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# MULTI-OBJECTIVES OPTIMIZATION AND CONVOLUTION FUZZY C-MEANS: CONTROL OF DIABETIC POPULATION DYNAMIC

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**Abstract.** The optimal control models proposed in the literature to control a population of diabetics are all single-objective which limits the identification of alternatives and potential opportunities for different reasons: the minimization of the total does not necessarily imply the minimization of different terms and two patients from two different compartments may not support the same intensity of exercise or the same severity of regime. In this work, we propose a multi-objectives optimal control model to control a population of diabetics taking into account the specificity of each compartment such that each objective function involves a single compartment and a single control. In addition, the Pontryagin's maximum principle results in expansive control that devours all resources because of max-min operators and the control formula is very complex and difficult to assimilate by the diabetologists. In our case, we use a multi-objectives heuristic method, NSGA-II, to estimate the optimal control based on our model. Since the objective functions are conflicting, we obtain the Pareto optimal front formed by the non-dominated solutions and we use fuzzy C-means to determine the important main strategies based on a typical characterization. To limit human intervention, during the control period, we use the convolution operator to reduce hyper-fluctuations using kernels with different size. Several experiments were conducted and the proposed system highlights four feasible control strategies capable of mitigating socio-economic damages for a reasonable budget.

**Mathematics Subject Classification.** 90C20, 90C29, 90C90, 93E20.

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## 1. Introduction

The most recent report from the World Diabetes Federation indicates that the number of diabetics is climbing exponentially and is expected to continue to do so in the upcoming years [14, 27]. Type 2 diabetes remains the most common type of diabetes in the world. A variety of conditions contribute to the development of type 2 diabetes: unhealthy diet, inactive lifestyle and sedentary behaviors. It is extremely important to implement effective strategies (based on a balanced diet and physical exercise) to control this phenomenon with adverse

*Keywords.* Diabetes, optimal control, multi-objectives optimization, Non-dominated Sorting Genetic Algorithm II (NSGA-II), fuzzy C-means, kernel convolution.

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human and material consequences based on simple and personalized mathematical models and using intelligent methods to offer simple recommendations to experts with average mathematical knowledge.

In this work, we propose a new system to control diabetes in order to mitigate the socio-economic damages based on an original multi-objectives dynamic systems model, with two economic functions, and on NSGA-II algorithm and on fuzzy C-means clustering method.

Various mathematical models have been developed to study the population dynamics by considering the different stages of diabetic individuals [2–4, 19, 20, 22, 25]. The basic system of ordinary differential equations was proposed by Derouich *et al.* [8]. Basing on three compartments pre-diabetics, diabetics and diabetics peoples with complications. To control the second and the third compartiment, they introduce a single control in thier system. T. T. Yusuf proposed an optimal control strategy on a mathematical model of the diabetic population divided into two compartments, the first is the diabetic people without complication, and the second is the diabetic people with complication [28].In 2018 Permatasari *et al.* [23] also proposed an optimal control approach to reduce the burden of pre-diabetes, they also took into account people who became disabled due to diabetes. They built a mathematical model on the healthy population, pre-diabetics and diabetics without complications, diabetics with complications and diabetics who became disabled. Another study developed by Daud *et al.* [6] that takes pregnancy into account. They have built a mathematical model on non-pregnant non-diabetic women, non-pregnant diabetic women, pregnant diabetic women and pregnant diabetic women with complication. In 2019 kouidere *et al.* [15] proposed an optimal control model with the discrete-time approach, modeling the evolution from pre-diabetes to diabetes with and without complications and the effect of the living environment. In 2020 Kouidere *et al.* [16] proposed a mathematical model of diabetic population divided into six compartments, pre-diabetics by the effects of genetics and pre-diabetics by the effects of behavioral factors, diabetics without complications, diabetics with treatable complications and diabetics with severe complications. To reduce the number of diabetics the authors proposed to introduce four controls which represent the awareness program by education and media, treatment and psychological support with follow-up. In 2021 Kouidere *et al.* [17] studied a continuous mathematical model of discrete age that describes the dynamics of diabetics, they also studied the suffering of diabetic patients with the negative effect of the socio-environment according to age groups and how it negatively influences diabetics, the authors proposed a strategy of optimal control to protect diabetic patients from the negative impact of a lifestyle that leads them to complications. Almost of the cited works investigate the existence and uniqueness of an optimal control and determined it explicitly using the Pontryagin's maximum principle. Unfortunately, considering only one objective function leads to controls that strongly minimize some compartments to the detriment of others and produces controls that are very costly in human and material resources. Furthermore, this type of solution results in a single strategy that may be appropriate in one context and not in others. In addition, the formulas of the controls provided by Pontryagin's maximum principle are very difficult to assimilate by the diabologists.

