RAIRO-Oper. Res. 56 (2022) 3789–3800
https://doi.org/10.1051/ro/2022169

RAIRO Operations Research
www.rairo-ro.org

FLEXIBLE FRACTIONAL TRANSPORTATION PROBLEM WITH MULTIPLE GOALS: A PENTAGONAL FUZZY CONCEPT

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Abstract. We present the framework of the multiobjective fractional transportation problem in the form of pentagonal fuzzy supply and demand. The ideal transportation model is set up to match the decision makers' preferences in competing for the criteria, and transportation costs, delivery time, degradation, environmental and social concerns are the objectives. We employed flexible fuzzy goal programming to handle the Model's complexity to improve the reasonable compromise. The real-world problem of wind turbine blades is used to validate the superiority and effectiveness of the technique.

Mathematics Subject Classification. 90C32, 46N10.

Received January 15, 2022. Accepted September 24, 2022.

1. Introduction

We are all aware that the transportation problem (TP) is a multiobjective decision-making problem. The TPs are designed to transport numerous items to various locations at a lower cost, shorter transportation time, less production cost, etc. Assume a distribution centre wants to find the best transportation strategy for transporting identical commodities from M sources (also known as origin, supply, or capacity centres) to N locations (also called destination, demand or requirement centres). Furthermore, every source has materials to deliver to various destinations, and all destinations have a predicted requirement for goods to be received from sources. The TP aims to calculate the ideal quantities to transport from source to all destinations while minimizing production costs, transportation costs, and delivery time. When it comes to maximizing the relationship between physical and/or economic factors, fractional programming is beneficial as programming models can better fit real-world situations. Linear fractional programming (LFP) problems are important because many real-world problems are based on relative ratios, such as financial and corporate planning (debt/equity ratios), production planning (inventory/sales, output/employee ratios), and health care and hospital planning (cost/patient, nurse/patient ratios). Several strategies for solving LFP problems have been explored in previous research. The real-world problems represented by fractional functions are frequently met in the following situations: return on investment, current ratio, and risk assets to capital, foreign loans to total loan, residential mortgages to total mortgages, for finance or corporate planning, and in transportation such as actual cost to standard cost, etc. In the managerial domain, the ratio functions profit and cost quality to be maximized are essentially competing;

Keywords. Multiobjective optimization, fractional programming, transportation problem, flexible fuzzy goals, pentagonal fuzzy number.

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such problems are fundamentally multiobjective LFP problems. That is present in the various applications of the operations research field, such as resource allocation, transportation, production, finance, location theory, stochastic process, game theory, etc. The multiobjective fractional problem with multiple conflicting linearities under the given linear constraints is called the multiobjective LFP problem.

2. Literature review

TP is one of the oldest applications of the Linear Programming Problem. TP deals with problems concerning the function's effectiveness when we associate several origins with each possibility of different destinations. TP may also involve the movement of a product from plant to warehouses, warehouses to wholesalers, wholesalers to retailers and retailers to customers. It is a circumstance in which a product or products are to be carried from several sources to multiple sinks to save costs, travel time, and production costs, among other things. Hitchcock [1] created the basic TPs. To solve the TP, Dantzig and Thapa [2] described the simplex approach. Ezekiel and Edeki [3] recently discussed a modified Vogel's approximation technique to tackle the TP. The FTP was initially proposed by Swarup [4], Jain and Saksena [5] studied the time minimizing TP with fractional bottleneck objective function (TMTP-FBOF). Liu [6] investigated the FTPs where fuzzy parameters have represented the cost coefficients and right-hand sides, whereas Bhurjee and Panda [7] developed a novel multiobjective fractional programming problem with objective function parameters and interval constraints. Cetin and Tiryaki [8] developed a multi-level fractional transportation problem (MLFTP) and discussed a fuzzy approach for obtaining a Pareto-optimal compromise solution for this problem. Narayanamoorthy and Anukokila [9] examined the fractional solid TP, in which supply, demands, and conveyance capacities were assumed to be fuzzy variables. Radhakrishnan and Anukokila [10] used a compensating approach to resolve the fuzzy fractional TP. Based on the fuzzy goal programming (FGP) technique, KailashLachhwani [11] presented a modified method for solving multi-level multiobjective LFP problems. Pramanik and Banerjee [12] presented a chance-constrained capacitated multiobjective TP with the fuzzy objectives, and a compromise solution was established. Gupta *et al.* [13] developed an FGP framework for minimizing the negative deviational variables in the multiobjective TP. Examples of fractional programming objectives for TPs are minimizing actual cost/standard cost, actual time/standard time, actual deterioration/standard deterioration, and risk assets/capital. The minimum distance approaches for fuzzy programming and lexicographic GP are used to obtain a compromise solution to the multiobjective fractional transportation problem (MOFTP) with mixed constraints by Sadia *et al.* [14]. Costa [15] developed a weighted sum new method for optimizing the linear fractional objective functions.

