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THE SUPER-CONNECTIVITY OF DOUBLE GENERALIZED PETERSEN GRAPHS

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Abstract. The super–connectivity of a graph G is the minimum number of vertices whose removal disconnects the graph without isolating a vertex. In this paper, we prove that the super–connectivity of double generalized Petersen graph $DP(n, k)$ is equal to four when $n \geq 4$, $k \geq 1$ and $n \neq 2k$.

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1. Introduction

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The *neighbourhood* of $v \in V(G)$, denoted by $N_G(v)$, is the set of vertices adjacent to v in G. The *degree* of v, denoted by $\deg_G(v)$ is the cardinality of $N_G(v)$. Let $\delta(G)$ denote the *minimum vertex degree* in G. If $\deg_G(v) = r$ for every $v \in V(G)$, then G is called *r-regular*. For any vertex set $S \subseteq V(G)$, let $G[S]$ denote the subgraph induced by S.

A graph G is connected if there exists a path between any two vertices; otherwise G is disconnected. A component of G is a maximal connected subgraph of G. For a non-complete connected graph G, a vertex-cut S is a subset of $V(G)$ such that $G - S$ is disconnected. The connectivity of G, denoted by $\kappa(G)$, is the minimum size of a vertex–cut if G is not a complete graph, and $\kappa(G) = |V(G)| - 1$ if otherwise. Analogously, for a connected graph G, a disconnecting set F is a subset of $E(G)$ such that $G - F$ is disconnected. The edge-connectivity of G, denoted by $\lambda(G)$, is the minimum size of a disconnecting set. The connectivity and the edge–connectivity are two of the most important parameters to measure the reliability of networks. It is well known that $\kappa(G) \leq \lambda(G) \leq \delta(G)$ for any graph G. Harary [16] introduced the conditional connectivity and the conditional edge-connectivity by imposing some conditions on each component of the resulting graph. Motivated by this study, many researchers have studied various types of the conditional connectivity on several graph classes. The super–connectivity and super edge–connectivity are introduced in [2, 10] and have been studied extensively for several graph classes, such as circulant graphs [3], hypercubes [14, 15, 25, 26], split-star networks [17], generalized Petersen graphs [4], Kneser graphs [5, 6], line graphs [24].

A vertex-cut $S \subset V(G)$ is called a super vertex-cut if the resulting graph $G - S$ does not have an isolated vertex, that is, each component of $G - S$ has at least two vertices. The super–connectivity of G, denoted by

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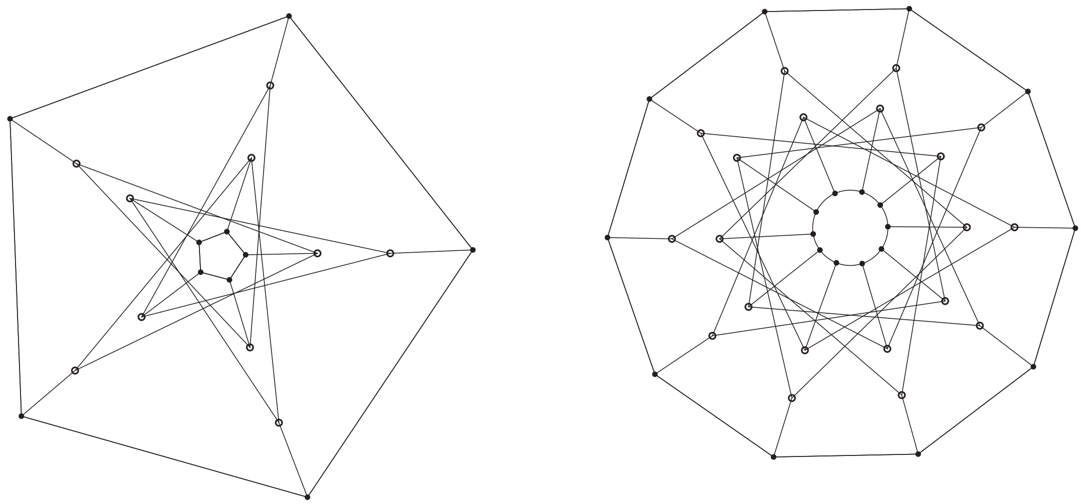

FIGURE 1. The double generalized Petersen graphs $DP(5,2)$ and $DP(8,3)$. Note that the vertices in $\{x_i, y_i \,|\, i \in \mathbb{Z}_n\}$ are represented by black filled circles and the vertices in $\{u_i, v_i \,|\, i \in \mathbb{Z}_n\}$ are represented by empty circles.

