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OPTIMAL PMU PLACEMENT PROBLEM IN OCTAHEDRAL NETWORKS

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Abstract. Power utilities must track their power networks to respond to changing demand and availability conditions to ensure effective and efficient operation. As a result, several power companies employ phase measuring units (PMUs) to check their power networks continuously. Supervising an electric power system with the fewest possible measurement equipment is precisely the vertex covering graph-theoretic problem, in which a set D is defined as a power dominating set (PDS) of a graph if it supervises every components (vertices and edges) in the system (with a couple of rules). The $\gamma_p(G)$ is the minimal cardinality of a PDS of a graph G. In this present study, the PDS is identified for octahedral networks.

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1. Introduction

A power network is made up of electrical hubs and transmitting cables that connect them. Electric power companies must constantly track the condition of their systems. The magnitude of the voltage at loads and the system process at generators must both be monitored. The placement of PMUs at certain locations within the device must be controlled. Due to the increasing price of PMUs, it's critical to employ as little as possible while still being able to track the entire system. This Power system observability with minimal PMU placement is a problem that Haynes *et al.* [19] introduced as a theoretical graph problem and dubbed the power dominating set (PDS) problem after it was demonstrated in [3].

Let G be a connected network with vertex set $V(G)$ and edge set $E(G)$. For any $v \in V(G)$ an open r-neighborhood of v is $N_r(v) = \{u \in V(G) : d_G(u,v) = r\}$, where $d_G(u,v)$ is the distance (number of edges in u–v path) between the u to v. Similarly $N_r[v] = N_r(v) \cup \{v\}$. The degree of v in $V(G)$ is $d_G(v) = |N_1(v)|$. Denote $[\alpha] := \{1, 2, \ldots, \alpha\}$ and $[\alpha]_0 := \{0, 1, 2, \ldots, \alpha\}$. Also to denote $\{\alpha+1, \alpha+2, \ldots, 2\alpha+1\}$ we use the notation $[2\alpha+1] - [\alpha]$.

Keywords. Dominating set, phase measurement unit, private neighbor, power domination, octahedral network.

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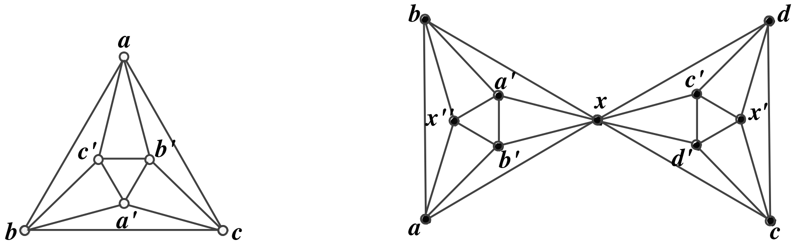


(a) (b)

FIGURE 1. (a) Unit octahedron; (b) Twin octahedron.

A dominant subset is a proper subset D of $V(G)$ if any node in $V(G)$ that is not in D must have at least one adjacent node in D. The least cardinality of all possible D is $\gamma(G)$. If all nodes of $V(G)$ can be recursively observed by either domination or propagation, a subset D is called PDS (power dominating set).

(1) **Rule (i): (Domination)**

$$M(D) \leftarrow D \cup N(D).$$

(2) **Rule (ii): (Propagation)**

$$\exists,\ \ v \in M(D) \text{ s.t. } (V(G) - M(D)) \cap N(v) = \{w\}$$
$$M(D) \leftarrow M(D) \cup \{w\}.$$

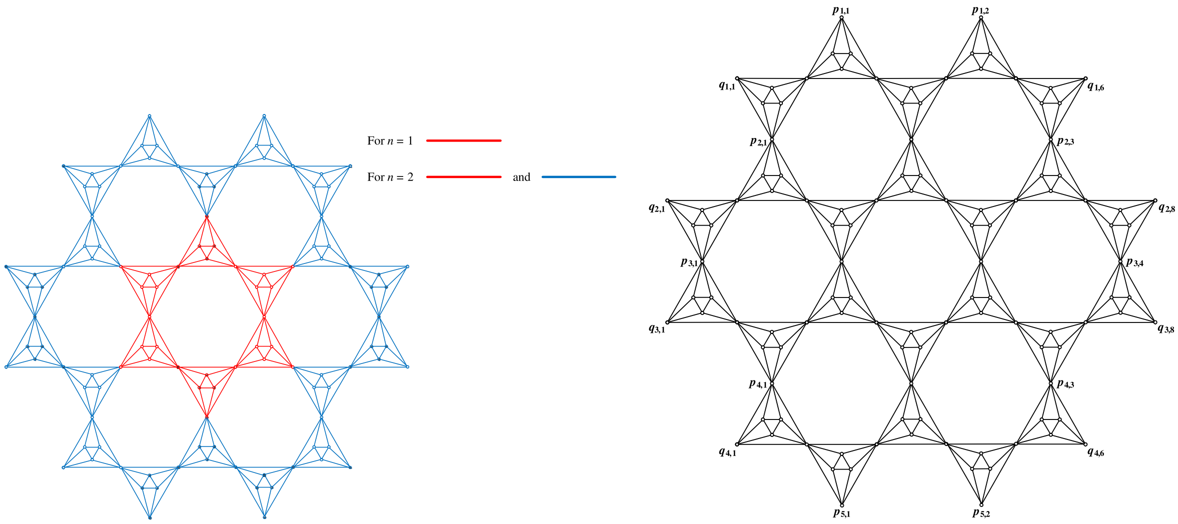
A subset D is named as PDS for G with $M(D) = V(G)$. The least positive integer representing a PDS is $\gamma_p(G)$.