Decision-makers (DM) desire a more realistic technique to deal with uncertainty in real-world situations based on the effectiveness of fuzzy sets in real-world applications. As a result, Atanassov [16] proposed a new set known as the intuitionistic fuzzy set (IFS). In decision-making problems [17], intuitionistic FGP has been widely used. Some researchers have also employed IFS to solve various transportation issues [18,19]. IFS consider both the degree of falsity and truth, but it does not consider indeterminacy. As a result, it cannot deal with indeterminacy in the real world. To address the limitations of the fuzzy set and IFS, Smarandache [20] proposed the neutrosophic set (NS) structure for developing a reasonable solution to any real-world situation. The IFS is extended or generalized by NS. It effectively and efficiently portrays real-world problems by considering all components of decision scenarios Abdel-Basset *et al.* [21]. Nafei and Nasseri [22] investigated integer neutrosophic programming. Edalatpanah [23] designed a nonlinear neutrosophic programming framework. Rizk-Allah *et al.* [24] created a neutrosophic-based compromise optimal solution framework for the multiobjective TP. Ye [25] established some basic neutrosophic number operations and a neutrosophic function before building a neutrosophic number linear programming (NNLP) scheme to deal with the neutrosophic numbers. Ahmad [26] recently solved multiobjective optimization using the robust neutrosophic programming approach. Panda and Pal [27] discussed the pentagonal fuzzy number (PFN) properties and arithmetic operations. Das *et al.* [28] discussed the real-life multiobjective fractional programming problem and proposed the ranking approach between two fuzzy triangular numbers. Das [29] described the TP under the single-valued pentagonal neutrosophic numbers and introduced the arithmetic operations. Midya *et al.* [30] presented a multiobjective fixed-charged TP in a rough decision-making framework, and a transformation procedure is modified to convert the non-linear problem into its linear form. Kumar *et al.* [31] discussed a computing procedure for solving the fuzzy Pythagorean TP under three different models. Kane *et al.* [32] proposed a two-step method for solving the TP where the parameters are presented in the non-negative triangular fuzzy numbers with a numerical illustration. Pratihar *et al.* [33] discussed modifying the classical Vogel's approximation method for solving the interval type 2 fuzzy TP. Gadhi *et al.* [34] discussed the fractional multiobjective optimization problem and gave the necessary optimality condition for convexity and the Karush–Kuhn–Tucker multipliers. Kane *et al.* [35] discussed the fully fuzzy TP involving trapezoidal fuzzy numbers for the transportation costs and values of the supply and demand. They proposed a two-step method for solving the fuzzy TP where fuzzy triangular numbers represent the problem's parameters. Joshi *et al.* [36] described the neutrosophic environment's multiobjective linear fractional TP. Pathinathan and Ponnivalavan [37] described the PFN, including addition, subtraction, and arithmetic operations. Chang [38] discussed the mixed binary problem with GP; Ramzannia *et al.* [39] presented flexible fuzzy goals (FFGs) and constraints for multichoice goal programming. Haq *et al.* [40] described how to convert a nonlinear programming problem into a binary goal programming (BGP) problem and solve it using the FFG programming approach.

This article examines an FFG model for the multiobjective fractional TP. The supply and demand of the TP are in the form of PFN with flexible fuzzy type goals. The crisp form of the problem is converted using the defuzzified method to formulate it into the integer Nonlinear Programming Problem. The formulated problem is transformed into a BGP problem, and the solution is produced by utilizing the multichoice GP idea with LINGO software. The numerical example is also examined in terms of its practical use.

3. Mathematical model of MOFTP and pentagonal fuzzy set

This section discussed the MOFTP's general form and Pentagonal fuzzy set. The general model of the MOFTP is given as

Model 1

$$\text{Min } Z_k = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} n_{ij}^{(k)} x_{ij}}{\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}^{(k)} x_{ij}} = \frac{N_k(x)}{D_k(x)}, \quad k = 1,2,\ldots,K$$

$$\text{subject to} \begin{cases} \sum_{i=1}^{I} x_{ij} \geq b_j; \ \ \sum_{j=1}^{J} x_{ij} \leq a_i \\ x_{ij} \geq 0, \ \ j = 1,2,\ldots,J; \ \ i = 1,2,\ldots,I \end{cases}$$

where $n_{ij}^{(k)}$ is the coefficient vector for the numerator and $d_{ij}^{(k)}$ is the coefficient vector for the denominator. a_i, b_j are the available product at ith source and demand at the jth destination.

The constraint ensures the restrictions on the goods delivered due to finite availability, while the demand restrictions ensure that product demand is met at each destination.

(a) Supply and demand: linear pentagonal fuzzy number

Data on some independent factors is rarely accurate in real-life case studies; therefore, there is some uncertainty in the data. As a result, in light of the new information on vagueness, model 1 has been revised.

