are discussed as:

1. the interdependency among the three pillars of sustainability such as economic, social and environmental aspects is studied in the proposed model,
2. this study investigated the effect of COVID-19 pandemic on an inventory management in socially and economically,
3. emissions of $SO_2$ gas is taken into account in this proposed model as $SO_2$ gas emits mostly during production and it can affect both human beings health and environment,
4. to reduce greenhouse gases emissions due to deterioration, production and transportation of products, PT investment, GT investment and carbon cap- and -trade policy are applied.

### 7.2. Special cases

Some special cases of the proposed study are as follows:

1. If this work does not consider green level dependent demand and effect of COVID-19 pandemic on demand rate then this study is similar to Datta [12]. The present study can increase 10% profit than the profit of Datta [12] by observing the impact of customers’ choice on green product during the present pandemic period.
2. If this study has no discussion of sustainable development and products’ deterioration then this study becomes similar to the traditional EPQ model established by Wee et al. [54] with partially back-ordering shortages.
3. By solving one numerical experiment of the work developed by Taleizadeh et al. [50], it is implied that this study gives 8% more profit than Taleizadeh et al. [50] using PT and GT investments.

### 8. Sensitivity analysis

In this Section, the effects of emergency level $\omega$ during the lockdown on the total profit and total greenhouse gases ($CO_2$ and $SO_2$) emissions have discussed. Those effects has reflected in Table 8. A sensitivity analysis is performed to show the consequences of the major parameters by changing their values from $-10\%$ to $+20\%$. The sensitivity analysis is given in Table 9. Furthermore, effects of emergency level $\omega$ during COVID-19 pandemic on selling price $s_p$, greening level $g$, and PT investment $\beta$ are shown in Figures 10, 11 and 12, respectively. The following observations can be made from the sensitivity Tables 8 and 9 and Figures 10–19.

1. When the emergency level $\omega = 0$, then there is no lockdown. At this time, delivery of raw materials, workers’ availability, etc. are normal. Hence, the total profit is much high. With the increasing in $\omega$, greening level and PT investment decrease. Thus, demand rate for a product decreases, for which the total emissions as well as the total profit decreases. Meanwhile, Figure 10 shows that the product’s selling price sometimes is growing and sometimes it is reducing. With our greatest information, it is concluded that this fact is a realistic truth fact throughout the current COVID-19 occurrence.
2. From Table 9, when the potential market demand $D_0$ rises, the greening level ($g$) and PT investment $\beta$ increase. Consequently, the selling price ($s_p$) decreases. In contrast, the total profit ($TPF$) increases.
3. As production rate is dependent on the demand rate (it is assumed in this work), so if market demand rate $D_0$ increases then production rate increases. Hence, total emissions ($TGE$) increases.
Table 8. Effects of $\omega$ on $\text{TGF}^*$ and $\text{TGE}^*$ for Experiment 7.1.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>Total emissions ($\text{TGE}^*$) kg</th>
<th>Total profit ($\text{TPF}^*$)($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1081.84</td>
<td>61,481.00</td>
</tr>
<tr>
<td>1</td>
<td>1045.30</td>
<td>60,209.75</td>
</tr>
<tr>
<td>2</td>
<td>1022.8</td>
<td>58,941.30</td>
</tr>
<tr>
<td>3</td>
<td>1001.62</td>
<td>57,671.50</td>
</tr>
<tr>
<td>4</td>
<td>980.43</td>
<td>56,401.10</td>
</tr>
<tr>
<td>5</td>
<td>959.25</td>
<td>55,130.60</td>
</tr>
</tbody>
</table>

Figure 10. Selling price utilization during emergency levels for pandemic situation.

Figure 11. Green level utilization during emergency levels for pandemic time.

(4) Figure 13 shows that investments in both GT and PT always increase the total profit of a sustainable inventory model during COVID-19 pandemic.

(5) Figures 14 and 15 indicate that when the emissions trading prices ($\text{TX}_c$ and $\text{TX}_s$) increase, the total profit and total emissions decrease, respectively. In the case of increased emissions trading prices, the manufacturer has to invest more in GT, thus increasing the selling price to compensate for these expenses.