The problem of finding PDS is NP-complete in general. Even for graph classes like chordal, split and bipartite, it remains NP-complete [19]. In [1,18,19], various algorithms for computing the PDS for a specific class of graphs were described. An improved algorithms with complexity results were reported in [18]. This problem is studied for block graphs [34], circular-arc graphs [25], hypercubes [8,11], grids [16], generalized Petersen graphs [4,9,21,33], permutaion graphs [32], planar graphs with small diameter [36], maximal planar graphs [15], Knodel graphs and Hanoi graphs [20], de Bruijn graphs and Kautz graphs [23], regular claw-free graphs [27], and certain chemical graphs [30]. This problem is also discussed for Cartesian product of graphs in [4,22], tensor and strong product in [13], corona product and join of graphs in [35], and for some other graph products were discussed in [5]. The lower bounds for this problem is discussed in [17]. An upper bound for one component graph with $n > 4$ is presented, and few extremal graphs concerning PDS are characterized in [37]. The Nordhaus–Gaddum results of this problem were reported in [5].

Straight forward generalization of PDS problem is done in [9] as the k-PDS problem. It is trivial to note that when $k = 1$, this problem converges to the original PDS problem and when $k = 0$ it is a traditional domination problem. This k-PDS problem is discussed for Sierpiński graphs [12], block graphs [31], regular graphs [14], certain interconnection networks [29], and weighted trees [10]. In [7], the complexity of power dominating throttling is discussed. The infectious power domination is introduced in [6], along with a general bound for determining the influence of particular hypergraph operations.

2. Octahedral and its derived networks

The octahedral structures are introduced in [2]. A platonic solid's corresponds to a polyhedral graph, which is called an octahedron graph. The unit octahedron contains 6 vertices and 12 edges. We define a twin octahedron in OH(n) as two octahedrons sharing exactly one common vertex. The structural graph of the unit octahedron and twin octahedron and the vertex representations are depicted in Figure 1. For more information on these



(a) (b)

FIGURE 2. (a) Construction of OH(2); (b) Addressing scheme of OH(2).

graphs, it can be seen from [24,28]. Different ways of connecting unit octahedron derive the varieties of octahedral structures.

2.1. Octahedral network

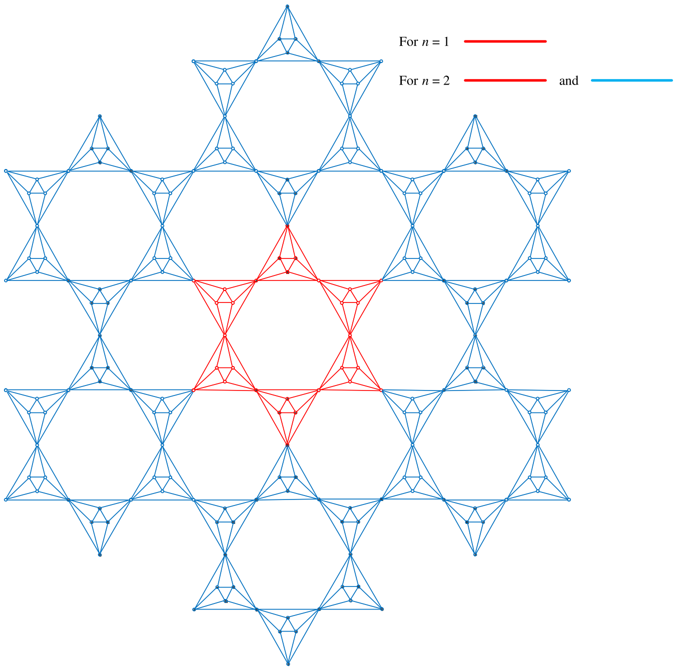
An n-dimensional octahedral network has $27n^2 + 3n$ vertices and $72n^2$ edges. It is noted that OH(n) has $6n^2$ unit octahedron and $3n^2$ edge-disjoint twin octahedron. Figure 2a gives an idea about the extension of OH(n) network and its addressing scheme is depicted in Figure 2b. $V(\text{OH}(n)) = \bigcup_{i=1}^{2} P_i \cup \bigcup_{i=1}^{2} Q_i$ where, $P_1 = \{p_{i,j} : i \in [n+1] \ \& \ j \in [n+i-1]\}$, $P_2 = \{p_{i,j} : i \in [2n+1] - [n+1] \ \& \ j \in [3n-i+1]\}$, $Q_1 = \{q_{i,j} : i \in [n] \ \& \ j \in [2(n+i)]\}$, and $Q_2 = \{q_{i,j} : i \in [2n] - [n] \ \& \ j \in [2(3n-i+1)]\}$.