$\kappa'(G)$, is the size of a minimum super vertex-cut. If there is no super vertex-cut in G, then we write $\kappa'(G) = +\infty$. Analogously, a disconnecting set $F \subset V(G)$ is called a super edge-cut if the resulting graph $G - F$ does not have an isolated vertex, that is, each component of $G - F$ has at least two vertices. The super edge-connectivity of G, denoted by $\lambda'(G)$, is the size of a minimum super edge-cut. If there is no super edge-cut in G, then we write $\lambda'(G) = +\infty$.

If every minimum vertex-cut (resp. disconnecting set) isolates a vertex, then G is super–connected (resp. super edge-connected). It is clear that if G is super–connected, then $\kappa'(G) > \kappa(G)$, otherwise $\kappa'(G) = \kappa(G)$. Similarly, if G is super edge-connected, then $\lambda'(G) > \lambda(G)$, otherwise $\lambda'(G) = \lambda(G)$. Thus, it is a natural question to ask the super–connectivity (resp. super edge-connectivity) of G if it is known to be super–connected (resp. super edge-connected).

The class of generalized Petersen graphs, introduced by Coxeter [7] in 1950 and named by Watkins [23] in 1969, is a natural generalization of the well-known Petersen graph. Given an integer $n \geq 3$ and $k \in \mathbb{Z}_n - \{0\}$, $2 \leq 2k < n$, the generalized Petersen graph $GP(n,k)$ is defined to have the vertex set $\{u_i, v_i | i \in \mathbb{Z}_n\}$ and the edge set $O \cup I \cup S$, where

$O = \{\{u_i, u_{i+1}\} \,|\, i \in \mathbb{Z}_n\}$ (the outer edges)
$I = \{\{v_i, v_{i+k}\} \,|\, i \in \mathbb{Z}_n\}$ (the inner edges)
$S = \{\{u_i, v_i\} \,|\, i \in \mathbb{Z}_n\}$ (the spokes)

The generalized Petersen graph $GP(n,k)$ has been investigated thoroughly in literature [1, 9, 12, 13, 22]

In [28], Zhou and Feng defined the double generalized Petersen graphs to classify cubic vertex-transitive non-Cayley graphs of order $8p$, for any prime p. Given an integer $n \geq 3$ and $k \in \mathbb{Z}_n - \{0\}$, $2 \leq 2k < n$, the double generalized Petersen graph $DP(n,k)$ is defined to have the vertex set $\{x_i, y_i, u_i, v_i \,|\, i \in \mathbb{Z}_n\}$ and the edge set $O \cup I \cup S$, where

$O = \{\{x_i, x_{i+1}\}, \{y_i, y_{i+1}\} \,|\, i \in \mathbb{Z}_n\}$ (the outer edges)
$I = \{\{u_i, v_{i+k}\}, \{v_i, u_{i+k}\} \,|\, i \in \mathbb{Z}_n\}$ (the inner edges)
$S = \{\{x_i, u_i\}, \{y_i, v_i\} \,|\, i \in \mathbb{Z}_n\}$ (the spokes).

Note that $DP(n,k)$ is defined by modifying the generalized Petersen graph construction such that the subgraph induced by the outer edges is a union of two disjoint n-cycles (See Fig. 1 for $DP(5,2)$ and $DP(8,3)$). Due to its interesting properties, the class of double generalized Petersen graphs has received increasing attention in recent years. In [29], all vertex-transitive graphs and all non-Cayley vertex-transitive graphs are determined among this class. Later, both Sakamoto [20] and Wang [21] proved that all $DP(n, k)$ graphs are Hamiltonian. In [8], the determining number of this class is determined by Das [8]. Recently, the canonical double covers [19] and the Hamilton laceability [18] of this class are investigated.

In this paper we focus our attention on the reliability of double generalized Petersen graphs. We determine the super–connectivity of $DP(n, k)$ when $n \geq 4$, $k \geq 1$ and $n \neq 2k$. We also obtain the super–edge–connectivity of $DP(n, k)$ as a conclusion of the main result.

2. The super–connectiviy of DP(n,k)

Let the vertex set $V(DP(n, k))$ have a partition $V(DP(n, k)) = X \cup Y \cup U \cup V$, where $X = \{x_i \,|\, i \in \mathbb{Z}_n\}$, $Y = \{y_i \,|\, i \in \mathbb{Z}_n\}$, $U = \{u_i \,|\, i \in \mathbb{Z}_n\}$ and $V = \{v_i \,|\, i \in \mathbb{Z}_n\}$.

