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ON GENERATING FUZZY PARETO SOLUTIONS IN FULLY FUZZY MULTIOBJECTIVE LINEAR PROGRAMMING *VIA* A COMPROMISE METHOD

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Abstract. In the present paper, it is unified and extended recent contributions on fully fuzzy multi-objective linear programming, and it is proposed a new method for obtaining fuzzy Pareto solutions of a fully fuzzy multiobjective linear programming problem. For its formulation, triangular fuzzy numbers and variables are combined with fuzzy partial orders and fuzzy arithmetic, and no ranking functions are required. By means of solving related crisp multiobjective linear problems, it is provided algorithms to generate fuzzy Pareto solutions; in particular, to generate compromise fuzzy Pareto solutions, what is a novelty in this field.

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1. Introduction

Fuzzy sets are a well known tool to deal with uncertainty on data, in such a way that its combination with linear programming produces a very useful field, called fuzzy linear programming. This later offers researchers an instrument to model decision making in fuzzy environment [9,12,15,20,22,35,36]. In a fuzzy linear programming problem, although it is not necessary that all incomes and outcomes are fuzzy numbers, it is interesting and convenient to propose a general model in which all elements can be fuzzy, what is called fully fuzzy linear programming problem ((FFLP), for short). In a similar manner, there exists a variety of models about the use of equalities, inequalities, and requirements on some properties of the triangular numbers involved in the problem. In this regard, Lofti *et al.* [34] offered a method to obtain the fuzzy optimal solution of (FFLP) with equality constraints with symmetric fuzzy numbers. Kumar *et al.* [31] offered a new method for locating the fuzzy optimal solutions of (FFLP) with equality constraints, using ranking functions (see, for instance, [9], as well as the bibliography there in). To that model and method, Najafi and Edalatpanah [41] made corrections. Khan *et al.* [29] studied (FFLP) problems with inequalities, and they also use ranking functions to compare the objective function values (see also [14, 30]). The methods provided by Lofti *et al.* [34] and Kumar *et al.* [31] where recovered by Ezzati *et al.* [21] to deal with a multiobjective programming problem with equality constraints. Some limitations of the existing method to solve (FFLP) have been pointed out by Liu and Gao [33]. As applications, Chakraborty *et al.* [16] locate fuzzy optimal solutions in fuzzy transportation problems. Recently, Arana-Jiménez [4] have provided a novel method to find fuzzy optimal (nondominated) solutions of

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(FFLP) problems with inequality constraints, with triangular fuzzy numbers and not necessarily symmetric, *via* solving a crisp multiobjective linear programming problem. This method does not require ranking functions, and has been extended to linear programming problem with parameterized fuzzy numbers by Arana-Jiménez and Sánchez-Gil [8]. Reader can find very recent applications and extensions to solve fully fuzzy minimax mixed integer linear programming and maximal covering location problems in Arana-Jiménez *et al.* [6, 10].

As an extension of that commented above, some models require decision maker to address not only one objective, but several objectives at the same time. That is, a model with two or more objectives, which have to be optimized, with conflicts among the objectives, what derives a multiobjectve programming problem. The Pareto optimality in multiobjective programming is a well known concept of a solution to this type of problem, with important applications in optimal control, economics, engineering, decision theory, among others (see [3,7]).

Recently, in a conference paper, Arana-Jiménez [5] has advanced a natural extension of such model to fully fuzzy multiobjective linear programming, with the introduction of the fuzzy Pareto solutions. To this matter, Author has proposed a method to generate fuzzy Pareto solutions by means of related crisp multiobjective programming problems. The proposal does not require ranking functions, and then is different from that given by Bharati *et al.* [13], who proposed the concept of Pareto-optimal solution suggested by Jimenez and Bilbao [27]. Some applications can be found in Data Envelopment Analysis by Mehlawat *et al.* [38]. In that conference paper [5], Author comments that proofs and examples are omitted and will be presented in a paper (extended version). In this work, and as an extension of the proposals advanced by Arana-Jiménez [5], we address the challenge of studying a linear optimization model where all variables and data can be fuzzy numbers, that is, a fully fuzzy multiobjective linear programming problem ((FFMLP), for short). To this matter, and with no ranking functions, we proof results that derive a method to to get a set of fuzzy Pareto solutions. Furthermore, since the decision maker can require a very reduced set of fuzzy Pareto solutions, even only one in some cases, we propose a new method based on a compromise method, as well as corresponding algorithm to get such fuzzy Pareto solution.

The structure is as follows. In next section, we present notations, arithmetic and partial orders on fuzzy numbers. Later, in Section 3, we formulate the fully fuzzy multiobjective linear programming problem, and relate its fuzzy Pareto solutions to Pareto solutions of auxiliary crisp multiobjective programming problems, as advanced in Arana-Jiménez [5]. Then, and based on the previous relations, in Section 4, we provide algorithms to generate fuzzy Pareto solutions; in particular, an algorithm to attain a compromise fuzzy Pareto solution for (FFMLP). To illustrate this latter, in Section 5 we present a numerical application. Finally, we conclude the paper and present future works.