2.2. Dominated octahedral network

The structural graph of n dimensional dominated octahedral network DOH$(n), n \geq 2$ and its addressing scheme is portrayed in Figure 3. It has $81n^2 - 75n + 24$ vertices and $216n^2 - 216n + 72$ edges. It contains $18n^2 - 18n + 6$ unit octahedron and hence $9n^2 - 9n + 3$ twin octahedron. The $V(\text{DOH}(n)) = \bigcup_{i=1}^{3} P_i \cup \bigcup_{i=1}^{3} Q_i$, where $P_1 = \{p_{i,j} : i \in [n] \ \& \ j \in [3i-2]\}$, $P_2 = \left\{p_{i,j} : i \in [3n-1] - [n] \ \& \ j \in \left[\frac{((-1)^{i-n-1} - 1)}{2} + 3n - 1\right]\right\}$, $P_3 = \{p_{i,j} : i \in [4n-1] - [3n-1] \ \& \ j \in [12n-3i-2]\}$, $Q_1 = \{q_{i,j} : i \in [n-1] \ \& \ j \in [6i-2]\}$, $Q_2 = \{q_{i,j} : i \in [3n-1] - [n-1] \ \& \ j \in [6n-2]\}$, and $Q_3 = \{q_{i,j} : i \in [4n-2] - [3n-1] \ \& \ j \in [24n-6i-8]\}$.

2.3. Rectangular octahedral network of Type I and Type II

In line with [26], we introduce the rectangular octahedral network in this subsection. The rectangular octahedral network of Type I denoted by ROH$^1(m,n)$ is derived by arranging octahedron in a two-dimensional plane so that the first octahedron whose apex is facing down. Rectangular octahedral network of Type II denoted by ROH$^2(m,n)$ is derived by presenting octahedrons in a two-dimensional plane similar to ROH$^1(m,n)$ with the condition that the first octahedron, which is the north west corner most unit octahedron whose apex should be facing the top. Rectangular octahedral network of Type I has $\frac{1}{2}(9mn + 2m + n - 1)$ vertices and Type II has $\frac{1}{2}(9mn+2m+n+1)$ vertices for m even and n odd. For other cases, both Type I and Type II has $\frac{1}{2}(9mn+2m+n)$



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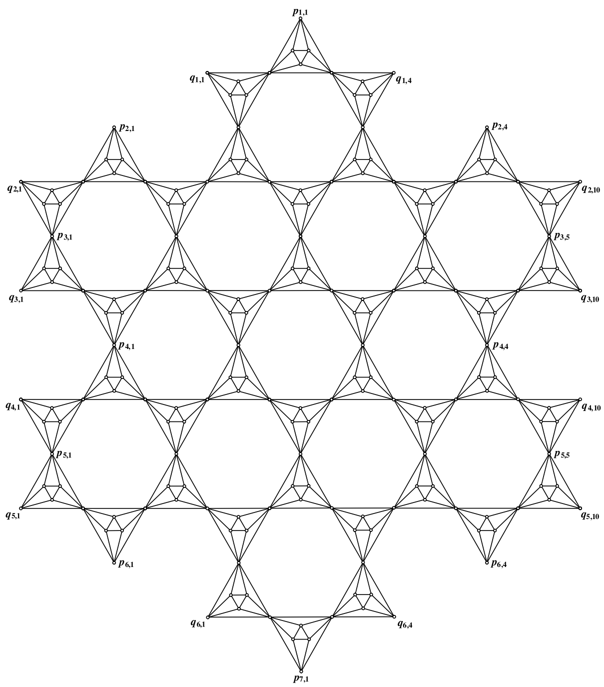


(b)

FIGURE 3. (a) Construction of DOH(2); (b) Addressing Scheme of DOH(2).

