RAIRO-Oper. Res. 56 (2022) 3941–3953
https://doi.org/10.1051/ro/2022163

RAIRO Operations Research
www.rairo-ro.org

INVENTORY POOLING AND PRICING DECISIONS IN MULTIPLE MARKETS WITH STRATEGIC CUSTOMERS

Xuantao Wang[1], Zhiming Chen[2], Shaorui Zhou[3,*], Mingfang Hu[4] and Jianjie Ke[3]

Abstract. This study considers the pricing and inventory decisions for the retailer selling in multiple markets with strategic customers. The impact of strategic customer behavior on inventory pooling is examined. The equilibrium decisions, in which strategic customers tend to purchase early, are subsequently characterized in the pooled/non-pooled systems. Our results highlight the role of the strategic customer in each market. Compared with myopic customers, the retailer is prone to reduce its inventory for strategic customers. The retailer's optimal inventory for high-profit products is lower in the pooled system than in the non-pooled system. However, the result is reversed for low-profit products. The analytical and numerical results simultaneously demonstrate that, in the high-profit condition, the retailers' inventory in the pooled system increases with the correlation coefficient of different markets, while the retailers' profit decreases with the correlation coefficient. There is an opposite relationship in the low-profit condition. When the markets are less correlated, the retailer owns low inventory but high profit. Moreover, the retailers' profit is always higher in the pooled system than in the non-pooled system.

Mathematics Subject Classification. 49N90, 62C12, 91A15.

Received February 9, 2022. Accepted September 18, 2022.

1. Introduction

One of the primary goals of an inventory management system is to protect the firm against demand uncertainty. In the case of distributed demand, retailers may pool inventory at centralized locations instead of holding separate stocks for each demand. In the situation of random demand, combining multiple markets can reduce aggregate uncertainties and better match supply and demand, thus increasing the retailers' profit. While firm's inventory pooling capabilities were rare in the early 2000s, many consumers now expect firms to follow inventory pooling strategy; Forrester Research reports that cross-channel fulfillment programs by inventory pooling are a top strategic priority for retailers, and moreover, 62% of retailers invest in such capabilities mainly because

Keywords. Inventory pooling, newsvendor model, rational expectations equilibrium, strategic customers.

[1] School of Business Administration, Guangdong University of Finance, Guangdong 510521, P.R. China.
[2] School of Credit Management, Guangdong University of Finance, Guangzhou 510521, P.R. China.
[3] School of Intelligent Systems Engineering, Sun Yat-sen University, Guangzhou 510275, P.R. China.
[4] School of Business Administration, Guangxi University, Nanning 530004, P.R. China.
*Corresponding author: zshaorui@gmail.com; zhoushr5@mail.sysu.edu.cn

of consumer expectations [1]. Thus, inventory pooling, *i.e.*, serving multiple markets from a single stock of inventory, has received a lot of attention among academics and practitioners.

This study is primarily motivated to solve the inventory pooling problem faced by a Chinese retailer that we work with. With the development of e-commerce, inventory pooling has been widely used. For example, 75 branches of Avon retailers were recently established in China, along with 75 warehouses. Avon's survey results showed that the warehouses were scattered, and the information was unimpeded, making the turnover days of goods inventory accumulate higher and higher. In addition, scattered in 75 large and small warehouses everywhere, Avon had to invest a lot of labor costs in warehousing, cashier, billing, and other operations. This "branch warehouse" is the center of the logistics mode of large consumption, slow speed, complex management, and the inability to keep up with the sales pace. As a result, Avon canceled dozens of warehouses and set up eight logistic centers in the major cities of China. This centralized inventory increased Avon sales by up to 45% each year. As a result, Avon's operating costs have decreased from 8% to 6%. At the same time, the growing use of the internet provides consumers an opportunity to gather information on companies' pricing policies and respond strategically. As a result, more consumers may attempt to time their purchases to maximize their benefits. These consumers may begin studying sellers' promotions and wait to purchase new products with the lowest price. At the end of the selling season, Avon salvaged the leftover unit to a discount or outlet store (or any other selling channel) to serve a distinct set of consumers. Thus, it's very important for Avon to explore the impact of strategic customer behavior on its inventory pooling and pricing decisions.

In this study, we investigate the behavior of these consumers and extend the inventory pooling model to a case where a retailer selling seasonal products decides the price and inventory to maximize its profit. We assume the consumers are strategic. They recognize that the product may be available on the salvage market and will delay their purchase until the salvage period to maximize the expected consumer surplus.

We consider two inventory cases: the non-pooled system and the pooled system. In the non-pooled system, an individual inventory is committed to an individual market. In the pooled system, the demands of different markets are simultaneously satisfied by centralized inventory. We study the retailer's best response price and inventory level with strategic consumers for both systems. Moreover, we compare the retailer's optimal decisions and profits between the two systems.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the inventory pooling problem with strategic customer behavior. Section 4 investigates the value of inventory pooling in the presence of strategic customer behavior. Section 5 presents numerical examples to compare the optimal strategies and profits between the pooled and non-pooled systems. Finally, Section 6 presents conclusions and future research directions.

2. Literature review

There are two streams of literature related to our paper. The first stream involves inventory pooling, and the second involves strategic customer behavior.

2.1. Inventory pooling

The first stream of relevant research focuses mainly on inventory pooling, which has been extensively studied in operations management. Eppen [2] explores a multi-location newsboy problem with normal-distributed demand at each location. Federgruen and Zipkin [3] consider a finite-horizon inventory pooling with an expanded range of demand distribution and nonidentical retailers. Corbett and Rajaram [4] extend the model of Eppen [2] to general dependent demand distributions and find that inventory pooling is more valuable when demands are less positively dependent. Mak and Shen [5] study the relationship between risk-pooling benefits and the dependence of demand in general two-tiered supply chains. Bimpikis and Markakis [6] find that the benefit from inventory pooling decreases as the tail of the underlying demand becomes heavier. Gerchak [7] analyses the consequences of a non-cooperative game with a modified scheme that benefits all parties relative to a no-pooling situation. Amrani and Khmelnitsky [8] study a partial pooling model where the total amount of inventory is fixed. Gerchak

