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OPTIMAL STRATEGY FOR A PERIODIC REVIEW INVENTORY SYSTEM WITH DISCOUNTED VARIABLE COST AND FINITE ORDERING CAPACITY

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Abstract. We study a periodic review inventory system with finite ordering capacity, and assume the variable ordering cost would be discounted when the ordering quantity is the full capacity. Applying the concept of strong CK-concavity, we show that the optimal pricing and ordering strategy could be partially characterized by an (S, S', p) strategy in four regions depending on the starting inventory level per period. Numerical experiments verify the proposed strategy.

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1. Introduction

Limiting purchase quantity and providing discounted prices are two of the most common sales strategies. On the one hand, when the products sell well, which indicates that demand exceeds supply, suppliers are likely to sell goods by limiting purchase quantity. On the other hand, when the products are unsaleable, which indicates that supply exceeds demand, suppliers are likely to stimulate sales by providing discounted prices. In traditional business practices, suppliers will not adopt the two sales strategies simultaneously.

However, in the live streaming market, finite ordering capacity and price discounts have been perfectly unified. On live streaming marketing platforms in China such as Taobao Live and Tik Tok Live, suppliers usually provide limited quantity of products, aiming to promote products but not make profits, while live streaming stores in China, especially popular ones, can often attract hundreds of thousands, millions or even tens of millions of people watching the live stream at the same time. In order to promote their products to more audiences and give more people the opportunity to buy the goods, suppliers often limit purchase quantity for each buyer. At the same time, in order to show that the products sell well and many people are rushing to buy them, suppliers always provide low prices, which are much lower than prices offered by physical stores as well as traditional online stores. Moreover, many suppliers will provide quantity discounts in order to motivate each audience member to buy as many goods as possible so as to enjoy the quantity discounts. For example, one product is

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sold by the following rule: buy one at the original price, the second at half price, the third at CNY 0.1, and the fourth free. Sometimes, even if the quantity discount is given only when the buyers order up to the capacity limit, products will still be sold out quickly (which is the case we investigate in this paper). Therefore, on these live streaming marketing platforms, finite ordering capacity and price discounts can be unified.

Furthermore, we observe many other cases in the supply chain that finite ordering capacity and price discounts coexist. On the one hand, suppliers usually set a quantity limit on an order due to production, inventory or transportation capacity, or to avoid buyer dependence. On the other hand, the suppliers prefer large-volume orders, and would like to give buyers a discount when they purchase more than a specified number of products. For example, Xiaoxmi, a famous mobile phone manufacturer in China, often adopts "hunger marketing" strategies for its products; it restricts the upper bound of allocated quantity of products to every distributor, and also offers quantity discounts to them. When considering both finite ordering capacity and price discounts, for the buyer, is the classical (s, S) policy still optimal? The answer is negative.

Several studies have been conducted to analyze and calculate the optimal procurement (inventory replenishment) policy of an inventory system with price discounts and finite supply capacity. Both Manerba and Mansini [1] and Tan and Alp [2] considered a procurement problem of a company that needs to purchase products from a set of capacitated suppliers. The suppliers offer total quantity discounts, which are independent of the ordering capacity. These models only consider a single period, and do not theoretically construct the optimal procurement strategy.

In this paper, we study a periodic review inventory system in which the ordering quantity is limited. We assume the variable cost would be discounted if the order quantity is full. The objective is to maximize the expected total discounted profit over the whole planning horizon. Supported by the concepts of strong CK-concavity, we point out that the optimal pricing and ordering strategy for the system is partially characterized by an (S, S', p) policy in four regions of the starting inventory level per period separately.

Our model has a close relationship with the system in terms of setup cost and finite capacity. Several studies have been conducted to analyze the optimal strategy of an inventory system with setup cost and finite capacity. If there is no setup cost, Federgruen and Zipkin [3, 4] pointed out that the optimal strategy for the system is regarded as the modified base-stock policy. However, when the setup cost is fixed, the optimal strategy is very complicated. Chen and Lambrecht [5] and Chen [6] have shown that the optimal ordering policy could be partially characterized in the form of $X-Y$ bands. Then Gallego and Scheller-Wolf [7] refined the policy between the bands into four regions by two thresholds s and s'. In Gallego and Toktay [8], the ordering quantity could only be either zero or the maximum capacity. Chao *et al.* [9] considered a periodic review inventory system with finite ordering capacity and setup cost.