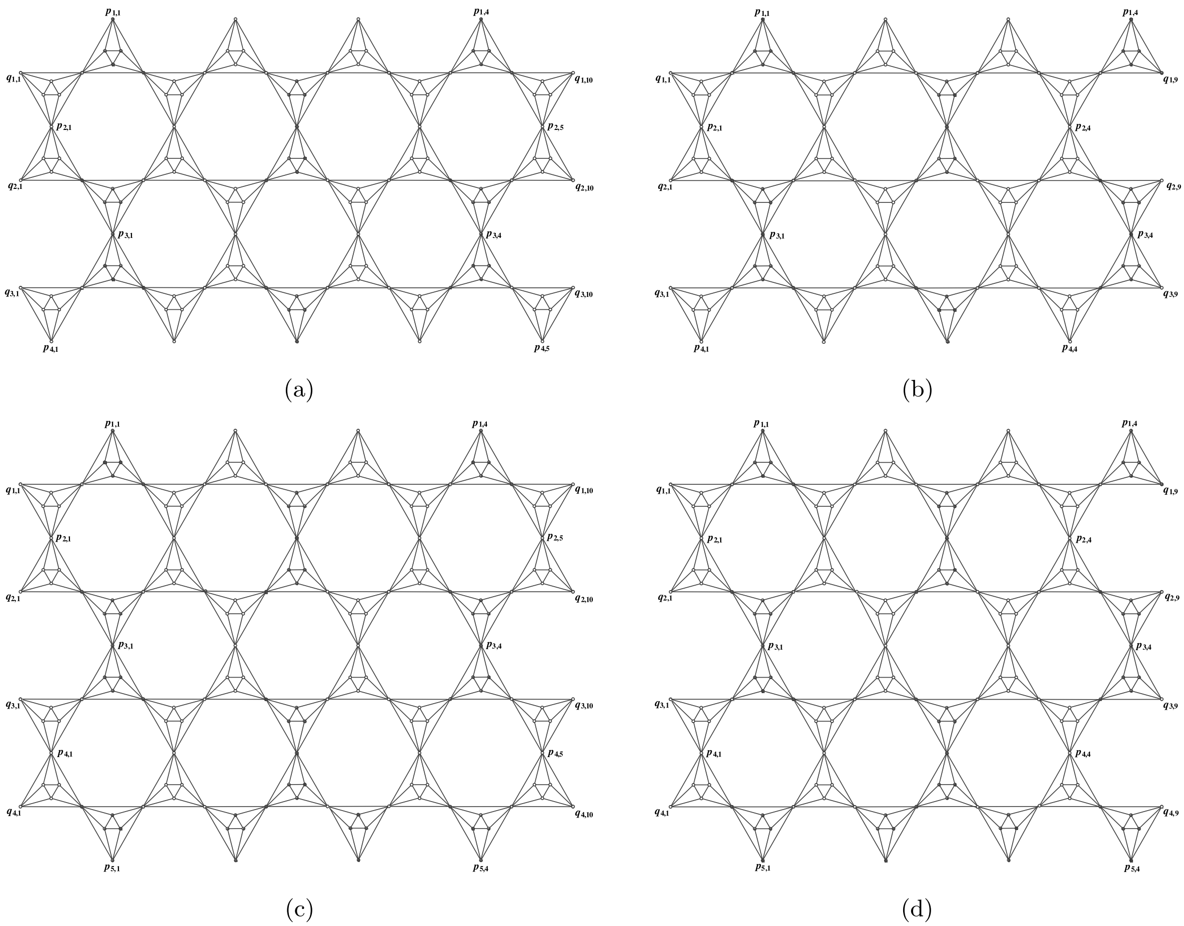


FIGURE 4. (a) $\mathrm{ROH}^1(3,9)$; (b) $\mathrm{ROH}^1(3,8)$; (c) $\mathrm{ROH}^1(4,9)$; (d) $\mathrm{ROH}^1(4,8)$.

vertices. Both types of these structures have $12mn$ edges. Different cases of $\mathrm{ROH}^1(m,n)$ and their addressing schemes are portrayed in Figure 4. It is clear that $\mathrm{ROH}^1(m,n) \cong \mathrm{ROH}^2(m,n)$ for all m and n except m even and n odd. The non-isomorphic case is depicted in Figure 5. The vertex set of Type I rectangular octahedral network $V\big(\mathrm{ROH}^1(m,n)\big) = P \cup Q$, where, $P = \left\{p_{i,j} : i \in [m+1] \ \& \ j \in \left[\frac{n}{2}\right]\right\}$, $Q = \{q_{i,j} : i \in [m] \ \& \ j \in [n+1]\}$, for n even and $P = \left\{p_{i,j} : i \in [m+1] \ \& \ j \in \left[\frac{n+(-1)^i}{2}\right]\right\}$, $Q = \{q_{i,j} : i \in [m] \ \& \ j \in [n+1]\}$. for n odd.