We first prove the following useful lemma to show that deleting at most three vertices from $X \cup Y$ or from $U \cup V$ does not disconnect the graph.

Lemma 2.1. *Let $\mathcal{G} = DP(n, k)$ for $n \geq 4$, $k \geq 1$ and $n \neq 2k$. Let $S \subset V(\mathcal{G})$ such that $|S| \leq 3$. If S is contained either in $X \cup Y$ or in $U \cup V$, then $\mathcal{G} - S$ is connected.*

Proof. Let $S \subset V(\mathcal{G})$, where $r = |S| \leq 3$. Suppose to the contrary that S is a vertex–cut such that $S \subset X \cup Y$ or $S \subset U \cup V$.

It is easy to check that $\mathcal{G}$ does not have a cut–vertex, that is, G does not have a vertex–cut of size 1. Thus, $r \geq 2$. There are two cases to consider:

Case 1. We first let S be contained in $X \cup Y$. Then there are two subcases to consider.

(i) Let S be either in X or in Y, without loss of generality, say in X. Thus, the cycle induced by Y is intact. Note that each vertex of U has two neighbours in V and each vertex of V has a neighbour in Y in $\mathcal{G} - S$. It is easy to see that each vertex $x_i \in X - S$ is adjacent to $u_i \in U$ in $\mathcal{G} - S$. Hence, the remaining graph $\mathcal{G} - S$ is connected, a contradiction.

(ii) Let $S \cap X \neq \emptyset$ and $S \cap Y \neq \emptyset$. Since r is either 2 or 3 by the assumption, at least one of X and Y contains exactly one vertex from S. Without loss of generality, assume that $S \cap X = \{x_\alpha\}$. The vertices in $X - \{x_\alpha\}$ lies on a path of length $n - 1$ and each $u_i \in U - \{u_\alpha\}$ is adjacent to $x_i \in X - \{x_\alpha\}$ in $\mathcal{G} - S$. Thus, all the vertices of $(X - \{x_\alpha\}) \cup (U - \{u_\alpha\})$ are in the same component of $\mathcal{G} - S$, say C. Note that each vertex of V has at least one neighbour in $U - \{u_\alpha\}$, that is, each vertex of V has a neighbour in C. This means that the vertex set V is contained in C. Note also that each vertex of $Y - S$ has a neighbour in V, that is, each vertex of $Y - S$ has a neighbour in C. Hence, all the vertices of $(X - \{x_\alpha\}) \cup (U - \{u_\alpha\}) \cup V \cup (Y - S)$ are in the same component, C. We now consider the vertex u_α. Since u_α has two neighbours in V, the resulting graph $\mathcal{G} - S$ is connected, a contradiction.

Case 2. We now let S be contained in $U \cup V$. Then there are two cases to consider:

(i) Let S be either in U or in V, without loss of generality, say in U. The cycles induced by X and Y are intact. Since each $v_i \in V$ is adjacent to $y_i \in Y$ in $\mathcal{G} - S$, all the vertices of $V \cup Y$ are in the same component of $\mathcal{G} - S$. Note also that each $u_i \in U - S$ is adjacent to $x_i \in X$ in $\mathcal{G} - S$, thus all the vertices of $(U - S) \cup X$ are in the same component of $\mathcal{G} - S$. Since $|U| = n \geq 4$ and $r \leq 3$ by the assumption, we have $|U - S| \geq 1$. Each $u_i \in U - S$ has two neighbours in V in $\mathcal{G} - S$. That is, $\mathcal{G} - S$ is connected, a contradiction.

(ii) Let $S \cap U \neq \emptyset$ and $S \cap V \neq \emptyset$. Since r is either 2 or 3 by the assumption, at least one of U and V contains exactly one vertex from S. Without loss of generality, say V. This subcase can be proved similarly as in the previous case, due to the fact that each $u_i \in U - S$ has at least one neighbour in $V - S$ in the resulting graph $\mathcal{G} - S$.

□

Theorem 2.2. *Let $n \geq 3$, $k \geq 1$ and $n \neq 2k$. The connectivity of the graph $DP(n,k)$ is three.*

Proof. Let $\mathcal{G} = DP(n,k)$. The connectivity $\kappa(\mathcal{G}) \leq \delta(\mathcal{G}) = 3$. Since $\mathcal{G}$ does not have a cut–vertex, we have $\kappa(\mathcal{G}) \geq 2$. In order to finish the proof of the theorem, it is enough to show that there is no vertex–cut of size two. If $n = 3$, then there is exactly one case to consider, that is, $k = 1$. It is easy to check that the size of a minimum vertex–cut is three for $DP(3,1)$. Thus, in the rest of the proof, we consider the case when $n \geq 4$.