2. Preliminaries on arithmetic and partial order on fuzzy numbers

As usual in the literature, we consider a fuzzy set on $\mathbb{R}^n$ as a mapping $u : \mathbb{R}^n \to [0,1]$. Each fuzzy set u has associated a family of α-level sets, which are described as $[u]^\alpha = \{x \in \mathbb{R}^n \mid u(x) \geq \alpha\}$ for any $\alpha \in (0,1]$, and its support as $supp(u) = \{x \in \mathbb{R}^n \mid u(x) > 0\}$. The 0-level of u is defined as the closure of $supp(u)$, that is, $[u]^0 = cl(supp(u))$. A very useful type of fuzzy set to model parameters and variables are the fuzzy numbers. Following Dubois and Prade [18, 19], a fuzzy set u on $\mathbb{R}$ is said to be a fuzzy number if u is normal, this is there exists $x_0 \in \mathbb{R}$ such that $u(x_0) = 1$, upper semi-continuous function, convex, and (iv) $[u]^0$ is compact. $\mathcal{F}_C$ denotes the family of all fuzzy numbers. The α-levels of a fuzzy number can be represented by means of real interval, that is, $[u]^\alpha = [\underline{u}_\alpha, \overline{u}_\alpha] \in \mathcal{K}_C$, $\underline{u}_\alpha, \overline{u}_\alpha \in \mathbb{R}$, with $\mathcal{K}_C$ is the set of real compact intervals. There exist many families of fuzzy numbers that have been applied to model uncertainty in different situations. some of the most popular are the L-R, triangular, trapezoidal, polygonal, gaussian, quasi-quadric, exponential, and singleton fuzzy numbers. The reader is referred to Báez-Sánchez *et al.* [11], Hanss [26] and Stefanini *et al.* [42] for a complete description of these families and their representation properties. Among them, we point out triangular fuzzy numbers, because of their easy modeling and interpretation (see, for instance, [18,28,29,34,42]), and whose definition is as follows.

Definition 1. Given a fuzzy number $\tilde{a} = (a^-, \hat{a}, a^+)$ whose membership function is

$$\tilde{a}(x) = \begin{cases} \frac{x-a^-}{\hat{a}-a^-}, & \text{if } a^- \leq x \leq \hat{a}, \\ \frac{a^+-x}{a^+-\hat{a}}, & \text{if } \hat{a} < x \leq a^+, \\ 0, & \text{otherwise,} \end{cases}$$

then, it is said to be a triangular fuzzy number (TFN for short).

In terms of α-levels, if we consider a triangular fuzzy number $\tilde{a} = (a^-, \hat{a}, a^+)$, then its α-levels are as follows:

$$[\tilde{a}]^\alpha = [a^- + (\hat{a} - a^-)\alpha, a^+ - (a^+ - \hat{a})\alpha],$$

for all $\alpha \in [0, 1]$. This means that triangular fuzzy number are well determined by three real numbers $a^- \leq \hat{a} \leq a^+$. A unique triangular fuzzy number is characterized by means of the previous formulation of α-levels, such as Goestschel and Voxman [24] established. The set of all TFNs is denoted as $\mathcal{T}_\mathcal{F}$.

Many optimization problems requires conditions about the nonpositivity or nonnegativity on some parameters and variables involeved. To this matter, a fuzzy number $\tilde{a}$ is said to be nonnegative or nonpositive if $\underline{\tilde{a}}_0 \geq 0$ or $\overline{\tilde{a}}_0 \leq 0$, repectively. So, in the case that $\tilde{a}$ is a TFN, then $\tilde{a}$ nonnegative (nonpositive, respectively) if and only if $a^- \geq 0$ ($a^+ \leq 0$, respectively).

Classical arithmetic operations on intervals are well known, and can be referred to Moore [39,40] and Alefeld and Herzberger [1]. A natural extension of these arithmetic operations to fuzzy numbers $u, v \in \mathcal{F}_C$ can be found described in Liu [32] and Ghaznavi *et al.* [23], where the membership function of the operation $u * v$, with $* \in \{+, \cdot\}$, is defined by

$$(u * v)(z) = \sup_{z=x*y} \min\{u(x), v(y)\}. \tag{1}$$

Furthermore, the previous arithmetic operations can be provided by means of their α-levels as follows (see, [23], Thm. 2.6). For any $\alpha \in [0, 1]$:

$$[u + v]^\alpha = [\underline{u}_\alpha + \underline{v}_\alpha,\ \overline{u}_\alpha + \overline{v}_\alpha], \tag{2}$$

$$[\lambda u]^\alpha = [\min\{\lambda\underline{u}_\alpha, \lambda\overline{u}_\alpha\},\ \max\{\lambda\underline{u}_\alpha, \lambda\overline{u}_\alpha\}], \tag{3}$$

$$\begin{aligned} [uv]^\alpha &= [u]^\alpha \times [v]^\alpha \\ &= [\min\ \{\underline{u}_\alpha\underline{v}_\alpha, \overline{u}_\alpha\overline{v}, \underline{u}_\alpha\overline{v}_\alpha, \overline{u}_\alpha\underline{v}_\alpha\},\ \max\ \{\underline{u}_\alpha\underline{v}_\alpha, \overline{u}_\alpha\overline{v}, \underline{u}_\alpha\overline{v}_\alpha, \overline{u}_\alpha\underline{v}_\alpha\}]. \end{aligned} \tag{4}$$

$\mathcal{T}_\mathcal{F}$ is closed under addition and multiplication by scalar. The above operations (2) and (3) are straightforward particularized to triangular fuzzy number as follows. Given $\tilde{a} = (a^-, \hat{a}, a^+)$, $\tilde{b} = (b^-, \hat{b}, b^+) \in \mathcal{T}_\mathcal{F}$ and $\lambda \in \mathbb{R}$, then

$$\tilde{a} + \tilde{b} = \left(a^- + b^-, \hat{a} + \hat{b}, a^+ + b^+\right), \tag{5}$$

$$\lambda\tilde{a} = \begin{cases} (\lambda a^-, \lambda\hat{a}, \lambda a^+) & \text{if } \lambda \geq 0, \\ (\lambda a^+, \lambda\hat{a}, \lambda a^-) & \text{if } \lambda < 0. \end{cases} \tag{6}$$