To overcome this shortcoming, we propose a new diabetic control system based on: (a) an original multi-objectives dynamic model, (b) NSGA-II method, (c) fuzzy C-means, and (d) a post-treatment phase using m-by-m kernel convolution. Each objective function of our model is focused on only a single compartment and a single control. Since NSGA-II produces a solution front, we use fuzzy C-means to structure the strategy space to assist diabologists in choosing those that meet their requirements; to this end, we characterize the controls basing on several aspects. To correct the fluctuations, intrinsic to any approximation, we use convolution masks of several sizes. Compared to other single-objective control models, the proposed control system has shown an unprecedented ability to produce cost-effective continuous controls capable of mitigating socio-economic damages with a reasonable budget.

This article is organized as follows: the second section presents the proposed multi-objectives diabetic control model. In the third section, we give a short description of NSGA-II. The fourth section presents the proposed features extraction procedure and give, in short, the principle of fuzzy C-means. The fifth section deals with the experimental results obtained with the proposed system using matlab. Finally, we give some conclusions and future extensions of our system.

## 2. Multi-objectives diabete control model

### 2.1. Mono-objective diabete control model

Let's $E(t)$, $D(t)$, and $C(t)$ are the pre-diabetic compartment, the uncomplicated diabetic compartment and the complicated diabetic compartment, respectively. Boutayeb *et al.* [3]. proposed the following controlled model :

$$S(u):\begin{cases}\frac{dE}{dt}=I-(\mu+(\beta_3+\beta_1)(1-u(t)))E(t)\\ \frac{dD}{dt}=\beta_1(1-u(t))E(t)-(\mu+\beta_2(1-u(t)))D(t)+\gamma C(t)\\ \frac{dC}{dt}=\beta_3(1-u(t))E(t)+\beta_2(1-u(t))D(t)-(\mu+\gamma+\nu+\delta)C(t).\end{cases}$$

Where $u$ is a control, $I(t)$ is the incidence of pre-diabetes, $\mu$ is the natural mortality rate, $\beta_1$ is the probability of developing diabetes, $\beta_2$ is the probability of a diabetic person developing a complication, $\beta_3$ is the probability of developing diabetes at stage of complications, $\gamma$ is the rate at which complications are cured, $\nu$ is the rate at which patients with complications become severely disabled, $\delta$ is the mortality rate due to complications.$\Gamma(u) = \int_0^T (D(t)+C(t)+Au^2(t))dt$.

Where $A$ is a positive number. we denote by $U$ the set of the control fo the system $S(u)$.

**Problems:**

(1) For a decision $u$, the quantities $\int_0^T (D(t)+C(t)+Au^2(t))dt$ and $\int_0^T D(t)dt$ may be minimal while $\int_0^T C(t)dt$ is maximal because the terms of $\Gamma$ are contradictory.
(2) Practically, it is very easy to find a constant $A$ that realizes a compromise between the compartments $D$ and $u$; but it is difficult to find a constant that realizes a compromise between $D$, $C$, and $u$ at the same time.
(3) The profiles of patients in compartment $D$ are different from those of patients in compartment $C$; therefore the control of our system requires strategies personalized to different compartments.
(4) Pontryagins principal produces a control that take 1 on the interval $]0,T]$ which yieled to an expansive control that depleting all resources.
(5) The principal of Pontryagin rise to very complicated mathematical formulas that difficult to assimilate and implement by the diabologists.

### 2.2. The proposed multi-objectives diabete control model

To act on $E$ and $D$ with two different tactics, we propose a multi-objectives mathematical model to control the dynamic of Morocco diabetic population.