Our research is also related to the studies on quantity-dependent variable costs, especially when the ordering cost functions are concave. Porteus [10] analyzed inventory systems with concave ordering costs. Fox *et al.* [11] studied a periodic review inventory system with two suppliers, one with a high variable cost but low fixed cost, and the other with a low variable cost but high fixed cost. In Chao and Zipkin [12], if the order quantity is greater than the contracted volume, the fixed cost could be incurred. Chen *et al.* [13] applied a new preservation property of quasi-K-concavity to study single-product periodic review inventory models with concave ordering cost. Caliskan-Demirag *et al.* [14] studied a periodic review inventory model in which the order quantity decides the fixed setup cost.

Finally, our work is partly related to the problem of the coordination of pricing and inventory replenishment. Whitin [15] was the first to study the newsvendor problem in which the demand depends on the price. Petruzzi and Dada [16] conducted a further study on the basis of Whitin. Thomas [17] studied a periodic review inventory system with finite horizon and fixed-ordering cost and assumed that the demand is price-dependent. Pekelman [18] considered a joint pricing and production optimization model in which the demand is linear. On the basis of Pekelman [18], Feichtinger and Hartl [19] extended the system into a model in which the demand is non-linear. Federgruen and Heching [20] considered the optimal pricing and inventory strategy for both the infinite and finite horizon problems, which could be considered as a base-stock list-price strategy. Polatoglu and Sahin [21] investigated a model which is similar to the one in Thomas [17], and Polatoglu and Sahin [21] studied a lost-sales system.

The rest of this paper is organized as follows. In Section 2, we present our model. Then, we provide the analytical results in Section 3. The numerical results are provided in Section 4. Finally, we present the conclusion.

2. Model

We study a periodic review inventory system, in which the ordering quantity per period cannot exceed C. Assume there are N periods, from period 1 to period N. In each period, the variable ordering cost per unit is c when the ordering quantity is less than C. Otherwise, the variable ordering cost per unit is c_1, where $c_1 < c$. And we assume the ordering leadtime is zero, which implies that, before the demand is realized, an order placed would be fulfilled immediately at the beginning of a period. We further assume that the random demand is price-dependent, which indicates that $D_n(p_n)$ is determined by p_n. Similar to Petruzzi and Dada [16] and Chen and Simchi-Levi [22], we consider the additive demand. Assume the selling price is p_n in period n, the demand in the same period is given as follows

$$D_n(p_n) = d(p_n) + \epsilon_n,$$

where ϵ_n is a random variable, and the mean value of ϵ_n is zero. The selling price p_n in period n is defined over $[\underline{p}, \bar{p}]$, and it is a decision variable. $d(p_n)$ is the average demand, which is decreasing in p_n. Therefore, when p_n decreases from $\bar{p}$ to $\underline{p}$, $d(p_n)$ will increase from $d(\bar{p})$ to $d(\underline{p})$. For simplicity, let $\bar{d}$ denote $d(\bar{p})$, $\underline{d}$ denote $d(\underline{p})$, and $p = p(d)$ denote the inverse function of $d(p)$, which indicates that $p(d)$ will decrease as d increases. Then the expected revenue is given as

$$R(d) = d \cdot p(d).$$

We assume $d(p)$ is a concave continuous function with a continuous inverse, therefore, the expected revenue $R(d)$ is also concave. For example, if $d(p_n)$ is a linear decreasing function of p_n, then it is clear that $R(d)$ is concave in d.

The sequence of events happening in one period is as follows: (1) review the inventory level; (2) release the replenishment order; (3) the replenishment order arrives, (4) set the selling price; (5) the random demand is realized, and (6) all costs are collected.