A fuzzy number can represent the parameters' uncertainty. As a result, after making the following changes to all of the assumptions, Model 1 can be recast as follows:

Model 1a

$$\text{Min } Z_k = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} n_{ij}^{(k)} x_{ij}}{\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}^{(k)} x_{ij}} = \frac{N_k(x)}{D_k(x)}, \quad k = 1,2,\ldots,K$$

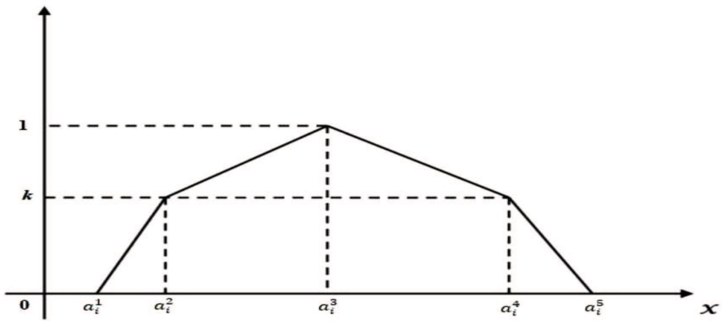

Figure 1. Linear PFN.

$$\text{subject to} \begin{cases} \sum_{i=1}^{I} x_{ij} \geq \tilde{b}_j; \;\; \sum_{j=1}^{J} x_{ij} \leq \tilde{a}_i, \\ x_{ij} \geq 0, \quad j = 1, 2, \ldots, J; \;\; i = 1, 2, \ldots, I. \end{cases}$$

The items' demand and production units (Supply) are linear PFNs. Then, the Model 1a will be as follows:

Model 1b

$$\text{Min}\, Z_k = \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} n_{ij}^{(k)} x_{ij}}{\sum_{i=1}^{I} \sum_{j=1}^{J} d_{ij}^{(k)} x_{ij}} = \frac{N_k(x)}{D_k(x)}, \quad k = 1, 2, \ldots, K$$

$$\text{subject to} \begin{cases} \sum_{i=1}^{I} x_{ij} \geq \left(b_j^1,\ b_j^2,\ b_j^3,\ b_j^4,\ b_j^5;\ k\right), \\ \sum_{j=1}^{J} x_{ij} \leq \left(a_i^1,\ a_i^2,\ a_i^3,\ a_i^4,\ a_i^5;\ k\right), \\ x_{ij} \geq 0, \quad j = 1, 2, \ldots, J; \;\; i = 1, 2, \ldots, I. \end{cases}$$

(b) Defuzzification PFN based on removal of area method for linear pentagonal fuzzy number

Model 1b cannot be solved directly; we use the defuzzification methodology [41] based on the removal of area method for linear PFNs to convert it into an equivalent crisp form. The defuzzification approach was applied for crisp form.

Let $\tilde{a}_i = (a_i^1, a_i^2, a_i^3, a_i^4, a_i^5, k)$ be a linear PFN, as shown in Figure 1. We used the following expression for calculating its magnitude:

$$D(\tilde{a}_i, k) = \frac{1}{10}\left[k\left(a_i^1 - 2a_i^3 + 2a_i^4 + a_i^5\right) + \left(a_i^2 + 4a_i^3 + a_i^4\right)\right]. \tag{1}$$

4. Methodology for MOFTP under linear PFN

In this section, we discussed the solving procedure for the MOFTP problems. Using equation (1), we convert Model 1b into crisp form. The Model 1b will be changed as follows:

Model 2

$$\text{Min}\, Z_k = \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} n_{ij}^{(k)} x_{ij}}{\sum_{i=1}^{I} \sum_{j=1}^{J} d_{ij}^{(k)} x_{ij}} = \frac{N_k(x)}{D_k(x)}, \quad k = 1, 2, \ldots, K$$

$$\text{subject to} \begin{cases} \sum_{i=1}^{I} x_{ij} \geq \frac{1}{10}\left[k\left(b_j^1 - 2b_j^3 + 2b_j^4 + b_j^5\right) + \left(b_j^2 + 4b_j^3 + b_j^4\right)\right], \\ \sum_{j=1}^{J} x_{ij} \leq \frac{1}{10}\left[k\left(a_i^1 - 2a_i^3 + 2a_i^4 + a_i^5\right) + \left(a_i^2 + 4a_i^3 + a_i^4\right)\right], \\ x_{ij} \geq 0, \;\; j = 1, 2, \ldots, J; \;\; i = 1, 2, \ldots, I. \end{cases}$$

A) Objective function as multiple-goal value: flexible FGP

Model 3

$$\text{Min } Z_k = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} n_{ij}^{(k)} x_{ij}}{\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}^{(k)} x_{ij}} = \frac{N_k(x)}{D_k(x)} \le (Z_{k1} \text{ or } \mathrm{Z}_{k2} \text{ or } \dots \text{ or } \mathrm{Z}_{kT}), \quad k = 1, 2, \dots, K$$

$$\text{subject to } \begin{cases} \sum_{i=1}^{I} x_{ij} \ge \frac{1}{10}\left[k\left(b_j^1 - 2b_j^3 + 2b_j^4 + b_j^5\right) + \left(b_j^2 + 4b_j^3 + b_j^4\right)\right], \\ \sum_{j=1}^{J} x_{ij} \le \frac{1}{10}\left[k\left(a_i^1 - 2a_i^3 + 2a_i^4 + a_i^5\right) + \left(a_i^2 + 4a_i^3 + a_i^4\right)\right], \\ x_{ij} \ge 0, \;\; j = 1, 2, \dots, J; \;\; i = 1, 2, \dots, I. \end{cases}$$

The aspiration levels (ALs) are considered for the kth goal, as $Z_{k1}, Z_{k2}, \dots, Z_{kT}$. We minimize deviations from the kth goals.