Suppose to the contrary that S is a vertex–cut of $\mathcal{G}$ such that $|S| = 2$. Note that S cannot be contained in $X \cup Y$ or $U \cup V$, by Lemma 2.1. Thus, $|S \cap (X \cup Y)| = 1$ and $|S \cap (U \cup V)| = 1$.

Since $|S \cap (X \cup Y)| = 1$, we have either $|S \cap X| = 1$ or $|S \cap Y| = 1$. Without loss of generality, assume that $|S \cap X| = 1$. We need to consider the following two cases.

Case 1. Let $|U \cap S| = 1$. First note that the cycle induced by Y is intact and each vertex of V is connected to this outer cycle by a spoke. Note also that every remaining vertex of $U - S$ is adjacent to two vertices of V. Thus, all the vertices in $Y \cup V \cup (U - S)$ are in the same component of $\mathcal{G} - S$, say C. Let $X \cap S = \{x_\alpha\}$ and $U \cap S = \{u_\beta\}$. If $\alpha = \beta$, then each vertex $x_i \in X - S$ is adjacent to $u_i \in U - S$. Since the vertices of $U - S$ are in C, the resulting graph $\mathcal{G} - S$ is connected, a contradiction. If $\alpha \neq \beta$, then each vertex $x_i \in X - \{x_\alpha, x_\beta\}$ is adjacent to $u_i \in U - \{u_\alpha, u_\beta\}$. We only need to consider x_β. Note that at least one of $x_{\beta-1}$ and $x_{\beta+1}$ is in $X - S$. Thus, there is a path between x_β and a vertex from C in $\mathcal{G} - S$. That is, $\mathcal{G} - S$ is connected, a contradiction.

Case 2. Let $|V \cap S| = 1$. First note that the cycle induced by Y is intact and each remaining vertex in $V - S$ is connected to this cycle by a spoke. Since $|V \cap S| = 1$, each vertex of U has at least one neighbour in $V - S$. Since each remaining vertex $x_i \in X - S$ is adjacent to $u_i \in U$, it is easy to see that $\mathcal{G} - S$ is connected, a contradiction.

The case when $|S \cap Y| = 1$ can be proved similarly, thus it is omitted. □

In Theorem 2.3, we prove that the super–connectivity of $\mathcal{G} = DP(n,k)$ is four when $n \geq 4$, $k \geq 1$ and $n \neq 2k$. We first show that if S is a vertex set of order three, then $\mathcal{G} - S$ is either connected or contains an isolated vertex. In order to finish the proof, we present a super vertex–cut of order four.

Note that the theorem is not true when $n = 3$. If $S = U$, then the remaining graph $\mathcal{G} - S$ is disconnected and does not have an isolated vertex. Thus, when $n = 3$, there exists a super vertex–cut of order κ, that is, $DP(n,k)$ is not super–κ for $n = 3$.

Theorem 2.3. *Let $n \geq 4$, $k \geq 1$ and $n \neq 2k$. The super–connectivity of the graph $DP(n,k)$ is four.*

Proof. For any graph G, we know that $\kappa'(G) \geq \kappa(G)$. Letting $\mathcal{G} = DP(n,k)$, we have $\kappa'(\mathcal{G}) \geq \kappa(\mathcal{G}) = 3$.

We first suppose that S is a super vertex–cut of order three, that is, $\mathcal{G} - S$ is disconnected and does not have an isolated vertex. By Lemma 2.1, the vertex–cut S can not be contained in $X \cup Y$ or in $U \cup V$. Thus, $S \cap (X \cup Y) \neq \emptyset$ and $S \cap (U \cup V) \neq \emptyset$, that is, S contains at least one inner vertex and one outer vertex. There are two cases to consider:

Case 1. Let $|S \cap (X \cup Y)| = 1$ and $|S \cap (U \cup V)| = 2$. Without loss of generality, assume that $|S \cap X| = 1$ and thus $|S \cap Y| = 0$. Let $S \cap X = \{x_\alpha\}$. Note that the cycle induced by Y is intact and $X - \{x_\alpha\}$ induces a path $\mathcal{P}$ of length $n - 1$. We need to consider the following three subcases:

(*i*) Let $|S \cap U| = 2$ where $S \cap U = \{u_\beta, u_\gamma\}$.
Each vertex $v_i \in V$ is adjacent to $y_i \in Y$ and each vertex $u_i \in U - S$ is adjacent to two vertices in V. Thus, all the vertices in $Y \cup V \cup (U - S)$ are in the same component of $\mathcal{G} - S$, say C. Since $n \geq 4$, there exists a vertex $x_\theta \in X - S$ lying on the path $\mathcal{P}$ such that $\theta \notin \{\alpha, \beta, \gamma\}$. Note that x_θ is adjacent to $u_\theta \in C$. Thus, the remaining graph $\mathcal{G} - S$ is connected, a contradiction.

(ii) Let $|S \cap V| = 2$ where $S \cap V = \{v_\beta, v_\gamma\}$.
Note that each vertex $v_i \in V - \{v_\beta, v_\gamma\}$ is adjacent to $y_i \in Y$. Thus, all the vertices in $Y \cup (V - S)$ are in the same component of $\mathcal{G} - S$, say C.
We first suppose that $N_{\mathcal{G}}(v_\beta) \cap N_{\mathcal{G}}(v_\gamma) = \emptyset$. Each vertex in U has at least one neighbour in $V - S$. Thus, the vertices of U are also in C. Since any vertex $x_i \in X - \{x_\alpha\}$ is adjacent to $u_i \in U$, the remaining graph $\mathcal{G} - S$ is connected, a contradiction.
We then suppose that $N_{\mathcal{G}}(v_\beta) \cap N_{\mathcal{G}}(v_\gamma) = \{u_\theta\}$. If $\alpha = \theta$, then the resulting graph $\mathcal{G} - S$ has an isolated vertex, a contradiction. If $\alpha \neq \theta$, then we know that each vertex $u_i \in U - \{u_\theta\}$ is adjacent to a vertex from $V - S$. Thus, all the vertices in $Y \cup (U - \{u_\theta\}) \cup (V - S)$ are in the same component of $\mathcal{G} - S$, say C. Note that u_θ is adjacent to a vertex of $\mathcal{P}$, namely x_θ. On the other hand, each vertex $x_i \in X - \{x_\alpha, x_\theta\}$ is adjacent to u_i in $\mathcal{G} - S$. Thus, the remaining graph $\mathcal{G} - S$ is connected, a contradiction.

(iii) Let $|S \cap U| = 1$ and $|S \cap V| = 1$, where $S \cap U = \{u_\beta\}$ and $S \cap V = \{v_\gamma\}$.
Each vertex $v_i \in V - S$ is adjacent to $y_i \in Y$. Moreover, each vertex $u_i \in U - \{u_\beta\}$ is adjacent to at least one vertex from $V - \{v_\gamma\}$. Thus, all the vertices in $Y \cup (V - S) \cup (U - S)$ are in the same component of $\mathcal{G} - S$, say C. Since $n \geq 4$, there exists a vertex $x_\theta \in X - S$ lying on the path $\mathcal{P}$ such that $\theta \in \mathbb{Z}_n - \{\alpha, \beta\}$. Note that x_θ is adjacent to $u_\theta \in C$. Thus, the remaining graph $\mathcal{G} - S$ is connected, a contradiction.

The case when $|S \cap X| = 0$ and $|S \cap Y| = 1$ can be proved similarly, thus it is omitted.

Case 2. Let $|S \cap (X \cup Y)| = 2$ and $|S \cap (U \cup V)| = 1$. Without loss of generality, assume that $|S \cap U| = 1$ and $|S \cap V| = 0$. Let $S \cap U = \{u_\gamma\}$. We need to consider the following three subcases:

(i) Let $|S \cap X| = 2$, where $S \cap X = \{x_\alpha, x_\beta\}$. Note that the cycle induced by Y is intact. Since $S \cap V = \emptyset$, the vertices in $Y \cup V$ are all in the same component of $\mathcal{G} - S$, say C. Each vertex of $U - S$ is adjacent to two vertices in V, thus the vertices in $U - S$ are also in C.
If $\gamma \in \{\alpha, \beta\}$, then note that each vertex $x_i \in X - S$ is adjacent to u_i in $\mathcal{G} - S$. Thus, the remaining graph $\mathcal{G} - S$ is connected, a contradiction.
If $\gamma \notin \{\alpha, \beta\}$, then we first consider the vertex x_γ. If $N_{\mathcal{G}}(x_\gamma) \cap X = \{x_\alpha, x_\beta\}$, then x_γ is an isolated vertex in the remaining graph $\mathcal{G} - S$, a contradiction. Otherwise, if $N_{\mathcal{G}}(x_\gamma) \cap X \neq \{x_\alpha, x_\beta\}$, then note that at least one of $x_{\gamma-1}$ and $x_{\gamma+1}$ is in $X - S$. Thus, there is a path between x_γ and a vertex from C in $\mathcal{G} - S$. That is, $x_\gamma \in C$. Note also that each vertex $x_i \in X - \{x_\alpha, x_\beta, x_\gamma\}$ is adjacent to u_i in $\mathcal{G} - S$. Thus, $\mathcal{G} - S$ is connected, a contradiction.