The vertex set $V\big(\mathrm{ROH}^2(m,n)\big) = P \cup Q$, where $P = \left\{p_{i,j} : i \in [m+1] \ \& \ j \in \left[\frac{n-(-1)^i}{2}\right]\right\}$ and $Q = \{q_{i,j} : i \in [m] \ \& \ j \in [n+1]\}$ for non-isomorphic case of Type II rectangular octahedral network.

3. MAIN RESULTS

Theorem 3.1. *Let G be any octahedral structure, H be the edge disjoint subgraph of G isomorphic to twin octahedron and D be the PDS of G. Then $V(H) \cap D \neq \emptyset$.*

Proof. Since H is an edge-disjoint subgraph of G, the nodes a, b, c, d are the only nodes through which the rest of the graph is connected to H. See Figure 6. Suppose the contrary that there exists an H such that $D \cap V(H) = \emptyset$.

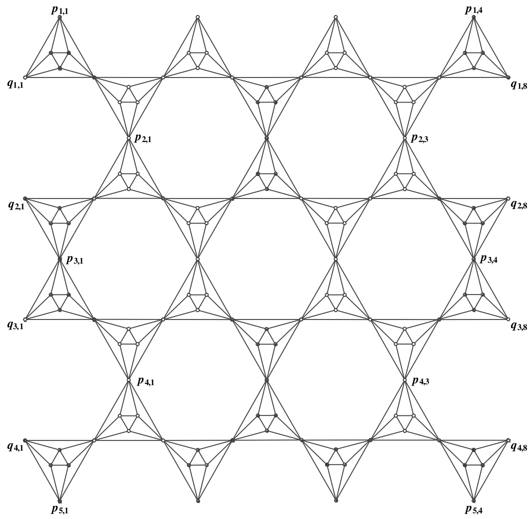


FIGURE 5. $\mathrm{ROH}^2(4, 7)$.

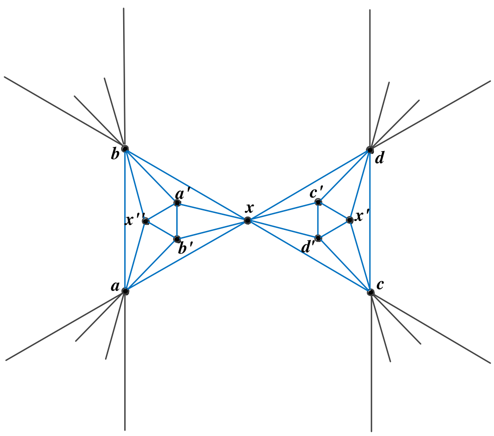


FIGURE 6. Edge disjoint subgraph H of G.

Case 1. $N(D) \cap V(H) = \emptyset$.

Here $\{a, b, c, d\}$ are not dominated. Suppose that they are observed by propagation, then the vertices $\{x, x', x'', a', b', c', d'\}$ cannot be observed by propagation as at least three vertices of this set are adjacent to each vertex in $\{a, b, c, d\}$. This is the reason for failure of propagation at $\{a, b, c, d\}$. Therefore D must contain some vertices of H. A contradiction.

Case 2. $N(D) \cap V(H) \neq \emptyset$.

Here $\{a, b, c, d\}$ are dominated. But due to the fact that $d_G(u) \geq 4$, for $u \in \{a, b, c, d\}$, the propagation at u fails. There fore the vertices $\{x, x', x'', a', b', c', d'\}$ are not observed. This contradicts the assumption of D.

□

3.1. Power domination in octahedral structures

Theorem 3.2. *For* $n \geq 1$, $\gamma_p(\mathrm{OH}(n)) = 3n^2$.

Proof. Let D be the PDS of $\mathrm{OH}(n)$. Since there are $3n^2$ copies of twin octahedron in $\mathrm{OH}(n)$. By Theorem 3.1, D must have at least $3n^2$ vertices. Therefore $|D| \geq 3n^2$. The reverse inequality is clear from the following choices.

Case 1. n odd.
For this case the PDS $D = \bigcup_{i=1}^{2} D_i^P \cup \bigcup_{i=1}^{2} D_i^Q$, where $D_1^P = \{p_{i,n+j} : i \in [n+1]-\{1\} \ \&\ j \in [i-1]\}$, $D_2^P = \{p_{n+i+1,n+j} : i \in [n-1] \ \&\ j \in [n-i]\}$, $D_1^Q = \{q_{i,2j} : i \in [n] \ \&\ j \in [n]\}$, and $D_2^Q = \{q_{n+i,2j} : i \in [n] \ \&\ j \in [n]\}$. See Figure 7a.