Suppose the inventory level on hand is x_n, and the inventory level on hand plus on order is y_n at the beginning of period n, and unsatisfied demands are fully backlogged. Because the ordering quantity is constrained, we have

$$x_n \le y_n \le x_n + C.$$

At the end of each period, assume the inventory level is z, a cost $G(z)$ is generated, which is a convex function of z, and would be the inventory holding cost when $z \ge 0$, and be the shortage cost when $z < 0$. Given that the expected demand for period n is D_n, the expected holding and shortage cost in period n is given as $G(y_n - D_n)$. Then in period n, the expected total cost can be written as

$$c(y_n - x_n)1[x_n \le y_n < x_n + C] + c_1 C 1[y_n = x_n + C] + E[G(y_n - D_n)],$$

where $1[A]$ is the indicate function taking value 1 if statement A is true and 0 otherwise.

Let $W_n(x_n)$ be the maximum expected total discounted profit from period n to the last period, when the inventory level before the replenishment order is x_n. Assume α be the discount factor per period, $\alpha \in [0, 1]$. The optimality equation is given as follows

$$\begin{aligned} W_n(x_n) = \max_{x_n \le y_n \le x_n + C} \max_{d_n \in [\underline{d}, \bar{d}]} \{ & R(d_n) - c(y_n - x_n)1[x_n \le y_n < x_n + C] \\ & - c_1 C 1[y_n = x_n + C] - E[G(y_n - d_n - \epsilon_n)] + \alpha E[W_{n+1}(y_n - d_n - \epsilon_n)] \}, \end{aligned}$$

where the objective is to develop the optimal pricing and ordering strategy that maximizes the expected total discounted profit within the planning horizon. The terminal condition is $W_{N+1}(x) \equiv 0$.

3. Analytical results

To find the optimal strategy, we need to make some new assumptions and notations firstly. Note that the unit cost will be discounted if the order quantity is full, which means that the cost will be reduced by $(c-c_1)C$ when the ordering quantity is the full capacity. For convenience, we assume $(c-c_1)C = K$. Then, the optimization equation $W_n(x_n)$ will change to:

$$\begin{aligned} W_n(x_n) = \max_{x_n \le y_n \le x_n + C} \max_{d_n \in [\underline{d}, \bar{d}]} \{ & R(d_n) - K1[y_n < x_n + C] + K - c(y_n - x_n) \\ & - E[G(y_n - d_n - \epsilon_n)] + \alpha E[W_{n+1}(y_n - d_n - \epsilon_n)]\}. \end{aligned}$$

Suppose $u_n = -x_n - C$, $v_n = -y_n$ and $L_n(u_n) = W_n(-u_n - C)$. Then we will have

$$\begin{aligned} L_n(u_n) = K - c(u_n + C) + \max_{u_n \le v_n \le u_n + C} \max_{d_n \in [\underline{d}, \bar{d}]} \{ & R(d_n) - K1[v_n > u_n] + cv_n \\ & - E[G(-v_n - d_n - \epsilon_n)] + \alpha E[L_{n+1}(v_n - C + d_n + \epsilon_n)]\}. \end{aligned}$$

Then we will introduce the definition of strong CK-concave which is crucial in finding the optimal strategy in this inventory system (see [9] for detail).

Definition. A function $g(.) : \mathbb{R} \to \mathbb{R}$ is strong CK-concave if, for all $a \ge 0$,$b > 0$ and $z \in [0, C]$, we have

$$\frac{z}{b} g(y-a) + g(y) \ge \frac{z}{b} g(y-a-b) + g(y+z) - K. \tag{1}$$

According to Chao *et al.* [9], there are some properties on strong CK-concave. *E.g.*, K-concavity is a special case of strong CK-concavity when $C = \infty$ and $b = 0$. Concavity is also strong CK-concavity for any nonnegative C and K. If G is a strong CK-concave function, then it is also a strong DL-concave function, if $0 \le D \le C$ and $L \ge K$. If G_i is a strong CK_i-concave function $(i = 1, 2)$, then $\alpha G_1 + \beta G_2$ is also a strong $C(\alpha K_1 + \beta K_2)$-concave function, if $\alpha, \beta \ge 0$. And if G is a strong CK-concave function, then $E[G(y - X)]$ is also a strong CK-concave function for any random variable X such that the expectation exists.

