

A BRANCH-AND-PRICE ALGORITHM FOR A ROUTING AND SCHEDULING PROBLEM FROM ECONOMIC AND ENVIRONMENTAL PERSPECTIVES

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Abstract. This paper addresses a routing and scheduling problem from two different perspectives: economic and environmental. From economic perspective, we aim to optimize the vehicle routing plan to reduce the operating cost, but from environmental perspective, we aim to optimize the vehicle routing and speed decisions to reduce the carbon emissions. This research can provide two different decision plans under these two different perspectives, and the comparison of the results from the two different perspectives will be very meaningful and helpful to the logistics decision-makers. We formulate the problem using two mixed-integer programming (MIP) models with different objectives. However, this problem is very challenging, with medium-sized instances already difficult for the MIP solver. In order to solve it with larger scale instances, we propose an exact branch-and-price (BAP) algorithm. The BAP algorithm relies on efficiently solving the pricing sub-problem with different objectives. We design two different tailored labeling algorithms to solve it. Extensive computational experiments demonstrate the effectiveness of the proposed BAP algorithm, comparing with the MIP formulation solved by CPLEX with a time limit of 2 h.

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1. INTRODUCTION

With the vigorous development of the e-commerce, logistics and freight transportation are particularly important. The main role of logistics in the digital world is to use real-time data to generate vehicle routes, so as to reduce the negative consequences such as congestion, safety and environment [32]. This paper studies such a logistics problem: a large number of customers who has demand are usually distributed in a town, a village or a city. Each customer has a different service time horizon (also called time window in the paper) and a different demand quantity. These customers will be serviced by several homogeneous trucks with the same capacity. Therefore, to a certain extent, the problem studied in this paper is similar to a vehicle routing problem with time window (VRPTW) considering carbon emissions [20, 30].

Generally, the transportation cost will influence the optimization decisions of logistics company, and the goal is to design a valid route plan to reach the best operations cost [10, 28]. Except the economic factors of transportation, sustainable transportation is an active and important research topic today, which refers to any green means of transportation with little impact on the environment [22, 23]. Generally, the transportation

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sectors are responsible for a large portion of the pollution for Green House Gas (GHG) emissions worldwide [17]. According to the work reports by the US Environmental Protection Agency (EPA), the transportation sector contributes 28% of national GHG emissions [8]. In other words, the transportation sector of this routing and scheduling problem is to transform the driver to support the customers' required goods by cars or other means of transportation [9]. Among these logistics activities, GHG, especially carbon dioxide (CO₂) emissions, are the most concerning because CO₂ emissions have direct influences on people's health [1]. Based on this motivation, this study develops a routing and scheduling model under economic and environmental perspectives to design the route plan for the logistics company.

In this paper, the aim is to design a reasonable logistics route plan under some constraints, and the objective is to minimize operating cost. Moreover, we also test the objective of minimizing the total carbon emissions, which has a positive linear relationship with fuel consumption [24]. Generally, this goal can reduce environmental pollution while optimizing operating costs for the logistics company. In order to verify the rationality of this goal, this paper compares the routing plan of some instances under different goals and compares their operating costs.

The carbon emissions along a route are calculated by a formulation based on the traveling speed. In this paper, we set several speed data for selecting by the vehicles' drivers. Therefore, this paper not only studies the route of visiting customers, but also determines the speed of the vehicles. The consideration of carbon emissions makes the problem be a route and speed joint optimization problem. There is no doubt that our scheduling problem is more complicated than the classical VRPTW. Since VRPTW is NP-hard problem [30], our scheduling problem is of course NP-hard. In order to solve the problem, an exact branch-and-price (BAP) algorithm is proposed in this paper. Experimental results highlight the efficiency of the proposed approach thanks to a comparison of the results with the mixed-integer programming (MIP) formulations solved by CPLEX.

The main contributions of this paper can be denoted as follows: (1) this paper addresses a routing and scheduling problem from economic perspective and environmental protection perspective, and the studied problem has a different objective under different perspectives; (2) an exact branch-and-price approach is proposed to solve the problem, in which two tailored labeling algorithms are used to solve the pricing sub-problem; (3) the experimental results show the effectiveness of the proposed BAP algorithm and can be used as a benchmark for future research of related problem.

The rest of this paper is organized as follows. The relevant literature is investigated in Section 2. Section 3 introduces the problem and mathematical model. Section 4 illustrates the column generation algorithm for its relaxation problem. Section 5 develops a BAP approach in order to solve the problem. The computational experiments are described in Section 6. Section 7 concludes the paper.

2. LITERATURE REVIEW

With the rapid development of environmental awareness in modern logistics industry, more and more scholars studied vehicle routing problem (VRP) with the green issues [2]. Bektaş and Laporte [1] first introduced the pollution-routing problem (PRP) in 2011, which is a variant of the VRP. This problem considered the relationship between carbon emissions and speed, was modeled as a mixed-integer linear programming (MILP) model. In 2012, Demir *et al.* [6] proposed an adaptive large neighborhood search (ALNS) for the PRP to minimize a function combining fuel, emission and driver costs. After that, Kwon *et al.* [18] considered carbon emission in logistics systems, and proposed a Tabu search (TS) algorithm to minimize the sum of variable operation costs and carbon emission trades. In 2014, Tajik *et al.* [31] presented a new model for PRP with pickup and delivery, and they introduced a robust counterpart of the MILP model to deal with uncertainty. Based on the previous works, Lin *et al.* [19] in 2014 systematically reviewed the green VRP (GVRP) from energy saving, emissions reduction and reverse logistics; moreover, they provided a classification of GVRP. Qian and Eglese [25] in 2016 produced routes and schedules for a fleet of delivery vehicles in order to minimize the fuel emissions in a time-varying road network, and they proposed a column generation based TS algorithm to solve this problem. In 2017, Dabia *et al.* [5] developed an exact branch-and-price (BAP) algorithm for a variant of the PRP, in which

the speed of each arc is assumed to be the same. Teoh *et al.* [32] proposed a data driven multi-objective differential evolution (MODE) algorithm to solve the safe capacitated vehicle routing problems (CVRP) by minimizing the GHG emissions and hazardous risk in 2018. Fathollahi-Fard *et al.* [10] in 2019 studied the environmental pollution or green emissions into the home health care (HHC) routing and scheduling problems, and proposed a new modified simulated annealing (SA) algorithm to solve this problem. Cai *et al.* [2] in 2021 developed a hybrid particle swarm optimization (HPSO) algorithm to solve a low-carbon VRP. Luo *et al.* [22] in 2021 studied a routing and scheduling problem with the consideration of carbon emissions, and they proposed an ant colony optimization (ACO) heuristic for the problem.

Based on the above studies related to VRP with green issues, we can find that it is meaningful to study green logistics. However, when studying green logistics, we should not only focus on energy consumption or exhaust emissions. We also have to pay attention to operational costs. In my opinion, it is very valuable to study green logistics, especially when considering operating costs, it is still possible to conduct research on green logistics. Therefore, this paper considers two objectives, and conducts two independent experiments. One is with the objective of operating cost (in the form of transportation costs), the other is with the objective of carbon emissions. This research can provide two different decision plans under these two different perspectives, which is very interesting and meaningful for a logistics organization. In order to solve the studied problem, we develop an exact BAP approach. There is no doubt that exact algorithm will be very interesting and have theoretical meaning.

Related to the VRP with green issues, some good exact algorithms have been developed in the literature. For example, Qian and Eglese [25] used a column generation based TS algorithm to solve a green VRP in 2016. They designed a column generation method to solve the master problem, and the pricing sub-problem was solved by a TS heuristic. Dabia *et al.* [5] in 2017 developed an exact BAP algorithm for a variant of the PRP, which is the first time that scholars have not used the algorithm based on meta-heuristics to solve VRP with green issues. In this problem, the master problem is a set-partitioning problem solved by means of column generation, and the pricing sub-problem is a speed-and start-time elementary shortest path problem with resource constraints solved by a tailored labeling algorithm. New dominance rules are developed to discard unpromising labels. In 2019, Yu *et al.* [34] developed a BAP algorithm for the heterogeneous fleet green VRP with time windows. In this study, in order to solve the pricing sub-problem, a multi-vehicle approximate dynamic programming (MVADP) algorithm based on the labeling algorithm is developed. This research is similar to our study. However, only one speed is considered in this research. multiple speed selections make our study more complicated. In 2021, Wang *et al.* [33] proposed a BAP algorithm to solve a green location routing problem with multi-type charging infrastructure, where initial feasible columns are given by a hybrid heuristic algorithm, the pricing sub-problems are solved by the label-setting algorithm, and the global lower bound is raised by the Lagrangian lower bound.

3. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

3.1. Problem description

Throughout this paper, we work on the delivery version of the routing and scheduling problem with homogeneous vehicles from economic perspective and environmental protection perspective. The aim is to optimize the daily routes plan in order to minimize operating cost or carbon emissions. The problem can be defined as follows. Let $G = (N, A)$ be a directed graph with a set of nodes $N = \{0, 1, \dots, n, n + 1\}$ and a set of arcs $A = \{(i, j) | i, j \in N, i \neq j\}$. Node 0 and node $n + 1$ represent the star depot and the end laboratory, respectively. Nodes $P = \{1, 2, \dots, n\}$ represent the customers who has demand from the logistics company.

Each customer $i \in P$ has a demand q_i , and each vehicle has the same load capacity Q . Each customer $i \in P$ is associated with a service duration s_i . Each customer $i \in P$ has a service time window $[a_i, b_i]$, where a_i represents the earliest time and b_i represents the latest time for visiting the customers. Each driver is allowed to arrive before the earliest time a_i , but the driver must wait until the time is available for the customer and the driver can start working. The driver is prohibited to arrive after the latest time b_i . The start working time at customer i is denoted by τ_i . The start depot and the end depot have the same time window, meaning the drivers must

leave from the start depot and return to the end depot between the earliest time and latest time. There is a fixed planning horizon $[0, T]$ for the logistics company.

The distance between i and j is denoted as d_{ij} . The carbon emission between i and j is calculated by a formulation denoted in Section 3.2 based on the distance and speed between i and j . The speed v_{ij} between i and j is an integer variable in the paper. Based on the speed v_{ij} , it is very easy to calculate the travel time d_{ij}/v_{ij} between i and j .

The problem is developed to determine a set of routes in order to minimize the objective (operating cost or carbon emissions) under the constraints of time windows, capacity, and the following assumptions: (1) each vehicle leaves from the start depot, deliver the samples to the end depot, and at last returns to the depot, and visits each customer at most once; (2) we set the cost from the start depot to end depot as the fixed cost, which is not considered in the paper; (3) the speed of the vehicle is assumed to be an average speed in each arc.

3.2. Carbon emission function

The research addressed in this paper aims to minimize the carbon emissions by optimizing vehicle speeds and routes. Carbon emissions is used to provide an estimate of the GHGs generated by vehicles. In general, the carbon emissions are measured by calculating the fuel consumption, and then multiplying by the carbon emissions conversion factor, namely the carbon emissions have a linear relationship with fuel consumption [24]. The carbon emission per unit distance traveled (kilogram per kilometer, kg/km) at speed v is $f(v)$, which is developed by the United Kingdom Transport Research Laboratory [15]. The emissions function has been used by many researchers, such as Jabali *et al.* [16], Teoh *et al.* [32] and so on, which can demonstrate the effectiveness of the emission function. The emissions function $f(v)$ is provided as follows:

$$f(v) = (e + e_1v + e_2v^2 + e_3v^3 + e_4v^{-1} + e_5v^{-2} + e_6v^{-3})/1000 \quad (3.1)$$

where v is the speed of the vehicle in km/h, and the coefficients $e, e_1, e_2, e_3, e_4, e_5$ and e_6 will be different under the vehicles with different types and sizes. The coefficients in this paper are adopted the settings in Hickman *et al.* [15], and the values of $e, e_1, e_2, e_3, e_4, e_5$ and e_6 are 765, -7.04 , 0, 0.006320, 8334, 0, 0, respectively. The vehicle will emit $f(v)$ kg/km carbon dioxide (CO₂) when the vehicle is driven at the speed v . It is easy to know that the emissions formula is a function of speed v . Based on a survey of the speed limit of French cities, we set three speed selections in this paper, respectively 30 km/h, 45 km/h and 60 km/h.

3.3. Mathematical model

As mentioned before, this work focuses on the delivery version of the routing and scheduling problem with homogeneous vehicles. The aim is to optimize the daily routes decisions in order to minimize operating cost or carbon emissions. Based on the problem description, this paper develops a mixed-integer programming (MIP) formulation in this section. In order to model the problem clearly, we introduce two important binary variables. One is the most widely used three-index binary variable which is presented as follows:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels at arc } (i, j), \text{ in which } i \neq j; \\ 0, & \text{otherwise.} \end{cases}$$

The other is a variable associated to speed. Based on the above notations, the speed used on arc (i, j) is undoubtedly represented by $x_{ijk}v_{ij}$. However, v_{ij} is a variable, and x_{ijk} is also a variable, so it is obvious that the function is non-linear. In order to linearize the mathematical model, we generate a new binary decision variable z_{ijk_r} which is denoted as follows:

$$z_{ijk_r} = \begin{cases} 1, & \text{if vehicle } k \text{ travels at arc } (i, j) \text{ with speed level } r \in R; \\ 0, & \text{otherwise} \end{cases}$$

where R is the discrete speed levels $R = \{1, 2, \dots, r, \dots\}$. In this paper, we use three speeds, namely 30 km/h, 45 km/h, and 60 km/h, which corresponding to the speed level 1, 2 and 3. So $R = \{1, 2, 3\}$.

TABLE 1. List of notations used in the MIP model.