Case 2. n even.
For this the PDS is $D = \bigcup_{i=1}^{2} D_i^P \cup \bigcup_{i=1}^{2} D_i^Q$, where $D_1^P = \{p_{i,j+1} : i \in [n+1]-\{1\} \ \&\ j \in [i-1]\}$, $D_2^P = \{p_{n+i+1,j+1} : i \in [n-1] \ \&\ j \in [n-i]\}$, $D_1^Q = \left\{q_{i,2j}, q_{i,n+2i+2j-1} : i \in [n] \ \&\ j \in \left[\dfrac{n}{2}\right]\right\}$, and $D_2^Q = \left\{q_{n+i,2j}, q_{n+i,4n-2i-2j+3} : i \in [n] \ \&\ j \in \left[\frac{n}{2}\right]\right\}$. See Figure 7b.

∎

Theorem 3.3. *For* $n \geq 1$, $\gamma_p(\mathrm{DOH}(n)) = 9n^2 - 9n + 3$.

Proof. Let $D \subseteq V(\mathrm{DOH}(n))$ be the PDS.

There are $9n^2 - 9n + 3$ copies of twin octahedron in $\mathrm{DOH}(n)$. By Theorem 3.1, every twin octahedron must contribute one vertex to D. Hence $|D| \geq 9n^2 - 9n + 3$.

Now let us prove the reverse inequality by exhibiting the PDS of cardinality $9n^2 - 9n + 3$.

The PDS of dominated octahedral network DOH(2) is depicted in Figure 8. The set $D = \bigcup_{i=1}^{3} D_i^P \cup \bigcup_{i=1}^{3} D_i^Q$ exhibits the PDS for $\mathrm{DOH}(n)$, where $D_1^P = \{p_{i,3j-1} : i \in [n]-\{1\} \ \&\ j \in [i-1]\}$, $D_2^P = \{p_{n+2i-1,3j-2} : i \in [n] \ \&\ j \in [n]\} \cup \{p_{n+2i,3j-1} : i \in [n-1] \ \&\ j \in [n-1]\}$, $D_3^P = \{p_{3n+i-1,3j-1} : i \in [n-1] \ \&\ j \in [n-i]\}$, $D_1^Q = \{q_{i,3j} : i \in [n-1] \ \&\ j \in [2i-1]\}$, $D_2^Q = \{q_{i,3j} : i \in [3n-1]-[n-1] \ \&\ j \in [2n-1]\}$, and $D_3^Q = \{q_{i,3j} : i \in [4n-2]-[3n-1] \ \&\ j \in [8n-2i-3]\}$. ∎

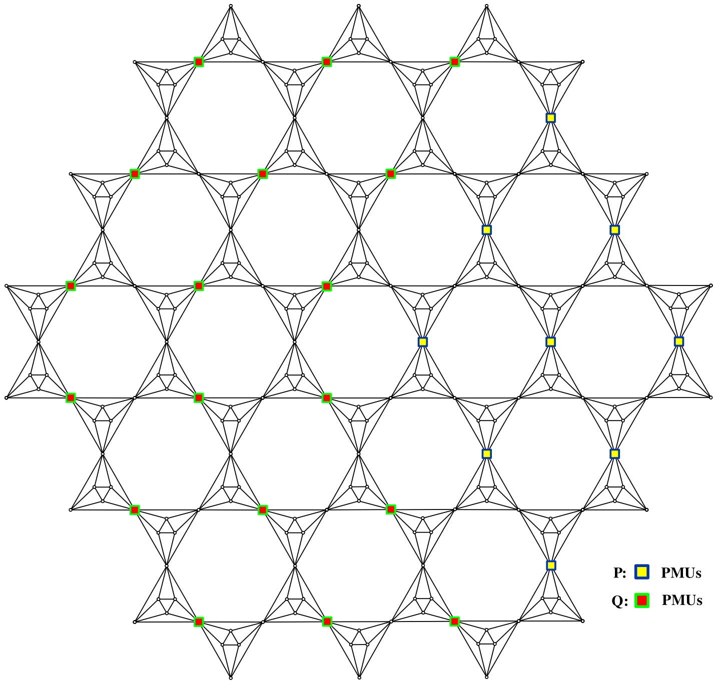
Theorem 3.4. *Let* G *be a rectangular octahedron network* $\mathrm{ROH}^1(m,n)$, $m,n \geq 1$. *Then* $\gamma_p(G) = \begin{cases} \dfrac{mn+1}{2} & : m \text{ odd and } n \text{ odd} \\ \dfrac{mn}{2} & : \text{otherwise.} \end{cases}$

Proof. The proof is arranged as follows.