However, $\mathcal{T}_\mathcal{F}$ is not closed under the multiplication operation (4) (see, for instance, the examples in [2]). To avoid this situation, it is usual to apply a different multiplication operation between TFNs, such as those referenced in Kaufmann and Gupta [28], Kumar *et al.* [31], Khan *et al.* [29] and Arana-Jiménez [4], which can be considered as an approximation to the multiplication given in (1). To this regard, in Arana-Jiménez [4] readers can find a discussion. Then, we provide the following multiplication operation, which is used throughout the text:

$$\begin{aligned} \tilde{a}\tilde{b} &= \left(\left(\tilde{a}\tilde{b}\right)^-, \widehat{\left(\tilde{a}\tilde{b}\right)}, \left(\tilde{a}\tilde{b}\right)^+\right) \\ &= \left(\min\left\{a^-b^-, a^-b^+, a^+b^-, a^+b^+\right\}, \hat{a}\hat{b},\ \max\left\{a^-b^-, a^-b^+, a^+b^-, a^+b^+\right\}\right). \end{aligned} \tag{7}$$

In the case that $\tilde{a}$ or $\tilde{b}$ is a nonnegative TFN, then the previous multiplication is reduced (see, for instance, [28,31]). For instance, if $\tilde{b}$ is nonnegative, then

$$\tilde{a}\tilde{b} = \begin{cases} \left(a^-b^-, \hat{a}\hat{b}, a^+b^+\right), & \text{if } a^- \geq 0, \\ \left(a^-b^+, \hat{a}\hat{b}, a^+b^+\right), & \text{if } a^- < 0,\ a^+ \geq 0, \\ \left(a^-b^+, \hat{a}\hat{b}, a^+b^-\right), & \text{if } a^+ < 0. \end{cases} \tag{8}$$

And if $\tilde{a}$ and $\tilde{b}$ are nonnegative, then

$$\tilde{a}\tilde{b} = \left(a^-b^-, \hat{a}\hat{b}, a^+b^+\right). \tag{9}$$

To compare two fuzzy numbers, there exist several definitions based on interval binary relations (see *e.g.*, [25]) which provides partial orders in fuzzy sets (see, *e.g.*, [43,45]).

Definition 2. Given $u, v \in \mathcal{F}_C$, it is said that:

(i) $\mu \prec \nu$ if and only if $\underline{\mu}_\alpha < \underline{\nu}_\alpha$ and $\overline{\mu}_\alpha < \overline{\nu}_\alpha$, for all $\alpha \in [0,1]$,
(ii) $\mu \leqq \nu$ if and only if $\underline{\mu}_\alpha \leq \underline{\nu}_\alpha$ and $\overline{\mu}_\alpha \leq \overline{\nu}_\alpha$, for all $\alpha \in [0,1]$.

In a minimization process, and through the paper, we refer $(\mu \prec \nu)\mu \leqq \nu$ as a fuzzy number ν is (strictly) dominated by a fuzzy number μ, or equivalently, μ (strictly) dominates ν. In a similar way, we define $\succ$, $\geqq$. In case of TFNs, the previous definition can be really reduced, as recently Arana-Jiménez and Blanco [6] have proved:

Theorem 1. *Given* $\tilde{a} = (a^-, \hat{a}, a^+)$, $\tilde{b} = (b^-, \hat{b}, b^+) \in \mathcal{T}_\mathcal{F}$, *then:*

(i) $\tilde{a} \prec \tilde{b}$ *if and only if* $a^- < b^-$, $\hat{a} < \hat{b}$ *and* $a^+ < b^+$,
(ii) $\tilde{a} \leqq \tilde{b}$ *if and only if* $a^- \leq b^-$, $\hat{a} \leq \hat{b}$ *and* $a^+ \leq b^+$.

The relations $\succ$, $\geqq$ are obtained in a similar manner. Note that to say that $\tilde{a}$ is nonnegative (previously defined) is equivalent to write $\tilde{a} \geqq \tilde{0} = (0,0,0)$.

3. Fully fuzzy multiobjective linear problem

Consider a fuzzy vector $\tilde{z} = (\tilde{z}_1, \ldots, \tilde{z}_p) \in \mathcal{T}_\mathcal{F} \times \cdots \times \mathcal{T}_\mathcal{F} = (\mathcal{T}_\mathcal{F})^p$, with $p \in \mathbb{N}$. For the sake of simplicity, we write $\tilde{z} = (\tilde{z}_i)_{i=1}^p$. In a same manner, $x = (x_1^-, \hat{x}_1, x_1^+, \ldots, x_n^-, \hat{x}_n, x_n^+) \in \mathbb{R}^{3n}$ can be written as $x = (x_j^-, \hat{x}_j, x_j^+)_{j=1}^n$, and so on. Following Arana-Jiménez [5], let us present a formulation of a Fully Fuzzy Multiobjective Linear Problem, as well as a concept for its solutions.

$$\text{(FFMLP) Min/Max } \tilde{z} = (\tilde{z}_i)_{i=1}^p = \left(\sum_{j=1}^n \tilde{c}_{ij}\tilde{x}_j\right)_{i=1}^p \tag{10}$$

$$\text{s.t. } \sum_{j=1}^n \tilde{a}_{rj}\tilde{x}_j \leqq \tilde{b}_r, \qquad r = 1, \ldots, m, \tag{11}$$

$$\tilde{x}_j \geqq \tilde{0}, \qquad j = 1, \ldots, n, \tag{12}$$

where $\tilde{z}$ is the fuzzy-valued vector objective function, each $\tilde{c}_i = (\tilde{c}_1, \ldots, \tilde{c}_n) \in (\mathcal{T}_\mathcal{F})^n$ is the fuzzy vector with the coefficients of the ith component of the fuzzy-valued vector function, $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ is the fuzzy vector with the fuzzy decision variables, and $\tilde{a}_{rj}$ and $\tilde{b}_r$ are the fuzzy technical coefficients. Since we deal with (FFMLP) without any kind of ranking function, it is necessary to define a nondominated fuzzy solution concept, as follows.