Notation	Explanation
Z_1	The objective value, the total travel distance (operating cost)
Z_2	The objective value, the total carbon emissions
x_{ijk}	A binary variable, vehicle k travels at arc (i, j) , in which $i \neq j$
z_{ijk_r}	A binary variable, vehicle k travels at arc (i, j) using the speed level r , in which $i \neq j$
τ_i	A decision variable, the start working time at vertex i
u_{ij}	A continuous variable, the total load up to arc (i, j)
K	The set of all vehicles
N	The set of all nodes $\{0, 1, \dots, n, n + 1\}$, including the start depot $\{0\}$ and the end depot $\{n + 1\}$
A	The set of arcs, $A = \{(i, j) i, j \in N, i \neq j\}$
P	The set of all customers $\{1, \dots, n\}$.
Q	The capacity of each vehicle
d_{ij}	The distance from node i to node j
q_i	The demand of customer i
s_i	The service duration for node i
$[a_i, b_i]$	The availability time window of customer i
v_r	The speed with level r
$f(v)$	The carbon emissions function
e, e_1, \dots, e_6	The coefficients of carbon emissions function

Consequently, the variable v_{ij} is represented by the binary decision variable z_{ijk_r} and a new speed parameter v_r , namely $z_{ijk_r}v_r$. And the relationship between decision variables z_{ijk_r} and x_{ijk} is presented as follows:

$$\sum_{r \in R} z_{ijk_r} = x_{ijk}, \quad \forall i \in N, j \in P, k \in K, i \neq j. \tag{3.2}$$

As mentioned in the previous chapter, this paper studies two different objectives and conducts two independent experiments. One objective is the operating cost, and it is in the form of transportation costs. The equation of this objective is presented as follows:

$$Z_1 = \min \sum_{(i,j) \in A} \sum_{k \in V} \sum_{r \in R} d_{ij} z_{ijk_r} \tag{3.3}$$

where the objective is the total travel distance, which is the most common objective optimized in routing and scheduling problems. The other objective studied in this paper is carbon emissions, which have a linear relationship with fuel consumption. To a certain extent, The carbon emissions can also reflect operating cost. The formulation of this objective is denoted as follows:

$$Z_2 = \min \sum_{(i,j) \in A} \sum_{k \in V} \sum_{r \in R} f(v_r) d_{ij} z_{ijk_r}. \tag{3.4}$$

It is clear that the difference between two objectives is the carbon emission function. Before introducing the MIP formulation, this paper summarizes the notations used in the MIP model, which are shown in Table 1.

Then, we will introduce the constraints of the mathematical models, and the formulations are presented as follows:

$$\text{s.t. } \sum_{k \in V} \sum_{j \in N'} x_{ijk} = 1, \quad \forall i \in P \tag{3.5a}$$

$$\sum_{j \in N} x_{jik} - \sum_{j \in N} x_{ijk} = 0, \quad \forall i \in P, k \in K \quad (3.5b)$$

$$\sum_{j \in N} x_{0jk} \leq 1, \quad \forall k \in K \quad (3.5c)$$

$$\sum_{i \in N} x_{i(n+1)k} \leq 1, \quad \forall k \in K \quad (3.5d)$$

$$\sum_{i \in N} u_{ji} - \sum_{i \in N} u_{ij} = q_j, \quad \forall j \in P \quad (3.5e)$$

$$u_{ij} \leq Q \sum_{k \in V} x_{ijk}, \quad \forall (i, j) \in A \quad (3.5f)$$

$$\tau_i - \tau_j + s_i + d_{ij}/v_r \leq M(1 - z_{ijk_r}), \quad \forall i \in N, j \in P, k \in K, r \in R, i \neq j \quad (3.5g)$$

$$a_i \leq y_i \leq b_i, \quad \forall i \in N \quad (3.5h)$$

$$x_{ijk} \in \{0, 1\}, \quad \forall (i, j) \in A, k \in K \quad (3.5i)$$

$$z_{ijk_r} \in \{0, 1\}, \quad \forall (i, j) \in A, k \in K, r \in R \quad (3.5j)$$

$$\sum_{r \in R} z_{ijk_r} = x_{ijk}, \quad \forall (i, j) \in A, k \in K \quad (3.5k)$$

$$u_{ij} \geq 0, \quad \forall (i, j) \in A \quad (3.5l)$$

$$\tau_i \geq 0, \quad \forall i \in P. \quad (3.5m)$$

The objective (3.3) and constraints ((3.5a)–(3.5m)) constitute the first MIP model, and we name it as MIP1, namely:

$$\begin{aligned} \text{MIP1: } Z_1 = \min & \sum_{(i,j) \in A} \sum_{k \in V} \sum_{r \in R} d_{ij} z_{ijk_r} \\ \text{s.t. } & (3.5a) - (3.5m). \end{aligned}$$

The other objective (3.4) and constraints ((3.5a)–(3.5m)) constitute the second MIP model, and we name it as MIP2, namely:

$$\begin{aligned} \text{MIP2: } Z_2 = \min & \sum_{(i,j) \in A} \sum_{k \in V} \sum_{r \in R} f(v_r) d_{ij} z_{ijk_r} \\ \text{s.t. } & (3.5a) - (3.5m). \end{aligned}$$

It should be noted that the objective of MIP1 is distance, but not about speed. Therefore, we only need to set the fastest speed, then we can meet all the constraints and get the best operating cost (or total distance).

In these two MIP models, constraint (3.5a) guarantees that each customer is visited only once. Constraint (3.5b) ensures the flow balance of the vehicles, *i.e.*, the driver visits the customer and then will leave the customer. Constraints (3.5c) and (3.5d) ensure that the vehicles start at the start depot and end at the end depot. Constraint (3.5e) is the flow equation for the demand of customers, and constraint (3.5f) is the capacity constraints. Constraint (3.5g) denotes that the vehicle k cannot arrive at j before $\tau_i + s_i + d_{ij}/v_r$, the reason is that the vehicle k needs the service duration s_i and travel time from i to j . Constraint (3.5h) ensures the time window of the customer i . Constraints (3.5i) and (3.5j) ensure that the decision variables x_{ijk} and z_{ijk_r} are binary. Constraint (3.5k) is relationship between two decision variables x_{ijk} and z_{ijk_r} . Constraints (3.5l) and (3.5m) ensure the non-negative.

4. COLUMN GENERATION

The MIP formulation of the studied problem can be directly solved by commercial optimization solvers. However, the computational efficiency of these algorithms degrades significantly as the problem scale or the instance size increases. To overcome this difficulty, we develop a column generation algorithm to solve the relation of routing and scheduling problem in this section. Firstly, we give a set-partitioning-based formulation of the problem; then, we develop the column generation to solve the linear programming (LP) relaxation problem; Finally, we present the labeling algorithm and several effective heuristics to solve the pricing sub-problem.

4.1. Set-partitioning formulation for the studied problem

We define Ω as the set of feasible paths. A path is feasible if it satisfies capacity and time window constraints. Let y_p be a binary variable deciding whether path p is included in the optimal solution or not, define c_p as the cost (operating cost or carbon emissions) of the path p , and let σ_{ip} be a binary variable that denotes the customer i is visited by the path p or not. We formulate the studied problem as a set partitioning model, which is presented as follows:

$$Z_3 = \min \sum_{p \in \Omega} c_p y_p \quad (4.1a)$$

$$\text{s.t. } \sum_{p \in \Omega} \sigma_{ip} y_p = 1, \quad \forall i \in P \quad (4.1b)$$

$$y_p \in \{0, 1\}, \quad \forall p \in \Omega \quad (4.1c)$$

where the objective function Z_2 (4.1a) minimizes the operating cost or carbon emissions of the chosen paths, constraint (4.1b) guarantees that each customer $i \in P$ is visited only once, and constraint (4.1c) ensures that the decision variables are binary.

We define the LP relaxation of the set-partitioning model (4.1) as the master problem (MP). We use column generation [7] to solve the MP with a small subset $\Omega' \subseteq \Omega$ of feasible paths. The MP with the subset Ω' is denoted as the restricted master problem (RMP), and the RMP can be formulated as follows:

$$Z_4 = \min \sum_{p \in \Omega'} c_p y_p \quad (4.2a)$$

$$\text{s.t. } \sum_{p \in \Omega'} \sigma_{ip} y_p \geq 1, \quad \forall i \in P \quad (4.2b)$$

$$y_p \geq 0, \quad \forall p \in \Omega' \quad (4.2c)$$

where the sub-problem that adds feasible routes (also called columns) to the RMP is denoted as the pricing problem.

4.2. The pricing sub-problem

The pricing sub-problem constructs a feasible route with a minimum reduced cost, using the dual values obtained from the LP solution of the RMP. If the constructed route has negative reduced cost, its corresponding column is added to the RMP. Otherwise, the LP procedure will be terminated with an optimal solution to the continuous relaxation of the MP. The pricing problem searches for the routes with a negative reduced cost, and its objective function is defined as follows:

$$\min_{p \in \Omega'} \bar{c}_p = c_p - \sum_{i \in P} \pi_i \sigma_{ip} \quad (4.3)$$

where \bar{c}_p is the reduced cost of path p , and π_i is the dual variable associated with the formulation (4.1b).

4.2.1. The pricing problem with the objective of operating cost

As mentioned in Section 3.3, the objective is operating cost, and we use the total distance to present the operating cost, which is not about speed. Therefore, we only need to set the fastest speed, then we can meet all the constraints and get the best operating cost (total distance). In other words, we only need to set one speed level (the fastest speed), which is enough to meet all the constraints and have no influences on the total distance. In order to solve this problem, we design a labeling algorithm, which will be introduced in the next section.

4.2.2. The pricing problem with the objective of carbon emissions

For the pricing problem with the objective of carbon emissions, it involves a speed optimization problem (SOP). Then, we will introduce the SOP in detail. For a feasible path $p = (i_0, i_1, \dots, i_h)$, the problem that calculates the optimal carbon emissions c_p is defined as a SOP. Let $N_p := \{i_m | m = 0, 1, \dots, h\}$ be the vertex sets of path p , let $\text{Arc}_p := \{(i_m, i_{m+1}) | l = 0, 1, \dots, h - 1\}$ be the arc sets of the path p , and let z_{ijr} be a three-index binary variable that denotes the arc $(i, j) \in \text{Arc}_p$ is traveled with a speed level $r \in R$. As mentioned above, we use three speeds, namely 30, 45, and 60 km/h, which corresponding to the speed level 1, 2 and 3, namely $R = \{1, 2, 3\}$. τ_i is a decision variable that presents the start working time at vertex $i \in N_p$. The SOP is formulated as a MIP formulation, we name the MIP formulation as MIP3, which is presented as follows:

$$\text{MIP3: } Z_5 = \min \sum_{(i,j) \in \text{Arc}_p} \sum_{r \in R} f(v_r) d_{ij} z_{ijr} \quad (4.4a)$$

$$\text{s.t. } \tau_i - \tau_j + d_{ij}/v_r + s_i \leq M(1 - z_{ijr}), \quad \forall i, j \in N_p, (i, j) \in \text{Arc}_p, r \in R \quad (4.4b)$$

$$a_i \leq \tau_i \leq b_i, \quad \forall i \in N_p \quad (4.4c)$$

$$z_{ijr} \in \{0, 1\}, \quad \forall (i, j) \in \text{Arc}_p, r \in R \quad (4.4d)$$

where the objective function Z_5 (4.4a) minimizes the carbon emissions of the path p , constraints (4.4b) and (4.4c) guarantee no violation for the time windows, and constraint (4.4d) ensures that the decision variables are binary. Since the SOP is optimized for the feasible solutions, capacity constraint need not be considered.

This problem is highly related to time, it is very hard to find a best speed for each edge. Therefore, the SOP is difficult to be solved only by using a dynamic programming algorithm. In this paper, we solve the SOP by solving the MIP3 by using a commercial solver. The MIP solver is enough to solve this problem.

4.3. The labeling algorithm

The labeling algorithm has been successfully applied into many VRP-related researches [5, 13, 21, 27]. In general, the labeling algorithm is performed where the labels are extended from the start depot (*i.e.*, node 0) to its successors. Recently, many scholars have been using bidirectional search to speed up the labeling algorithm, namely labels are extended both forward from the start depot (*i.e.*, node 0) to its successors, and backward from the end depot (*i.e.*, node $n + 1$) to its predecessors [4]. It should be noted that the efficiency of labeling algorithm largely depends on the dominance rules.

In this paper, in order to solve these two different pricing problems, we design two different labeling algorithms. For the pricing problem with the objective of operating cost, we design a bidirectional labeling algorithm to solve it. For the pricing problem with the objective of carbon emissions, we propose a forward labeling algorithm to solve it. The detailed introductions are below.

4.3.1. A tailored bidirectional labeling algorithm for the pricing problem with the objective of operating cost

As mentioned before, the cost of path is distance. Thus, we can set only one speed level (the fastest speed, namely 60 km/h in this paper) to meet all constraints, which will not influence the distance. In this section, we design a bidirectional labeling algorithm to solve the pricing problem. In the bidirectional labeling algorithm, we set a fixed time $t_m = T/2$ as the critical resource, and the forward and backward labels are not allowed to extend beyond t_m . Then we will introduce the bidirectional labeling algorithm in detail.

Firstly, we define the notations of forward label L_f . A forward label consists of $L_f = (i, q, t, c, \bar{c}, V, \bar{V})$. The related attributes are presented as follows:

- i : The last node visited by L_f ;
- q : Sum of demand when leaving node i in label L_f ;
- t : Departure time using the fastest speed at node i in label L_f ;
- c : The distance along label L_f ;
- \bar{c} : Reduced cost of label L_f ;
- V : Set of nodes visited by vehicle in order along label L_f ;
- \bar{V} : Set of nodes that are unreachable from node i in label L_f .

$$\bar{V} = V \cup \{j \in N \mid t > b_j + s_j \vee q + q_j > Q\}. \tag{4.5}$$

The forward labeling algorithm stores all of the feasible routes extended over the start depot ($i = 0$) to its successors. The search is restricted to elementary paths by discarding extensions to any vertex $j \in V$. When the label $L_f = (i, q, t, c, \bar{c}, V, \bar{V})$ is extended forward to a vertex $j \in N \setminus \bar{V}$, a new label $L'_f = (j, q', t', c', \bar{c}', V', \bar{V}')$ is computed by using the following rules:

$$\begin{aligned} q' &= q + q_j; \\ t' &= \begin{cases} t + d_{ij}/v + s_j, & t + d_{ij}/v \geq a_j; \\ a_j + s_j, & t + d_{ij}/v < a_j; \end{cases} \\ V' &= V \cup \{j\}; \\ \bar{V}' &= V' \cup \{k \in N \mid t' > b_k + s_k \vee q' + q_k > Q\}; \\ c' &= c + d_{ij}; \\ \bar{c}' &= \bar{c} + d_{ij} - \pi_j. \end{aligned}$$

The initial forward label is $L_f = (0, 0, 0, 0, 0, \{0\}, N \setminus \{0\})$. In the process of forward extension, all labels should not be dominated labels. The forward dominance rule is denoted as follows.

Dominance 1 (Forward Dominance rule). A Label $L_f^2 = (i^2, q^2, t^2, c^2, \bar{c}^2, V^2, \bar{V}^2)$ is dominated by the other label $L_f^1 = (i^1, q^1, t^1, c^1, \bar{c}^1, V^1, \bar{V}^1)$ if

$$i^1 = i^2, \tag{4.6a}$$

$$q^1 \leq q^2, \tag{4.6b}$$

$$\bar{V}^1 \subseteq \bar{V}^2, \tag{4.6c}$$

$$t^1 \leq t^2, \tag{4.6d}$$

$$\bar{c}^1 \leq \bar{c}^2, \tag{4.6e}$$

and at least one of the above inequalities is strictly satisfied.