Based on the definition of strong CK-concave, Lemmas 1 and 2 in Chao *et al.* [9] show that if $W_n(y)$ is strong CK-concave, then so is

$$g(y) = \max_{d_n \in [\underline{d}, \bar{d}]} \{r_n(d_n) + W_n(y - d_n)\},$$

where $r_n(d)$ is concave. Furthermore, Theorem 1 in Chao *et al.* [9] shows that the optimality equation

$$V_n(x) = cx + \max_{x \le y \le x + C} \{-K1[y > x] - cy\} + \max_{d_n \in [\underline{d}, \bar{d}]} \{r(d_n) + W_n(y - d_n)\}$$

is also strong CK-concave, and the structure of the optimal inventory and pricing policy for each period is characterized by an (s, s', p) policy in four regions of the starting inventory level.

Based on Lemmas 1, 2 and Theorem 1 in Chao *et al.* [9], it is easy to obtain the following lemma.

Lemma 1. *Given that $R(d_n)$ is concave in d_n, and $G(z)$ is convex in z, then $L_n(u_n)$ is CK-concave, $n = 1, \ldots, N$.*

Proof. Suppose $L_{n+1}(u_{n+1})$ is strong CK-concave, we can show that $L_n(u_n)$ is also strong CK-concave. Note that $R(d_n)$ is concave in d_n, and $G(z)$ is convex in z. Assume

$$L'_n(v_n) = \max_{d_n \in [\underline{d}, \bar{d}]} \{R(d_n) - E[G(-v_n - d_n - \epsilon_n)] + \alpha E[L_{n+1}(v_n - C + d_n + \epsilon_n)]\}.$$

According to Lemmas 1 and 2 in Chao *et al.* [9], it is clear that $L'_n(v_n)$ is also strong CK-concave. Note that,

$$L_n(u_n) = K - c(u_n + C) + \max_{u_n \le v_n \le u_n + C} \{-K1[v_n > u_n] + cv_n + L'_n(v_n)\}.$$

Hence, based on the proof of Theorem 1 in Chao *et al.* [9], $L_n(u_n)$ is also strong CK-concave. Because $W_{N+1}(x) \equiv 0$, it is obvious $L_{N+1}(u) \equiv 0$, which indicates that $L_{N+1}(u_{N+1})$ is strong CK-concave. Therefore, $L_n(u_n)$ is strong CK-concave, where $n = 1, \ldots, N$. □

We then characterize the optimal strategy of the system in the following theorem.

Theorem 1. *Given that $L_n(u_n)$ is CK-concave. Suppose x_n is the inventory level on hand, and y_n is the inventory level on hand plus on order at the beginning of period n. The optimal pricing and ordering strategy of $W_n(x_n)$ is characterized by the optimal pricing $p^*_n(y_n)$ and two threshholds S_n and S'_n, $p^*_n(y_n)$ depends on y_n and $S_n \ge S'_n \ge 0$. If $S'_n + C \ge S_n$, then the optimal ordering strategy is given as*

(i) *order nothing if $x_n > S'_n$;*
(ii) *order at most up to S'_n if $S_n - C < x_n \le S'_n$;*
(iii) *order either capacity C or at most up to S'_n if $S'_n - C < x_n \le S_n - C$; and*
(iv) *order capacity C if $x_n \le S'_n - C$.*

Else if $S'_n + C < S_n$, then the optimal ordering strategy is given as

(i') *order nothing if $x_n > S_n - C$;*
(ii') *order either capacity C or nothing if $S'_n < x_n \le S_n - C$;*
(iii') *order either capacity C or at most up to S'_n if $S'_n - C < x_n \le S'_n$; and*
(iv') *order capacity C if $x_n \le S'_n - C$.*

Proof. Because $L_n(u_n)$ is $CK - concave$, according to Theorem 1 in Chao *et al.* [9], the optimal strategy of $L_n(u_n)$ is characterized by the optimal pricing $p^*_n(v_n)$ and two threshholds s_n and s'_n, such that $p^*_n(v_n)$ depends on the post-order inventory level v_n and $s_n \le s'_n$. If $s'_n - C \le s_n$, the optimal ordering strategy is

(i) place an order of a quantity of C if $u_n < s'_n - C$;
(ii) place an order for an amount to bring the inventory level at least up to s'_n if $s'_n - C \le u_n < s_n$;
(iii) no order is placed or place an order for an amount to bring the inventory level at least up to s'_n if $s_n \le u_n < s'_n$; and
(iv) no order is placed if $u_n \ge s'_n$.