(6) By analyzing the sensitivity of back-ordering rate $\delta$, Figure 16 implies that the average increment in the total profit of the case with GT and PT investments is 14% from the case without GT and PT investments.

(7) Figure 17 implies that percentage change in cost parameters like holding cost, deterioration cost, and back-ordering cost leads to a negative change in the current value of the total profit.

(8) From Figure 18, when the unit greenhouse gases emissions rates increase then total emissions fine increases. This result shows a decrease in total profit. In contrast, this greatly increases total emissions which is shown in Figure 19.
(9) Suppose the deterioration rate ($\lambda_0$) increases. This decreases the selling price and the total profit, while the greening level remains the same. On the contrary, investment in PT increases. The increasing deterioration rate declines the total profit due to increasing product loss. To prevent the deterioration rate, the manufacturer decreases the product’s selling price to sell the product quickly to avoid any unavoidable circumstance.

(10) When the manufacturer’s production cost is growing up, the total profit decreases and the selling price increases. However, the PT investment, greening level, and total amount of emissions remain unchanged.

(11) From Table 9 it is concluded that when $\alpha_1 = 0$, i.e., buyers are not concerned about green products then the greening level $g$ is much less. Due to the reason GT investment is also less. Now, with the increasing...
in $\alpha_1$, the greening level $g$ increases. Thus, the emission reduction percentage ($\theta$) increases, as a result, the buyers’ demands as well as the manufacturer’s total profit increase.

(12) The efficiency ($\gamma$) of GT helps the manufacturer to invest less money in GT to make more green product as a higher value of $\gamma$ increases the product’s greening level. For which buyers demands and total profit increase. However, it does not change the investment in PT significantly.

(13) When workers’ production rates ($f_1, f_2$) increase, then the total profit increases because it causes lower labour cost. Moreover, it causes a lower health allowance for $SO_2$ gas inhalation and a lower additional cost $m_c$ to keep workers’ healthy during the current COVID-19 pandemic.
9. Managerial insights

Some important insights that would be helpful for sustainable manufacturing businesses can be drawn from this investigation. Those insights are as follows:

(1) This study reveals that the rate of deterioration has a significant effect on the total profit and total emissions. It is similar to the result showed in the previous study, such as Mashud et al. [28]. For this reason, a manufacturing company must keep the deterioration rate as low as possible. To reduce the deterioration rate of products, PT can be used.

(2) The most important pollutants that have an effect on the planet, people, and various industries’ profit are CO\(_2\) and SO\(_2\) gases. So, the company can significantly decrease emissions fines, SO\(_2\) inhalation quantity and consequently health costs by reducing emissions. The GT investment and emissions cap- and -trade policy always refer to some relaxation to the company in curbing greenhouse gases emissions. The GT investment also attracts more buyers to the company for purchasing more green products.

(3) It is a necessity for a company to consider sustainability because considering environmental and social aspects with economic aspect prevent workers’ physical and mental health. In addition, it increases total profit. It also increases efficiency, health, satisfaction, and security during the current COVID-19 pandemic.

(4) By using the proposed model, an industrial manager can easily take a decision on the optimum selling price to achieve a safe margin in total profit during the present pandemic situation.