The pseudo-code of the forward labeling is described in Algorithm 1. Before describing the algorithm, we need to define two sets \mathcal{L}_f^e and \mathcal{L}_f^p . Let \mathcal{L}_f^e be the set of forward labels waiting for extension, and define \mathcal{L}_f^p as the set of forward labels waiting for merging. The termination condition of the forward labeling algorithm is to determine whether \mathcal{L}_f^e is an empty set. In order to speed up the algorithm, we set the departure time as a strict judgment resources. If the departure time is larger than $T/2$, we will not extend this label (Lines 6, 7). In the labeling algorithm, we ensure that the newly extended labels and all labels in the \mathcal{L}_f^e and \mathcal{L}_f^p are not dominated. If any label is dominated, then the label will be discarded (Lines 15–18). The output of the labeling algorithm is set \mathcal{L}_f^p .

The difference of backward labeling and forward labeling is that the backward labeling starts at the end depot, namely $n + 1$ to its predecessors. Similarly, we define $L_b = (i, q, t, c, \bar{c}, V, \bar{V})$ as the backward label. The related attributes are presented as follows:

Algorithm 1: The forward labeling algorithm.

```

1 Initialize the forward label  $L_f = (0, 0, 0, 0, 0, \{0\}, N \setminus \{0\})$ ;
2 Set  $\mathcal{L}_f^e \leftarrow \{L_f\}$ ,  $\mathcal{L}_f^p \leftarrow \emptyset$ ;
3 while  $\mathcal{L}_f^e \neq \emptyset$  do
4   Choose a label  $L_f \in \mathcal{L}_f^e$  with the minimum reduced cost;
5    $\mathcal{L}_f^e \leftarrow \mathcal{L}_f^e \setminus \{L_f\}$ ,  $\mathcal{L}_f^p \leftarrow \mathcal{L}_f^p \cup \{L_f\}$ ;
6   if Departure time  $t \geq T/2$  then
7     | Continue;
8   else
9     for  $j \in N \setminus \bar{V}$  do
10      | Extend the label  $L_f$  to node  $j$ ;
11      | if violate the constraints (time window or capacity) then
12        | | Continue;
13      | else
14        | | Update the information of new label  $L'_f$  with the last node  $j$  based on the rules;
15        | | if  $L'_f$  is not dominated by any label in  $\mathcal{L}_f^e \cup \mathcal{L}_f^p$  based on the dominance rule then
16          | | |  $\mathcal{L}_f^e \leftarrow \mathcal{L}_f^e \cup \{L'_f\}$ ;
17          | | | if any label in  $\mathcal{L}_f^e$  or  $\mathcal{L}_f^p$  is dominated by  $L'_f$  based on the dominance rule then
18            | | | | Delete the label from the set  $\mathcal{L}_f^e$  or  $\mathcal{L}_f^p$ ;
19 Output  $\mathcal{L}_f^p$ .

```

- i : The first node visited by L_b ;
 q : Sum of demand when departing from node i in label L_b ;
 t : Departure time using the fastest speed at node i in label L_b ;
 c : The distance along label L_b ;
 \bar{c} : Reduced cost of label L_b ;
 V : Set of nodes visited by vehicle in order along label L_b ;
 \bar{V} : Set of nodes that are unreachable from node i in label L_f .

$$\bar{V} = V \cup \{j \in N \vee t > a_j + s_j \vee q + q_j > Q\}. \quad (4.7)$$

The initial backward label is $L_b = (n+1, 0, T, 0, 0, \{n+1\}, N \setminus \{n+1\})$. When the label $L_b = (i, q, t, c, \bar{c}, V, \bar{V})$ is extended forward to a vertex $j \in N \setminus \bar{V}$, a new label $L'_b = (j, q', t', c', \bar{c}', V', \bar{V}')$ is computed by using the following rules:

$$\begin{aligned}
 q' &= q + q_j; \\
 t' &= \begin{cases} t - d_{ij}/v - s_j, & t - d_{ij}/v - s_j \leq b_j \\ b_j + s_j, & t - d_{ij}/v - s_j > b_j \end{cases}; \\
 V' &= V \cup \{j\}; \\
 \bar{V}' &= V' \cup \{k \in N \vee t' < a_k + s_k \vee q' + q_k > Q\}; \\
 c' &= c + d_{ij}; \\
 \bar{c}' &= \bar{c} + d_{ij} - \pi_j.
 \end{aligned}$$

Similarly, in the process of backward extension, all labels should not be dominated labels. The backward dominance rule is denoted as follows.

Dominance 2 (Backward Dominance rule). A Label $L_b^2 = (i^2, q^2, t^2, c^2, \bar{c}^2, V^2, \bar{V}^2)$ is dominated by the other label $L_f^1 = (i^1, q^1, t^1, c^1, \bar{c}^1, V^1, \bar{V}^1)$ if

$$i^1 = i^2, \tag{4.8a}$$

$$q^1 \leq q^2, \tag{4.8b}$$

$$\bar{V}^1 \subseteq \bar{V}^2, \tag{4.8c}$$

$$t^1 \geq t^2, \tag{4.8d}$$

$$\bar{c}^1 \leq \bar{c}^2, \tag{4.8e}$$

and at least one of the above inequalities is strictly satisfied.

The extension process of the backward labeling algorithm is similar to Algorithm 1, there are two sets \mathcal{L}_b^e and \mathcal{L}_b^p , respectively are the set of backward labels waiting for extension, and the set of backward labels waiting for merging. The line 6 in Algorithm 1 will be modified as $t \leq T/2$. After get the set of \mathcal{L}_b^p , then we can merge the forward labels and backward labels.

Algorithm 2: The merging process.

```

1 Input  $\mathcal{L}_f^p$  and  $\mathcal{L}_b^p$ ;
2 for  $L_f \in \mathcal{L}_f^p$  do
3   for  $L_b \in \mathcal{L}_b^p$  do
4     if  $i_{L_f} = i_{L_b} \ \&\& \ V_{L_f} \cap V_{L_b} = \{i_{L_f}\}$  then
5       Merge  $L_f$  and  $L_b$ , and new label is  $L$ ;
6       if the merging path of label  $L$  is feasible, namely  $q_{L_f} + q_{L_b} - q_{i_{L_f}} \leq Q \ \&\& \ t_{L_f} \leq t_{L_b}$  then
7         if The reduced cost  $\bar{c}_L < 0$  then
8           Insert  $L$  into  $\mathcal{L}^p$ ;
9 Output  $\mathcal{L}^p$ .
```

We define the fixed label as L , and let \mathcal{L}^p be the set of fixed paths with negative reduced cost. The process of the merging is presented in Algorithm 2. There are some constraints about merging as follows:

- The last visited node of L_f and the first visited node of L_b is same, and the same node between L_f and L_b can only be this node.
- The new path L will not violate the capacity and time window constraints.
- The reduced cost of new path L will be smaller than 0, in which $\bar{c}_L = c_{L_f} + c_{L_b} + \pi_{i_{L_f}}$.

4.3.2. A tailored forward labeling algorithm for the pricing problem with the objective of carbon emissions

In this section, a tailored forward labeling algorithm is proposed for the pricing problem with the objective of carbon emissions. Because the pricing problem involves a SOP, which is closely related to the departure time. If we get a backward label and the departure time of the first node has changed, the whole path will be calculated again. Therefore, in this section, we only use forward labeling algorithm to solve the pricing problem.

We first define some notations related to a label. Given a label $L_f = (i, q, t_b, t_f, c, \bar{c}, V, \bar{V})$, where the tuple represents the state of the path associated with the label L_f . The related attributes are presented as follows:

- i : The last node visited by L_f ;
- q : Sum of demand of customers when leaving node i in label L_f ;
- t_b : Departure time of best carbon emissions at node i in label L_f ;
- t_f : Departure time using the fastest speed at node i in label L_f ;

- c : Best carbon emissions along label L_f ;
 \bar{c} : Reduced cost of label L_f ;
 V : Set of nodes visited by vehicle in order along label L_f ;
 \bar{V} : Set of nodes that are unreachable from node i in label L_f .

$$\bar{V} = V \cup \{j \in N \vee t_f > b_j + s_j \vee q + q_j > Q\}. \quad (4.9)$$

Before presenting the extension of the labels, we review the relationship between speed and carbon emissions. As mentioned in Section 3.2, three speeds can be selected by the caregiver, namely 30 km/h, 45 km/h and 60 km/h. According to the parameters setting in Section 3.2, we can calculate that $f(30) = 8.0432$, $f(45) = 26.546$, and $f(60) = 62.974$. Therefore, this paper concludes that $f(30) < f(45) < f(60)$. In this paper, we use three notations to denote this three speeds, and shown as follows: $v_{\min} = 30$ km/h, $v_{\text{mid}} = 45$ km/h and $v_{\max} = 60$ km/h. Thus, it concludes that $f(v_{\min}) < f(v_{\text{mid}}) < f(v_{\max})$ in this paper.

The forward labeling algorithm stores all of the feasible routes extended over the start depot to its successors. The search is restricted to elementary paths by discarding extensions to any vertex $j \in V$. When the label $L_f = (i, q, t_b, t_f, c, \bar{c}, V, \bar{V})$ is extended forward to a vertex $j \in P \setminus \bar{V}$, a new label $L'_f = (j, q', t'_b, t'_f, c', \bar{c}', V', \bar{V}')$ is computed by using the following rules:

$$\begin{aligned}
 q' &= q + q_j; \\
 t'_f &= \begin{cases} t_f + d_{ij}/v_{\max} + s_j, & t_f + d_{ij}/v_{\max} \geq a_j, \\ a_j + s_j, & t_f + d_{ij}/v_{\max} < a_j, \end{cases} \\
 V' &= V \cup \{j\}; \\
 \bar{V}' &= V' \cup \{k \in N \vee t'_f > b_k + s_k \vee q' + q_k > Q\}.
 \end{aligned}$$

Before updating the other notations tuple (t'_b, c', \bar{c}') , this paper first proposes a proposition, which is presented as follows.

Proposition 4.1. *For a label $L_f = (i, q, t_b, t_f, c, \bar{c}, V, \bar{V})$, we define $t_b = a_i + s_i$ as the earliest departure time. The earliest departure time corresponds to the minimal carbon emissions of the path up to vertex i . The vertex i in label L_f with the earliest departure time t_b and minimal carbon emissions is thus as optimal vertex.*

Proof. The departure time $t \geq a_i + s_i$, and $t_b = a_i + s_i$, so the departure time t_b is the earliest departure time. t_b is the departure time of best carbon emissions at vertex i in label L_f , which means it is impossible to reduce carbon emissions in this path up to vertex i . Therefore, no matter what vertex the label extends after vertex i , the carbon emissions of the path up to vertex i will not decrease nor increase, and the earliest departure time will also not be changed. Thus, in label L_f , the vertex i seems a new “depot” for the vertex expanded after i . So the vertex i is thus as *optimal vertex*. \square

Based on the Proposition 4.1, the updating of tuple (t'_b, c', \bar{c}') is shown as follows.

Proposition 4.2. *For a label $L_f = (i, q, t_b, t_f, c, \bar{c}, V, \bar{V})$ extends to a vertex $j \in N \setminus \bar{V}$.*

If $t_b + d_{ij}/v_{\min} \leq b_j$, then $t'_b = \begin{cases} t_b + d_{ij}/v_{\min} + s_j, & t_b + d_{ij}/v_{\min} \geq a_j \\ a_j + s_j, & t_b + d_{ij}/v_{\min} < a_j \end{cases}$, $c' = c + d_{ij}f(v_{\min})$, $\bar{c}' = \bar{c} + d_{ij}f(v_{\min}) - \pi_j$.

If $t_b + d_{ij}/v_{\min} > b_j$, then the tuple (t'_b, c', \bar{c}') should be re-optimized for the route from the last optimal vertex to vertex j using MIP3 ((4.4a)–(4.4d)).

Proof. c is the best carbon emissions of the label L_f , and the related best departure is t_b . And as mentioned before, $f(v_{\min}) < f(v_{\text{mid}}) < f(v_{\max})$. So the minimal carbon emissions of arc (i, j) is $d_{ij}f(v_{\min})$. Thus if $t_b + d_{ij}/v_{\min} \leq b_j$ holds, it is easy to get that the minimal carbon emissions of the path up to vertex j is

$c' = c + d_{ij}f(v_{\min})$, and other notations t'_b and \bar{c}' will be updated as presented in Proposition 4.2. As for the other situation, if $t_b + d_{ij}/v_{\min} > b_j$, then it will not be allowed to use the speed v_{\min} on the arc (i, j) . Therefore, we cannot decide to adjust the speed of the path up to vertex i or arc (i, j) in order to achieve the lowest overall carbon emissions up to vertex j . \square

If $t_b + d_{ij}/v_{\min} > b_j$, the speed of the path up to vertex j should be re-optimized. In this paper, the problem is solved by a MIP formulation ((4.4a)–(4.4d)) using CPLEX. We can get the best carbon emissions c' and the corresponding t'_b . The reduced cost is updated by the formulation $\bar{c}' = c' - \sum_{i \in P} \pi_i \sigma_{ip}$, where p is the current path up to vertex j .

After generating the new labels, the dominance rules are used to reduce the unpromising labels that cannot lead to the optimal solution in order to save the storage space and calculation time of the computer. The dominance rule is shown as follows:

Dominance 3 (Dominance rule). The Label $L_f^2 = (i^2, q^2, t_b^2, t_f^2, c^2, \bar{c}^2, V^2, \bar{V}^2)$ is dominated by the other label $L_f^1 = (i^1, q^1, t_b^1, t_f^1, c^1, \bar{c}^1, V^1, \bar{V}^1)$ if

$$i^1 = i^2, \tag{4.10a}$$

$$q^1 \leq q^2, \tag{4.10b}$$

$$\bar{V}^1 \subseteq \bar{V}^2, \tag{4.10c}$$

$$t_b^1 \leq t_b^2 \vee t_f^1 \leq t_f^2, \tag{4.10d}$$

$$\bar{c}^1 \leq \bar{c}^2, \tag{4.10e}$$

and at least one of the above inequalities is strictly satisfied.

The pseudo code of the forward labeling algorithm is similar to Algorithm 1. The difference is that Lines 6, 7 in Algorithm 1 are deleted. Finally, we will extend all the labels in \mathcal{L}_f^p with negative reduced cost to the end depot, and add the paths to the column generation process.

5. BRANCH-AND-PRICE ALGORITHM

5.1. The framework of BAP algorithm

To optimally solve the routing and scheduling problem, we develop a branch-and-price (BAP) algorithm, which is the leading exact algorithm for the routing and scheduling problems [3, 12, 14, 26]. The BAP algorithm is composed by embedding the column generation procedure into the branch-and-bound framework. That is because that the objective value of the RMP may not be integers, but it can provide the lower bound (LB) of the studied problem at the node of the search tree. The flowchart of the BAP algorithm is shown as follows.