[9] considers partial pooling as a combination of physical pooling and decentralized storage at retail locations. Gerchak and Mossman [10] explore analytically cardinal effects of the extent of demand randomness on optimal inventory levels and the associated expected costs. The results indicate that risk pooling might neither reduce inventories nor move them closer to the mean or median demand. Zhong *et al.* [11] show that, with allocation flexibility, the amount of safety stock needed in a system with independent and identically distributed demands does not grow with the number of customers but instead diminishes to zero and eventually becomes negative as the number of customers grows sufficiently large. Silbermayr and Gerchak [12] find that retailers can achieve higher profits with partial pooling than with no pooling or complete pooling. Yang and Schrage [13] explore the inventory pooling when there is demand substitution or risk pooling, and show the general result that right skewness of the demand distribution is the important feature that causes the anomaly. Furthermore, even in the absence of right skewness, the anomaly may arise if there is partial risk pooling. Bhatnagar and Bing [14] study joint transshipment and production control policies for multi-location production/inventory systems, in which items are manufactured and stocked at each location to meet demand. Only Aflaki and Swinney [15] analyze the value of inventory pooling for a firm selling to rational consumers with two markets. None of the above studies investigate the effect of consumer behavior on retailers' inventory pooling and pricing decisions with multiple markets. In this study, we will explore the impact of strategic customer behavior on inventory pooling and pricing decisions and possess both operational and behavioral value in inventory pooling.

2.2. Strategic customer behavior

Recent research on strategic consumer behavior has explored and developed various pricing and inventory strategies to counteract strategic consumer behavior. The influence of strategic consumer behavior on pricing strategy is reviewed firstly in the following. Coase [16] shows that the durable good monopolist must sell products with marginal cost and end up earning zero profits because as long as prices are above marginal cost, strategic consumers will anticipate and wait for the price reduction. Su and Zhang [17] find that when the sale price is exogenously set, the retailer can reduce inventory and set a low price to induce strategic consumers to purchase. Levina *et al.* [18] use an aggregating algorithm to study a monopolistic retailer who can dynamically price a perishable product and simultaneously learn the market. Wu *et al.* [19] propose a consumer heuristic model for estimating markdown prices. Chen and Wang [20] find that the retailer's ability to learn consumers' valuations can be undermined by strategic waiting behavior. Huang *et al.* [21] investigate the relationship between demand learning and preference learning under mass customizations. Aviv *et al.* [22] study the potential benefits of responsive pricing and demand learning for sellers. Dong and Wu [23] examine the impact of strategic customer behavior in two-period pricing in a quick response system. Huang and Wang [24] consider a closed-loop supply chain consisting of a manufacturer and a third party in which the manufacturer licenses the third party to undertake remanufacturing activities in the presence of strategic consumers. Peng *et al.* [25] study the price guarantee policies of a seller adopting the advance-selling strategy in the presence of preorder-dependent social learning. Yu *et al.* [26] develop a dynamic pricing model to examine how strategic consumers affect the strategic interaction among firms under three dual-channel formats.

The second stream studies the impact of consumer behavior on inventory strategy. The inventory decision of retailers can affect customer purchasing behavior in many ways. Liu and Van Ryzin [27] demonstrate that capacity rationing can mitigate strategic waiting behavior. Zhao and Stecke [28] and Prasad *et al.* [29] study how advanced selling benefits a newsvendor retailer. Wei and Zhang [30] explore a preorder-contingent production strategy in which the retailer's production decision is contingent on pre-order quantities. Altug and Aydinliyim [31] study the impact of the return policy on strategic customer behavior. Courty and Nasiry [32] suggest that the retailer could also produce less to intentionally undersupply consumers when consumers are sufficiently uncertain about their valuations and discount their future surplus to a certain extent. Zhang *et al.* [33] consider the inventory decision of a retailer facing strategic customers to develop a behavioral theory that accounts for reference dependence. Based on a unique setting from Amazon lightning deals, Cui *et al.* [34] explore whether and how consumers learn from inventory availability information. Wang *et al.* [35] address the optimal inventory decision for a retailer considering strategic and myopic consumers with and without a quick response. Indeed,

TABLE 1. Notations.

Symbol	Description
p_i	Unit selling price in the market i
s	Unit salvage value
v	Valuation of the product by the consumer
Q_i	Inventory procurement, $i = 1, 2, \cdots, n, T$
π_i	Expected profit of the retailer in the market i
c	Unit production cost
D_i	Individual market demand, with mean μ_i and variance σ_i^2
D_T	Total market demands
$\phi(\cdot)$	PDF of standard normal distribution
$\Phi(\cdot)$	CDF of standard normal distribution
σ_{ij}^2	Covariance between D_i and D_j
ρ_{ij}	Correlation coefficient between D_i and D_j
φ_i	Probability of obtaining salvaged product for the consumer in the market i
r_i	Reservation price of consumers (their willingness to pay in the selling period)
X	Standard normal random variable

the vast majority of the strategic customer behavior literature ignores inventory pooling; an exception is Aflaki and Swinney [15], who studies the impact of inventory pooling in channels and strategic customer responses, but they only consider two markets. Moreover, most of these papers only consider one or two markets. However, in this study, we examine the inventory pooling in multiple markets with strategic consumers and explore the impact of the correlation between the different markets on inventory pooling and pricing decisions.

The key contributions of this study lie in the following aspects. First, we provide new insights on inventory pooling and pricing decisions in a non-pooled/pooled system. Compared with myopic customers, the retailer is prone to reduce its inventory for strategic customers. The retailer's optimal inventory for high-profit products is lower in the pooled system than in the non-pooled system. However, the result is reversed for low-profit products. Second, we consider the strategic consumer's behavior in multiple different markets. Only a few studies combine inventory pooling with strategic consumers. For high-profit products, if the markets are highly correlated, the retailer owns high inventory but low profit. However, the result is reversed for low-profit products. Finally, we compare the retailer's optimal price, inventory, and profit in the non-pooled and pooled systems.

3. Model description

We start with the description of the model assumptions. Table 1 summarizes the notation used in this paper. Then, we present the models and analytical results.