(5) According to the obtained results, it is seen that a sustainable EPQ model with full back-ordering rate gives more profit than a model with partially back-ordering rate. But, in reality, all companies perform in
Table 9. Sensitivity analysis.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>(TGE*)</th>
<th>(TPF*)</th>
<th>$s_p^*$</th>
<th>$g^*$</th>
<th>$\beta^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0$</td>
<td>600</td>
<td>429.643</td>
<td>23295.8</td>
<td>93.7141</td>
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<td></td>
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<td>747.407</td>
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<td>88.5759</td>
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<td>61481.1</td>
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<td>85.04</td>
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<td>26.452</td>
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<tr>
<td></td>
<td>1800</td>
<td>1700.77</td>
<td>99467.9</td>
<td>84.5</td>
<td>0.9607</td>
<td>30.231</td>
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<tr>
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<td>0.15</td>
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</tr>
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<td>7.5</td>
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<td>63810.2</td>
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<td>0.64</td>
<td>25.41</td>
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<td>10</td>
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<td>25.41</td>
</tr>
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<td>59151.9</td>
<td>89.950</td>
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<td>25.41</td>
</tr>
<tr>
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<td>56822</td>
<td>90.4541</td>
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<td>25.41</td>
</tr>
<tr>
<td>$p$</td>
<td>50, 75</td>
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<td>61485.8</td>
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</tr>
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<td>61481.6</td>
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<td>61481.1</td>
<td>85.57</td>
<td>0.64</td>
<td>25.41</td>
</tr>
<tr>
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<td>61480.6</td>
<td>85.1</td>
<td>0.64</td>
<td>23.0279</td>
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<td>125, 187.5</td>
<td>1081.84</td>
<td>61480.1</td>
<td>84.802</td>
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<td>0.6312</td>
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<tr>
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<td>18</td>
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<td>61481.1</td>
<td>85.57</td>
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<td>25.41</td>
</tr>
<tr>
<td></td>
<td>22</td>
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<td>61506.4</td>
<td>84.56</td>
<td>0.651</td>
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<td>25</td>
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</tr>
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</tr>
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<td>61481.2</td>
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<td>0.654</td>
<td>25.56</td>
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<tr>
<td></td>
<td>0.75</td>
<td>1080.5</td>
<td>61481.5</td>
<td>85.08</td>
<td>0.66</td>
<td>25.8</td>
</tr>
</tbody>
</table>

a competitive situation and in many cases, full back-ordering is impossible. For this reason, a sustainable EPQ partially back-ordering model will be a more reasonable model that adds social and environmental effects to an economic aspect.

(6) This study investigates that the emergency levels of lockdown during COVID-19 pandemic have a negative effect on the total profit by decreasing demands. Hence, a company should consider the demand rate that is dependent on emergency level $\omega$ and as a result, the company will be able to predict the demand rate during the current pandemic.

(7) Figures 10–12 have shown that the total profit of a production company can be optimized at maximum level during a pandemic by increasing the greening level, PT investment and controlling the selling price.

10. Conclusions

Global climate change is intensified by excessive greenhouse gases emissions, such as CO$_2$ and SO$_2$ gases. As a result, lowering greenhouse gases emissions has become a common objective. Excessive emissions can
affect human health. In this inquiry, a multi-objective sustainable EPQ inventory model has been proposed. In this model, all aspects of sustainability, namely environmental impact and social impact during the lockdown for COVID-19 pandemic have been included in addition to the economic aspect. The joint influence of social and environmental factors has been examined. To curb greenhouse gases emissions GT investment policy and emissions cap- and -trade policy have been applied by taking greening level dependent demand. This research provides a concrete evidence of the impact of a greening level on environmentally concerned shoppers’ decisions to purchase sustainable items. These effects will have a favourable impact on customers’ green consciousness in their social everyday lives. When many people purchased more items with a high green rating, manufacturers have shown more interest to produce green items that reduce greenhouse gases emissions.

During the pandemic, demand decreases and hence, deterioration rate increases for some transporting products such as cooking oil, chemical product and full-fat dairy product, etc. To prevent the deterioration rate, PT has been used in the proposed sustainable model. Furthermore, the proposed study has investigated a controllable production rate with the fluctuating demand due to the emergency levels determined by a government to control the transmission of the novel coronavirus.

Optimal solutions of the proposed multi-objective model have been obtained by using the fuzzy GP approach. The model has been illustrated by four numerical experiments. Two numerical results revealed that an EPQ model with GT and PT investments has more profit and lower emissions than the model without GT and PT investments. According to the obtained results, it has been seen that a sustainable EPQ model with full back-ordering rate gives more profit than a model with partially back-ordering rate. But, in reality, all companies perform in a competitive situation and in many cases, full back-ordering is impossible. For this reason, a sustainable EPQ partially back-ordering model with GT and PT investment will be a more reasonable model that adds social and environmental effects to an economic aspect.