Else if $s'_n - C > s_n$, then the optimal ordering strategy is

(i') place an order of a quantity of C if $u_n < s_n$;
(ii') no order is placed or place an order of a quantity of C if $s_n \le u_n < s'_n - C$;
(iii') no order is placed or place an order for an amount to bring the inventory level at least up to s'_n if $s'_n - C \le u_n < s'_n$; and
(iv') no order is placed if $u_n \ge s'_n$.

This theorem can be proved based on the optimal policy of $L_n(u_n)$. We only characterize the optimal strategy of $W_n(x_n)$ in the scenario of $S'_n + C \ge S_n$ and $x_n > S'_n$, and the proof is similar in other scenarios. If $s'_n - C \le s_n$ and $u_n < s'_n - C$, the optimal ordering strategy of $L_n(u_n)$ is to place an order of a quantity of C, which indicates $v^*_n = u^*_n + C$. Note that $u_n = -x_n - C$ and $v_n = -y_n$, it implies that when $-x_n - C < s'_n - C$, the optimal ordering strategy is $-y^*_n = -x^*_n$, which means that the optimal ordering strategy is no order. Furthermore, $u_n < s'_n - C$ implies that $x_n > -s'_n$, therefore, when $s'_n - C \le s_n$ and $x_n > -s'_n$, the optimal ordering strategy of $L_n(x_n)$ is no order. Moreover, the optimal pricing $p^*_n(y_n)$ depends on y_n. For convenience, we assume $S'_n = -s'_n$ and $S_n = -s_n$. Therefore, the optimal ordering strategy of $W_n(x_n)$ is no order when $S'_n + C \ge S_n$ and $x_n > S'_n$. □

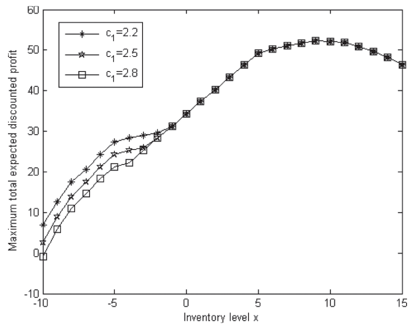

FIGURE 1. Maximum total expected discounted profit for different c_1.

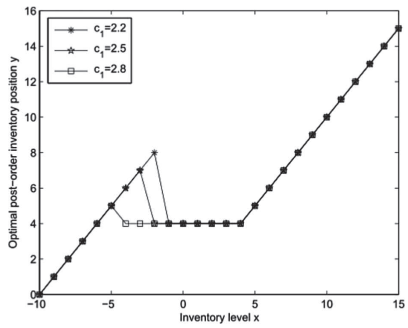

FIGURE 2. Optimal post-order inventory level y for different c_1.

4. Numerical experiments

In this section, we conduct several numerical experiments for this periodic review inventory system, where the variable unit ordering cost is discounted when the ordering quantity is the full capacity. We further investigate the effects of various parameters on the control strategy and the profit.

To validate our analytical results, we consider a 4-period inventory problem, and set the discount factor α to 0.9. In period n, the demand is given as $D_n(p_n) = d(p_n)+\epsilon_n$, where $d(p_n) = 10-p_n$, and ϵ_n is a random variable, with the probability mass function $P\{\epsilon_n = 1\} = P\{\epsilon_n = -1\} = 0.5, n = 1,\ldots,4$. Therefore, $R(d) = d(10-d)$. Other parameters are given as follows: the unit holding cost $h = 2$, the unit shortage cost $b = 4$, the ordering capacity $C = 10$, the variable unit ordering cost $c = 3$ for the case when the ordering capacity is less than C, and the variable unit ordering cost $c_1 = 2.5$ for the case when the ordering capacity is equal to C.

4.1. Effects of the discounted variable ordering cost

In this subsection, we illustrate the optimal pricing and ordering strategy for different values of the discounted variable unit ordering cost c_1. The results are shown in Figures 1–3, where the x-axis is the initial inventory level on hand x. Here, the value of x increases from -10 to 15, with the increment of 1. In Figure 1, the y-axis displays the maximum expected total discounted profit. In Figure 2, the y-axis displays the optimal post-order inventory level y. In Figure 3, the y-axis displays the optimal average selling quantity $d(x)$.