Model 3a

$$\text{Min} \sum_{k=1}^{K} |Z_k - (Z_{k1} \text{ or } \mathrm{Z}_{k2} \text{ or } \dots \text{ or } \mathrm{Z}_{kT})|, \quad k = 1, 2, \dots, K$$

$$\text{subject to } \begin{cases} \sum_{i=1}^{I} x_{ij} \ge \frac{1}{10}\left[k\left(b_j^1 - 2b_j^3 + 2b_j^4 + b_j^5\right) + \left(b_j^2 + 4b_j^3 + b_j^4\right)\right], \\ \sum_{j=1}^{J} x_{ij} \le \frac{1}{10}\left[k\left(a_i^1 - 2a_i^3 + 2a_i^4 + a_i^5\right) + \left(a_i^2 + 4a_i^3 + a_i^4\right)\right], \\ x_{ij} \ge 0, \;\; j = 1, 2, \dots, J; \;\; i = 1, 2, \dots, I. \end{cases}$$

Therefore,

Model 3b

$$\text{Min} \sum_{k=1}^{K} w_k\left(d_k^- + d_k^+\right)$$

$$\text{subject to } \begin{cases} Z_k - d_k^- + d_k^+ \;\; = \sum_{\mathrm{t}=1}^{\mathrm{T}} Z_{kt} S_{kt}(b), \\ \sum_{i=1}^{I} x_{ij} \ge \frac{1}{10}\left[k\left(b_j^1 - 2b_j^3 + 2b_j^4 + b_j^5\right) + \left(b_j^2 + 4b_j^3 + b_j^4\right)\right], \\ \sum_{j=1}^{J} x_{ij} \le \frac{1}{10}\left[k\left(a_i^1 - 2a_i^3 + 2a_i^4 + a_i^5\right) + \left(a_i^2 + 4a_i^3 + a_i^4\right)\right], \\ x_{ij} \ge 0, \;\; k = 1, 2, \dots, K; \;\; j = 1, 2, \dots, J; \;\; i = 1, 2, \dots, I \end{cases}$$

where $S_{kt}(b)$ is the binary variables term that defines the AL for kth objective; d_k^+ and d_k^- are over and underachievement of the kth goal.

B) The FFG membership function

The nature of objectives is flexible and fuzzy. The objectives' membership function is as follows:

$$\mu(G_j(X) \prec_{ff} Z_{k1} \text{ or } \dots \text{ or } Z_{kS_k}) = \frac{Z_{k1} - Z_k}{P_{k1}} + 1 \text{ or } \dots \text{ or } \frac{Z_{kT} - Z_k}{P_{kT}} + 1, \quad k = 1, \dots, K \tag{2}$$

where $P_{kt}(k = 1, 2, \dots, K), (t = 1, 2, \dots, T)$ is the tolerance quantity for the target value Z_{kt} of the objectives.

C) The crisp goal value for FFG

If λ_k is the minimal quantity of membership degree which fulfils the objective, *i.e.*

$$\mu(Z_k(X) \prec_{ff} Z_{k1} \text{ or } \ldots \text{ or } Z_{kT}) = \frac{Z_{k1} - Z_k}{P_{k1}} + 1 \geq \lambda_k \text{ or } \ldots \text{ or } \frac{Z_{kT} - Z_k}{P_{kT}} + 1 \geq \lambda_k, \quad k = 1, \ldots, K.$$

The FFG will be converted into the crisp objective as follows:

$$Z_k \leq Z_{k1} + P_{k1}(1 - \lambda_k) \text{ or } \ldots \text{ or } Z_{kT} + P_{kT}(1 - \lambda_k), \quad k = 1, \ldots, K. \tag{3}$$

Model 4

$$\text{Min}\, Z_k = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} n_{ij}^{(k)} x_{ij}}{\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}^{(k)} x_{ij}} = \frac{N_k(x)}{D_k(x)} \prec_{ff} Z_{k1} \text{ or } \ldots \text{ or } Z_{kT}; \quad k = 1, 2, \ldots, K$$

subject to

$$\sum_{j=1}^{J} x_{ij} \leq \frac{1}{10}\left[k\left(a_i^1 - 2a_i^3 + 2a_i^4 + a_i^5\right) + \left(a_i^2 + 4a_i^3 + a_i^4\right)\right],$$

$$\sum_{i=1}^{I} x_{ij} \geq \frac{1}{10}\left[k\left(b_j^1 - 2b_j^3 + 2b_j^4 + b_j^5\right) + \left(b_j^2 + 4b_j^3 + b_j^4\right)\right],$$

$$x_{ij} \geq 0, \; j = 1, 2, \ldots, J; \; i = 1, 2, \ldots, I.$$

Using the principle of MCGP, the above MCGP problem with several ALs may be transformed into a BGP problem, and the LINGO software is used to achieve the solution.