As shown in Figure 1, it is clear that there are some initial columns for the RMP. In this paper, we use the route Depot-Customer-Depot as the initial columns. Therefore, the number of the initial columns is the number of customers of the instance. At next, we will introduce the branching and search strategies of the BAP algorithm.

5.2. Branching and search strategies

After the column generation, we first use the integer branching strategy to save computation time. The best integer solution at the root node is obtained by solving the RMP as a 0–1 integer programming problem. The objective value of the integer solution will be the initial upper bound of the studied problem. If the upper bound is equal to the lower bound. The problem is solved and accordingly the proposed BAP algorithm terminates.

In the proposed BAP algorithm, the branch-and-bound tree is explored according to a best bound first strategy. As analysed in Feillet [11], branching on a fractional y_p variable poses some difficulties and is impractical,

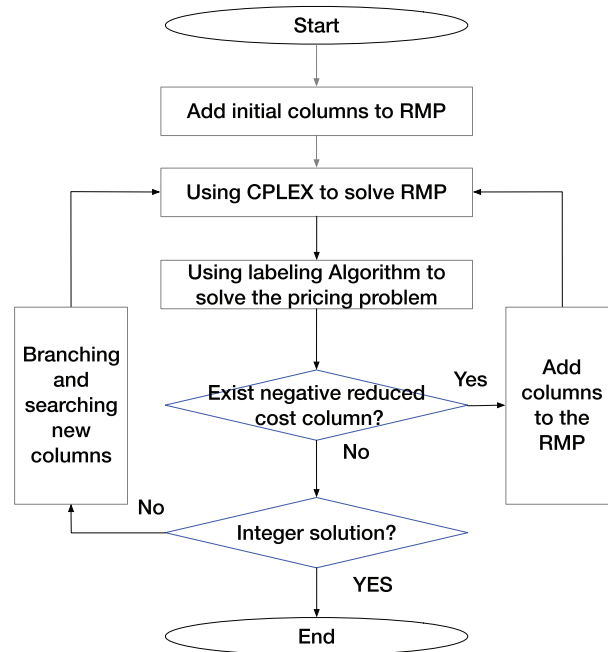


FIGURE 1. The flowchart of BAP algorithm.

and branching on $y_p = 0$ would need a more complicated modification of the pricing problem in order to check column p . In contrast, including or forbidding arcs in the solution of the pricing problem will be easily achieved. According to Reihaneh and Ghoniem [26], branching on arcs is given higher priority, this empirically results in the best improvement in the lower bound. Therefore, a branching strategy on arcs is adopted in this paper [21].

Let H_{ij} be the set of all columns that contain arc $(i, j) \in A, i, j \in P$. The sum of the flows on arc (i, j) is equal to $\sum_{p \in H_{ij}} y_p$. If there exists at least an arc (i, j) with fractional $\sum_{p \in H_{ij}} y_p$, then we branch on the value $\sum_{p \in H_{ij}} y_p$ which is the closest to the midpoint 0.5. Two new child nodes are generated accordingly by forcing arc (i, j) in one node and forbidding arc (i, j) in the other node. In the former case, all columns containing arcs (i, j') and (i', j) with $i' \neq i$ and $j' \neq j$ are deleted when solving the pricing problem. In the latter case, columns using arc (i, j) have to be removed when solving the pricing problem.

6. COMPUTATIONAL EXPERIMENTS

We conduct extensive computational experiments of the proposed BAP algorithm, and compare its performance with that of a MIP formulation (provided in the Sect. 3.3) solved by a state-of-the-art optimization solver CPLEX. All of the algorithms in this paper are coded in C++ programming language. The RMPs and MIPs are both solved by CPLEX 12.10.0. Computational experiments are conducted on a PC with an Inter(R) Core(TM) i7-7700 CPU @3.60 GHz and a 16 GB RAM, under a Linux operating system. Computation times are reported in seconds on this machine. Each instance is optimal or not solved within a time limit of 7200 Cpus.

6.1. Problem instances and experimental setup

In this paper, we generate the test instances based on the classical Solomon VRPTW benchmark instances [30]. The Solomon VRPTW instances are very famous and widely used by a large number of scholars such as Dabia *et al.* [5], Shi *et al.* [29], Yu *et al.* [34], etc. According to the geographical distribution characteristics of the instances, the Solomon VRPTW instances can be divided into three categories namely C-type instances

(clustered customers), R-type instances (uniformly distributed customers) and RC-type instances (a mix of R and C types). In this paper, we set up three scale instances, with 10, 25, and 50 customers respectively, and we name them small-scale, medium-scale and large-scale instances.

In order to guarantee the correctness of the new instances, we modified the Solomon's benchmark instances with reference to the works of Dabia *et al.* [5] and Yu *et al.* [34]. In the basis of the Solomon VRPTW benchmark instances, the rules of generating the test instances of the proposed problems are as follows:

- we set the coordinate of the end depot as (30, 40);
- the planning horizon was set as 12 hour (h), and all customers' time windows were scaled accordingly using the coefficient $12/b_0$, therefore, the time window $[a_i, b_i]$ of customer i in the Solomon instances was modified as $[a_i * (12/b_0), b_i * (12/b_0)]$;
- in the tests, the distance d_{ij} was not changed, but we set the unit of distance as kilometers (km);
- the service time was set to 0.75 h for all customers;
- the other parameters were not changed.

As for the speed settings of the test instances, we set three kinds of speed namely 30 km/h, 45 km/h and 60 km/h to test the influence of different speed to the carbon emissions.

In this paper, we use the proposed BAP algorithm and MIP formulation to do two different experiments. *Experiment 1* is conducted from the economic perspective, and with the objective of operating cost (distance). *Experiment 2* is conducted from the environmental protection perspective, and with the objective of carbon emissions. We also compare some results with these different objectives.

6.2. Results of *Experiment 1* and *Experiment 2*

As mentioned in Section 6.1, this paper conducts two different experiments from the economic perspective and environmental protection perspective, respectively. This section will present the experimental results of these two experiments.

Tables 2–4 report our computational results of *Experiment 1* for solving the base MIP formulation in Section 3.3 (using CPLEX *vs.* solving the set partitioning formulation using the proposed BAP algorithm). Tables 5–7 report our computational results of *Experiment 2* for solving the base MIP formulation in Section 3.3 (using CPLEX *vs.* solving the set partitioning formulation using the proposed BAP algorithm). In these 6 tables, Columns 1 and 2 report the information of instances, and respectively are the name and the size of instance. For CPLEX, columns 3–5, respectively, report the best upper bound (BUB1) obtained by the CPLEX solver, be optimal or just an upper bound, the Cpu time (CpuT1), and the optimality gap (Gap1) when the calculation is terminated. For the proposed BAP algorithm, columns 6–9 focus on the performance of the proposed BAP algorithm at root node stage and report the following: (i) the lower bound (LB) solved by the column generation; (ii) upper bound (UB) corresponding to the best integer solution at the root node by solving the RMP as a 0–1 integer programming problem using CPLEX; (iii) the optimality gap (Gap2 = $(UB - LB)/LB * 100\%$) at the root node based on the UB and LB; and (iv) the Cpu time (CpuT2) used at the root node stage. Columns 10–14 report the overall performance of the proposed BAP algorithm, which is presented as the following: (i) the best upper bound (BUB2) of the proposed BAP algorithm; (ii) the optimality gap (Gap3 = $(BUB2 - LB)/LB * 100\%$) compared with the LB at termination or within a time limit of 7200 Cpus; (iii) the optimality gap (Gap4 = $(BUB2 - BUB1)/BUB1 * 100\%$) compared with the BUB1 at termination or within a time limit of 7200 Cpus; (iv) the number of explored branch-and-bound tree nodes (Nodes); and (v) the total Cpu time (CpuT3).

An entry “–” in the table means that the algorithm is not able to obtain an associated bound within the time limit of 7200s. As for the proposed BAP algorithm, when the time limit 7200s is reached, the algorithm will not be terminated until it finishes processing the current branch-and-bound node. In these 6 tables, in column “BUB2”, there are many “–”, which means that the best upper bound not be found in branch-and-bound explored stage, and we set the UB at the root node by solving the RMP as a 0–1 integer programming problem using CPLEX as the BUB.

TABLE 2. Computational results on 10-customers instances of *Experiment 1*.

Instance		BAP algorithm											
Name	NP	MIP			Root node				BUB2	Gap3 (%)	Gap4 (%)	Nodes	CpuT3 (s)
		BUB1	Gap1 (%)	CpuT1 (s)	LB	UB	Gap2 (%)	CpuT2 (s)					
C101	10	71.41	0.00	0.10	71.41	71.41	0.00	0.01	71.41	0.00	0.00	1	0.01
C102	10	69.85	0.00	1.93	69.85	69.85	0.00	0.03	69.85	0.00	0.00	1	0.03
C103	10	69.85	0.00	1.88	69.85	69.85	0.00	0.02	69.85	0.00	0.00	1	0.02
C104	10	68.38	0.00	4.18	68.38	68.38	0.00	0.03	68.38	0.00	0.00	1	0.03
C105	10	71.41	0.00	0.07	71.41	71.41	0.00	0.01	71.41	0.00	0.00	1	0.01
C106	10	71.41	0.00	0.05	71.41	71.41	0.00	0.01	71.41	0.00	0.00	1	0.01
C107	10	70.58	0.00	0.47	70.58	70.58	0.00	0.01	70.58	0.00	0.00	1	0.01
C108	10	70.58	0.00	1.37	70.58	70.58	0.00	0.01	70.58	0.00	0.00	1	0.01
C109	10	70.58	0.00	1.39	70.58	70.58	0.00	0.01	70.58	0.00	0.00	1	0.01
C201	10	227.15	0.00	0.17	193.67	227.15	17.28	0.01	227.15	17.28	0.00	11	0.36
C202	10	174.96	0.00	17.14	170.82	174.96	2.43	0.07	174.96	2.43	0.00	4	0.12
C203	10	174.96	0.00	17.69	172.10	174.96	1.66	0.15	174.96	1.66	0.00	3	0.24
C204	10	127.50	0.00	3.44	127.50	127.50	0.00	0.13	127.50	0.00	0.00	1	0.13
C205	10	175.53	0.00	0.07	169.00	175.53	3.86	0.01	175.53	3.86	0.00	3	0.03
C206	10	149.67	0.00	0.37	149.67	149.67	0.00	0.01	149.67	0.00	0.00	1	0.01
C207	10	218.77	0.00	0.24	187.56	218.77	16.64	0.01	218.77	16.64	0.00	7	0.06
C208	10	149.67	0.00	0.48	149.66	149.66	0.00	0.01	149.66	0.00	0.00	1	0.01
R101	10	259.07	0.00	0.05	259.07	259.07	0.00	0.01	259.07	0.00	0.00	1	0.01
R102	10	220.05	0.00	3.01	220.05	220.05	0.00	0.01	220.05	0.00	0.00	1	0.01
R103	10	220.05	0.00	3.04	220.05	220.05	0.00	0.01	220.05	0.00	0.00	1	0.01
R104	10	198.25	0.00	3.55	198.25	198.25	0.00	0.01	198.25	0.00	0.00	1	0.01
R105	10	259.07	0.00	0.12	259.07	259.07	0.00	0.01	259.07	0.00	0.00	1	0.01
R106	10	220.05	0.00	1.79	219.29	220.05	0.35	0.01	220.05	0.35	0.00	3	0.03
R107	10	220.05	0.00	1.78	219.29	220.05	0.35	0.01	220.05	0.35	0.00	3	0.03
R108	10	198.25	0.00	3.27	198.25	198.25	0.00	0.01	198.25	0.00	0.00	1	0.01
R109	10	237.52	0.00	1.38	237.08	237.52	0.19	0.01	237.52	0.19	0.00	3	0.03
R110	10	200.72	0.00	2.65	200.72	200.72	0.00	0.01	200.72	0.00	0.00	1	0.01
R111	10	212.52	0.00	2.33	212.52	212.52	0.00	0.01	212.52	0.00	0.00	1	0.01
R112	10	191.70	0.00	1.26	191.58	191.70	0.06	0.01	191.70	0.06	0.00	3	0.04
R201	10	259.07	0.00	0.04	259.07	259.07	0.00	0.01	259.07	0.00	0.00	1	0.01
R202	10	220.05	0.00	1.90	220.05	220.05	0.00	0.01	220.05	0.00	0.00	1	0.01
R203	10	220.05	0.00	1.93	220.05	220.05	0.00	0.01	220.05	0.00	0.00	1	0.01
R204	10	198.25	0.00	2.07	198.25	198.25	0.00	0.01	198.25	0.00	0.00	1	0.01
R205	10	237.52	0.00	0.87	237.08	237.52	0.19	0.01	237.52	0.19	0.00	3	0.03
R206	10	198.25	0.00	1.63	198.25	198.25	0.00	0.01	198.25	0.00	0.00	1	0.01
R207	10	198.25	0.00	1.65	198.25	198.25	0.00	0.01	198.25	0.00	0.00	1	0.01
R208	10	198.25	0.00	2.40	197.33	198.25	0.47	0.01	198.25	0.47	0.00	3	0.03
R209	10	222.55	0.00	1.97	221.15	222.55	0.63	0.01	222.55	0.63	0.00	3	0.04
R210	10	212.52	0.00	1.96	212.52	212.52	0.00	0.01	212.52	0.00	0.00	1	0.01
R211	10	191.70	0.00	1.57	191.70	191.70	0.00	0.01	191.70	0.00	0.00	1	0.01
RC101	10	184.79	0.00	0.40	184.78	184.78	0.00	0.01	184.78	0.00	0.00	1	0.01
RC102	10	163.36	0.00	4.12	163.36	163.36	0.00	0.02	163.36	0.00	0.00	1	0.02
RC103	10	163.36	0.00	4.15	163.36	163.36	0.00	0.02	163.36	0.00	0.00	1	0.02
RC104	10	158.97	0.00	12.67	158.97	158.97	0.00	0.01	158.97	0.00	0.00	1	0.01
RC105	10	172.86	0.00	3.67	172.86	172.86	0.00	0.01	172.86	0.00	0.00	1	0.01
RC106	10	172.36	0.00	1.91	172.36	172.36	0.00	0.01	172.36	0.00	0.00	1	0.01
RC107	10	160.42	0.00	6.01	158.00	160.42	1.53	0.03	160.42	1.53	0.00	5	0.1
RC108	10	160.32	0.00	31.74	160.32	160.32	0.00	0.04	160.32	0.00	0.00	1	0.04
RC201	10	184.79	0.00	0.05	184.78	184.78	0.00	0.01	184.78	0.00	0.00	1	0.01
RC202	10	163.16	0.00	36.30	163.16	163.16	0.00	0.01	163.16	0.00	0.00	1	0.01
RC203	10	163.16	0.00	34.95	163.16	163.16	0.00	0.01	163.16	0.00	0.00	1	0.01
RC204	10	158.97	0.00	126.30	158.97	158.97	0.00	0.02	158.97	0.00	0.00	1	0.02
RC205	10	176.18	0.00	1.58	176.18	176.18	0.00	0.01	176.18	0.00	0.00	1	0.01
RC206	10	175.39	0.00	1.47	172.36	175.39	0.00	0.01	175.39	0.00	0.00	1	0.01
RC207	10	172.36	0.00	2.99	172.36	172.36	0.00	0.01	172.36	0.00	0.00	1	0.01
RC208	10	150.79	0.00	86.07	148.78	150.79	1.35	0.03	150.79	1.35	0.00	5	0.15
AVERAGE			0.00	7.99			0.84	0.02		0.84	0.00	1.80	0.03

TABLE 3. Computational results on 25-customers instances of *Experiment 1*.