**Decision functions (controls):**
To control the compartment $C$, we introduce the decision function $u_1 : [0\ T] \longrightarrow [0\ 1]$ which represents the intensity degree of the strategy, at each instant, applied to control the behavior of the compartment $C$.
To control the compartment $D$, we introduce the decision function $u_2 : [0\ T] \longrightarrow [0\ 1]$ which represents the intensity degree of the strategy, at each instant, applied to control the behavior of the compartment $D$.

**Objective functions:**
The first objective function implements the compartment $C$ and the control $u_1$. As the values taken by $C$ are in the order of 1000 and $0 \le u_1 \le 1$, it is necessary to use a parameter, which we note $A$, to make a compromise between these two quantities. Moreover, to ensure the control of $C$ over the whole control period, we consider all the values that this compartment can take, so we obtain the following economic function $\Gamma_1(u_1,u_2) = \int_0^T (C(t)+Au_1^2(t))dt$

A similar analysis leads to the second objective function given by: $\Gamma_1(u_1, u_2) = \int_0^T (D(t) + Au_2^2(t))dt$.

**Constraints:**
To prevent a percentage of $(1 - u_1)\%$ from moving from compartment $E$ to compartment $D$, we multiply $E$ by $(1 - u_1)$. And to prevent a percentage of $(1 - u_2)\%$ from going from compartment $D$ to compartment $C$, we multiply $D$ by $(1 - u_2)$ which leads to the following system of constraints [3]:$S(u_1, u_2)$ :

$$\begin{cases} \frac{dE}{dt} = I - (\mu + (\beta_3 + \beta_1)(1 - u_1(t)))E(t) \\ \frac{dD}{dt} = \beta_1(1 - u_1(t))E(t) - (\mu + \beta_2(1 - u_2(t)))D(t) + \gamma C(t) \\ \frac{dC}{dt} = \beta_3(1 - u(t))E_1(t) + \beta_2(1 - u_2(t))D(t) - (\mu + \gamma + \nu + \delta)C(t). \end{cases}$$

The multi-objectives problem that we propose in this work is given by:

$$(P) : \begin{cases} Min\ \Gamma_1(u_1, u_2) \quad Min\ \Gamma_2(u_1, u_2) \\ \quad \texttt{Subject to} \\ \qquad (u_1, u_2) \in U_{1,2} \\ \qquad (E, D, C)\ solution\ of\ S(u_1, u_2). \end{cases}$$

Where $U_{1,2}$ is the set of the controls of the system $S(u_1, u_2)$. Our model has a practical interest more than a theoretical one because it offers us more flexibility to have exploded the control in two sub-controls, one for the compartment $E$ and the other for the control of the compartment $D$. So we are not going to detail too much in the demonstrations of the results which concern the invariance, the existence of the solution of the system $S(u_1, u_2)$ for each couple of control $(u_1, u_2)$.

## 2.3. Theoretical proprieties of the proposed multi-objectives model

**Invariance :**
By applying the Gronwall inequality to the different equations of the system $S(u_1, u_2)$ [2, 4], we prove that $E(t) > 0$, $D(t) > 0$, $C(t) > 0$, and $N(t) = E(t) + D(t) + C(t) \leq I/\mu$.

**Existence:**
Concerning the existence of the solution of the system $S(u_1, u_2)$ for each pair of control $(u_1, u_2)$.Following the same steps in [2, 4], we show that the function formed by the members on the right of the system $S(u_1, u_2)$ is libchetzian. It should be noted that the only difference between the demonstrations given in [4] and [2] is the formula of the Libchetz constants, in our case these constants implement the two controls $u_1$ and $u_2$ and the results, given in [4] and [2], remain valid.