3.1. Model assumptions

We consider a retailer dealing with multiple distinct markets. The retailer can adopt either centralized or decentralized inventory to satisfy demand. Without loss of generality, the demand of each market is denoted by $D_i, i = 1, 2, \cdots, n$, and follows a normal distribution with mean μ_i and variance σ_i^2. The covariance and correlation coefficient of demands D_i and D_j are σ_{ij}^2 and ρ_{ij}, respectively. According to a multivariate normal assumption, the total demand $D_T = D_1 + D_2 + \cdots + D_n$ is also normally distributed with mean $\mu_T = \sum_{i=1}^n \mu_i$ and variance $\sigma_T^2 = \Sigma_{i,j=1}^n \sigma_{ij}^2$. We refer to $\Phi(\cdot)$ and $\phi(\cdot)$ as the distribution and density functions of the standard normal distribution, respectively.

The retailer can choose the non-pooled system or pooled system. In the non-pooled system, suppose there is no transshipment, consumer search, or product substitution. An individual inventory is committed to an individual market and cannot be used to satisfy demands in other markets. The retailer decides the inventory level Q_i for the ith market and obtains the expected profit $\pi_i(Q_i) i = 1, 2, \cdots, n$. In the pooled system, all demands are satisfied by a centralized inventory. The retailer decides the total inventory level Q_T and obtains the expected profit $\pi_T(Q_T)$. We frequently use generic inventory and profit expressions, $\pi_i(Q_i)$, which may allude to markets $1, 2, \cdots, n$, in the non-pooled system ($i = 1, 2, \cdots, n$) or the "pooled market" ($i = T$) in the pooled system, as necessary.

In both systems, the product is sold over a finite season. Before the beginning of the selling season, given the goal of profit maximization, the retailer makes inventory procurement $Q_i (i = 1, 2, \cdots, n, T)$ with unit cost, c, and then decides the unit selling price p_i. At the end of the selling season, the leftover units can be sold in an exogenous salvage market at s per unit, where $s < c < p$. We assume that any amount of unsold inventory at the end of the selling season can be salvaged to a discount or outlet store (or any other selling channel) at a price s to serve a distinct set of consumers, which we refer to as bargain hunters. This assumption is made to ensure that all inventory can be cleared at the end of the season, which will enable us to derive closed-form expressions for firm inventory and profit (see also [15,17,36]). Each product is valued by customers at υ, which can be considered the customer's utility from consuming the product.

There are strategic customers in the markets. Since the product will ultimately be salvaged, they will likely delay their purchase until the price decreases. At the beginning of the selling season, after observing the selling price p_i, each consumer chooses to either purchase the product immediately or delay their purchase until the sales promotion. If the customer determines to purchase at a normal price p_i, she will obtain the product for certain. However, if the customer delays purchasing at salvaged price s, she will obtain the product with a probability $\varphi_i (i = 1, 2, \cdots, n, T)$. In the purchase decision, each customer tends to compare the surplus $\upsilon - p_i$ from the immediate purchase with the surplus $\varphi_i(\upsilon - s)$ from the delayed purchase. Consumers will choose to purchase at a price that yields a greater surplus. Moreover, if customers are indifferent between the two schemes, they are assumed to buy at the price p_i immediately. Because those customers who eagerly wait for the sales promotion have made adequate preparations, it is reasonable to assume that they possess the highest priority in obtaining the product.

The interaction between the retailer and the strategic consumers is a game. The customers do not know the inventory level but can observe the selling price. The reservation price $r_i = \upsilon - \varphi_i(\upsilon - s)$ will be formed by the customers as the highest price they are willing to pay in the selling season. During the game, the retailer makes inventory procurements subject to the belief about consumer behavior, while strategic consumers choose to purchase or wait. The retailer will privately form its beliefs of $\hat{r}_i$ based on the reservation price of customers. Then the retailer optimally chooses the price $p_i = \hat{r}_i$ and procurement quantities Q_i to maximize its expected profit as follows:

$$\pi_i(Q_i, p_i) = (p_i - s) E \min(D_i, Q_i) - (c - s) Q_i, \quad (i = 1, 2, \cdots, n, T). \tag{1}$$

3.2. Model solution

In equilibrium, both the retailer's belief about the reservation price of customers and the customers' beliefs in obtaining the product on the salvage market must be consistent with outcomes. Thus, we derive an equilibrium in which all strategic consumers purchase early.

Definition 1. The equilibrium $(p_i, Q_i, r_i, \varphi_i, \hat{r}_i)$ satisfies the following conditions for $i = 1, 2, \ldots, n$, in the non-pooled system and $i = T$ in the pooled system:

(a) $r_i = \upsilon - (\upsilon - s)\varphi_i$.
(b) $Q_i = \arg\max_{Q_i} \pi_i(Q_i, p_i)$, $p_i = \hat{r}_i$.
(c) $\varphi_i = \Phi_i(Q_i)$.
(d) $\hat{r}_i = r_i$.

Conditions (a) and (b) assert that under the expectations of φ_i and $\hat{r}_i$ the retailer and all strategic consumers will rationally choose appropriate actions to maximize their utilities. Conditions (c) and (d) require that expectations be consistent with outcomes. In condition (c), the expectation of φ_i concur with the actual probability of obtaining the product if an individual strategic consumer waits for the sale and the probability φ_i is endogenous. In equilibrium, the retailer prices the product with the value of strategic consumers' reservation price, so all strategic consumers will buy the product. Therefore, if an individual strategic consumer decides to wait, then this consumer will obtain the product if and only if $D_i \leq Q_i$, which occurs with probability $\Phi_{\text{i}}(Q_i)$ as shown in condition (c). In condition (d), the retailer correctly anticipates strategic consumers' reservation prices.

Proposition 1.