Definition 3. Let $\tilde{\bar{x}}$ be a feasible solution for (FFMLP), and $\tilde{\bar{z}}$ the fuzzy-valued objective function at $\tilde{\bar{x}}$. $\tilde{\bar{x}}$ is said to be a fuzzy Pareto solution of (FFMLP) in the minimization (maximization) case if there does not exist a feasible solution $\tilde{x}$, with $\tilde{z}$ the fuzzy-valued objective function at $\tilde{x}$, such that $\tilde{\bar{z}}$ is dominated by $\tilde{z}$, *i.e.*, $\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_j \preceqq (\succeqq) \sum_{j=1}^{n}\tilde{c}_{ij}\tilde{\bar{x}}_j$ for all $i = 1, \ldots, p$, with $\sum_{j=1}^{n}\tilde{c}_{i_0j}\tilde{x}_j \neq \sum_{j=1}^{n}\tilde{c}_{i_0j}\tilde{\bar{x}}_j$ for some $i_0 \in \{1, \ldots, p\}$.

Following the notation of TFNs, we have:

$$
\begin{aligned}
\tilde{z}_i &= \left(z_i^-, \hat{z}_i, z_i^+\right), && i = 1, \ldots, p,\\
\tilde{x}_j &= \left(x_j^-, \hat{x}_j, x_j^+\right), && j = 1, \ldots, n,\\
\tilde{c}_{ij} &= \left(c_{ij}^-, \hat{c}_{ij}, c_{ij}^+\right), && i = 1, \ldots, p, \quad j = 1, \ldots, n,\\
\tilde{a}_{rj} &= \left(a_{rj}^-, \hat{a}_{rj}, a_{rj}^+\right), && r = 1, \ldots, m, \; j = 1, \ldots, n,\\
\tilde{b}_r &= \left(b_r^-, \hat{b}_r, b_r^+\right), && r = 1, \ldots, m.
\end{aligned}
$$

Let us remark that $\tilde{x}_j$ is a nonnegative TFN, and so the multiplication role is given by (8). This means that $\tilde{c}_{ij}\tilde{x}_j$ is computed by one of the three expressions in (8), which only depends on $\tilde{c}_{ij}$. Since the fuzzy coefficients $\tilde{c}_{ij}$ are known, then the expressions of $\tilde{c}_{ij}\tilde{x}_j = ((\tilde{c}_{ij}\tilde{x}_j)^-, \widehat{(\tilde{c}_{ij}\tilde{x}_j)}, (\tilde{c}_{ij}\tilde{x}_j)^+)$ are also known. The same occurs to $\tilde{a}_{rj}\tilde{x}_j$.

Problem (FFMLP) has associated the following crisp multiobjective problem:

$$
\text{(CMLP) Min/Max } f(x) = \left(\sum_{j=1}^{n}(\tilde{c}_{ij}\tilde{x}_j)^-, \sum_{j=1}^{n}\widehat{(\tilde{c}_{ij}\tilde{x}_j)}, \sum_{j=1}^{n}(\tilde{c}_{ij}\tilde{x}_j)^+\right)_{i=1}^{p} \tag{13}
$$

$$
\text{s.t. } \sum_{j=1}^{n}(\tilde{a}_{rj}\tilde{x}_j)^- \leq b_r^-, \qquad r = 1, \ldots, m, \tag{14}
$$

$$
\sum_{j=1}^{n}\widehat{(\tilde{a}_{rj}\tilde{x}_j)} \leq \hat{b}_r, \qquad r = 1, \ldots, m, \tag{15}
$$

$$
\sum_{j=1}^{n}(\tilde{a}_{rj}\tilde{x}_j)^+ \leq b_r^+, \qquad r = 1, \ldots, m, \tag{16}
$$

$$
x_j^- - \hat{x}_j \leq 0, \qquad j = 1, \ldots, n, \tag{17}
$$

$$
\hat{x}_j - x_j^+ \leq 0, \qquad j = 1, \ldots, n, \tag{18}
$$

$$
x_j^- \geq 0, \qquad j = 1, \ldots, n. \tag{19}
$$

$f = (f_{11}, f_{12}, f_{13}, \ldots, f_{p1}, f_{p2}, f_{p3}) : \mathbb{R}^{3n} \to \mathbb{R}^{3p}$ is a vector function, with the variable $x = (x_j^-, \hat{x}_j, x_j^+)_{j=1}^{n} \in \mathbb{R}^{3n}$, with f_{is} linear functions, with $(i, s) \in \{1, \ldots, p\} \times \{1, 2, 3\}$. And since all constraints are represented as linear inequalities on the variable x, then (CMLP) is a multiobjective linear programming problem. Recall that a feasible point $\bar{x} \in \mathbb{R}^{3n}$ of (CMLP) in the minimization (maximization) case is said to be a Pareto solution if there does not exist another feasible point x such that $f_{is}(\bar{x}) \leqq (\geqq) f_{1s}(x)$, for all $(is) \in \{1, \ldots, p\} \times \{1, 2, 3\}$, and $f_{i_0s_0}(\bar{x}) < (>) f_{i_0s_0}(x)$, for some $(i_0, s_0) \in \{1, \ldots, p\} \times \{1, 2, 3\}$. The relationship between the fuzzy Pareto solutions of (FFMLP) and the Pareto solutions of (CMLP) was recently advanced in a conference with no proof by Arana-Jiménez [5], and following we present such result and we prove it.