**Theorem 2.1.** *Consider the following control problem:*

$$\begin{cases} Min\ \Gamma_1(u_1, u_2) \quad Min\ \Gamma_2(u_1, u_2) \\ \quad \textit{Subject to} \\ \qquad (u_1, u_2) \in U_{1,2} \\ \qquad (E, D, C)\ solution\ of\ S(u_1, u_2). \end{cases}$$

*There exists a Pareto front formed by non-dominate optimal controls (that we call optimal Pareto control).*

*Proof.* The existence of optimal Pareto control can be obtained using result by Fleming and Rishel [9] and the following steps:

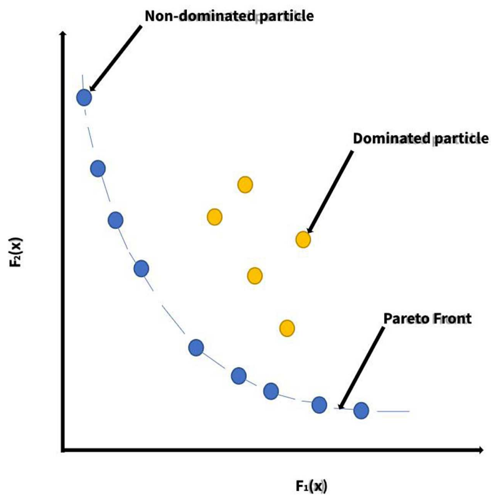



FIGURE 1. Pareto front concept.

(1) In a first step we will check that the control set is non-empty, for this we implement the theorem 1.1.1 of [5], which shows easily the result. Let $E'(t) = F_E(t)$, $D'(t) = F_D(t)$ and $C'(t) = F_C(t)$ with $F_E,\ F_D and F_C$ are the second term functions. Let $u$ be a constant, all parameters are constants and $E(t), D(t), C(t)$ with $t \geq 0$ are continuous, then this shows that $F_E,\ F_D\ and\ F_C$ are continuous. $\frac{\partial F_E}{\partial E}$, $\frac{\partial F_D}{\partial D}$ and $\frac{\partial F_C}{\partial C}$ are continuous, consequently, the existence and the uniqueness of a solution which verifies the initial condition. Hence the control set is non empty.
(2) $U_{1,2}$ is convex;
(3) $\Gamma_1(u_1, u_2) = \int_0^T (C(t) + Au_1^2(t))dt$ and $\Gamma_1(u_1, u_2) = \int_0^T (D(t) + Au_2^2(t))dt$ are convex in considering $(u_1, u_2)$;
(4) The right hand sides of the system equations $S(u_1, u_2)$ are continuous, bounded and can be written as a linear function of $u_1$ and $u_2$ with coefficients depending on time and state;
(5) The integrand in the objective functions, $C(t) + Au_1^2(t)$ and $D(t) + Au_2^2(t)$, are clearly convex on $U_{1,2}$;
(6) We have $C(t) + Au_1^2(t) \geq \alpha_{1,1} + \alpha_{1,2}\|u_1\|^2$ and $D(t) + Au_2^2(t) \geq \alpha_{2,1} + \alpha_{2,2}\|u_2\|^2$ where $\alpha_{1,1} = \inf_{t\in[0,T]} D(t)$ and $\alpha_{2,1} = \inf_{t\in[0,T]} C(t)$ and $\alpha_{1,2} = \alpha_{2,2} = A$.

Then from Fleming and Rishel [9], we conclude that there exists an optimal Pareto controls. □

## 3. MULTI-OBJECTIVES GENETIC ALGORITHM

### 3.1. Pareto front

The Pareto front is the curvature constituted by the not fully ordered cost decisions.

In other words, by moving on the Pareto front, we cannot improve one objective function without degrading the others.

A Pareto front can be local only, that is to say that some points of our front can not dominate the solutions located nearby while they can exercise their dominance on solutions located very far from their positions in the search space.

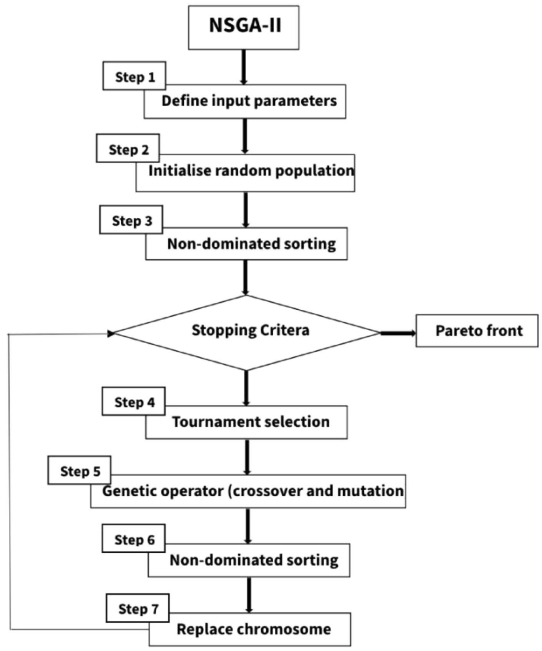



FIGURE 2. NSGA II.