(1.1) *In the non-pooled system, the retailer's optimal responsive price and inventory level, which make strategic consumers purchase during the selling season, are characterized as follows:*

$$p_i^* = \sqrt{(c-s)(\upsilon-s)} + s,\ Q_i^* = \mu_i + \sigma_i z^*,\ z^* = \Phi^{-1}\left(1 - \sqrt{\frac{c-s}{\upsilon-s}}\right) \quad i = 1, 2, \cdots, n.$$

(1.2) *In the pooled system, the retailer's optimal responsive price and inventory level, which make strategic consumers purchase during the selling season, are characterized as follows:*

$$p_T^* = \sqrt{(c-s)(\upsilon-s)} + s,\ Q_T^* = \mu_T + \sigma_T z^*,\ z^* = \Phi^{-1}\left(1 - \sqrt{\frac{c-s}{\upsilon-s}}\right) \quad i = 1, 2, \cdots, n.$$

Proof. The conditions for the rational expectations equilibrium in Definition 1 can be reduced to a pair of equations in p_i and Q_i only: $p_i = \upsilon - (\upsilon - s)\Phi_i(Q_i)$ and $Q_i = \arg\max_{Q_i} \pi_i(Q_i, p_i), (i = 1, 2, \cdots n, T)$, which can be reduced to $p_i = \upsilon - (\upsilon - s)F_i(Q_i)$ and $\overline{\Phi}(Q_i) = \frac{c-s}{p_i - s}$, respectively. Solving the two equations yields the above results. □

From Proposition 1, we can see that the prices in both systems are the same. In both systems, strategic consumers' behavior forces the retailer to price below υ. In the non-pooled system, the optimal inventory level of each market is different. We also find that $\sum_{i=1}^{n} \sigma_i z^*$ in the optimal non-pooled inventory $\sum_{i=1}^{n} \mu_i + \sum_{i=1}^{n} \sigma_i z^*$ and $\sigma_T z^*$ in the optimal pooled inventory level $\mu_T + \sigma_T z^*$ are respectively proportional to the sum of the standard deviation $\sum_{i=1}^{n} \sigma_i$ and the standard deviation of total demand σ_T with the same proportionality constant. Now, we compare the optimal inventory level in both systems as follows:

Proposition 2.

(2.1) *Given strategic customers, the optimal inventory level of the retailer in the non-pooled system is higher than that in the pooled system, i.e.,* $Q_1^* + Q_2^* + \cdots + Q_n^* > Q_T^*$ *if* $1 - \sqrt{\frac{c-s}{\upsilon-s}} > 0.5$.

(2.2) *Given strategic customers, the optimal inventory level of the retailer in the non-pooled system is lower than that in the pooled system, i.e.,* $Q_1^* + Q_2^* + \cdots + Q_n^* \leq Q_T^*$ *if* $1 - \sqrt{\frac{c-s}{\upsilon-s}} \leq 0.5$.

Proof. If $1 - \sqrt{\frac{c-s}{\upsilon-s}} > 0.5$, we can show that

$$\Phi^{-1}\left(1 - \sqrt{\frac{c-s}{\upsilon-s}}\right) > 0. \tag{2}$$

Note that $\sum_{i=1}^{n} \mu_i = \mu_T$ and $\sigma_T \leq \sum_{i=1}^{n} \sigma_i$ with equality if and only if all the demands from all markets are perfectly correlated (*i.e.*, $\rho_{ij} = 1$ for all i, j).

Thus, we can obtain

$$Q_1^* + Q_2^* + \ldots + Q_n^* = \sum_{i=1}^{n} \mu_i + \sum_{i=1}^{n} \sigma_i z^* > Q_T^* = \mu_T + \sigma_T z^*. \tag{3}$$

If $1-\sqrt{\frac{c-s}{\upsilon-s}} \le 0.5$, then $\Phi^{-1}\left(1-\sqrt{\frac{c-s}{\upsilon-s}}\right) \le 0$.

Thus we have

$$Q_1^* + Q_2^* + \ldots + Q_n^* \le Q_T^*. \tag{4}$$

□

Schweitzer and Cachon [37] define a product as a high/low-profit product when the profit-maximizing order quantity is greater/lower than the mean of demand. Note that $Q_i^* = \mu_i + \sigma_i z^* > \mu_i$ when $1-\sqrt{\frac{c-s}{\upsilon-s}} > 0.5$. Thus, when $1-\sqrt{\frac{c-s}{\upsilon-s}} > 0.5$, the product can be considered as the high-profit product. The retailer's optimal inventory Q_T^* in the pooled system is lower than the optimal inventory $Q_1^* + Q_2^* + \ldots + Q_n^*$ in the non-pooled system. By contrast, when $1-\sqrt{\frac{c-s}{\upsilon-s}} \le 0.5$, then $Q_i^* \le \mu_i$. Similarly, when $1-\sqrt{\frac{c-s}{\upsilon-s}} \le 0.5$, the product can be considered as a low-profit product. The retailer's optimal inventory level Q_T^* in the pooled system is higher than $Q_1^* + Q_2^* + \ldots + Q_n^*$ in the non-pooled system.

Furthermore, it is meaningful to compare the price and inventory level in the presence of myopic customers. These myopic customers are willing to pay their valuation υ for the product in the selling season without considering future purchasing opportunities. Consequently, the retailer can optimally choose the price $p_i^\circ = \upsilon$ and quantity $Q_i^\circ = \arg\max_{Q_i} \pi_i(Q_i, p_i)$, where $\pi_i(Q_i, p_i) = (p_i - s)E\min(D_i, Q_i) - (c-s)Q_i$.

Thus, with myopic consumers, the retailer's optimal responsive price and inventory level in both systems are characterized by the following

$$p_i^\circ = \upsilon;\ Q_i^\circ = \mu_i + \sigma_i z^\circ;\ z^\circ = \Phi^{-1}\left(1 - \frac{c-s}{\upsilon-s}\right);\quad i = 1, 2, \cdots, n, T.$$

We compare the optimal inventory level in both systems as follows.

Proposition 3. *With myopic customers, the optimal inventory level of the retailer in the non-pooled system is higher than that in the pooled system* ($Q_1^\circ + Q_2^\circ + \cdots + Q_n^\circ > Q_T^\circ$) *if* $1 - \frac{c-s}{\upsilon-s} > 0.5$. *With myopic customers, the optimal inventory level of the retailer in the non-pooled system is lower than that in the pooled system, i.e.,* ($Q_1^\circ + Q_2^\circ + \cdots + Q_n^\circ \le Q_T^\circ$) *if* $1 - \frac{c-s}{\upsilon-s} \le 0.5$.