Model 5

$$\text{Min}\, Z = \sum_{k=1}^{K} d_j^{+} + \sum_{k=1}^{K} d_j^{-}$$

subject to

$$Z_k - d_k^{+} + d_k^{-} = \sum_{t=1}^{T}(Z_{kt} + P_{kt}(1 - \lambda_k))S_{kt}(b),$$

$$\sum_{j=1}^{J} x_{ij} \leq \frac{1}{10}\left[k\left(a_i^1 - 2a_i^3 + 2a_i^4 + a_i^5\right) + \left(a_i^2 + 4a_i^3 + a_i^4\right)\right],$$

$$\sum_{i=1}^{I} x_{ij} \geq \frac{1}{10}\left[k\left(b_j^1 - 2b_j^3 + 2b_j^4 + b_j^5\right) + \left(b_j^2 + 4b_j^3 + b_j^4\right)\right],$$

$$x_{ij} \geq 0, \; k = 1, \ldots, K; \; j = 1, 2, \ldots, J; \; i = 1, 2, \ldots, I$$

where $S_{kt}(b)$ is the binary variables term that defines the AL for kth objective; d_k^{+} and d_k^{-} are over and underachievement of the kth goal.

5. Numerical illustration

Consider a wind turbine manufacturing business with four different operations in different regions and shift wind turbine blades to four regions. All of the facilities produced the same style of a wind turbine blade. However, wind turbine blades are priced differently due to various circumstances in different regions. The ith source can deliver a_i units of a particular item, whereas the jth destination needs b_j units of the identical item. Thus, when supplying wind turbine blades to other regions, the industrial firm emphasizes three key goals: minimizing the total cost, delivery time, and deterioration. As a result, the goals are to reduce the actual/standard cost, actual/standard deterioration, and actual/standard delivery time involved in moving wind turbine blades from various origin sites to various destinations. PFNs are active for the restricted availability at the source and the minimal demand to be supplied at the destinations to fulfil the requirements.

(1) Total transportation cost: the wind turbine blade's primary goal is to decrease the total transportation cost from source point i to endpoint j. Wind turbine blades have a breadth, a tall height, and a heavy weight. As a result, these blades are tough to transport by rail, road, and other means. Moreover, the costs of transporting the blades are estimated based on the source point and endpoints.

$$\text{Min } Z_1 = \frac{\text{Transportation actual cost } (C_a)}{\text{Transportation standard cost } (C_s)} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} n_{ij}^{(1)} x_{ij}}{\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}^{(1)} x_{ij}}.$$

(2) Total deterioration: the second goal is to reduce overall blade deterioration from ith source to the jth destination.

$$\text{Min } Z_2 = \frac{\text{Actual deteriaration } (D_a)}{\text{Standard deteriaration } (D_s)} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} n_{ij}^{(2)} x_{ij}}{\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}^{(2)} x_{ij}}.$$

(3) Total delivery time: the last goal is to reduce the distribution time of wind turbine blades from ith source point to the jth endpoint. The journey time is carefully calculated due to the dimensions of the wind turbine plate.

$$\text{Min } Z_3 = \frac{\text{Delivery actual time } (T_a)}{\text{Delivery standard time } (T_s)} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} n_{ij}^{(3)} x_{ij}}{\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}^{(3)} x_{ij}}.$$

This work optimizes the TP of the wind turbine blade with the flexible FGP model as the multiple objectives of the fractional TP. As a result, the constraints are assumed to be sharp. Notably, the MOFTP model presented above provides a compromise solution if the total supply available at all sources equals the total demand at all destinations.

Problem and data description: a manufacturer of wind turbine blades [42] sells the product from the ith ($i = 1, 2, 3, 4$) source to the jth ($j = 1, 2, 3, 4$) destination. Due to some intention, the same type of wind turbine blade is marketed at varying prices in different regions.