### 3.2. Non-dominated Sorting Genetic Algorithm II (NSGA-II)

To estimate the best controls of our multi-objectives system, we use elitist genetic algorithm denote by NSGA-II [7] and [26]. This method is based on two principles (a) Encouraging individuals with the best fitness values (obtained by ranking strategy) and (b) encouraging individuals able to ensure a certain degree of diversity despite their poor qualities ; this diversity allows to explore promising regions and avoids the convergence towards local fronts of poor quality. Diversity is ensured by controlling elite individuals in the population as the research evolves. The Pareto fraction and the distance measure function are the tools used to manage elitism. The Pareto fraction restricts the size of the population at the front (elite individuals). The distance function, determined by the proximity measure, helps maintain diversity at the front by favoring individuals that are moderately distant from the front. The search is concluded if the margin, an evaluation of the front slip, is sufficiently small. Figure 2 shows the different steps of the NSGA-II.

Among the problems that users may encounter when implementing NSGA-II to solve a well-defined real-world problem are the evaluation of the front sliding and the choice of the proximity function, and the size of the front.

## 4. Pareto controls characterization

The optimal Pareto front contains a very height number of controls (48 pair of controls) that we cannot all test to know their effects on the different compartments. A good solution is to structure this set of controls into subgroups and to represent each of these groups by an average control. To this end, first, we characterize the controls based on four aspects: fluctuation level, strategy cost, geometrical form, and compartments qualities. Lets $m$ be the size of the Pareto front. We estimate these controls on several points, said $d$ from the control interval [0,10 years] (generally $d \gg m$).

**Fluctuation features:**
To extract fluctuation features, first, we remove the linear trend from the controls $u_1$ and $u_2$ using Fast Fourier Transform (FFT) processing [10], we obtain the corrected controls $u_1'$ and $u_2'$. Thus two fluctuation features are extracted in this phase : $\|u_1 - u_1\|$ and $\|u_2 - u_2\|$.

**Cost features:**
The total of the resources mobilized to control the compartment $D$ and $C$ is estimated here by the sum of the control along the control duration. For example, if a patient is able to walk 2 hours per day, and our strategy recommends $u_i(t)\%$ during $t_{i+1} - t_i$, we have consumed $2 * u_i(t)\%$ of resources. Thus two cost features are extracted in this phase: $sum(u_1(t)/t \in [0, T])$ and $sum(u_2(t)/t \in [0, T])$.

**Geometrical features:**
To obtain the geometric features, we interpolate the two controls using polynomials of suitable degrees (in our case, we have opted for a degree of 5). Eleven geometric features are obtained in this phase: The six coefficients of the $u_1$ polynomials and the six coefficients of the $u_2$ polynomials.

**Quality compartments features:**
To group the components of the optimal Pareto controls, it is necessary to estimate the quality of the controls effects on different compartments. The quality of the control is measured by $DIST(X, X_u) = \|X - X_u\|$; where $X$ is non-controlled compartment and $X_u$ is the compartment $X$ controlled by $u$. We obtain tree compartments features : $DIST(E, E_u)$, $DIST(D, D_u)$, and $DIST(C, C_u)$. Note that the larger $DIST$ is, the better the control is.

**Short note on fuzzy C-means:**
At the end of this section, it will be very interesting to give, in short, the principle of the clustering method that we will use to decompose the Pareto front into subgroups.