Theorem 2. *$\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ with $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+) \in \mathcal{T}_{\mathcal{F}}$, $j = 1, \ldots, n$, is a fuzzy Pareto solution of (FFMLP) in the minimization (maximization) case if and only if $x = (x_1^-, \hat{x}_1, x_1^+, \ldots, x_n^-, \hat{x}_n, x_n^+) \in \mathbb{R}^{3n}$ is a Pareto solution of (CMLP) in the minimization (maximization) case.*

Proof. Firstly, let us prove that $x = (x_1^-, \hat{x}_1, x_1^+, \ldots, x_n^-, \hat{x}_n, x_n^+) \in \mathbb{R}^{3n}$ is a feasible solution for (CMLP) if and only if $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ with $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+) \in \mathcal{F}_C$, $j = 1, \ldots, n$, is a feasible solution for (FFMLP). To this purpose, if $x = (x_1^-, \hat{x}_1, x_1^+, \ldots, x_n^-, \hat{x}_n, x_n^+) \in \mathbb{R}^{3n}$ is a feasible solution for (CMLP), then the conditions (17)–(19) are held. These conditions are equivalent to $0 \leq x_j^- \leq \hat{x}_j \leq x_j^+$, and in consequence, they are equivalent to state that $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+)$ is a nonnegative fuzzy triangular number, for all $j = 1, \ldots, n$. Furthermore, by the direct application Theorem 1, it follows that the remaining feasibility conditions on x, (14)–(16), are equivalent to the feasibility conditions (11) on $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ in (FFMLP). Therefore, it is derived that x is a feasible solution for (CMLP) if and only if $\tilde{x}$ is a feasible solution for (FFMLP).

Now, let us consider only the minimization case, and so the related definitions of Pareto solutions. Let us suppose that x is a Pareto solution of (CMLP), and, following, we prove that the feasible solution $\tilde{x}$ is a fuzzy Pareto solution of (FFMLP). To this end, suppose the contrary, that is, there exists a feasible solution $\tilde{y} = (\tilde{y}_1, \ldots, \tilde{y}_n)$ for (FFMLP), with $\tilde{y}_j = (y_j^-, \hat{y}_j, y_j^+)$, $j = 1, \ldots, n$, such that

$$\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{y}_j \leqq \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}, \quad \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{y}_j \neq \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_j, \quad i = 1, \ldots, p. \tag{20}$$

By Theorem 1, condition (20) is equivalent to

$$\sum_{j=1}^{n} (\tilde{c}_{ij}\tilde{y}_j)^- \leq \sum_{j=1}^{n} (\tilde{c}_{ij}\tilde{x}_j)^-, \ \sum_{j=1}^{n} \widehat{(\tilde{c}_{ij}\tilde{y}_j)} \leq \sum_{j=1}^{n} \widehat{(\tilde{c}_{ij}\tilde{x}_j)}, \ \sum_{j=1}^{n} (\tilde{c}_{ij}\tilde{y}_j)^+ \leq \sum_{j=1}^{n} (\tilde{c}_{ij}\tilde{x}_j)^+, \tag{21}$$

for all $i = 1, \ldots, p$, with at least one strict inequality. Since $\tilde{y}$ is feasible for (FFMLP), it follows that $y = (y_1^-, \hat{y}_1, y_1^+, \ldots, y_n^-, \hat{y}_n, y_n^+) \in \mathbb{R}^{3n}$ is feasible for (CMLP). But by (21), x is not a Pareto solution of (CMLP), what is a contradiction. Therefore, it follows that if x is a Pareto solution of (CMLP), then $\tilde{x}$ is a fuzzy Pareto solution of (FFMLP). Conversely, in a similar manner as before, let us suppose that $\tilde{x}$ is a fuzzy Pareto solution of (FFMLP), and let us prove that x is a Pareto solution of (CMLP). Since $\tilde{x}$ is feasible for (FFMLP), it follows that x is feasible for (CMLP). Let us suppose that x is not a Pareto solution of (CMLP). This means that there exists a feasible solution $y = (y_1^-, \hat{y}_1, y_1^+, \ldots, y_n^-, \hat{y}_n, y_n^+) \in \mathbb{R}^{3n}$ for (CMLP) such that $f_{is}(y) \leq f_{is}(x)$, for $(i, s) \in \{1, \ldots, p\} \times \{1, 2, 3\}$, with at least one strict inequality, what implies (21), with $\tilde{y} = (\tilde{y}_1, \ldots, \tilde{y}_n)$ for (FFLP), with $\tilde{y}_j = (y_j^-, \hat{y}_j, y_j^+)$, $j = 1, \ldots, n$. And (21) is equivalent to (20), which means that $\tilde{x}$ is a fuzzy Pareto solution of (FFMLP), what is a contradiction with our initial assumptions. Therefore, it is proved that if $\tilde{x}$ is a fuzzy Pareto solution of (FFMLP), then x is a Pareto solution of (CMLP). For the maximization case we proceed in a similar way. In consequence, the proof is complete. □

4. A proposal to generate fuzzy Pareto solutions for (FFMLP)

In the literature, we can find several methods to generate Pareto solutions of a multiobjective linear problem (see [3] and the bibliography therein). Most popular methods are based on scalarization, such as the weighted problems. These usually produce a set of solutions, whose extension can be partially controlled by the election of a set of weights. The compromise methods allow us to reduce and orient the election of Pareto solutions.