Case 1. n odd.
Let $V(G) = \left\{p_{i,j} : i \in [m+1], j \in \left[\frac{n+(-1)^i}{2}\right]\right\} \cup \{q_{i,j} : i \in [m], j \in [n+1]\}$.

Case 1.1. m is odd.
Here we have $\frac{mn-1}{2}$ edge-disjoint twin octahedron and one edge-disjoint unit octahedron. By Theorem 3.1, we have $\gamma_p(G) \geq \frac{mn-1}{2} + 1$. To prove the reverse inequality by exhibiting PDS in G. The following set $D = D^P \cup D^Q \cup \left\{p_{m+1,\frac{n+1}{2}}\right\}$ exhibits the PDS, where $D^P = \left\{p_{2i,j} : i \in \left[\frac{m-1}{2}\right], j = \frac{n+1}{2}\right\}$ and $D^Q = \left\{q_{i,2j} : i \in [m], j \in \left[\frac{n-1}{2}\right]\right\}$.

Case 1.2. m is even.
Since m is even, there are $\frac{mn}{2}$ twin octahedron. By Theorem 3.1, we have $\gamma_p(G) \geq \frac{mn}{2}$. The PDS $D = \left\{p_{2i,j} : i \in \left[\frac{m}{2}\right], j = \frac{n+1}{2}\right\} \cup \left\{q_{i,2j} : i \in [m], j \in \left[\frac{n-1}{2}\right]\right\}$ is an evidence for the reverse inequality $\gamma_p(G) \leq \frac{mn}{2}$.



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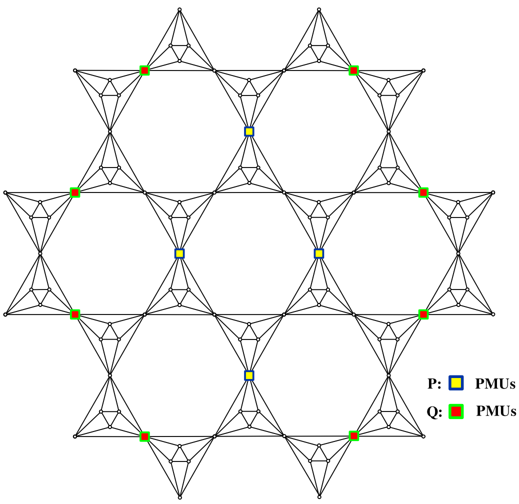

(b)

FIGURE 7. Power dominating set in octahedral network (a) OH(3); (b) OH(2).