Instance		BAP algorithm											
Name	NP	MIP			Root node				BUB2	Gap3 (%)	Gap4 (%)	Nodes	CpuT3 (s)
		BUB1	Gap1 (%)	CpuT1 (s)	LB	UB	Gap2 (%)	CpuT2 (s)					
C101	25	213.90	0.00	0.49	213.90	213.90	0.00	0.35	213.90	0.00	0.00	1	0.35
C102	25	212.34	0.00	524.32	212.34	212.34	0.00	5.664	212.34	0.00	0.00	1	5.66
C103	25	207.51	8.59	7200.00	207.51	207.51	0.00	4.8	207.51	0.00	0.00	1	4.80
C104	25	206.31	11.10	7200.00	203.31	203.77	0.23	11.03	203.77	0.23	-1.23	3	23.54
C105	25	209.66	0.00	0.77	209.66	209.66	0.00	0.3	209.66	0.00	0.00	1	0.30
C106	25	213.90	0.00	1.02	213.90	213.90	0.00	0.33	213.90	0.00	0.00	1	0.33
C107	25	209.66	0.00	133.99	209.66	209.66	0.00	0.4	209.66	0.00	0.00	1	0.40
C108	25	209.66	0.00	646.92	209.66	209.66	0.00	0.81	209.66	0.00	0.00	1	0.81
C109	25	255.88	26.38	7200.00	209.66	209.66	0.00	1.26	209.66	0.00	-18.06	1	1.26
C201	25	418.25	0.00	291.64	394.32	422.93	7.26	0.83	418.25	6.07	0.00	9	7.57
C202	25	373.07	28.97	7200.00	324.62	339.18	4.49	0.78	335.43	3.33	-10.09	169	51.67
C203	25	338.33	40.29	7200.00	334.58	338.33	1.12	1.03	338.33	1.12	0.00	249	167.34
C204	25	297.02	36.58	7200.00	293.54	297.02	1.19	0.93	297.02	1.19	0.00	287	207.12
C205	25	340.99	0.00	512.30	320.23	351.10	9.64	1.33	340.99	6.48	0.00	135	36.87
C206	25	311.89	10.64	7200.00	291.89	316.61	8.47	0.61	311.89	6.85	0.00	439	282.30
C207	25	329.30	15.52	7200.00	293.41	323.47	10.24	0.93	316.69	7.93	-3.83	253	160.19
C208	25	265.04	8.23	7200.00	251.58	265.04	5.35	0.92	265.04	5.35	0.00	289	231.94
R101	25	598.39	0.00	0.64	598.39	598.39	0.00	0.35	598.39	0.00	0.00	1	0.35
R102	25	499.71	12.85	7200.00	499.71	499.71	0.00	0.23	499.71	0.00	0.00	1	0.23
R103	25	455.16	19.35	7200.00	449.82	450.20	0.08	0.48	450.20	0.08	-1.09	3	0.87
R104	25	415.19	16.57	7200.00	407.80	408.55	0.18	0.55	408.55	0.18	-1.60	3	1.01
R105	25	511.58	0.00	82.36	511.58	511.58	0.00	0.89	511.58	0.00	0.00	1	0.89
R106	25	462.50	15.01	7200.00	453.05	453.41	0.08	0.34	453.41	0.08	-1.97	3	0.63
R107	25	438.18	20.37	7200.00	413.36	417.05	0.89	0.48	415.30	0.47	-5.22	7	1.68
R108	25	391.12	13.25	7200.00	379.81	379.81	0.00	0.56	379.81	0.00	-2.89	1	0.56
R109	25	442.61	6.62	7200.00	440.49	442.60	0.48	0.21	442.60	0.48	0.00	3	0.41
R110	25	424.09	21.13	7200.00	410.20	410.20	0.00	0.38	410.20	0.00	-3.28	1	0.38
R111	25	420.51	15.53	7200.00	418.44	418.44	0.00	0.4	418.44	0.00	-0.49	1	0.40
R112	25	403.42	19.30	7200.00	369.00	372.70	1.00	0.39	372.70	1.00	-7.61	5	1.17
R201	25	512.25	0.00	3.45	512.25	512.25	0.00	0.18	512.25	0.00	0.00	1	0.18
R202	25	470.77	7.07	7200.00	462.59	470.77	1.77	0.47	470.77	1.77	0.00	27	18.67
R203	25	434.80	25.90	7200.00	416.39	434.80	4.42	0.83	434.80	4.42	0.00	86	73.15
R204	25	401.62	21.70	7200.00	381.94	401.62	5.15	0.52	401.62	5.15	0.00	39	31.46
R205	25	445.89	0.00	272.90	445.37	445.89	0.12	0.29	445.89	0.12	0.00	3	0.59
R206	25	414.24	14.60	7200.00	412.88	414.24	0.33	0.35	414.24	0.33	0.00	3	0.64
R207	25	417.84	21.86	7200.00	405.89	406.68	0.19	0.58	406.68	0.19	-2.67	3	1.06
R208	25	391.12	20.64	7200.00	366.32	369.93	0.99	0.54	369.93	0.99	-5.42	3	1.35
R209	25	430.00	13.07	7200.00	424.89	424.89	0.00	0.46	424.89	0.00	-1.19	1	0.46
R210	25	421.57	8.45	7200.00	421.57	421.57	0.00	1.17	421.57	0.00	0.00	1	1.17
R211	25	402.07	18.36	7200.00	384.93	388.77	1.00	0.55	387.38	0.64	-3.65	7	1.83
RC101	25	437.15	0.00	239.54	407.89	445.81	9.30	0.87	437.14	7.17	0.00	79	62.24
RC102	25	338.53	6.81	7200.00	338.53	338.53	0.00	0.26	338.53	0.00	0.00	1	0.26
RC103	25	320.48	11.17	7200.00	320.48	320.48	0.00	0.41	320.48	0.00	0.00	1	0.41
RC104	25	293.16	2.82	7200.00	293.16	293.16	0.00	0.65	293.16	0.00	0.00	1	0.65
RC105	25	386.54	11.56	7200.00	386.54	386.54	0.00	0.17	386.54	0.00	0.00	1	0.17
RC106	25	339.74	8.45	7200.00	335.73	335.73	0.00	0.15	335.73	0.00	-1.18	1	0.15
RC107	25	289.76	1.71	7200.00	289.76	289.76	0.00	0.46	289.76	0.00	0.00	1	0.46
RC108	25	287.04	2.98	7200.00	287.04	287.04	0.00	0.7	287.04	0.00	0.00	1	0.70
RC201	25	405.57	0.00	2.32	380.99	408.09	7.11	0.11	405.56	6.45	0.00	13	1.17
RC202	25	339.06	14.88	7200.00	334.57	339.06	1.34	0.61	339.06	1.34	0.00	109	73.44
RC203	25	321.33	26.16	7200.00	314.23	317.64	1.09	0.89	317.64	1.09	-1.15	68	57.68
RC204	25	301.61	25.08	7200.00	297.71	297.71	0.00	0.75	297.71	0.00	-1.29	1	0.75
RC205	25	352.72	0.00	2107.74	352.72	352.72	0.00	0.24	352.72	0.00	0.00	1	0.24
RC206	25	338.76	0.00	1830.38	335.73	338.76	0.90	0.27	338.76	0.90	0.00	4	1.31
RC207	25	335.51	37.17	7200.00	311.80	311.80	0.00	0.36	311.80	0.00	-7.07	1	0.36
RC208	25	321.92	36.13	7200.00	292.36	292.36	0.00	0.72	292.36	0.00	-9.18	1	0.72
AVERAGE			12.19	5261.62			1.51	0.91		1.27	-1.61	41.59	27.18

TABLE 4. Computational results on 50-customers instances of *Experiment 1*.

Instance		MIP				BAP algorithm							
Name	NP	BUB1	Gap1 (%)	CpuT1 (s)	LB	UB	Gap2 (%)	CpuT2 (s)	BUB2	Gap3 (%)	Gap4 (%)	Nodes	CpuT3 (s)
C101	50	362.48	0.00	1.91	362.48	362.48	0.00	1.61	362.48	0.00	0.00	1	1.61
C102	50	430.87	20.23	7200.00	364.89	377.29	3.40	3.32	368.16	0.90	-14.56	318	625.82
C103	50	-	-	7200.00	376.26	394.17	4.76	11.11	-	-	-	-	7200.00
C104	50	637.36	49.40	7200.00	367.35	382.94	4.24	6.63	-	-	-	-	7200.00
C105	50	360.72	0.00	6.99	360.72	360.72	0.00	2.10	360.72	0.00	0.00	1	2.10
C106	50	362.48	0.00	3.90	362.48	362.48	0.00	1.55	362.48	0.00	0.00	1	1.55
C107	50	363.65	3.93	7200.00	359.14	363.65	1.26	6.06	363.65	1.26	0.00	24	73.16
C108	50	456.14	28.37	7200.00	357.04	368.46	3.20	4.90	368.46	3.20	-19.22	117	289.11
C109	50	1000.10	67.82	7200.00	354.88	374.51	5.53	4.20	367.18	3.47	-63.29	81	183.55
C201	50	686.11	15.44	7200.00	625.18	686.11	9.75	2.24	686.11	9.75	0.00	1154	2731.82
C202	50	-	-	7200.00	671.82	695.38	3.51	4.16	-	-	-	-	7200.00
C203	50	-	-	7200.00	683.69	717.53	4.95	3.96	717.53	4.95	-	2671	4755.19
C204	50	-	-	7200.00	607.53	638.49	5.10	5.73	638.49	5.10	-	275	746.28
C205	50	592.96	20.10	7200.00	569.13	592.96	4.19	3.10	592.96	4.19	0.00	377	861.19
C206	50	765.72	50.80	7200.00	614.11	627.58	2.19	6.16	627.58	2.19	-18.04	192	472.13
C207	50	1078.12	65.55	7200.00	665.40	705.92	6.09	2.93	-	-	-	-	7200.00
C208	50	1002.56	65.69	7200.00	594.36	627.38	5.56	5.93	619.73	4.27	-38.19	392	723.53
R101	50	1012.74	0.00	2.51	1009.66	1012.74	0.31	0.46	1012.74	0.31	0.00	5	1.11
R102	50	1146.17	44.85	7200.00	895.58	907.53	1.34	6.32	907.53	1.34	-20.82	43	107.69
R103	50	1345.07	59.80	7200.00	784.63	838.52	6.87	9.85	-	-	-	-	7200.00
R104	50	-	-	7200.00	782.80	788.37	0.71	3.25	788.37	0.71	-	8	18.63
R105	50	913.76	11.27	7200.00	866.03	866.64	0.07	0.85	866.64	0.07	-5.16	3	1.74
R106	50	-	-	7200.00	846.86	861.79	1.76	3.37	861.79	1.76	-	37	97.28
R107	50	-	-	7200.00	758.05	787.63	3.90	6.08	-	-	-	-	7200.00
R108	50	-	-	7200.00	729.04	762.49	4.59	4.00	-	-	-	-	7200.00
R109	50	1372.17	57.80	7200.00	738.99	747.77	1.19	2.57	747.77	1.19	-45.50	65	78.65
R110	50	-	-	7200.00	676.87	703.93	4.00	5.27	703.93	4.00	-	120	455.99
R111	50	-	-	7200.00	700.60	725.92	3.61	4.82	-	-	-	-	7200.00
R112	50	1950.36	74.70	7200.00	627.89	656.71	4.59	6.96	656.71	4.59	-66.33	307	1182.25
R201	50	890.03	0.00	3216.25	874.57	904.70	3.45	1.17	890.03	1.77	0.00	389	1077.24
R202	50	-	-	7200.00	753.13	768.53	2.04	7.52	768.53	2.04	-	82	137.61
R203	50	-	-	7200.00	835.93	881.85	5.49	6.14	-	-	-	-	7200.00
R204	50	2060.14	79.07	7200.00	773.59	837.27	8.23	5.95	-	-	-	-	7200.00
R205	50	1301.62	51.93	7200.00	808.86	837.27	3.51	1.34	837.27	3.51	-35.67	614	1638.25
R206	50	-	-	7200.00	735.63	769.53	4.61	2.44	751.36	2.14	-	833	2581.62
R207	50	802.24	44.40	7200.00	685.85	728.21	6.18	2.42	713.15	3.98	-11.10	562	1372.56
R208	50	804.49	46.64	7200.00	670.64	706.91	5.41	2.58	706.91	5.41	-12.13	371	824.18
R209	50	-	-	7200.00	766.84	796.62	3.88	2.62	-	-	-	-	7200.00
R210	50	1309.06	62.87	7200.00	774.62	803.57	3.74	4.83	-	-	-	-	7200.00
R211	50	-	-	7200.00	678.99	716.31	5.50	2.88	705.62	3.92	-	1384	3729.63
RC101	50	1034.82	33.10	7200.00	856.50	917.32	7.10	0.31	917.32	7.10	-11.36	2707	4138.15
RC102	50	-	-	7200.00	802.93	837.83	4.35	2.79	822.05	2.38	-	1967	3511.56
RC103	50	-	-	7200.00	718.04	749.84	4.43	6.86	-	-	-	-	7200.00
RC104	50	1873.42	72.31	7200.00	672.94	727.61	8.12	2.17	709.84	5.48	-62.11	3155	5932.25
RC105	50	-	-	7200.00	835.45	865.26	3.57	5.31	-	-	-	-	7200.00
RC106	50	-	-	7200.00	805.39	830.34	3.10	1.07	830.34	3.10	-	508	573.64
RC107	50	-	-	7200.00	709.85	738.21	3.99	2.40	738.21	3.99	-	149	394.35
RC108	50	2749.86	81.29	7200.00	652.02	681.20	4.48	5.76	681.20	4.48	-75.23	-	279.44
RC201	50	863.70	11.90	7200.00	806.93	862.08	6.83	0.81	862.08	6.83	-0.19	289	302.79
RC202	50	-	-	7200.00	677.26	695.45	2.69	5.16	687.63	1.53	-	73	108.33
RC203	50	-	-	7200.00	706.54	768.27	8.74	3.17	-	-	-	-	7200.00
RC204	50	-	-	7200.00	735.51	784.27	6.63	3.62	756.59	2.87	-	695	968.12
RC205	50	962.75	52.53	7200.00	691.28	691.79	0.07	1.57	691.79	0.07	-28.14	3	4.84
RC206	50	-	-	7200.00	662.72	674.44	1.77	2.00	674.44	1.77	-	27	53.20
RC207	50	-	-	7200.00	642.16	642.16	0.00	3.54	642.16	0.00	-	1	3.54
RC208	50	-	-	7200.00	568.93	568.93	0.00	7.13	568.93	0.00	-	1	7.13
AVERAGE			39.06	6614.85			3.83	3.98		2.82	-21.08	500.05	2661.59

TABLE 5. Computational results on 10-customers instances of *Experiment 2*.