In Figure 1, when the initial inventory level on hand x is small, the maximum expected total discounted profit is increasing in x. However, after the maximum expected total discounted profit reaches the maximum value, it will decrease in x. Obviously, the profit is not concave in the initial inventory level x. In addition, when x is large enough, the maximum total expected discounted profit will not change with respect to the discounted

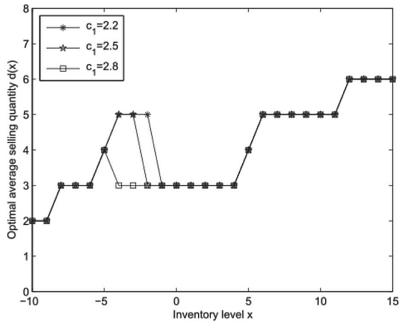

FIGURE 3. Optimal average selling quantity $d(x)$ for different c_1.

variable ordering cost c_1. That is because when x is large enough, the optimal ordering quantity will be always less than the full capacity. Therefore, c_1 will not affect the profit.

In Figure 2, the optimal post-order inventory level y is increasing in the initial inventory level on hand x when x is small. This is because when x is small, the optimal ordering strategy is ordering the full capacity. Then y will decrease when x increases to $S_n - C + 1$, and the reason is that the optimal ordering strategy changes to ordering at most up to S'_n. Then y is still non-decreasing in x when $x > S_n - C$, because the optimal ordering strategy is ordering at most up to S'_n when $S_n - C < x \leq S'_n$, and the optimal strategy is ordering nothing when $x > S'_n$.

In Figure 3, when the initial inventory level on hand $x \leq S_n - C$, the optimal average selling quantity $d(x)$ is increasing in x. However, $d(x)$ will decline when $x = S_n - C + 1$. Then $d(x)$ keeps non-decreasing in x again when $x > S_n - C$. Furthermore, compared with Figure 2, we find that when the optimal post-order inventory level y keeps constant, $d(x)$ also keeps constant, the reason is that the optimal selling price $p^*_n(y)$ depends on both y and $d(x)$.

In addition, from Figures 1 to 3, we could see that the maximum total expected discounted profit, the optimal post-order inventory level y, and the optimal average selling quantity $d(x)$ are all non-increasing in c_1. First, it is obvious that the profit should be non-decreasing in the cost c_1. Second, when ordering the full capacity, there will be more cost saving if c_1 is smaller, therefore, if the initial inventory level on hand is given, it is better to order the full capacity when c_1 is smaller. Third, the average selling quantity $d(x)$ is non-decreasing in y, therefore, $d(x)$ is also non-increasing in c_1.

Finally, we find that S'_1 is always equal to S_1 in all these cases, and both S_1 and S'_1 are non-increasing in c_1, which is consistent with our intuition, and the more the discount is, the more we prefer to order. These findings are shown in Figure 4.

4.2. Effects of the ordering capacity

In this subsection, we investigate the effects of ordering capacity C on the optimal pricing and ordering strategy. The results are shown in Figure 5–7, where the x-axis is the initial inventory level on hand x. Here, the value of x increases from -10 to 15, with the increment of 1. In Figure 5, the y-axis displays the maximum expected total discounted profit. In Figure 6, the y-axis displays the optimal post-order inventory level y. In Figure 7, the y-axis displays the optimal average selling quantity $d(x)$.

In Figure 5, the maximum total expected discounted profit is also increasing in the initial inventory level x when x is small. Then it is decreasing in x after the maximum expected total discounted profit reaches its largest value. This observation is similar to what is shown in Figure 1. However, different values of C could make different effects on the maximum expected total discounted profit when x is small.

In Figure 6, when the initial inventory level on hand $x \leq -s_n - C$, the optimal post-order inventory level y is increasing in x, because $y = x + C$ always holds in this case. When $x = -s_n - C + 1$, the optimal post-order

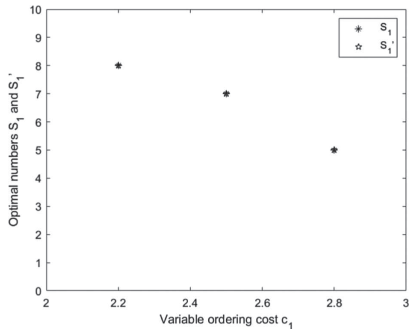

FIGURE 4. Optimal numbers S_1 and S_1' for different c_1.