Fuzzy C-means (FCM) is a clustering method that maps a data set into two or several clusters. This method initiated by Dunn [24] and improved by Bezdek [11] is frequently used in pattern recognition. It is based on the minimization of the cost function defined by:

$$J(\mu, c) = \sum_{i=1}^{N} \sum_{j=1}^{\boldsymbol{K}} (\mu_{ij})^m \|z_i - c_j\|^2$$

where $x_i$ is the $ith$ sample from $IR^d$, $m \in ]1 \ +\infty[$, $\mu_{ij}$ informs us how much the sample $x_i$ is in the group $j$, and $c_j$ is the center of the $jth$ cluster.

Fuzzy partitioning is carried out through an iterative optimization of the objective function, given above, to determine the centers and the membership functions [11].

## 5. Experiments results

In this section, we will estimate the two controls (defined by model (P)) at several points in the interval $[0\ T]$ with $T = 10 years$ using multi-objectives genetic algorithm. To this end, we will follow the steps below:

**Estimation of the dynamic model parameters**. Initially, a set of values of the three compartments is used to estimate the parameters of the basic dynamic model using the least squares method [12]. Several tests can show that the values of these parameters are contained in intervals: $\mu \in [0.01\ 0.03]$, $\nu \in [0.04\ 0.06]$, $\gamma \in [0.078\ 0.85]$, $I = 2000000$, $\beta_1 \in [0.43\ 0.56]$, $\beta_2 \in [0.03\ 0.23]$, and $\beta_3 \in [0.2\ 0.7]$. The three compartments start their evolution as follows $E(0) = 6.66 \times 10^6$, $D(0) = 10.2 \times 10^6$, and $C(0) = 5.5 \times 10^6$.

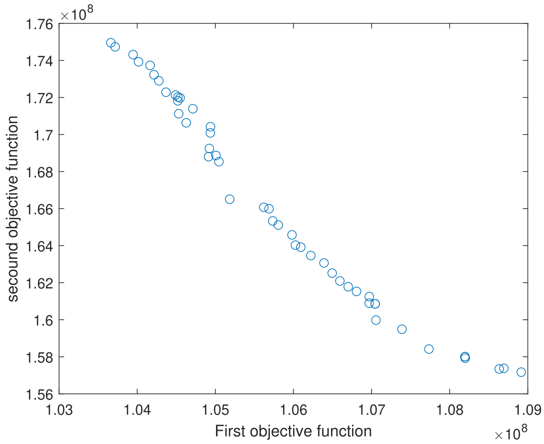



FIGURE 3. Pareto curve formed the first objective function *vs.* the second objective function.

**NSGA-II algorithm configuration**. To approximate the two controls $u_1$ and $u_2$ on a number of points, 100 points for example, of the interval $[0\ T]$, we use the genetic algorithm named NSGA-II configured as follows: creation Function (quasi-random Sobol sequence to generate a well-dispersed initial population), crossover function (Child=parent1+rand*Ratio*(parent2-parent1)), crossover fraction (0.8), maximum generations (60), and mutation function (mutation adaptive feasible).

**Pareto front**. Since the objective functions are conflicting, we obtain the Pareto optimal front formed by the non-dominated controls. The Figure 3 gives the obtained front, the first and the second axes correspond to the values of the first and the second objective function for an optimal control$(u_1^*, u_2^*)$ respectively.

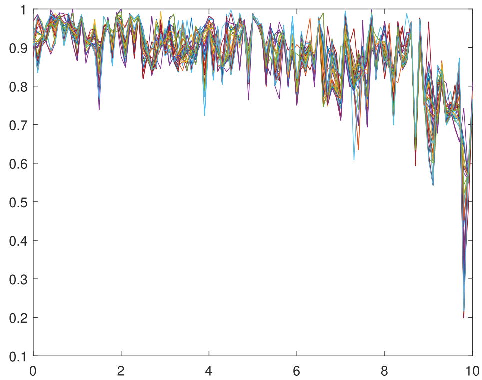
The regularity of the Pareto front proves that the controls produced are acceptable and offer several interesting strategies. These strategies are given by the Figures 4a and 4b. This richness offers the user several strategies to control the different compartments according to his priorities and requirements.