The standard cost (C_s) and the total actual cost (C_a) are

$$C_s = \begin{bmatrix} 18 & 16 & 19 & 12 \\ 20 & 15 & 15 & 18 \\ 16 & 12 & 15 & 10 \\ 13 & 12 & 16 & 14 \end{bmatrix} \qquad C_a = \begin{bmatrix} 20 & 18 & 18 & 13 \\ 19 & 13 & 16 & 18 \\ 15 & 11 & 17 & 12 \\ 14 & 14 & 16 & 13 \end{bmatrix}.$$

Then, the mathematical formulation of the standard cost (C_s) and the total actual cost (C_a) will be as follows:

$$\begin{aligned} C_a &= 20x_{11} + 18x_{12} + 18x_{13} + 13x_{14} + 19x_{21} + 13x_{22} + 16x_{23} + 18x_{24} \\ &\quad + 15x_{31} + 11x_{32} + 17x_{33} + 12x_{34} + 14x_{41} + 14x_{42} + 16x_{43} + 13x_{44} \\ C_s &= 18x_{11} + 16x_{12} + 19x_{13} + 12x_{14} + 20x_{21} + 15x_{22} + 15x_{23} + 18x_{24} \\ &\quad + 16x_{31} + 12x_{32} + 15x_{33} + 10x_{34} + 13x_{41} + 12x_{42} + 16x_{43} + 14x_{44}. \end{aligned} \tag{4}$$

TABLE 1. The objective goals (Z_{kt}) and the tolerance amount (P_{kt}).

t	Z_{1t}	Z_{2t}	Z_{3t}	P_{1t}	P_{2t}	P_{3t}
1	0.9153488	0.9616571	0.9014567	0.0008979	0.0000419	0.0003765
2	0.9156627	0.9616840	0.9016092	0.0012118	0.0000688	0.0005290
3	0.9158004	0.9617834	0.9014218	0.0013495	0.0001682	0.0003416
4	0.9153675	0.9618001	0.9011599	0.0009166	0.0001849	0.0000797
5	0.9152941	0.9617304	0.9013786	0.0008432	0.0001152	0.0002984

The standard time (T_s) and the total actual time (T_a) are

$$T_s = \begin{bmatrix} 28 & 33 & 32 & 35 \\ 25 & 25 & 32 & 28 \\ 22 & 20 & 25 & 28 \\ 21 & 20 & 26 & 30 \end{bmatrix} \qquad T_a = \begin{bmatrix} 30 & 34 & 34 & 34 \\ 26 & 24 & 31 & 29 \\ 21 & 20 & 26 & 29 \\ 21 & 22 & 25 & 31 \end{bmatrix}.$$

Then, the mathematical formulation of the standard time (T_s) and the total actual time (T_a) will be as follows:

$$\begin{aligned} T_a &= 30x_{11} + 34x_{12} + 34x_{13} + 34x_{14} + 26x_{21} + 24x_{22} + 31x_{23} + 29x_{24} \\ &\quad + 21x_{31} + 20x_{32} + 26x_{33} + 29x_{34} + 21x_{41} + 22x_{42} + 25x_{43} + 31x_{44} \\ T_s &= 28x_{11} + 33x_{12} + 32x_{13} + 35x_{14} + 22x_{21} + 25x_{22} + 32x_{23} + 28x_{24} \\ &\quad + 22x_{31} + 20x_{32} + 25x_{33} + 28x_{34} + 21x_{41} + 20x_{42} + 26x_{43} + 30x_{44}. \end{aligned} \tag{5}$$

The standard deterioration (D_s) and the total actual deterioration (D_a) are

$$D_s = \begin{bmatrix} 38 & 35 & 35 & 32 \\ 38 & 30 & 34 & 36 \\ 40 & 36 & 32 & 37 \\ 33 & 35 & 32 & 32 \end{bmatrix} \qquad D_a = \begin{bmatrix} 40 & 34 & 37 & 28 \\ 38 & 28 & 37 & 40 \\ 42 & 39 & 30 & 41 \\ 29 & 38 & 32 & 32 \end{bmatrix}.$$

Then, the mathematical formulation of the standard deterioration (D_s) and the total actual deterioration (D_a) will be as follows:

$$\begin{aligned} D_a &= 40x_{11} + 34x_{12} + 37x_{13} + 28x_{14} + 38x_{21} + 28x_{22} + 37x_{23} + 40x_{24} \\ &\quad + 42x_{31} + 39x_{32} + 30x_{33} + 41x_{34} + 29x_{41} + 38x_{42} + 32x_{43} + 32x_{44} \\ D_s &= 38x_{11} + 35x_{12} + 35x_{13} + 32x_{14} + 38x_{21} + 30x_{22} + 34x_{23} + 36x_{24} \\ &\quad + 40x_{31} + 36x_{32} + 32x_{33} + 37x_{34} + 33x_{41} + 35x_{42} + 32x_{43} + 32x_{44}. \end{aligned} \tag{6}$$

The goal of each objective (Z_{kt}) and tolerance amount (P_{kt}) is shown in Table 1.