Proof. When $1 - \frac{c-s}{\upsilon-s} > 0.5$, then $\Phi^{-1}\left(1 - \frac{c-s}{\upsilon-s}\right) > 0$. Note that $\sigma_T \le \sum_{i=1}^n \sigma_i$ with equality if and only if all the demands are perfectly correlated (*i.e.*, $\rho_{ij} = 1$ for all i, j).

Therefore, we can obtain

$$Q_1^\circ + Q_2^\circ + \ldots + Q_n^\circ = \sum_{i=1}^n \mu_i + \sum_{i=1}^n \sigma_i z^\circ > Q_T^\circ = \mu_T + \sigma_T z^\circ; \tag{5}$$

when $1 - \frac{c-s}{\upsilon-s} \le 0.5$, then $\Phi^{-1}\left(1 - \frac{c-s}{\upsilon-s}\right) \le 0$.

Thus we have

$$Q_1^\circ + Q_2^\circ + \ldots + Q_n^\circ \le Q_T^\circ. \tag{6}$$

□

When $1 - \frac{c-s}{\upsilon-s} > 0.5$, then $Q_i^\circ > \mu_i$. In this case, the product is a high-profit product. Thus, under the high-profit condition, the retailer's optimal order $Q_1^\circ + Q_2^\circ + \ldots + Q_n^\circ$ in the non-pooled system is larger than Q_T° in the pooled system. By contrast, when $1 - \frac{c-s}{\upsilon-s} \le 0.5$, then $Q_i^\circ \le \mu_i$. In this case, the product is a low-profit product. Under the low-profit condition, the retailer's optimal order $Q_1^\circ + Q_2^\circ + \ldots + Q_n^\circ$ in the non-pooled system is smaller than Q_T° in the pooled system.

From Propositions 2 and 3, we find that the optimal inventory level in the non-pooled system is higher than that in the pooled system in the high-profit condition. Moreover, the optimal inventory level in the non-pooled system is lower than that in the pooled system in the low-profit condition, regardless of consumer type (strategic or myopic).

However, the two cases for the products that are either high-profit or low-profit in Propositions 2 and 3 are different. Moreover, due to $z^\circ > z^*$, we find the optimal inventory level $Q_i^* < Q_i^\circ$, $i = 1, 2, \cdots, n, T$. The retailer keeps low inventory level with strategic consumers in both systems. The reason is that the retailer wants to reduce the probability of consumers waiting for sales promotions. And the retailer hopes to encourage strategic customers to buy during the selling period by reducing the inventory level in each market to reduce the probability of strategic consumers waiting for products and encourages customers to purchase during the selling period.

4. Benefit of inventory pooling

Now we analyze how the retailer's profit is influenced by inventory pooling when consumers are strategic. We aim to answer the question: Does the retailer's profit increase in the pooled system? The following result provides sufficient conditions for pooling benefits.

Proposition 4. *Suppose that the following inequalities hold, that is, the market volatility of aggregate demand is at least equal to the market volatility in any location but lower than the sum of all locations' market volatility.*

(4.1) $\sigma_T \le \sum_{i=1}^n \sigma_i$.
(4.2) $\sigma_T \ge \sigma_i$, *for every* $i = 1, \ldots, n$.

Then, we have $\Pi_T^*(Q_T^*) \ge \sum_{i=1}^n \Pi_i^*(Q_i^*)$. *This inequality means inventory pooling increases the retailer's expected profit when consumers are all strategic.*

Proof. We examine the two conditions of the proposition in detail. Condition (4.1) always holds, with equality if and only if the demands of all locations are perfectly correlated with $\rho_{ij} = 1$. Condition (4.2) shows that the standard deviation of aggregate demand is at least as large as the standard deviation in any location, which is likely to hold in reality (see [38], [39]). From Proposition 1, the retailer's expected profit in the non-pooled system is

$$\sum_{i=1}^n \Pi_i^*(Q_i^*) = \sum_{i=1}^n (p_i^* - s) E \min(D_i, Q_i^*) - (c - s) Q_i^*. \tag{7}$$

From Proposition 1, the retailer's expected profit in the pooled system is

$$\Pi_T^*(Q_T^*) = (p_T^* - s) E \min(D_T, Q_T^*) - (c - s) Q_T^*. \tag{8}$$

Instead of choosing the stocking quantities Q_i^*, we choose the standardized stocking quantities $z_i^* = \frac{(Q_i^* - \mu_i)}{\sigma_i}$. From Proposition 1, $z_i^* = z_T^* = z^*, i = 1, 2, \cdots, n$. The profit function over z_i^* should satisfy $\widetilde{\Pi}_i(z_i^*) = \Pi_i(Q_i^*)$, so we have

$$\widetilde{\Pi}_i(z_i^*) = (p^* - s)\sigma_i E \min(X, z^*) + (p^* - c)\mu_i - (c - s)\sigma_i z^*, \quad i = 1, 2, \ldots, n, T, \tag{9}$$

where X is a standard normal random variable. Combining the conclusions leads to

$$\begin{aligned}
\Pi_T^*(Q_T^*) - \sum_{i=1}^n \Pi_i^*(Q_i^*) &= \widetilde{\Pi}_T(z_T^*) - \sum_{i=1}^n \widetilde{\Pi}_i(z_i^*) \\
&= (p^* - s)\left(\sigma_T - \sum_{i=1}^n \sigma_i\right) E \min(X, z^*) + (p^* - c)\left(\mu_T - \sum_{i=1}^n \mu_i\right) - (c - s)\left(\sigma_T - \sum_{i=1}^n \sigma_i\right) z^*
\end{aligned}$$

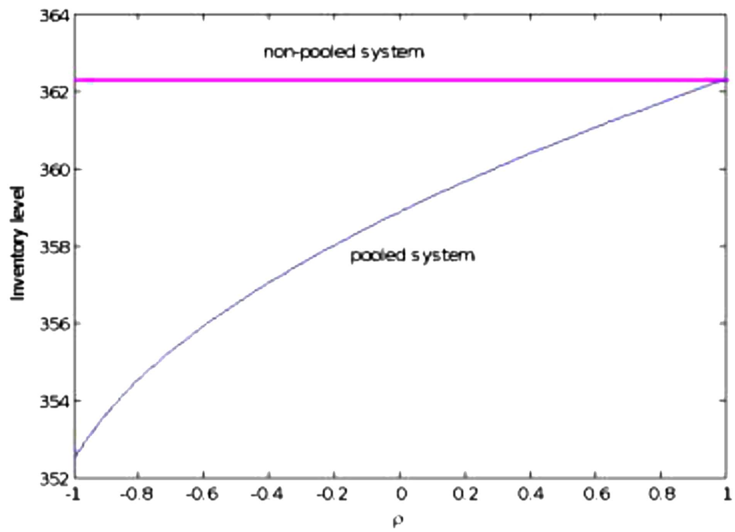

FIGURE 1. Inventory level with correlation coefficient ρ in both systems.