**Decomposition of the Pareto front**. To assist the user in choosing a suitable control, we need to structure the two-dimensional control space generated by NSGA-II. For this purpose, we have presented each pair of controls based on several aspects (see the sub-section entitled Fuzzy C-means to group the Pareto controls into homogeneous controls'): the controls fluctuation, the cost of the controls, the geometrical character of the controls, and the quality of the controls. Then, we determine the number of clusters based on the criterion of choice named Calinski Harabasz [21]. The Figure 5 gives the value of this criterion for different values of clusters (between 2 and 6); this criterion informs us that the best number of clusters is 4.

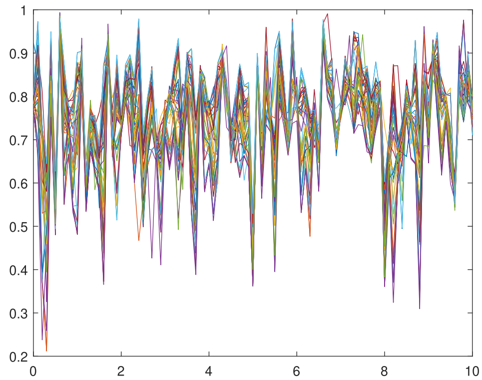
The fuzzy C-means method gives us, then, four centers (control pairs) which represent the means of the controls of each group.

**Post processing of the controls using convolution operator**. To remedy the fluctuation problem, intrinsic to any approximation, we proceed by a post-processing. In this regard, we use different controls with a specified kernel whose elements define how to remove or enhance features of the original data. We tested kernels with different sizes ($3 \times 3$, $4 \times 4$, $5 \times 5$, $6 \times 6$, $7 \times 7$, $8 \times 8$, $9 \times 9$, and $10 \times 10$) and we consider the one that provide the best fluctuation correction ($10 \times 10$ convolution filter).

The Figure 6 represents the controlsobtained by applying convolution of $10 \times 10$ kernel mask to different controls of different clusters. In this figure, each column gives the control $(u_1, u_2)$ of each clusters, we have four clusters and therefore four pairs of controls. We notice in this sense that two acts that follow each other have almost the same intensity, which allows the experts to plan a progressive strategy over a very long time.




(A)




(B)

FIGURE 4. Grapes formed the controls produced by NSGA-II. (a) Grap of controls $u_1$. (b) Grap of controls $u_2$.

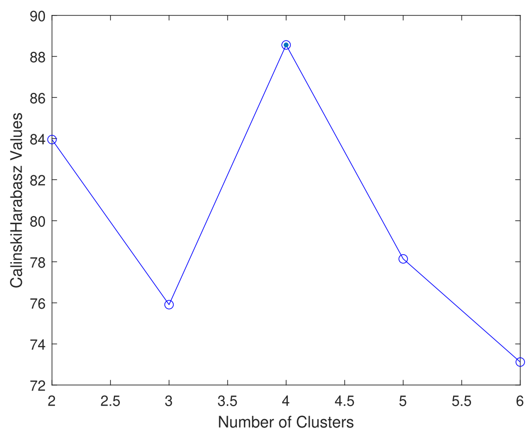



FIGURE 5. Calinski Harabasz criterion value considering the number of clusters.

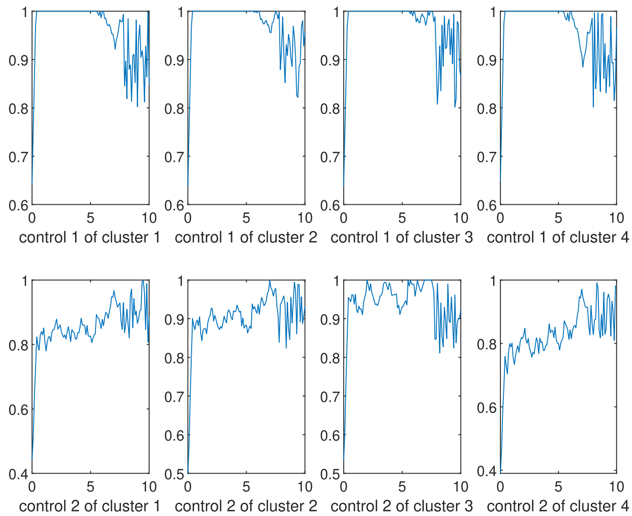



FIGURE 6. Controls obtained by applying convolution of 10X10 kernel mask to different controls of different clusters.