The supplies (a_i) and demands (b_j) are in the form of PFN as:

$$a_i = \begin{bmatrix} (20, 22, 25, 27, 30; k) \\ (25, 28, 30, 32, 35; k) \\ (26, 29, 32, 35, 37; k) \\ (23, 25, 28, 30, 33; k) \end{bmatrix} \qquad b_j = \begin{bmatrix} (6, 8, 10, 12, 15; k) \\ (9, 12, 14, 16, 18; k) \\ (18, 20, 22, 25, 28; k) \\ (12, 15, 18, 20, 24; k) \end{bmatrix}.$$

Procedure: the fractional TP will be solved by simultaneously optimizing the objectives with the given restrictions. The problem can be represented in a mathematical model as follows:

$$\text{Min } Z_1 = \frac{C_a}{C_s};\ \text{Min } Z_2 = \frac{D_a}{D_s};\ \text{Min } Z_3 = \frac{T_a}{T_s}$$

subject to

$$x_{11}+x_{12}+x_{13}+x_{14} \le (20,22,25,27,30;k)$$
$$x_{21}+x_{22}+x_{23}+x_{24} \le (25,28,30,32,35;k)$$
$$x_{31}+x_{32}+x_{33}+x_{34} \le (26,29,32,35,37;k)$$
$$x_{41}+x_{42}+x_{43}+x_{44} \le (23,25,28,30,33;k)$$
$$x_{11}+x_{21}+x_{31}+x_{41} \ge (6,8,10,12,15;k)$$
$$x_{12}+x_{22}+x_{32}+x_{42} \ge (9,12,14,16,18;k)$$
$$x_{13}+x_{23}+x_{33}+x_{43} \ge (18,20,22,25,28;k)$$
$$x_{14}+x_{24}+x_{34}+x_{44} \ge (12,15,18,20,24;k)$$
$$(x_{ij} \ge 0) \in \text{integer} \ \ \forall j=1,2,3,4;\ \ i=1,2,3,4.$$

Using equation (1), the problem will be converted into crisp form. Then, the problem will be

$$\text{Min } Z_1 = \frac{C_a}{C_s};\ \text{Min } Z_2 = \frac{D_a}{D_s};\ \text{Min } Z_3 = \frac{T_a}{T_s}$$

subject to

$$x_{11}+x_{12}+x_{13}+x_{14} \le {}^{[54k+149]}/_{10}; \quad x_{21}+x_{22}+x_{23}+x_{24} \le {}^{[64k+180]}/_{10}$$
$$x_{31}+x_{32}+x_{33}+x_{34} \le {}^{[69k+192]}/_{10}; \quad x_{41}+x_{42}+x_{43}+x_{44} \le {}^{[60k+167]}/_{10}$$
$$x_{11}+x_{21}+x_{31}+x_{41} \ge {}^{[25k+60]}/_{10}; \quad x_{12}+x_{22}+x_{32}+x_{42} \ge {}^{[31k+84]}/_{10}$$
$$x_{13}+x_{23}+x_{33}+x_{43} \ge {}^{[52k+133]}/_{10}; \quad x_{14}+x_{24}+x_{34}+x_{44} \ge {}^{[40k+107]}/_{10}$$
$$k \in [0,1];\ (x_{ij} \ge 0) \in \text{integer} \ \ \forall j=1,2,3,4;\ \ i=1,2,3,4.$$

We have switched to BGP using the FFG membership function. Then, the problem will be represented as:

$$\text{Minimize } Z = \sum_{k=1}^{3} d_j^+ + \sum_{k=1}^{3} d_j^-$$

subject to

$$\frac{C_a}{C_s} - d_1^+ + d_1^- = \sum_{l=1}^{T} (Z_{1t} + P_{1t}(1-\lambda_1))b_{1t};$$
$$\frac{D_a}{D_s} - d_2^+ + d_2^- = \sum_{l=1}^{T} (Z_{2t} + P_{2t}(1-\lambda_2))b_{2t};$$
$$\frac{T_a}{T_s} - d_3^+ + d_3^- = \sum_{l=1}^{T} (Z_{3t} + P_{3t}(1-\lambda_3))b_{3t};$$
$$x_{11}+x_{12}+x_{13}+x_{14} \le {}^{[54k+149]}/_{10}; \quad x_{21}+x_{22}+x_{23}+x_{24} \le {}^{[64k+180]}/_{10}$$
$$x_{31}+x_{32}+x_{33}+x_{34} \le {}^{[69k+192]}/_{10}; \quad x_{41}+x_{42}+x_{43}+x_{44} \le {}^{[60k+167]}/_{10}$$
$$x_{11}+x_{21}+x_{31}+x_{41} \ge {}^{[25k+60]}/_{10}; \quad x_{12}+x_{22}+x_{32}+x_{42} \ge {}^{[31k+84]}/_{10}$$
$$x_{13}+x_{23}+x_{33}+x_{43} \ge {}^{[52k+133]}/_{10}; \quad x_{14}+x_{24}+x_{34}+x_{44} \ge {}^{[40k+107]}/_{10}$$
$$k \in [0,1];\ (x_{ij} \ge 0) \in \text{integer} \ \ \forall j=1,2,3,4;\ \ i=1,2,3,4$$
$$b_{kt} \text{ is binary number and } \sum_{t=1}^{T} b_{kt} = 1, \quad k=1,2,3.$$

TABLE 2. Compromise solution.