$$= \sqrt{(c-s)(v-s)}\left(\sigma_T - \sum_{i=1}^{n}\sigma_i\right)\int_{-\infty}^{z^*} x\varphi(x)\,\mathrm{d}x - (c-s)\left(\sigma_T - \sum_{i=1}^{n}\sigma_i\right)z^* \geq 0. \tag{10}$$

Note that the inequality holds due to $\sigma_T \leq \sum_{i=1}^{n}\sigma_i, \int_{-\infty}^{z^*} x\varphi(x)\,\mathrm{d}x < 0$, and $p_i^* = \sqrt{(c-s)(v-s)} + s, i = 1, 2, \cdots n, T$. □

Proposition 4 shows that inventory pooling can increase the retailer's profit in the pooled system. Notice that $\sigma_T = \sum_{i=1}^{n}\sigma_i$ if and only if all demands are perfectly correlated (*i.e.*, $\rho_{ij} = 1$ for all i, j), which implies that the retailer's profits in both systems are the same if and only if $\rho_{ij} = 1$ for all i, j. The impact of pooling is positive because it reduces demand uncertainty and minimizes supply-demand mismatch. Because of the reduction of aggregate demand uncertainty in the pooled system, the retailer can procure inventory more precisely, lowering the risk of salvaging leftover inventory.

Meanwhile, the strategic customer's probability of obtaining the product at the sale price decreases. Therefore, they tend not to wait for the salvage price, which increases the retailer's profit in the pooled system.

5. Numerical examples

In this section, we present the results of numerical experiments for high-profit and low-profit products, respectively. In order to verify our results, we consider two distinct markets and use the same parameters as that in Aflaki and Swinney [15].

For high-profit products, the related parameters are assumed as follows: $v = 11, c = 6, s = 4.5$. Demands in both markets are normally distributed: $D_1 \sim N\left(150, 100^2\right), D_2 \sim N\left(200, 150^2\right)$. The correlation coefficient ρ for the two markets is set from -1 to 1, and the results for each correlation coefficient are recorded. According to Proposition 1, we can get the optimal inventory level for the non-pooled system and pooled system in Figure 1 below.

In this case, since $1 - \sqrt{\frac{6-4.5}{11-4.5}} \approx 0.5196 > 0.5$, the products are high-profit type. Figure 1 describes how the retailer's optimal inventory varies with the correlation coefficient ρ in both systems. It is shown that the retailer's optimal inventory is always lower in the pooled system than in the non-pooled system; that is, inventory pooling can decrease the retailer's inventory level. It can also be found that the inventory level in the pooled system

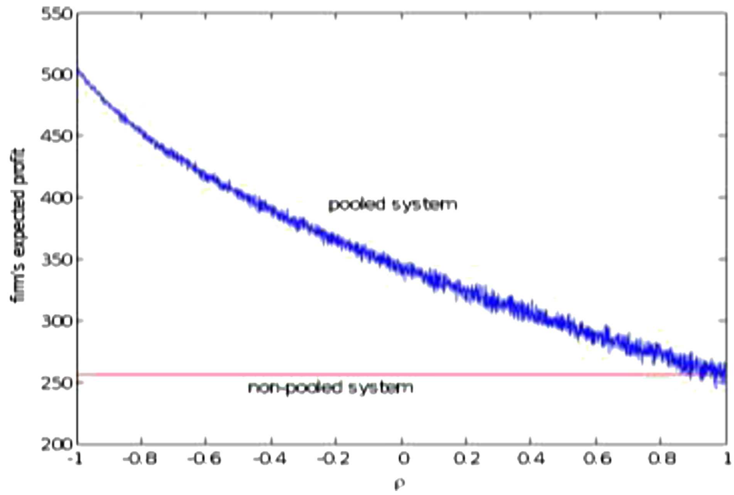

FIGURE 2. Expected profit with correlation coefficient ρ in both systems.

increases with the correlation coefficient ρ, which means if the two markets are more correlated, the retailer's inventory is higher in the pooled system.

According to equation (1), we can obtain the firm's expected profit of non-pooled and pooled systems for each correlation coefficient ρ in Figure 2.

Figure 2 describes how the retailer's optimal profit varies with the correlation coefficient ρ in both systems when products are high-profit types. Figure 2 shows that the retailer's profit is always higher in the pooled system than in the non-pooled system. On the other hand, it can be seen that the retailer's profit decreases with the correlation coefficient ρ in the pooled system, which means if the two markets are more correlated, the retailer's profit is lower in the pooled system. Figures 1 and 2 also demonstrate that the retailer should adopt the inventory pooling strategy when the two markets are less correlated. The retailer has low inventory but high profit when the two markets are less correlated.

For low-profit products, the related parameters are assumed as follows: $v = 8$, $c = 6$, $s = 3.5$. Demands in both markets are normally-distributed with means of 150, 200, and standard deviations of 100 and 150. The correlation coefficient ρ for the two markets is set from -1 to 1, and the results for each correlation coefficient are recorded. According to Proposition 1, we can get the optimal inventory level for the non-pooled system and pooled system in Figure 3.

In this case, because $1 - \sqrt{\frac{6-3.5}{8-3.5}} \approx 0.2546 < 0.5$, the products are low-profit type. Figure 3 describes how the retailer's optimal inventory varies with the correlation coefficient ρ in both systems when products are low-profit types. It is shown that the retailer's optimal inventory in the pooled system is always higher than that in the non-pooled system; that is, inventory pooling can increase the retailer's inventory level. It is also indicated that the inventory level in the pooled system decreases with the correlation coefficient ρ, which means if the two markets are more correlated, the retailer's inventory is lower in the pooled system. According to equation (1), we conducted a set of experiments to examine how the retailer's optimal profit varies with the correlation coefficient ρ in both system when product are low-profit products. We recorded the results in Figure 4 for each correlation coefficient ρ.