Instance		BAP algorithm											
Name	NP	MIP			Root node				BUB2	Gap3 (%)	Gap4 (%)	Nodes	CpuT3 (s)
		BUB1	Gap1 (%)	CpuT1 (s)	LB	UB	Gap2 (%)	CpuT2 (s)					
C101	10	71.57	0.00	0.09	71.57	71.57	0.00	0.03	71.57	0.00	0.00	1	0.03
C102	10	70.00	0.00	12.54	70.00	70.00	0.00	8.10	70.00	0.00	0.00	1	8.10
C103	10	70.00	0.00	12.47	70.00	70.00	0.00	8.48	70.00	0.00	0.00	1	8.48
C104	10	69.16	0.00	29.46	68.61	69.84	1.79	7.11	69.16	0.80	0.00	3	27.19
C105	10	71.57	0.00	0.15	71.57	71.57	0.00	0.03	71.57	0.00	0.00	1	0.03
C106	10	71.57	0.00	0.08	71.57	71.57	0.00	0.03	71.57	0.00	0.00	1	0.03
C107	10	71.57	0.00	2.69	71.57	71.57	0.00	0.14	71.57	0.00	0.00	1	0.14
C108	10	70.74	0.00	3.98	70.74	70.74	0.00	0.15	70.74	0.00	0.00	1	0.15
C109	10	70.74	0.00	4.78	70.74	70.74	0.00	1.08	70.74	0.00	0.00	1	1.08
C201	10	231.66	0.00	0.40	231.66	231.66	0.00	0.05	231.66	0.00	0.00	1	0.05
C202	10	176.11	0.00	134.99	176.11	176.11	0.00	3.42	176.11	0.00	0.00	1	3.42
C203	10	176.11	0.00	135.94	176.11	176.11	0.00	3.35	176.11	0.00	0.00	1	3.35
C204	10	137.15	0.00	30.59	135.51	139.14	2.68	5.95	137.15	1.21	0.00	2	13.40
C205	10	175.93	0.00	0.29	175.93	175.93	0.00	0.05	175.93	0.00	0.00	1	0.05
C206	10	174.25	0.00	1.39	174.25	174.25	0.00	0.09	174.25	0.00	0.00	1	0.09
C207	10	220.06	0.00	0.46	220.06	220.06	0.00	0.04	220.06	0.00	0.00	1	0.05
C208	10	150.00	0.00	1.36	150.00	150.00	0.00	0.12	150.00	0.00	0.00	1	0.12
R101	10	262.18	0.00	0.16	262.18	262.18	0.00	0.04	262.18	0.00	0.00	1	0.05
R102	10	228.75	0.00	121.55	228.75	228.75	0.00	1.91	228.75	0.00	0.00	1	1.91
R103	10	228.75	0.00	124.83	228.75	228.75	0.00	1.89	228.75	0.00	0.00	1	1.89
R104	10	198.70	0.00	19.26	197.85	201.46	1.82	7.53	198.70	0.43	0.00	2	14.95
R105	10	259.65	0.00	0.19	259.65	259.65	0.00	0.06	259.65	0.00	0.00	1	0.06
R106	10	220.54	0.00	76.04	220.54	220.54	0.00	3.67	220.54	0.00	0.00	1	3.67
R107	10	220.54	0.00	71.35	220.54	220.54	0.00	3.71	220.54	0.00	0.00	1	3.71
R108	10	198.70	0.00	10.08	198.70	198.70	0.00	5.98	198.70	0.00	0.00	1	5.98
R109	10	238.05	0.00	1.49	238.05	238.05	0.00	0.25	238.05	0.00	0.00	1	0.25
R110	10	207.52	0.00	8.41	207.22	207.52	0.15	1.27	207.52	0.14	0.00	2	2.32
R111	10	214.09	0.00	53.11	214.09	214.09	0.00	3.66	214.09	0.00	0.00	1	3.66
R112	10	192.13	0.00	8.62	192.13	192.13	0.00	18.10	192.13	0.00	0.00	1	18.10
R201	10	259.65	0.00	0.11	259.65	259.65	0.00	0.04	259.65	0.00	0.00	1	0.04
R202	10	220.54	0.00	110.07	220.54	220.54	0.00	2.90	220.54	0.00	0.00	1	2.90
R203	10	220.54	0.00	110.24	220.54	220.54	0.00	3.04	220.54	0.00	0.00	1	3.04
R204	10	198.70	0.00	13.80	198.70	198.70	0.00	20.25	198.70	0.00	0.00	1	20.26
R205	10	238.05	0.00	2.47	238.05	238.05	0.00	0.30	238.05	0.00	0.00	1	0.30
R206	10	201.23	0.00	16.86	201.23	201.23	0.00	6.71	201.23	0.00	0.00	1	6.71
R207	10	201.23	0.00	16.73	201.23	201.23	0.00	6.88	201.23	0.00	0.00	1	6.89
R208	10	198.70	0.00	29.88	198.70	198.70	0.00	8.32	198.70	0.00	0.00	1	8.32
R209	10	229.50	0.00	7.03	229.50	229.50	0.00	0.53	229.50	0.00	0.00	1	0.53
R210	10	218.28	0.00	77.69	218.28	218.28	0.00	2.65	218.28	0.00	0.00	1	2.66
R211	10	209.96	0.00	41.39	208.26	209.96	0.82	5.66	209.96	0.82	0.00	1	7.94
RC101	10	189.69	0.00	1.27	189.69	189.69	0.00	0.04	189.69	0.00	0.00	1	0.04
RC102	10	165.39	0.00	14.58	165.39	165.39	0.00	7.68	165.39	0.00	0.00	1	7.68
RC103	10	165.39	0.00	13.80	165.39	165.39	0.00	7.72	165.39	0.00	0.00	1	7.72
RC104	10	159.33	0.00	48.50	159.33	159.33	0.00	17.14	159.33	0.00	0.00	1	17.14
RC105	10	176.56	0.00	71.42	176.56	176.56	0.00	2.79	176.56	0.00	0.00	1	2.79
RC106	10	172.75	0.00	14.91	172.75	172.75	0.00	0.44	172.75	0.00	0.00	1	0.44
RC107	10	160.78	0.00	31.52	160.78	160.78	0.00	3.85	160.78	0.00	0.00	1	3.85
RC108	10	160.68	0.00	39.64	160.68	160.68	0.00	8.16	160.68	0.00	0.00	1	8.16
RC201	10	185.20	0.00	0.21	185.20	185.20	0.00	0.03	185.20	0.00	0.00	1	0.03
RC202	10	163.52	13.04	7200.00	163.52	163.52	0.00	29.90	163.52	0.00	0.00	1	29.90
RC203	10	163.52	13.04	7200.00	163.52	163.52	0.00	28.84	163.52	0.00	0.00	1	28.84
RC204	10	159.33	16.75	7200.00	159.33	159.33	0.00	37.40	159.33	0.00	0.00	1	37.40
RC205	10	176.57	0.00	74.72	176.57	176.57	0.00	0.55	176.57	0.00	0.00	1	0.55
RC206	10	175.78	0.00	3.80	175.78	175.78	0.00	0.25	175.78	0.00	0.00	1	0.25
RC207	10	172.75	0.00	44.98	172.75	172.75	0.00	0.59	172.75	0.00	0.00	1	0.59
RC208	10	160.68	18.16	7200.00	160.68	160.68	0.00	29.80	160.68	0.00	0.00	1	29.81
AVERAGE			1.09	543.79			0.13	5.69		0.06	0.00	1.09	6.38

TABLE 6. Computational results on 25-customers instances of *Experiment 2*.

Instance		BAP algorithm											
Name	NP	MIP			Root node				BUB2	Gap3 (%)	Gap4 (%)	Nodes	CpuT3 (s)
		BUB1	Gap1 (%)	CpuT1 (s)	LB	UB	Gap2 (%)	CpuT2 (s)					
C101	25	240.39	0.00	2.84	240.39	240.39	0.00	1.31	240.39	0.00	0.00	1	1.31
C102	25	238.82	1.15	7200.00	238.82	238.82	0.00	36.43	238.82	0.00	0.00	1	36.46
C103	25	283.28	29.63	7200.00	233.68	238.81	2.20	57.15	235.20	0.65	-16.97	106	589.51
C104	25	248.83	22.20	7200.00	211.89	219.26	3.48	54.15	213.15	0.59	-14.34	47	278.15
C105	25	220.75	0.00	4.18	220.75	220.75	0.00	4.03	220.75	0.00	0.00	1	4.03
C106	25	241.22	0.00	2.48	241.22	241.22	0.00	2.54	241.22	0.00	0.00	1	2.54
C107	25	214.38	0.00	6755.31	214.38	214.38	0.00	9.57	214.38	0.00	0.00	1	9.65
C108	25	215.77	8.99	7200.00	213.76	216.85	1.44	7.95	214.38	0.29	-0.65	11	24.34
C109	25	314.12	41.42	7200.00	208.95	213.76	2.30	17.63	210.13	0.56	-33.10	95	356.45
C201	25	440.71	0.00	908.10	440.71	440.71	0.00	1.02	440.71	0.00	0.00	1	1.02
C202	25	383.45	29.11	7200.00	379.53	388.50	2.36	39.51	385.97	1.70	0.66	53	261.63
C203	25	418.33	49.99	7200.00	309.75	328.15	5.94	45.33	317.11	2.38	-24.20	164	682.52
C204	25	385.31	50.13	7200.00	289.75	303.72	4.82	61.52	295.62	2.03	-23.28	331	1034.63
C205	25	365.50	19.58	7200.00	365.50	365.50	0.00	7.04	365.50	0.00	0.00	1	7.08
C206	25	336.98	25.79	7200.00	334.95	347.52	3.75	13.52	341.55	1.97	1.36	16	40.59
C207	25	391.60	26.92	7200.00	342.64	342.64	0.00	56.43	342.64	0.00	-12.50	1	56.44
C208	25	315.09	26.18	7200.00	308.03	308.03	0.00	40.78	308.03	0.00	-2.24	1	40.79
R101	25	619.51	0.00	0.59	619.51	619.51	0.00	0.31	619.51	0.00	0.00	1	0.31
R102	25	534.73	22.82	7200.00	528.65	541.96	2.52	48.85	530.15	0.28	-0.86	91	379.29
R103	25	570.82	38.74	7200.00	457.22	457.22	0.00	37.32	457.22	0.00	-19.90	1	37.33
R104	25	597.03	43.86	7200.00	428.61	447.18	4.33	48.16	434.72	1.43	-27.19	134	528.15
R105	25	532.26	0.00	1049.93	532.26	532.26	0.00	13.23	532.26	0.00	0.00	1	13.27
R106	25	500.42	24.58	7200.00	470.82	482.01	2.38	48.63	475.57	1.01	-4.97	78	353.68
R107	25	479.84	28.93	7200.00	422.15	438.63	3.90	38.43	427.61	1.29	-10.88	73	306.15
R108	25	554.27	41.45	7200.00	401.72	401.72	0.00	29.63	401.72	0.00	-27.52	1	29.66
R109	25	464.81	13.56	7200.00	446.90	465.66	4.20	38.45	455.06	1.83	-2.10	39	138.66
R110	25	513.82	35.90	7200.00	426.56	442.81	3.81	32.14	430.97	1.03	-16.12	17	87.23
R111	25	466.28	26.83	7200.00	428.77	428.77	0.00	43.16	428.77	0.00	-8.04	1	43.19
R112	25	459.14	29.36	7200.00	397.65	397.65	0.00	32.05	397.65	0.00	-13.39	1	32.07
R201	25	540.32	0.00	6.90	540.32	540.32	0.00	1.72	540.32	0.00	0.00	1	1.73
R202	25	489.80	19.51	7200.00	469.63	487.15	3.73	43.52	474.43	1.02	-3.14	137	579.46
R203	25	462.58	29.09	7200.00	417.63	433.24	3.74	51.53	425.00	1.77	-8.12	106	468.53
R204	25	545.45	43.39	7200.00	408.52	426.22	4.33	43.62	417.73	2.26	-23.42	367	1336.25
R205	25	464.41	2.86	7200.00	464.41	464.41	0.00	25.83	464.41	0.00	0.00	1	25.85
R206	25	421.66	18.64	7200.00	416.60	416.60	0.00	57.92	416.60	0.00	-1.20	1	57.94
R207	25	466.79	31.13	7200.00	416.60	416.60	0.00	53.47	416.60	0.00	-10.75	1	53.50
R208	25	446.13	32.22	7200.00	387.93	403.17	3.93	33.18	393.77	1.50	-11.74	117	387.62
R209	25	454.87	21.78	7200.00	429.63	437.15	1.75	37.15	431.12	0.35	-5.22	84	268.10
R210	25	435.54	22.55	7200.00	426.14	426.14	0.00	43.62	426.14	0.00	-2.16	1	43.63
R211	25	521.08	38.88	7200.00	395.76	418.62	5.78	47.33	402.97	1.82	-22.67	167	433.82
RC101	25	452.48	0.00	2076.17	452.48	452.48	0.00	6.44	452.48	0.00	0.00	1	6.47
RC102	25	419.74	27.93	7200.00	346.01	346.01	0.00	47.26	346.01	0.00	-17.57	232	764.03
RC103	25	325.77	14.07	7200.00	324.83	324.83	0.00	59.32	324.83	0.00	-0.29	1	59.33
RC104	25	342.80	20.03	7200.00	295.62	305.13	3.22	51.33	297.08	0.49	-13.34	81	246.40
RC105	25	498.06	35.79	7200.00	393.91	393.91	0.00	68.62	393.91	0.00	-20.91	1	68.64
RC106	25	426.52	29.32	7200.00	344.80	344.80	0.00	22.33	344.80	0.00	-19.16	1	22.34
RC107	25	349.55	20.67	7200.00	286.12	302.03	5.56	37.92	292.52	2.24	-16.31	136	673.49
RC108	25	333.02	16.96	7200.00	289.63	293.63	1.38	26.32	291.10	0.51	-12.59	105	439.51
RC201	25	408.70	0.00	24.07	408.70	408.70	0.00	6.51	408.70	0.00	0.00	1	6.55
RC202	25	347.19	26.32	7200.00	340.91	340.91	0.00	38.53	340.91	0.00	-1.81	1	38.54
RC203	25	464.88	52.01	7200.00	317.62	330.29	3.99	43.63	322.56	1.56	-30.61	301	934.27
RC204	25	405.77	44.50	7200.00	290.37	308.21	6.14	57.13	304.32	4.81	-25.00	263	783.93
RC205	25	389.19	23.85	7200.00	356.43	362.18	1.61	7.46	360.77	1.22	-7.30	6	23.37
RC206	25	384.93	27.73	7200.00	341.95	344.58	0.77	9.52	343.09	0.33	-10.87	5	21.38
RC207	25	397.70	50.06	7200.00	301.53	319.72	6.03	11.53	312.50	3.64	-21.42	77	238.10
RC208	25	456.59	55.81	7200.00	289.63	311.53	7.56	58.63	293.83	1.45	-35.65	362	1218.66
AVERAGE			23.97	6107.69			1.91	33.17		0.75	-10.31	68.34	260.31

TABLE 7. Computational results on 50-customers instances of *Experiment 2*.