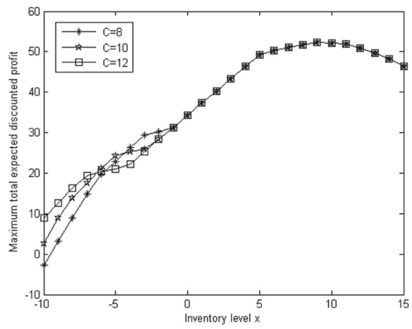

FIGURE 5. Maximum total expected discounted profit for different C.

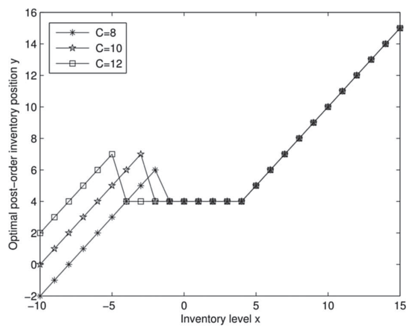

FIGURE 6. Optimal post-order inventory level y for different C.

inventory level y suddenly drops. After that y starts to be non-decreasing in x when $x > -s_n - C$. And there also exist some quantities that buyers will never order. These observations are also similar with what is shown in Figure 2. However, different values of C could also make different effects on the optimal post-order inventory level y when x is small.

In Figure 7, when $C = 10$ and $C = 12$, the observations are similar as what it is shown in Figure 3. When the initial inventory level on hand $x \leq -s_n - C$, the optimal average selling quantity $d(x)$ is increasing in x. At the point when $x = -s_n - C + 1$, $d(x)$ suddenly drops. Then $d(x)$ starts to be non-decreasing in x again when $x > -s_n - C$. When $C = 8$, the observation has a little difference that the curve drops twice before

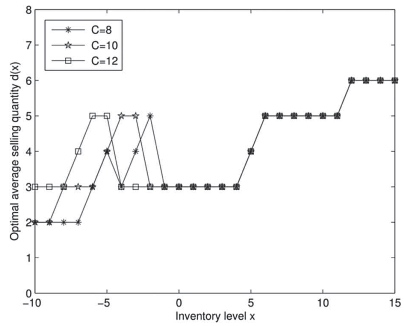

FIGURE 7. Optimal average selling quantity $d(x)$ for different C.

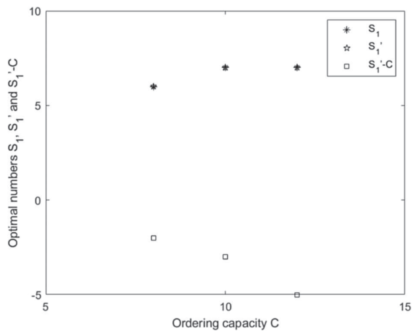

FIGURE 8. Optimal numbers S_1 and S_1' for different C.

it continues to be non-decreasing in x. Furthermore, compared with Figure 6, $d(x)$ keeps constant when the optimal post-order inventory level y keeps unchanged.

In this subsection, we also find that S_1' is always equal to S_1 in all these cases. Both S_1 and S_1' are non-decreasing in C. Moreover, $S_1' - C$, the upper limit for the full capacity order, is decreasing in C, which indicates that the more we have to order to enjoy the discount, the less we prefer to order the full capacity. These findings are shown in Figure 8.

5. Conclusion

In this paper, we study a periodic review inventory system with finite ordering capacity. In addition, if the system orders the full capacity, the variable ordering cost will be discounted, and both the price and the ordering quantity are decision variables. The optimal pricing and ordering strategy is partially characterized by an (S, S', p) strategy in four regions, which depends on the initial inventory level per period. We find that S and S' turn out to be the same in the numerical results. Moreover, both S and S' are increasing in c_1 and decreasing in C. Future research directions include designing effective algorithms to calculate values of S and S'.

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