As shown in Figure 4, it is found that the retailer's profit is always higher in the pooled system than in the non-pooled system. Moreover, the retailer's profit decreases with the correlation coefficient ρ in the pooled system, which means if the two markets are more correlated, the retailer's profit is lower in the pooled system.

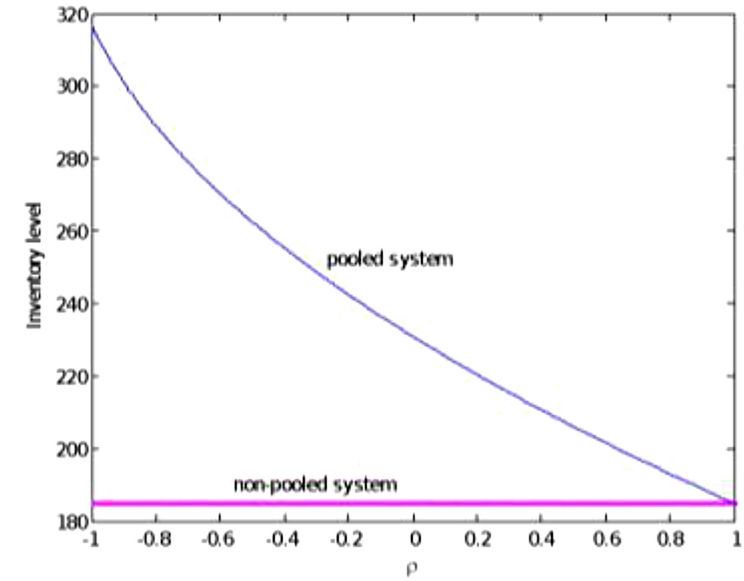

FIGURE 3. Inventory level with correlation coefficient ρ in both systems.

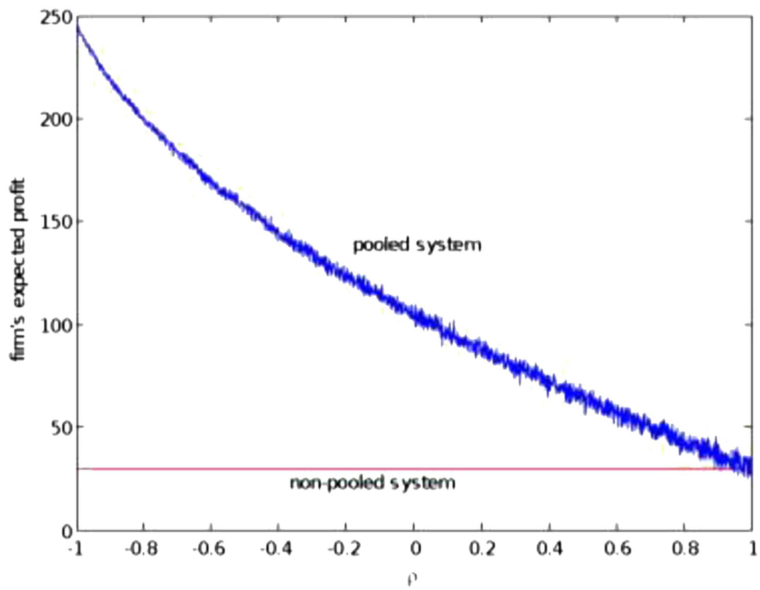

FIGURE 4. Expected profit with correlation coefficient ρ in both systems.

6. CONCLUSIONS

This paper studies the pricing and inventory decisions considering inventory pooling for multiple markets with strategic customers. We compare the retailer's optimal pricing and inventory decisions in the non-pooled system with those in the pooled system. The prices in each market are the same for both systems, which arises from the same belief of strategic customers about the probability of getting the product in the salvage market.

Facing strategic customers, the retailer tends to reduce its inventory compared to myopic customers in both systems. The reason is that it wants to decrease the probability of strategic consumers waiting for products and encourage them to buy early.

Moreover, in the non-pooled system, the optimal inventory level in each market is different. Specifically, when selling high-profit products, the retailer tends to keep a larger total inventory in the non-pooled system than in the pooled system. When selling low-profit products, the finding is reversed. The reason is that the market volatility of aggregate demand is lower than the sum of market volatility at all locations. We also show that inventory pooling increases retailer profits when consumers are all strategic. Furthermore, the numerical study indicates that, in the high-profit condition, the retailer's inventory level in the pooled system increases with the correlation coefficient, while the retailer's profit decreases with the correlation coefficient. There is an opposite relationship in the low-profit condition. Besides, when the product is high-profit, and the two markets are less correlated, the retailer has low inventory but high profit. When the product is low-profit, and the two markets are more correlated, the retailer's profit is lower in the pooled system.

In future research, the study can be extended in several directions. On the one hand, this paper only considers strategic or myopic customers in the market. Future studies can consider strategic consumers and myopic consumers coexisting in the market. On the other hand, future studies can study decentralized settings considering supply chain coordination.

Acknowledgements. This work is supported by the National Natural Science Foundation of China (Grant nos. 71802057, 71701221, 72271251) and the Natural Science Foundation of Guangdong Province (Grant nos. 2018A030313424, 2019A1515011127).

References

[1] Forrester Research, Customer desires vs. retailer capabilities: minding the omnichannel commerce gap. Tech. rept. Forrester Consulting (2014).

[2] G.D. Eppen, Effects of centralization on expected costs in a multi-location newsboy problem. *Manage. Sci.* **25** (1979) 498–501.

[3] A. Federgruen and P. Zipkin, Approximations of dynamic, multi-location production and inventory problems. *Manage. Sci.* **30** (1984) 69–84.

[4] C.J. Corbett and K. Rajaram, A generalization of the inventory pooling effect to nonnormal dependent demand. *M&SOM-Manuf. Serv. Oper.* **8** (2006) 351–358.