One can extend this model by incorporating inflation and time value of money. This study can be extended by utilizing trade credit policy for two or multiple products as discussed by Pervin et al. [38] which would make it more realistic in future research. Various types of deterioration rate can be investigated in this proposed model to make it more practical. One can analyze the proposed model by considering the effect of greening level on selling price and production cost (as studied in the model developed by Paul et al. [37]). One can add the effect of transportation system into the proposed model because CO₂ gas emits due to transportation of raw materials and finished products (as considered in the model which developed by Ghosh and Roy [15]). Furthermore, this model can be expanded by incorporating a non linear stock-dependent holding function as proposed by Mishra et al. [31]. In future, this model can be solved by using Gradient-based Grey Wolf Optimizer technique as discussed by Khalilpourazari et al. [23].

**APPENDIX A.**

Total greenhouse gases emissions of the system without GT is as follows:

\[
\text{Total CO}_2 \text{ emissions} + \text{Total SO}_2 \text{ emissions} = \text{CE} + \text{SE}, \quad \text{where}
\]

\[
\begin{align*}
\text{CE} &= c_{fp} + c_{vp} \times (\text{Total production quantity}) + c_{vd} \times (\text{Total deteriorate quantity}) \\
&\quad + c_{vh} \times (\text{Total holding quantity}) \\
&= c_{fp} + c_{vp} PT_2 + c_{vd} \lambda(\beta) P(T_2 - T_1) + c_{vh} \int_{T_1}^{T_2} I_2(t) dt + \int_{T_2}^{T_3} I_3(t) dt \\
&= c_{fp} + c_{vp} PT_2 + c_{vd} \lambda(\beta) P(T_2 - T_1) + c_{vh} \frac{P}{\lambda(\beta)} \left( T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right) \\
&\quad + c_{vh} \frac{D}{\lambda(\beta)} \left( T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3-T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} \right) \\
&= c_{fp} + c_{vp} PT_2 + c_{vd} \lambda(\beta) P(T_2 - T_1) + c_{vh} \frac{P}{\lambda(\beta)} \left( T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right) \\
&\quad + c_{vh} \frac{D}{\lambda(\beta)} \left( T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3-T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} \right)
\end{align*}
\]

(A.1)

and

\[
\text{SE} = s_{fp} + s_{vp} \times (\text{Total production quantity}) = s_{fp} + s_{vp} PT_2.
\]

(A.2)
APPENDIX B.

Total greenhouse gases emissions after investing in GT is stated as:

\[
TGE = CE(1 - \theta) + SE(1 - \theta)
\]

\[
= c_{ch} \frac{P}{\lambda(\beta)} \left( T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right)(1 - \theta)
\]

\[
+ c_{hv} \frac{D}{\lambda(\beta)} \left( T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3-T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} \right)(1 - \theta)
\]

\[
+(c_{fp} + c_{vp}PT_2 + c_{vd}\lambda(\beta)P(T_2 - T_1))(1 - \theta) + (s_{fp} + s_{vp}PT_2)(1 - \theta),
\]

where CE and SE are given in equations (A.1) and (A.2), respectively.

APPENDIX C.

\[
TEC = [CE(1 - \theta) - \rho_c]TX_c + [SE(1 - \theta) - \rho_s]TX_s
\]

\[
= c_{ch} \frac{D}{\lambda(\beta)} \left( T_1 - T_3 + \frac{e^{\lambda(\beta)(T_3-T_2)}}{\lambda(\beta)} - \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} \right)(1 - \theta)TX_c
\]

\[
+c_{hv} \frac{P}{\lambda(\beta)} \left( T_2 - T_1 + \frac{e^{\lambda(\beta)(T_2-T_1)}}{\lambda(\beta)} - \frac{1}{\lambda(\beta)} \right)(1 - \theta)TX_c
\]

\[
+(c_{fp} + c_{vp}PT_2 + c_{vd}\lambda(\beta)P(T_2 - T_1))(1 - \theta)TX_c
\]

\[
+(s_{fp} + s_{vp}PT_2)(1 - \theta)TX_s - \rho_cTX_c - \rho_sTX_s,
\]

where CE and SE are given in equations (A.1) and (A.2), respectively.

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Conflicts of interest. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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