Instance		MIP				BAP algorithm							
Name	NP	BUB1	Gap1 (%)	CpuT1 (s)	Root node				BUB2	Gap3 (%)	Gap4 (%)	Nodes	CpuT3 (s)
					LB	UB	Gap2 (%)	CpuT2 (s)					
C101	50	416.00	0.00	9.90	416.00	416.00	0.00	6.87	416.00	0.00	0.00	1	6.98
C102	50	-	-	7200.00	385.07	417.62	8.45	89.13	395.40	2.68	-	482	1492.16
C103	50	-	-	7200.00	380.27	407.27	7.10	242.62	-	-	-	-	7200.00
C104	50	1539.88	79.38	7200.00	372.27	405.14	8.83	192.16	-	-	-	-	7200.00
C105	50	376.21	0.00	404.20	376.21	376.21	0.00	25.26	376.21	0.00	0.00	1	25.75
C106	50	417.24	0.00	67.81	417.24	417.24	0.00	27.49	417.24	0.00	0.00	1	28.09
C107	50	433.95	19.24	7200.00	360.57	369.14	2.37	133.56	363.29	0.75	-16.28	452	1235.78
C108	50	792.69	58.59	7200.00	357.06	374.51	4.89	145.13	385.29	1.75	-54.17	537	1732.15
C109	50	-	-	7200.00	377.96	391.16	3.49	168.19	385.35	1.96	-	936	3761.51
C201	50	823.08	31.44	7200.00	749.34	783.57	4.57	249.26	766.24	2.25	-6.91	831	3985.46
C202	50	1155.90	65.15	7200.00	744.12	768.16	3.23	215.92	758.35	1.91	-34.39	1395	6715.63
C203	50	-	-	7200.00	727.37	760.91	4.61	298.73	-	-	-	-	7200.00
C204	50	2359.89	87.03	7200.00	723.35	764.26	5.66	279.15	753.81	4.21	-68.06	1538	6937.58
C205	50	757.11	44.77	7200.00	660.32	675.55	2.31	257.39	-	-	-	-	7200.00
C206	50	892.87	61.03	7200.00	646.93	671.63	3.82	286.45	653.22	0.97	-26.84	1391	5837.63
C207	50	-	-	7200.00	653.20	669.36	2.47	317.63	-	-	-	-	7200.00
C208	50	-	-	7200.00	554.51	570.27	2.84	281.35	562.79	1.49	-	732	3468.62
R101	50	1028.78	0.00	8.17	1028.78	1028.78	0.00	7.19	1028.78	0.00	0.00	1	7.32
R102	50	1007.39	37.25	7200.00	921.26	947.35	2.83	237.24	938.40	1.86	-6.85	1325	5437.68
R103	50	-	-	7200.00	941.95	972.58	3.25	271.74	946.52	0.49	-	581	3104.62
R104	50	1863.48	72.95	7200.00	718.82	749.45	4.26	216.52	739.93	2.94	-60.29	1075	4873.44
R105	50	959.85	14.65	7200.00	876.31	913.51	4.24	247.62	896.01	2.25	-6.65	937	4351.23
R106	50	-	-	7200.00	872.07	907.25	4.03	261.36	888.38	1.87	-	792	3913.53
R107	50	-	-	7200.00	926.54	945.12	2.01	317.23	-	-	-	-	7200.00
R108	50	1944.52	74.70	7200.00	879.72	907.52	3.16	305.61	892.94	1.50	-54.08	1137	4724.81
R109	50	1254.15	53.05	7200.00	912.53	954.65	4.62	197.62	926.83	1.57	-26.10	892	3892.52
R110	50	-	-	7200.00	925.77	967.44	4.50	263.43	938.55	1.38	-	1524	6843.83
R111	50	-	-	7200.00	876.46	894.72	2.08	274.13	-	-	-	-	7200.00
R112	50	-	-	7200.00	892.91	915.84	2.57	267.46	905.33	1.39	-	1473	6715.65
R201	50	916.80	0.00	1742.75	916.80	916.80	0.00	76.82	916.80	0.00	0.00	1	76.84
R202	50	-	-	7200.00	853.65	871.33	2.07	236.19	862.75	1.07	-	734	3842.68
R203	50	-	-	7200.00	894.01	913.67	2.20	271.67	-	-	-	-	7200.00
R204	50	-	-	7200.00	871.59	884.63	1.50	253.74	-	-	-	-	7200.00
R205	50	950.95	35.30	7200.00	801.83	837.56	4.46	287.47	816.57	1.84	-14.13	873	4358.61
R206	50	-	-	7200.00	817.27	836.21	2.32	311.57	-	-	-	-	7200.00
R207	50	-	-	7200.00	813.63	846.83	4.08	291.36	826.93	1.64	-	1273	6107.47
R208	50	2238.00	81.00	7200.00	829.72	858.13	3.42	278.31	832.85	0.38	-62.79	1473	7162.72
R209	50	1487.62	65.08	7200.00	843.35	878.28	4.14	273.92	-	-	-	-	7200.00
R210	50	-	-	7200.00	766.18	791.23	3.27	291.25	772.82	0.87	-	704	3107.51
R211	50	-	-	7200.00	847.15	868.35	2.50	303.16	-	-	-	-	7200.00
RC101	50	1079.45	35.77	7200.00	736.50	757.82	2.90	291.29	743.47	0.95	-31.13	1053	5627.16
RC102	50	-	-	7200.00	692.64	728.63	5.20	305.63	724.68	4.62	-	896	4976.21
RC103	50	-	-	7200.00	729.03	766.49	5.14	338.36	-	-	-	-	7200.00
RC104	50	-	-	7200.00	681.09	703.44	3.28	276.18	694.45	1.96	-	1607	7051.93
RC105	50	-	-	7200.00	703.05	726.47	3.33	307.61	-	-	-	-	7200.00
RC106	50	-	-	7200.00	766.94	796.53	3.86	273.15	773.25	0.82	-	1368	6614.73
RC107	50	-	-	7200.00	735.92	761.47	3.47	278.62	753.46	2.38	-	1273	6155.49
RC108	50	-	-	7200.00	742.57	774.95	4.36	296.72	-	-	-	-	7200.00
RC201	50	1001.95	27.46	7200.00	744.07	768.18	3.24	315.32	749.69	0.76	-25.18	917	4837.52
RC202	50	-	-	7200.00	732.55	751.26	2.55	305.27	-	-	-	-	7200.00
RC203	50	-	-	7200.00	819.17	841.63	2.74	362.71	-	-	-	-	7200.00
RC204	50	-	-	7200.00	756.00	768.13	1.60	301.21	760.38	0.58	-	1137	5212.95
RC205	50	-	-	7200.00	780.88	809.46	3.66	294.24	796.28	1.97	-	704	3716.13
RC206	50	-	-	7200.00	763.83	785.49	2.84	328.32	-	-	-	-	7200.00
RC207	50	3508.91	90.82	7200.00	784.92	816.55	4.03	294.08	797.02	1.54	-77.29	1708	7162.65
RC208	50	-	-	7200.00	764.95	791.54	3.48	308.82	-	-	-	-	7200.00
AVERAGE			43.11	6597.01			3.35	247.08		1.48	-27.20	912.30	5212.55

TABLE 8. Summary of the comparative results.

Instance		MIP				BAP algorithm							
Name	NP	Solved		Gap1	CpuT1	Solved		Gap2	CpuT2	Gap3	Gap4	Nodes	CpuT3
		N/T	Ratio1 (%)	(%)	(s)	N/T	Ratio2 (%)	(%)	(s)	(%)	(%)		(s)
<i>Experiment 1</i>													
C1	10	9/9	100.00	0.00	1.27	9/9	100.00	0.00	0.02	0.00	0.00	1.00	0.02
C2	10	8/8	100.00	0.00	4.95	8/8	100.00	5.23	0.05	5.23	0.00	3.88	0.12
R1	10	12/12	100.00	0.00	2.02	12/12	100.00	0.08	0.01	0.08	0.00	1.67	0.02
R2	10	11/11	100.00	0.00	1.64	11/11	100.00	0.12	0.01	0.12	0.00	1.55	0.02
RC1	10	8/8	100.00	0.00	8.08	8/8	100.00	0.19	0.02	0.19	0.00	1.50	0.03
RC2	10	8/8	100.00	0.00	36.21	8/8	100.00	0.17	0.01	0.17	0.00	1.50	0.03
Total	10	56/56	100.00	0.00	7.99	56/56	100.00	0.84	0.02	0.84	0.00	1.80	0.03
C1	25	6/9	66.67	5.12	2545.28	9/9	100.00	0.03	2.77	0.03	-2.14	1.22	4.16
C2	25	2/8	25.00	17.53	5500.49	8/8	100.00	5.97	0.92	4.79	-1.74	228.75	143.13
R1	25	2/12	16.67	13.33	6006.92	12/12	100.00	0.23	0.44	0.19	-2.01	2.50	0.72
R2	25	2/11	18.18	13.79	5916.03	11/11	100.00	1.27	0.54	1.24	-1.18	15.82	11.87
RC1	25	1/8	12.50	5.69	6329.94	8/8	100.00	1.16	0.46	0.90	-0.15	10.75	8.13
RC2	25	3/8	37.50	17.43	4992.56	8/8	100.00	1.31	0.49	1.22	-2.34	24.75	16.96
Total	25	16/56	28.57	12.19	5261.62	56/56	100.00	1.51	0.91	1.27	-1.61	41.59	27.18
C1	50	3/9	33.33	21.22	4801.42	7/9	77.78	2.49	4.61	1.26	-13.87	77.57	1730.77
C2	50	0/8	0.00	43.52	7200.00	6/8	75.00	5.17	4.28	5.07	-14.06	843.50	3086.27
R1	50	1/12	8.33	41.40	6600.21	8/12	66.67	2.74	4.48	1.75	-27.56	73.50	2561.95
R2	50	1/11	9.09	47.49	6837.84	7/11	63.64	4.73	3.63	3.25	-14.73	605.00	3651.01
RC1	50	0/8	0.00	62.23	7200.00	6/8	75.00	4.89	3.33	4.42	-49.56	1697.20	3653.67
RC2	50	0/8	0.00	32.22	7200.00	7/8	87.50	3.34	3.38	1.87	-14.17	155.57	1080.99
Total	50	5/56	8.93	39.06	6614.85	41/56	73.21	3.83	3.98	2.82	-21.08	500.05	2661.59
<i>Experiment 2</i>													
C1	10	9/9	100.00	0.00	7.36	9/9	100.00	0.20	2.79	0.09	0.00	1.22	5.03
C2	10	8/8	100.00	0.00	38.18	8/8	100.00	0.33	1.63	0.15	0.00	1.13	2.57
R1	10	12/12	100.00	0.00	41.26	12/12	100.00	0.16	4.00	0.05	0.00	1.17	4.71
R2	10	11/11	100.00	0.00	38.75	11/11	100.00	0.07	5.21	0.07	0.00	1.00	5.42
RC1	10	8/8	100.00	0.00	29.46	8/8	100.00	0.00	5.98	0.00	0.00	1.00	5.98
RC2	10	4/8	50.00	7.62	3615.46	8/8	100.00	0.00	15.92	0.00	0.00	1.00	15.92
Total	10	52/56	92.86	1.09	543.79	56/56	100.00	0.13	5.69	0.06	0.00	1.09	6.38
C1	25	4/9	44.44	11.49	4751.65	9/9	100.00	1.05	21.20	0.23	-7.23	29.33	144.72
C2	25	1/8	12.50	28.46	6413.51	8/8	100.00	2.11	33.14	1.01	-7.53	71.00	265.59
R1	25	2/12	16.67	25.50	6087.54	12/12	100.00	1.76	34.20	0.57	-10.91	36.50	162.42
R2	25	1/11	9.09	23.64	6546.08	11/11	100.00	2.11	39.90	0.79	-8.04	89.36	332.40
RC1	25	1/8	12.50	20.60	6559.52	8/8	100.00	1.27	39.94	0.40	-12.52	69.75	285.03
RC2	25	1/8	12.50	35.04	6303.01	8/8	100.00	3.26	29.12	1.62	-16.58	127.00	408.10
Total	25	10/56	17.86	23.97	6107.69	56/56	100.00	1.91	33.17	0.75	-10.31	68.34	260.31
C1	50	3/9	33.33	26.20	4853.55	7/9	77.78	3.90	114.49	1.02	-14.09	344.29	2520.27
C2	50	0/8	0.00	57.88	7200.00	5/8	62.50	3.69	273.23	2.17	-34.05	1177.40	6068.11
R1	50	1/12	8.33	42.10	6600.68	10/12	83.33	3.13	238.93	1.52	-25.66	973.70	4855.38
R2	50	1/11	9.09	45.35	6703.89	6/11	54.55	2.72	261.41	0.96	-25.64	843.00	5514.17
RC1	50	0/8	0.00	35.77	7200.00	3/8	37.50	3.94	295.95	2.15	-31.13	1239.40	6503.19
RC2	50	0/8	0.00	59.14	7200.00	4/8	50.00	3.02	313.75	1.21	-51.23	1116.50	6216.16
Total	50	5/56	8.93	43.11	6597.01	35/56	62.50	3.35	247.08	1.48	-27.20	912.30	5212.55

In these tables, we calculate all the average gaps and CPU times. It is clear that the proposed BAP algorithm has a better performance compared with the MIP model on these two experiments. However, it is difficult for readers to read these 6 tables. Therefore, in the next section, we summarize all these experiments in Table 8 and analyze all the experimental results.