(ii) Let $|S \cap Y| = 2$, where $S \cap Y = \{y_\alpha, y_\beta\}$.
Note that the cycle induced by X is intact. Each vertex of $u_i \in U - S$ is adjacent to $x_i \in X$. Note that each vertex in V has at least one neighbour in $U - S$. Thus, the vertices in $X \cup (U - S) \cup V$ are all in the same component of $\mathcal{G} - S$, say C. Note also that each vertex in $Y - \{y_\alpha, y_\beta\}$ is adjacent to a vertex in V. Thus, the remaining graph $\mathcal{G} - S$ is connected, a contradiction.

(iii) Let $|S \cap X| = 1$ and $|S \cap Y| = 1$, where $S \cap X = \{x_\alpha\}$ and $S \cap Y = \{y_\beta\}$. The set $Y - S$ induces a path of length $n - 1$, say $\mathcal{P}$ and each vertex $v_i \in V - \{v_\beta\}$ is adjacent to a vertex $y_i \in Y - \{y_\beta\}$. Note that each vertex in $U - S$ is adjacent to at least one vertex in $V - \{v_\beta\}$. The vertex v_β has at least one neighbour in $U - S$, say u_θ, which is adjacent to a vertex in $V - \{v_\beta\}$. Thus, the vertices in $(Y - S) \cup V \cup (U - S)$ are in the same component of $\mathcal{G} - S$, say C. Now we only need to consider the vertices in $X - S$. If $\alpha = \gamma$, then each vertex $x_i \in X - S$ is adjacent to $u_i \in U - S$ and thus the remaining graph $\mathcal{G} - S$ is connected, a contradiction. If $\alpha \neq \gamma$, then each vertex $x_i \in X - S$ is adjacent to $u_i \in U - S$ except x_γ. Since $|S \cap X| = 1$, at least one of $x_{\gamma-1}$ and $x_{\gamma+1}$ is in $\mathcal{G} - S$. Thus, $\mathcal{G} - S$ is connected, a contradiction.

The case when $|S \cap U| = 0$ and $|S \cap V| = 1$ can be proved similarly, thus it is omitted.

Hence it is not enough to delete three vertices from $\mathcal{G}$ to disconnect it without isolating a vertex, that is, $\kappa'(\mathcal{G}) > 3$. Consider the endvertices of an edge $e \in E(\mathcal{G})$, say $e = x_0x_1$. The set $N_{\mathcal{G}}(x_0) \cup N_{\mathcal{G}}(x_1) - \{x_0, x_1\} = \{x_2, x_{n-1}, u_0, u_1\}$ forms a super vertex-cut of order four in $\mathcal{G}$. Thus, $\kappa'(\mathcal{G}) \leq 4$ and this finishes the proof. □

By Theorems 2.2 and 2.3, the graph $DP(n, k)$ is obviously super–connected. In 2010, Zhou and Feng [27] proved that the only super–connected but not super-edge-connected graph with minimum degree 3 is the Ladder graph of order 6. Thus, $DP(n,k)$ is super–edge–connected. Since the connectivity and edge-connectivity are equal for a cubic graph, we have $\lambda'(DP(n,k)) > \lambda(DP(n,k)) = 3$. On the other hand, the minimum edge degree of a graph G is defined as $\xi(G) = min\{\xi_G(e) \,|\, e \in E(G)\}$, where $\xi_G(e) = \deg_G(x) + \deg_G(y) - 2$ for $e = xy \in E(G)$. In [11], it is proved that if a connected graph G of order at least 4 is not a star $K_{1,n-1}$, then $\lambda'(G) \leq \xi(G)$. Thus, the corollary below follows from our main result.

Corollary 2.4. *Let $n \geq 4$, $k \geq 1$ and $n \neq 2k$. The super–edge connectivity of the graph $DP(n,k)$ is four.*

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