Techniques	Objectives	Decision variables
FGP	$Z_1 = 1.0196$; $Z_2 = 1.0320$; $Z_3 = 1.0248$	$x_{12} = 10,\ x_{13} = 5,\ x_{14} = 10,\ x_{21} = 8,\ x_{22} = 6,\ x_{23} = 1,$ $x_{24} = 1,\ x_{32} = 7,\ x_{34} = 25,\ x_{41} = 1,\ x_{43} = 26,\ x_{44} = 1$ $x_{11} = x_{31} = x_{33} = x_{42} = 0,$
GP	$Z_1 = 1.0230$; $Z_2 = 1.0085$; $Z_3 = 0.9997$	$x_{11} = 5,\ x_{13} = 4,\ x_{14} = 16,\ x_{21} = 8,\ x_{22} = 6,\ x_{23} = 11,$ $x_{24} = 5,\ x_{31} = 8,\ x_{32} = 7,\ x_{33} = 17,\ x_{41} = 1,\ x_{42} = 11,$ $x_{44} = 16,\ x_{12} = x_{34} = x_{43} = 0,$
Intuitionistic FGP	$Z_1 = 0.9875$; $Z_2 = 0.9748$; $Z_3 = 0.9842$	$x_{14} = 25,\ x_{21} = 21,\ x_{22} = 7,\ x_{24} = 4,$ $x_{31} = 20,\ x_{33} = 4,\ x_{34} = 8,\ x_{43} = 28,$ $x_{11} = x_{12} = x_{13} = x_{23} = x_{32} = x_{41} = x_{42} = x_{44} = 0$
Neutrosophic GP	$Z_1 = 0.9789$; $Z_2 = 0.9874$; $Z_3 = 0.9601$	$x_{13} = 3,\ x_{14} = 22,\ x_{21} = 4,\ x_{22} = 24, x_{23} = 2,$ $x_{31} = 18,\ x_{33} = 14,\ x_{43} = 13,\ x_{44} = 15$ $x_{11} = x_{12} = x_{24} = x_{32} = x_{34} = x_{41} = x_{42} = 0,$
Flexible FGP	$Z_1 = 0.9631512$; $Z_2 = 0.9650924$; $Z_3 = 0.9466048$	$x_{14} = 12,\ x_{22} = 20,\ x_{31} = 4,\ x_{41} = 3,\ x_{44} = 15$ $x_{11} = x_{12} = x_{13} = x_{21} = x_{23} = x_{24} = x_{32} = 0,$ $x_{33} = x_{34} = x_{42} = x_{43} = 0$

6. Results and discussion

The suggested flexible FGP is tested using a numerical example. In this sense, the FGP model for the MOFTP is developed using ALs and tolerance levels to find the optimal solution. The flexible FGP problems are then solved using the LINGO software. For case study problems, the resultant solution is compared to the GP, FGP, intuitionistic FGP, and neutrosophic GP techniques presented in Table 2. Table 2 shows that the first objective function values are 1.0230, 0.0196, 0.9875, 0.9789, and 0.9631512 for FGP, GP, intuitionistic FGP, neutrosophic GP, and flexible FGP, respectively. For FGP, GP, intuitionistic FGP, neutrosophic GP, and flexible FGP, the second objective function values are 1.0085, 1.032, 0.9748, 0.9874, and 0.9650924, respectively. Similarly, the third objective function value for FGP, GP, intuitionistic FGP, neutrosophic GP, and flexible FGP is 0.9997, 1.0248, 0.9842, 0.9601 and 0.9466048, respectively. It demonstrates that the flexible FGP technique provides the least objective value, implying that the flexible FGP technique is used to obtain the compromise solution. As a result, flexible FGP provides an improved result than the FGP, GP, intuitionistic FGP, and neutrosophic GP methods. This method can be used efficiently utilizing to solve the other optimization problem. Overall, the proposed technique is better suited the multiobjective structural issues.

7. Motivation and contribution

This study is motivated by flexible fractional programming using fuzzy pentagonal numbers to capture the potential of decision-makers. The following are the contributions of the study:

(i) It serves as an additional contribution to the literature on the fractional TP.
(ii) A case study is provided in which solution procedures for multiobjective multi-product problem formulation are reported.
(iii) Fuzzy concepts of Pentagonal are used in fractional TPs.
(iv) This study has applied a new approach based on Flexible Fractional Programming.
(v) The approach is compared with GP, FGP, Intuitionistic FGP and Neutrosophic GP the result proves to be better.

8. Conclusion

The best compromise solution for MOFTP, a flexible FGP framework, is created. A numerical problem has been used to demonstrate the effectiveness of the approach. Finally, this study demonstrates that the flexible FGP approach outperforms existing approaches such as FGP, GP, intuitionistic FGP, and neutrosophic GP in terms of compromise results. The MOFTP model was developed with the assistance of a decision-maker to overcome the challenges experienced in real-world scenarios. Five alternative strategies were used to identify the best compromise solution. A fuzzy pentagonal number is used for the supply & demand parameter. All the objectives are minimization types which is the major limitation of the proposed technique. Different fuzzy numbers are used to solve the optimization problem in future research work. The multichoice parameter can also be used for solving the MOFTP with the help of different techniques.

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