6.3. Comprehensive analysis of these two experiments

In order to facilitate the analysis of the calculation results, Table 8 summarizes all calculation results. Columns 1 and 2 report the information of instances, and respectively are the type and the size of instance. Columns 3–6 summarize the performance of CPLEX for the MIP formulation and report as follows: (i) the number of solved instance and total instance (N/T); (ii) the ratio (Ratio1) between solved instances and total instances; (iii) the

average gap (Gap1); and (iv) the average Cpu calculation time. Columns 7–14 summarize the performance of the proposed BAP algorithm and report in the following: (i) the number of solved instance and total instance (N/T); (ii) the ratio (Ratio2) between solved instances and total instances; (iii) the average gap (Gap2) between UB and LB; (iv) the average Cpu time (CpuT2) used at the root node stage; (v) the average gap (Gap3) between BUB2 and LB; (vi) the average gap (Gap4) between BUB2 and BUB1; (vii) the average number of explored branch-and-bound tree nodes (Nodes); and (viii) the average total Cpu time (CpuT3).

As for *Experiment 1* with the instance scale of 10 customers, it is clear that the MIP solver can solve all the instances optimally, and the gap between the proposed BAP algorithm and MIP model is 0, which illustrates that the proposed BAP algorithm can also solve all the instances with 10 customers optimally. Besides, the average Cpu time of MIP solver is 7.99s, but the Cpu time of the BAP algorithm is only 0.03s, which fully demonstrates the efficiency of the BAP algorithm. The experimental results of the small-scale instances are very illustrative, because the small-scale instances can accurately test the accuracy and effectiveness of the proposed algorithm. With the increases of the scale of increases, the solving ability of the MIP solver becomes weaker. The MIP solver can only solve 16 out of 56 instances (28.57%) with the scale of 25. However, the proposed MIP model can solve all the instances with this scale, and the gap between the proposed BAP algorithm and MIP model is -1.61% , which highlights the solving ability and efficiency of the BAP algorithm. This conclusion is more obvious in 50-scale instances. We can see that the MIP solver can only solve 5 out of 56 instances (8.93%), and the average Cpu time is 6614.85s, which denotes that the MIP solver has difficulty in handling large-scale instances. But the proposed BAP algorithm solves 41 out of 56 instances (73.12%), the gap between the proposed BAP algorithm and MIP model is -21.08% , and the Cpu time of BAP algorithm is 2661.59s, which obtains a great advantage in comparison with MIP solver. To sum up, the proposed BAP algorithm is effective and efficient for solve the studied problem from economic perspective, and has the ability to solve the instances with relatively large scale up to 50.

Similarly, three scale instances are used to test the proposed BAP algorithm in *Experiment 2*. As shown in Table 8, for the small-scale instances with 10 customers, the MIP solver solves 52 instances out of 56 instances (92.86%), and the average Gap1 is 1.09%, so the results of MIP solver are very accurate and comparable. From Table 8, the proposed BAP algorithm solves all the small-scale instances with 10 customers, and the average Gap4 between the BUB2 and BUB1 is 0, which illustrates the efficiency and effectiveness of the proposed BAP algorithm. As for the medium-scale instances with 25 customers, the overall BAP algorithm itself produced global optimal solutions in Cpu times that are significantly shorter than the MIP solver, and the MIP solver can only solve 10 out of 56 instances (17.86%) while the proposed BAP can solve all medium-scale instances, which further demonstrates the efficiency and effectiveness of the proposed BAP algorithm. As for the large-scale instances, the MIP solver seems very weak and can only solve 5 out of 56 instances (8.93%), even the MIP solver cannot give a UB for many large-scale instances (32 out of 56 instances, which is shown in Tab. 8). While the proposed BAP algorithm can solve 35 out of 56 instances (62.5%), which demonstrates the solving ability of the proposed BAP algorithm for large-scale cases. And the average gap of root node (Gap2) and overall gap (Gap3) are 3.35% and 1.48%, respectively. For the large-scale instances, the gaps are relatively small and acceptable, which further illustrates the effectiveness of the proposed BAP algorithm.

Besides, via the comparison between *Experiment 1* and *Experiment 2*, we can see that the proposed BAP algorithm can both solve many instances, but there are large differences in the Cpu time. The solution time in *Experiment 1* is much faster than that in *Experiment 2*. There are two main reasons. First but also most important is that the problem in *Experiment 2* is complicated than the problem in *Experiment 1*. The BAP algorithm in *Experiment 2* involves a SOP solved by MIP solver. Thus the BAP algorithm will call CPLEX to solve MIP3 model for many times, which will take a lot of solution time. Second is that the BAP algorithm in *Experiment 1* uses bidirectional search to speed up the labeling algorithm, but in *Experiment 1* only uses the forward labeling algorithm.

For a logistics company or organization, reducing operating cost is the most important, though environmental issues deserve everyone's attention. The research in this paper gives the scheduling plan from two perspectives, which can give decision makers more choices when making the scheduling plan.

TABLE 9. Management decision analysis.

Instance scale	Cost comparison			Carbon comparison		
	<i>Experiment 1</i>	<i>Experiment 2</i>	Ratio (%)	<i>Experiment 1</i>	<i>Experiment 2</i>	Ratio (%)
10	173.34	175.54	1.27	320.10	175.93	-45.04
25	356.36	361.97	1.57	658.05	368.84	-43.95
50	681.38	724.52	6.33	1270.36	733.36	-42.27

6.4. Management decision analysis

In this paper, we design a BAP algorithm to solve the routing and scheduling problem from two different perspectives. A large number of computational and experimental results show the effectiveness of the proposed algorithm. In order to make readers better understand the differences in transportation costs and carbon emissions between the two different perspectives, this section compares the results from different perspectives, and the comparison results are presented in Table 9.

As can be seen from Table 9, from the perspective of environment, carbon emissions can be reduced by 45.04%, 43.95% and 42.27% when the transportation cost is only increased by 1.27%, 1.57% and 6.33%. This result shows that the mathematical model from the perspective of environment can achieve more balanced results than the mathematical model from the perspective of economic cost. Moreover, this conclusion shows that the mathematical models from different perspectives proposed in this paper can better compare the differences of transportation cost and carbon emission, and enable decision makers to make better decisions suitable for the operation of the company.

7. CONCLUSIONS

In this paper, we studied a routing and scheduling problem from economic and environmental perspective. From economic perspective, we aim to optimize the vehicle routing plan to reduce the operating cost, but from environmental perspective, we aim to optimize the vehicle routing and speed decisions to reduce the carbon emissions. This research can provide both a vehicle scheduling plan with the minimal operating cost and an environment-friendly scheduling plan. We formulate the problem as two MIP models with different objectives, and try to use CPLEX solver to solve the MIP model. However, this problem is very challenging, with medium-sized instances already difficult for the MIP solver. In order to solve the studied problem with larger scale instances, we propose an effective BAP algorithm to precisely solve this problem, where the master problem and the pricing sub-problem are solved by a column generation algorithm and a labeling algorithm, respectively. The BAP algorithm relies on efficiently solving the pricing sub-problem. As for the pricing problem with operating cost objective, we design a tailored bidirectional labeling algorithm to solve it. As for the pricing problem with environmental objective, we design a tailored forward labeling algorithm to solve it. Extensive computational results show that the proposed BAP algorithm outperforms a state-of-the-art MIP optimization solver, which highlights the effectiveness and efficiency of the proposed BAP algorithm.

This research can provide two different decision plans under these two different perspectives, and the comparison of the results from the two different perspectives shows that the mathematical model from the perspective of environment can achieve more balanced results than the mathematical model from the perspective of economic cost. The comparison of the results from the two perspectives will be very meaningful and helpful to the decision-makers.

At last, we discuss some future research directions. One direction is to consider traffic information into our model. Indeed, traffic has a huge impact on vehicle speed. The modeling and algorithmic questions in this direction are both challenging. Another direction is consider some uncertainty factors to increase the interest of this research.

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REFERENCES

- [1] T. Bektaş and G. Laporte, The pollution-routing problem. *Transp. Res. Part B: Methodol.* **45** (2011) 1232–1250.
- [2] L. Cai, W. Lv, L. Xiao and Z. Xu, Total carbon emissions minimization in connected and automated vehicle routing problem with speed variables. *Expert Syst. App.* **165** (2021) 113910.
- [3] C.A. Cruz, P. Munari and R. Morabito, A branch-and-price method for the vehicle allocation problem. *Comput. Ind. Eng.* **149** (2020) 106745.
- [4] S. Dabia, S. Ropke, T.V. Woensel and T. De Kok, Branch and price for the time-dependent vehicle routing problem with time windows. *Transp. Sci.* **47** (2013) 380–396.
- [5] S. Dabia, E. Demir and T.V. Woensel, An exact approach for a variant of the pollution-routing problem. *Transp. Sci.* **51** (2017) 607–628.
- [6] E. Demir, T. Bektaş and G. Laporte, An adaptive large neighborhood search heuristic for the pollution-routing problem. *Eur. J. Oper. Res.* **223** (2012) 346–359.
- [7] G. Desaulniers, J. Desrosiers and M.M Solomon, Column Generation. Vol. 5. Springer Science & Business Media (2006).
- [8] S. Erdoğan and E. Miller-Hooks, A green vehicle routing problem. *Transp. Res. Part E: Logistics Transp. Rev.* **48** (2012) 100–114.
- [9] A.M. Fathollahi-Fard, M. Hajiaghahi-Keshteli and R. Tavakkoli-Moghaddam, A bi-objective green home health care routing problem. *J. Cleaner Prod.* **200** (2018) 423–443.
- [10] A.M. Fathollahi-Fard, K. Govindan, M. Hajiaghahi-Keshteli and A. Ahmadi, A green home health care supply chain: new modified simulated annealing algorithms. *J. Cleaner Prod.* **240** (2019) 118200.
- [11] D. Feillet, A tutorial on column generation and branch-and-price for vehicle routing problems. *4or* **8** (2010) 407–424.
- [12] M.W. Friske, L.S. Buriol and E. Camponogara, A branch-and-price algorithm for a compressor scheduling problem. *Comput. Ind. Eng.* **116** (2018) 72–81.
- [13] M. Gendreau, G. Ghiani and E. Guerriero, Time-dependent routing problems: a review. *Comput. Oper. Res.* **64** (2015) 189–197.
- [14] A. Gharaei and F. Jolai, A branch and price approach to the two-agent integrated production and distribution scheduling. *Comput. Ind. Eng.* **136** (2019) 504–515.
- [15] J. Hickman, D. Hassel, R. Joumard, Z. Samaras and S. Sorenson, Methodology for calculating transport emissions and energy consumption (1999).
- [16] O. Jabali, T. Van Woensel and A.G. De Kok, Analysis of travel times and CO₂ emissions in time-dependent vehicle routing. *Prod. Oper. Manage.* **21** (2012) 1060–1074.
- [17] J. Kirkinen, T. Palosuo, K. Holmgren and I. Savolainen, Greenhouse impact due to the use of combustible fuels: life cycle viewpoint and relative radiative forcing commitment. *Environ. Manage.* **42** (2008) 458.
- [18] Y.-J. Kwon, Y.-J. Choi and D.-H. Lee, Heterogeneous fixed fleet vehicle routing considering carbon emission. *Transp. Res. Part D: Transp. Environ.* **23** (2013) 81–89.
- [19] C. Lin, K.L. Choy, G.T.S. Ho, S.H. Chung and H.Y. Lam, Survey of green vehicle routing problem: past and future trends. *Expert Syst. App.* **41** (2014) 1118–1138.
- [20] R. Liu and Z. Jiang, A constraint relaxation-based algorithm for the load-dependent vehicle routing problem with time windows. *Flexible Serv. Manuf. J.* **31** (2019) 331–353.
- [21] Z. Luo, H. Qin, and A. Lim, Branch-and-price-and-cut for the multiple traveling repairman problem with distance constraints. *Eur. J. Oper. Res.* **234** (2014) 49–60.
- [22] H. Luo, M. Dridi and O. Grunder, An ACO-based heuristic approach for a route and speed optimization problem in home health care with synchronized visits and carbon emissions. *Soft Comput.* **25** (2021) 14673–14696.
- [23] M.A. Moktadir, S.M. Ali, R. Rajesh and S.K. Paul, Modeling the interrelationships among barriers to sustainable supply chain management in leather industry. *J. Cleaner Prod.* **181** (2018) 631–651.
- [24] J. Qian and R. Eglese, Finding least fuel emission paths in a network with time-varying speeds. *Networks* **63** (2014) 96–106.
- [25] J. Qian and R. Eglese, Fuel emissions optimization in vehicle routing problems with time-varying speeds. *Eur. J. Oper. Res.* **248** (2016) 840–848.
- [26] M. Reihaneh and A. Ghoniem, A branch-and-price algorithm for a vehicle routing with demand allocation problem. *Eur. J. Oper. Res.* **272** (2) 2019 523–538.
- [27] G. Righini and M. Salani, Symmetry helps: bounded bi-directional dynamic programming for the elementary shortest path problem with resource constraints. *Discrete Optim.* **3** (2006) 255–273.
- [28] Y. Shi, T. Boudouh and O. Grunder, A hybrid genetic algorithm for a home health care routing problem with time window and fuzzy demand. *Expert Syst. App.* **72** (2017) 160–176.
- [29] Y. Shi, T. Boudouh and O. Grunder, A robust optimization for a home health care routing and scheduling problem with consideration of uncertain travel and service times. *Transp. Res. Part E: Logistics Transp. Rev.* **128** (2019) 52–95.
- [30] M.M. Solomon, Algorithms for the vehicle routing and scheduling problems with time window constraints. *Oper. Res.* **35** (1987) 254–265.

- [31] N. Tajik, R. Tavakkoli-Moghaddam, B. Vahdani and S.M. Mousavi, A robust optimization approach for pollution routing problem with pickup and delivery under uncertainty. *J. Manuf. Syst.* **33** (2014) 277–286.
- [32] B.E. Teoh, S.G. Ponnambalam and N. Subramanian, Data driven safe vehicle routing analytics: a differential evolution algorithm to reduce CO₂ emissions and hazardous risks. *Ann. Oper. Res.* **270** (2018) 515–538.
- [33] M. Wang, L. Miao and C. Zhang, A branch-and-price algorithm for a green location routing problem with multi-type charging infrastructure. *Transp. Res. Part E: Logistics Transp. Rev.* **156** (2021) 102529.
- [34] Y. Yu, S. Wang, J. Wang and M. Huang, A branch-and-price algorithm for the heterogeneous fleet green vehicle routing problem with time windows. *Transp. Res. Part B: Methodol.* **122** (2019) 511–527.

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