4.1. Fuzzy Pareto solutions *via* weighted problems

The formulation of a weighted problem can be as follows. Given (CMLP) and $w = (w_{11}, w_{12}, w_{13}, \ldots, w_{p1}, w_{p2}, w_{p3}) \in \mathbb{R}^{3p}$, $w_{is} > 0$, $\sum_{i=1}^{p} \sum_{s=1}^{3} w_{is} = 1$, we define its related weighted problem as

$$(\text{CMLP})_w \ \text{Min/Max} \ \sum_{i=1}^{p} \sum_{s=1}^{3} w_{is} f_{is}(x)$$
$$\text{s.t. (14)–(19).}$$

In Arana-Jiménez [5], it was advanced that we can generate a set of fuzzy Pareto solutions of (FFMLP) by means of optimal solutions of the previous weighted problems. Such result was presented, but no proof was provided. Thus, following, we write the result with a proof.

Theorem 3. *Given* $w = (w_{11}, w_{12}, w_{13}, \ldots, w_{p1}, w_{p2}, w_{p3}) \in \mathbb{R}^{3p}$, *with* $w_{is} > 0$, $\sum_{i=1}^{p} \sum_{s=1}^{3} w_{is} = 1$, *if* $x = (x_j^-, \hat{x}_j, x_j^+)_{j=1}^n \in \mathbb{R}^{3n}$ *is an optimal solution in the minimization (maximization) case of the weighted optimization problem* $(CMLP)_w$, *then* $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ *with* $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+) \in \mathcal{T}_{\mathcal{F}}$, $j = 1, \ldots, n$, *is a fuzzy Pareto solution of (FFMLP) in its minimization (maximization) case.*

Proof. Let us consider the minimization case in both problems. If $x = (x_1^-, \hat{x}_1, x_1^+, \ldots, x_n^-, \hat{x}_n, x_n^+) \in \mathbb{R}^{3n}$ is an optimal solution of the weighted optimization problem $(\text{CMLP})_w$, then, it follows that x is a Pareto solution of (CMLP) (see [3], for instance). Then, by Theorem 2, we have that $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+) \in \mathcal{T}_{\mathcal{F}}$, $j = 1, \ldots, n$, is a fuzzy Pareto solution of (FFMLP). For the maximization case we proceed similarly, and the proof is complete. □

The previous result allows us to outline a method to get fuzzy Pareto solutions for (FFMLP) problem in the minimization (maximization) case of (FFMLP) by means of the solutions of the minimization (maximization) case of the weighed problems $(\text{CMLP})_{w_h}$. Therefore, given $k \in \mathbb{N}$ and a set of weights S_W, it is obtained a set D of fuzzy Pareto solutions for (FFMLP) problem.

4.2. Compromise fuzzy Pareto solution

By now, and thanks to the previous method, we provide a set D of fuzzy Pareto solutions for (FFMLP) to the decision maker. Now, the decision maker can apply additional criterion to the set D to choose some elements if necessary. In the literature on multiobjective optimization, we find some criterion to select 'the best' objective function value among the nondominated set (usually, the images of the Pareto or weakly Pareto solutions). Thanks to Theorem 2, we can perform a similar method for (FFMLP) by means of (MCLP). In this way, and among the different methods to do this, we can use compromise programming to get a compromise solution [46]. In this method, the procedure for obtaining a compromise solution is to minimize a distance between the potential optimal solution and the utopia or ideal score in the criterion space (see also [17, 37, 46]), where distances are defined on R^n. Let us recall that the utopia score is obtained by the optimization (minimization or maximization in (MCLP)) of each component of the objective function, which usually is not attained by any feasible point (see [3]). For further information on compromise solution methods, we refer Marler and Arora [37], who offer a survey of methods to compute Pareto solutions in multiobjective optimization.

On the other hand, let us consider the lexicographic weighted Tchebycheff method. It is considered a min-max method, and not a compromise method, such as reader can verify in [37]. This method depends on a collection of weights. However, the lexicographic weighted Tchebycheff method provides a modification by Tind and Wiecek [44], in which all weights are equal, eliminates the possibility of non-unique solutions and guarantees a Pareto solution (see [44]). Such as Marler and Arora [37] describe, and in summary, in this particular case the method is as follows. First, calculate the utopia objective function value, and then minimize the l_∞ distance between the non-dominated scores and the utopia score for the multiobjective optimization program. Then, include this result as a new constraint, and minimize l_1 distance between the non-dominated scores and the utopia score. In order to not confuse this particular case with the general lexicographic weighted Tchebycheff method, we will refer as Tind-Wiecek lexicographic Tchebycheff method from now on. Observe that this method, in essence, provides a solution that is as close as possible (by means of distances) to the utopia point, what links with the definition of compromise solution. Then, in our opinion, we can refer the solution given by the Tind-Wiecek lexicographic Tchebycheff method as a compromise solution.

The previous steps of the Tind-Wiecek lexicographic Tchebycheff method are determined by means of the following algorithm, depending of the optimization (minimization or maximization) case in (FFMLP), as follows in Algorithms (1 and 2, respectively).