[5] H.Y. Mak and Z.J.M. Shen, Pooling and dependence of demand and yield in multiple-location inventory systems. *M&SOM-Manuf. Serv. Oper.* **16** (2014) 263–269.

[6] K. Bimpikis and M.G. Markakis, Inventory pooling under heavy-tailed demand. *Manage. Sci.* **62** (2016) 1800–1813.

[7] Y. Gerchak, Allocating pooled inventory according to contributions and entitlements. *Decis. Making. Manuf. Serv.* **9** (2015) 51–54.

[8] H. Amrani and E. Khmelnitsky, Optimal division of inventory between depot and bases. *Nav. Res. Log.* **64** (2017) 3–18.

[9] Y. Gerchak, On the viability of partial inventory pooling. *Int. J. Inventory Res.* **4** (2017) 44–60.

[10] Y. Gerchak and D. Mossman, On the effect of demand randomness on inventories and costs. *Oper. Res.* **40** (1992) 804–807.

[11] Y. Zhong, Z. Zheng, M.C. Chou and C.-P. Teo, Resource pooling and allocation policies to deliver differentiated service. *Manage. Sci.* **64** (2018) 1555–1573.

[12] L. Silbermayr and Y. Gerchak, Partial pooling by independent firms with allocation according to contribution to pool. *Int. J. Prod. Econ.* **218** (2019) 375–385.

[13] H. Yang and L. Schrage, Conditions that cause risk pooling to increase inventory. *Eur. J. Oper. Res.* **192** (2009) 837–851.

[14] R. Bhatnagar and B. Lin, The joint transshipment and production control policies for multi-location production/inventory systems. *Eur. J. Operat. Res.* **275** (2019) 957–970.

[15] A. Aflaki and R. Swinney, Inventory integration with rational consumers. *Oper. Res.* **69** (2021) 1025–1043.

[16] R.H. Coase, Durability and monopoly. *J. Law Econ.* **15** (1972) 143–149.

[17] X. Su and F. Zhang, Strategic customer behavior, commitment, and supply chain performance. *Manage. Sci.* **54** (2008) 1759–1773.

[18] T. Levina, Y. Levin, J. McGill and M. Nediak, Dynamic pricing with online learning and strategic consumers: an application of the aggregating algorithm. *Oper. Res.* **57** (2009) 327–341.

[19] S. Wu, Q. Liu and R.Q. Zhang, The reference effects on a retailer's dynamic pricing and inventory strategies with strategic consumers. *Oper. Res.* **63** (2015) 1320–1335.

[20] X. Chen and Z. Wang, Bayesian dynamic learning and pricing with strategic customers. *SSRN Electron. J.* (2016). https://doi.org/10.2139/ssrn.2715730

[21] T. Huang, C. Liang and J. Wang, The value of "Bespoke": demand learning, preference learning, and customer behavior. *Manage. Sci.* **64** (2018) 3129–3145.

[22] Y. Aviv, M.M. Wei and F. Zhang, Responsive pricing of fashion products: the effects of demand learning and strategic consumer behavior. *Manage. Sci.* **65** (2019) 2982–3000.

[23] J. Dong and D.D. Wu, Two-period pricing and quick response with strategic customers. *Int. J. Prod. Econ.* **215** (2019) 165–173.

[24] Y. Huang and Z. Wang, Pricing and production decisions in a closed-loop supply chain considering strategic consumers and technology licensing. *Int. J. Prod. Res.* **57** (2019) 2847–2866.

[25] W. Peng, Z. Tian and Y. Wang, Price guarantee for advance selling in the presence of preorder-dependent social learning. *Int. J. Prod. Econ.* **219** (2020) 115–122.

[26] D. Yu, M. Wan and C. Luo, Dynamic pricing and dual-channel choice in the presence of strategic consumers. *Manag. Decis. Econ.* **1** (2022) 1–17.

[27] Q. Liu and G.J. van Ryzin, Strategic capacity rationing to induce early purchases. *Manage. Sci.* **54** (2008) 1115–1131.

[28] X. Zhao and K.E. Stecke, Pre-orders for new To-be-released products considering consumer loss aversion. *Prod. Oper. Manag.* **19** (2010) 198–215.

[29] A. Prasad, K.E. Stecke and X. Zhao, Advance selling by a newsvendor retailer. *Prod. Oper. Manag.* **20** (2011) 129–142.

[30] M.M. Wei and F. Zhang, Advance selling to strategic consumers: pre-order contingent production strategy with advance selling target. *Prod. Oper. Manag.* **27** (2018) 1221–1235.

[31] M.S. Altug and T. Aydinliyim, Counteracting strategic purchase deferrals: the impact of online retailers' return policy decisions. *M&SOM-Manuf. Serv. Oper.* **18** (2016) 376–392.

[32] P. Courty and J. Nasiry, Product launches and buying frenzies: a dynamic perspective. *Prod. Oper. Manag.* **25** (2016) 143–152.

[33] Y. Zhang, B. Mantin and Y. Wu, Inventory decisions in the presence of strategic customers: theory and behavioral evidence. *Prod. Oper. Manag.* **28** (2019) 374–392.

[34] R. Cui, D.J. Zhang and A. Bassamboo, Learning from inventory availability information: evidence from field experiments on amazon. *Manage. Sci.* **65** (2019) 1216–1235.

[35] X.T. Wang, P. Ma and Y.L. Zhang, Pricing and inventory strategies under quick response with strategic and myopic consumers. *Int. T. Oper. Res.* **27** (2020) 1729–1750.

[36] G.P. Cachon and R. Swinney, The value of fast fashion: quick response, enhanced design, and strategic consumer behavior. *Manage. Sci.* **57** (2011) 778–795.

[37] M.E. Schweitzer and G.P. Cachon, Decision bias in the newsvendor problem with a known demand distribution: experimental evidence. *Manage. Sci.* **46** (2000) 404–420.

[38] X. Su, Bounded rationality in newsvendor models. *M&SOM-Manuf. Serv. Oper.* **10** (2008) 566–589.

[39] S. Zhou, H. Zhang, N. Shi, Z. Xu and F. Wang, A new convergent hybrid learning algorithm for two-stage stochastic programs. *Eur. J. Oper. Res.* **283** (2020) 33–46.