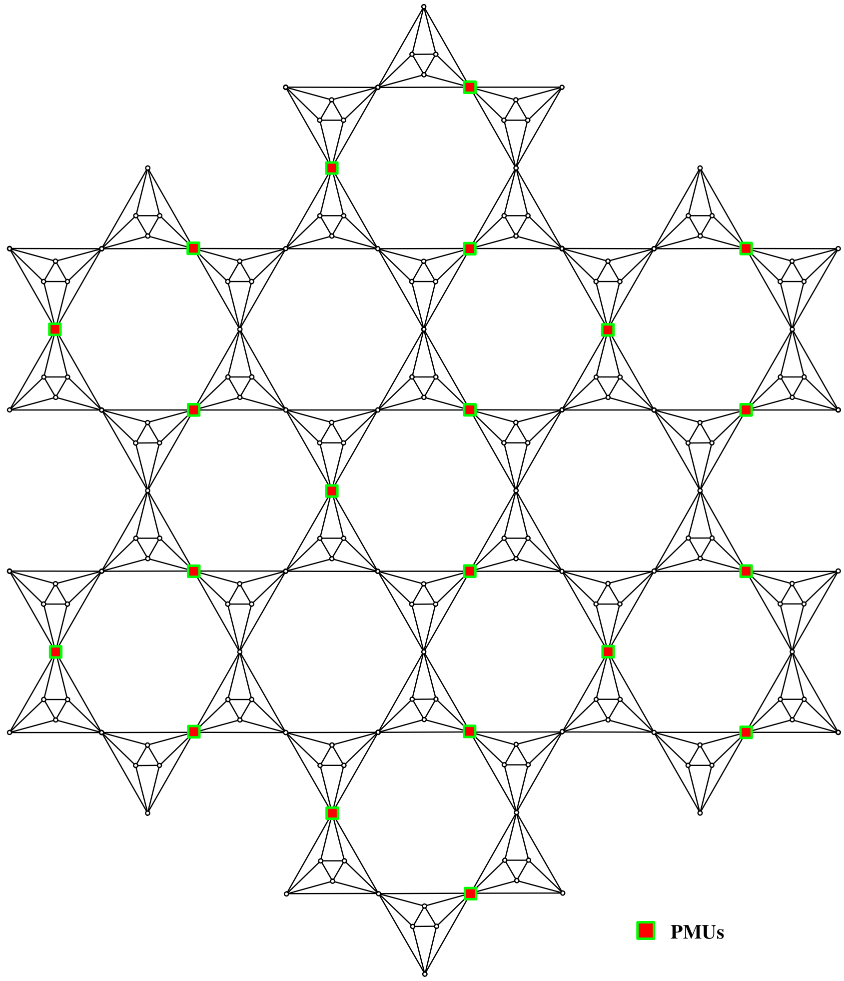

FIGURE 8. Power dominating set in dominated octahedral network DOH(2).

Case 2. n even.

Let $V(G) = \{p_{i,j} : i \in [m+1], j \in \left[\frac{n}{2}\right]\} \cup \{q_{i,j} : i \in [m], j \in [n+1]\}$ with cardinality $\frac{1}{2}(9mn + 2m + n)$ and $12mn$ edges. The proof is similar to the rest of the choice of values of m and n along with the PDS $D = \left\{q_{i,2j} : i \in [m], j \in \left[\frac{n}{2}\right]\right\}$.

□

Theorem 3.5. *Let G be a rectangular octahedron network* $\mathrm{ROH}^2(m,n)$*, $m, n \geq 1$ with m even and n odd. Then* $\gamma_p(G) = \dfrac{mn+2}{2}$.

Proof. Let $V(G) = \left\{p_{i,j} : i \in [m+1], j \in \left[\frac{n-(-1)^i}{2}\right]\right\} \cup \{q_{i,j} : i \in [m], j \in [n+1]\}$.

Since n is odd, there are $\frac{n-1}{2}$ twin octahedrons in the first and last row of the $V(G)$ and one unit octahedron in each of these rows. Hence there are $\frac{mn-2}{2}$ twin octahedrons in $\mathrm{ROH}^2(m,n)$ say TO_1, TO_2, $\ldots \mathrm{TO}_{\frac{mn-2}{2}}$. The $V(G) \setminus \bigcup_{i=1}^{\frac{mn-2}{2}} V(\mathrm{TO}_i) = S_1 \cup S_2$, where $S_1,\ S_2 \in \left\{\{N[p_{1,1}] \cup N[q_{1,1}] \setminus \{q_{1,2}\},\right.$

$\left\{N\left[p_{1,\frac{n+1}{2}}\right] \cup N[q_{1,n+1}] \setminus \{q_{1,n}\}\right\}, \{N[p_{m+1,1}] \cup N[q_{m,1}] \setminus \{q_{m,2}\}\}, \left\{N\left[p_{m+1,\frac{n+1}{2}}\right] \cup N[q_{m,n+1}] \setminus \{q_{m,n}\}\right\}$. Assume $D = \left\{p_{2i+1,j} : i \in \left[\frac{m}{2} - 1\right] \ \& \ j = \frac{n+1}{2}\right\} \cup \left\{q_{i,2j} : i \in [m] \ \& \ j \in \left[\frac{n-1}{2}\right]\right\}$, such that $M[D] = V(G) \setminus \{S_1 \cup S_2\}$. Since $S_1 \cap S_2 = \emptyset$, we have $\gamma_p(G) = |D| + 2$. Which results $\gamma_p(G) \leq \frac{mn+2}{2}$.

To prove the reverse inequality $\gamma_p(G) \geq \frac{mn+2}{2}$, assume the contrary that $\gamma_p(G) = \frac{mn}{2}$. It is clear by structure, there are $\frac{mn}{2} - 1$ twin octahedrons. By Theorem 3.1, any PDS D must contain $\frac{mn}{2} - 1$ vertices. The remaining one vertex is not sufficient to power dominate the vertices of S_1 and S_2 due to the fact that $S_1 \cap S_2 = \emptyset$, a contradiction. □

4. Conclusion

This investigation offers exhaustive work on the power domination problem of various networks. It also discusses the optimal PMU placement problem in octahedral networks. The lower bound is attained using edge-disjoint subgraph-technique, and the upper bound is obtained by exhibiting the PDS in an octahedral network. Using these techniques, the tasks mentioned above of placing optimal PMU placement problem with more accurate, optimal and reliable. The problem of placing optimal PMUs for other networks are under investigation.

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