TABLE 1. Algorithm for minimization.

Algorithm 1 (minimization case)	
Step 1	Compute $f_{is}^{ut} = \text{Min}\,\{f_{is}(x) :\ (14)–(19)\}$, for $(i,s) \in \{1,\ldots,p\} \times \{1,2,3\}$
Step 2	Compute $\lambda^* = \text{Min}\,\left\{\lambda : f_{is}(x) - f_{is}^{ut} \leq \lambda, (14)–(19), \forall (i,s) \in \{1,\ldots,p\} \times \{1,2,3\}\right\}$
Step 3	Solve Min $\left\{\sum_{i=1}^{p}\sum_{s=1}^{3} f_{is}(x) : f_{is}(x) - f_{is}^{ut} \leq \lambda^*, (14)–(19), \forall (i,s) \in \{1,\ldots,p\} \times \{1,2,3\}\right\}$ $\rightarrow \bar{x} = (\bar{x}_1^-, \hat{\bar{x}}_1, \bar{x}_1^+, \ldots, \bar{x}_n^-, \hat{\bar{x}}_n, \bar{x}_n^+) \in \mathbb{R}^{3n}$
Step 4	$\tilde{\bar{x}}_j \leftarrow (\bar{x}_j^-, \hat{\bar{x}}_j, \bar{x}_j^+) \in \mathcal{T}_{\mathcal{F}},\ j = 1,\ldots n$ $\tilde{\bar{x}} \leftarrow (\tilde{\bar{x}}_1, \tilde{\bar{x}}_2, \ldots \tilde{\bar{x}}_n) \in (\mathcal{T}_{\mathcal{F}})^n$
Step 5	End

TABLE 2. Algorithm for maximization.

Algorithm 2 (maximization case)	
Step 1	Compute $f_{is}^{ut} = \text{Max}\,\{f_{is}(x) : (14)–(19)\}$, for $(i,s) \in \{1,\ldots,p\} \times \{1,2,3\}$
Step 2	Compute $\lambda^* = \text{Min}\,\left\{\lambda : f_{is}^{ut} - f_{is}(x) \leq \lambda, (14)–(19), \forall (i,s) \in \{1,\ldots,p\} \times \{1,2,3\}\right\}$
Step 3	Solve Max $\left\{\sum_{i=1}^{p}\sum_{s=1}^{3} f_{is}(x) : f_{is}^{ut} - f_{is}(x) \leq \lambda^*, (14)–(19), \forall (i,s) \in \{1,\ldots,p\} \times \{1,2,3\}\right\}$ $\rightarrow \bar{x} = (\bar{x}_1^-, \hat{\bar{x}}_1, \bar{x}_1^+, \ldots, \bar{x}_n^-, \hat{\bar{x}}_n, \bar{x}_n^+) \in \mathbb{R}^{3n}$
Step 4	$\tilde{\bar{x}}_j \leftarrow (\bar{x}_j^-, \hat{\bar{x}}_j, \bar{x}_j^+) \in \mathcal{T}_{\mathcal{F}},\ j = 1,\ldots n$ $\tilde{\bar{x}} \leftarrow (\tilde{\bar{x}}_1, \tilde{\bar{x}}_2, \ldots \tilde{\bar{x}}_n) \in (\mathcal{T}_{\mathcal{F}})^n$
Step 5	End

Thus, we refer the computed output by the previous algorithm as a compromise fuzzy Pareto solution for (FFMLP). To prove that such output is really a fuzzy Pareto solution for (FFMLP), we provide the following result.

Theorem 4. *Consider (FFMLP) in the minimization (maximization) case. If Algotithm 1 (2) in Table 1 (Tab. 2) is applied to (FFMLP) , and $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ with $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+) \in \mathcal{T}_{\mathcal{F}}$, $j = 1, \ldots, n$, is an output of such application, then $\tilde{x}$ is a fuzzy Pareto solution of (FFMLP).*

Proof. If $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ with $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+) \in \mathcal{T}_{\mathcal{F}}$, $j = 1, \ldots, n$, is an output in the application of Algorithm 1 (2), it means that $x = (x_1^-, \hat{x}_1, x_1^+, \ldots, x_n^-, \hat{x}_n, x_n^+) \in \mathbb{R}^{3n}$ is a feasible solution for (CMLP) obtained by the Tind-Wiecek lexicographic Tchebycheff method, and then, by Tind and Wiecek [44], it follows that x is a Pareto solution for (MCLP). Therefore, by Theorem 2, we have that $\tilde{x}_j = (x_j^-, \hat{x}_j, x_j^+) \in \mathcal{T}_{\mathcal{F}}$, $j = 1, \ldots, n$, is a fuzzy Pareto solution of (FFMLP), and the proof is complete. □

5. NUMERICAL APPLICATION

To illustrate the previous algorithm to compute a compromise fuzzy Pareto solution, let us consider the following fully fuzzy multiobjective programming problem, from another used by Khan *et al.* [29, 30], also by Arana-Jiménez [4], where two fuzzy objective functions have been included.

$$\text{(FFMLP1) Max } f(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) = \left(\left(\frac{7}{5}, 4, \frac{43}{7}\right)\tilde{x}_1 + (5, 7, 12)\tilde{x}_2 + \left(\frac{39}{4}, 11, \frac{33}{2}\right)\tilde{x}_3,\right.$$

TABLE 3. Application of Algorithm 2.