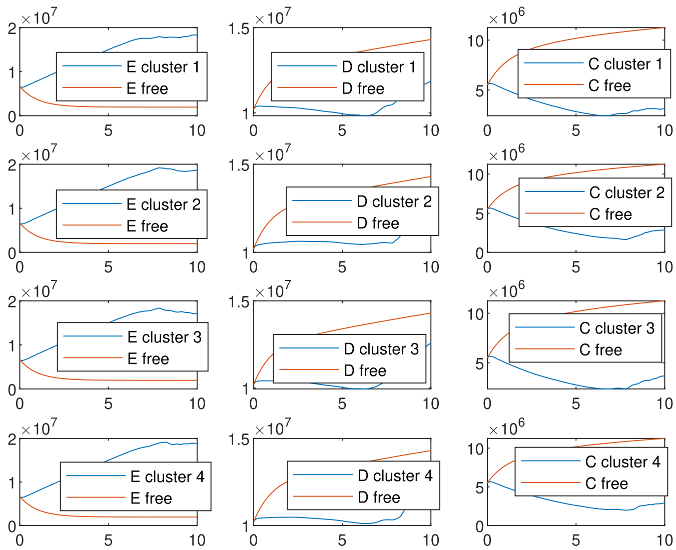



FIGURE 7. Compartments behavior before and after applying the controls of different clusters.

**Controls realization**. The four strategies (which correspond to the four couples of sub-controls), obtained after convolution with $10 \times 10$ kernels, were implemented to control the dynamics of the Morocco diabetic population. The Figure 7 gives the evolution of the three compartments for each pair of controls and this for each cluster. Each line of this figure shows the three compartments obtained by each of the four strategies. We notice, in this sense, that the three compartments did not escape the controls and the evolution obtained by the different strategies are satisfactory. Therefore, the multi-objectives solution, which we propose in this paper, provides personalized recommendations to the different compartments while adopting the right reflexes at the right times without exhausting all the available resources.

Towards the end of this analysis, it will be very useful to know how the approximations offered by the genetic algorithm can be practically translated into each of the 100 points of the interval $[0; 10]$. First of all, the control can be a magical combination of the following three inputs: physical exercise, a well-balanced diet, and a very personalized diet. In this sense, let us say that we are at time $t_i$ of the interval $[0\ T]$, the degree of severity of the inputs must be maintained at the values $u_{1,i}$ and $u_{2,i}$ over the whole interval $[t_i, t_{i+1}]$. What is interesting here is that exercise at an intensity level of $u_{2,i}\%$ may be preferred for patients in the compartment $C$ and a diet with a low glycemic load at an intensity level of $u_{1,i}\%$ for patients in the compartment $D$, or the reverse. In addition, in order, for practitioners, to choose the controls that are right for them, among the four clusters, they have to decide at the beginning which of the two compartments $C$ or $D$ or the available budget they are most interested in.

## 6. Conclusion

The optimal control of population dynamics in different domains is truly multi-criteria and a mono-objective strategy limits the identification of alternatives and potential opportunities. In this paper, we proposed a multi-criteria optimal control strategy, to control the dynamics of the diabetic of Morocco population, based on an original multi-objectives model that takes into account different aspects of the compartments. To overcome the limitations of the expansive controls produced by the Pontryagin's maximum principle, which is difficult to exploit by diabetologists, we used a multi-objectives heuristic method, called NSGA-II, to solve the proposed model. To determine the major strategies important in the Pareto front, we first proposed a vector characterization of the controls, and then, we highlight the different groups using fuzzy C-means. In addition, we used the convolution operator to reduce hyper-fluctuations, which involves human interventions. We were able to highlight four types of strategies capable of mitigating socio-economic damages for a reasonable budget and we left it up to the user of our system to choose the most suitable ones.

The main shortcoming of the proposed system is that when we refine the discretization of the time interval, the NSGA-II algorithm needs a long time to give a good solution. To overcome this problem, we will adopt parallel computing techniques.

In the future, we will propose a multi-objectives fractional dynamic control model for diabetics and we will introduce a hybrid version of the newest heuristic methods to estimate the controls.

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