Steps	Outputs from Algorithm 2 (maximization case)
Step 1	$(f_{is}^{ut}) = (2.457143, 19.354839, 36.562500, 2.400000, 14.516129, 16.666667)$
Step 2	$\lambda^* = 3.022473$
Step 3	$\bar{x} = (0.0000000, 1.6149387, 2.2223681, 0.0000000, 0.3650752, 0.3650752,$ $0.4000000, 0.4761280, 0.4761280)$
Step 4	$\tilde{\bar{x}}_1 = (0.0000000, 1.6149387, 2.2223681) \in \mathfrak{T}_{\mathcal{F}}$ $\tilde{\bar{x}}_2 = (0.0000000, 0.3650752, 0.3650752) \in \mathfrak{T}_{\mathcal{F}}$ $\tilde{\bar{x}}_3 = (0.4000000, 0.4761280, 0.4761280) \in \mathfrak{T}_{\mathcal{F}}$

$$\begin{aligned}
&(3,4,6)\tilde{x}_1 + \left(\frac{10}{3}, 5, 9\right)\tilde{x}_2 + (4,5,10)\tilde{x}_3\Big) \\
\text{s.t. } &(2,5,8)\tilde{x}_1 + \left(3, \frac{41}{6}, 10\right)\tilde{x}_2 + \left(5, \frac{31}{3}, 18\right)\tilde{x}_3 \preceqq \left(6, \frac{50}{3}, 30\right), \\
&\left(4\frac{32}{3}, 12\right)\tilde{x}_1 + \left(5, \frac{73}{6}, 20\right)\tilde{x}_2 + \left(7, \frac{105}{6}, 30\right)\tilde{x}_3 \preceqq (10, 30, 50), \\
&(3,5,7)\tilde{x}_1 + (5,15,20)\tilde{x}_2 + (5,10,15)\tilde{x}_3 \preceqq \left(2, \frac{145}{6}, 30\right), \\
&\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \succeqq 0.
\end{aligned}$$

We consider that the decision maker is interested in a compromise fuzzy Pareto solution, instead of a set of solutions. Then, firstly it is obtained its related crisp multiobjective problem, where $x = (x_1^-, \hat{x}_1, x_1^+, \ldots, x_3^-, \hat{x}_3, x_3^+)$, and $f = (f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}) : \mathbb{R}^9 \to \mathbb{R}^6$ a vector-valued function, with $f_{11}(x) = \frac{7}{5}x_1^- + 5x_2^- + \frac{39}{4}x_3^-$, $f_{12}(x) = 4\hat{x}_1 + 7\hat{x}_2 + 11\hat{x}_3$, $f_{13}(x) = \frac{43}{7}x_1^+ + 12x_2^+ + \frac{33}{2}x_3^+$, $f_{21}(x) = 3x_1^- + \frac{10}{3}x_2^- + 4x_3^-$, $f_{22}(x) = 4\hat{x}_1 + 5\hat{x}_2 + 5\hat{x}_3$, and $f_{23}(x) = 6x_1^+ + 9x_2^+ + 10x_3^+$. Then, it is applied Algorithm 2, in the maximization case, and then the results are obtained, step by step, given in Table 3.

Note that in Step 1 in Table 3, the obtained vector with utopia scores can be interpreted as two fuzzy utopia scores $(2.457143, 19.354839, 36.562500)$ and $(2.400000, 14.516129, 16.666667)$. By the remaining steps, the fuzzy value of the fuzzy-valued objective function f at $(\tilde{\bar{x}}_1, \tilde{\bar{x}}_2, \tilde{\bar{x}}_3)$ is $(\tilde{\bar{z}}_1, \tilde{\bar{z}}_2) = f(\tilde{\bar{x}}_1, \tilde{\bar{x}}_2, \tilde{\bar{x}}_3)$, with

$$\begin{aligned}
\tilde{\bar{z}}_1 &= (2.457143, 16.343755, 33.540027) \in \mathfrak{T}_{\mathcal{F}}, \\
\tilde{\bar{z}}_2 &= (2.40000, 11.49366, 15.47613) \in \mathfrak{T}_{\mathcal{F}}.
\end{aligned}$$

Note that the two fuzzy utopia scores are less than or equal to (in the fuzzy sense $\preceqq$) the corresponding $\tilde{\bar{z}}_1$ and $\tilde{\bar{z}}_2$. Furthermore, each lower extreme of the two fuzzy utopia scores coincides with the corresponding lower extreme of $\tilde{\bar{z}}_1$ and $\tilde{\bar{z}}_2$, respectively. Then, the compromise fuzzy Pareto solution for the fully fuzzy multiobjective programming problem is given by $(\tilde{\bar{x}}_1, \tilde{\bar{x}}_2, \tilde{\bar{x}}_3)$, and the fuzzy value of the fuzzy-valued objective function is $(\tilde{\bar{z}}_1, \tilde{\bar{z}}_2)$.

The computations have been made in R (see `https://www.r-project.org`), and using the `lpSolve` package for solving Linear Programs.

6. CONCLUSIONS

An equivalence between a (FFMLP) problem and a crisp multiobjective lineal programming problem is established, without loss of information and without ranking functions. As results, methods to obtain fuzzy

Pareto solutions for (FFMLP) has been provided; in particular, a compromise fuzzy Pareto solution is obtained by an algorithm which considers an adaptation of lexicographic weighted Tchebycheff method.

As future works, the techniques presented will be extended to generate fuzzy Pareto solutions in interval and fuzzy fractional programming with applications to economy, among others, as well as to inverse Data Envelopment Analysis with fuzzy data in inputs and outputs.

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